Buffon's Needle

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1 π from needles

There are several ways to obtain π from equations and experiments. Buffon's Needle is one of them. In the program I use the simplest libraries as possible to make the code clear for the reader and the ones who wants to practice basic C programming.

2 Process

We have an plane (800,800)$\in \mathbb{Q}$. Then the program throws a needle. Say its middle point lands x_0, y_0 and the length L. Now remember we have to lines, L_1 which is on the line (y_1, x) where y is constant, L_2 which is on the line (y_1, x) where y is constant. Lines are parallel and there is a constant distance d between them. Now we can see that $|y_0 - y_1| \leq \frac{d}{2}$ and $|y_0 - y_2| \leq \frac{d}{2}$. Now without loss of generality we can pick the minimum of the distances, say X . Since we throw a needle random, it will have an angle with the lines $\theta \leq \frac{\pi}{2}$. Now the points of the needle is $y_0 + \sin\theta\frac{L}{2}$ or $y_0 - \sin\theta\frac{L}{2}$. Now, if $X \leq \sin\theta\frac{L}{2}$, the needle bisects the one of the lines. So the probability:

$$\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta\frac{L}{2}} \frac{4}{\pi d}\, dx\, d\theta$$

and by this P(A) is equal to

$$\frac{2L}{\pi d}$$

, with that result we can derive the π.