Population growth and structures

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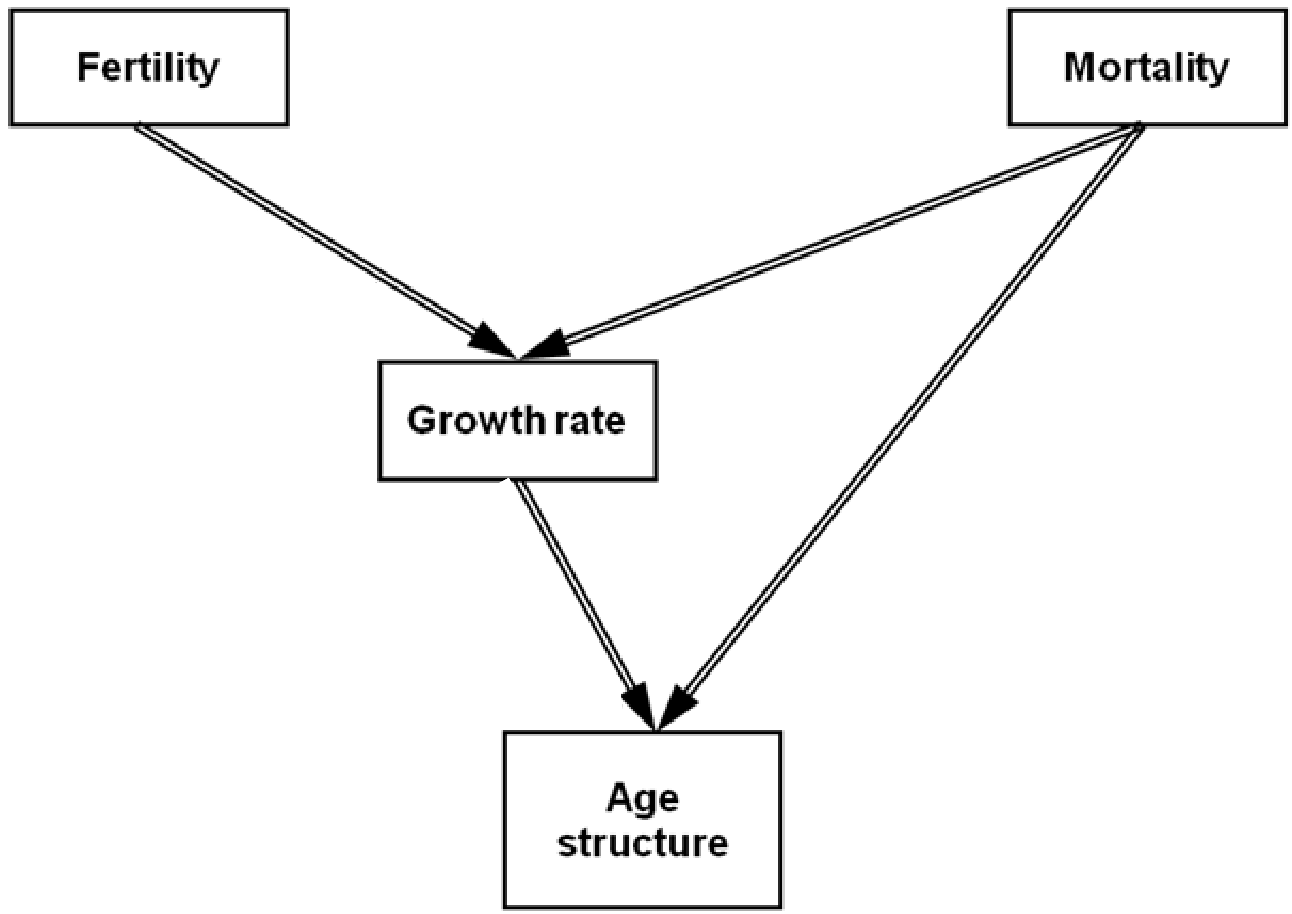
Outline

- Motivation
- Measures of population growth
- Models of population growth
- Typology of population structures
- Measures of population structures
- Examples, exercises

Demographic processes vary with the individual's age, sex as well as with other attributes (marital status, educational level, social or ethnic group, religion etc.). These attributes describe population structures.



Source: [2]

Population growth: measures

$$P_{t+1} = P_t + B_t - D_t + I_t - E_t \qquad \textit{(balancing equation)}$$

There are two different types of entries and departures to natural populations: birth and immigration and death and emigration. Difference between birth and death is **natural increase**, difference between immigration and emigration is **net migration**. In **closed populations** there is no migration. Closed populations are useful in demographic modeling.

Above mentioned measures of population growth are absolute. Relating them to population size leads to relative measures of population growth (next slide).

