Constructing a Capacitor and Measuring Capacitance

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The capacitance of a parallel plate capacitor has been built and studied experimentally for the determination of the capacitance. Capacitance was tested using two different methods with varying dielectrics. The measured values were not accurate, however they were relatively precise. The inclusion of dielectric materials into the construction of the capacitor altered the capacitance in the expected manner. It was shown that realizing a rough capacitor can be done with easily accessible materials and that the measurement of the capacitance can be achieved in a number of ways.

I. INTRODUCTION

A. History

The capacitor has gone through many iterations through the years. When capacitors began to hit the academic scene in the mid 1700's, they were often in the form of glass jars with metal coating on the inside insulated with metal on the outside [2]. This design acted more as a proof of concept for future scientists and engineers to use as they moved forward with their studies. It was not until Michael Faraday utilized a modified plate capacitor that capacitance received its current units of farads.

B. Theory

Capacitors are electrical components that hold a certain amount of charge after being charged by a difference in voltage. All capacitors share the same general equation:

$$C = \frac{Q}{V} \tag{1}$$

where Q is charge, V is voltage, and C is the constant of capacitance. Capacitors have the unit of Farads (F) which has the unit of Coulomb/volt [3].

The most fundamental design of the capacitor is the parallel plate. As the name implies, two conducting metal plates are placed a certain space d, apart. When this space has a voltage difference applied between the two plates, one will assume a positive charge, and the other a negative charge. Capacitors hold their charge by forcing electrons away from the positive plate towards the negative plate. Manipulating Eq. 1, one can derive capacitance of a parallel plate capacitor in terms of the dimensions of the gap in between the plates as

$$C = \frac{\epsilon_0 A}{d} \tag{2}$$

This equation is assuming that the space between the plates is a vacuum with nothing in between the plates. Any medium that is placed between the plates is referred to as a dielectric. Dielectrics can be used to increase the capacitance without actually modifying the design. In practice the equation is

$$C = k\frac{\epsilon_0 A}{d} \tag{3}$$

where k is the dielectric constant associated with the medium.

Within the context of circuits, capacitors have different behaviors depending on the time that the capacitor has been charging and the type of current flowing through (i.e. AC or DC). When exposed to an active current, the capacitor will slowly reach its maximum charge found in accordance with the constant value of capacitance (Eq. 1). The capacitor is considered to be in steady state when no current can pass through. At this point, the circuit would be analyzed with an open circuit branch with the same voltage as the capacitor before it reached steady state. As a capacitor begins to charge more and more, the resistance values will change.

In an AC circuit, the electrons oscillate rapidly back and forth depending on the frequency of the source. Capacitors exposed to AC current will gain impedance in the imaginary domain, meaning that it can be expressed by the use of complex numbers. This impedance is given by the equation

$$Z = \frac{1}{i\omega C} \tag{4}$$

which states that there is an inverse relationship between the impedance (Z) and the frequency of the source (ω).

Due to the capacitor's behavior in AC circuits, it can be used to manipulate waveforms. Diodes are an electrical element in which one direction has minimal resistance, but the other direction has very high resistance. Using four diodes and a capacitor, it is possible to make a waveform that is relatively close to DC from a sinusoidal AC source [1]. This works by having two sets of two diodes in series – one set for the negative portion of the wave, and another for the positive portion of the wave. The capacitor acts as a way to remove the oscillation of the now positive waveform, making it a relatively straight line. Small oscillations can be seen on the output due to the rapid charging and discharging of the capacitor.

Capacitors have the ability to act as an active circuit element via their ability to charge and discharge. A charging circuit only requires that there be an active current between the positive and negative plates on the capacitor. The rate of charge is given by the equation

$$V(t) = V_0(1 - e^{\frac{t}{\tau}}) \tag{5}$$

where τ = RC. When multiplied together, the units of resistance and capacitance yield seconds. A single time constant here will result in the capacitor being charged 63.2% of its maximum. The simplest example of a discharging circuit is a series circuit with one capacitor and one resistor. When a charged capacitor is disconnected from an independent power source, the capacitor will create its own current. The rate of discharge is found via the equation

$$V(t) = V_0(e^{\frac{-t}{\tau}}) \tag{6}$$

where a single time constant will result in a 36.8% discharge of the current voltage value.
Bryant

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II. EXPERIMENTS

Experimentation in this project involved building a parallel plate capacitor. The capacitor was tested against theoretical values to determine its quality. Next, the capacitor was used to determine the dielectric constant of two materials.

