Reflection and Refraction

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Both reflection and refraction can be utilized as methods by which the index of refraction of a material can be determined. This report demonstrates this capability by deriving the index of refraction of a glass cube and an acrylic plate utilizing each method. The results show that both methods derive similar indices of refraction for the same material.

I. INTRODUCTION

Reflection and refraction are optical phenomena which can be described by geometric optics. As light is incident on a smooth surface both phenomena occur, except for certain angles of incidence. Refraction and reflection are utilized in a wide array of optical instruments; for example, refraction is used to modify the focal length of optical lenses and reflection is the "phenomenon responsible for guiding of light in optical fibers"[1].

A. Reflection

Reflection was described as early as 300 BCE by Euclid, a Greek mathematician[3]. Reflection is the phenomenon which occurs when light incident on a surface is "thrown back". Similar to an an elastic collision, light "bounces" off the surface it is incident on. The law of reflection states that the angle of reflection, θ_r, is equal to the angle of incidence, θ_i, when both are measured with respect to the normal of the surface:

$$\theta_i = \theta_r \tag{1}$$

B. Refraction

Refraction describes the phenomenon which occurs when light changes direction after entering crossing the interface between any two media of different indices of refraction. The law of refraction, described mathematically by Willebrord Snellius in 1621[2], states that the sine of the angle of refraction, θ_t, is proportional to the angle of incidence, θ_i, where both are measured with respect to the normal of the surface:

$$n_i \sin\theta_i = n_t \sin\theta_t \tag{2}$$

The refractive index,n, of a material is defined by this relationship. By convention, the refractive index of air is defined as 1. All other refractive indices are derived from this relationship and this convention. When light enters a medium of greater refractive index, the angle of refraction is smaller than the angle of incidence and vice versa.

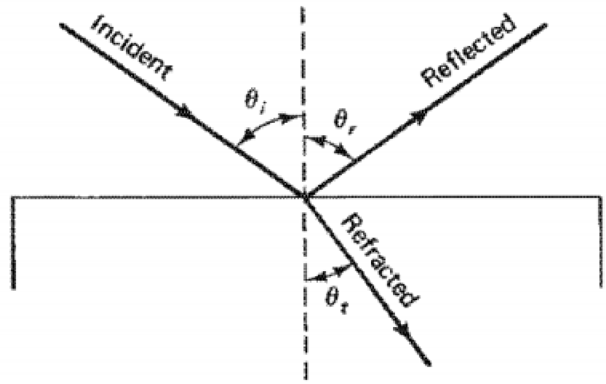

FIG. 1. Refraction and Reflection

C. Lateral Displacement

Refraction leads to the phenomenon of lateral displacement. Lateral displacement is defined as the separation width between a ray of light which passes into and out of a medium of greater refractive index and the same ray if it had not passed through the medium (figure 2). The relationship between the lateral displacement, d_t, and the angle of refraction can be derived as follows:

$$\sin(\theta_1 - \theta_2) = d_t/l \tag{3}$$

$$\cos\theta_1 = t/l \tag{4}$$

$$\therefore d_t = \frac{t\sin(\theta_1 - \theta_2)}{\cos theta_1} \tag{5}$$

* Code base for this research is publicly available at https://github.com/UladKasach/Academic/tree/master/...
...Physics/Phys401/reflection_and_refraction

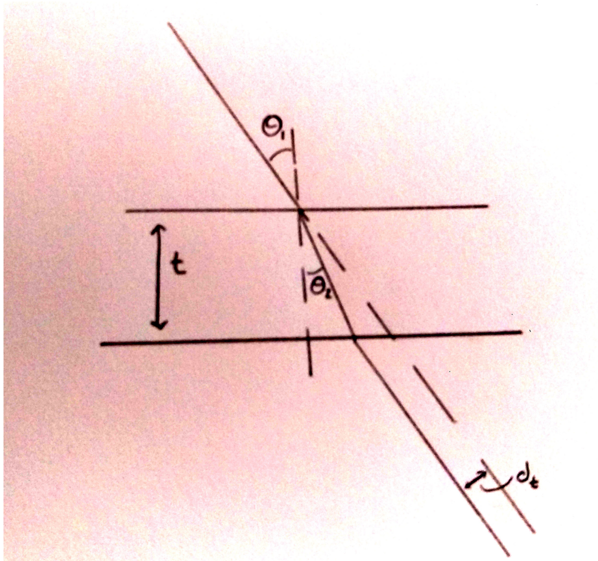

FIG. 2. Lateral Displacement

D. Separation of Reflected Beams

Reflection in combination with refraction leads to the phenomenon of separated reflected beams. The separation is measured between a ray that is reflected at incidence and a ray that is first refracted upon entry into a material, then reflected, and then refracted again upon exit (figure 3). The relationship between the separation of the reflected beams, d_r, and the angle of refraction can be derived as follows:

