Frequency Responses in RLC Circuits

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(Dated: February 7, 2018)

The bandpass filter and the notch filter, the 'inverse' of the bandpass filter, are circuits for valued for their utility in attenuating specific frequencies of a signal. In this paper we create the two filters, in the forms of resistor-inductor-capacitor (RLC) circuits, and examine both circuits from experimental and theoretical views. The filters are shown to accurately attenuated selective frequencies based on chosen component values. The results demonstrate that these filters are highly predictable: the theoretical and experimental values match very well even in non-ideal circumstances.

I. INTRODUCTION AND BACKGROUND

Filters are commonly used circuits for the selective attenuation of specific frequencies. A band-pass filter is designed to attenuate all frequencies except those lying within a certain band. A notch filter, or band-reject filter, is the 'inverse' of the band pass filter; all frequencies are passed in a notch filter except for those lying with a certain range, the 'notch'. An RLC circuit is a simple yet effective method for the construction of both of these common and powerful filters.

A. RLC Filter Theory

The RLC circuit of a band-pass filter is shown in figure 1. This filter attenuates all signals except for those lying within a region of frequencies center around a specific frequency known as the resonance frequency,f_o. Resonance occurs when the impedance, a function of applied frequency, of the inductor and the capacitor cancel one another and the total impedance of the circuit is entirely due to the resistor. The equations for the inductive reactance and capacitive reactance are given in equations 1 and 2 respectively, where j is the imaginary number, ω is the angular frequency (2πfrequency), L is the inductance, and C is the capacitance.

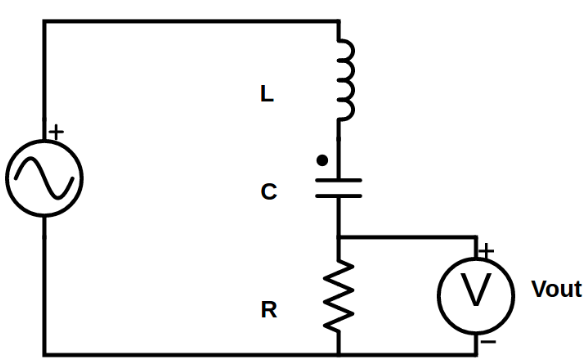

FIG. 1. Bandpass Filter

$$X_L = j\omega L \tag{1}$$

$$X_C = -1/(j\omega C) \tag{2}$$

Equations 1 and 2 determine the impedance of the inductor and capacitor, the analogue of resistance for the transient components of the two. At a specific value of ω designated by ω_o, the two terms will be equal and opposite. Using Ohm's law, the voltage found on the capacitor and inductor will sum to zero and thus V_{source} will be equivalent to the voltage found across the resistor. The notch filter works on the same principles except the output voltage will be taken across the inductor capacitor component pair.

$$Q = \omega_o L/R = \frac{f_o}{f_2 - f1} \tag{3}$$

$$a(w) = V_o/V_s = \frac{R}{\sqrt{R^2 + (\omega L - 1/\omega C)^2}} \tag{4}$$

The bandwidth of a filter is often described by its Q-Factor, defined in equation 3, where a larger Q-Factor implies a narrower bandwidth and a smaller the opposite. The Q-Factor is both a function of the circuit components, the inductor and resistor, and the angular resonance frequency, ω_o. At the same time, the Q-Factor can additionally be found by dividing the resonance frequency, $_o$ by the frequencies above and below resonance where the gain, equation 4, is half.

II. EXPERIMENTS

Experimentation in this project involved defining the values of the components we were working with followed by the frequency response analysis of both the bandpass and notch filter. The component values were used for calculating theoretical values and the frequency response analysis defined our experimental results - used for determining the resonance frequency and bandwidth.

A. Measurement of Component Values

In order to calculate theoretical values for our circuits we needed to define the actual values of the components of our RLC circuits.

