Amplification, Integration, and Differentiation with Operational Amplifiers

Uladzimir Kasacheuski and Daulton Eaton
Indiana University - Purdue University Indianapolis
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Signal processing is an important part of analog electronics. The operational amplifier (op-amp) is ubiquitous in signal processing aand instrumentation design due to its highly configurable nature,wide area of applicability and robust performance. In this paper, we use the op-amp to amplify, integrate, differentiate and generate a standing square wave signal. For the amplifying circuit in particular we analyze features of different amplification configurations including the op-amps internal input and output impedance, its frequency response and gain characteristics. The results demonstrate that while the output of op-amps can be reliably predicted, their characterizing features depend on too many factors to be accurately predicted.

I. INTRODUCTION

The operational amplifier (op-amp) began its service in the analog electronics era, used popularly for mathematical computations in analog computers [3]. Today, it still finds itself as one of the most used integrated circuits in analog electronics and widely used in signal processing [4].

The op-amp can be reliably used to process signals through amplification, integration, and differentiation. An amplifying circuit is a circuit which increases or decreases the magnitude of a signal by a chosen gain. An integrating circuit is one which produces a signal which is the mathematical integral of the signal which is input. The differentiating circuit can be considered the inverse of the integrating circuit, producing a differentiated signal as an output. Configurations utilizing feedback components are capable of producing these desired processing results. An op amp can even be used to generate waves without input, such as a square wave, by utilizing a configuration called a relaxation oscillator. The op-amp is a simple, reliable, and powerful method for constructing these signal processing circuits.

A. Basic Op-Amp Theory

The op-amp is an integrated circuit with two signal inputs, two power supply inputs, and an output. Referring to figure 1, V_{s+} and V_{s-} are the DC voltages supplied to the chip. The chosen DC voltages are important as they decide the allowable amplification range. For the rest of this paper V_{s+} and V_{s-} remain constant, a constant within the operating specifications defined by the op-amp manufacturer, and will not be adjusted. V_+ and V_- are the positive and negative input of the op-amp respectively. Intuitively, V_{out} is the amplified signal output generated by the op-amp. V_{out} is fundamentally proportional to the difference between the positive and negative input voltages. This is expressed by equation 1, where A is the amount of amplification experienced.

$$V_{out} = A(V_+ - V_-) \tag{1}$$

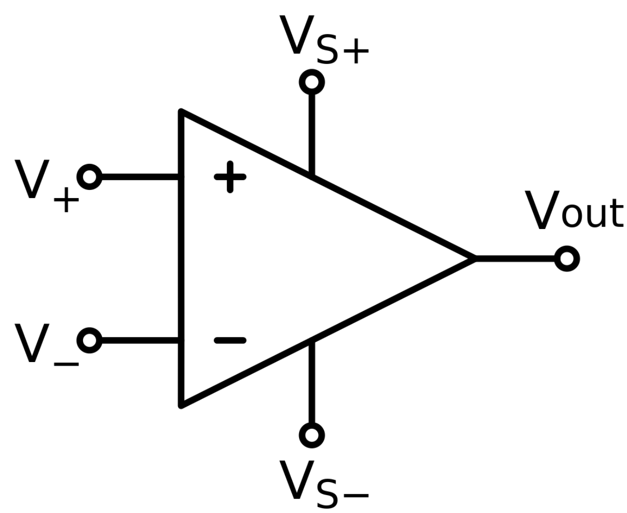

FIG. 1. Operational Amplifier

There are a few rules fundamental to op-amps which enable us to accurately calculate V_{out} for a circuit knowing V_+ and V_- [1]. They are the following :

1. The op amp has infinite open-loop gain.
2. The input impedance of the +/ inputs is infinite.

These two rules can be expressed mathematically by equations 2 and 3.

$$(V_+ - V_-) = V_{out}/A = 0 \implies V_+ = V_- \tag{2}$$

$$I_{in} = V/R = 0 \tag{3}$$

II. EXPERIMENTS

Experimentation in this project involved characterizing the properties of amplifying circuit as well as evaluating an integrator circuit, a differentiator circuit, and a