$$\sin(90 - \theta_1) = \cos\theta_1 = \frac{d_r}{2w} \tag{6}$$

$$\tan\theta_2 = w/t \tag{7}$$

$$\therefore d_r = 2t\cos\theta_1 \tan\theta_2 \tag{8}$$

II. METHODS AND PROCEDURES

The refractive index of both the glass cube and the acrylic is determined by utilizing both the lateral displacement and the separation of reflected beams phenomena. The experimentation described here requires a optical track, a He-Ne laser, an angular rotation table, a viewing screen, a glass cube, and an acrylic plate. The thickness of the glass cube is $53mm$ and the thickness of the acrylic plate is $19.5mm$.

The optical track is utilized to secure the angular rotation table and He-Ne laser inline with each other as shown in figure 4. The angular rotation table is secured

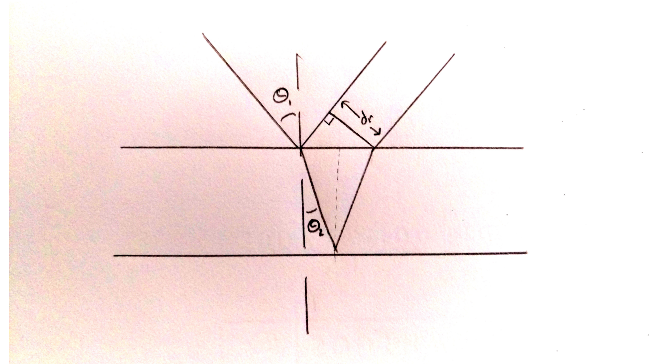

FIG. 3. Separation of Reflected Beams

to the optical track by magnetism while the laser is secured with elastic compression.

A. Utilizing Lateral Displacement

The lateral displacement relationship (equation 5) can be used to express the angle of refraction, θ_2, in terms of the angle of incidence, θ_1, the thickness of the material, t, and the displacement distance, d_s (equation 9). The angle of refraction and angle of incidence can then be used in Snell's law (equation 2) to determine the refractive index (equation 10).

$$\tan(\theta_2) = \frac{\sin(\theta_1) - d_s/t}{\cos(\theta_1)} \tag{9}$$

$$n_2 = \frac{\sin\theta_1[(d_s/t)^2 - 2(d_s/t)\sin\theta_1 + 1]^{1/2}}{\sin\theta_1 - (d_s/t)} \tag{10}$$

In order to increase the precision of our measurements of the angle of incidence, the laser was adjusted and secured into a position where the ray beam was horizontally parallel to the optical track. This was done by ensuring that the beam was incident at the same horizontal position on the viewing screen close to the laser and at the end of the track.

In order to measure displacement of the beam the position of the non-refracted beam on the viewing screen is required. The optical system was setup in such a way that the metric scale on the viewing screen read "zero" when there was no refraction.

In order to calibrate the angle of incidence measurement on our angular rotation table, the block was positioned in such a way that when the angular rotation read zero degrees the lateral displacement of the system was zero.

With this experimental setup, the incident angle and lateral displacement can accurately be measured and the refractive index of any translucent material with a smooth surface can be determined. The results of this