The measurements of the resistor and the capacitor were very straight forward due to the availability of a modern multi-meter capable of explicitly measuring both. Measuring the inductance of the chosen inductor was more complicated and required the construction of the circuit shown in Figure 2. Equation 5 is the rearrangement of Ohm's law where f is the applied frequency, V_{source} is the applied voltage in root-mean-square(rms) and i is the current measured from the test circuit, which enables us to calculate the value of the inductor. Table II A shows the measured values for the components used in the construction of both filters.

$$L = V_{source}/2\pi f i \tag{5}$$

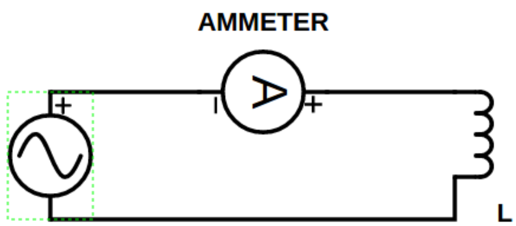

FIG. 2. Inductance Measurement Test Circuit

Measured Component Values	
Resistor(R)	344Ω
Inductor(L)	10.3mH
Capacitor(C)	3.7nF

B. Bandpass Filter Frequency Response Analysis

The circuit shown in figure 1 was constructed using the previously measured component values. The applied voltage was swept through a wide array of frequencies and the values of V_{out} and V_{source} were measured at each applied frequency. The results of this experiment are plotted in figure 3.

The phase shift induced by the inductor and capacitor were then separately monitored, again sweeping through a similar range of frequencies and noting manually the phase change.

Our next point of interest dealt with how V_o would respond to varying types of input wave function, such as triangle and square. The results were surprising and are shown in figures 4 - 6

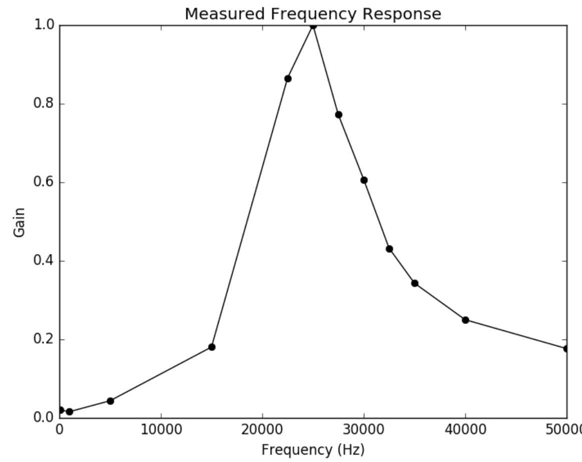

FIG. 3. Measured Frequency Response Curve

A more advanced method of measuring phase shift was then additionally implemented by by using the built in math function of the oscilloscope, Ch1-Ch2. This ability yielded a notable but predicted result. Simultaneously, this opportunity was used to measure the voltage across the inductor and capacitor separately. These values were used as one more method of obtaining a Q-Factor value experimentally, as shown in equation 6.

