Polarization

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The polarization of electromagnetic (EM) waves can theoretically analyzed and manipulated with mathematics. In this report the theoretical predictions of polarization are compared to experimental results for incandescent and laser light. Polarizer systems consisting of combinations of linear polarizers, both dichroismic and reflective, and retardation plates are explored.

I. INTRODUCTION

The topic of polarization has several components of analytic interest: exploring properties of polarized light, exploring the production of polarized light, and exploring the difference between linearly polarized light and elliptically polarized light.

A. Polarized Light

Polarization is a property specific to transverse waves such as EM waves. Polarization describes, in the case of EM waves, the direction of the electric field vector $\vec{E}$. The amplitude of an EM wave can be analyzed with a Jones vector in which the total amplitude is evaluated in terms of its $\hat{x}$ and $\hat{y}$ components, as seen in equation 1. We note that because of the phase difference, we will expect the total amplitude of the electric field to rotate through the $\hat{x}$ and $\hat{y}$ directions. In fact, the most general polarized state is denoted as "elliptically polarized" and is completely described by the Jones vector. In the case of elliptical polarization, as expected, the electric field rotates throughout the plane perpendicular to the direction of propagation and traces out an ellipse with the tip of the resultant $\vec{E}$ vector.

All other cases of polarization can be described as super positions or special cases of this general mode of polarization. For example, circular polarization, where the tip of the electric field draws a circle instead of an ellipse, is represented by equation 2; in this case, the $\pm$ is determined by whether the rotation is clockwise or counterclockwise, with the positive direction defined as the counterclockwise direction. The case of circularly polarized light occurs with a phase difference of $\Delta\phi = \pi/2$ and $\Delta\phi = 3\pi/2$ between the phases of the electric field amplitude components and the amplitude of each component is equal. Linearly polarized light, the case where the amplitude components are precisely in phase or out of phase, $\Delta\phi = 0$ and $\Delta\phi = \pi$, is described by equation 3. We note that this case can also be expressed as a super position of left-circular and right-circular polarized light. We also note that equation 3 describes only the case of linear polarization in the $\hat{x}$ direction; however, transforming this vector with the rotation matrix defined in equation 4 can produce a vector representation of linearly polarized light in any direction.

Light produced by ordinary circumstances, such as incandescent light bulbs, fire, or the sun, is produced by many independent sources which are not synchronized in amplitude, frequency, or polarization and the thus net direction of electric field is random and this kind of light is described as randomly polarized.

$$\vec{E} = \begin{bmatrix} E_{0x}e^{i\phi_x} \\ E_{0y}e^{i\phi_y} \end{bmatrix} = \begin{bmatrix} A \\ be^{i\Delta\phi} \end{bmatrix} = \begin{bmatrix} A \\ B \pm iC \end{bmatrix} \tag{1}$$

$$\vec{E} = A \begin{bmatrix} 1 \\ \pm i \end{bmatrix} \tag{2}$$

$$\vec{E} = A \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{A}{2}(\begin{bmatrix} 1 \\ +i \end{bmatrix} + \begin{bmatrix} 1 \\ -i \end{bmatrix}) \tag{3}$$

B. Polarizers

As the polarization of light is described in vector notation by the Jones vector, equation 1, it is possible to mathematically define transformations which manipulate the polarization of light. For each of these transformations there exists an physical manifestation which enables us to conduct each of transformations in real life.

The rotator matrix, defined in equation 4, rotates the polarization of light an angle of β in the counterclockwise direction. As noted earlier, the rotation transformation enables us to describe direction of polarization distinctly from the mode of polarization. Rotation of polarization can be physically conducted with techniques such as Faraday rotators, Prism rotators, and more.

The linear polarizer is defined by equation 5; as expected, upon input of arbitrarily polarized light it produces linearly polarized light of a particular direction. Physically this is realized with materials which exhibit dichroism: these substances absorb light in in varying degrees depending on the direction and in optimal cases can produce nearly completely linearly polarized light.

