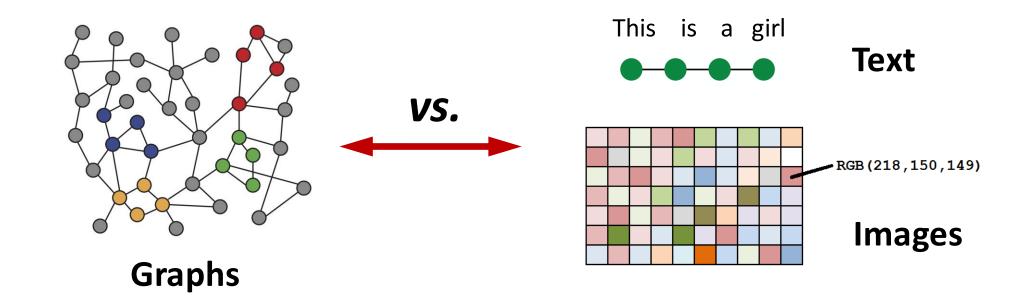
Graph Learning Basics

Jiaxuan You
Assistant Professor at UIUC CDS



CS598: Deep Learning with Graphs, 2024 Fall

Recap: Machine Learning with Graphs is Hard

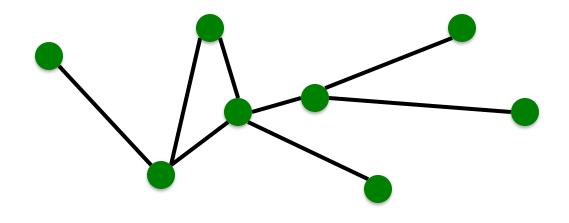


- Arbitrary size and topological structure
- Nodes have no fixed ordering

Graph Learning Basics

Graph Representation Basics

Components of a Network



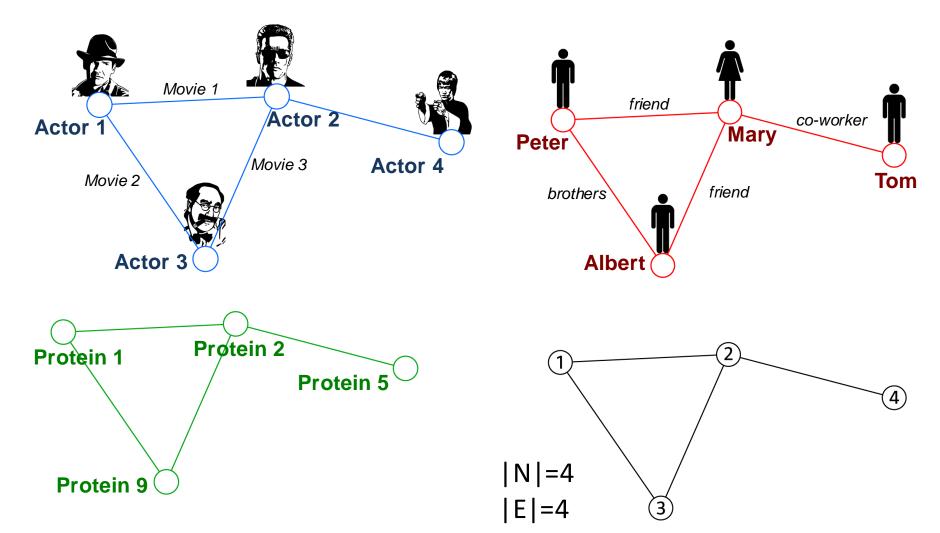
- Objects: nodes, vertices
- Interactions: links, edges
- System: network, graph

N

E

G(N,E)

Graphs: A Common Language



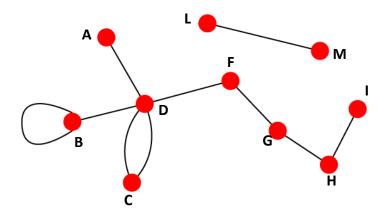
How do you define a graph?

- How to build a graph:
 - What are nodes?
 - What are edges?
- Choice of the proper network representation of a given domain/problem determines our ability to use networks successfully:
 - In some cases, there is a unique, unambiguous representation
 - In other cases, the representation is by no means unique
 - The way you assign links will determine the nature of the question you can study

Directed vs. Undirected Graphs

Undirected

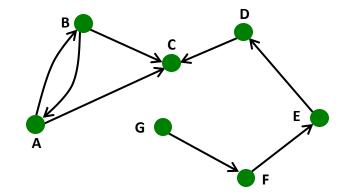
Links: undirected (symmetrical, reciprocal)



- Examples:
 - Collaborations
 - Friendship on Facebook
 - Pairs of positive/negative samples in contrastive learning

Directed

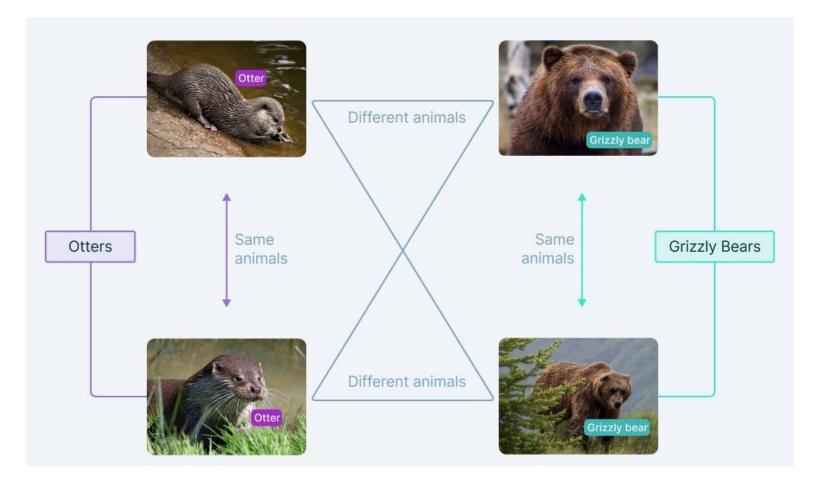
Links: directed (arcs)



- Examples:
 - Phone calls
 - Following on Twitter
 - Computational graphs in deep learning