$$Q = V_s/V_L = V_s/V_C \tag{6}$$

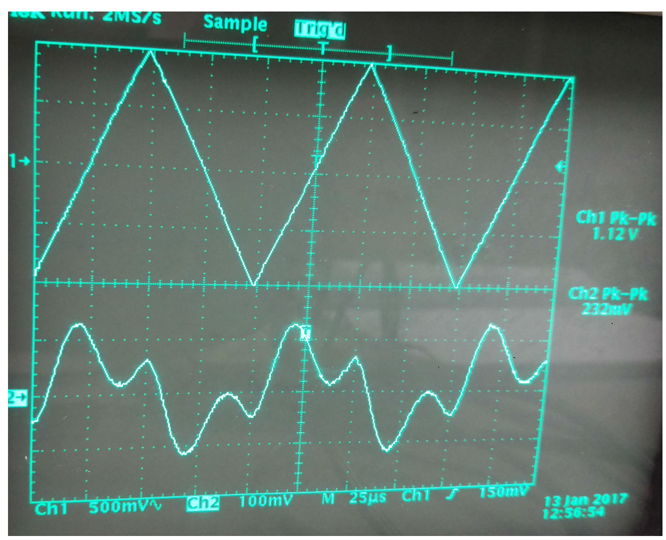

FIG. 4. Triangle Wave Response at 10kHz

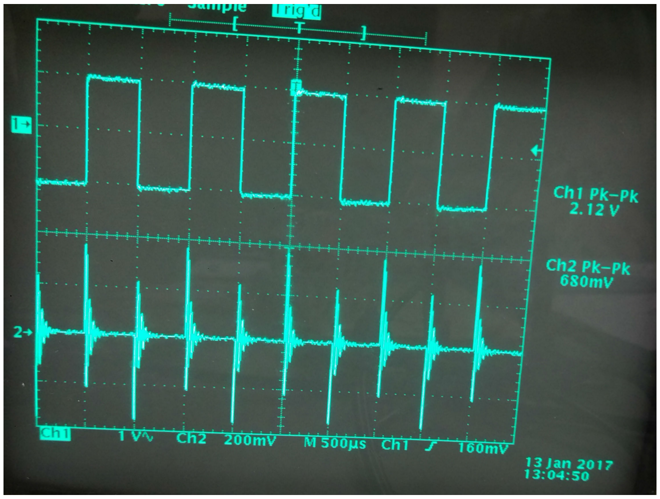

FIG. 5. Square Wave Response at 1kHz

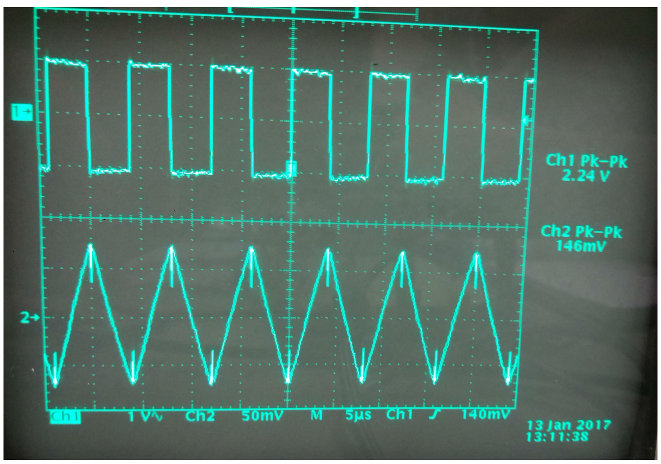

FIG. 6. Square Wave Response at 50kHz

C. Notch Filter Frequency Response Analysis

A notch filter, the RLC circuit for whom is shown in figure 7, was then constructed and a similar frequency response analysis as for the bandpass filter was applied. In this case, resonance frequency will correspond to the frequency at which attenuation occurs and the result should be straight forward to find. Sweeping through the frequencies generated a full and agreeable analysis like before. The results are shown in figure 8.

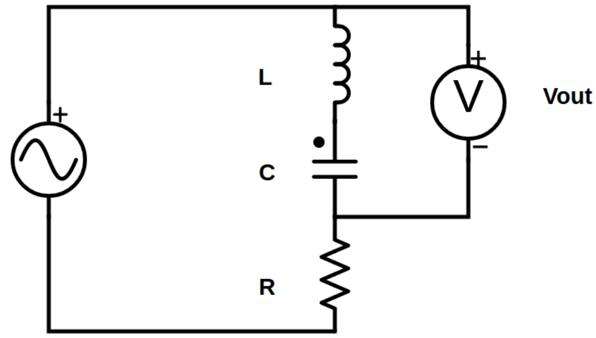

FIG. 7. Notch Filter

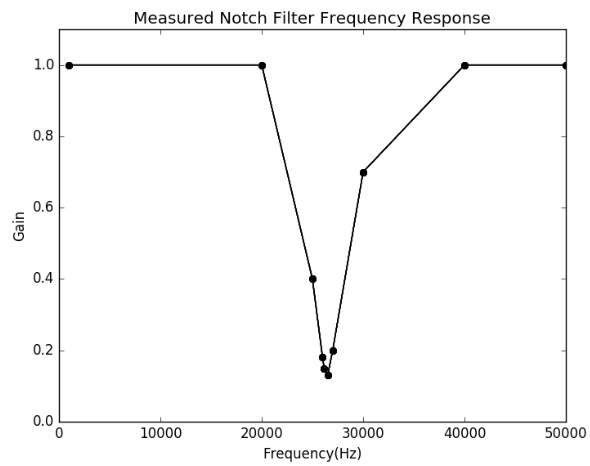

FIG. 8. Measured Notch Filter Frequency Response Curve

III. DISCUSSION

A. Band-pass Filter

Using Kirchoff's Voltage Law (KVL) on the circuit shown in Figure 1 yields equation 7. Where ω is the angular frequency ($2\pi f$) and j is the imaginary number.

$$V_{source} = i(j\omega L - 1/j\omega C + R) \tag{7}$$

Using Ohm's law on the resistor yields equation 8. Equations 7 and 8 can be combined to find the transfer function of the circuit as a function of the applied frequency. This relation is shown in equation 9. The plot of the magnitude of the theoretical transfer function is shown in figure 9. The plot of the magnitude of the transfer function experimentally measured is shown in figure 3. The experimentally measured f_o is found to be at approximately 25 KHz.

$$V_{out} = iR \tag{8}$$

$$\frac{V_{out}}{V_{source}} = T(\omega) = \frac{R}{(j\omega L - 1/j\omega C + R)} \tag{9}$$

$$f_o = 1/(2\pi\sqrt{LC}) \tag{10}$$

Calculating f_o with equation 10, referencing the actual values for the RLC components as specified in table II A yields a resonance frequency of 25.8Khz. This matches closely with the experimentally determined value of resonance. Error in the measured values can be attributed to the internal resistance of the inductor and general instrumentation errors. More accuracy would likely be achieved by further increasing the resolution of applied frequencies.