Source: [1]

Population growth: measures

Rate of natural increase = difference between crude birth rate and crude death rate:

$$\frac{B_t^v}{P_t^{1.7}} - \frac{D_t}{P_t^{1.7}} = \frac{B_t^v - D_t}{P_t^{1.7}} = \frac{B_t^v - D_t}{(P_t^{1.1} + P_{t+1}^{1.1})/2}$$

Net migration rate = difference between crude rate of immigration and crude rate of emigration:

$$\frac{I_t}{P_t^{1.7}} - \frac{E_t}{P_t^{1.7}} = \frac{I_t - E_t}{P_t^{1.7}} = \frac{I_t - E_t}{(P_t^{1.1} + P_{t+1}^{1.1})/2}$$

Growth rate = ratio of the total increase to the average population size:

$$\frac{B_t^v - D_t + I_t - E_t}{P_t^{1.7}} = \frac{P_{t+1} - P_t}{(P_t^{1.1} + P_{t+1}^{1.1})/2}$$

Vital index = ratio of live birth to death: $\frac{B_t^v}{D_t}$

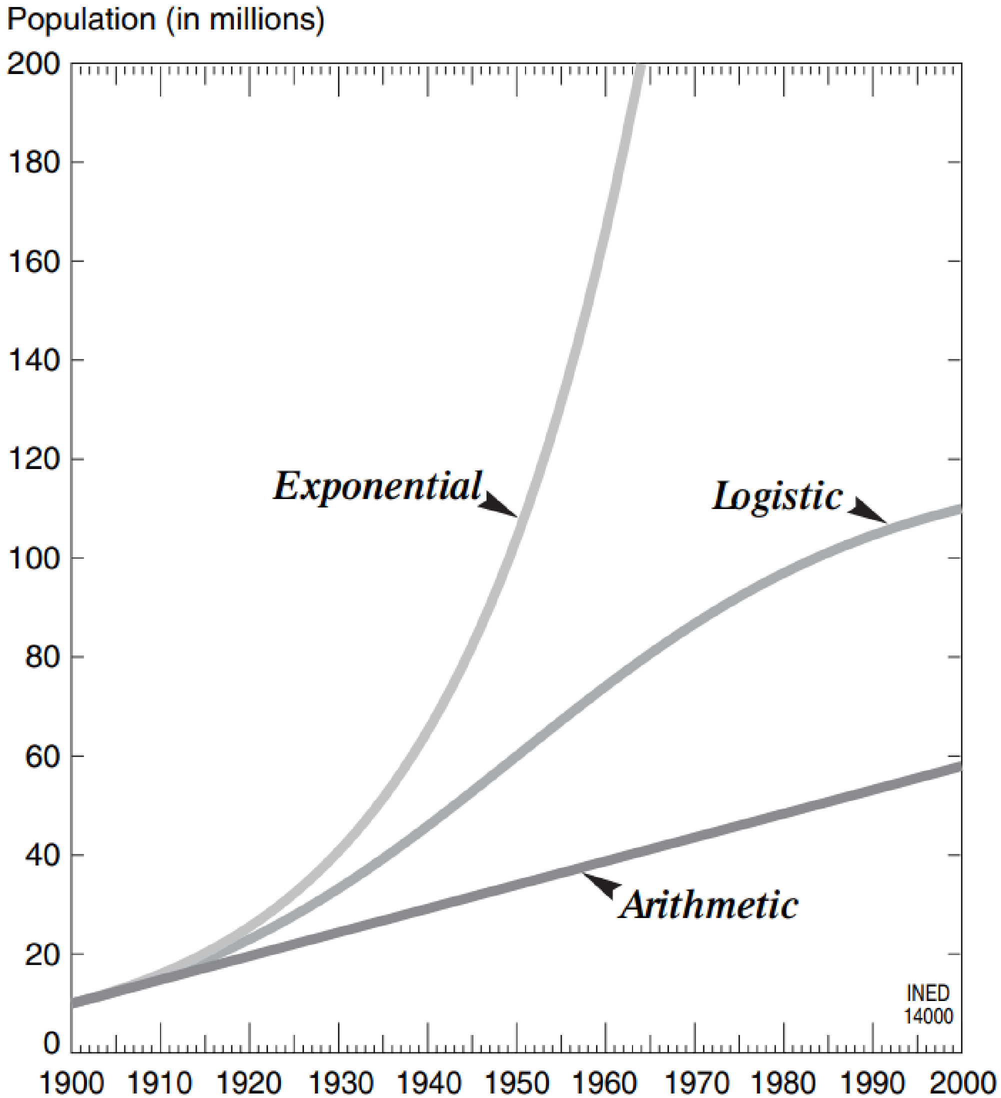
Source: [1]

Population increase/decrease: theory

- First attempts of producing population projections were based on distinct models of population growth
- By a **model of population growth** we mean mathematical sequences of numbers, which population size may follow during its development
- Basic mathematical sequences are f. e: Fibonacci, arithmetic, geometric, exponential…
- From that, we distinguish
  - **Arithmetic population increase/decrease** (*or linear*)
  - **Geometric population increase/decrease**
  - **Exponential population increase/decrease**
  - **Logistic population increase/decrease**

Fibonacci sequence: https://en.wikipedia.org/wiki/Fibonacci_number

Population increase/decrease: theory



← Evolution over a period of 100 years of a population of 10 million inhabitants with the same initial growth rate of around 5% per annum but following three different models of growth [1].