Undirected Graph Example in DL

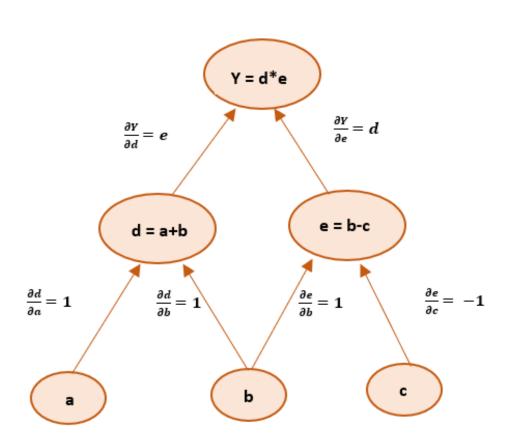
Contrastive learning



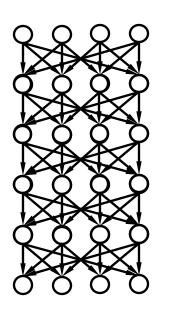
Directed Graph Examples in DL

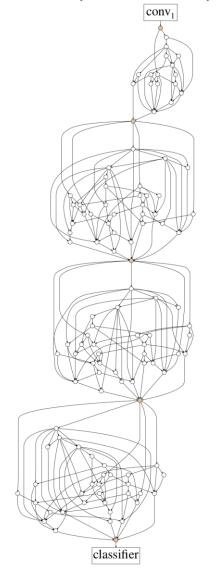
RandWire, Xie et al., 2019

Computational graphs



A 5-layer Neural network

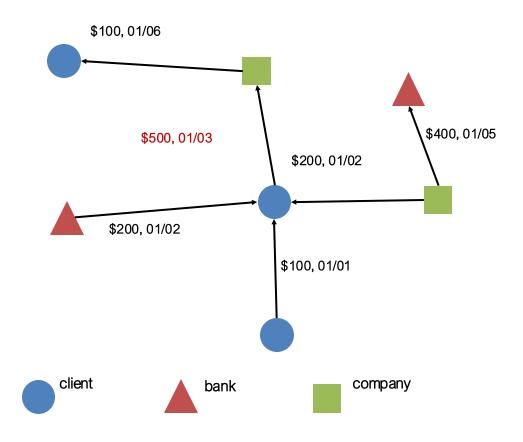




Dynamic Graphs

- Dynamic graph representation option 1: Graph + timestamp G = (V, E, T)
 - Nodes $v_i \in V$
 - Edges $(v_i, v_j) \in E$
 - Timestamps $T(v_i)$, $T(v_i, v_j)$
- Dynamic graph representation option 2: Snapshots of graphs
 - Each snapshot is a standard graph G_t
 - A dynamic graph is a series of graph snapshots $G = (G_1, ..., G_T)$

Dynamic Graph Example: Financial Networks



- Transaction-based approach
 - "On 01/03, Client A sends Company B \$500"
- Graph-based approach
 - Represent a transaction in a much broader context
 - A dynamic network, changing over time

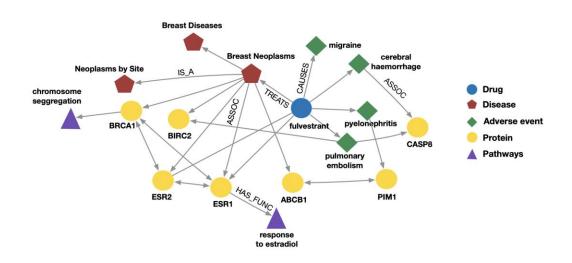
Heterogeneous Graphs

A heterogeneous graph is defined as

$$G = (V, E, \tau, \phi)$$

- Nodes with node types $v \in V$
 - Node type for node v: $\tau(v)$
- Edges with edge types $(u, v) \in E$
- An edge can be described as a pair of nodes
- Edge type for edge (u, v): $\phi(u, v)$
- **Relation type** for edge e is a tuple: r(u, v)
 - $= (\tau(u), \phi(u, v), \tau(v))$

Many Graphs are Heterogeneous Graphs



Biomedical Knowledge Graphs

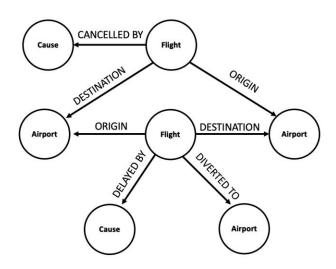
Example node: Migraine

Example relation: (fulvestrant,

Treats, Breast Neoplasms)

Example node type: Protein

Example edge type: Causes



Event Graphs

Example node: SFO

Example relation: (UA689, Origin, LAX)

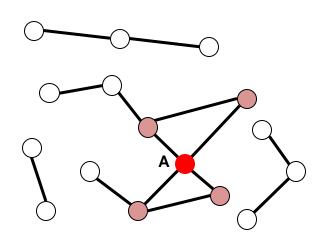
Example node type: Flight

Example edge type: Destination

Node Degrees

Undirected

- Node degree, k_i : the number of edges adjacent to node i, $k_A = 4$
- Avg. degree: $\overline{k} = \langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i = \frac{2E}{N}$

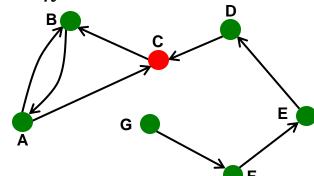


Directed

In directed networks we define an indegree and out-degree.

The (total) degree of a node is the sum of in- and out-degrees.

 $k_C^{in} = 2, k_C^{out} = 1, k_C = 3$ $\overline{k} = \frac{E}{N}, \overline{k^{in}} = \overline{k^{out}}$



Source: Node with $k^{in} = 0$

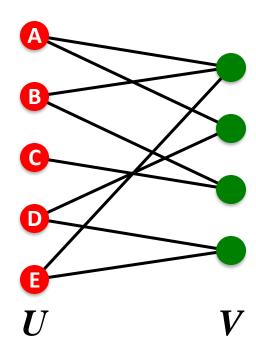
Sink: Node with $k^{out} = 0$

Bipartite Graph

Bipartite graph is a graph whose nodes can be divided into two disjoint sets U and V such that every link connects a node in U to one in V; that is, U and V are independent sets.

