

Solutions to Skill-Assessment Exercises

**To Accompany
Control Systems Engineering
3rd Edition**

**By
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John Wiley & Sons

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Solutions to Skill-Assessment Exercises

Chapter 2

2.1.

The Laplace transform of t is $\frac{1}{s^2}$ using Table 2.1, Item 3. Using Table 2.2, Item 4,

$$F(s) = \frac{1}{(s+5)^2}.$$

2.2.

Expanding $F(s)$ by partial fractions yields:

$$F(s) = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{(s+3)^2} + \frac{D}{(s+3)}$$

where,

$$A = \frac{10}{(s+2)(s+3)^2} \Big|_{s \rightarrow 0} = \frac{5}{9} \quad B = \frac{10}{s(s+3)^2} \Big|_{s \rightarrow -2} = -5 \quad C = \frac{10}{s(s+2)} \Big|_{s \rightarrow -3} = \frac{10}{3}, \text{ and}$$

$$D = (s+3)^2 \frac{dF(s)}{ds} \Big|_{s \rightarrow -3} = \frac{40}{9}$$

Taking the inverse Laplace transform yields,

$$f(t) = \frac{5}{9} - 5e^{-2t} + \frac{10}{3}te^{-3t} + \frac{40}{9}e^{-3t}$$

2.3.

Taking the Laplace transform of the differential equation assuming zero initial conditions yields:

$$s^3C(s) + 3s^2C(s) + 7sC(s) + 5C(s) = s^2R(s) + 4sR(s) + 3R(s)$$

Collecting terms,

$$(s^3 + 3s^2 + 7s + 5)C(s) = (s^2 + 4s + 3)R(s)$$

Thus,

$$\frac{C(s)}{R(s)} = \frac{s^2 + 4s + 3}{s^3 + 3s^2 + 7s + 5}$$

2.4.

$$G(s) = \frac{C(s)}{R(s)} = \frac{2s + 1}{s^2 + 6s + 2}$$

Cross multiplying yields,

$$\frac{d^2c}{dt^2} + 6\frac{dc}{dt} + 2c = 2\frac{dr}{dt} + r$$

2.5.

$$C(s) = R(s)G(s) = \frac{1}{s^2} * \frac{s}{(s+4)(s+8)} = \frac{1}{s(s+4)(s+8)} = \frac{A}{s} + \frac{B}{(s+4)} + \frac{C}{(s+8)}$$

where

$$A = \frac{1}{(s+4)(s+8)} \Big|_{s \rightarrow 0} = \frac{1}{32} \quad B = \frac{1}{s(s+8)} \Big|_{s \rightarrow -4} = -\frac{1}{16}, \text{ and } C = \frac{1}{s(s+4)} \Big|_{s \rightarrow -8} = \frac{1}{32}$$

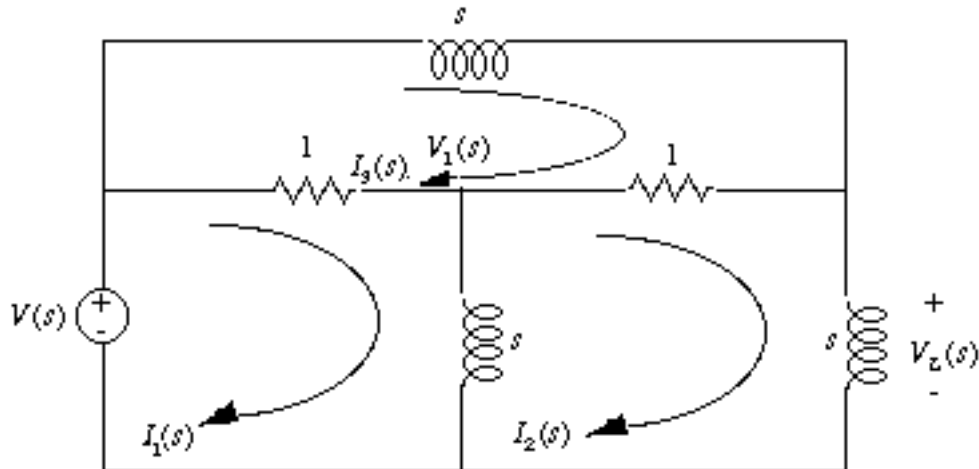
Thus,

$$c(t) = \frac{1}{32} - \frac{1}{16}e^{-4t} + \frac{1}{32}e^{-8t}$$

2.6.

Mesh Analysis

Transforming the network yields,



Now, writing the mesh equations,

$$(s+1)I_1(s) - sI_2(s) - I_3(s) = V(s)$$

$$-sI_1(s) + (2s+1)I_2(s) - I_3(s) = 0$$

$$-I_1(s) - I_2(s) + (s+2)I_3(s) = 0$$

Solving the mesh equations for $I_2(s)$,

$$I_2(s) = \frac{\begin{vmatrix} (s+1) & V(s) & -1 \\ -s & 0 & -1 \\ -1 & 0 & (s+2) \end{vmatrix}}{\begin{vmatrix} (s+1) & -s & -1 \\ -s & (2s+1) & -1 \\ -1 & -1 & (s+2) \end{vmatrix}} = \frac{(s^2 + 2s + 1)V(s)}{s(s^2 + 5s + 2)}$$

But, $V_L(s) = sI_2(s)$

Hence,

$$V_L(s) = \frac{(s^2 + 2s + 1)V(s)}{(s^2 + 5s + 2)}$$

or

$$\frac{V_L(s)}{V(s)} = \frac{s^2 + 2s + 1}{s^2 + 5s + 2}$$

Nodal Analysis

Writing the nodal equations,

$$\left(\frac{1}{s} + 2\right)V_1(s) - V_L(s) = V(s)$$

$$-V_1(s) + \left(\frac{2}{s} + 1\right)V_L(s) = \frac{1}{s}V(s)$$

Solving for $V_L(s)$,

$$V_L(s) = \frac{\begin{vmatrix} \left(\frac{1}{s} + 2\right) & V(s) \\ -1 & \frac{1}{s}V(s) \end{vmatrix}}{\begin{vmatrix} \left(\frac{1}{s} + 2\right) & -1 \\ -1 & \left(\frac{2}{s} + 1\right) \end{vmatrix}} = \frac{(s^2 + 2s + 1)V(s)}{(s^2 + 5s + 2)}$$

or

$$\frac{V_L(s)}{V(s)} = \frac{s^2 + 2s + 1}{s^2 + 5s + 2}$$

2.7.**Inverting**

$$G(s) = -\frac{Z_2(s)}{Z_1(s)} = \frac{-100000}{(10^5 / s)} = -s$$

Noninverting

$$G(s) = \frac{[Z_1(s) + Z_2(s)]}{Z_1(s)} = \frac{(\frac{10^5}{s} + 10^5)}{(\frac{10^5}{s})} = s + 1$$

2.8.

Writing the equations of motion,

$$(s^2 + 3s + 1)X_1(s) - (3s + 1)X_2(s) = F(s)$$

$$-(3s + 1)X_1(s) + (s^2 + 4s + 1)X_2(s) = 0$$

Solving for $X_2(s)$,

$$X_2(s) = \frac{\begin{vmatrix} (s^2 + 3s + 1) & F(s) \\ -(3s + 1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + 3s + 1) & -(3s + 1) \\ -(3s + 1) & (s^2 + 4s + 1) \end{vmatrix}} = \frac{(3s + 1)F(s)}{s(s^3 + 7s^2 + 5s + 1)}$$

Hence,

$$\frac{X_2(s)}{F(s)} = \frac{(3s + 1)}{s(s^3 + 7s^2 + 5s + 1)}$$

2.9.

Writing the equations of motion,

$$(s^2 + s + 1)\theta_1(s) - (s + 1)\theta_2(s) = T(s)$$

$$-(s + 1)\theta_1(s) + (2s + 2)\theta_2(s) = 0$$

where $\theta_1(s)$ is the angular displacement of the inertia.Solving for $\theta_2(s)$,

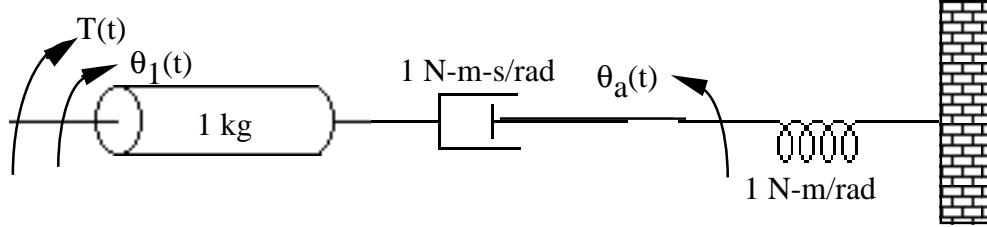
$$\theta_2(s) = \frac{\begin{vmatrix} (s^2 + s + 1) & T(s) \\ -(s + 1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + s + 1) & -(s + 1) \\ -(s + 1) & (2s + 2) \end{vmatrix}} = \frac{(s + 1)F(s)}{2s^3 + 3s^2 + 2s + 1}$$

From which, after simplification,

$$\theta_2(s) = \frac{1}{2s^2 + s + 1}$$

2.10.

Transforming the network to one without gears by reflecting the 4 N-m/rad spring to the left and multiplying by $(25/50)^2$, we obtain,



Writing the equations of motion,

$$(s^2 + s)\theta_1(s) - s\theta_a(s) = T(s)$$

$$-s\theta_1(s) + (s+1)\theta_a(s) = 0$$

where $\theta_1(s)$ is the angular displacement of the 1-kg inertia.

Solving for $\theta_a(s)$,

$$\theta_a(s) = \frac{\begin{vmatrix} (s^2 + s) & T(s) \\ -s & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + s) & -s \\ -s & (s+1) \end{vmatrix}} = \frac{sT(s)}{s^3 + s^2 + s}$$

From which,

$$\frac{\theta_a(s)}{T(s)} = \frac{1}{s^2 + s + 1}$$

$$\text{But, } \theta_2(s) = \frac{1}{2}\theta_a(s).$$

Thus,

$$\frac{\theta_2(s)}{T(s)} = \frac{1/2}{s^2 + s + 1}$$

2.11.

First find the mechanical constants.

$$J_m = J_a + J_L\left(\frac{1}{5} * \frac{1}{4}\right)^2 = 1 + 400\left(\frac{1}{400}\right) = 2$$

$$D_m = D_a + D_L\left(\frac{1}{5} * \frac{1}{4}\right)^2 = 5 + 800\left(\frac{1}{400}\right) = 7$$

Now find the electrical constants. From the torque-speed equation, set $\omega_m = 0$ to find stall torque and set $T_m = 0$ to find no-load speed. Hence,

$$T_{stall} = 200$$

$$\omega_{no-load} = 25$$

which,

$$\frac{K_t}{R_a} = \frac{T_{stall}}{E_a} = \frac{200}{100} = 2$$

$$K_b = \frac{E_a}{\omega_{no-load}} = \frac{100}{25} = 4$$

Substituting all values into the motor transfer function,

$$\frac{\theta_m(s)}{E_a(s)} = \frac{\frac{K_T}{R_a J_m}}{s(s + \frac{1}{J_m}(D_m + \frac{K_T K_b}{R_a}))} = \frac{1}{s(s + \frac{15}{2})}$$

where $\theta_m(s)$ is the angular displacement of the armature.

Now $\theta_L(s) = \frac{1}{20} \theta_m(s)$. Thus,

$$\frac{\theta_L(s)}{E_a(s)} = \frac{1/20}{s(s + \frac{15}{2})}$$

2.12.

Letting

$$\theta_1(s) = \omega_1(s) / s$$

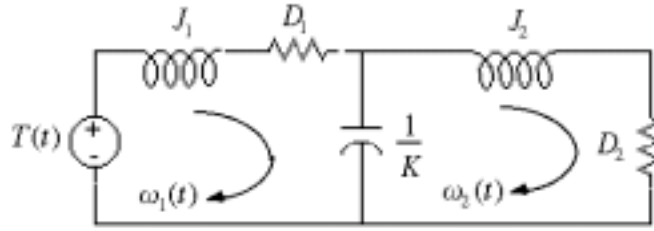
$$\theta_2(s) = \omega_2(s) / s$$

in Eqs. 2.127, we obtain

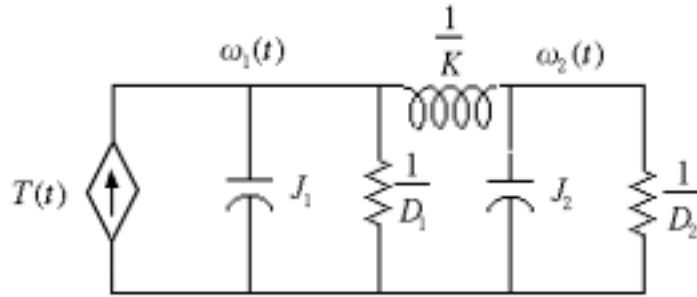
$$(J_1 s + D_1 + \frac{K}{s})\omega_1(s) - \frac{K}{s}\omega_2(s) = T(s)$$

$$-\frac{K}{s}\omega_1(s) + (J_2 s + D_2 + \frac{K}{s})\omega_2(s)$$

From these equations we can draw both series and parallel analogs by considering these to be mesh or nodal equations, respectively.



Series analog



Parallel analog

2.13.

Writing the nodal equation,

$$C \frac{dv}{dt} + i_r - 2 = i(t)$$

But,

$$C = 1$$

$$v = v_o + \delta v$$

$$i_r = e^{v_r} = e^v = e^{v_o + \delta v}$$

Substituting these relationships into the differential equation,

$$\frac{d(v_o + \delta v)}{dt} + e^{v_o + \delta v} - 2 = i(t) \quad (1)$$

We now linearize e^v .

The general form is

$$f(v) - f(v_o) \approx \left. \frac{df}{dv} \right|_{v_o} \delta v$$

Substituting the function, $f(v) = e^v$, with $v = v_o + \delta v$ yields,

$$e^{v_o + \delta v} - e^{v_o} \approx \left. \frac{de^v}{dv} \right|_{v_o} \delta v$$

Solving for $e^{v_o + \delta v}$,

$$e^{v_o + \delta v} = e^{v_o} + \left. \frac{de^v}{dv} \right|_{v_o} \delta v = e^{v_o} + e^{v_o} \delta v$$

Substituting into Eq. (1)

$$\frac{d\delta v}{dt} + e^{v_o} + e^{v_o} \delta v - 2 = i(t) \quad (2)$$

Setting $i(t) = 0$ and letting the circuit reach steady state, the capacitor acts like an open circuit. Thus, $v_o = v_r$ with $i_r = 2$. But, $i_r = e^{v_r}$ or $v_r = \ln i_r$.

Hence, $v_o = \ln 2 = 0.693$. Substituting this value of v_o into Eq. (2) yields

$$\frac{d\delta v}{dt} + 2\delta v = i(t)$$

Taking the Laplace transform,

$$(s + 2)\delta v(s) = I(s)$$

Solving for the transfer function, we obtain

$$\frac{\delta v(s)}{I(s)} = \frac{1}{s + 2}$$

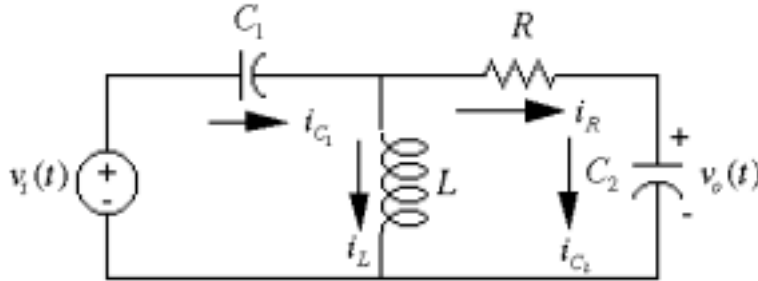
or

$$\frac{V(s)}{I(s)} = \frac{1}{s + 2} \text{ about equilibrium.}$$

Chapter 3

3.1.

Identifying appropriate variables on the circuit yields



Writing the derivative relations

$$\begin{aligned} C_1 \frac{dv_{C_1}}{dt} &= i_{C_1} \\ L \frac{di_L}{dt} &= v_L \\ C_2 \frac{dv_{C_2}}{dt} &= i_{C_2} \end{aligned} \quad (1)$$

Using Kirchhoff's current and voltage laws,

$$\begin{aligned} i_{C_1} &= i_L + i_R = i_L + \frac{1}{R}(v_L - v_{C_2}) \\ v_L &= -v_{C_1} + v_i \\ i_{C_2} &= i_R = \frac{1}{R}(v_L - v_{C_2}) \end{aligned}$$

Substituting these relationships into Eqs. (1) and simplifying yields the state equations as

$$\begin{aligned} \frac{dv_{C_1}}{dt} &= -\frac{1}{RC_1}v_{C_1} + \frac{1}{C_1}i_L - \frac{1}{RC_1}v_{C_2} + \frac{1}{RC_1}v_i \\ \frac{di_L}{dt} &= -\frac{1}{L}v_{C_1} + \frac{1}{L}v_i \\ \frac{dv_{C_2}}{dt} &= -\frac{1}{RC_2}v_{C_1} - \frac{1}{RC_2}v_{C_2} + \frac{1}{RC_2}v_i \end{aligned}$$

where the output equation is

$$v_o = v_{C_2}$$

Putting the equations in vector-matrix form,

$$\dot{\mathbf{x}} = \begin{bmatrix} -\frac{1}{RC_1} & \frac{1}{C_1} & -\frac{1}{RC_1} \\ -\frac{1}{L} & 0 & 0 \\ -\frac{1}{RC_2} & 0 & -\frac{1}{RC_2} \end{bmatrix} \mathbf{x} + \begin{bmatrix} \frac{1}{RC_1} \\ \frac{1}{L} \\ \frac{1}{RC_2} \end{bmatrix} v_i(t)$$

$$y = [0 \quad 0 \quad 1] \mathbf{x}$$

3.2.

Writing the equations of motion

$$\begin{aligned} (s^2 + s + 1)X_1(s) - sX_2(s) &= F(s) \\ -sX_1(s) + (s^2 + s + 1)X_2(s) - X_3(s) &= 0 \\ -X_2(s) + (s^2 + s + 1)X_3(s) &= 0 \end{aligned}$$

Taking the inverse Laplace transform and simplifying,

$$\begin{aligned} \ddot{x}_1 &= -\dot{x}_1 - x_1 + \dot{x}_2 + f \\ \ddot{x}_2 &= \dot{x}_1 - \dot{x}_2 - x_2 + x_3 \\ \ddot{x}_3 &= -\dot{x}_3 - x_3 + x_2 \end{aligned}$$

Defining state variables, z_i ,

$$z_1 = x_1; \quad z_2 = \dot{x}_1; \quad z_3 = x_2; \quad z_4 = \dot{x}_2; \quad z_5 = x_3; \quad z_6 = \dot{x}_3$$

Writing the state equations using the definition of the state variables and the inverse transform of the differential equation,

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= \ddot{x}_1 = -\dot{x}_1 - x_1 + \dot{x}_2 + f = -z_2 - z_1 + z_4 + f \\ \dot{z}_3 &= \dot{x}_2 = z_4 \\ \dot{z}_4 &= \ddot{x}_2 = \dot{x}_1 - \dot{x}_2 - x_2 + x_3 = z_2 - z_4 - z_3 + z_5 \\ \dot{z}_5 &= \dot{x}_3 = z_6 \\ \dot{z}_6 &= \ddot{x}_3 = -\dot{x}_3 - x_3 + x_2 = -z_6 - z_5 + z_3 \end{aligned}$$

The output is z_5 . Hence, $y = z_5$. In vector-matrix form,

$$\dot{\mathbf{z}} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & -1 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} f(t); y = [0 \ 0 \ 0 \ 0 \ 1 \ 0] \mathbf{z}$$

3.3.

