**Sabancı University**
**Faculty of Engineering and Natural Sciences**

**CS301 – Algorithms**

**Homework 1**

Due: October 20, 2020 @ 23.59
(Upload to SUCourse+ - **no late submission**)

**PLEASE NOTE**:

- Provide only the requested information and nothing more. Unreadable, unintelligible and irrelevant answers will not be considered.
- You can collaborate with your `TA/INSTRUCTOR ONLY` and discuss the solutions of the problems. However you have to write down the solutions on your own.
- Plagiarism will not be tolerated.

1. Consider the following divide-and-conquer algorithm: (15pt)

```
float myFunc(X){
    n = X.length;
    if (n==1){
        return X[0];
    }
    // let X1,X2 be arrays of size n/2
    for (i=0; i <= (n/2)-1; i++){
        X1[i] = A[i];
        X2[i] = A[n/2 + i];
    }

    for (i=0; i<=(n/2)-1; i++){
        for (j=0; j<=(n/2)-1; j++){
            if (X1[i] == X2[j])
                X2[j] = 0;
        }
    }
    r1 = myFunc(X1);
    r2 = myFunc(X2);
    return max(r1,r2);
}
```

(a) Obtain a recurrence relation for the algorithm's basic operation count. (5pt)

(b) Solve the recurrence relation found in part (a) to find a tight bound. (10pt)

Note that you don't need to consider what does the algorithm compute; just focus on the basic operations.

2. Consider the following statements as True or False. If True, explain; if False, give a counterexample. (20pt)
   (a) $f(n) = \Theta(f(n/2))$
   (b) $f(n) + o(f(n)) = \Theta(f(n))$
   (c) $f(n) = O((f(n))^2)$
   (d) $f(n) + g(n) = \Theta(min(f(n), g(n)))$
   (e) $f(n) = O(g(n))$ implies $\lg(f(n)) = O(\lg(g(n)))$, where $\lg(g(n)) >= 1$ and $f(n) >= 1$ for all sufficiently large n.

3. Find an asymptotically tight bound for the following recurrence relation using substitution method. (25pt)

   $T(n) = T(n-5) + 2n^2$

4. Find the tight bound for the following recurrence by changing the variables: (20pt)

   $T(n) = 2T(\sqrt{n}) + \log n$

5. Find the time complexity for the following recurrence using the master theorem. (20pt)
   (a) $T(n) = 4T(n/2) + n^2$
   (b) $T(n) = 16T(n/4) + n$
   (c) $T(n) = 0.75T(n/4) + 1/3n$
   (d) $T(n) = 3T(n/4) + n \log n$