Determination of the Capacitance of Rectangular Parallel Capacitor Using Finite Difference Method

Article · April 2014

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International Journal of Emerging Technology & Research
Volume 1, Issue 3, Mar-Apr, 2014 (www.ijetr.org) ISSN (E): 2347-5900 ISSN (P): 2347-6079

Determination of the Capacitance of Rectangular Parallel-Plate Capacitor Using Finite Difference Method

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Abstract

In this paper a numerical method is presented for determining the static charge distribution, and capacitance per unit length of a rectangular parallel plate capacitor in vacuum including fringing effect. Capacitances were determined as a function of separation distance between plates held at a potential of 2 Volts defined inside a relatively large conducting box at zero potential. The method involves solving Laplace equation numerically using Finite Difference Scheme for calculating potential distribution in and outside the capacitor. The charge distribution on the plates and electric fields everywhere within the box were calculated from the potential distribution. Charge density and capacitance per unit length were extracted for the chosen grid sizes. The Richardson's extrapolation method was applied to get the more accurate results. The results obtained agree very well with the calculated analytical values. A comparison of the results with the analytical values is also presented in the paper.

Keywords: *Finite Difference Method, Capacitance, Charge density, Electric Potential.*

1. Introduction

In electrostatics many problems have boundaries that do not coincide with the coordinate systems. For these cases, analytical methods may not be available, and numerical methods must be used. The finite difference method (FDM) is one of the powerful methods for solving Laplace or Poisson's equations subject to conditions on boundary surfaces of arbitrary shapes. Determining capacitance of parallel plate capacitors accurately is important in many ways. According to literature survey determining capacitance of parallel plates has been done from time to time by using several methods. The method of subareas has been used by Reitan[1] to obtain an accurate capacitance of rectangular parallel plate capacitors. The projection method has been used by Harrington[2]. The point-matching method has been applied by Adams and Mautz[3]. A new projective method has been used by Benedek[4] to determine capacitance with the presence of dielectrics. But, Finite Difference Method also can be used to determine more accurate capacitance of rectangular parallel plate capacitor. The main purpose of this paper is to apply FDM to determine capacitance per unit length for parallel-plate capacitor in vacuum based on the determined potential. The charge density on the plates and electric fields within and outside the capacitor will also be calculated from the estimated potential distribution.

2. Finite Difference Solution

In this work a parallel plate capacitor were defined in the middle of a $3 \times 3\ m^2$ square with plates parallel to the coordinate plane. The potential of the upper plate, bottom plate and edges of the square were Dirichlet boundaries fixed at $+1\ V$, $-1\ V$, and $0\ V$ respectively. The potential satisfies Laplace's equation $\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 0$ in two dimensions. Uniform grid of points was considered in both x and y directions. The Laplace equation was solved numerically using FDM for potentials[5] at all free grid points inside the square varying the grid sizes and the ratio width, d to length, a of the capacitor. Here we required a 10^{-6} tolerance as iteration stopping criteria. The iteration was done using well known Gauss-Seidal method [6]. The programming was done using C++ programming language. As for example, Figure 1 shows the histogram of potential distribution in the entire square region after the iteration was completed for particular d/a. We visually see the potential distribution is symmetric around zero and

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that lies between $1V$ and $-1V$. Thus again we make sure the solution is accurate and nothing is wrong with the iteration convergence. Then we obtained equipotential lines and electric field distribution calculated from the potential. The distribution of equipotential lines and the electric field lines are shown in the superimposed plot shown in Figure 2. It can be seen that the potential and electric fields are uniformly distributed in the middle of the capacitor. They are somewhat bending in the edges of the capacitor as we expect [7]. The value of the electric field is also greater in the edges of the plates. In addition, electric field lines exist in both surfaces of the plates.

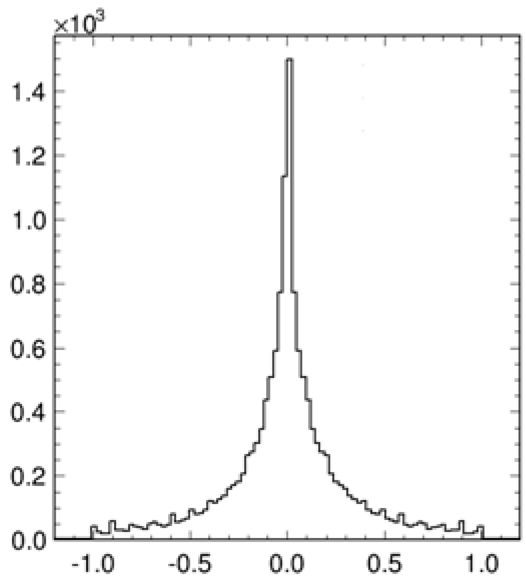

Figure 1 Potential Variation

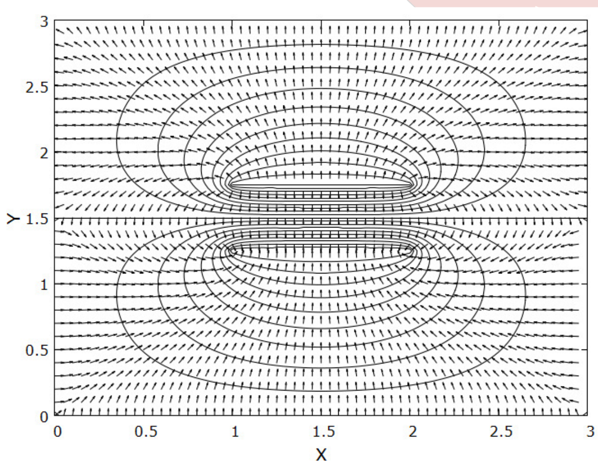

Figure 2 Potential and electric field distribution in the capacitor

3. Charge Distribution

Once the iteration process was finished the surface charge density at nodes on each side of the plates was found by using the potentials. While the surface charge density lying on the top surface was estimated numerically by

$$\sigma_s^t = \epsilon_0 E_y = -\epsilon_0 (V(x, y + \Delta) - V(x, y))/\Delta \,, \qquad (1)$$

and on the bottom surface was estimated by

$$\sigma_s^b = -\epsilon_0 E_y = \epsilon_0 (V(x, y) - V(x, y - \Delta))/\Delta \qquad (2)$$