First derive the state equations for the transfer function without zeros.

$$\frac{X(s)}{R(s)} = \frac{1}{s^2 + 7s + 9}$$

Cross multiplying yields

$$(s^2 + 7s + 9)X(s) = R(s)$$

Taking the inverse Laplace transform assuming zero initial conditions, we get

$$\ddot{x} + 7\dot{x} + 9x = r$$

Defining the state variables as,

$$x_1 = x$$

$$x_2 = \dot{x}$$

Hence,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{x} = -7\dot{x} - 9x + r = -9x_1 - 7x_2 + r$$

Using the zeros of the transfer function, we find the output equation to be,

$$c = 2\dot{x} + x = x_1 + 2x_2$$

Putting all equation in vector-matrix form yields,

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -9 & -7 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r$$

$$c = [1 \ 2] \mathbf{x}$$

3.4.

The state equation is converted to a transfer function using

$$G(s) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B} \quad (1)$$

where

$$\mathbf{A} = \begin{bmatrix} -4 & -1.5 \\ 4 & 0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}, \text{ and } \mathbf{C} = [1.5 \quad 0.625].$$

Evaluating $(s\mathbf{I} - \mathbf{A})$ yields

$$(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s+4 & 1.5 \\ -4 & s \end{bmatrix}$$

Taking the inverse we obtain

$$(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^2 + 4s + 6} \begin{bmatrix} s & -1.5 \\ 4 & s+4 \end{bmatrix}$$

Substituting all expressions into Eq. (1) yields

$$G(s) = \frac{3s + 5}{s^2 + 4s + 6}$$

3.5.

Writing the differential equation we obtain

$$\frac{d^2x}{dt^2} + 2x^2 = 10 + \delta f(t) \quad (1)$$

Letting $x = x_o + \delta x$ and substituting into Eq. (1) yields

$$\frac{d^2(x_o + \delta x)}{dt^2} + 2(x_o + \delta x)^2 = 10 + \delta f(t) \quad (2)$$

Now, linearize x^2 .

$$(x_o + \delta x)^2 - x_o^2 = \left. \frac{d(x^2)}{dx} \right|_{x_o} \delta x = 2x_o \delta x$$

from which

$$(x_o + \delta x)^2 = x_o^2 + 2x_o \delta x \quad (3)$$

Substituting Eq. (3) into Eq. (1) and performing the indicated differentiation gives us the linearized intermediate differential equation,

$$\frac{d^2\delta x}{dt^2} + 4x_o \delta x = -2x_o^2 + 10 + \delta f(t) \quad (4)$$

The force of the spring at equilibrium is 10 N. Thus, since $F = 2x^2$,

$$10 = 2x_o^2$$

from which

$$x_o = \sqrt{5}$$

Substituting this value of x_o into Eq. (4) gives us the final linearized differential equation.

$$\frac{d^2 \delta x}{dt^2} + 4\sqrt{5} \delta x = \delta f(t)$$

Selecting the state variables,

$$x_1 = \delta x$$

$$x_2 = \dot{\delta x}$$

Writing the state and output equations

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \ddot{\delta x} = -4\sqrt{5}x_1 + \delta f(t)$$

$$y = x_1$$

Converting to vector-matrix form yields the final result as

$$\dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ -4\sqrt{5} & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \delta f(t)$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}$$

Chapter 4

4.1.

For a step input

$$C(s) \downarrow \frac{10(s+4)(s+6)}{s(s+1)(s+7)(s+8)(s+10)} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{s+7} + \frac{D}{s+8} + \frac{E}{s+10}$$

Taking the inverse Laplace transform,

$$c(t) = A + Be^{-t} + Ce^{-7t} + De^{-8t} + Ee^{-10t}$$

4.2.

$$\text{Since } a = 50, T_c = \frac{1}{a} = \frac{1}{50} = 0.02 \text{ s}; T_s = \frac{4}{a} = \frac{4}{50} = 0.08 \text{ s}; \text{ and } T_r = \frac{2.2}{a} = \frac{2.2}{50} = 0.044 \text{ s}.$$

4.3.

a. Since poles are at $-6 \pm j19.08$, $c(t) = A + Be^{-6t} \cos(19.08t + \phi)$.

b. Since poles are at -78.54 and -11.46 , $c(t) = A + Be^{-78.54t} + Ce^{-11.4t}$.

c. Since poles are double on the real axis at -15 , $c(t) = A + Be^{-15t} + Cte^{-15t}$.

d. Since poles are at $\pm j25$, $c(t) = A + B \cos(25t + \phi)$.

4.4.

a. $\omega_n = \sqrt{400} = 20$ and $2\zeta\omega_n = 12$; $\therefore \zeta = 0.3$ and system is underdamped.

b. $\omega_n = \sqrt{900} = 30$ and $2\zeta\omega_n = 90$; $\therefore \zeta = 1.5$ and system is overdamped.

c. $\omega_n = \sqrt{225} = 15$ and $2\zeta\omega_n = 30$; $\therefore \zeta = 1$ and system is critically damped.

d. $\omega_n = \sqrt{625} = 25$ and $2\zeta\omega_n = 0$; $\therefore \zeta = 0$ and system is undamped.

4.5.

$$\omega_n = \sqrt{361} = 19 \text{ and } 2\zeta\omega_n = 16; \therefore \zeta = 0.421.$$

$$\text{Now, } T_s = \frac{4}{\zeta\omega_n} = 0.5 \text{ s and } T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}} = 0.182 \text{ s}.$$

From Figure 4.16, $\omega_n T_r = 1.4998$. Therefore, $T_r = 0.079 \text{ s}$.

$$\text{Finally, } \%os = e^{\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}} * 100 = 23.3\%$$

4.6.

a. The second-order approximation is valid, since the dominant poles have a real part of -2 and the higher-order pole is at -15 , i.e. more than five-times further.

b. The second-order approximation is not valid, since the dominant poles have a real part of -1 and the higher-order pole is at -4 , i.e. not more than five-times further.

4.7.

a. Expanding $G(s)$ by partial fractions yields $G(s) = \frac{1}{s} + \frac{0.8942}{s+20} - \frac{1.5918}{s+10} - \frac{0.3023}{s+6.5}$.

But -0.3023 is not an order of magnitude less than residues of second-order terms (term 2 and 3). Therefore, a second-order approximation is not valid.

b. Expanding $G(s)$ by partial fractions yields $G(s) = \frac{1}{s} + \frac{0.9782}{s+20} - \frac{1.9078}{s+10} - \frac{0.0704}{s+6.5}$.

But 0.0704 is an order of magnitude less than residues of second-order terms (term 2 and 3). Therefore, a second-order approximation is valid.

4.8.

See Figure 4.31 in the textbook for the Simulink block diagram and the output responses.

4.9.

a. Since $s\mathbf{I} - \mathbf{A} = \begin{bmatrix} s & -2 \\ 3 & s+5 \end{bmatrix}$, $(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^2 + 5s + 6} \begin{bmatrix} s+5 & 2 \\ -3 & s \end{bmatrix}$. Also,

$$\mathbf{B}\mathbf{U}(s) = \begin{bmatrix} 0 \\ 1/(s+1) \end{bmatrix}.$$

The state vector is $\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}[\mathbf{x}(0) + \mathbf{B}\mathbf{U}(s)] = \frac{1}{(s+1)(s+2)(s+3)} \begin{bmatrix} 2(s^2 + 7s + 7) \\ s^2 - 4s - 6 \end{bmatrix}$.

The output is $Y(s) = [1 \quad 3]\mathbf{X}(s) = \frac{5s^2 + 2s - 4}{(s+1)(s+2)(s+3)} = -\frac{0.5}{s+1} - \frac{12}{s+2} + \frac{17.5}{s+3}$.

Taking the inverse Laplace transform yields $y(t) = -0.5e^{-t} - 12e^{-2t} + 17.5e^{-3t}$.

b. The eigenvalues are given by the roots of $|s\mathbf{I} - \mathbf{A}| = s^2 + 5s + 6$, or -2 and -3 .

4.10.

a. Since $(s\mathbf{I} - \mathbf{A}) = \begin{bmatrix} s & -2 \\ 2 & s+5 \end{bmatrix}$, $(s\mathbf{I} - \mathbf{A})^{-1} = \frac{1}{s^2 + 5s + 4} \begin{bmatrix} s+5 & 2 \\ -2 & s \end{bmatrix}$. Taking the Laplace

transform of each term, the state transition matrix is given by

$$\Phi(t) = \begin{bmatrix} \frac{4}{3}e^{-t} - \frac{1}{3}e^{-4t} & \frac{2}{3}e^{-t} - \frac{2}{3}e^{-4t} \\ -\frac{2}{3}e^{-t} + \frac{2}{3}e^{-4t} & -\frac{1}{3}e^{-t} + \frac{4}{3}e^{-4t} \end{bmatrix}.$$

b. Since $\Phi(t - \tau) = \begin{bmatrix} \frac{4}{3}e^{-(t-\tau)} - \frac{1}{3}e^{-4(t-\tau)} & \frac{2}{3}e^{-(t-\tau)} - \frac{2}{3}e^{-4(t-\tau)} \\ -\frac{2}{3}e^{-(t-\tau)} + \frac{2}{3}e^{-4(t-\tau)} & -\frac{1}{3}e^{-(t-\tau)} + \frac{4}{3}e^{-4(t-\tau)} \end{bmatrix}$ and $\mathbf{Bu}(\tau) = \begin{bmatrix} 0 \\ e^{-2\tau} \end{bmatrix}$,

$$\Phi(t - \tau)\mathbf{Bu}(\tau) = \begin{bmatrix} \frac{2}{3}e^{-\tau}e^{-t} - \frac{2}{3}e^{2\tau}e^{-4t} \\ -\frac{1}{3}e^{-\tau}e^{-t} + \frac{4}{3}e^{2\tau}e^{-4t} \end{bmatrix}.$$

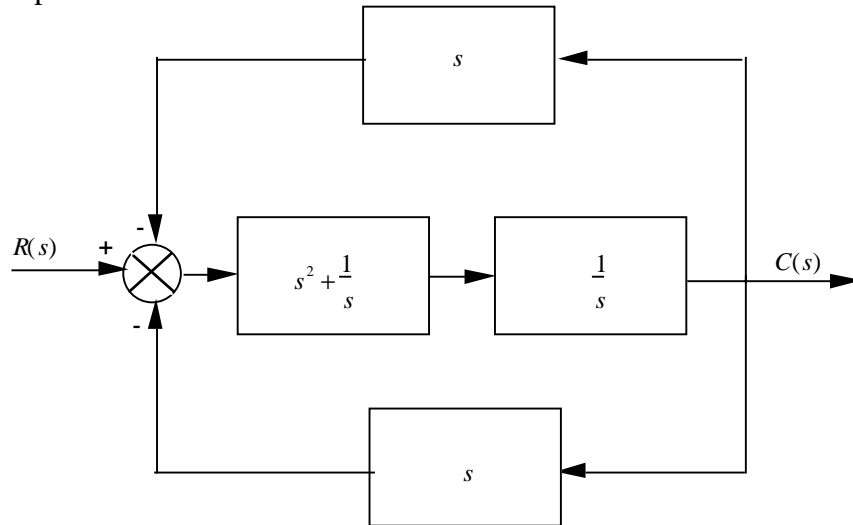
$$\text{Thus, } \mathbf{x}(t) = \Phi(t)\mathbf{x}(0) + \int_0^t \Phi(t - \tau)\mathbf{Bu}(\tau)d\tau = \begin{bmatrix} \frac{10}{3}e^{-t} - e^{-2t} - \frac{4}{3}e^{-4t} \\ -\frac{5}{3}e^{-t} + e^{-2t} + \frac{8}{3}e^{-4t} \end{bmatrix}.$$

c. $y(t) = \begin{bmatrix} 2 & 1 \end{bmatrix} \mathbf{x} = 5e^{-t} - e^{-2t}$

Chapter 5

5.1.

Combine the parallel blocks in the forward path. Then, push $\frac{1}{s}$ to the left past the pickoff point.



Combine the parallel feedback paths and get $2s$. Then, apply the feedback formula, simplify, and get, $T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$.

5.2.

Find the closed-loop transfer function, $T(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{16}{s^2 + as + 16}$,

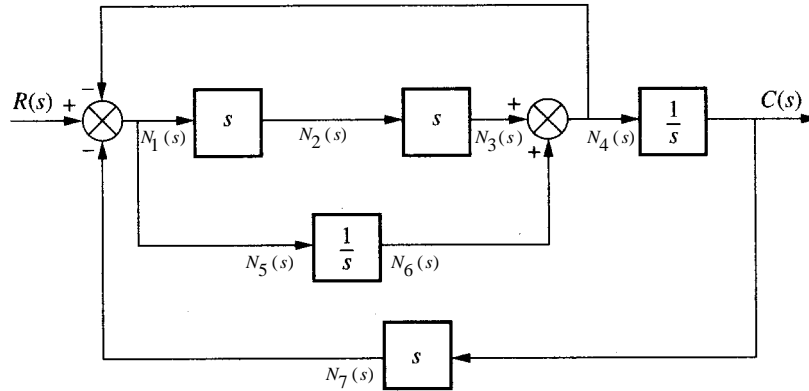
where $G(s) = \frac{16}{s(s+a)}$ and $H(s) = 1$. Thus, $\omega_n = 4$ and $2\zeta\omega_n = a$, from which

$$\zeta = \frac{a}{8}. \text{ But, for 5\% overshoot, } \zeta = \frac{-\ln(\frac{\%}{100})}{\sqrt{\pi^2 + \ln^2(\frac{\%}{100})}} = 0.69. \text{ Since, } \zeta = \frac{a}{8},$$

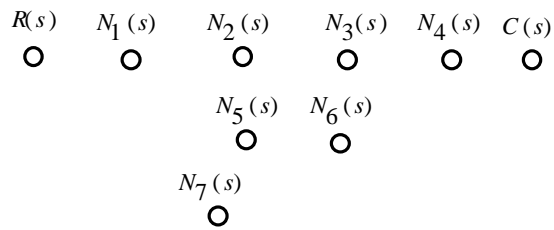
$$a = 5.52.$$

5.3.

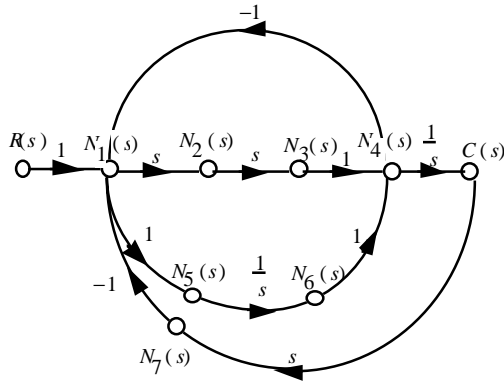
Label nodes.



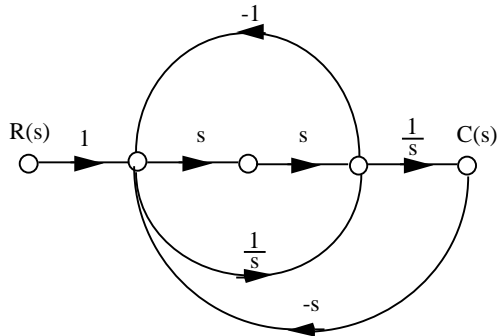
Draw nodes.



Connect nodes and label subsystems.



Eliminate unnecessary nodes.



5.4.

Forward-path gains are $G_1 G_2 G_3$ and $G_1 G_3$.

Loop gains are $-G_1G_2H_1$, $-G_2H_2$, and $-G_3H_3$.

Nontouching loops are $[-G_1G_2H_1][-G_3H_3] = G_1G_2G_3H_1H_3$

and $[-G_2H_2][-G_3H_3] = G_2G_3H_2H_3$.

Also, $\Delta = 1 + G_1G_2H_1 + G_2H_2 + G_3H_3 + G_1G_2G_3H_1H_3 + G_2G_3H_2H_3$.

Finally, $\Delta_1 = 1$ and $\Delta_2 = 1$.

Substituting these values into $T(s) = \frac{C(s)}{R(s)} = \frac{\sum_k T_k \Delta_k}{\Delta}$ yields

$$T(s) = \frac{G_1(s)G_3(s)[1 + G_2(s)]}{[1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)][1 + G_3(s)H_3(s)]}$$

5.5.

The state equations are,

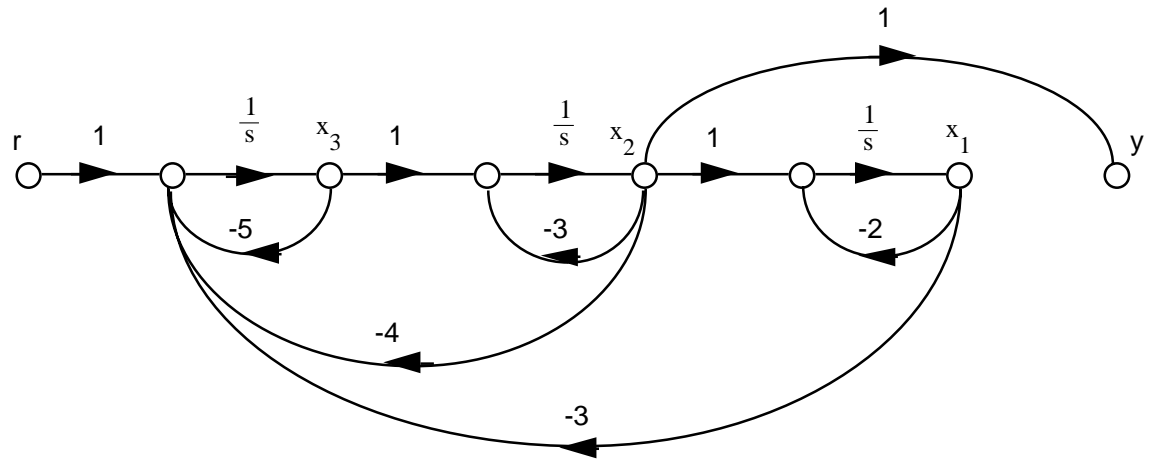
$$\dot{x}_1 = -2x_1 + x_2$$

$$\dot{x}_2 = -3x_2 + x_3$$

$$\dot{x}_3 = -3x_1 - 4x_2 - 5x_3 + r$$

$$y = x_2$$

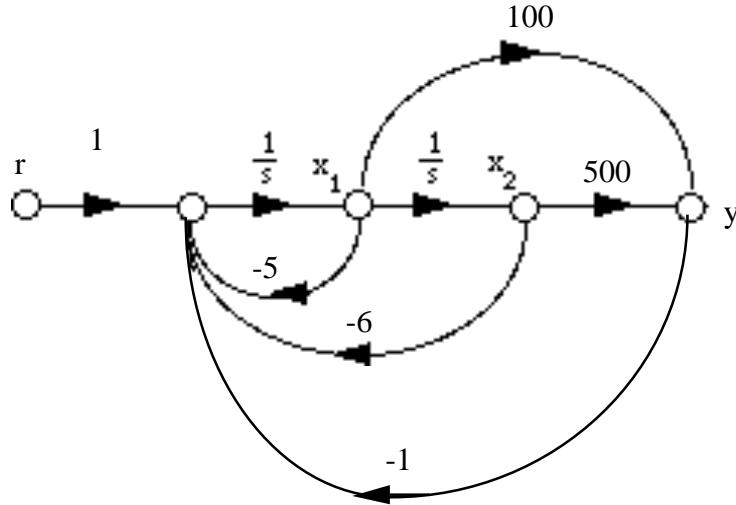
Drawing the signal-flow diagram from the state equations yields



5.6.