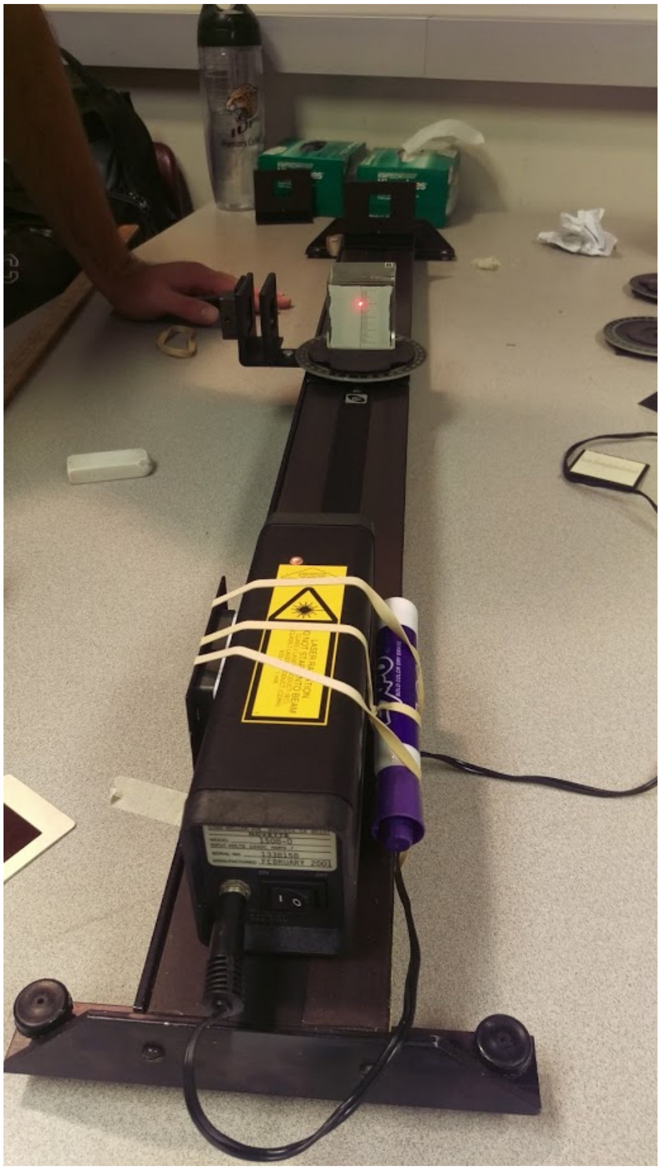

FIG. 4. Setup of Inline Laser and Rotation Table

experiment for the glass cube and the acrylic plate are shown in tables I and II respectively.

B. Utilizing Separation of Reflected Beams

The separation of reflected beams relationship (equation 8) can be used to express the angle of refraction, θ_2, in terms of the angle of incidence, θ_1, the thickness of the material, t, and the separation distance, d_r (equation 11). The angle of refraction and angle of incidence can then be used in Snell's law (equation 2) to determine the refractive index (equation 12).

$$\tan(\theta_2) = \frac{d_r}{2t\cos(\theta_1)} \tag{11}$$

θ_1	d_t	d_t
5	1.5	1.5
10	3.0	3.5
15	4.5	4.5
20	3.5	3.5
25	9.5	9.0
30	11.0	11.0
35	15	13
degrees	mm	mm

TABLE I. Glass cube lateral displacement for two trials

θ_1	d_t	d_t	d_t	d_t
5	0.5	0.5	0.5	0.5
10	1.0	1.0	1.0	1.0
15	1.5	1.5	1.5	1.5
20	2.0	2.0	2.0	2.5
25	3.0	3.0	3.0	3.0
30	3.5	3.5	3.5	3.5
35	4.5	4.5	4.5	4.5
degrees	mm	mm	mm	mm

TABLE II. Acrylic plate lateral displacement for four trials

$$n_2 = \frac{\sin\theta_1[d_r^2 + 4t^2\cos^2\theta_1]^{1/2}}{d_r^2} \tag{12}$$

The laser beam must be kept parallel to the optical track, as was done for the lateral displacement setup, to increase the precision of the measurement of incidence angle. The angular rotation table must also be kept calibrated with the method utilized in the previous section in order to increase the precision of our measurements.

For this setup the metric scale was moved onto a viewing screen attached to the angular rotation table. The screen attached to the rotation table rotates completely around the surface. When taking measurements, it is important to attempt to make the rotation table perpendicular to the reflected beams in order to increase the precision of our measurements. The perpendicular distance is defined as d_r and any deviation from perpendicularity will result in errors.

With this experimental setup, the incident angle and separation of reflected beams can accurately be measured and the refractive index of any translucent material with a smooth surface can be determined.

III. DISCUSSION

Statistic analysis of the data generated through the experimentation is displayed in table V. In this experiment, the analysis reveals that the precision of measurements is more correlated with the material used rather than the method used. The analysis also reveals a significant difference between the refractive index generated with each method for the each material.

