Introduction to Quantum Mechanics

Homework 2

Due date: September 8, 2017

Problem 1

Consider the periodic function $f(x) = x$ for $x \in [-1, 1]$.

(a) Find the Fourier coefficients.

(b) Use your favorite program to plot the difference between the function and the partial sum of the Fourier series for n terms, with $n = 5, 10, 15, ...$

Problem 2

Consider the convolution of two functions

$$h(t) = f(t) \otimes g(t) = \int_{-\infty}^{\infty} d\tau f(\tau) g(t - \tau) \tag{1}$$

Let $f(t)$ be a square pulse of height 2, zero everywhere except between $1 < t < 2$ and $g(t)$ a pulse such that $g(t) = 2/5t$ for $1 < t < 5$ and 0 otherwise.

Analytically and numerically calculate and plot

(a) $h(t)$

(b) The Fourier transform of $h(t)$, $H(\omega)$.

Problem 3

An electron of energy E is "incident" from $-x$ on a potential barrier of height V_o.

(a) Calculate the transmission and reflection coefficients for $E > V_o$

(b) Calculate the probability that the electron is transmitted, eventually reaching large values of x.

(c) Find a numerical value for the transmission probability and comment on the agreement/disagreement with the classical values for

$$\begin{array}{rcrcrcr} V_o & = & 4\text{eV} & ; & E & = & 5\text{eV} \\ V_o & = & 4\text{eV} & ; & E & = & 3\text{eV} \\ V_o & = & \text{-}400\text{eV} & ; & E & = & 3\text{eV}. \end{array}$$

Only 55000

Problem 4

Solve the infinite potential well when it is positioned between $-a/2$ and $a/2$. Show that the energies are the same as the problem solved in class. Make an x-axis displacement to show the eigenfunctions of the Hamiltonian are the same.
Can you build a Gaussian wavepacket? What are the limitations?