From $G(s) = \frac{100(s+5)}{s^2 + 5s + 6}$ we draw the signal-flow graph in controller canonical

form and add the feedback.



Writing the state equations from the signal-flow diagram, we obtain

$$\dot{\mathbf{x}} = \begin{bmatrix} -105 & -506 \\ 1 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} r$$

$$y = [100 \quad 500] \mathbf{x}$$

5.7.

From the transformation equations,

$$\mathbf{P}^{-1} = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix}$$

Taking the inverse,

$$\mathbf{P} = \begin{bmatrix} 0.4 & -0.2 \\ 0.1 & -0.3 \end{bmatrix}$$

Now,

$$\mathbf{P}^{-1} \mathbf{A} \mathbf{P} = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} 0.4 & -0.2 \\ 0.1 & -0.3 \end{bmatrix} = \begin{bmatrix} 6.5 & -8.5 \\ 9.5 & -11.5 \end{bmatrix}$$

$$\mathbf{P}^{-1} \mathbf{B} = \begin{bmatrix} 3 & -2 \\ 1 & -4 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} -3 \\ -11 \end{bmatrix}$$

$$\mathbf{C} \mathbf{P} = [1 \quad 4] \begin{bmatrix} 0.4 & -0.2 \\ 0.1 & -0.3 \end{bmatrix} = [0.8 \quad -1.4]$$

Therefore,

$$\dot{\mathbf{z}} = \begin{bmatrix} 6.5 & -8.5 \\ 9.5 & -11.5 \end{bmatrix} \mathbf{z} + \begin{bmatrix} -3 \\ -11 \end{bmatrix} u$$

$$y = [0.8 \quad -1.4] \mathbf{z}$$

5.8.

First find the eigenvalues.

$$|\lambda \mathbf{I} - \mathbf{A}| = \begin{vmatrix} \lambda & 0 \\ 0 & \lambda \end{vmatrix} - \begin{vmatrix} 1 & 3 \\ -4 & -6 \end{vmatrix} = \begin{vmatrix} \lambda - 1 & -3 \\ 4 & \lambda + 6 \end{vmatrix} = \lambda^2 + 5\lambda + 6$$

From which the eigenvalues are -2 and -3 .

Now use $\mathbf{A}\mathbf{x}_i = \lambda\mathbf{x}_i$ for each eigenvalue, λ . Thus,

$$\begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \lambda \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

For $\lambda = -2$,

$$3x_1 + 3x_2 = 0$$

$$-4x_1 - 4x_2 = 0$$

Thus $x_1 = -x_2$

For $\lambda = -3$

$$4x_1 + 3x_2 = 0$$

$$-4x_1 - 3x_2 = 0$$

Thus $x_1 = -x_2$ and $x_1 = -0.75x_2$; from which we let

$$\mathbf{P} = \begin{bmatrix} 0.707 & -0.6 \\ -0.707 & 0.8 \end{bmatrix}$$

Taking the inverse yields

$$\mathbf{P}^{-1} = \begin{bmatrix} 5.6577 & 4.2433 \\ 5 & 5 \end{bmatrix}$$

Hence,

$$\mathbf{D} = \mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \begin{bmatrix} 5.6577 & 4.2433 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} 0.707 & -0.6 \\ -0.707 & 0.8 \end{bmatrix} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix}$$

$$\mathbf{P}^{-1}\mathbf{B} = \begin{bmatrix} 5.6577 & 4.2433 \\ 5 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 18.39 \\ 20 \end{bmatrix}$$

$$\mathbf{C}\mathbf{P} = \begin{bmatrix} 1 & 4 \end{bmatrix} \begin{bmatrix} 0.707 & -0.6 \\ -0.707 & 0.8 \end{bmatrix} = \begin{bmatrix} -2.121 & 2.6 \end{bmatrix}$$

Finally,

$$\dot{\mathbf{z}} = \begin{bmatrix} -2 & 0 \\ 0 & -3 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 18.39 \\ 20 \end{bmatrix} u$$

$$y = \begin{bmatrix} -2.121 & 2.6 \end{bmatrix} \mathbf{z}$$

Chapter 6

6.1.

Make a Routh table.

s^7	3	6	7	2
s^6	9	4	8	6
s^5	4.666666667	4.333333333	0	0
s^4	-4.35714286	8	6	0
s^3	12.90163934	6.426229508	0	0
s^2	10.17026684	6	0	0
s^1	-1.18515742	0	0	0
s^0	6	0	0	0

Since there are four sign changes and no complete row of zeros, there are four right half-plane poles and three left half-plane poles.

6.2.

Make a Routh table. We encounter a row of zeros on the s^3 row. The even polynomial is contained in the previous row as $-6s^4 + 0s^2 + 6$. Taking the derivative yields $-24s^3 + 0s$. Replacing the row of zeros with the coefficients of the derivative yields the s^3 row. We also encounter a zero in the first column at the s^2 row. We replace the zero with ϵ and continue the table. The final result is shown now as

s^6	1	-6	-1	6	
s^5	1	0	-1	0	
s^4	-6	0	6	0	
s^3	-24	0	0	0	ROZ
s^2	ϵ	6	0	0	
s^1	$144/\epsilon$	0	0	0	
s^0	6	0	0	0	

There is one sign change below the even polynomial. Thus the even polynomial (4^{th} order) has one right half-plane pole, one left half-plane pole, and 2 imaginary axis poles. From the top of the table down to the even polynomial yields one sign change. Thus, the rest of the polynomial has one right half-plane root, and one left

half-plane root. The total for the system is two right half-plane poles, two left half-plane poles, and 2 imaginary poles.

6.3.

$$\text{Since } G(s) = \frac{K(s+20)}{s(s+2)(s+3)}, \quad T(s) = \frac{G(s)}{1+G(s)} = \frac{K(s+20)}{s^3 + 5s^2 + (6+K)s + 20K}$$

Form the Routh table.

s^3	1	$(6+K)$
s^2	5	$20K$
s^1	$\frac{30-15K}{5}$	
s^0	$20K$	

From the s^1 row, $K < 2$. From the s^0 row, $K > 0$. Thus, for stability, $0 < K < 2$.

6.4.

First find

$$|s\mathbf{I} - \mathbf{A}| = \begin{vmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{vmatrix} - \begin{vmatrix} 2 & 1 & 1 \\ 1 & 7 & 1 \\ -3 & 4 & -5 \end{vmatrix} = \begin{vmatrix} (s-2) & -1 & -1 \\ -1 & (s-7) & -1 \\ 3 & -4 & (s+5) \end{vmatrix} = s^3 - 4s^2 - 33s + 51$$

Now form the Routh table.

s^3	1	-33
s^2	-4	51
s^1	-20.25	
s^0	51	

There are two sign changes. Thus, there are two rhp poles and one lhp pole.

Chapter 7

7.1.

a. First check stability.

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{10s^2 + 500s + 6000}{s^3 + 70s^2 + 1375s + 6000} = \frac{10(s + 30)(s + 20)}{(s + 26.03)(s + 37.89)(s + 6.085)}$$

Poles are in the lhp. Therefore, the system is stable. Stability also could be checked via Routh-Hurwitz using the denominator of $T(s)$. Thus,

$$15u(t): e_{step}(\infty) = \frac{15}{1 + \lim_{s \rightarrow 0} G(s)} = \frac{15}{1 + \infty} = 0$$

$$15tu(t): e_{ramp}(\infty) = \frac{15}{\lim_{s \rightarrow 0} sG(s)} = \frac{15}{\frac{10 * 20 * 30}{25 * 35}} = 2.1875$$

$$15t^2u(t): e_{parabola}(\infty) = \frac{15}{\lim_{s \rightarrow 0} s^2G(s)} = \frac{30}{0} = \infty, \text{ since } \mathcal{L}[15t^2] = \frac{30}{s^3}$$

b. First check stability.

$$\begin{aligned} T(s) &= \frac{G(s)}{1 + G(s)} = \frac{10s^2 + 500s + 6000}{s^5 + 110s^4 + 3875s^3 + 4.37e04s^2 + 500s + 6000} \\ &= \frac{10(s + 30)(s + 20)}{(s + 50.01)(s + 35)(s + 25)(s^2 - 7.189e - 04s + 0.1372)} \end{aligned}$$

From the second-order term in the denominator, we see that the system is unstable. Instability could also be determined using the Routh-Hurwitz criteria on the denominator of $T(s)$. Since the system is unstable, calculations about steady-state error cannot be made.

7.2.

a. The system is stable, since

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{1000(s + 8)}{(s + 9)(s + 7) + 1000(s + 8)} = \frac{1000(s + 8)}{s^2 + 1016s + 8063} \text{ and is of}$$

Type 0. Therefore,

$$K_p = \lim_{s \rightarrow 0} G(s) = \frac{1000 * 8}{7 * 9} = 127; K_v = \lim_{s \rightarrow 0} sG(s) = 0; \text{ and } K_a = \lim_{s \rightarrow 0} s^2G(s) = 0$$

$$\text{b. } e_{step}(\infty) = \frac{1}{1 + \lim_{s \rightarrow 0} G(s)} = \frac{1}{1 + 127} = 7.8e - 03$$

$$e_{ramp}(\infty) = \frac{1}{\lim_{s \rightarrow 0} sG(s)} = \frac{1}{0} = \infty$$

$$e_{parabola}(\infty) = \frac{1}{\lim_{s \rightarrow 0} s^2 G(s)} = \frac{1}{0} = \infty$$

7.3.

System is stable for positive K . System is Type 0. Therefore, for a step input

$$e_{step}(\infty) = \frac{1}{1 + K_p} = 0.1. \text{ Solving for } K_p \text{ yields } K_p = 9 = \lim_{s \rightarrow 0} G(s) = \frac{12K}{14 * 18}; \text{ from}$$

which we obtain $K = 189$.

7.4.

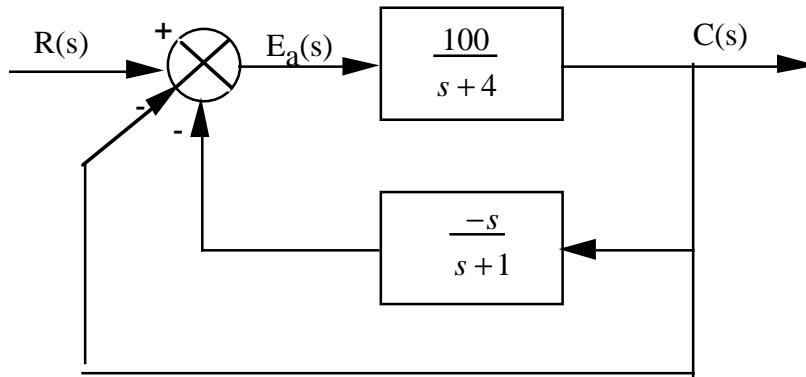
System is stable. Since $G_1(s) = 1000$, and $G_2(s) = \frac{(s+2)}{(s+4)}$,

$$e_D(\infty) = -\frac{1}{\lim_{s \rightarrow 0} \frac{1}{G_2(s)} + \lim_{s \rightarrow 0} G_1(s)} = -\frac{1}{2 + 1000} = -9.98e-04$$

7.5.

System is stable. Create a unity-feedback system, where $H_e(s) = \frac{1}{s+1} - 1 = \frac{-s}{s+1}$.

The system is as follows:



Thus,

$$G_e(s) = \frac{G(s)}{1 + G(s)H_e(s)} = \frac{\frac{100}{(s+4)}}{1 - \frac{100s}{(s+1)(s+4)}} = \frac{100(s+1)}{s^2 - 95s + 4}$$

Hence, the system is Type 0. Evaluating K_p yields

$$K_p = \frac{100}{4} = 25$$

The steady-state error is given by

$$e_{step}(\infty) = \frac{1}{1 + K_p} = \frac{1}{1 + 25} = 3.846e - 02$$

7.6.

$$\text{Since } G(s) = \frac{K(s+7)}{s^2 + 2s + 10}, \quad e(\infty) = \frac{1}{1 + K_p} = \frac{1}{1 + \frac{7K}{10}} = \frac{10}{10 + 7K}.$$

Calculating the sensitivity, we get

$$S_{e:K} = \frac{K}{e} \frac{\partial e}{\partial K} = \frac{K}{\left(\frac{10}{10 + 7K}\right)} \frac{(-10)7}{(10 + 7K)^2} = -\frac{7K}{10 + 7K}$$

7.7.

Given

$$\mathbf{A} = \begin{bmatrix} 0 & 1 \\ -3 & -6 \end{bmatrix}; \quad \mathbf{B} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}; \quad \mathbf{C} = [1 \quad 1]; \quad R(s) = \frac{1}{s}.$$

Using the final value theorem,

$$\begin{aligned} e_{step}(\infty) &= \lim_{s \rightarrow 0} sR(s)[1 - \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}] = \lim_{s \rightarrow 0} [1 - [1 \quad 1] \begin{bmatrix} s & -1 \\ 3 & s+6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}] \\ &= \lim_{s \rightarrow 0} [1 - [1 \quad 1] \frac{\begin{bmatrix} s+6 & 1 \\ -3 & s \end{bmatrix}}{s^2 + 6s + 3} \begin{bmatrix} 0 \\ 1 \end{bmatrix}] = \lim_{s \rightarrow 0} \frac{s^2 + 5s + 2}{s^2 + 6s + 3} = \frac{2}{3} \end{aligned}$$

Using input substitution,

$$\begin{aligned} e_{step}(\infty) &= 1 + \mathbf{C}\mathbf{A}^{-1}\mathbf{B} = 1 - [1 \quad 1] \begin{bmatrix} 0 & 1 \\ -3 & -6 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\ &= 1 + [1 \quad 1] \frac{\begin{bmatrix} -6 & -1 \\ 3 & 0 \end{bmatrix}}{3} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1 + [1 \quad 1] \begin{bmatrix} -\frac{1}{3} \\ 0 \end{bmatrix} = \frac{2}{3} \end{aligned}$$

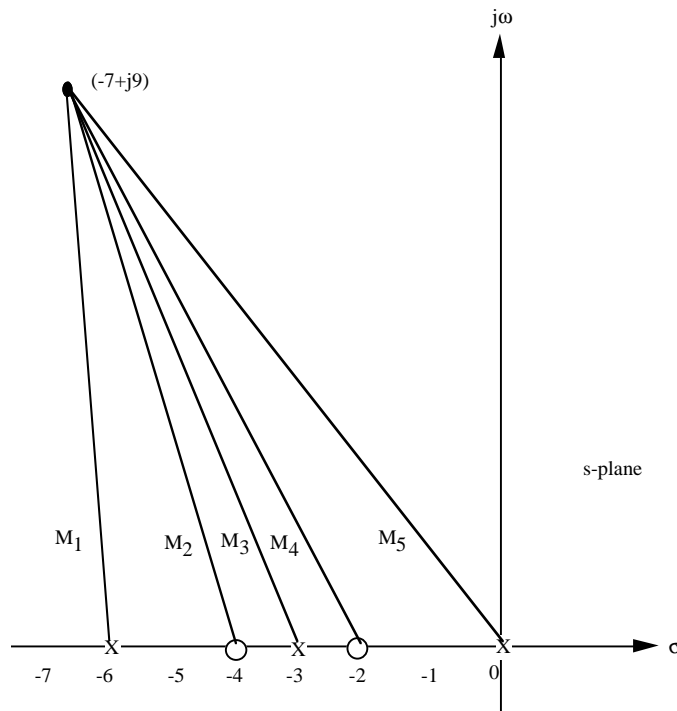
Chapter 8

8.1.

a.

$$\begin{aligned}
 F(-7 + j9) &= \frac{(-7 + j9 + 2)(-7 + j9 + 4)0.0339}{(-7 + j9)(-7 + j9 + 3)(-7 + j9 + 6)} = \frac{(-5 + j9)(-3 + j9)}{(-7 + j9)(-4 + j9)(-1 + j9)} \\
 &= \frac{(-66 - j72)}{(944 - j378)} = -0.0339 - j0.0899 = 0.096 \angle -110.7^\circ
 \end{aligned}$$

b. The arrangement of vectors is shown as follows:

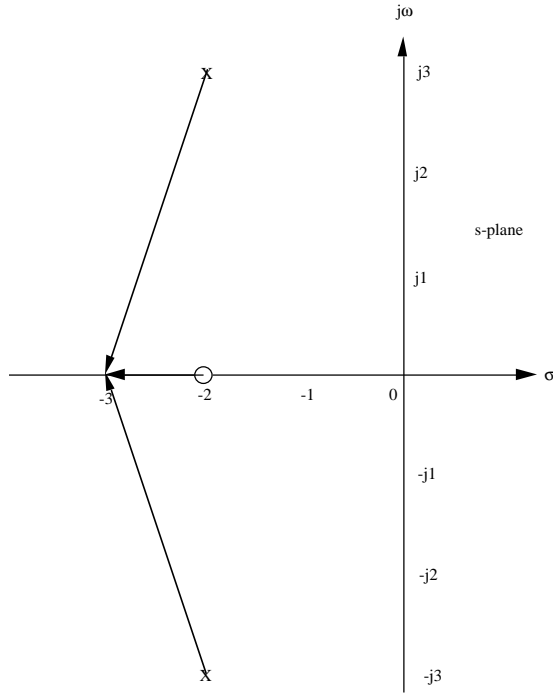


From the diagram,

$$\begin{aligned}
 F(-7 + j9) &= \frac{M_2 M_4}{M_1 M_3 M_5} = \frac{(-3 + j9)(-5 + j9)}{(-1 + j9)(-4 + j9)(-7 + j9)} \\
 &= \frac{(-66 - j72)}{(944 - j378)} = -0.0339 - j0.0899 = 0.096 \angle -110.7^\circ
 \end{aligned}$$

8.2.

a. First draw the vectors.



From the diagram,

$$\sum \text{angles} = 180^\circ - \tan^{-1}\left(\frac{-3}{-1}\right) - \tan^{-1}\left(\frac{-3}{1}\right) = 180^\circ - 108.43^\circ + 108.43^\circ = 180^\circ.$$

b. Since the angle is 180° , the point is on the root locus.

$$\text{c. } K = \frac{\Pi \text{ pole lengths}}{\Pi \text{ zero lengths}} = \frac{(\sqrt{1^2 + 3^2})(\sqrt{1^2 + 3^2})}{1} = 10$$

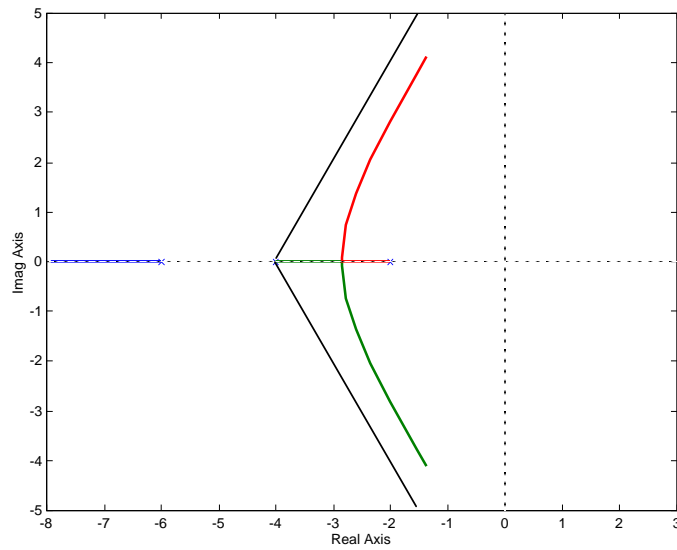
8.3.