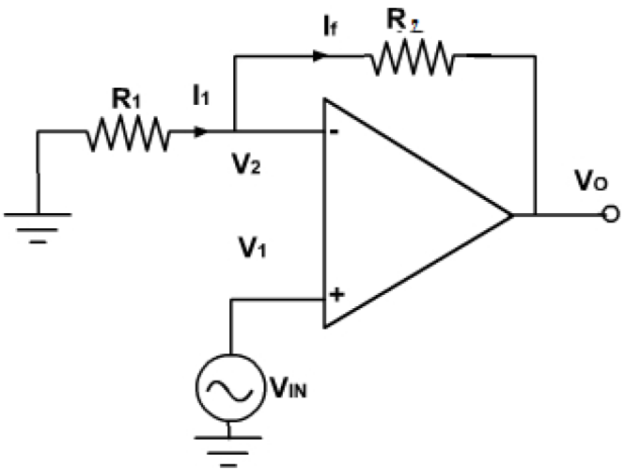

FIG. 2. Non-Inverting Amplifier

square wave oscillator circuit. The theoretical output is derived for each circuit to compare to observed output.

A. Non-Inverting Amplifier

A non-inverting amplifier modifies the magnitude of an input signal. The amplification can be calculated by using the two fundamental rules stated above yielding equation 4. This equation relates the impedance of the two resistors represented in figure 2 to the magnitude change, gain, of the signal.

$$V_{out} = (1 + R_2/R_1)V_{in} = GV_{in} \tag{4}$$

In this study we analyzed the characteristics of non-inverting amplifier circuits of several different gains: 1, 10, 30, 100, 200, and 500. For each amplification level, we approximated the theoretical gain as close as possible to our target amplifications by using equation 4 and manipulating the sizes of our resistors. After constructing each circuit, we then measured the following attributes: measured gain, bandwidth, input impedance (Z_{in}), and output impedance (Z_{out}).

Gain was measured by comparing the input and output voltages with an oscilloscope. Bandwidth is defined as the frequency at which the power is reduced by half. In other words, as expressed by equation 7, this is where $V_{out} = 1/\sqrt{2} * G * V_{in}$.

$$P = V^2/R \tag{5}$$

$$P' = P/2 = V'^2/R \tag{6}$$

$$V'^2 = PR/2 = V^2/2 \implies V' = V/\sqrt{2} \tag{7}$$

Theor. G	Meas. G	BW	A*BW	Z_{in}	Z_{out}	R_1	R_2
1	1	450	450	130.22	-0.907	0.993	0
11	10	45	450	749.241	0.697	0.993	9.82
33.73	34	27	918	1637	0.485	0.993	32.5
101	104.3	13	1355.9	1787	0.141	0.993	100.2
201	200	11.5	2300	408	3.018	0.993	200
506	500	1.85	925	1116.66	2.99	0.993	505.5
		kHz	kHz	kΩ	Ω	kΩ	kΩ

TABLE I. Non-Inverting Amplifier Circuit Characteristics

Input and output impedance were calculated utilizing a test resistor on the input and output respectively. For input impedance, a test resistor, R_t is placed between V_{in} and V_+. The input current, I_{in} can be calculated by measuring the voltage drop across R_t by Ohm's Law. Let the voltage after R_t be labeled V_2. Because we know the voltage drop across the op-amp must be V_2 and we also now know I_{in}, Z_{in} can be calculated by equation 9.

$$I_{in} = \frac{V_{in} - V_2}{R_t} \tag{8}$$

$$Z_{in} = \frac{V_2}{I_{in}} = \frac{V_2}{V_{in} - V_2} R_t \tag{9}$$

Output impedance, Z_{out}, is calculated using a similar methodology. We first calculate the output current, I_{out}, by measuring the voltage drop, V_{closed} across a test resistor, R_t, which is placed between V_{out} and ground. Because Z_{out} is a small quantity, the choice of R_t in this situation must be proportionately small. Additionally,V_{closed} will often be large enough to over saturate the op-amp. An ad hoc voltage divider was often necessary to be used as a pre-input stage for V_{in} to decrease the voltage below saturation levels; otherwise, V_{closed} can not be attained. Knowing both V_{closed} and I_{out} we now only require V_{out}, as usually measured, to calculate Z_{out}. This is demonstrated in equation 11.

$$I_{in} = \frac{V_{closed}}{R_t} \tag{10}$$

$$Z_{out} = \frac{V_{out} - V_{closed}}{I_{out}} = \frac{V_{out} - V_{closed}}{V_{closed}} R_t \tag{11}$$

The results of these analyses are displayed in table I. For furthur analysis of this circuit we tested our output offset voltage by shorting our input to ground. Had our op-amp had an output offset voltage, it would have been important to account for in our calculations. However, our op-amp had zero offset and no furthur calculations had to be done.