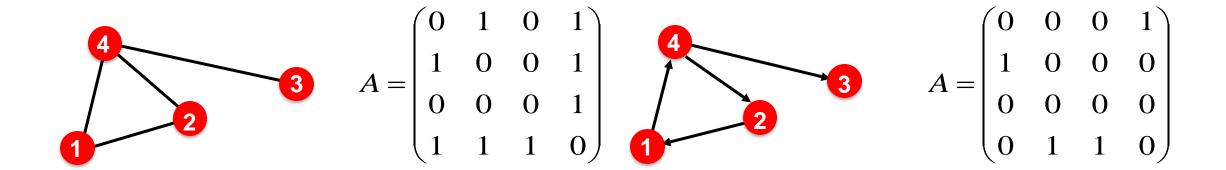
Examples:

- Authors-to-Papers (they authored)
- Actors-to-Movies (they appeared in)
- Users-to-Movies (they rated)
- Recipes-to-Ingredients (they contain)
- "Folded" networks:
 - Author collaboration networks
 - Movie co-rating networks



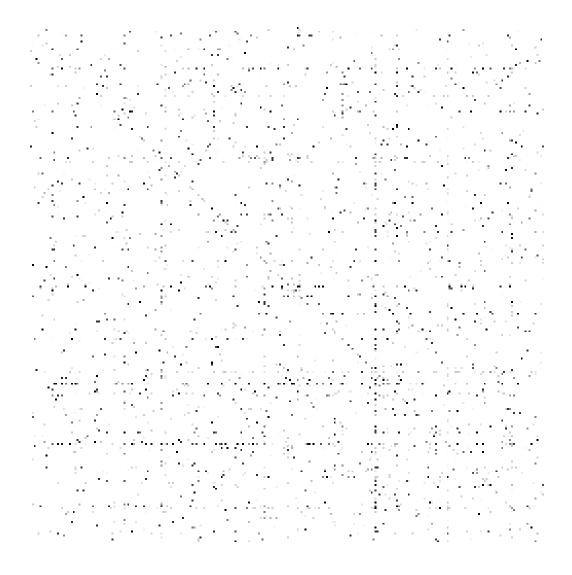
Representing Graphs: Adjacency Matrix

- $A_{ii} = 1$ if there is a link from node i to node j
- $\mathbf{A}_{ii} = \mathbf{0}$ otherwise



Note that for a directed graph (right) the matrix is not symmetric.

Adjacency Matrices are Sparse



Networks are Sparse Graphs

- Most real-world networks are sparse
- $E << E_{max}$ (or k << N-1)

			•		•	
NETWORK	NODES	LINKS	DIRECTED/ UNDIRECTED	N	L	<k></k>
Internet	Routers	Internet connections	Undirected	192,244	609,066	6.33
WWW	Webpages	Links	Directed	325,729	1,497,134	4.60
Power Grid	Power plants, transformers	Cables	Undirected	4,941	6,594	2.67
Phone Calls	Subscribers	Calls	Directed	36,595	91,826	2.51
Email	Email Addresses	Emails	Directed	57,194	103,731	1.81
Science Collaboration	Scientists	Co-authorship	Undirected	23,133	93,439	8.08
Actor Network	Actors	Co-acting	Undirected	702,388	29,397,908	83.71
Citation Network	Paper	Citations	Directed	449,673	4,689,479	10.43
E. Coli Metabolism	Metabolites	Chemical reactions	Directed	1,039	5,802	5.58
Protein Interactions	Proteins	Binding interactions	Undirected	2,018	2,930	2.90

- Consequence: Adjacency matrix is filled with zeros!
- (Density of the matrix (E/N^2): WWW=1.51x10⁻⁵, MSN IM = 2.27x10⁻⁸)

Node and Edge Attributes

Possible options:

- Weight (e.g., frequency of communication)
- Ranking (best friend, second best friend...)
- Type (friend, relative, co-worker)
- Sign: Friend vs. Foe, Trust vs. Distrust
- Properties depending on the structure of the rest of the graph: Number of common friends

More Types of Graphs

Unweighted (undirected)

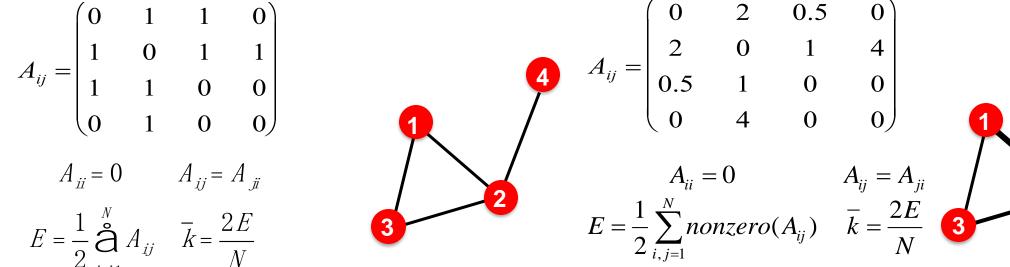
Weighted (undirected)

$$A_{ij} = \begin{pmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

$$A_{ii} = 0 \qquad A_{ij} = A_{ji}$$

$$E = \frac{1}{2} \mathop{a}_{i, \neq 1}^{N} A_{ij} \quad \overline{k} = \frac{2E}{N}$$

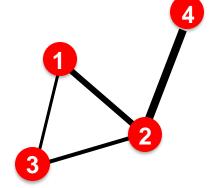
$$E = \frac{1}{2} \mathop{\mathring{\mathbf{a}}}_{i, \neq 1}^{N} A_{ij} \quad \overline{k} = \frac{2E}{N}$$



$$A_{ij} = \begin{bmatrix} 0 & 2 & 0.5 & 0 \\ 2 & 0 & 1 & 4 \\ 0.5 & 1 & 0 & 0 \\ 0 & 4 & 0 & 0 \end{bmatrix}$$

$$A_{ii} = 0$$
 $A_{ij} = A_{ji}$

$$E = \frac{1}{2} \sum_{i, j=1}^{N} nonzero(A_{ij}) \quad \overline{k} = \frac{2E}{N}$$



