

Tutorial problems for Lectures 3/4/5. Due Friday May 5, 2017.

**Problem 1:** Write a function that will shift an array by an arbitrary amount using a convolution (yes, I know there are easier ways to do this). The function should take 2 arguments - an array, and an amount by which to shift the array. Plot a gaussian that started in the centre of the array shifted by half the array length. (10)

**Problem 2:** The correlation function  $f \star g$  is  $\int f(x)g(x+y)dx$ . Through a similar proof, one can show  $f \star g = \text{ift}(\text{dft}(f) \cdot \text{conj}(\text{dft}(g)))$ . Write a routine to take the correlation function of two arrays. Plot the correlation function of a Gaussian with itself. (10)

**Problem 3:** Using the results of part 1 and part 2, write a routine to take the correlation function of a Gaussian (shifted by an arbitrary amount) with itself. How does the correlation function depend on the shift? Does this surprise you? (10)

**Problem 4:** The circulant (wrap-around) nature of the dft can sometimes be problematic. Write a routine to take the convolution of two arrays *without* any danger of wrapping around. You may wish to add zeros to the end of the input arrays. (10)

**Problem 5:** Complete the complex class definition to support  $-$ ,  $*$ , and  $/$  (`--sub--`, `--mul--`, and `--div--`). Recall that  $a/b = a \cdot \text{conj}(b) / (b \cdot \text{conj}(b))$ . Show from a few sample cases that your functions work. (10)

**Bonus 1:** You have a sample code that calculates an FFT of an array whose length is a power of 2. Using that routine as a guideline, write an FFT routine that works on an array whose length is a power of 3 (e.g. 9, 27, 81). Verify that it gives the same answer as `numpy.fft.fft` (10)

**Bonus 2:** Extend the complex class to also support arbitrary (i.e. non-integer) powers (keyword is `--pow--`). +3 if the routine works if  $a^b$  works for complex  $a$  and real  $b$ , +5 if it works for complex  $a$  and complex  $b$ . (you may ignore branch cuts).