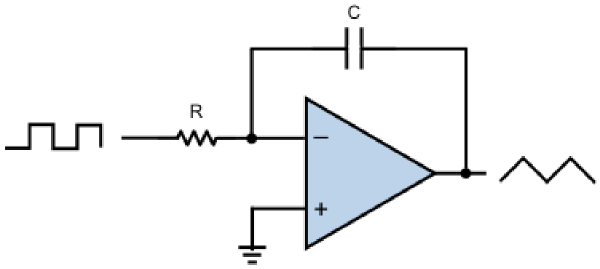

FIG. 3. Integrator Circuit

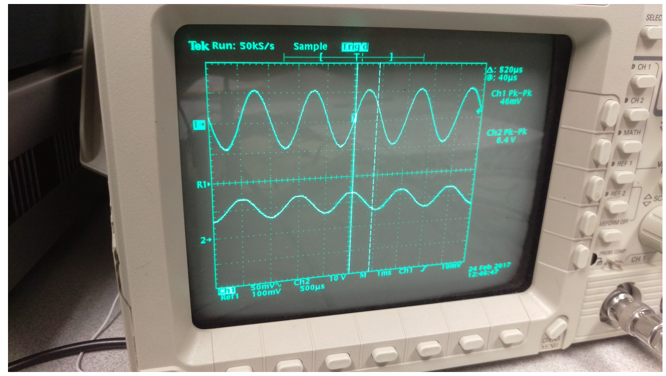

FIG. 4. Integrated Sinusoidal Signal

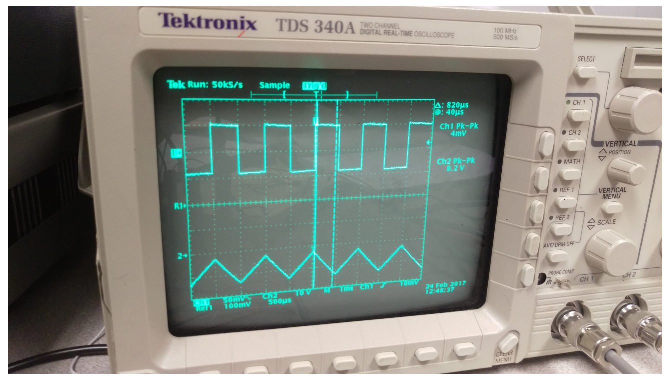

FIG. 5. Integrated Square Signal

B. Integrator Circuit

The integrator circuit, intuitively, outputs the integral of the input function. An example, displayed in figure 3, demonstrates how an input signal of a square wave would be transformed, integrated, into an output signal of a triangle wave. The output of this circuit can be mathematically expressed, as is done by equation 15. Equation 15 can be derived by using Kirchoff's voltage law(KVL) from the input, through the resistor and then to ground yielding equation 12. Equation 13 can be derived using KVL from the input, through the feedback capacitor to the output. Using equations 12, 13 and the current relationship for capacitors shown in 14 yields equation15. In this study we constructed the integrator circuit and analyzed its response for sinusoidal, square, and triangle input waves. For this analysis a resistor of $0.993k\Omega$ and a capacitor of $3.7nf$ was used.

$$V_R = iR = V_{in} \tag{12}$$

$$V_{in} = V_R + V_C + V_{out} \tag{13}$$

$$i = c\frac{dV_C}{dt} \tag{14}$$

$$V_{out} = \frac{-1}{RC}\int V_{in}dt \tag{15}$$

In the case of the sinusoidal wave, the integrator circuit produced another sinusoidal signal. This signal, however, had a 90 degree phase shift compared to the input signal. When analyzing the magnitude of the signal, we found that the magnitude of the signal decreased as the frequency increased. The generated sinusoidal wave was also found to be negated, in addition to its phase shift. An example of this can be found in figure 4, where the top wave is the input signal and the bottom is the output.

When the square wave was fed as the input signal, a triangle wave was produced by the integrator circuit. This triangle wave was the negative of a pure integrated square wave,instead of portraying a 'ramp up' at the 'on' edge of a square wave, it portrayed a 'ramp down' with a negative slope. Figure 5 demonstrates this result. The output wave, too, decreased in amplitude as frequency increased.