More Types of Graphs

Self-edges (self-loops) (undirected)

Multigraph (undirected)

Examples: Proteins, Hyperlinks

Examples: Communication, Collaboration

$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$A_{ij} = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

$$A_{ij} = \begin{pmatrix} 0 & 2 & 1 & 0 \\ 2 & 0 & 1 & 3 \\ 1 & 1 & 0 & 0 \\ 0 & 3 & 0 & 0 \end{pmatrix}$$

$$A_{ii} \neq 0$$

$$E = \frac{1}{2} \sum_{i,j=1,i\neq j}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii}$$

$$A_{ij} = A_{ji}$$

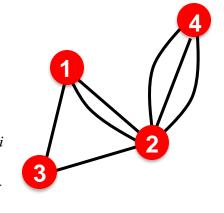
$$E = \frac{1}{2} \sum_{i,j=1,i\neq j}^{N} nonzero(A_{ij}) \quad \bar{k} = \frac{2E}{N}$$

$$A_{ii} \neq 0$$

$$E = \frac{1}{2} \sum_{i, j=1, i \neq j}^{N} A_{ij} + \sum_{i=1}^{N} A_{ii}$$

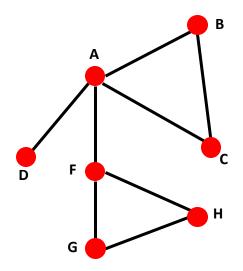
$$A_{ii} = 0 A_{ij} = 0$$

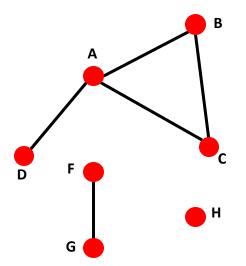
$$E = \frac{1}{2} \sum_{i=1}^{N} nonzero(A_{ij}) \bar{k} = 0$$



Connectivity of Undirected Graphs

- Connected (undirected) graph:
 - Any two vertices can be joined by a path
- A disconnected graph is made up by two or more connected components



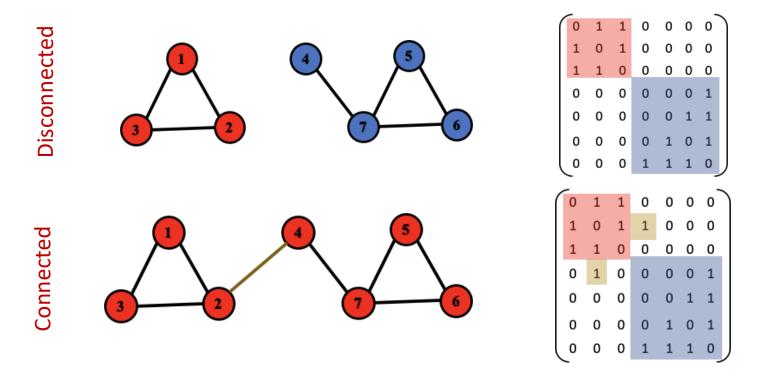


Largest Component: **Giant Component**

Isolated node (node H)

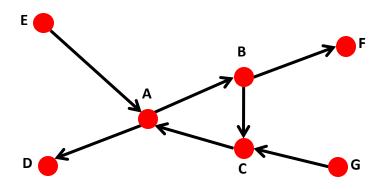
Connectivity: Example

The adjacency matrix of a network with several components can be written in a block- diagonal form, so that nonzero elements are confined to squares, with all other elements being zero:



Connectivity of Directed Graphs

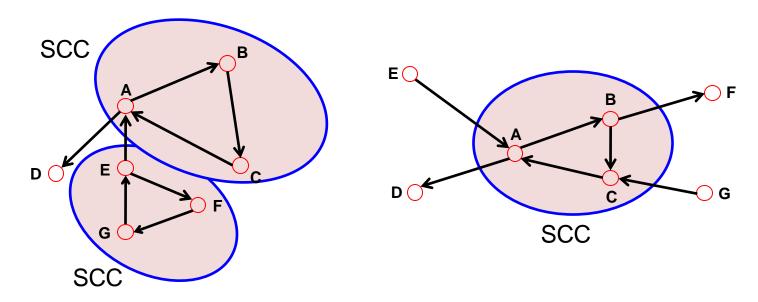
- Strongly connected directed graph
 - has a path from each node to every other node and vice versa (e.g., A-B path and B-A path)
- Weakly connected directed graph
 - is connected if we disregard the edge directions



Graph on the left is connected but not strongly connected (e.g., there is no way to get from F to G by following the edge directions).

Connectivity of Directed Graphs

 Strongly connected components (SCCs) can be identified, but not every node is part of a nontrivial strongly connected component.



In-component: nodes that can reach the SCC,

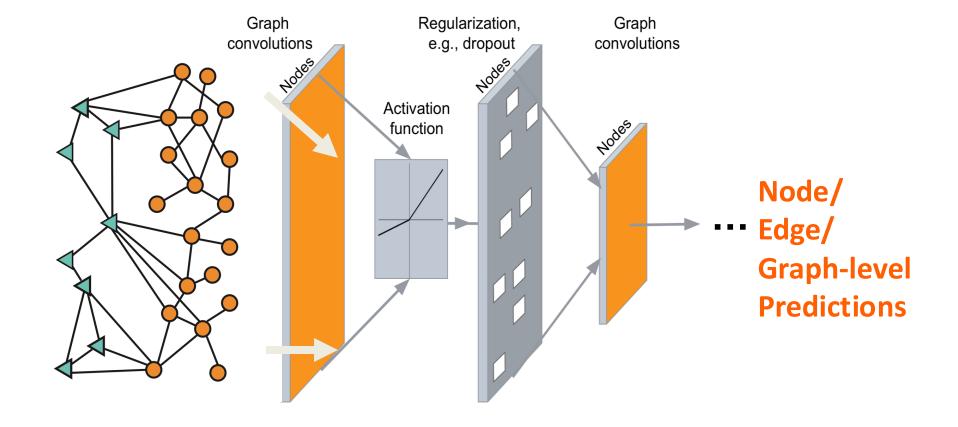
Out-component: nodes that can be reached from the SCC.