First, find the asymptotes.

$$\sigma_a = \frac{\sum \text{poles} - \sum \text{zeros}}{\# \text{poles} - \# \text{zeros}} = \frac{(-2 - 4 - 6) - (0)}{3 - 0} = -4$$

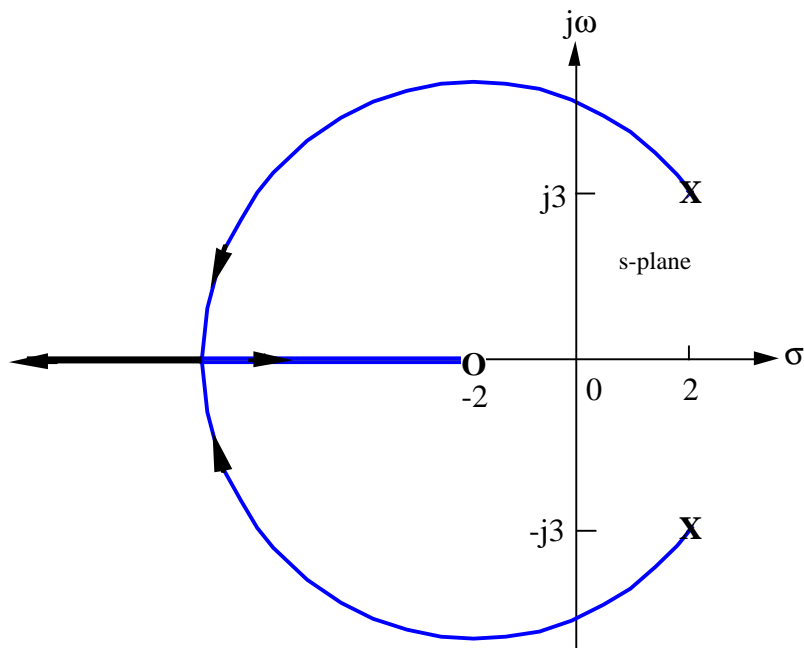
$$\theta_a = \frac{(2k+1)\pi}{3} = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$$

Next draw root locus following the rules for sketching.



8.4.

a.



b. Using the Routh-Hurwitz criteria, we first find the closed-loop transfer

$$\text{function. } T(s) = \frac{G(s)}{1 + G(s)} = \frac{K(s+2)}{s^2 + (K-4)s + (2K+13)}$$

Using the denominator of $T(s)$, make a Routh table.

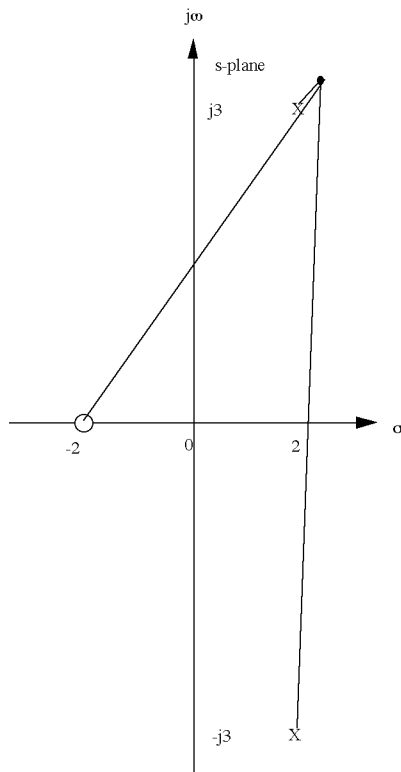
s^2	1	$2K+13$
s^1	$K-4$	0
s^0	$2K+13$	0

We get a row of zeros for $K = 4$. From the s^2 row with $K = 4$, $s^2 + 21 = 0$. From which we evaluate the imaginary axis crossing at $\sqrt{21}$.

c. From part (b), $K = 4$.

d. Searching for the minimum gain to the left of -2 on the real axis yields -7 at a gain of 18. Thus the break-in point is at -7 .

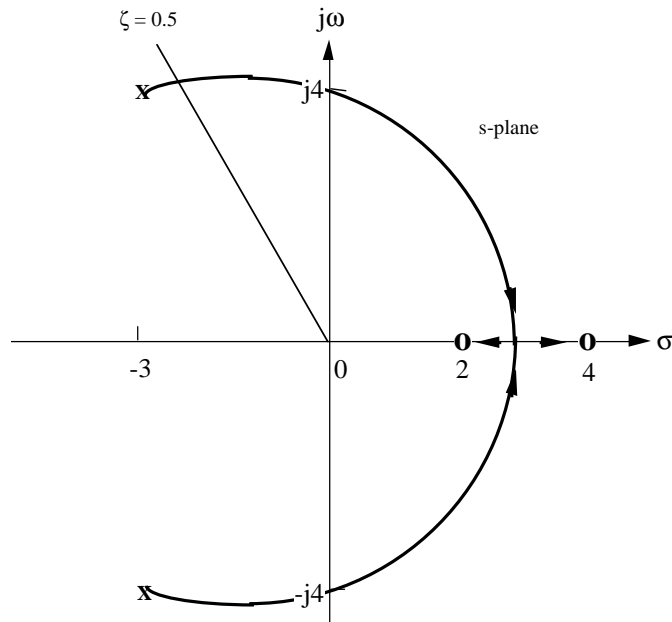
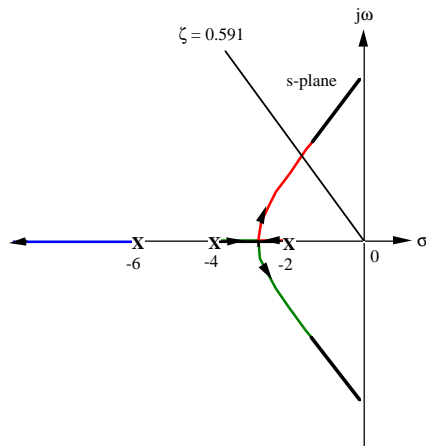
e. First, draw vectors to a point ϵ close to the complex pole.



At the point ϵ close to the complex pole, the angles must add up to zero. Hence, angle from zero – angle from pole in 4th quadrant – angle from pole in 1st quadrant

$$= 180^\circ, \text{ or } \tan^{-1}\left(\frac{3}{4}\right) - 90^\circ - \theta = 180^\circ. \text{ Solving for the angle of departure, } \theta = -$$

233.1.

8.5.**a.****b.** Search along the imaginary axis and find the 180° point at $s = \pm j4.06$.**c.** For the result in part (b), $K = 1$.**d.** Searching between 2 and 4 on the real axis for the minimum gain yields the break-in at $s = 2.89$.**e.** Searching along $\zeta = 0.5$ for the 180° point we find $s = -2.42 + j4.18$.**f.** For the result in part (e), $K = 0.108$.**g.** Using the result from part (c) and the root locus, $K < 1$.**8.6.****a.**

b. Searching along the $\zeta = 0.591$ (10% overshoot) line for the 180° point yields $-2.028 + j2.768$ with $K = 45.55$.

$$\text{c. } T_s = \frac{4}{|\text{Re}|} = \frac{4}{2.028} = 1.97 \text{ s}; \quad T_p = \frac{\pi}{|\text{Im}|} = \frac{\pi}{2.768} = 1.13 \text{ s};$$

$\omega_n T_r = 1.8346$ from the rise-time chart and graph in Chapter 4. Since ω_n is the radial distance to the pole, $\omega_n = \sqrt{2.028^2 + 2.768^2} = 3.431$. Thus, $T_r = 0.53 \text{ s}$;

since the system is Type 0, $K_p = \frac{K}{2 * 4 * 6} = \frac{45.55}{48} = 0.949$. Thus,

$$e_{step}(\infty) = \frac{1}{1 + K_p} = 0.51.$$

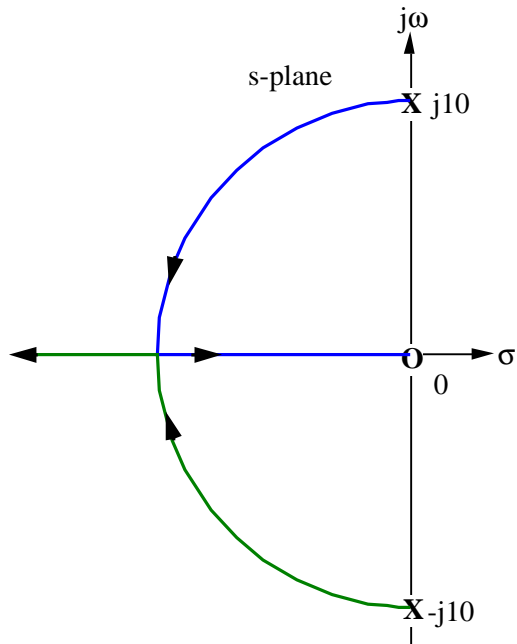
d. Searching the real axis to the left of -6 for the point whose gain is 45.55 , we find -7.94 . Comparing this value to the real part of the dominant pole, -2.028 , we find that it is not five times further. The second-order approximation is not valid.

8.7.

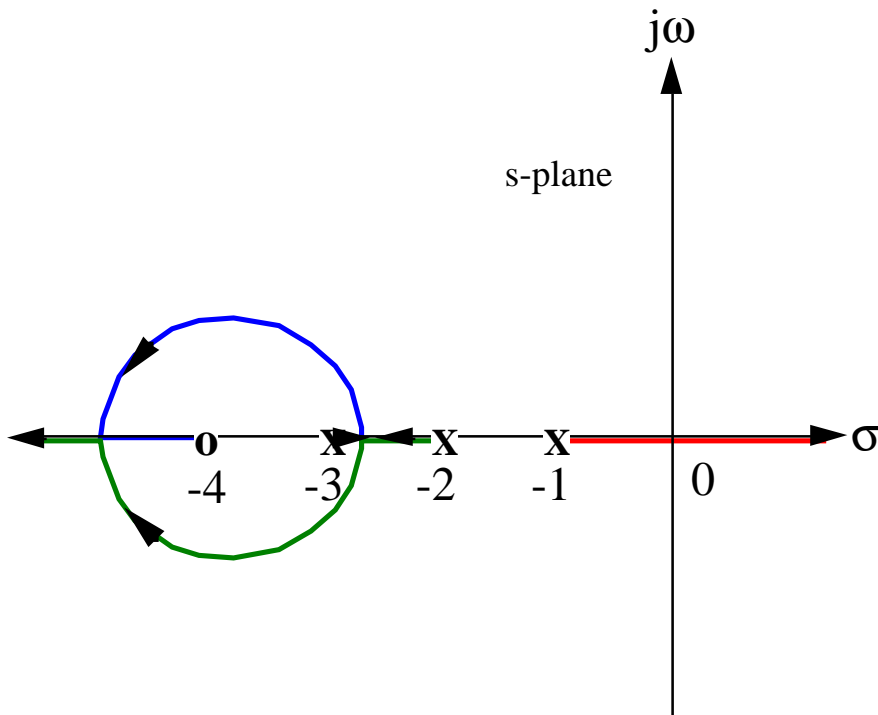
Find the closed-loop transfer function and put it the form that yields p_i as the root locus variable. Thus,

$$T(s) = \frac{G(s)}{1 + G(s)} = \frac{100}{s^2 + p_i s + 100} = \frac{100}{(s^2 + 100) + p_i s} = \frac{\frac{100}{s^2 + 100}}{1 + \frac{p_i s}{s^2 + 100}}$$

Hence, $KG(s)H(s) = \frac{p_i s}{s^2 + 100}$. The following shows the root locus.

**8.8.**

Following the rules for plotting the root locus of positive-feedback systems, we obtain the following root locus:



8.9.

The closed-loop transfer function is $T(s) = \frac{K(s+1)}{s^2 + (K+2)s + K}$. Differentiating the

denominator with respect to K yields

$$2s \frac{\partial s}{\partial K} + (K+2) \frac{\partial s}{\partial K} + (s+1) = (2s+K+2) \frac{\partial s}{\partial K} + (s+1) = 0$$

Solving for $\frac{\partial s}{\partial K}$, we get $\frac{\partial s}{\partial K} = \frac{-(s+1)}{(2s+K+2)}$. Thus, $S_{s:K} = \frac{K}{s} \frac{\partial s}{\partial K} = \frac{-K(s+1)}{s(2s+K+2)}$.

Substituting $K = 20$ yields $S_{s:K} = \frac{-10(s+1)}{s(s+11)}$.

Now find the closed-loop poles when $K = 20$. From the denominator of $T(s)$, $s_{1,2} = -21.05, -0.95$, when $K = 20$.

For the pole at -21.05 ,

$$\Delta s = s(S_{s:K}) \frac{\Delta K}{K} = -21.05 \left(\frac{-10(-21.05+1)}{-21.05(-21.05+11)} \right) 0.05 = -0.9975.$$

For the pole at -0.95 ,

$$\Delta s = s(S_{s:K}) \frac{\Delta K}{K} = -0.95 \left(\frac{-10(-0.95+1)}{-0.95(-0.95+11)} \right) 0.05 = -0.0025.$$

Chapter 9

9.1.

a. Searching along the 15% overshoot line, we find the point on the root locus at $-3.5 + j5.8$ at a gain of $K = 45.84$. Thus, for the uncompensated system, $K_v = \lim_{s \rightarrow 0} sG(s) = K / 7 = 45.84 / 7 = 6.55$.

Hence, $e_{ramp_uncompensated}(\infty) = 1 / K_v = 0.1527$.

b. Compensator zero should be 20x further to the left than the compensator pole.

Arbitrarily select $G_c(s) = \frac{(s + 0.2)}{(s + 0.01)}$.

c. Insert compensator and search along the 15% overshoot line and find the root locus at

$-3.4 + j5.63$ with a gain, $K = 44.64$. Thus, for the compensated

system, $K_v = \frac{44.64(0.2)}{(7)(0.01)} = 127.5$ and $e_{ramp_compensated}(\infty) = \frac{1}{K_v} = 0.0078$.

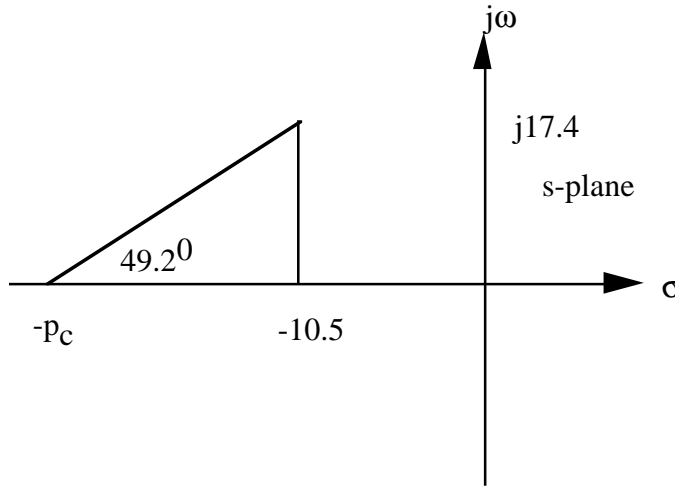
d. $\frac{e_{ramp_uncompensated}}{e_{ramp_compensated}} = \frac{0.1527}{0.0078} = 19.58$

9.2.

a. Searching along the 15% overshoot line, we find the point on the root locus at $-3.5 + j5.8$ at a gain of $K = 45.84$. Thus, for the uncompensated system,

$$T_s = \frac{4}{|\text{Re}|} = \frac{4}{3.5} = 1.143 \text{ s.}$$

b. The real part of the design point must be three times larger than the uncompensated pole's real part. Thus the design point is $3(-3.5) + j 3(5.8) = -10.5 + j17.4$. The angular contribution of the plant's poles and compensator zero at the design point is 130.8° . Thus, the compensator pole must contribute $180^\circ - 130.8^\circ = 49.2^\circ$. Using the following diagram,



we find $\frac{17.4}{p_c - 10.5} = \tan 49.2^\circ$, from which, $p_c = 25.52$. Adding this pole, we find

the gain at the design point to be $K = 476.3$. A higher-order closed-loop pole is found to be at -11.54 . This pole may not be close enough to the closed-loop zero at -10 . Thus, we should simulate the system to be sure the design requirements have been met.

9.3.

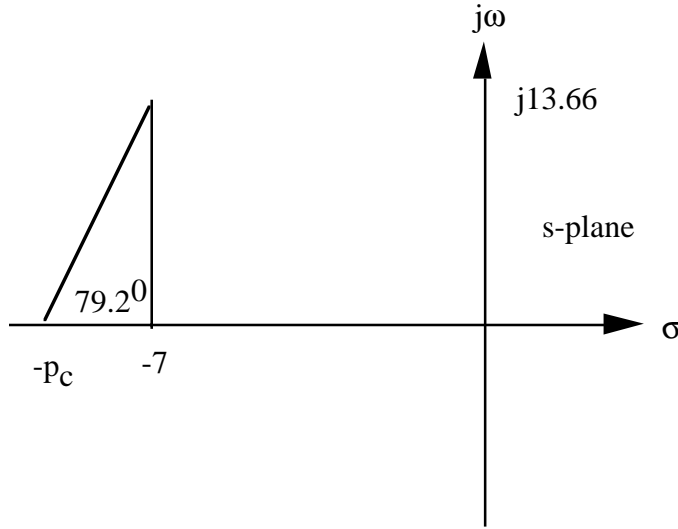
a. Searching along the 20% overshoot line, we find the point on the root locus at $-3.5 + j6.83$ at a gain of $K = 58.9$. Thus, for the uncompensated system,

$$T_s = \frac{4}{|\text{Re}|} = \frac{4}{3.5} = 1.143 \text{ s.}$$

b. For the uncompensated system, $K_v = \lim_{s \rightarrow 0} sG(s) = K / 7 = 58.9 / 7 = 8.41$. Hence,

$$e_{\text{ramp_uncompensated}}(\infty) = 1 / K_v = 0.1189.$$

c. In order to decrease the settling time by a factor of 2, the design point is twice the uncompensated value, or $-7 + j13.66$. Adding the angles from the plant's poles and the compensator's zero at -3 to the design point, we obtain -100.8° . Thus, the compensator pole must contribute $180^\circ - 100.8^\circ = 79.2^\circ$. Using the following diagram,



we find $\frac{13.66}{p_c - 7} = \tan 79.2^\circ$, from which, $p_c = 9.61$. Adding this pole, we find the

gain at the design point to be $K = 204.9$.

Evaluating K_v for the lead-compensated system:

$$K_v = \lim_{s \rightarrow 0} sG(s)G_{lead} = K(3) / [(7)(9.61)] = (204.9)(3) / [(7)(9.61)] = 9.138.$$

K_v for the uncompensated system was 8.41. For a 10x improvement in steady-state error, K_v must be $(8.41)(10) = 84.1$. Since lead compensation gave us $K_v = 9.138$, we need an improvement of $84.1/9.138 = 9.2$.

Thus, the lag compensator zero should be 9.2x further to the left than the

compensator pole. Arbitrarily select $G_c(s) = \frac{(s + 0.092)}{(s + 0.01)}$.