* Code base for this research is publicly available at https://github.com/UladKasach/Academic/tree/master/...
...Physics/Phys401/polarization

When polarized light passes through a linear polarizer, the intensity of the resultant light is $I = 1/2 * I_0$. When polarized light passes through a linear polarizer, the intensity of the resultant light can be calculated according to Malus' law, defined in equation 6. A unique method of producing linearly polarized light involves Brewster's Angle: when light is reflected off of a surface varying intensities of the incident beam are reflected and transmitted in the parallel and perpendicular direction. In particular, when light is incident on a surface of greater refractive index, such as from air to glass, at Brewster's angle, defined by equation 7 and which $\theta_p = 56.31$ in the case of air to glass, the component of the electric field perpendicular to the boundary plane reflected becomes zero. In this case, the reflected beam is linearly polarized parallel to the boundary plane.

Phase retarders introduce a phase difference between the amplitude components of the electric field, defined by equation 8. In practice, this provides the ability to convert linearly polarized light into circularly polarized light, circularly polarized light into elliptically polarized, or any combination of the three. Physically, this is realized by utilizing a material in which the refractive index in one direction of the material is different to the refractive index in a perpendicular direction. In that case, one component of light will travel more slowly through the material and a phase difference manifests.

$$\begin{bmatrix} \cos(\beta) & -\sin(\beta) \\ \sin(\beta) & \cos(\beta) \end{bmatrix} \tag{4}$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \tag{5}$$

$$I = I_0 \cos^2(\theta) \tag{6}$$

$$\theta_p = tan^{-1}(\frac{n_2}{n_1}) \tag{7}$$

$$\begin{bmatrix} e^{i\Delta\phi_x} & 0 \\ 0 & e^{i\Delta\phi_y} \end{bmatrix} \tag{8}$$

II. METHODS

Polarization of light was investigated for light both from an incandescent light source as well as a laser light source. The affects of individual polarizers and combinations of polarizers was upon the light in terms of intensity was investigated. In most experiments, unless stated otherwise, calibrated, purpose manufactured, linear polarizers were utilized. In the case of the laser, polarization was produced by reflection. The effect of a retarder on the light was also physically investigated in the case of a laser.

Instruments utilized in this experimentation include an optical track, an optical probe, several calibrated polarizers, a retarder, a mirror, a glass cube, a rotating platform, an incandescent light source, and a laser light source.

1. Light Stability

For each light source it is prudent to ensure that the intensity of light output by the light source is stable. This can be conducted by measuring the intensity of the light over a set period of time, for example 5 minutes every 30 seconds. It is important to note that the intensity and polarization of light can change as a light source "warms up". For this reason it is important to ensure the laser has been on for more than 2 hours.

2. Single Polarizer

The transmission coefficient of light through a single polarizer can be found by utilizing a calibrated polarizer and measuring the intensity of the transmitted light with the optical probe over angles of $\theta = [0, 360]$. This data will enable us to validate the predictions of Malus' law and may be used to derive additional information about each light source.

3. Two Polarizers

The transmission coefficient of light through a set of two polarizers is also of interest to be analyzed. By placing two polarizers on the optical track, turning one of the polarizers, we can analyze the intensity of the light transmitted through both polarizers. This data can be then compared to Malus' law.

4. Three Polarizers

A product of Malus' law that defies intuition is non zero transmission coefficient that is produced when three polarizers are set on an optical track with optical differences of $\Delta\theta = 45$ between each one. The first and last polarizers are offset by $\Delta\theta = 90$, which produces a transmission coefficient of zero. When the middle polarizer is added, however, the transmission coefficient is nonzero. This prediction can be validated with three polarizers in the setup described above.

5. Retardation Plate

It is possible to assess, efficiently in the case of a laser, the polarization of light transmitted through a retarder.

By passing the laser light through first a polarizer, then retarder, then an additional polarizer. In this case, we set the system up in a way that the first polarizer makes a $\Delta\theta = 45$ angle between the retarder and the polarizer. After analyzing the polarity of the light produced by the first polarizer and the retarder we can then rotate the retarder and assess the intensity of the resultant light. The light was reflected back towards the lazer before being assessed with a mirror.

6. Brewster's Angle

As noted previously, reflection can be utilized to generate linearly polarized light. By placing a calibrated polarizer in front of the laser we guarantee that the transmitted light is linearly polarized. Utilizing a cube on a rotating platform, we can evaluate the intensity of the reflected light visually. In particular, it is of interest to find the angle of incidence at which the reflected light is at a minimum intensity: this corresponds to the incidence angle which results in totally linearly polarized light. If the calibrated polarizer is set to pass light polarized in an angle perpendicular to the cube surface, the minimum intensity of the reflected light will be zero.