The phase shift due to the inductor and capacitor were monitored at a wide sweep of applied frequencies. The phase shift was well behaved, following the theoretically predicted values. At frequencies below f_o the voltage lead V_s, due to the capacitor, and at frequencies above f_o it lagged, due to the inductor. At f_o, as expected, V_o and V_s were in phase.

Using the math function of the oscilloscope as referenced before we were able to show that the phase difference between the current and voltage is completely frequency independent, exactly what theory predicts. The phase shift remained a constant $\pm\ \pi/2$ out of phase with the source. This result is exactly as expected.

The Q-Factor for the bandpass filter was calculated both theoretically as well as experimentally. Theoretically, using equation 3 and the values of table II A, we found the Q-Factor to be 4.72. Experimentally, however, we found a Q-Factor of 3 by using equation 6. Sources of error may include the internal resistance of the circuit.

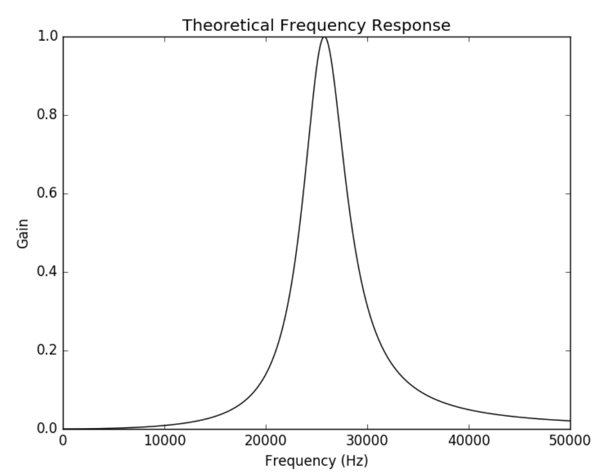

FIG. 9. Theoretical Frequency Response Curve

B. Notch Filter

The method of analysis for the Notch filter will be similar to the method used for the analysis of the Bandpass filter. V_{source} will remain as equation 7. $V_{outNotch}$ is derived using Ohm's law on the component pair of inductor and capacitor and is show in equation 11.

$$V_{outNotch} = ij(\omega L - 1/(\omega C)) \tag{11}$$

The transfer function is then formed from the ratio of equation 11 and 7 as shown in equation 12.

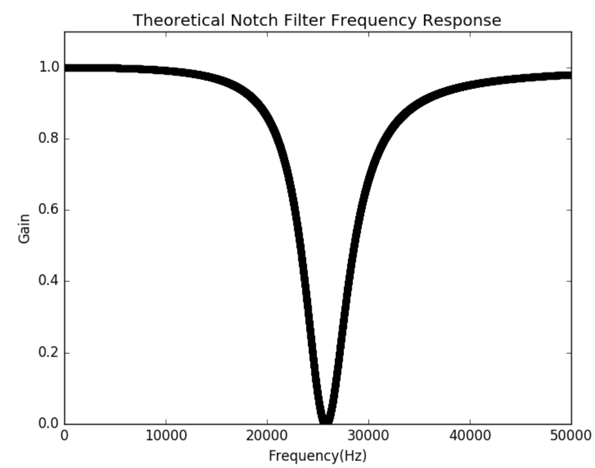

FIG. 10. Theoretical Notch Filter Frequency Response Curve

$$\frac{V_{source}}{V_{outNotch}} = T_{notch}(\omega) = \frac{j(\omega L - 1/(\omega C))}{(j\omega L - 1/j\omega C + R)} \tag{12}$$

It is evident from inspection that the frequency response yielded from the transfer function shown in figure 10 matches well with the experimentally found frequency response shown in figure 8. As for the theoretical and experimental f_o, due to retaining the same RLC components between the filters the theoretical and experimental values will remain constant and related as described above for the bandpass filter.

IV. CONCLUSION

Two different filter architectures were built using a series RLC circuit and tested at a wide sweep of frequencies. It was shown that even with a simple circuit construction utilizing less than ideal components that attenuation frequencies can be selected with relatively accurate precision. In the future, we would like to investigate methods of narrowing the bandwidth and otherwise controlling the Q-Factor.