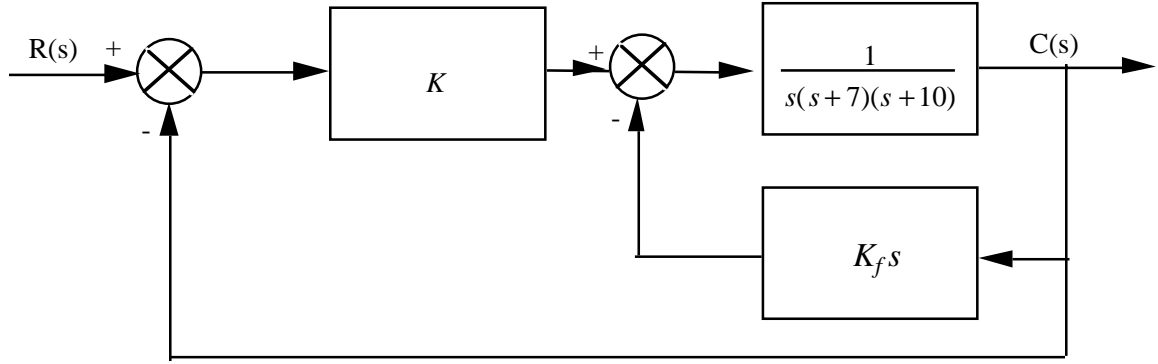
Using all plant and compensator poles, we find the gain at the design point to be $K = 205.4$. Summarizing the forward path with plant, compensator, and gain yields

$$G_e(s) = \frac{205.4(s + 3)(s + 0.092)}{s(s + 7)(9.61)(s + 0.01)}.$$

Higher-order poles are found at -0.928 and -2.6 . It would be advisable to simulate the system to see if there is indeed pole-zero cancellation.

9.4.

The configuration for the system is shown in the figure below.

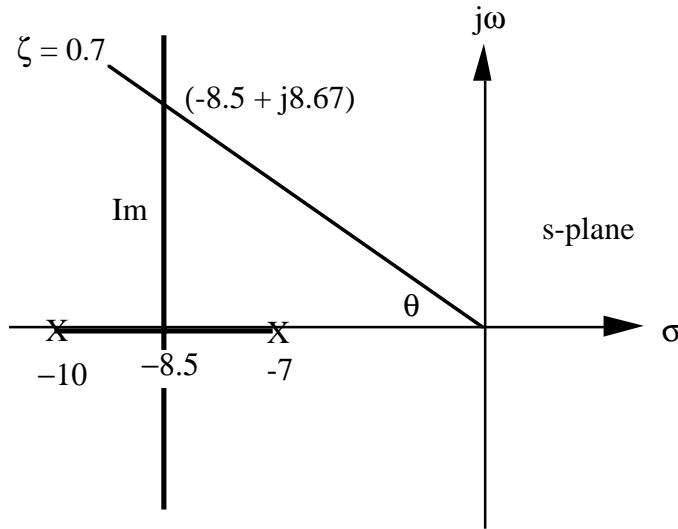


Minor-Loop Design:

For the minor loop, $G(s)H(s) = \frac{K_f}{(s+7)(s+10)}$. Using the following diagram, we

find that the minor-loop root locus intersects the 0.7 damping ratio line at $-8.5 + j8.67$. The imaginary part was found as follows: $\theta = \cos^{-1} \zeta = 45.57^\circ$. Hence,

$$\frac{\text{Im}}{8.5} = \tan 45.57^\circ, \text{ from which } \text{Im} = 8.67.$$



The gain, K_f , is found from the vector lengths as

$$K_f = \sqrt{1.5^2 + 8.67^2} \sqrt{1.5^2 + 8.67^2} = 77.42$$

Major-Loop Design:

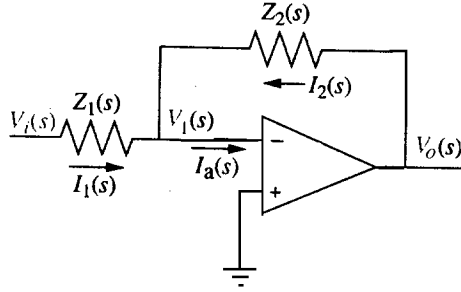
Using the closed-loop poles of the minor loop, we have an equivalent forward-path transfer function of

$$G_e(s) = \frac{K}{s(s+8.5+j8.67)(s+8.5-j8.67)} = \frac{K}{s(s^2+17s+147.4)}.$$

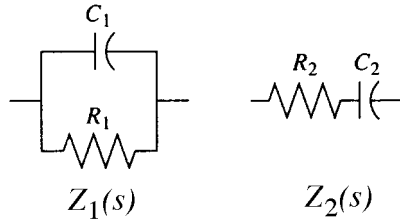
Using the three poles of $G_e(s)$ as open-loop poles to plot a root locus, we search along $\zeta = 0.5$ and find that the root locus intersects this damping ratio line at $-4.34 + j7.51$ at a gain, $K = 626.3$.

9.5.

a. An active PID controller must be used. We use the circuit shown in the following figure:



where the impedances are shown below as follows:



Matching the given transfer function with the transfer function of the PID controller yields

$$G_c(s) = \frac{(s+0.1)(s+5)}{s} = \frac{s^2 + 5.1s + 0.5}{s} = s + 5.1 + \frac{0.5}{s} = - \left[\left(\frac{R_2}{R_1} + \frac{C_1}{C_2} \right) + R_2 C_1 s + \frac{R_1 C_2}{s} \right]$$

Equating coefficients

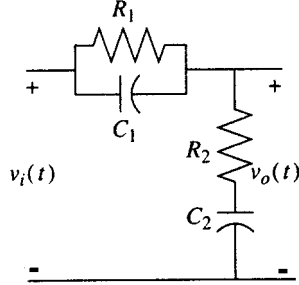
$$\frac{1}{R_1 C_2} = 0.5 \quad (1)$$

$$R_2 C_1 = 1 \quad (2)$$

$$\left(\frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = 5.1 \quad (3)$$

In Eq. (2) we arbitrarily let $C_1 = 10^{-5}$. Thus, $R_2 = 10^5$. Using these values along with Eqs. (1) and (3) we find $C_2 = 10 \mu\text{F}$ and $R_1 = 10^5 \text{ k}\Omega$.

b. The lag-lead compensator can be implemented with the following passive network, since the ratio of the lead pole-to-zero is the inverse of the ratio of the lag pole-to-zero:



Matching the given transfer function with the transfer function of the passive lag-lead compensator yields

$$G_c(s) = \frac{(s + 0.1)(s + 2)}{(s + 0.01)(s + 20)} = \frac{(s + 0.1)(s + 2)}{s^2 + 20.01s + 0.2} = \frac{\left(s + \frac{1}{R_1 C_1}\right)\left(s + \frac{1}{R_2 C_2}\right)}{s^2 + \left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right)s + \frac{1}{R_1 R_2 C_1 C_2}}$$

Equating coefficients

$$\frac{1}{R_1 C_1} = 0.1 \quad (1)$$

$$\frac{1}{R_2 C_2} = 2 \quad (2)$$

$$\left(\frac{1}{R_1 C_1} + \frac{1}{R_2 C_2} + \frac{1}{R_2 C_1}\right) = 20.01 \quad (3)$$

Substituting Eqs. (1) and (2) in Eq. (3) yields

$$\frac{1}{R_2 C_1} = 17.91 \quad (4)$$

Arbitrarily letting $C_1 = 100 \mu\text{F}$ in Eq. (1) yields $R_1 = 100 \text{ k}\Omega$.

Substituting $C_1 = 100 \mu\text{F}$ into Eq. (4) yields $R_2 = 558 \text{ k}\Omega$.

Substituting $R_2 = 558 \text{ k}\Omega$ into Eq. (2) yields $C_2 = 900 \mu\text{F}$.

Chapter 10

10.1.

a.

$$G(s) = \frac{1}{(s+2)(s+4)}; \quad G(j\omega) = \frac{1}{(8-\omega^2) + j6\omega}$$

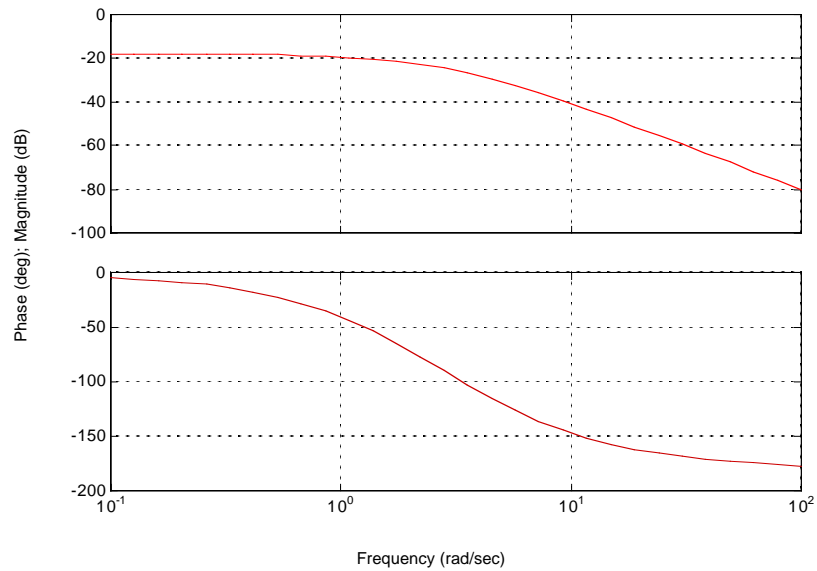
$$M(\omega) = \sqrt{(8-\omega^2)^2 + (6\omega)^2}$$

$$\text{For } \omega < \sqrt{8}, \phi(\omega) = -\tan^{-1}\left(\frac{6\omega}{8-\omega^2}\right).$$

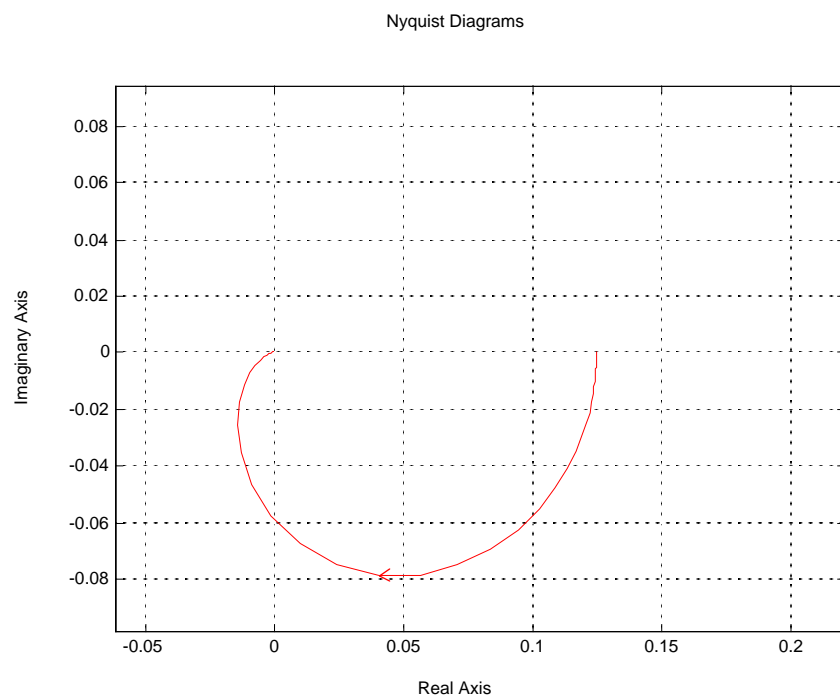
$$\text{For } \omega > \sqrt{8}, \phi(\omega) = -\left(\pi + \tan^{-1}\left[\frac{6\omega}{8-\omega^2}\right]\right).$$

b.

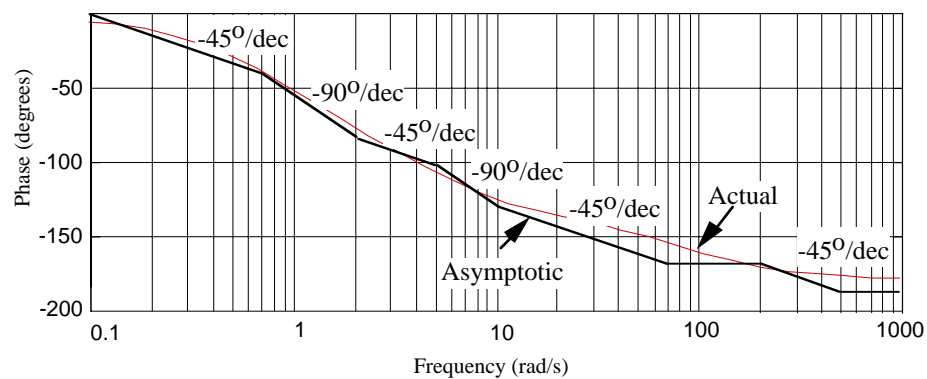
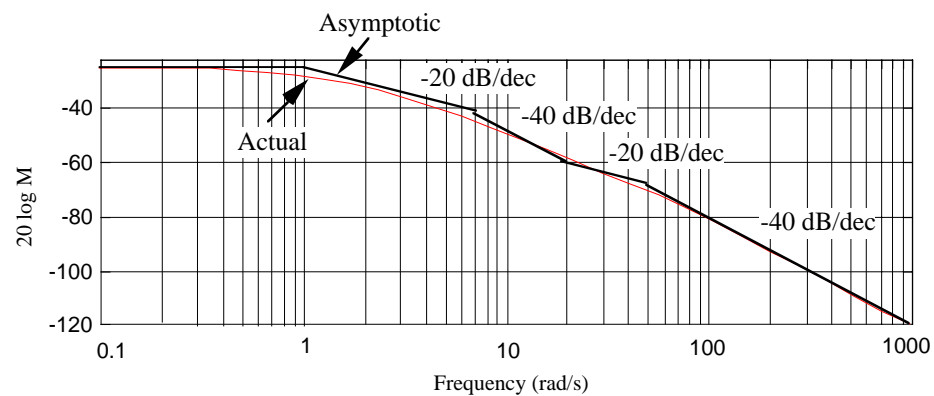
Bode Diagrams



c.

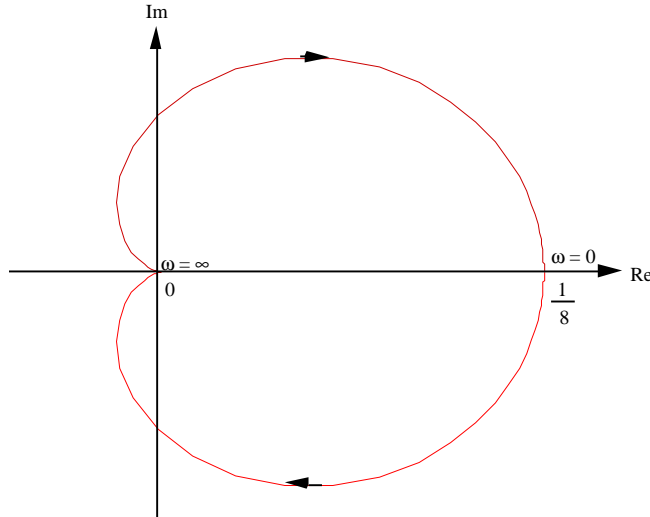


10.2.

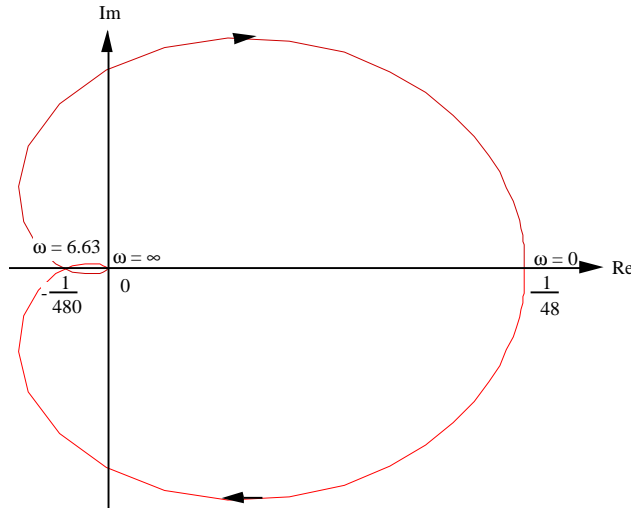


10.3.

The frequency response is $1/8$ at an angle of zero degrees at $\omega = 0$. Each pole rotates 90° in going from $\omega = 0$ to $\omega = \infty$. Thus, the resultant rotates -180° while its magnitude goes to zero. The result is shown below.

**10.4.**

a. The frequency response is $1/48$ at an angle of zero degrees at $\omega = 0$. Each pole rotates 90° in going from $\omega = 0$ to $\omega = \infty$. Thus, the resultant rotates -270° while its magnitude goes to zero. The result is shown below.



b. Substituting $j\omega$ into $G(s) = \frac{1}{(s+2)(s+4)(s+6)} = \frac{1}{s^3 + 12s^2 + 44s + 48}$ and

simplifying, we obtain $G(j\omega) = \frac{(48 - 12\omega^2) - j(44\omega - \omega^3)}{\omega^6 + 56\omega^4 + 784\omega^2 + 2304}$. The Nyquist

diagram crosses the real axis when the imaginary part of $G(j\omega)$ is zero. Thus, the Nyquist diagram crosses the real axis at $\omega^2 = 44$, or $\omega = \sqrt{44} = 6.63$ rad/s. At this frequency $G(j\omega) = -\frac{1}{480}$. Thus, the system is stable for $K < 480$.

10.5.

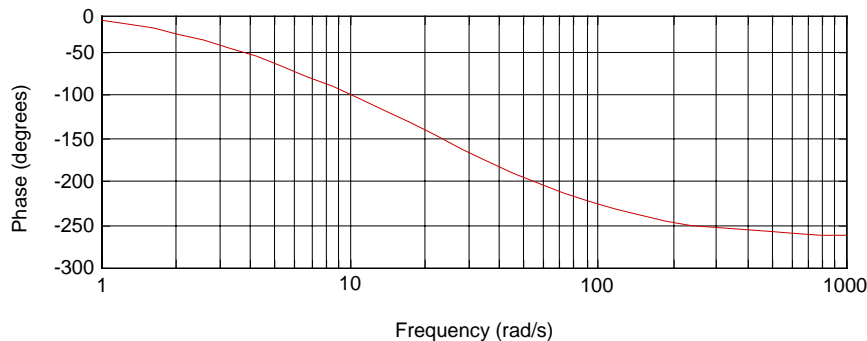
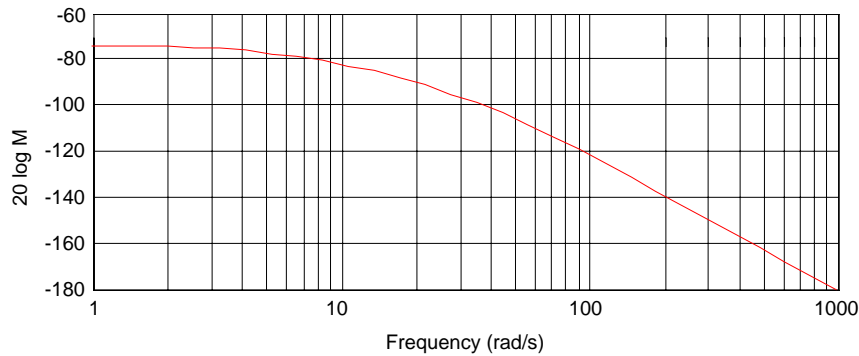
If $K = 100$, the Nyquist diagram will intersect the real axis at $-100/480$. Thus,

$$G_M = 20 \log \frac{480}{100} = 13.62 \text{ dB. From Skill-Assessment Exercise Solution 10.4, the}$$

180° frequency is 6.63 rad/s.