Graph Learning Basics

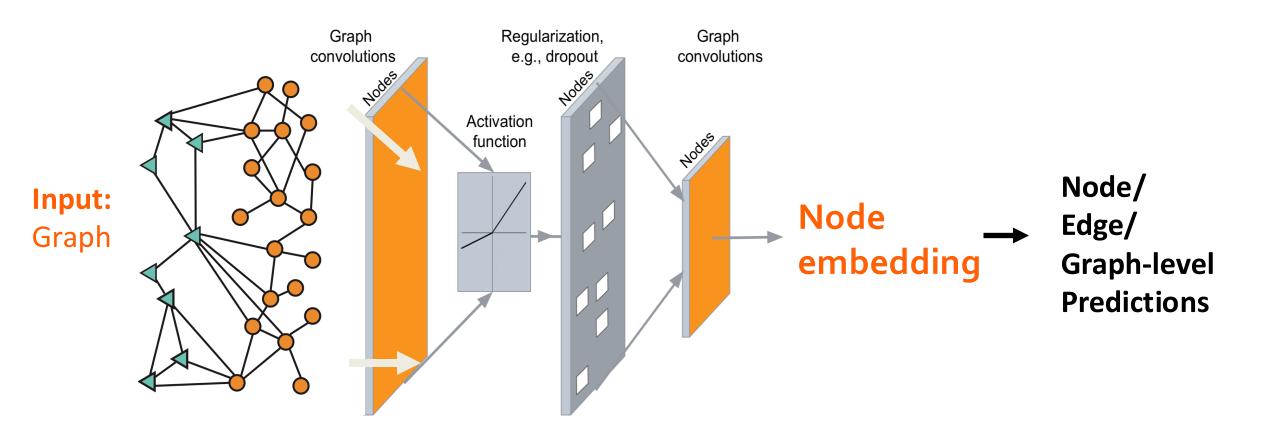
Graph Learning Prediction Tasks

Recap: Deep Learning with Graphs

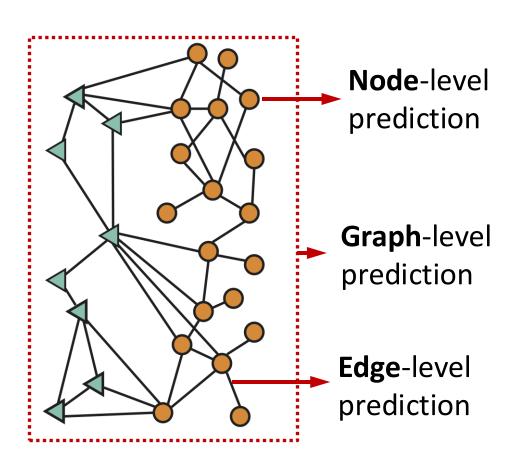
Input: Graph



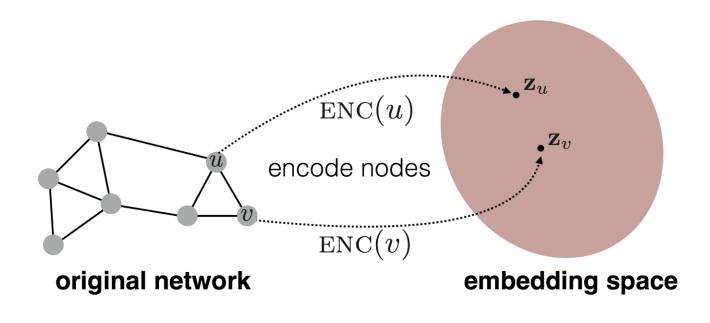
Deep Learning with Graphs



Graph ML Tasks



Key Idea: Node Embeddings

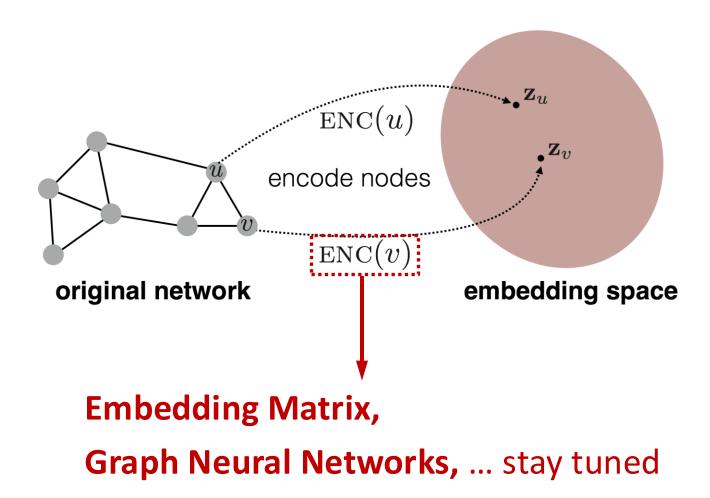


Intuition: Map nodes to d-dimensional embeddings such that similar nodes in the graph are embedded close together

Graph ML Tasks

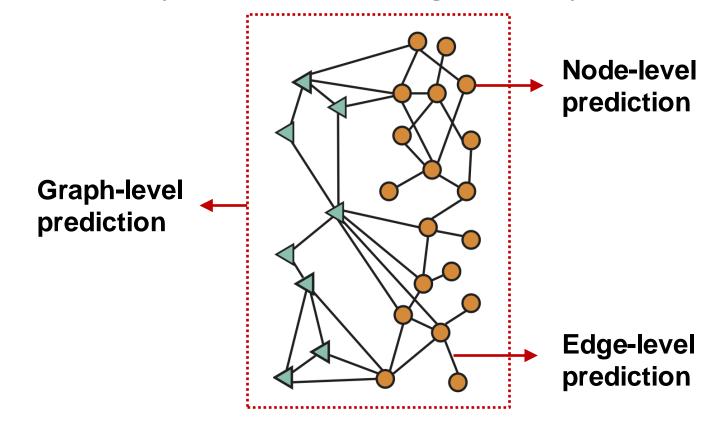
Node-level prediction **Graph**-level prediction Edge-level prediction