The triangle wave, when processed by the integrator circuit, yielded a periodic polynomial signal. As with the sinusoidal and square wave cases, the amplitude of the output signal decreases as frequency increases. This output signal is the negative integral of the triangle signal, i.e. where the rising ramp is x, the integrated circuit output is a function of $-x^2$ with integration constants removed for simplicity.

This can be seen by analyzing the positive ramp of the input signal: if you take the center of the ramp to be zero, the output signal peaks at zero and the values decrease from there to either direction. The same effect is seen with the negative ramp, where the resulting function is x^2. At the negative ramp of the input signal, the center of the input signal ramp marks the minimum of the output signal for that interval and values increase to either side.

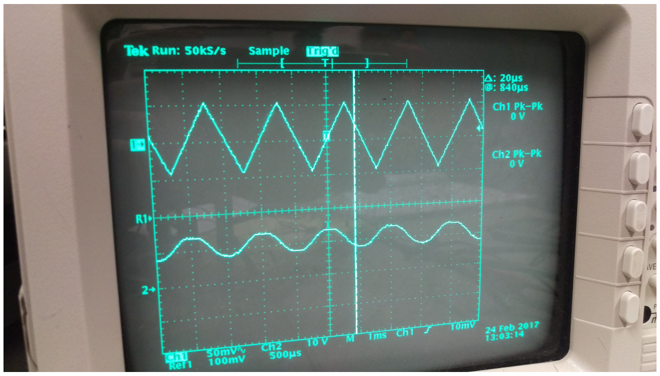

FIG. 6. Integrated Triangle Signal

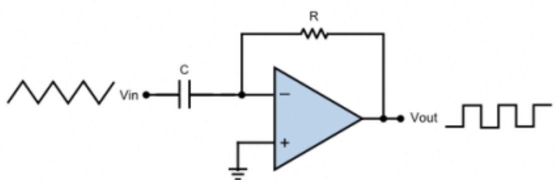

FIG. 7. Differentiator Circuit

C. Differentiator Circuit

In analyzing the differentiator circuit the same process as was used for the integrator circuit is employed. The output of the differentiator circuit can be expressed mathematically be equation 18. Equation 18 is easily shown by analyzing the op amp circuit shown in Figure 7 using Kirchoff's Voltage Law(KVL) and some basic ideal op-amp approximations. Following the first loop from V_{in} through the capacitor, across the two inputs to ground yields equation 16 where V_C is the voltage across the capacitor. Using KVL from input across the feedback resistor to the output yields equation 17 where i represents the input current, and R the value of the resistor. Using equations 16,17 and the relationship for the capacitor current shown in equation 14, we can solve for equation 18. It is plainly evident why the circuit is called a differentiator circuit. In many ways the differentiator circuit is the inverse of the integrator circuit; this includes not only functionality but the configuration of the circuit as well, as shown in figure 7, where the only difference in configuration is the position of the capacitor and resistor. For this analysis, too, a resistor of $0.993k\Omega$ and a capacitor of $3.7nf$ was used.