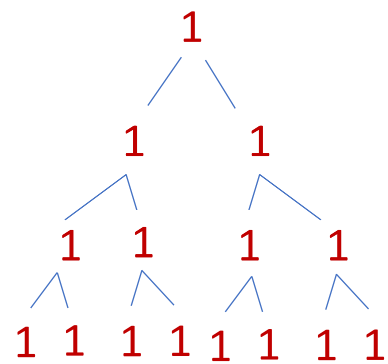
Source: [1]

Population increase/decrease: theory

Arithmetic **growth** (or linear) means, that between P_t and P_{t+1} there is the over time **constant difference**. Thus, in *n* years, the difference will be n-times its basic value.

Geometric population increase/decrease means, that each P_{t+n} is some **multiple** of previous population (P_t). This multiple remains **constant** over time.

Exponential population increase/decrease the amount of increase from a single generation keeps increasing.



Logistic population increase/decrease is characterized by growth rate, which changes over time. If a population follows logistic model, its size converges to some target size and does not grow unlimitedly.

Population increase/decrease: theory

	Rate of population increase	Projected population size	Doubling time of the population
Arithmetic growth	$r = \frac{P_{t+n} - P_t}{n \times \left(\frac{P_t + P_{t+n}}{2}\right)}$	$P_{t+n} = P_t + n(P_{t+1} - P_t)$	$t = \frac{P_t}{\left(\frac{P_{t+n} - P_t}{n}\right)}$
Geometric growth	$r = \sqrt[n]{\frac{P_{t+n}}{P_t}} - 1$	$P_{t+n} = P_t(1 + r)^n$	$t = \frac{ln(2)}{\ln(1 + r)}$
Exponential growth	$r = \frac{\ln\left(\frac{P_{t+n}}{P_t}\right)}{n}$	$P_{t+n} = P_t e^{rn}$	$t = \frac{\ln(2)}{r}$
Logistic growth*	*Fitting a logistic model to data. Logistic model is given by*: $P(t) = \frac{a}{1+e^{b-rt}}$		

*Logistic function is described by three parameters: **a** is a supremum of the values of the function (final population size, carrying capacity), **b** is location parameter (determines position of the function in respect to x-axis), **r** is growth rate (determines, how fast the curve converges to its supremum).

Population increase/decrease: theory

Logistic growth

a = carrying capacity
b = location parameter
r = growth rate

$$P(t) = \frac{a}{1 + e^{b-rt}}$$

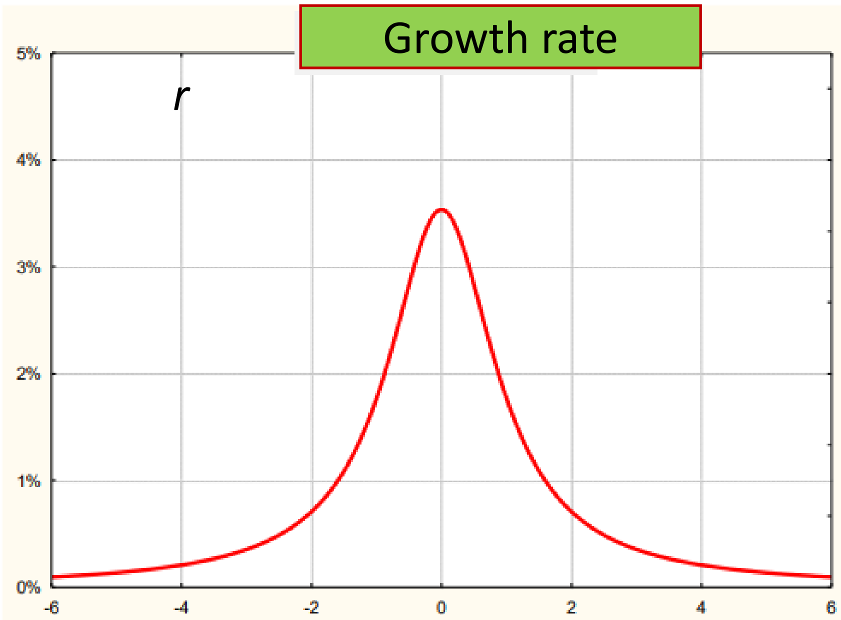
Fit can be done in several steps:

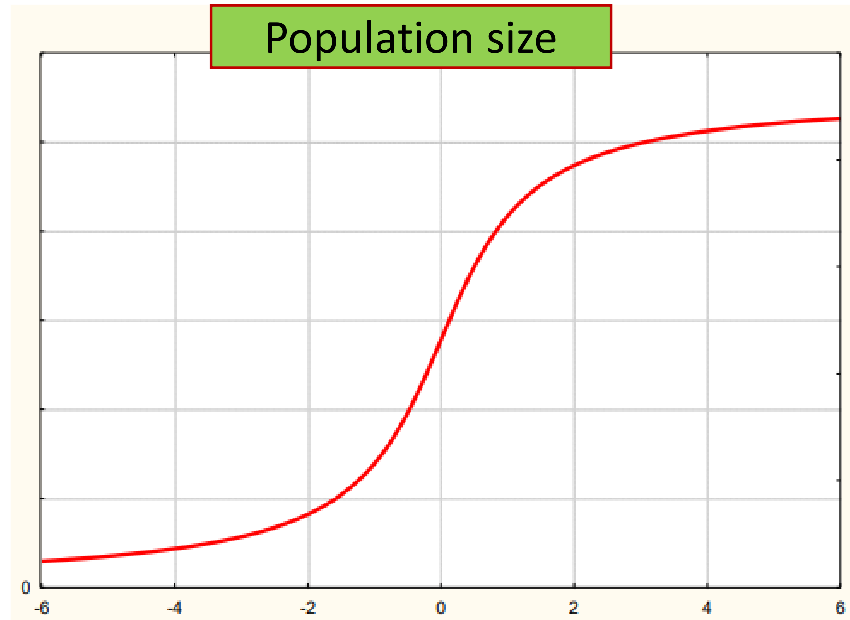
1. Set **a**
2. For **t** = 0 estimate **b** with known initial population **P(t)**
3. For **t**>0 estimate **r**, knowing **b**

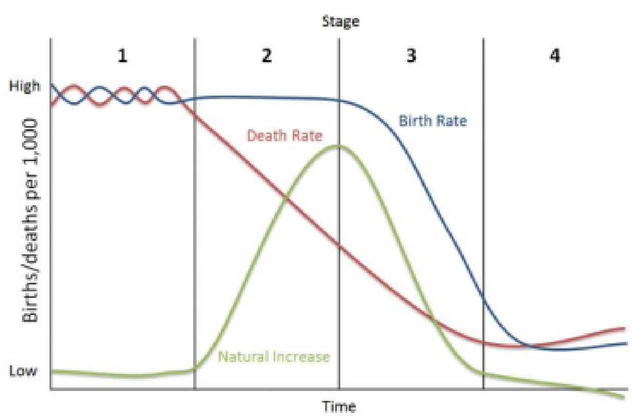
It follows, that **r** is a function of **t**. This function reaches its maximum at the same time point, in which inflection point of the logistic curve is located. Since that moment in time, the rate of population growth slows down. The model of logistic growth is implemented in the theory of demographic transition. The model was criticized for its subjectivity, which manifests to f.e setting *a priori* carrying capacity.