θ	t_1	t_2
0	0.475	0.425
15	0.475	0.412
30	0.475	0.412
45	0.450	0.388
60	0.438	0.375
75	0.450	0.388
90	0.438	0.388
105	0.438	0.388
120	0.463	0.400
135	0.475	0.412
150	0.475	0.412
165	0.475	0.425
180	0.487	0.425
195	0.487	0.412
210	0.463	0.412
225	0.450	0.400
240	0.438	0.375
255	0.438	0.375
270	0.438	0.375
285	0.450	0.375
300	0.463	0.375
315	0.475	0.388
330	0.475	0.412
345	0.487	0.412
degrees		

TABLE I. Measurements of Transmission Probability of Incandescent Light Passing Through Polarizer One and Two

III. EXPERIMENTATION

A. Incandescent Light

Before experimentation, light stability was evaluated in order to ensure that the incandescent light source had fully heated up and no changes of intensity or polarization would occur. The light source was found to be stable over the 5 minute interval it was evaluated on.

1. Single Polarizer

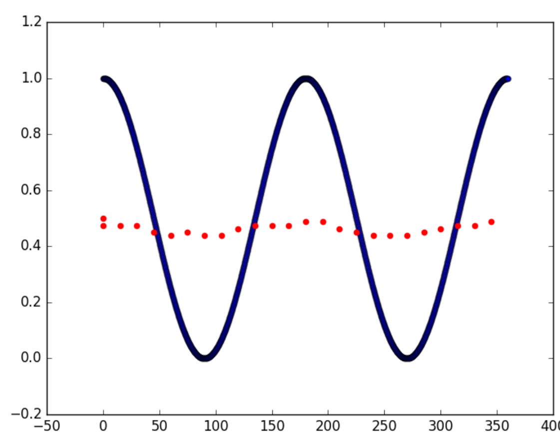

FIG. 1. Transmission Probability of Incandescent Light -vs- Malus' Law for Polarizer One

The intensity of incandescent light transmitted through a linear polarizer was analyzed for two distinct polarizers for angles of $\theta = [0, 360]$. The results are demonstrated in table I where t_1 and t_2 correspond to the transmission coefficient of light through polarizer one and polarizer two respectively. The data was then graphed and plotted in comparison to the expected values of Malus' law. The results are demonstrated in figures 1 and 2 for polarizers one and two respectively.

Note that the measured values of θ were shifted in order to align the maximum intensities at $\theta = 0$ and $\theta = 180$ as predicted by Malus' law for incident polarized light. This is acceptable as the values of theta recorded are relative.

2. Two Polarizers

The intensity of incandescent light transmitted through two linear polarizers offset by an angle of $\Delta\theta$ was analyzed for two distinct polarizers for angles of $\Delta\theta = [0, 360]$. The results are demonstrated in table II. The results were then plotted against the expected values predicted by Malus' law, producing figure 3. As before, the values of θ were shifted to align the maximum intensities at $\theta = 0$ and $\theta = 180$ as predicted by Malus'

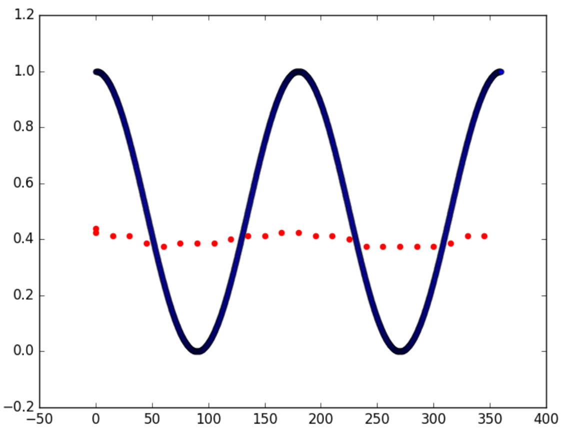

FIG. 2. Transmission Probability of Incandescent Light -vs- Malus' Law for Polarizer Two