$$V_{in} = V_C \tag{16}$$

$$V_{in} = V_{out} + V_C + iR \tag{17}$$

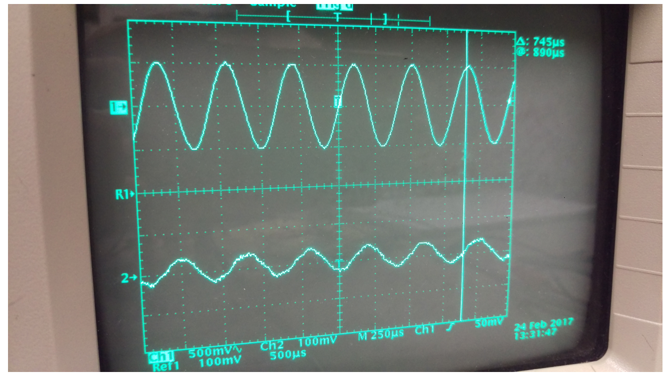

FIG. 8. Differentiated Sinusoidal Signal

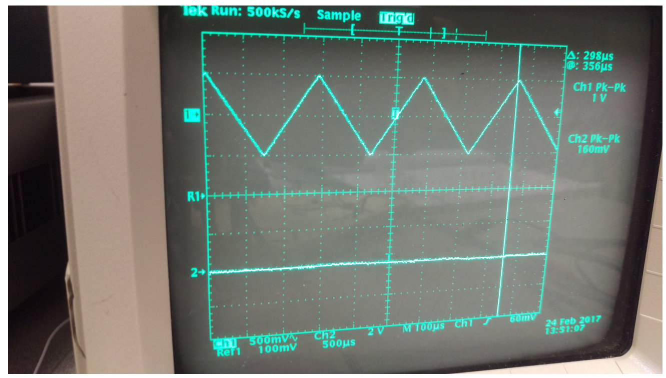

FIG. 9. Differentiated Triangle Signal

$$V_{out} = -RC\frac{dV_{in}}{dt} \tag{18}$$

The first wave form analyzed was the sinusoidal wave. The output of the differential circuit was again another sinusoidal wave with a 90 degree offset with a negation of the waveform. As seen in figure 8 in the case of the differentiator circuit the output waveform was 'fuzzy'. As in the integrator circuit, the amplitude of the output function is dependent on the frequency of the input signal; however, in this case a larger the input frequency correlated with a larger the output amplitude.

The square and triangle waves evaluated through the differentiator circuit return variations of a constant, DC like, output with a zero DC bias. The result of the differentiated triangle signal is shown in figure 9 and the result of the differentiated square signal is shown in figure 10. Because the input functions are effectively not periodical signals and are centered at zero DC offset, we see there is no frequency dependence of the output. For the triangle input signal we can can safely assume that the the result is the negated differentiated function for the same reason, as $0 * -1 = 0$. For the square input signal, however, we can see that the output function is indeed the negated function as the output signal is mostly negative when the wave turns 'on' and mostly negative when it turns 'off'.