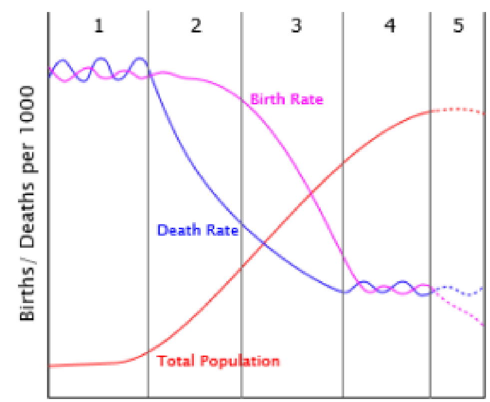
Population increase/decrease: theory

Logistic growth



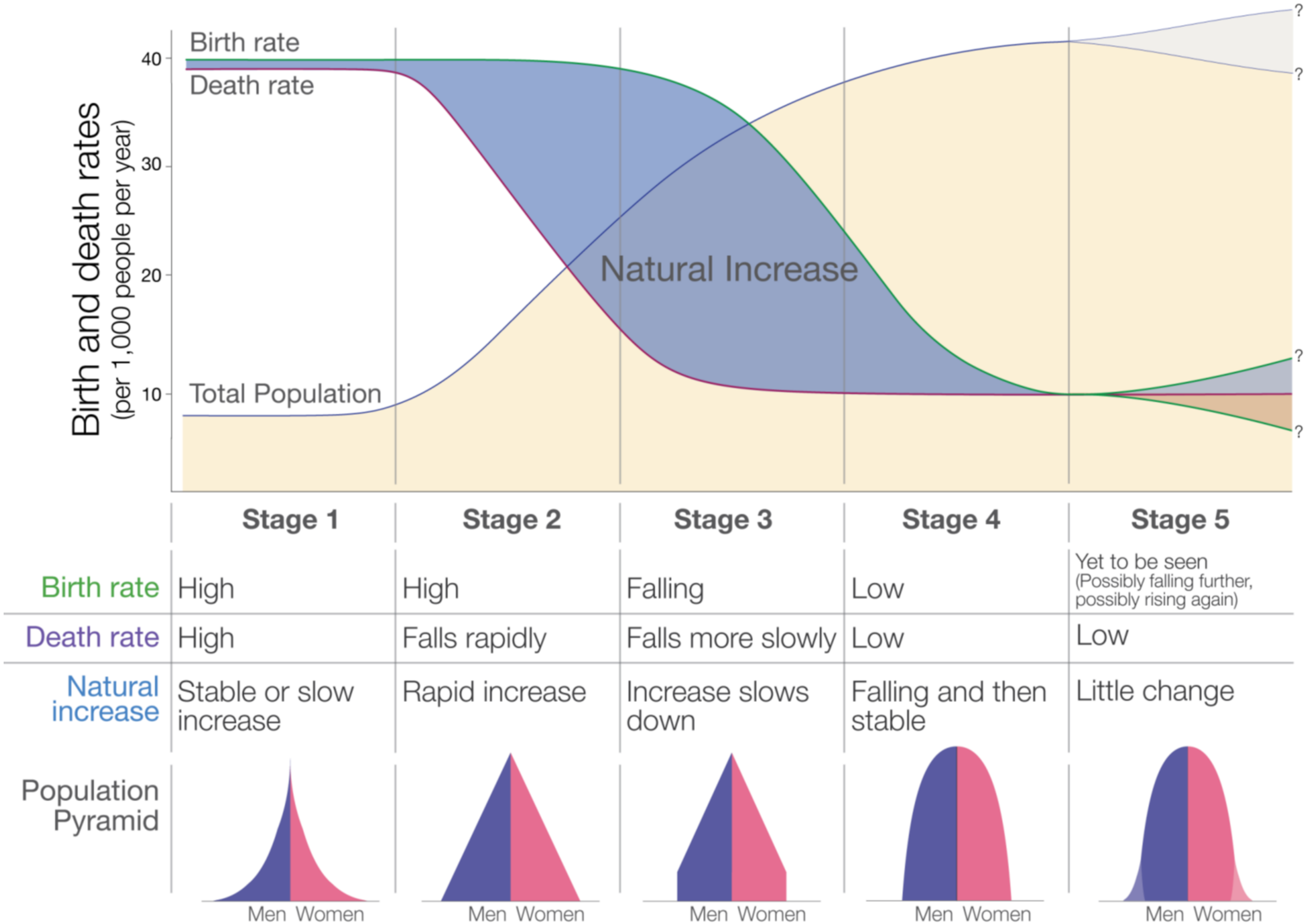






Population structures: typology

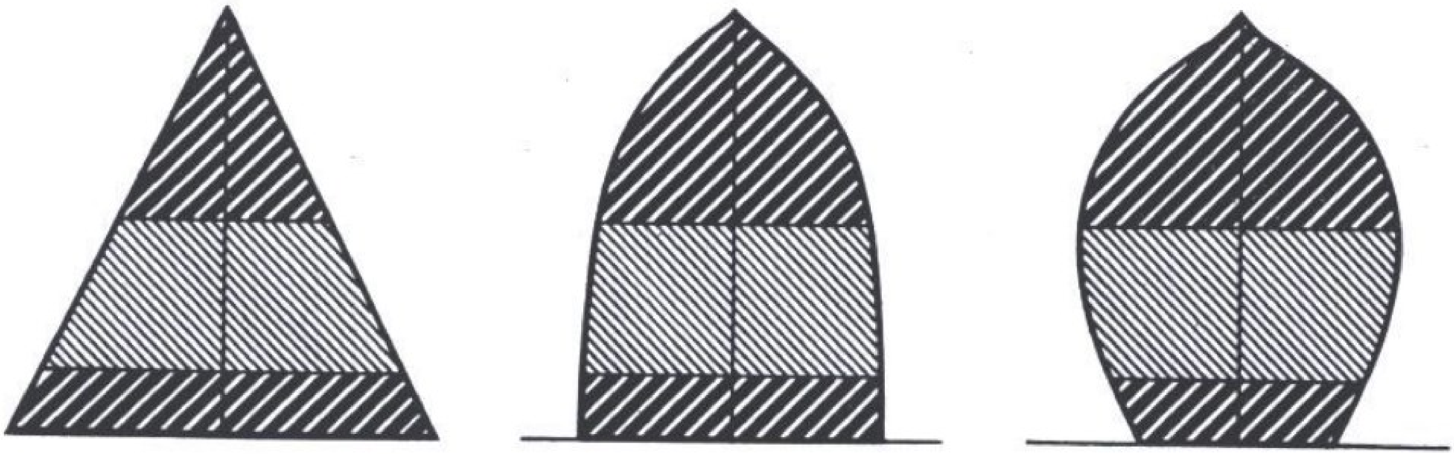
Types of the population structure depend on level of mortality and fertility (slide 2).