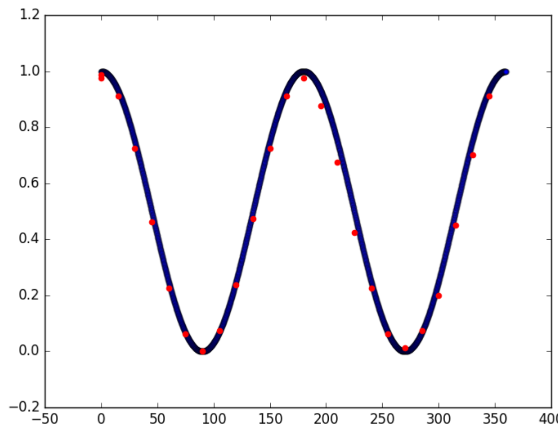

FIG. 3. Transmission Probability of Incandescent Light -vs- Malus' Law for Two Polarizers

θ	t	$t_{expected}$
0	0.975	1.000
0	0.988	1.000
15	0.912	0.933
30	0.725	0.750
45	0.463	0.500
60	0.225	0.250
75	0.062	0.067
90	0.000	0.000
105	0.075	0.067
120	0.237	0.250
135	0.475	0.500
150	0.725	0.750
165	0.912	0.933
180	0.975	1.000
195	0.875	0.933
210	0.675	0.750
225	0.425	0.500
240	0.225	0.250
255	0.062	0.067
270	0.013	0.000
285	0.075	0.067
300	0.200	0.250
315	0.450	0.500
330	0.700	0.750
345	0.912	0.933
degrees		

TABLE II. Measurements of Transmission Probability of Incandescent Light Passing Through Two Polarizers.

law.

B. Laser Light

Before experimentation, laser stability was evaluated in order to ensure that the laser had fully heated up and no sudden changes of intensity or polarization would occur. The laser light was found to be stable over the 5 minute period it was evaluated on.

It is important to point out that the laser light must be expanded in order to measure accurate intensity measurements from the optical probe. This can be done by passing the laser beam through a concave lens.

1. *Single Polarizer*

The intensity of laser light following a linear polarizer was analyzed for two distinct polarizers for angles of $\theta = [0, 360]$. The results are demonstrated in table III where t_1 and t_2 correspond to the transmission coefficient of light through polarizer one and polarizer two respectively.

The data was then plotted and compared to the expected value, given by equation 6. As before the values of θ were shifted to align the maximum intensities at $\theta = 0$ and $\theta = 180$ as predicted by Malus' law. The results are plotted in figures 4 and 6.

2. *Two Polarizers*

The intensity of laser light transmitted through two linear polarizers offset by an angle of $\Delta\theta$ was analyzed for two distinct polarizers for angles of $\Delta\theta = [0, 360]$. The results are demonstrated in table IV.

The data was then plotted and compared to the expected value, given by equation 6. As before the values of θ were shifted to align the maximum intensities at $\theta = 0$ and $\theta = 180$ as predicted by Malus' law. The results are plotted in figures **??**.

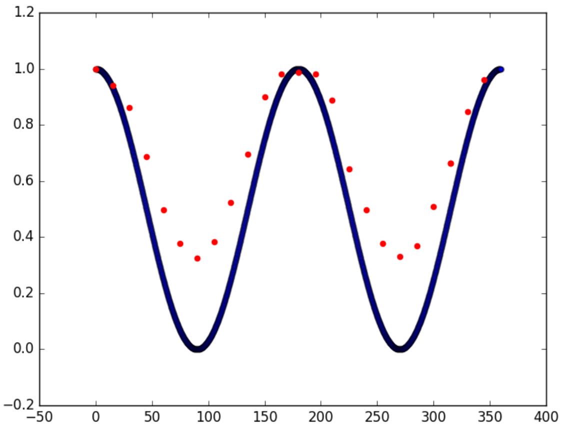

FIG. 4. Transmission Probability of Laser Light -vs- Malus' Law for Polarizer One

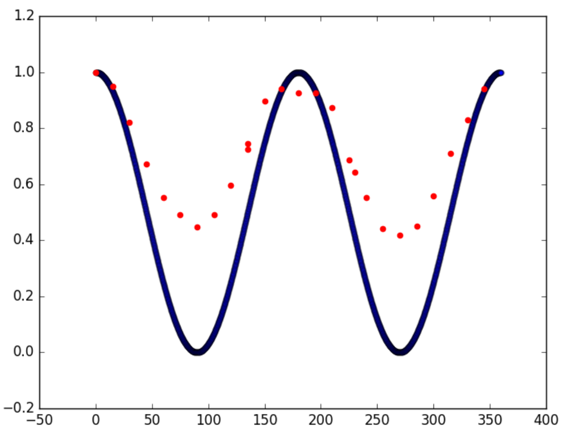

FIG. 5. Transmission Probability of Laser Light -vs- Malus' Law for Polarizer Two