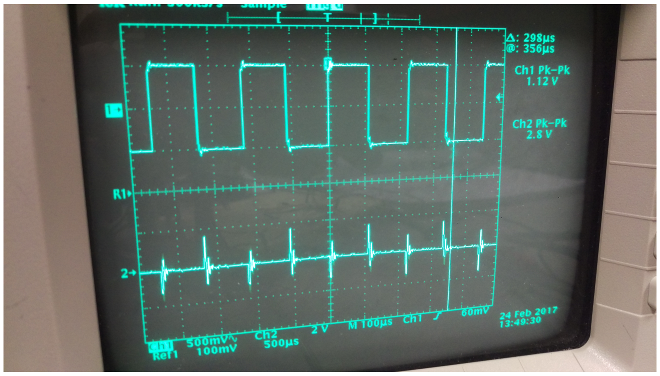

FIG. 10. Differentiated Square Signal

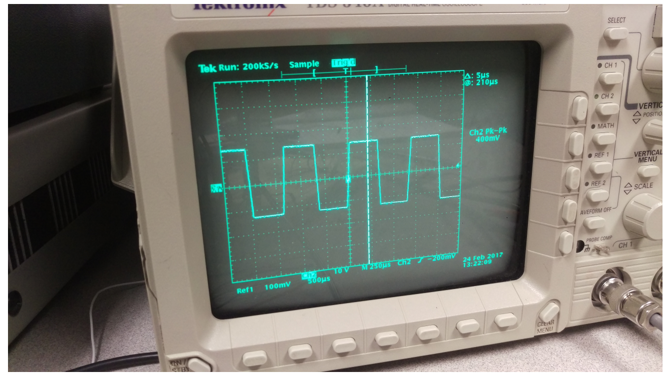

FIG. 12. Square Wave Oscillator Output

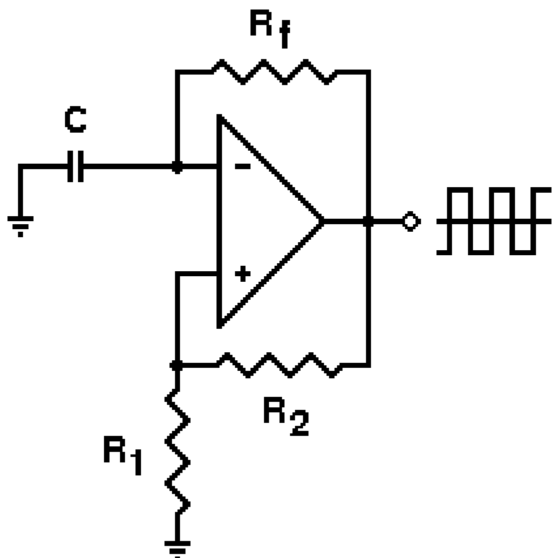

FIG. 11. Square Wave Oscillator Circuit

D. Square Wave Oscillator

A square wave oscillator can be created using a feedback loop with the op-amp as shown in figure 11. This oscillator works due to the exponential with respect to time charge and discharge cycles in the capacitor element. This effect is illustrated using an example case. Suppose V_{out} reaches positive saturation, current is being driven toward the inverting input to equalize input voltages. As this happens the capacitor charges. When the capacitor reaches max charge, the inputs have equalized and the capacitor starts to leak charge, driving the output towards negative saturation. This perpetually instability lends itself to another name for this configuration, Astable Multivibrator, as the circuit is multimodal and unstable in each state. For the oscillator we built we chose an output frequency in the audible hearing range of $1930.5Hz$. The output frequency can be predicted by equation 19. A capacitor of $3.7nf$ and $70k\Omega$ was used to achieve this theoretical value.

$$f_o = \frac{1}{RC} \tag{19}$$

Upon fully building the circuit we found it generated a strong square wave output as expected, shown in figure 12. We found that the wave width for on and off was $296\mu s$ and that the rise and fall time for the wave was $42\mu s$. The measured period of the wave is $700\mu s$ which means that the measured frequency of this wave is actually $1429Hz$. The slew rate, expressed by equation 20 [2], for this circuit is found to be $\frac{1}{3}V/\mu s$.

$$slew = V/\mu s \tag{20}$$

III. DISCUSSION

A. Non-Inverting Amplifier

Evaluating the results of table I we see that the theoretical and measured gain are remarkably close. Additionally, we see that, as expected, Z_{in} is typically a large value, measurable in megaohms. Also, as expected, Z_{out} is a small value, often less than one ohm. Notably, however, is the lack of linearity between all of the other circuit characteristics with respect to gain.

This non-linearity is upheld by all characteristics other than bandwidth; although, even then it is more of a downward trend than a linear response. This non-linearity is testament to the importance of being able to measure these circuit characteristics of the amplifiers.

An additional item of not was the 'negative' Z_{out} measured for unity gain circuit, the voltage follower. Unexpectedly, the V_{out} consistently increased when it was grounded across R_t, yielding V_{closed}. We believe that this may be a product of many 'moving parts' in our testing environment. Perhaps in a more controlled environment

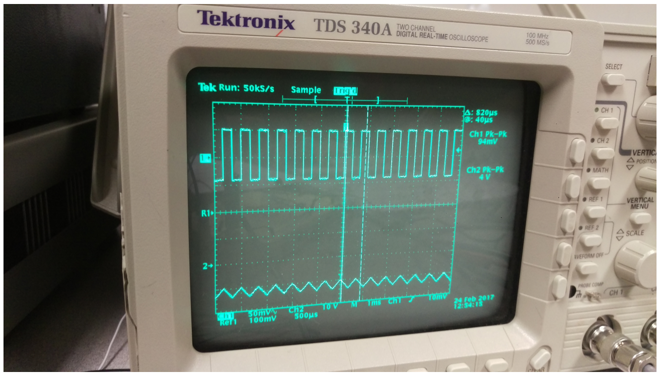

FIG. 13. Integrated Square Signal

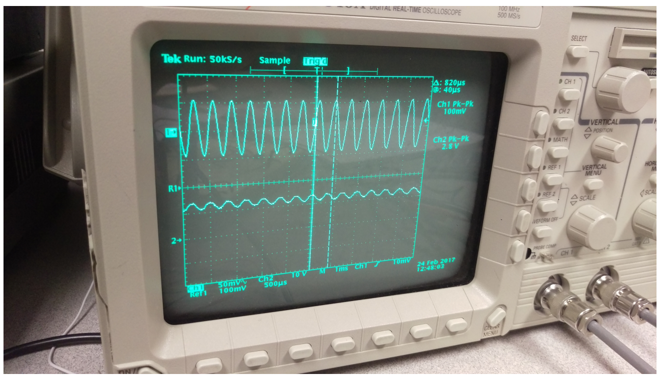

FIG. 14. Integrated Sinusoidal Signal

this phenomena could be eliminated. In our environment, however, this was experienced with multiple op-amps and two different 'rebuilds' of our circuit.