Here (x, y) are points on the plate and $V(x, y)$ are potentials on the plate. The same procedure was followed to estimate surface charge densities on the other plate as well. As for example Figure 3 shows the surface charge density variation in the inside surface of the capacitor plate of $+1V$.

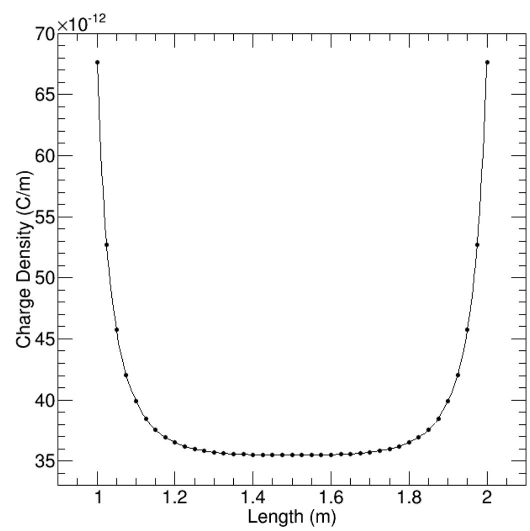

Figure 3 Static charge density variation of the capacitor plate

Then the total surface charge per unit length can be obtained by using the equation $\rho = \int_s \sigma_s \Delta x$. Integration was approximated numerically by implementing Trapezoidal rule [8].

4. Capacitance

Once the equivalent surface charge densities are determined, the capacitance can be numerically computed.

Capacitance per unit length was calculated as $C = \frac{\rho}{V}$ where the voltage V is the difference between the potential values on the two conductors. Average value of the total charges on the inner surfaces of two conductors was used for ρ in the calculation. We thus calculated a value for capacitance for each value of h for chosen d/a ratios as shown in first two columns in the Table 1. Finally, we applied Richardson extrapolation [9] to these capacitance values to arrive at our best estimate which is shown in column 4 in the Table 1. The C_0 values of the capacitance per unit length shown in the Table 1 were calculated by using the formula $C_0 = \epsilon_0 a/d$ where ϵ_0 is the permittivity of vacuum (or free space).

Table 1 Capacitance in Farads and percent difference as a function of plate spacing

$\frac{d}{a}$	h	C_{num} 10^{-11}	C_{ext} 10^{-11}	C_0 10^{-11}	% Error $\frac{(C_{ext} - C_0)}{C_0}$
0.05	0.05	17.708	17.814	17.708	0.601
	0.025	17.708			
	0.0125	17.748			
0.1	0.05	8.854	9.017	8.854	1.836
	0.025	8.894			
	0.0125	8.945			
0.2	0.05	4.468	4.631	4.427	4.601
	0.025	4.528			
	0.0125	4.568			
0.25	0.05	3.599	3.752	3.542	6.082
	0.025	3.652			
	0.0125	3.698			
0.5	0.05	1.887	2.023	1.771	14.227
	0.025	1.935			
	0.0125	1.974			
1	0.05	1.069	1.189	0.885	34.2
	0.025	1.112			
	0.0125	1.146			

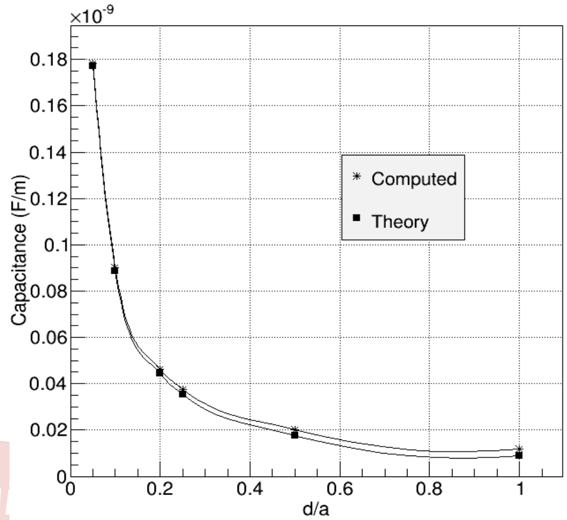

Figure 4 Capacitance versus ratio of d/a.

The last column in the Table 1 shows the percent excess fringing capacitance values or the percent error. We see that percent error become significance when the d/a ratio increases. For lower values of d/a, our result is in good agreement with the ideal value C_0 as the fringing field has little significance.

5. Conclusions

In this paper, static surface charge density, equipotential lines, electric fields, and capacitance per unit length were numerically calculated successfully based on the determined potential using FDM for rectangular parallel-plate capacitor.
Figure 4 shows the comparison of capacitances with respect to d/a ratio. Our result is very close to the exact values for lower values of d/a as fringing field is little significance in this case. For higher values of d/a, the result should be compared with the edge corrected capacitance values which should be investigated for further study.

Acknowledgments

The author would like to thank University of Ruhuna for the financial support given to publish this research paper.

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