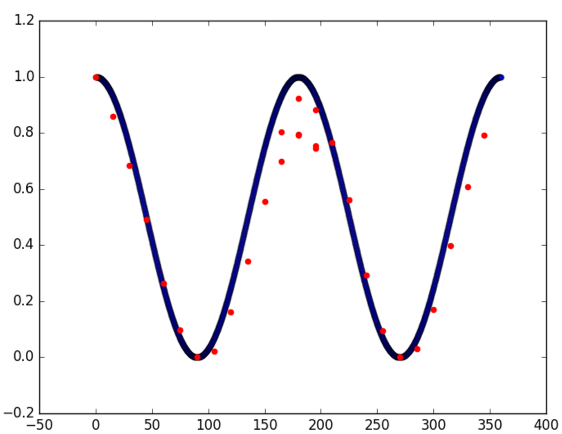

FIG. 6. Transmission Probability of Laser Light -vs- Malus' Law for Two Polarizers

θ	t_1	t_2
0	1.000	1.000
15	0.940	0.948
30	0.861	0.821
45	0.687	0.672
60	0.497	0.552
75	0.377	0.493
90	0.325	0.448
105	0.384	0.493
120	0.523	0.597
135	0.695	0.746
150	0.901	0.896
165	0.980	0.940
180	0.987	0.927
195	0.980	0.925
210	0.887	0.873
225	0.642	0.687
240	0.497	0.552
255	0.377	0.440
270	0.331	0.419
285	0.370	0.449
300	0.510	0.560
315	0.662	0.709
330	0.848	0.828
345	0.960	0.940
degrees		

TABLE III. Measurements of Transmission Probability of Laser Light Passing Through Polarizer One and Two

3. Three Polarizers

To compare the intensity of light produced by three polarizers offset by $\Delta\theta = 45$ with and without the middle polarizer and compare it to the expected values predicted by Malus' law we must first measure the intensity of light that passes through the first polarizer on its own. This value was found to be $I_f = 3.75$. When we add the last polarizer, we find an intensity of $I_{fl} = 0$, as predicted by Malus' law. When we add the center polarizer we find the intensity to be $I_{fcl} = 0.4$. We note that Malus's law predicts $I_{fcl} = I_f * 0.5 * 0.5 = 0.94$.

4. Retardation Plate

With the linear polarizer and the retardation plate offset by $\Delta\theta = 45$ the intensity of the transmitted beam further transmitted through an additional polarizer was analyzed with a viewing screen. It was found that the intensity did not vary as the analyzer polarizer was rotated. When analyzing the intensity of light transmitted through the polarizer retardation plate combination and reflected from a mirror at varying polarizer-retarder plate offsets it was found that there exists an angle at which the intensity of light is maximized and an angle at which the the intensity is minimized.

θ	t	$t_{expected}$
0	1.000	1.000
15	0.859	0.933
30	0.684	0.750
45	0.492	0.500
60	0.262	0.250
75	0.096	0.067
90	0.001	0.000
105	0.021	0.067
120	0.162	0.250
135	0.342	0.500
150	0.557	0.750
165	0.698	0.933
165	0.804	0.933
180	0.793	1.000
180	0.924	1.000
180	0.794	1.000
195	0.882	0.933
195	0.754	0.933
195	0.744	0.933
210	0.766	0.750
225	0.560	0.500
240	0.292	0.250
255	0.095	0.067
270	0.000	0.000
285	0.030	0.067
300	0.170	0.250
315	0.398	0.500
330	0.608	0.750
345	0.792	0.933
degrees		

TABLE IV. Measurements of Transmission Probability of Laser Light Passing Through Two Polarizers

5. *Brewster's Angle*

The theoretically predicted Brewster's angle can be validated utilizing a laser and a reflective surface. In this case, the reflective instrument utilized is a glass block with an approximate refractive index of $n = 1.5$. Plugging the refractive index of the block into equation 7 we find an expected value for the Brewster's angle of $\theta_p = 56.3$. The observed Brewster's angle, however, was $\theta_p = 57$.