B. Integrator Circuit

For all three input signal functions, the integrator circuit reliably produced the expected function. Interestingly, all three functions are frequency dependent - all decreasing in amplitude as frequency increases. Most notably, however, was the fact that all three output signals had a special type of DC offset bias. The bias can be seen in figure 5 where the output signal is clearly not centered at zero. 13 shows this phenomenon more clearly, but also comes with an interesting observation: when comparing figures 5 and figure 13 it is clear that the output signal is not centered at the bias offset: it is each signal's peak/minimum value in all frequencies. For sinusoidal and triangle inputs, the bias offset is the peak (figures 14 and 15). For square inputs, it is the signals minimum value (figure 13).

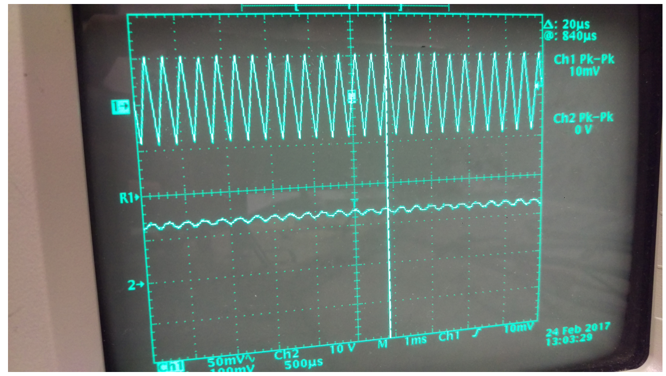

FIG. 15. Integrated Triangle Signal

C. Differentiator Circuit

The lack of frequency dependence of the differentiated square and triangle signals begs the question of why the sinusoidal signal is different. Upon analytically assessing the sinusoidal function the answer is clear. The sinusoidal function is explicitly frequency dependent, while the triangle and square waves are not.

Indeed, this frequency dependence of the amplitude can be quite accurately calculated for the differentiated sinusoidal function. This result is expressed in equation 23. Note, for our purposes the phase shift of the sinusoidal function makes no difference as we measure the peak to peak of the function and thus in equation 22 we do not distinguish between sine and cosine.

$$V_{in} = sin(\omega t) = sin(2\pi f t) \tag{21}$$

$$\frac{dV_{in}}{dt} = \frac{d(sin(2\pi f t))}{dt} = V_{in} 2\pi f \tag{22}$$

$$V_{out} = -RC\frac{dV_{in}}{dt} = -RCV_{in}2\pi f \tag{23}$$

When we passed a peak-to-peak input voltage of $1.04V$ into our differentiator circuit in a sinusoidal function, the output peak-to-peak amplitude we measured was $64mV$. For that setup, $f = 2.5kHz$. Calculating V_{out} with equation 23 correlates to a result with only a 5 percent error of the theoretical V_{out} when evaluated by equation 24. This result confirms that the frequency dependence of the differentiated sinusoidal function is due almost entirely by the inherent frequency dependence of the parent function.

$$error = \frac{|measured - theoretical|}{measured} \tag{24}$$

This also validates our intuition as to why the square and triangle differentiated signals are not frequency dependent: the input functions are not dependent on frequency. This assessment brings into question, however, why the square and triangle signals were frequency dependent in the integrator circuit. While clearly the input signals are not dependent on frequency, a frequency dependence is still seen in the integrated signals. Further analysis should be conducted to evaluate this phenomenon.

D. Square Wave Oscillator

When comparing the theoretical and the measured output frequency we quickly notice a large disparity between the two. In fact, the results evaluate to a measured error rate of 26 percent when evaluated with equation 25. This, combined with the relatively slow slew rate, leads to the conclusion that this square wave is not the most effective or ideal.

$$error = \frac{|theoretical - measured|}{theoretical} \tag{25}$$

IV. CONCLUSION

An operational amplifier is a powerful, modular, and reliable tool in analog electronics. This paper demonstrates how three different signal processing circuits and even a signal generating circuit can be constructed with high reliability using this inexpensive integrated circuit. While the output of these circuits is predictable, this paper shows that the characteristics of the circuits built can be unpredictable. In the future, we would like to investigate the relationship of square wave oscillator error dependent on frequency and investigate the cause of the DC bias offset of the integrator circuit output. We would also like to examine the effects of cascaded op-amps and investigate their potential applications.

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