A. Building a Capacitor

1. Theory

In order to evaluate the capacitor built in the lab, it was important that there be a theoretical model with which to compare it. For this reason, a parallel plate capacitor, for which the theoretical model is already well established, was chosen to be constructed. For an ideal parallel plate capacitor, one where $d << \sqrt{A}$, which can be easily derived from the basic relationships

$$C = QV \tag{7}$$

$$Q = \sigma * A \tag{8}$$

$$V = -\int_d^0 \vec{E} \cdot d\vec{l} = Ed \tag{9}$$

where E is the electric field between the plates which can be approximated to be double the field generated by an infinite plate

$$E_{one} = \frac{\sigma}{2\epsilon_0} \tag{10}$$

thus

$$V = 2E_{one}d = \frac{\sigma d}{\epsilon_0} \tag{11}$$

Using the relationship between Q and σ, this becomes

$$V = \frac{Qd}{\epsilon_0 A} \tag{12}$$

Now plugging this into the initial expression, it is found that capacitance is given by the expression

$$C = \frac{Q\epsilon_0 A}{Qd} = \frac{\epsilon_0 A}{d} \tag{13}$$

where A represents the area of the plates, d represents the distance between the plates, and ϵ_0 represents the permittivity of free space.

2. Methods

A clearly critical aspect in designing a parallel plate capacitor is that in order to work properly, the plates must be held parallel to each other. In our design, a cardboard and Styrofoam constraining system was created. In order to maintain symmetry in the XY axis, pieces of cardboard with slits spaced $5mm$ apart along a side were created. The plates slide into the slits and the distance between plates can thus be controlled. This can be seen in figure 1. In order to maintain azimuthal symmetry, a total of four of said slitted-cardboard pieces were used - spaced in the z direction by two pieces of Styrofoam of equal height, roughly $1cm$. This is demonstrated in figure 4.

3. Results

Two aluminum plates, $40mm^2$ in area, were utilized as the parallel plates. They were successfully, though tediously, installed into the holding apparatus. For all experiments in this paper, a separation distance of $5mm$ was used in order to maximize capacitance. From the ideal case approximation of capacitance as defined in equation 13, a theoretical capacitance of $2.83pF$ was calculated.

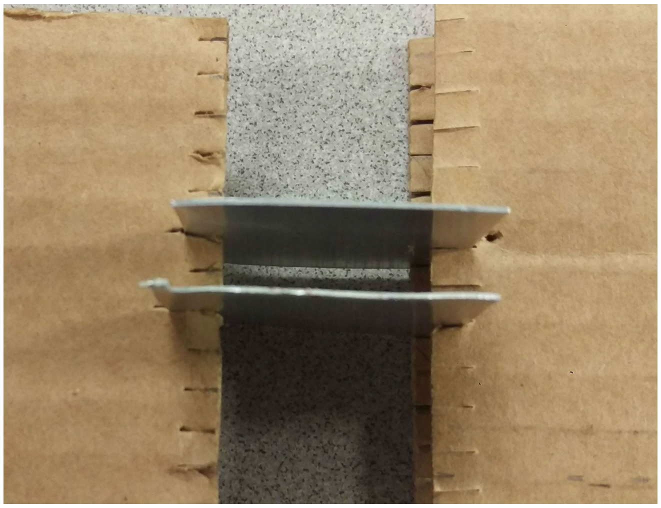

FIG. 1. XY plane perspective of the capacitor

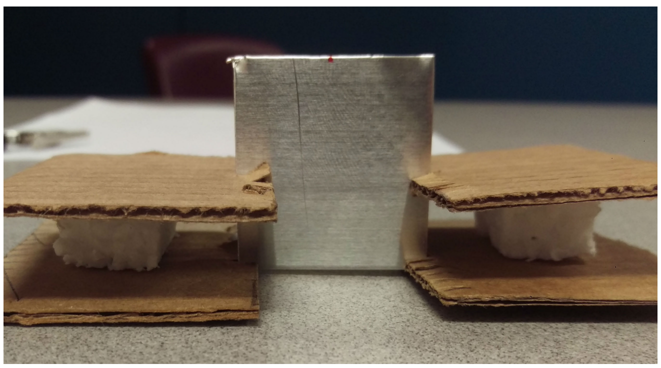

FIG. 2. XZ plane perspective of the capacitor

B. Dielectric Effect

The first dielectric we used was a cardboard insert between the two plates. This material was chosen because it is an insulator, which increases the capacitance by redistributing the charge. The second dielectric we used was a polymer piece of paper. This dielectric was chosen because it was also an insulator, but had a different shape and density. These variations were in line with the two principle mechanisms that effect the charge distributions, namely, stretching and rotating atoms. The two mechanisms are due to the fact that the charge stays attached to the atom or molecule in an insulator.