Key Idea: Node Embeddings



Graph Learning Prediction Heads

- Idea: Different task levels require different prediction heads
- Prediction head: map node embeddings to the predictions of interest

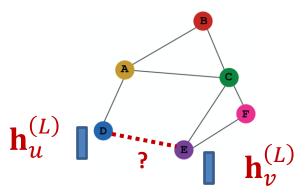


Prediction Heads: Node-level

- Node-level prediction: We can directly make prediction using node embeddings!
- Assuming we have d-dim node embeddings: $\{\mathbf{h}_v \in \mathbb{R}^d, \forall v \in G\}$
- Suppose we want to make k-way prediction
 - Classification: classify among k categories
 - Regression: regress on k targets
- $\hat{y}_v = \text{Head}_{\text{node}}(\mathbf{h}_v) = \mathbf{W} \mathbf{h}_v$
 - W $\in \mathbb{R}^{k*d}$: We map node embeddings from $\mathbf{h}_v \in \mathbb{R}^d$ to $\widehat{\mathbf{y}}_v \in \mathbb{R}^k$ so that we can compute the loss

Prediction Heads: Edge-level

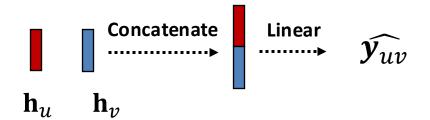
- Edge-level prediction: Make prediction using pairs of node embeddings
- Suppose we want to make k-way prediction
- $\widehat{\mathbf{y}}_{uv} = \operatorname{Head}_{\operatorname{edg}e}(\mathbf{h}_u, \mathbf{h}_v)$



• What are the options for $Head_{edge}(\mathbf{h}_u, \mathbf{h}_v)$?

Prediction Heads: Edge-level

- Options for $Head_{edge}(\mathbf{h}_u, \mathbf{h}_v)$:
- (1) Concatenation + Linear



- $\hat{y}_{uv} = W$ Concat(\mathbf{h}_u , \mathbf{h}_v)
- Here $\mathbf{W} \in \mathbb{R}^{k*2d}$ will map 2d-dimensional embeddings (since we concatenated embeddings) to k-dim embeddings (k-way prediction)
- W can be replace with deeper neural networks, e.g., MLP

Prediction Heads: Edge-level

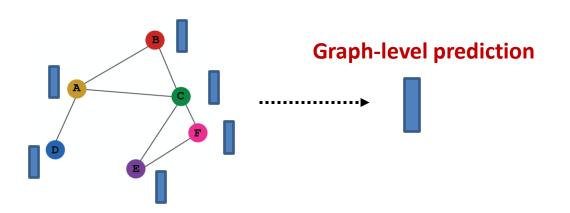
- Options for $Head_{edge}(\mathbf{h}_u, \mathbf{h}_v)$:
- (2) Dot product
 - $\widehat{\mathbf{y}}_{uv} = (\mathbf{h}_u)^T \mathbf{h}_v$
 - This approach only applies to 1-way prediction (e.g., link prediction: predict the existence of an edge)
 - Applying to k-way prediction:
 - Similar to multi-head attention: $\mathbf{W}^{(1)}$, ..., $\mathbf{W}^{(k)}$ trainable

$$\widehat{\mathbf{y}}_{uv}^{(1)} = (\mathbf{h}_u)^T \mathbf{W}^{(1)} \mathbf{h}_v$$

 $\widehat{\mathbf{y}}_{uv}^{(k)} = (\mathbf{h}_u)^T \mathbf{W}^{(k)} \mathbf{h}_v$ $\widehat{\mathbf{v}}_{uv} = \text{Concat}(\widehat{\mathbf{v}}_{uv}^{(1)}, \dots, \widehat{\mathbf{v}}_{uv}^{(k)}) \in \mathbb{R}^k$

Prediction Heads: Graph-level

- Graph-level prediction: Make prediction using all the node embeddings in our graph
- Suppose we want to make k-way prediction
- $\widehat{\boldsymbol{y}}_G = \operatorname{Head}_{\operatorname{graph}}(\{\mathbf{h}_v \in \mathbb{R}^d, \forall v \in G\})$



Prediction Heads: Graph-level

- Options for $\operatorname{Head}_{\operatorname{graph}}(\{\mathbf{h}_v \in \mathbb{R}^d, \forall v \in G\})$
- (1) Global mean pooling

$$\widehat{\mathbf{y}}_G = \operatorname{Mean}(\{\mathbf{h}_v \in \mathbb{R}^d, \forall v \in G\})$$

(2) Global max pooling

$$\widehat{\mathbf{y}}_G = \operatorname{Max}(\{\mathbf{h}_v \in \mathbb{R}^d, \forall v \in G\})$$

• (3) Global sum pooling

$$\widehat{\mathbf{y}}_G = \operatorname{Sum}(\{\mathbf{h}_v \in \mathbb{R}^d, \forall v \in G\})$$

Reading papers

Suggestions for Research

Sources of AI/ML papers

Recent major AI/ML conferences:

NeurlPS 2023:

https://openreview.net/group?id=NeurIPS.cc/2023/Conference#tab-accept-oral

ICML 2024:

https://openreview.net/group?id=ICML.cc/2024/Conference#tab-accept-oral

ICLR 2024:

https://openreview.net/group?id=ICLR.cc/2024/Conference#tab-accept-oral

LOG 2023 (Learning on graphs):

https://openreview.net/group?id=logconference.io/LOG/2023/Conference#tab-accept-oral

Sources of AI/ML papers

Latest Arxiv papers:

https://arxiv.org/list/cs.LG/pastweek?skip=0&show=25



Machine Learning

Authors and titles for recent submissions

- Fri, 30 Aug 2024
- Thu, 29 Aug 2024
- Wed, 28 Aug 2024
- Tue, 27 Aug 2024
- Mon, 26 Aug 2024

See today's new changes

Total of 600 entries : 1-25 26-50 51-75 76-100 ... 576-600 Showing up to 25 entries per page: fewer | more | all

Fri, 30 Aug 2024 (showing first 25 of 117 entries)

[1] arXiv:2408.16765 [pdf, html, other]