IV. ANALYSIS

A. Log Errors

Our theoretical expected values of intensity are found by utilizing Malus' law, defined in equation 6. Analysis of the imprecision of Malus' law when utilized with the scale and imprecision of the measurements taken can be conducted utilizing the log errors approach. The resulting relationship is demonstrated in equation 9. $\Delta\theta = 1$, the resolution of θ we are able to measure. For one measure of intensity at each angle in the set $\theta = [0, 360]$ in intervals of 15 degrees with an ideally measured intensity at that angle, the mean log error value is 0.60. This means that for each dataset, deviations in the measured value of transmission coefficient up to 0.60 can be explained by the imprecision in our measuring instruments for Malus's law. (!)(This does not make sense!)

$$\Delta I = -2I \tan(\theta)\Delta\theta \tag{9}$$

B. Incandescent Light

1. *Single Polarizer*

The results experimentally found for the transmission coefficient of unpolarized light through a polarizer matches closely with the expected value of $t = 0.5$, the average of Malus' law for all angles. The average measured intensity for both polarizers is $t_{avg} = 0.4315$ with a standard deviation of $\sigma = 0.0368$. This is consistently below the expected value but may be explained by attenuation of light caused by energy being lost in the polarizers.

2. *Two Polarizers*

The results experimentally found for the transmission coefficient of unpolarized light transmitted through two polarizers at various relative angles matches very well with the transmission coefficient predicted by Malus' law. The RMSE, defined by equation 10, for the data points is $RMSE = 0.034$. Given that the precision of the measuring instrument was 0.25, this is a very good precision.

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n}(y_i' - y_i)^2}{n}} \tag{10}$$

C. Laser Light

1. *Single Polarizer*

The results found for the single polarizer show that the laser is not fully polarized in one direction and instead that it is polarized in a range of directions. We find this conclusion by seeing that the intensity of the transmitted light never reduces to zero. We note that there is likely an error with the data captured because we did not record the absolute maximum intensity without transmission through a polarizer. It is very likely that the maximum intensity of light transmitted through the polarizer never reached I_0 and that the maximum transmission coefficient never reached $t = 1$.

2. Two Polarizers

The results experimentally found for the transmission of laser light through two polarizers matches less precisely than it did for the incandescent, unpolarized light. The RMSE for the datapoints was $RMSE = 0.1146$. This is most likely due to the the same reason that we were not able to get a consistent measured value of intensity in the range $\theta = 150$ to $\theta = 200$. Measurements were taken in this range of θ values at the beginning and end of experimentation. Although the laser was on for more than 5 days and the polarization was stable over the test period of 5 minutes, it seems that we experienced a shift in polarization nonetheless.

3. Three Polarizers

The difference between the expected value and the found value for the three polarizer system is significantly large: $\Delta I = 0.94 - 0.4 = 0.54$. It is relevant to note, however, that attenuation of light may be playing a significant role here. As the intensity of the laser unobstructed was $I_0 = 9$ an attenuation of $I_a = 0.54$ does not seem implausible. The decrease in transmitted light due to attenuation of light in this case would be $0.54/9 = 6\%$ We noted with the incandescent light that attenuation produced a, on average, a 6.8% decrease in intensity with only one lens.

4. Retardation Plate

Theoretically transmitting linearly polarized light through a retarder will create elliptically polarized light. We found that the intensity of light did not vary with the rotation of the analyzer polarizer. This is likely due to the fact that the oscillation of the polarized light is much faster than we can detect visually. This is to be expected as, at $\lambda = 632.8nm$, $f = c/\lambda \approx 4.74 * 10^{10}$. This rate of oscillation is surely impossible to see with the instrumentation available.

5. Brewster's Angle

The theoretical value and observed value for the angle of incidence at which total linear polarization occurs were found to be accurate within 1 degree. As the error in this case is lower than the resolution of the measurements available, this is a very agreeable value. Further, we can confidently assume that the block is of refractive index $n = 1.5$.

V. CONCLUSION

Mathematical representations of polarized light an polarizers can be utilized to predict experimentally validated measurements of polarized light with great accuracy. Although the theoretical analysis does not account for non-ideal phenomena such as the attenuation of light and non-stable light sources, it still does a very good job at predicting what occurs when light interacts with polarizers in the real world.