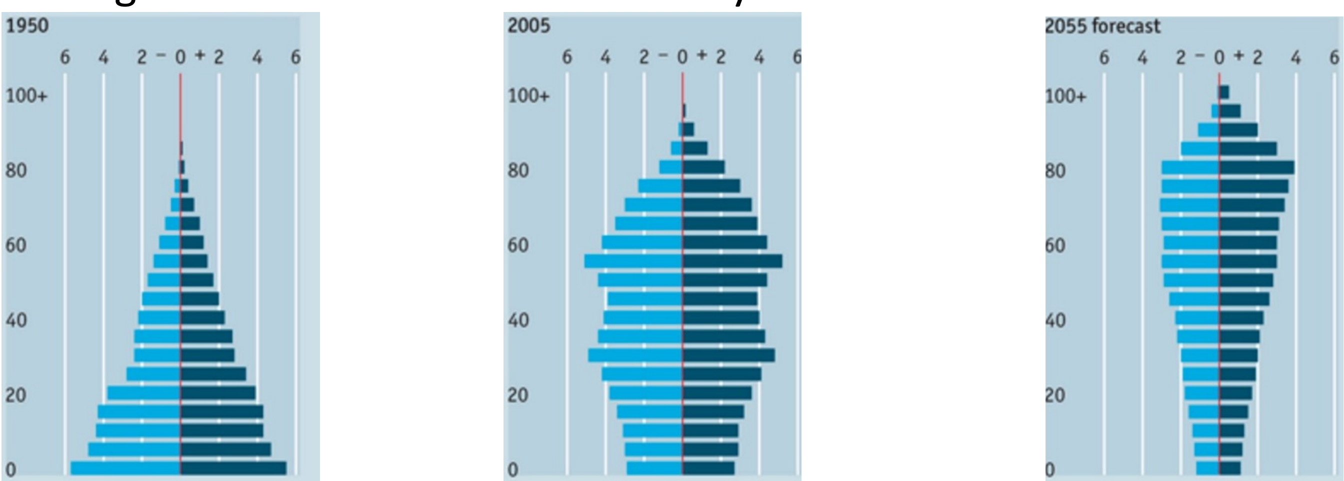
Source: https://ourworldindata.org/world-population-growth

Population structures: typology

Sundbärg (1907) age structure typology



Progressive Stationary Regressive



Japan’s population by age group

50% of population are in reproductive age and the rest is distributed among youth and old ones. The proportion of distribution of these 50% in non-reproductive ages determines the type of age structure.

Source: www.geo41.com

Population structures: measures

Measures of age-structure

Average age of the population $\bar{x} = \frac{\sum(x+0.5)\times P_x}{\sum P_x}$

Median age of the population *[linear interpolation of distribution function]*

Aging index P_{65+}/P_{0-14}

Old age dependency ratio P_{65+}/P_{15-64}

Young age dependency ratio P_{0-14}/P_{15-64}

(Total) age dependency ratio $(P_{65+}+P_{0-14})/P_{15-64}$

Measures of sex-structure

Index of masculinity P^m/P^f

Index of feminity P^f/P^m

Secondary sex ratio (index of masculinity) $N^{v,boys}/N^{v,girls} * 100$

Secondary sex ratio by birth order $N^{v,boys,i}/N^{v,girls,i} * 100$

Population structures: measures

Measures of age-structure

- Average age of the population $\bar{x} = \frac{\sum (x+0.5) \times P_x}{\sum P_x}$
- Median age of the population *[linear interpolation of distribution function]*
- Aging index P_{65+}/P_{0-14}
- Old age dependency ratio P_{65+}/P_{15-64}
- Young age dependency ratio P_{0-14}/P_{15-64}
- (Total) age dependency ratio $(P_{65+}+P_{0-14})/P_{15-64}$

Measures of sex-structure

- Index of masculinity P^m/P^f
- Index of feminity P^f/P^m
- Secondary sex ratio (index of masculinity) $N^{v,boys}/N^{v,girls} * 100$
- Secondary sex ratio by birth order $N^{v,boys,i}/N^{v,girls,i} * 100$

Other sources distinguish between biological and economic generations. **Economic generations** are defined: 0–19, 20–64, 65+. **Biological generations** are defined 0–14, 15–49, 50+. It implies, that dependency ratios can be computed in several ways depending on which age categories we use.