10.6.

a.



b. The phase angle is 180° at a frequency of 36.74 rad/s. At this frequency the gain is -99.67 dB. Therefore, $20 \log K = 99.67$, or $K = 96,270$. We conclude that the system is stable for $K < 96,270$.

c. For $K = 10,000$, the magnitude plot is moved up $20 \log 10,000 = 80$ dB.

Therefore, the gain margin is $99.67 - 80 = 19.67$ dB. The 180° frequency is 36.7

rad/s. The gain curve crosses 0 dB at $\omega = 7.74$ rad/s, where the phase is 87.1° . We calculate the phase margin to be $180^\circ - 87.1^\circ = 92.9^\circ$.

10.7.

Using $\zeta = \frac{-\ln(\% / 100)}{\sqrt{\pi^2 + \ln^2(\% / 100)}}$, we find $\zeta = 0.456$, which corresponds to 20% overshoot. Using $T_s = 2$, $\omega_{BW} = \frac{4}{T_s \zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 5.79$ rad/s.

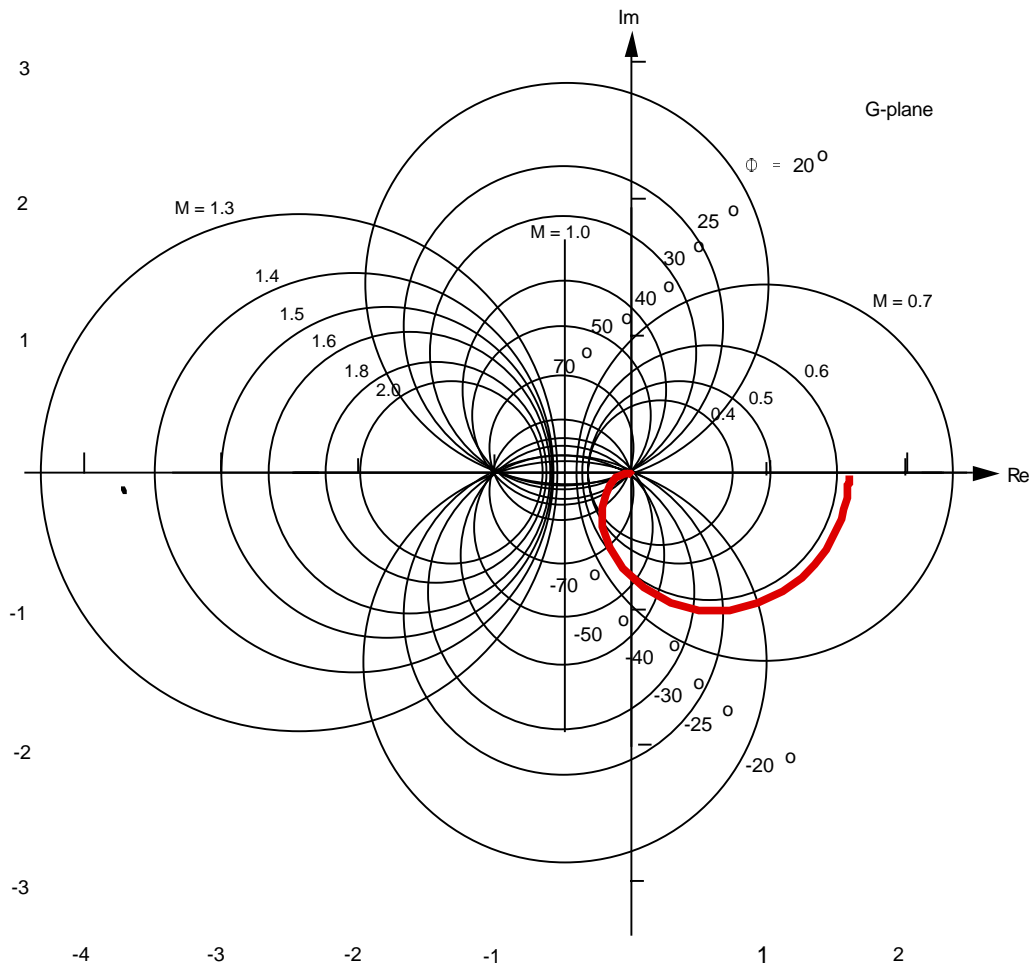
10.8.

For both parts find that

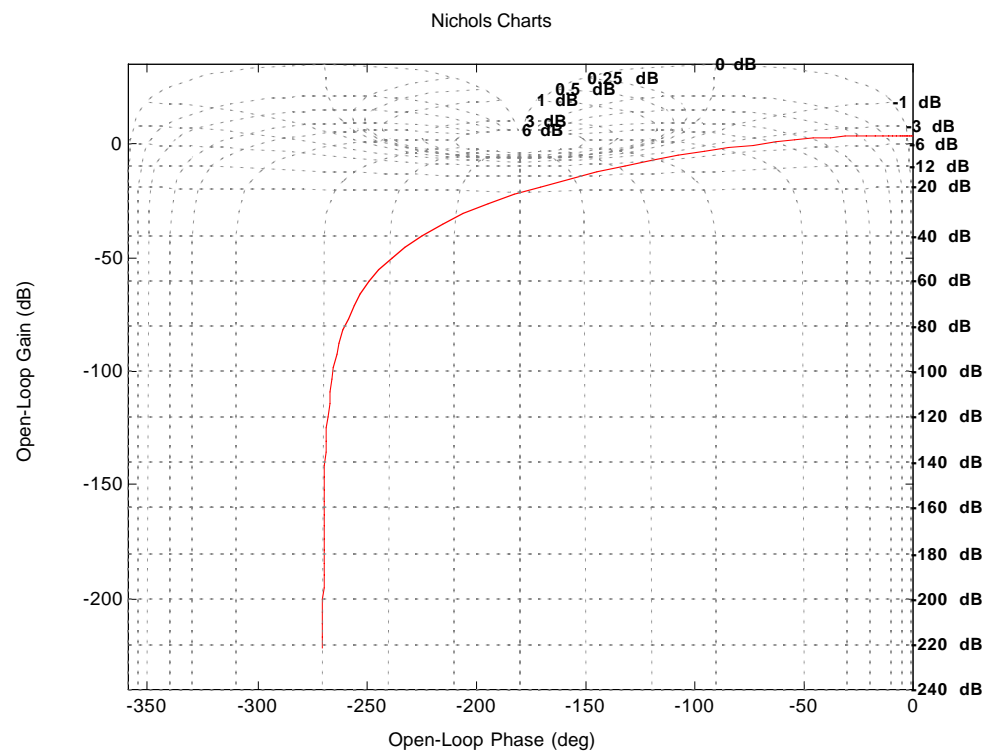
$$G(j\omega) = \frac{160}{27} * \frac{(6750000 - 101250\omega^2) + j1350(\omega^2 - 1350)\omega}{\omega^6 + 2925\omega^4 + 1072500\omega^2 + 250000000}.$$

For a range of values for ω , superimpose $G(j\omega)$ on the **a.** M and N circles, and on the **b.** Nichols chart.

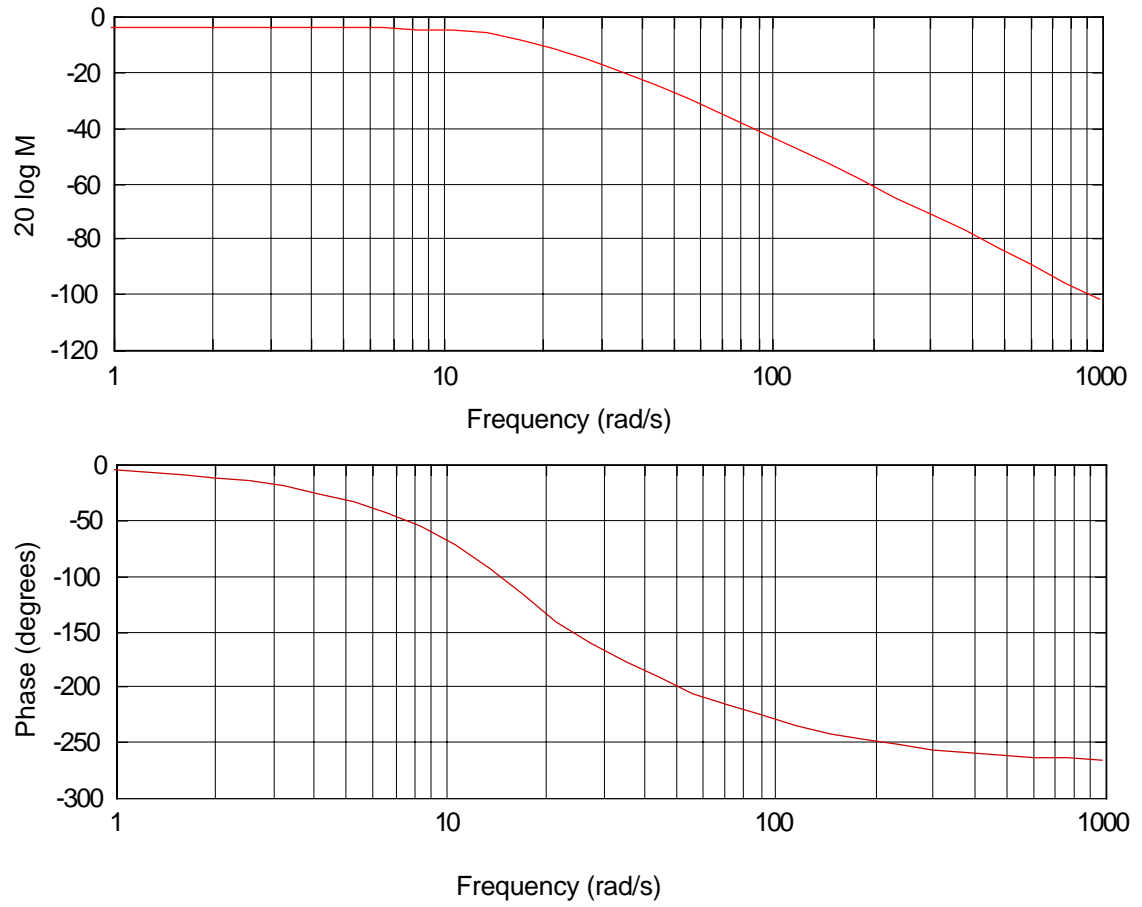
a.



b.

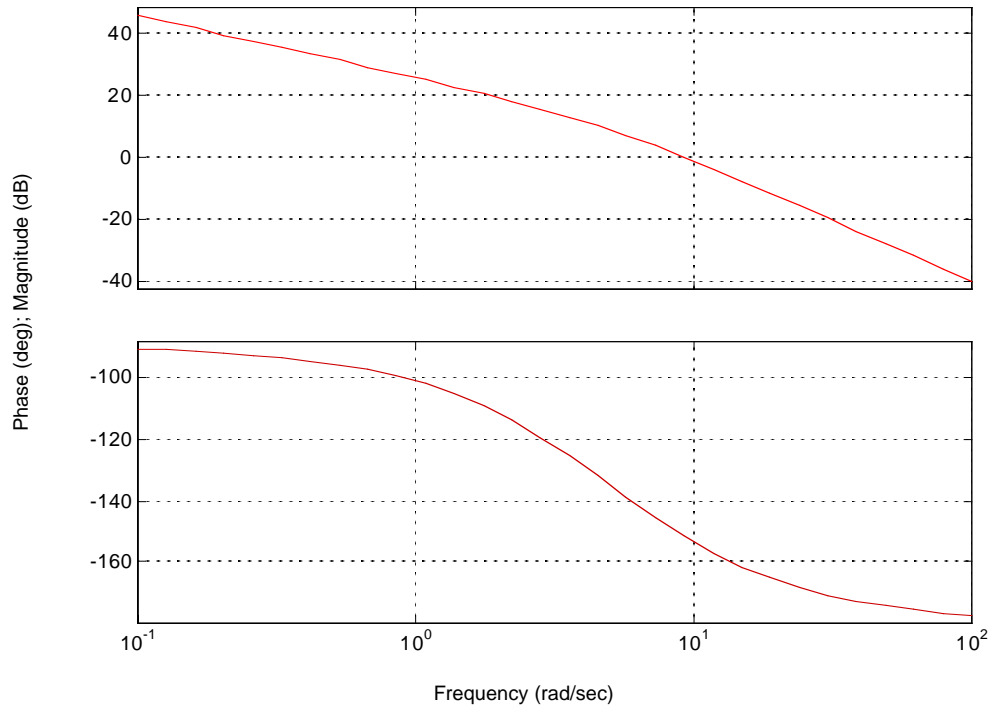


Plotting the closed-loop frequency response from **a.** or **b.** yields the following plot:

**10.9.**

The open-loop frequency response is shown in the following figure:

Bode Diagrams



The open-loop frequency response is -7 at $\omega = 14.5$ rad/s. Thus, the estimated bandwidth is $\omega_{BW} = 14.5$ rad/s. The open-loop frequency response plot goes through zero dB at a frequency of 9.4 rad/s, where the phase is 151.98° . Hence, the phase margin is $180^\circ - 151.98^\circ = 28.02^\circ$. This phase margin corresponds to

$$\zeta = 0.25. \text{ Therefore, } \%OS = e^{-\left(\frac{\zeta\pi}{\sqrt{1-\zeta^2}}\right)} \times 100 = 44.4\%,$$

$$T_s = \frac{4}{\omega_{BW}\zeta} \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 1.64 \text{ s and}$$

$$T_p = \frac{\pi}{\omega_{BW}\sqrt{1-\zeta^2}} \sqrt{(1-2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 0.33 \text{ s}$$

10.10.

The initial slope is 40 dB/dec. Therefore, the system is Type 2. The initial slope intersects 0 dB at $\omega = 9.5$ rad/s. Thus, $K_a = 9.5^2 = 90.25$ and $K_p = K_v = \infty$.

10.11.

a. Without delay, $G(j\omega) = \frac{10}{j\omega(j\omega + 1)} = \frac{10}{\omega(-\omega + j)}$, from which the zero dB

frequency is found as follows: $M = \frac{10}{\omega\sqrt{\omega^2 + 1}} = 1$. Solving for ω ,

$$\omega\sqrt{\omega^2 + 1} = 10, \text{ or after squaring both sides and rearranging, } \omega^4 + \omega^2 - 100 = 0.$$

Solving for the roots, $\omega^2 = -10.51, 9.51$. Taking the square root of the positive root, we find the 0 dB frequency to be 3.08 rad/s. At this frequency, the phase angle, $\phi = -\angle(-\omega + j) = -\angle(-3.08 + j) = -162^\circ$. Therefore the phase margin is $180^\circ - 162^\circ = 18^\circ$.

b. With a delay of 3 s,

$$\phi = -\angle(-\omega + j) - \omega T = -\angle(-3.08 + j) - (3.08)(3) = -162^\circ - 9.24^\circ = -171.24^\circ.$$

Therefore the phase margin is $180^\circ - 171.24^\circ = 8.76^\circ$.

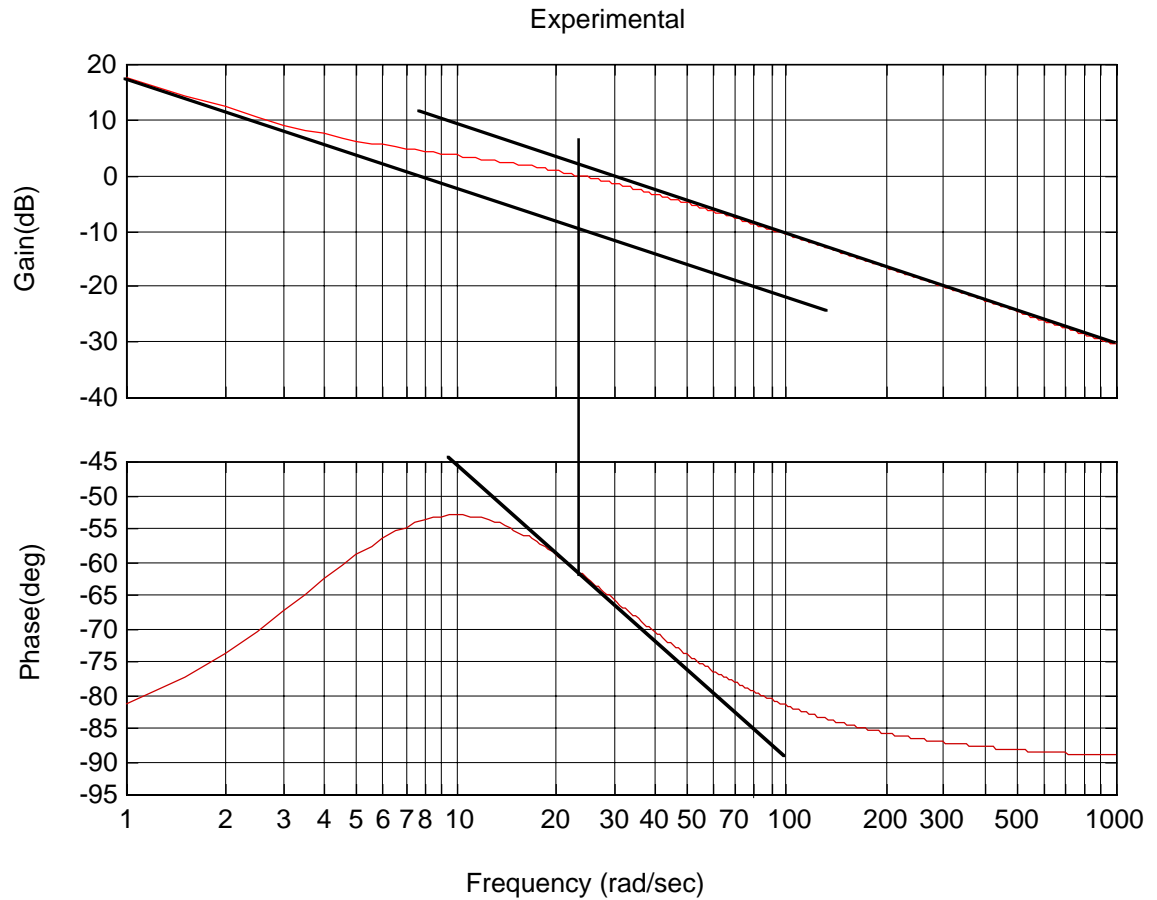
c. With a delay of 7 s,

$$\phi = -\angle(-\omega + j) - \omega T = -\angle(-3.08 + j) - (3.08)(7) = -162^\circ - 21.56^\circ = -183.56^\circ.$$

Therefore the phase margin is $180^\circ - 183.56^\circ = -3.56^\circ$. Thus, the system is unstable.

10.12.