A Score-Based Density Formula, with Applications in Diffusion Generative Models

Gen Li, Yuling Yan

Subjects: Machine Learning (cs.LG); Artificial Intelligence (cs.Al); Probability (math.PR); Statistics Theory (math.ST); Machine Learning (stat.ML)

[2] arXiv:2408.16717 [pdf, html, other]

A GREAT Architecture for Edge-Based Graph Problems Like TSP

Attila Lischka, Jiaming Wu, Morteza Haghir Chehreghani, Balázs Kulcsár

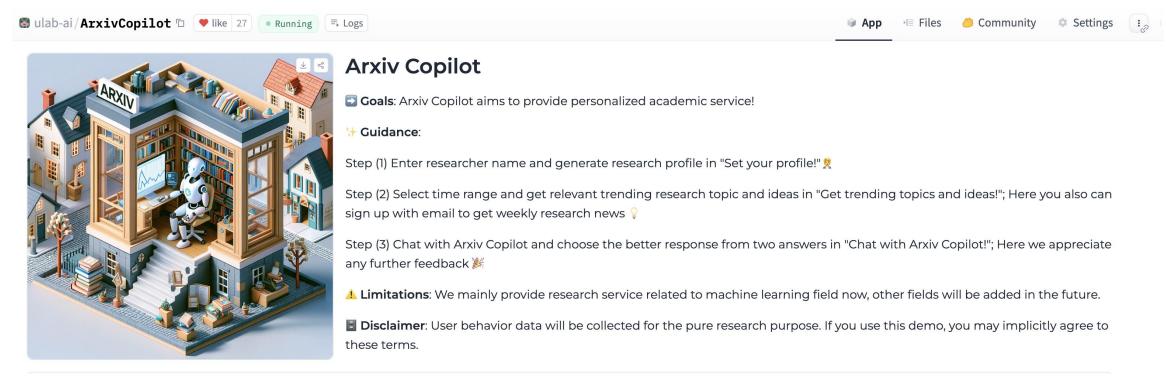
Comments: 15 pages, 7 figures

Subjects: Machine Learning (cs.LG); Artificial Intelligence (cs.Al)

Arxiv Copilot

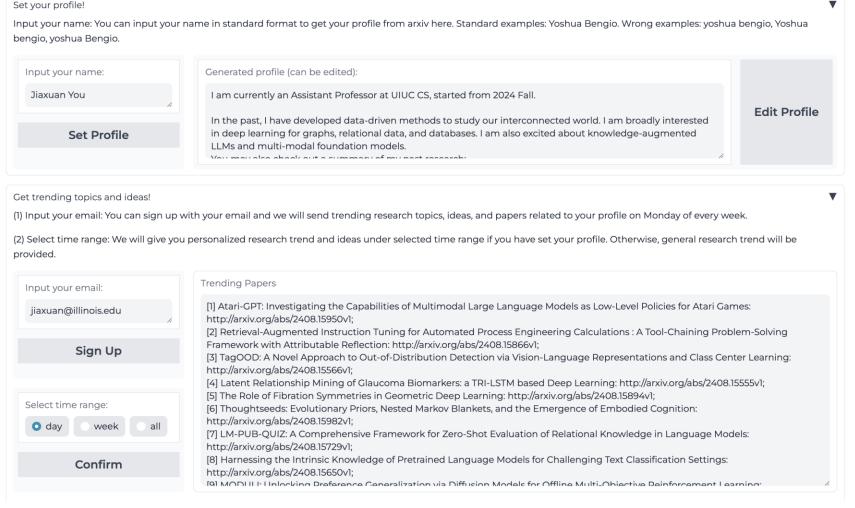
Still feeling it's too much work to track the recent papers?

- Try Arxiv Copilot, a personalized paper reading tool by U Lab!
- https://huggingface.co/spaces/ulab-ai/ArxivCopilot



Arxiv Copilot

Arxiv Copilot summarize latest Arxiv papers based on your profile



Arxiv Copilot is a research prototype demo. Feedback is welcomed!

Research behind Arxiv Copilot

Arxiv Copilot: A Self-Evolving and Efficient LLM System for Personalized Academic Assistance

Guanyu Lin¹ 2*, Tao Feng^{1*}, Pengrui Han¹ 3*, Ge Liu¹, Jiaxuan You¹

¹University of Illinois at Urbana-Champaign, ²Carnegie Mellon University, ³Carleton College

*Equal Contribution

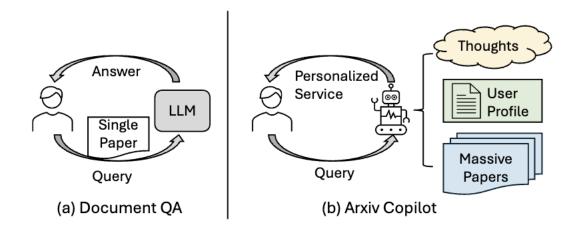


Figure 1: Comparison of (a) document Question Answering (QA) with our (b) Arxiv Copilot. Conven-

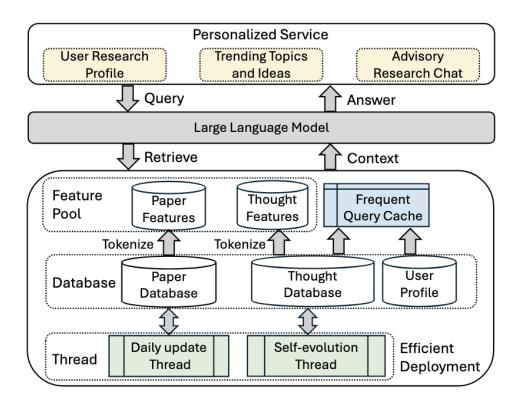


Figure 2: Architecture of Arxiv Copilot from bottom-to-up perspective. (a) In personalized service, Arxiv

Tips for Writing Papers

Highly recommend this webpage:

https://cs.stanford.edu/people/widom/paper-writing.html



Jennifer Widom

Frederick Emmons Terman Dean of the School of Engineering Fletcher Jones Professor in Computer Science and Electrical Engineering

Stanford University

The 5 Questions for A Good Paper

(Originally for writing paper introduction)

- What is the problem?
- Why is it interesting and important?
- Why is it hard? (E.g., why do naive approaches fail?)
- Why hasn't it been solved before? (Or, what's wrong with previous proposed solutions? How does mine differ?)
- What are the key components of my approach and results?
 Also include any specific limitations.