θ_1	d_r	d_r	d_r
4	6.5	6.5	6.5
8	11.5	11.0	11.5
12	16.0	16.5	15.5
16	20.5	20.5	20.5
20	25.0	24.5	25.0
24	28.5	28.5	28.5
28	32.0	31.5	32.0
32	35.0	35.0	35.5
36	38.0	37.5	
degrees	mm	mm	mm

TABLE III. Glass cube beam separation for three trials

θ_1	d_r	d_r	d_r
4	2.0	2.0	2.0
8	4.0	4.0	3.5
12	5.5	5.0	5.0
16	7.5	7.0	7.0
20	9.0	8.5	8.5
24	10.5	10.0	10.5
28	12.0	11.5	11.0
32	13.0	12.5	12.5
36	14.0	14.0	13.0
degrees	mm	mm	mm

TABLE IV. Acrylic plate beam separation for three trials

Possible sources of error may come from the resolution of separation distance and incident angle that we were able to take. Another possible source of error could come from reading the incident angle measurement with an angled viewpoint, leading to a misread incident angle. Error could also come from a de-calibration of the system: the laser could have been made not parallel or the material could have been bumped on the rotation table, either of which would have resulted in imprecise incidence angle measurements. Further, in the measurement of beam separation error could have come from the viewing screen not having been perfectly perpendicular to the reflected beams.

The method of log errors can be used to further assess the precision with which measurements are taken. The precision of the index of refraction, Δn_2 can be assessed by taking the log and then the derivative of both sides of equations 10 and 12 yielding the relationships of equation 13 and and equation 15 respectively. These relationships

method	object	mean	stdev	data points
L.D.	glass	1.55	0.1166	16
R.B.S	glass	1.35	0.1007	26
L.D.	acrylic	1.42	0.0394	28
R.B.S	acrylic	1.50	0.0718	27
		mm	mm	

TABLE V. Statistical analysis of the refractive index calculated from the data points generated in experimentation. L.D. stands for lateral displacement. R.B.S. stands for reflected beam separation.

method	object	mean error	stdev of error	data points
L.D.	glass	-0.18	0.6008	16
R.B.S	glass	0.18	0.1151	26
L.D.	acrylic	-1.54	2.7129	28
R.B.S	acrylic	0.38	0.2740	27
		mm	mm	

TABLE VI. Statistical analysis of errors of refractive index measurements. L.D. stands for lateral displacement. R.B.S. stands for reflected beam separation. Note, the relationship for L.D. is wrong and the values are nonsensical

were utilized to calculate the error of each refractive index measurement. A statistical analysis of the errors is presented in table VI.

Note, the relationship for L.D. is wrong and the values are nonsensical. The error should not be negative and it certainly should not be of the same order of magnitude as the actual measurement. There is likely a small math mistake being conducted in the derivation, availible upon request.

$$\frac{\Delta n_2}{n_2} = \theta_1 \Delta\theta_1 - \frac{\cos\theta_1 \Delta\theta_1 - \Delta(d_s/t)}{\sin\theta_1 - (d_s/t)} + \frac{\Delta(d_s/t)(1-\sin\theta_1) - (d_s/t)\cos\theta_1\Delta\theta_1}{(d_s/t)^2 - 2(d_s/t)\sin\theta_1} \tag{13}$$

$$\Delta(d_s/t) = \Delta d_s/t - s\Delta t/t^2 \tag{14}$$

$$\frac{\Delta n_2}{n_2} = \theta_1 \Delta\theta_1 - \frac{2\Delta d_r}{d_r} + \frac{d_r \Delta d_r + 4[\Delta t \cos\theta_1 - \Delta\theta_1 t \sin\theta_1]}{d_r^2 + 4t^2\cos^2\theta_1} \tag{15}$$

IV. CONCLUSION

The optical phenomena of reflection and refraction can be used to determine the index of refraction for a material. The work documented in this report demonstrates exactly how these two phenomena may be employed for this purpose. Future work would involve further assessing what properties lead to the discrepancies in index of refraction determined for a material between the two methods.

REFERENCES

[1] Govind P Agrawal. *Nonlinear fiber optics*. Academic press, 2007.

[2] Alistair Kwan, John Dudley, and Eric Lantz. "Who really discovered Snell's law?" In: *Physics World* 15.4 (2002), p. 64. URL: `http://stacks.iop.org/2058-7058/15/i=4/a=44`.

[3] *Pseudo-Euclid's On Mirrors (Catoptrics)*. 1972. URL: https://books.google.com/books?id=SlnRYgEACAAJ.