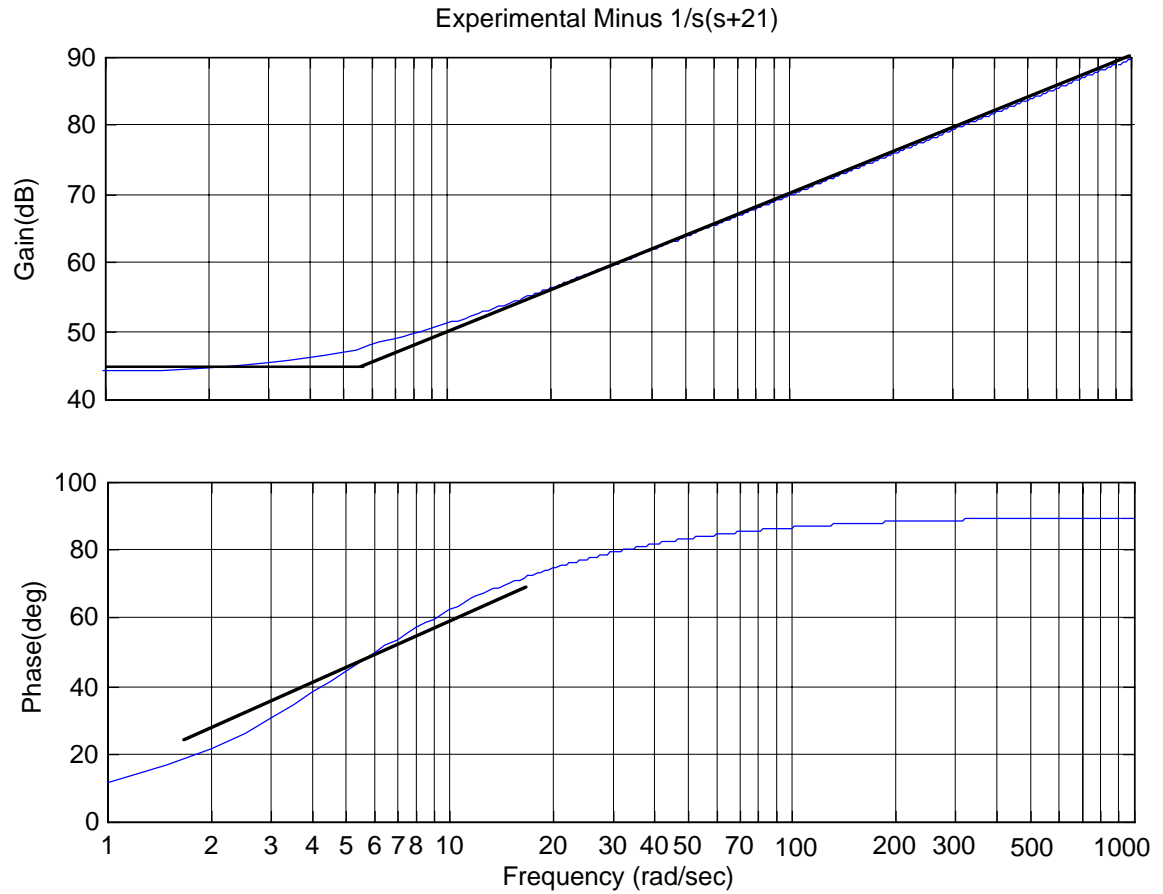
Drawing judiciously selected slopes on the magnitude and phase plot as shown below yields a first estimate.



We see an initial slope on the magnitude plot of -20 dB/dec. We also see a final -20 dB/dec slope with a break frequency around 21 rad/s. Thus, an initial estimate

$$\text{is } G_1(s) = \frac{1}{s(s + 21)}.$$

Subtracting $G_1(s)$ from the original frequency response yields the frequency response shown below.



Drawing judiciously selected slopes on the magnitude and phase plot as shown yields a final estimate. We see first-order zero behavior on the magnitude and phase plots with a break frequency of about 5.7 rad/s and a dc gain of about 44 dB $= 20\log(5.7K)$, or $K = 27.8$. Thus, we estimate $G_2(s) = 27.8(s + 7)$. Thus,

$$G(s) = G_1(s)G_2(s) = \frac{27.8(s + 5.7)}{s(s + 21)}.$$

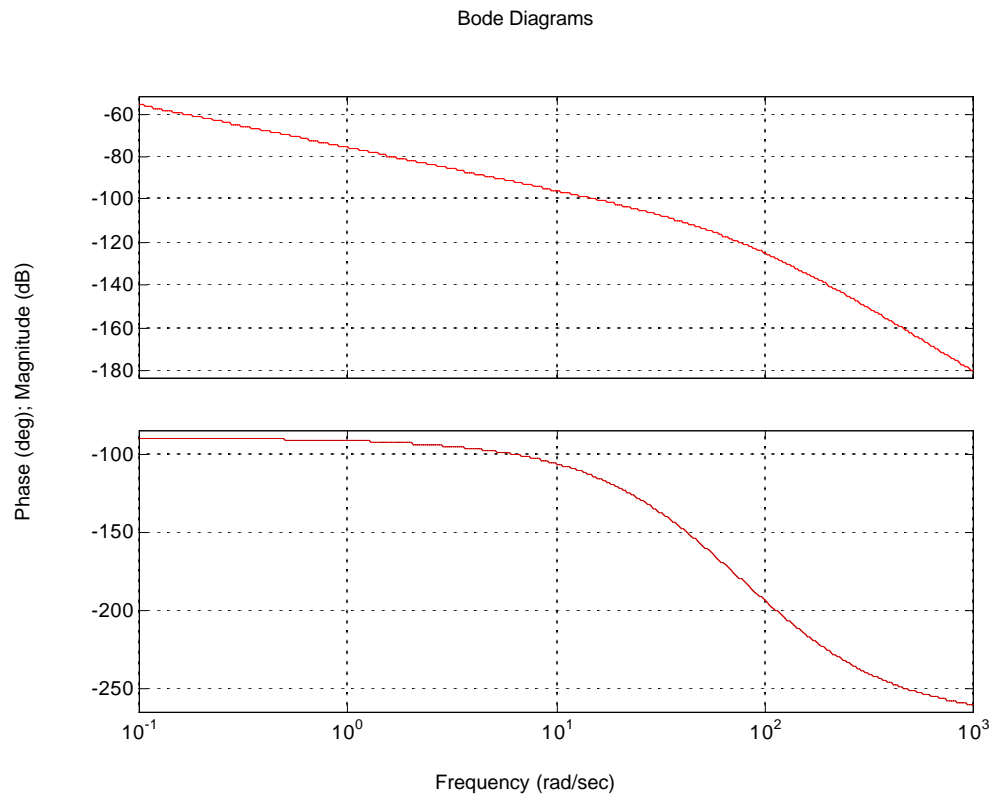
It is interesting to note that the original

problem was developed from $G(s) = \frac{30(s + 5)}{s(s + 20)}$.

Chapter 11

11.1.

The Bode plot for $K = 1$ is shown below.



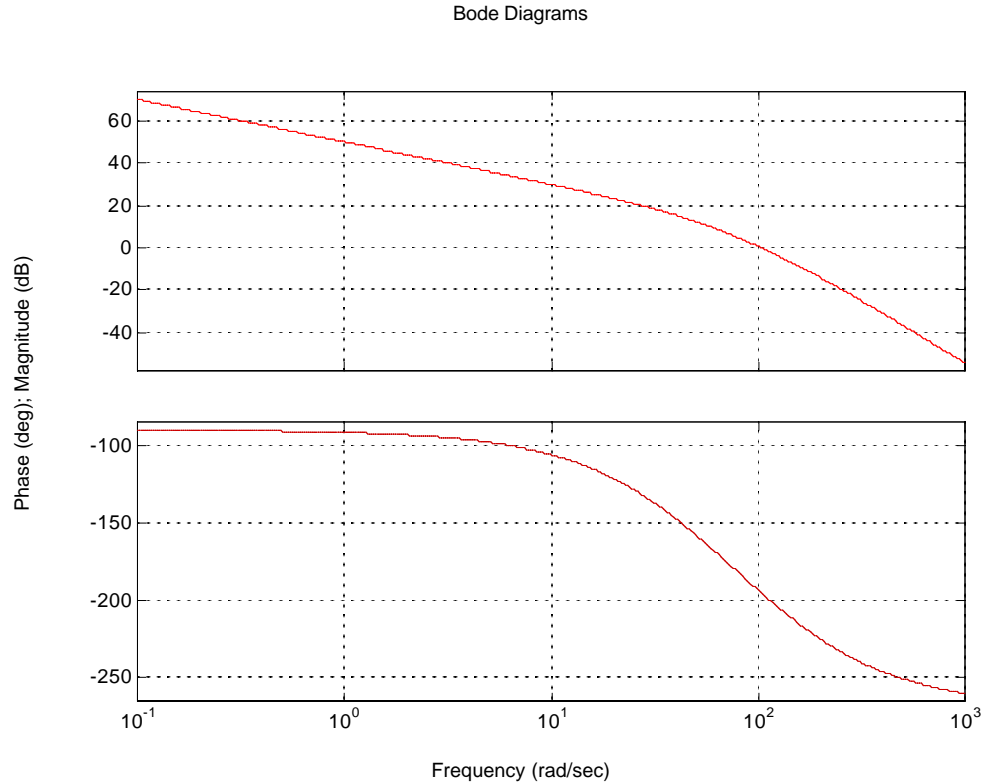
A 20% overshoot requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.456$. This damping ratio

implies a phase margin of 48.10, which is obtained when the $\angle = -180 + 48.10 = 131.9^\circ$. This phase angle occurs at $\omega = 27.6 \text{ rad/s}$. The magnitude at this frequency is 5.15×10^{-6} . Since the magnitude must be

$$\text{unity } K = \frac{1}{5.15 \times 10^{-6}} = 194,200.$$

11.2.

To meet the steady-state error requirement, $K = 1,942,000$. The Bode plot for this gain is shown below.



A 20% overshoot requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.456$. This damping ratio

implies a phase margin of 48.1° . Adding 10° to compensate for the phase angle contribution of the lag, we use 58.1° . Thus, we look for a phase angle of $-180^\circ + 58.1^\circ = -121.9^\circ$. The frequency at which this phase occurs is 20.4 rad/s. At this frequency the magnitude plot must go through zero dB. Presently, the magnitude plot is 23.2 dB. Therefore draw the high frequency asymptote of the lag compensator at -23.2 dB. Insert a break at $0.1(20.4) = 2.04$ rad/s. At this frequency, draw -20 dB/dec slope until it intersects 0 dB. The frequency of intersection will be the low frequency break or 0.141 rad/s. Hence the

compensator is $G_c(s) = K_c \frac{(s + 2.04)}{(s + 0.141)}$, where the gain is chosen to yield 0 dB at

low frequencies, or $K_c = 0.141 / 2.04 = 0.0691$. In summary,

$$G_c(s) = 0.0691 \frac{(s + 2.04)}{(s + 0.141)} \text{ and } G(s) = \frac{1,942,000}{s(s + 50)(s + 120)}.$$

11.3.

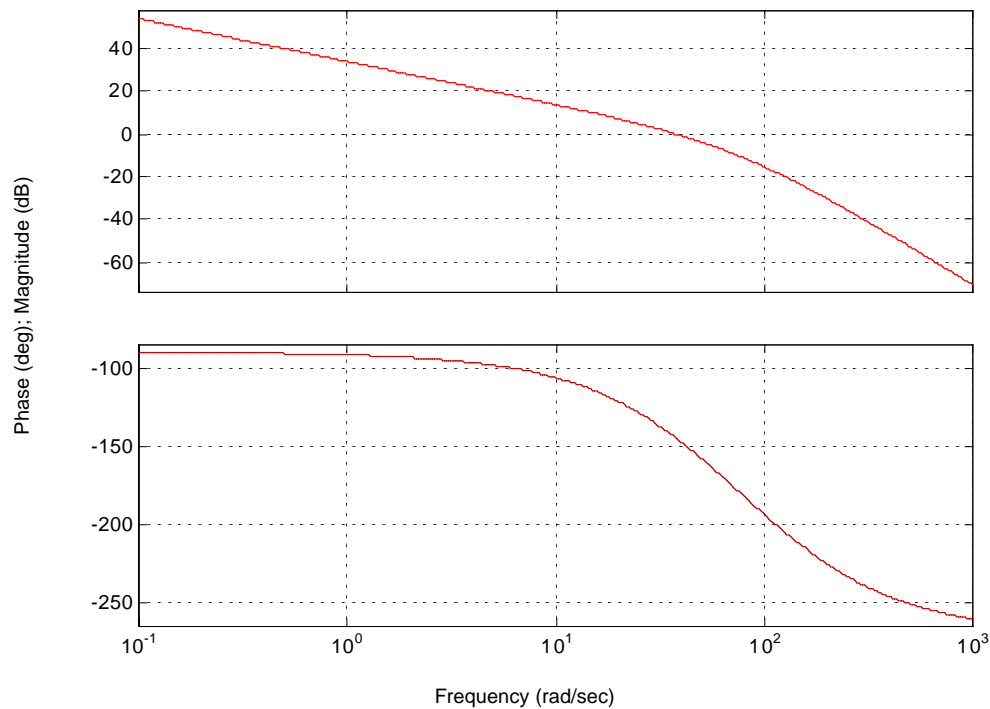
A 20% overshoot requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.456$. The required

bandwidth is then calculated as $\omega_{BW} = \frac{4}{T_s \zeta} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 57.9$

rad/s. In order to meet the steady-state error requirement of $K_v = 50 = \frac{K}{(50)(120)}$,

we calculate $K = 300,000$. The uncompensated Bode plot for this gain is shown below.

Bode Plot for K = 300000



The uncompensated system's phase margin measurement is taken where the magnitude plot crosses 0 dB. We find that when the magnitude plot crosses 0 dB, the phase angle is -144.8° . Therefore, the uncompensated system's phase margin is $-180^\circ + 144.8^\circ = 35.2^\circ$. The required phase margin based on the required damping ratio is $\Phi_M = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} = 48.1^\circ$. Adding a 10° correction factor, the

required phase margin is 58.1° . Hence, the compensator must contribute $\phi_{\max} =$

$58.1^\circ - 35.2^\circ = 22.9^\circ$. Using $\phi_{\max} = \sin^{-1} \frac{1-\beta}{1+\beta}$, $\beta = \frac{1 - \sin \phi_{\max}}{1 + \sin \phi_{\max}} = 0.44$. The

compensator's peak magnitude is calculated as $M_{\max} = \frac{1}{\sqrt{\beta}} = 1.51$. Now find the

frequency at which the uncompensated system has a magnitude $1/M_{\max}$, or -3.58 dB. From the Bode plot, this magnitude occurs at $\omega_{\max} = 50$ rad/s. The

compensator's zero is at $z_c = \frac{1}{T}$. But, $\omega_{\max} = \frac{1}{T\sqrt{\beta}}$. Therefore, $z_c = 33.2$. The

compensator's pole is at $p_c = \frac{1}{\beta T} = \frac{z_c}{\beta} = 75.4$. The compensator gain is chosen to

yield unity gain at dc. Hence, $K_c = 75.4 / 33.2 = 2.27$. Summarizing,

$$G_c(s) = 2.27 \frac{(s + 33.2)}{(s + 75.4)}, \text{ and } G(s) = \frac{300,000}{s(s + 50)(s + 120)}.$$

11.4.

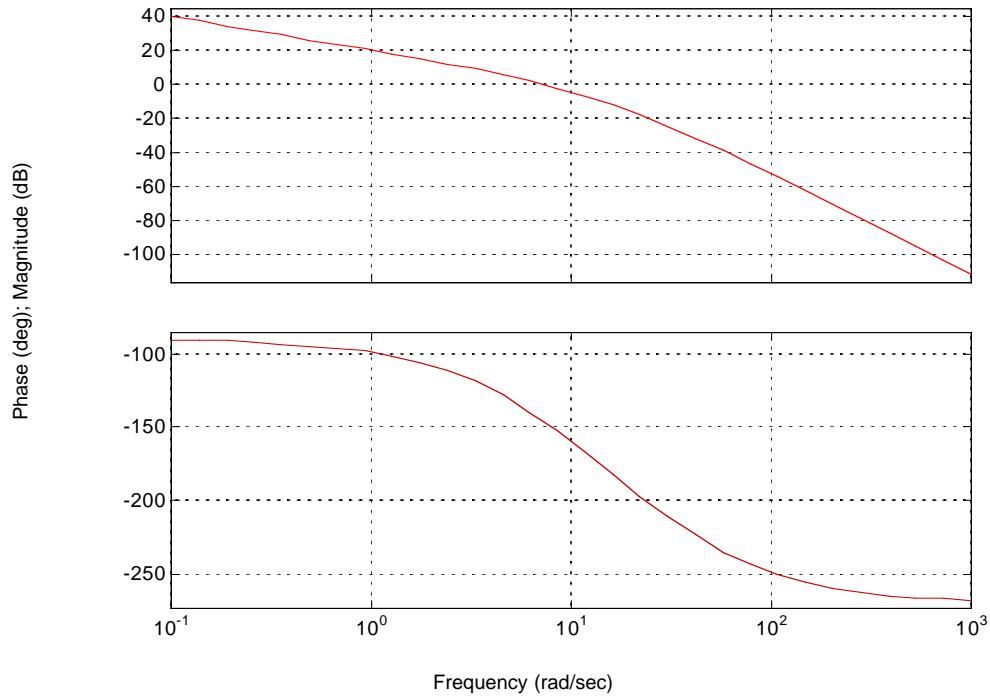
A 10% overshoot requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.591$. The required bandwidth

is then calculated as $\omega_{BW} = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} \sqrt{(1 - 2\zeta^2) + \sqrt{4\zeta^4 - 4\zeta^2 + 2}} = 7.53$ rad/s.

In order to meet the steady-state error requirement of $K_v = 10 = \frac{K}{(8)(30)}$, we

calculate $K = 2400$. The uncompensated Bode plot for this gain is shown below.

Bode Diagrams



Let us select a new phase-margin frequency at $0.8\omega_{BW} = 6.02$ rad/s. The required phase margin based on the required damping ratio

is $\Phi_M = \tan^{-1} \frac{2\zeta}{\sqrt{-2\zeta^2 + \sqrt{1 + 4\zeta^4}}} = 58.6^\circ$. Adding a 5° correction factor, the

required phase margin is 63.6° . At 6.02 rad/s, the new phase-margin frequency, the phase angle is -138.3° which represents a phase margin of $180^\circ - 138.3^\circ = 41.7^\circ$.

Thus, the lead compensator must contribute $\phi_{\max} = 63.6^\circ - 41.7^\circ = 21.9^\circ$. Using

$$\phi_{\max} = \sin^{-1} \frac{1 - \beta}{1 + \beta}, \beta = \frac{1 - \sin \phi_{\max}}{1 + \sin \phi_{\max}} = 0.456.$$

We now design the lag compensator by first choosing its higher break frequency one decade below the new phase-margin frequency, that is, $z_{lag} = 0.602$ rad/s. The lag compensator's pole is $p_{lag} = \beta z_{lag} = 0.275$. Finally, the lag compensator's gain is $K_{lag} = \beta = 0.456$.

Now we design the lead compensator. The lead zero is the product of the new phase margin frequency and $\sqrt{\beta}$, or $z_{lead} = 0.8\omega_{BW}\sqrt{\beta} = 4.07$. Also,

$$p_{lead} = \frac{z_{lead}}{\beta} = 8.93. \text{ Finally, } K_{lead} = \frac{1}{\beta} = 2.19. \text{ Summarizing,}$$

$$G_{lag}(s) = 0.456 \frac{(s + 0.602)}{(s + 0.275)}; \quad G_{lead}(s) = 2.19 \frac{(s + 4.07)}{(s + 8.93)}; \text{ and } K = 2400.$$

Chapter 12

12.1.

We first find the desired characteristic equation. A 5% overshoot

requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.69$. Also, $\omega_n = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} = 14.47$ rad/s. Thus, the

characteristic equation is $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 19.97s + 209.4$. Adding a pole at -10 to cancel the zero at -10 yields the desired characteristic equation,

$(s^2 + 19.97s + 209.4)(s + 10) = s^3 + 29.97s^2 + 409.1s + 2094$. The compensated system

matrix in phase-variable form is $\mathbf{A} - \mathbf{BK} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(k_1) & -(36 + k_2) & -(15 + k_3) \end{bmatrix}$. The

characteristic equation for this system is

$|s\mathbf{I} - (\mathbf{A} - \mathbf{BK})| = s^3 + (15 + k_3)s^2 + (36 + k_2)s + (k_1)$. Equating coefficients of this equation with the coefficients of the desired characteristic equation yields the gains as $\mathbf{K} = [k_1 \ k_2 \ k_3] = [2094 \ 373.1 \ 14.97]$.