GraphRNN: Generating Realistic Graphs with Deep Auto-regressive Models
J You, R Ying, X Ren, WL Hamilton, J Leskovec
International Conference on Machine Learning (ICML 2018)

1035 2018

My first paper during my PhD:p

Abstract: Modeling and generating graphs is fundamental for studying networks in biology, engineering, and social sciences. However, modeling complex distributions over graphs and then efficiently sampling from these distributions is challenging due to the non-unique, high-dimensional nature of graphs and the complex, non-local dependencies that exist between edges in a given graph. Here we propose GraphRNN, a deep autoregressive model that addresses the above challenges and approximates any distribution of graphs with minimal assumptions about their structure. GraphRNN learns to generate graphs by training on a representative set of graphs and decomposes the graph generation process into a sequence of node and edge formations, conditioned on the graph structure generated so far. In order to quantitatively evaluate the performance of GraphRNN, we introduce a benchmark suite of datasets, baselines and novel evaluation metrics based on Maximum Mean Discrepancy, which measure distances between sets of graphs. Our experiments show that GraphRNN significantly outperforms all baselines, learning to generate diverse graphs that match the structural characteristics of a target set, while also scaling to graphs 50 times larger than previous deep models.

What is the research problem?

By Jiaxuan in Aug 2024:

Modeling arbitrary distributions of graphs and sampling from it

By GPT-40 in Aug 2024 (included full abstract in the prompt): How can we model and generate complex graph distributions with minimal assumptions while efficiently sampling diverse and large-scale graphs?

Why is it interesting and important?

By Jiaxuan in Aug 2024:

Fundamental in studying graphs across science disciplines

By GPT-40 in Aug 2024 (included full abstract in the prompt):

It's important because it advances the ability to model and generate complex graphs, which is crucial for understanding networks in various fields, while also offering scalability and accuracy beyond existing methods.

Why is it hard? (E.g., why do naive approaches fail?)

By Jiaxuan in Aug 2024:

Graphs are unordered, high-dimensional, complex, and present non-local dependencies between edges

By GPT-40 in Aug 2024 (included full abstract in the prompt):

It's hard because graphs are high-dimensional, non-unique, and have complex, non-local dependencies between edges, making it challenging to model and sample from their distributions efficiently.

Why hasn't it been solved before? (Or, what's wrong with previous proposed solutions? How does mine differ?)

By Jiaxuan in Aug 2024:

Classic graph generative models are hand-engineered to model a particular family of graphs, while deep models are either limited to learning from a single graph or generating tiny graphs.

By GPT-40 in Aug 2024 (included full abstract + part of intro in the prompt):

Previous solutions to generative graph modeling have been limited by their reliance on hand-engineered assumptions, restricting them to specific graph families and failing to capture the full complexity of real-world graphs. Deep generative models have advanced the field but are constrained by their ability to either learn from a single graph or generate only small graphs, typically with fewer than 40 nodes

- What are the key components of my approach and results? Also include any specific limitations.
- By Jiaxuan in Aug 2024:

The first autoregressive graph generator, modeling graph generation as a sequence of additions of new nodes and edges, and a comprehensive suite of benchmark tasks and baselines for the graph generation problem

By GPT-40 in Aug 2024 (included full abstract + part of intro in the prompt): GraphRNN, is a deep autoregressive model designed to generate graphs by sequentially adding nodes and edges, capturing complex joint probabilities. Key components include: Hierarchical RNNs: A graph-level RNN generates nodes, and an edge-level RNN generates edges.

BFS Node-Ordering: Improves scalability and reduces computational complexity. MMD-Based Evaluation: Provides rigorous quantitative evaluation of generated graphs. Results: GraphRNN outperforms baselines, generating high-quality graphs up to 50×10^{-5} larger than previous models, with an 80%-90% reduction in MMD.

Project Writing Tasks

Week	Date	Knowledge learning	Research training	Events	Deadlines
1	Aug 28 Wed	Introduction	Paper reading & analysis		
	Aug 30 Fri	Graph learning tasks	Paper reading & analysis	Writing task, out	
2	Sept 04 Wed	"Shallow" graph learning	Paper reading & analysis		
	Sept 06 Fri	Graph neural networks: perspective	Paper reading & analysis		
3	Sept 11 Wed	Graph neural networks: model I	Paper reading & analysis		
	Sept 13 Fri	Graph neural networks: model II	Paper reading & analysis		Writing task due
4	Sept 18 Wed	Paper reading discussions	Ideate & discussion		
	Sept 20 Fri	Graph neural networks: objective	Ideate & discussion	Proposal task, out	
5	Sept 25 Wed	Graph neural networks: pipeline	Ideate & discussion		
	Sept 27 Fri	Graph neural networks: theory	Ideate & discussion		
6	Oct 02 Wed	Graph neural networks: add-ons	Ideate & discussion		
	Oct 04 Fri	GNN implementation: PyG & GraphGym	Ideate & discussion		Proposal due
7	Oct 09 Wed	Project idea discussions	Prototype implementation		
	Oct 11 Fri	Beyond simple graphs: heterogeneous graphs	Prototype implementation	Submission task, out	

Project Task – Paper Reading and Analysis

15% of final grade:

- Pick 3 recent research papers related to graphs in the suggested paper sources (NeurlPS 2023, ICML 2024, ICLR 2024, LOG 2023, Arxiv 2024), pick as diverse as possible
- Read the paper. Search / Ask LLM tool for relevant concepts to help you understand the paper, if needed.
- Answer the 5 questions for each paper
- Choose your favorite LLM tool, ask them to answer the question
- Short comment on how LLM benefits you and you can help LLM improve
- Suggested DDL: Sept 15
- We will announce the submission link next week.

Summary

- Choice of a graph representation:
 - Directed, undirected, bipartite, weighted, adjacency matrix
- Different types of tasks req:
 - Node level
 - Edge level
 - Graph level
- Tips for reading research papers