Source: [3]

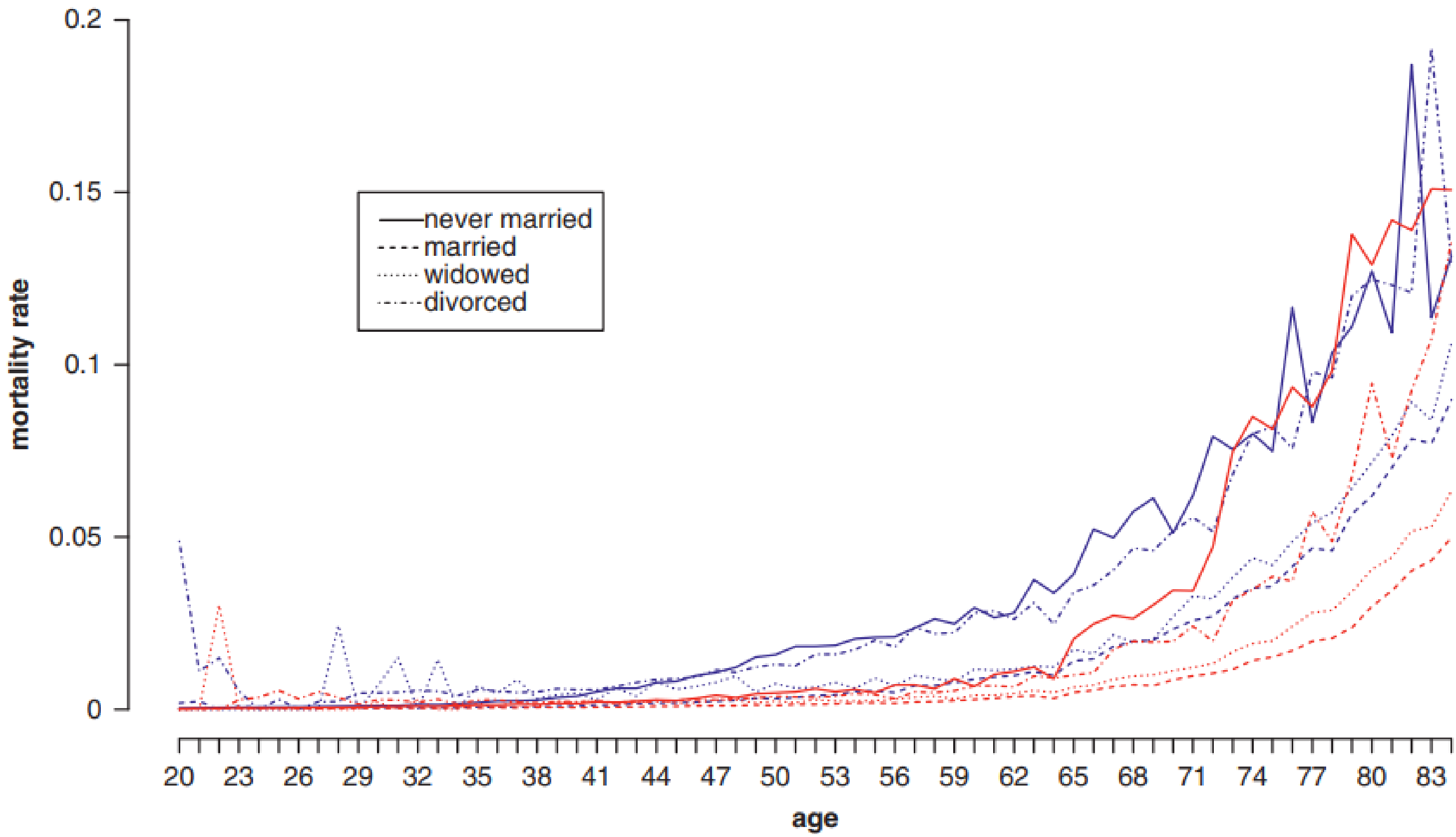
Population structures: measures

Apart from two **fundamental structures (by age and sex)** it is also **marital status** that affects demographic behavior.

Marital status (in Czechia): single, married, widowed, divorced

The intensity of mortality depends on marital status.