C. Measuring Capacitance

The measurement of capacitance was approached in two ways. The first, a fundamental and raw approach, was to use the discharging formula for a capacitor to determine the time constant, τ, of an RC circuit with a known resistance and use the relationship in equation 14 to solve for C. The second method was to utilize a multimeter with an ability to measure capacitance.

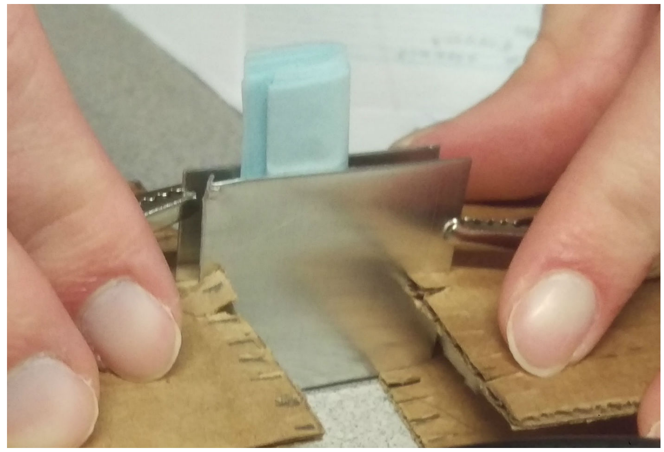

FIG. 3. "Mystery-Blue" dielectric in capacitor

FIG. 4. Cardboard dielectric in capacitor

$$\tau = RC \tag{14}$$

1. Theory

The time constant of an RC circuit can be derived from the time it takes to discharge the capacitor in said circuit in a straight forward manner. The equation of discharge for a capacitor is defined in equation 15, where V_s is the source potential and V_c is the potential difference across the capacitor. By definition, this means that there is a distinct potential across the capacitor, V_c, that occurs when $t = \tau$. Setting $t = \tau$ as done in equation 16 shows us that $t = \tau$ precisely when $V_c/V_s = 0.3679$. Thus by determining the time it takes for the capacitor to discharge from $V_c = V_s$ to $V_c = 0.3679 * V_s$, we determine the time constant of this RC circuit.

$$V_c = V_s * e^{-t/\tau} \tag{15}$$

$$\frac{V_c}{V_s} = e^{-1} = 0.3679 \tag{16}$$

2. Methods

Concerns with this approach materialize quickly after analyzing the theoretical capacitance of our constructed capacitor with respect to equation 14. For the best case scenario, dealing with resistor values on the scale of $M\Omega$, the theoretical time constant of our circuit will be on the order of microseconds. This conclusively rules out the usage of a multimeter or an oscilloscope without a recording option as tools for determining the time constant of the circuit.

To this end, we employed an Arduino Uno equipped with a voltage measuring script to measure the voltage and a computer with a serial port reading script to record the voltage. The recorded data was then analyzed with a script that detects when discharge begins to occur, maximizes the number of usable, valid data points (as opposed to the noise), and calculates average capacitance and standard deviation of capacitance based on this data. The code for this implementation can be found at https://github.com/uladkasach/Measure-Voltage.

The Arduino was able to capture voltage at a rate of 1 measurement every $190\mu s$. With the materials we had on hand, this was the best that could be manifested and although theoretically it will not be fast enough to measure the time constant this method was still attempted. A multimeter was then used to validate our results and confirm or deny their accuracy.

The charging and discharging of the circuit was conducted using the circuit displayed in figure 5. When switch S_1 is 'pointing west', the capacitor is charging and after a given amount of time the potential difference across it will reach V_s. When the switch is not connected to any path, the capacitor will stay charged. With the switch 'pointing south', the capacitor would discharge.

The capacitance depends on sizes, shapes, and insulating material between the two plates. The next part of the experiment dealt with using a dielectric insulating material to change the capacitance. First we placed a piece of cardboard between the two plates, and obtained readings using the same methods listed above. Next, after the cardboard was removed, a folded piece of polymer was inserted between the plates. The capacitance increases with the use of these materials, because the charge is redistributed within the insulating materials.

3. Results

Four 'different' capacitors were evaluated with these methods. First was a factory-made capacitor with a 'given' capacitance of $100nF$. The second, third, and fourth employed the home-made capacitor with an air, cardboard, and a blue-polymer dielectric respectively. Two values of resistance were employed in an attempt to see whether our results were precise. $R_1 = 2.85 * 10^6$, $R_2 = 5.65 * 10^6$. The capacitors were then charged and discharged several times to get a statistical approximation of the true capacitance values. The mean and standard deviation of capacitance was then calculated from this recorded data. Afterwards, the multimeter was used to further assess the data. These results are displayed in table I.

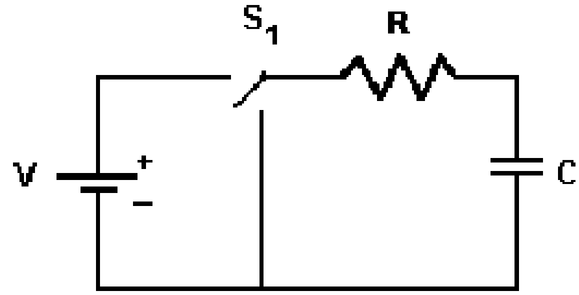

FIG. 5. RC circuit employed for charging and discharging the capacitor

Type	n_1	$Mean_1$	Std_1	n_2	$Mean_2$	Std_2	Multi M.
$100nF$	20/23	107.9nF	1.44nF	5/6	41.46nF	5.46nF	96.8nF
Air	29/67	23.6nF	2.76nF	6/17	15.57nF	15.14nF	3pF
Card.				14/30	12.90nF	2.92nF	13pF
B. Poly				21/44	11.9nF	1.29nF	16pF

TABLE I. Mean and standard deviation of capacitance measured with Arduino as well as capacitance measured by multimeter for each capacitor. n represents the number of data points used to generate the statistics divided by the total discharge cycle data points found. The subscripts represent which resistance value the RC circuit had.

III. DISCUSSION

A. Measuring Capacitance

We see that for the large, $100nF$, capacitor the Arduino method is able to calculate capacitance at a reasonable error rate, 7% from the capacitors estimated capacitance, when given enough data points. When only 5 data points were considered, even for the large factory capacitor, the data was not accurate. The number of statistics is clearly important in this case.

One may wonder why, given that the air capacitor for example had 29 data points, the capacitance of the air capacitor was so off; although the number of data used points was large, the number of non-usable discharge cycle data points was even larger, at 38. Reasons that data would not be taken into account is if the maximum voltage is not large enough, since we know we put in $4.7V$, or that a value less than or equal to $0.3697 * V_s$ was not found during that discharge cycle. Taking this into account, it is easy to see that the data software is calculating statistics on is very volatile. This is as expected, the measurements from the Arduino were not capable of producing accurate results for the home-made capacitor due to a time constant that is smaller than the rate at which it can capture the amount of data required for accuracy.

It is very likely that faster clock times could be attained by upgrading the version of Arduino used or optimizing settings on the baud rate of the Arduino. In the future, this should be considered more as it can be a viable method of calculating voltage at the speeds required for home made capacitors.

IV. CONCLUSION

The goal of this research was to design and implement a laboratory made parallel plate capacitor and measure its capacitance with several dielectric materials. The methods implemented to measure capacitance excelled on the factory made capacitor but struggled to yield meaningful results when tested on the laboratory built capacitor. The theoretical capacitance was expected to be small which lead to a rapid discharge rate; Surpassing the Arduino's max baud rate. However, the results obtained remained consistent and allowed for examination of the effect of placing a dielectric between the parallel plates. In consideration of future work, the capactior could benefit from a more precise construction, and the discharge measuring method of measuring capacitance could be further refined.

[1] Full wave rectifier, howpublished = `http://www.electronics-tutorials.ws/diode/diode_6.html`, note = Accessed: 2017-04-27.

[2] How capacitors work, howpublished = `http://electronics.howstuffworks.com/capacitor3.htm`, note = Accessed: 2017-04-27.

[3] D. J. Griffiths. *Introduction to electrodynamics.* Pearson, 4 edition, 2014.

[4] Kazuhiro Morita, El-Sayed Atlam, Masao Fuketra, Kazuhiko Tsuda, Masaki Oono, and Jun ichi Aoe. Word classification and hierarchy using co-occurrence word information. *Information Processing and Management*, 2003.