Empirical mortality rates, South Korea, 2010



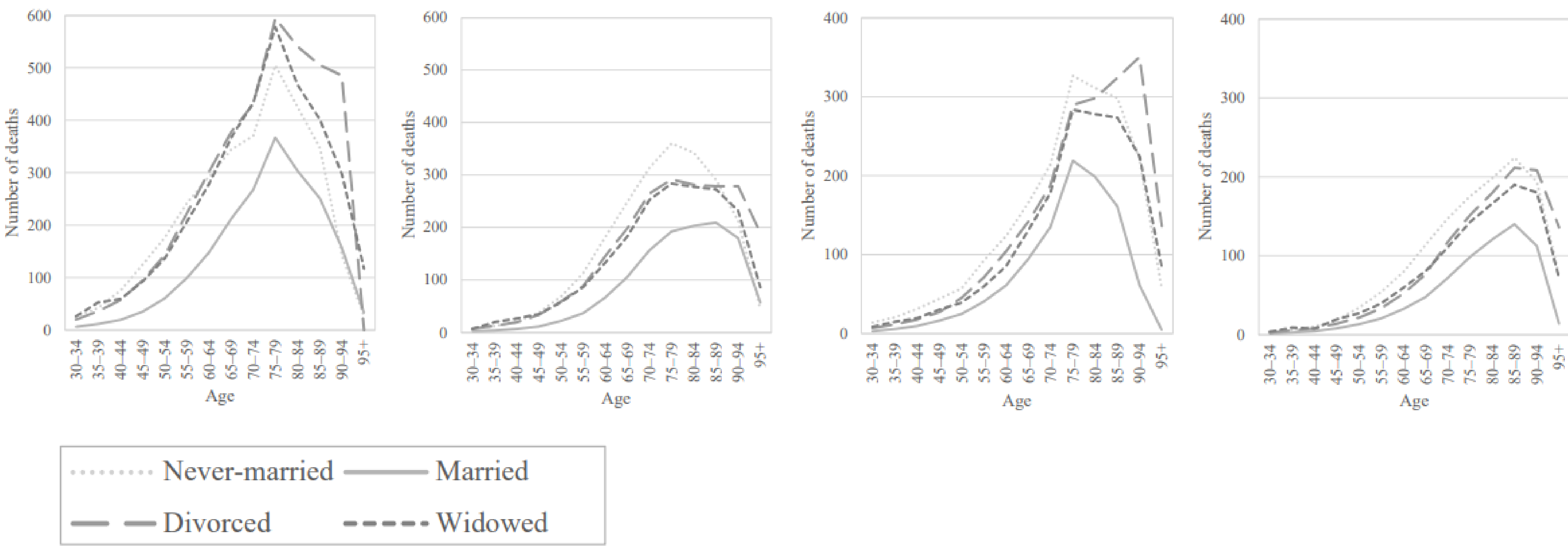
Source: [4]

Population structures: measures

Apart from two **fundamental structures (by age and sex)** it is also **marital status** that affects demographic behavior.
Marital status (in Czechia): single, married, widowed, divorced
The intensity of mortality depends on marital status.

Standardized death rates by marital status, Czechia

Males, 1990–1994 | Males, 2015–2019 | Females, 1990–1994 | Females, 2015–2019



Source: [5]

Population growth: facts

More info:
https://www.un.org/development/desa/pd/sites/www.un.org.development.desa.pd/files/wpp2022_summary_of_results.pdf

+ see reading list

Population structures: examples

More info:
https://population.un.org/wpp/Graphs/DemographicProfiles/Pyramid/682

+ see reading list

Population structures: exercise

Assignment from 30. 11. 2022

Population increase/decrease: exercise

Assignment 4

Download from Moodle *.xlsx* file (or use the one I have sent you before today lecture).

There are world’s population data and you should compute:

a. Average age of the population
b. Median age of the population
c. Population growth rate by linear model, geometric model, exponential model
d. Projected population for 10-year horizont for all three scenarios
e. Doubling time for all three scenarios

Deadline: 13. 12. 2022 4:30 am

Sources

[0] slides provided by dr. Burcin

[1] CASELLI, Graziella; VALLIN, Jacques; WUNSCH, Guillaume. *Demography: Analysis and Synthesis, Four Volume Set: A Treatise in Population*. Elsevier, 2005. Part II, Chapter 8. Pages 79–86.

[2] http://papp.iussp.org/

[3] LANGHAMROVÁ, Jitka a KAČEROVÁ, Eva. Demografie: materiály ke cvičení. 2. přeprac. vyd. Praha: Oeconomica, 2007. 91 s. ISBN 978-80-245-1224-2.

[4] Kwon, Hyuk-Sung. "Consideration of Marital Status in a Mortality Model and its Application for Mortality Risk Management " Asia-Pacific Journal of Risk and Insurance, vol. 10, no. 2, 2016, pp. 193-216. https://doi.org/10.1515/apjri-2015-0018

[5] Langhamrová, David Morávek–Jitka. "MORTALITY DIFFERENTIALS BY MARITAL STATUS IN CZECHIA 1990–2019." Available online: https://relik.vse.cz/2020/download/pdf/326-Moravek-David-paper.pdf