12.2.

The controllability matrix is $\mathbf{C}_M = [\mathbf{B} \ \mathbf{AB} \ \mathbf{A}^2\mathbf{B}] = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 4 & -9 \\ 1 & -1 & 16 \end{bmatrix}$. Since $|\mathbf{C}_M| = 80$,

\mathbf{C}_M is full rank, that is, rank 3. We conclude that the system is controllable.

12.3.

First check controllability. The controllability matrix is

$\mathbf{C}_{M_z} = [\mathbf{B} \ \mathbf{AB} \ \mathbf{A}^2\mathbf{B}] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -17 \\ 1 & -9 & 81 \end{bmatrix}$. Since $|\mathbf{C}_{M_z}| = -1$, \mathbf{C}_{M_z} is full rank, that is, rank

3. We conclude that the system is controllable.

We now find the desired characteristic equation. A 20% overshoot

requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.456$. Also, $\omega_n = \frac{4}{\zeta T_s} = 4.386$ rad/s. Thus, the

characteristic equation is $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 4s + 19.24$. Adding a pole at -6 to cancel the zero at -6 yields the resulting desired characteristic equation,

$$(s^2 + 4s + 19.24)(s + 6) = s^3 + 10s^2 + 43.24s + 115.45.$$

Since $G(s) = \frac{(s + 6)}{(s + 7)(s + 8)(s + 9)} = \frac{s + 6}{s^3 + 24s^2 + 191s + 504}$, we can write the phase-

variable representation as $\mathbf{A}_p = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -504 & -191 & -24 \end{bmatrix}$; $\mathbf{B}_p = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$; $\mathbf{C}_p = [6 \quad 1 \quad 0]$.

The compensated system matrix in phase-variable form is

$\mathbf{A}_p - \mathbf{B}_p \mathbf{K}_p = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -(504 + k_1) & -(191 + k_2) & -(24 + k_3) \end{bmatrix}$. The characteristic equation for

this system is $|s\mathbf{I} - (\mathbf{A}_p - \mathbf{B}_p \mathbf{K}_p)| = s^3 + (24 + k_3)s^2 + (191 + k_2)s + (504 + k_1)$. Equating coefficients of this equation with the coefficients of the desired characteristic equation yields the gains as $\mathbf{K}_p = [k_1 \quad k_2 \quad k_3] = [-388.55 \quad -147.76 \quad -14]$.

We now develop the transformation matrix to transform back to the z -system.

$\mathbf{C}_{Mz} = [\mathbf{B}_z \quad \mathbf{A}_z \mathbf{B}_z \quad \mathbf{A}_z^2 \mathbf{B}_z] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -17 \\ 1 & -9 & 81 \end{bmatrix}$ and

$\mathbf{C}_{Mp} = [\mathbf{B}_p \quad \mathbf{A}_p \mathbf{B}_p \quad \mathbf{A}_p^2 \mathbf{B}_p] = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -24 \\ 1 & -24 & 385 \end{bmatrix}$.

Therefore, $\mathbf{P} = \mathbf{C}_{Mz} \mathbf{C}_{Mp}^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & -17 \\ 1 & -9 & 81 \end{bmatrix} \begin{bmatrix} 191 & 24 & 1 \\ 24 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 7 & 1 & 0 \\ 56 & 15 & 1 \end{bmatrix}$

Hence,

$$\mathbf{K}_z = \mathbf{K}_p \mathbf{P}^{-1} = \begin{bmatrix} -388.55 & -147.76 & -14 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 49 & -15 & 1 \end{bmatrix} = \begin{bmatrix} -40.23 & 62.24 & -14 \end{bmatrix}.$$

12.4.

For the given system $\dot{\mathbf{e}}_x = (\mathbf{A} - \mathbf{LC})\mathbf{e}_x = \begin{bmatrix} -(24 + l_1) & 1 & 0 \\ -(191 + l_2) & 0 & 1 \\ -(504 + l_3) & 0 & 0 \end{bmatrix} \mathbf{e}_x$. The characteristic

polynomial is given by $|\mathbf{sI} - (\mathbf{A} - \mathbf{LC})| = s^3 + (24 + l_1)s^2 + (191 + l_2)s + (504 + l_3)$. Now we find the desired characteristic equation. The dominant poles from Skill-Assessment Exercise 12.3 come from $(s^2 + 4s + 19.24)$. Factoring yields $(-2 + j3.9)$ and $(-2 - j3.9)$. Increasing these poles by a factor of 10 and adding a third pole 10 times the real part of the dominant second-order poles yields the desired characteristic polynomial, $(s + 20 + j39)(s + 20 - j39)(s + 200) = s^3 + 240s^2 + 9921s + 384200$. Equating coefficients of the desired characteristic equation to the system's characteristic

equation yields $\mathbf{L} = \begin{bmatrix} 216 \\ 9730 \\ 383696 \end{bmatrix}$.

12.5.

The observability matrix is $\mathbf{O}_M = \begin{bmatrix} \mathbf{C} \\ \mathbf{CA} \\ \mathbf{CA}^2 \end{bmatrix} = \begin{bmatrix} 4 & 6 & 8 \\ -64 & -80 & -78 \\ 674 & 848 & 814 \end{bmatrix}$, where

$\mathbf{A}^2 = \begin{bmatrix} 25 & 28 & 32 \\ -7 & -4 & -11 \\ 77 & 95 & 94 \end{bmatrix}$. The matrix is of full rank, that is, rank 3, since $|\mathbf{O}_M| = -1576$.

Therefore the system is observable.

12.6.

The system is represented in cascade form by the following state and output equations:

$$\dot{\mathbf{z}} = \begin{bmatrix} -7 & 1 & 0 \\ 0 & -8 & 1 \\ 0 & 0 & -9 \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \mathbf{z}$$

The observability matrix is $\mathbf{O}_{Mz} = \begin{bmatrix} \mathbf{C}_z \\ \mathbf{C}_z \mathbf{A}_z \\ \mathbf{C}_z \mathbf{A}_z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 49 & -15 & 1 \end{bmatrix}$, where

$$\mathbf{A}_z^2 = \begin{bmatrix} 49 & -15 & 1 \\ 0 & 64 & -17 \\ 0 & 0 & 81 \end{bmatrix}. \text{ Since } G(s) = \frac{1}{(s+7)(s+8)(s+9)} = \frac{1}{s^3 + 24s^2 + 191s + 504}, \text{ we}$$

can write the observable canonical form as

$$\dot{\mathbf{x}} = \begin{bmatrix} -24 & 1 & 0 \\ -191 & 0 & 1 \\ -504 & 0 & 0 \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u$$

$$y = [1 \quad 0 \quad 0] \mathbf{x}$$

The observability matrix for this form is $\mathbf{O}_{Mx} = \begin{bmatrix} \mathbf{C}_x \\ \mathbf{C}_x \mathbf{A}_x \\ \mathbf{C}_x \mathbf{A}_x^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -24 & 1 & 0 \\ 385 & -24 & 1 \end{bmatrix}$, where

$$\mathbf{A}_x^2 = \begin{bmatrix} 385 & -24 & 1 \\ 4080 & -191 & 0 \\ 12096 & -504 & 0 \end{bmatrix}.$$

We next find the desired characteristic equation. A 10% overshoot

requires $\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.591$. Also, $\omega_n = \frac{4}{\zeta T_s} = 67.66$ rad/s. Thus, the

characteristic equation is $s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 80s + 4578.42$. Adding a pole at -400 , or 10 times the real part of the dominant second-order poles, yields the resulting desired characteristic equation,

$$(s^2 + 80s + 4578.42)(s + 400) = s^3 + 480s^2 + 36580s + 1.831 \times 10^6.$$

For the system represented in observable canonical form

$$\dot{\mathbf{e}}_x = (\mathbf{A}_x - \mathbf{L}_x \mathbf{C}_x) \mathbf{e}_x = \begin{bmatrix} -(24 + l_1) & 1 & 0 \\ -(191 + l_2) & 0 & 1 \\ -(504 + l_3) & 0 & 0 \end{bmatrix} \mathbf{e}_x. \text{ The characteristic polynomial is given}$$

by $|s\mathbf{I} - (\mathbf{A}_x - \mathbf{L}_x \mathbf{C}_x)| = s^3 + (24 + l_1)s^2 + (191 + l_2)s + (504 + l_3)$. Equating coefficients of the desired characteristic equation to the system's characteristic equation yields

$$\mathbf{L}_x = \begin{bmatrix} 456 \\ 36,389 \\ 1,830,496 \end{bmatrix}.$$

Now, develop the transformation matrix between the observer canonical and cascade forms.

$$\begin{aligned} \mathbf{P} = \mathbf{O}_{Mz}^{-1} \mathbf{O}_{Mx} &= \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 49 & -15 & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 & 0 \\ -24 & 1 & 0 \\ 385 & -24 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 7 & 1 & 0 \\ 56 & 15 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -24 & 1 & 0 \\ 385 & -24 & 1 \end{bmatrix} \\ &= \begin{bmatrix} 1 & 0 & 0 \\ -17 & 1 & 0 \\ 81 & -9 & 1 \end{bmatrix}. \end{aligned}$$

$$\text{Finally, } \mathbf{L}_z = \mathbf{P} \mathbf{L}_x = \begin{bmatrix} 1 & 0 & 0 \\ -17 & 1 & 0 \\ 81 & -9 & 1 \end{bmatrix} \begin{bmatrix} 456 \\ 36,389 \\ 1,830,496 \end{bmatrix} = \begin{bmatrix} 456 \\ 28,637 \\ 1,539,931 \end{bmatrix} \approx \begin{bmatrix} 456 \\ 28,640 \\ 1,540,000 \end{bmatrix}.$$

12.7.

We first find the desired characteristic equation. A 10% overshoot requires

$$\zeta = \frac{-\log\left(\frac{\%}{100}\right)}{\sqrt{\pi^2 + \log^2\left(\frac{\%}{100}\right)}} = 0.591$$

Also, $\omega_n = \frac{\pi}{T_p \sqrt{1 - \zeta^2}} = 1.948 \text{ rad/s}$. Thus, the characteristic equation is

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 2.3s + 3.79. \text{ Adding a pole at } -4, \text{ which corresponds to the original system's zero location, yields the resulting desired characteristic equation, } (s^2 + 2.3s + 3.79)(s + 4) = s^3 + 6.3s^2 + 13s + 15.16.$$

$$\text{Now, } \begin{bmatrix} \dot{\mathbf{x}} \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} (\mathbf{A} - \mathbf{BK}) & \mathbf{BK}_e \\ -\mathbf{C} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ 1 \end{bmatrix} r; \text{ and } y = [\mathbf{C} \quad 0] \begin{bmatrix} \mathbf{x} \\ x_N \end{bmatrix},$$

where

$$\mathbf{A} - \mathbf{BK} = \begin{bmatrix} 0 & 1 \\ -7 & -9 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -7 & -9 \end{bmatrix} - \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(7 + k_1) & -(9 + k_2) \end{bmatrix}$$

$$\mathbf{C} = [4 \quad 1]$$

$$\mathbf{B}k_e = \begin{bmatrix} 0 \\ 1 \end{bmatrix} k_e = \begin{bmatrix} 0 \\ k_e \end{bmatrix}$$

$$\text{Thus, } \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_N \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -(7+k_1) & -(9+k_2) & k_e \\ -4 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r; \quad y = \begin{bmatrix} 4 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_N \end{bmatrix}.$$

Finding the characteristic equation of this system yields

$$\begin{aligned} \left| s\mathbf{I} - \begin{bmatrix} (\mathbf{A} - \mathbf{BK}) & \mathbf{BK}_e \\ -\mathbf{C} & 0 \end{bmatrix} \right| &= \left| \begin{bmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ -(7+k_1) & -(9+k_2) & k_e \\ -4 & -1 & 0 \end{bmatrix} \right| \\ &= \left| \begin{bmatrix} s & -1 & 0 \\ (7+k_1) & s+(9+k_2) & -k_e \\ 4 & 1 & s \end{bmatrix} \right| = s^3 + (9+k_2)s^2 + (7+k_1+k_e)s + 4k_e \end{aligned}$$

Equating this polynomial to the desired characteristic equation,

$$s^3 + 6.3s^2 + 13s + 15.16 = s^3 + (9+k_2)s^2 + (7+k_1+k_e)s + 4k_e$$

Solving for the k 's,

$$\mathbf{K} = \begin{bmatrix} 2.21 & -2.7 \end{bmatrix} \quad \text{and} \quad k_e = 3.79.$$

Chapter 13

13.1.

$$f(t) = \sin(\omega kT); f^*(t) = \sum_{k=0}^{\infty} \sin(\omega kT) \delta(t - kT);$$

$$F^*(s) = \sum_{k=0}^{\infty} \sin(\omega kT) e^{-kTs} = \sum_{k=0}^{\infty} \frac{(e^{j\omega kT} - e^{-j\omega kT}) e^{-kTs}}{2j} = \frac{1}{2j} \sum_{k=0}^{\infty} (e^{T(s-j\omega)})^{-k} - (e^{T(s+j\omega)})^{-k}$$

$$\text{But, } \sum_{k=0}^{\infty} x^{-k} = \frac{1}{1-x^{-1}}$$

Thus,

$$\begin{aligned} F^*(s) &= \frac{1}{2j} \left[\frac{1}{1 - e^{-T(s-j\omega)}} - \frac{1}{1 - e^{-T(s+j\omega)}} \right] = \frac{1}{2j} \left[\frac{e^{-Ts} e^{j\omega T} - e^{-Ts} e^{-j\omega T}}{1 - (e^{-Ts} e^{j\omega T} - e^{-Ts} e^{-j\omega T}) + e^{-2Ts}} \right] \\ &= e^{-Ts} \left[\frac{\sin(\omega T)}{1 - e^{-Ts} 2 \cos(\omega T) + e^{-2Ts}} \right] = \frac{z^{-1} \sin(\omega T)}{1 - 2z^{-1} \cos(\omega T) + z^{-2}} \end{aligned}$$

13.2.

$$F(z) = \frac{z(z+1)(z+2)}{(z-0.5)(z-0.7)(z-0.9)}$$

$$\frac{F(z)}{z} = \frac{(z+1)(z+2)}{(z-0.5)(z-0.7)(z-0.9)}$$

$$= 46.875 \frac{1}{z-0.5} - 114.75 \frac{1}{z-0.7} + 68.875 \frac{1}{z-0.9}$$

$$F(z) = 46.875 \frac{z}{z-0.5} - 114.75 \frac{z}{z-0.7} + 68.875 \frac{z}{z-0.9},$$

$$f(kT) = 46.875(0.5)^k - 114.75(0.7)^k + 68.875(0.9)^k$$

13.3.

$$\text{Since } G(s) = (1 - e^{-Ts}) \frac{8}{s(s+4)},$$

$$G(z) = (1 - z^{-1}) z \left\{ \frac{8}{s(s+4)} \right\} = \frac{z-1}{z} z \left\{ \frac{A}{s} + \frac{B}{s+4} \right\} = \frac{z-1}{z} z \left\{ \frac{2}{s} + \frac{2}{s+4} \right\}.$$

$$\text{Let } G_2(s) = \frac{2}{s} + \frac{2}{s+4}. \text{ Therefore, } g_2(t) = 2 - 2e^{-4t}, \text{ or } g_2(kT) = 2 - 2e^{-4kT}.$$

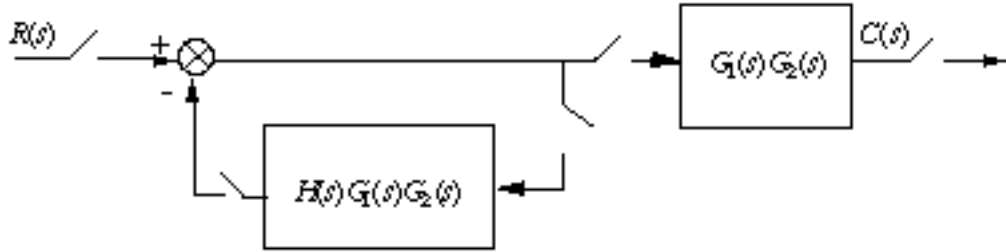
$$\text{Hence, } G_2(z) = \frac{2z}{z-1} - \frac{2z}{z - e^{-4T}} = \frac{2z(1 - e^{-4T})}{(z-1)(z - e^{-4T})}.$$

Therefore, $G(z) = \frac{z-1}{z} G_2(z) = \frac{2(1-e^{-4T})}{(z-e^{-4T})}$.

For $T = \frac{1}{4}$ s, $G(z) = \frac{1.264}{z-0.3679}$.

13.4.

Add phantom samplers to the input, feedback after $H(s)$, and to the output. Push $G_1(s)G_2(s)$, along with its input sampler, to the right past the pickoff point and obtain the block diagram shown below.



Hence, $T(z) = \frac{G_1 G_2(z)}{1 + H G_1 G_2(z)}$.

13.5.

Let $G(s) = \frac{20}{s+5}$. Let $G_2(s) = \frac{G(s)}{s} = \frac{20}{s(s+5)} = \frac{4}{s} - \frac{4}{s+5}$. Taking the inverse

Laplace transform and letting $t = kT$, $g_2(kT) = 4 - 4e^{-5kT}$. Taking the z-transform

yields $G_2(z) = \frac{4z}{z-1} - \frac{4z}{z-e^{-5T}} = \frac{4z(1-e^{-5T})}{(z-1)(z-e^{-5T})}$.

Now, $G(z) = \frac{z-1}{z} G_2(z) = \frac{4(1-e^{-5T})}{(z-e^{-5T})}$. Finally, $T(z) = \frac{G(z)}{1+G(z)} = \frac{4(1-e^{-5T})}{z-5e^{-5T}+4}$.

The pole of the closed-loop system is at $5e^{-5T} - 4$. Substituting values of T , we find that the pole is greater than 1 if $T > 0.1022$ s. Hence, the system is stable for $0 < T < 0.1022$ s.

13.6.

Substituting $z = \frac{s+1}{s-1}$ into $D(z) = z^3 - z^2 - 0.5z + 0.3$, we obtain

$D(s) = s^3 - 8s^2 - 27s - 6$. The Routh table for this polynomial is shown below.

s^3	1	-27
s^2	-8	-6
s^1	-27.75	0
s^0	-6	0

Since there is one sign change, we conclude that the system has one pole outside the unit circle and two poles inside the unit circle. The table did not produce a row of zeros and thus, there are no $j\omega$ poles. The system is unstable because of the pole outside the unit circle.

13.7.

Defining $G(s)$ as $G_1(s)$ in cascade with a zero-order-hold,

$$G(s) = 20(1 - e^{-Ts}) \left[\frac{(s+3)}{s(s+4)(s+5)} \right] = 20(1 - e^{-Ts}) \left[\frac{3/20}{s} + \frac{1/4}{(s+4)} - \frac{2/5}{(s+5)} \right].$$

Taking the z-transform yields

$$G(z) = 20(1 - z^{-1}) \left[\frac{(3/20)z}{z-1} + \frac{(1/4)z}{z - e^{-4T}} - \frac{(2/5)z}{z - e^{-5T}} \right] = 3 + \frac{5(z-1)}{z - e^{-4T}} - \frac{8(z-1)}{z - e^{-5T}}.$$

Hence for $T = 0.1$ second, $K_p = \lim_{z \rightarrow 1} G(z) = 3$, $K_v = \frac{1}{T} \lim_{z \rightarrow 1} (z-1)G(z) = 0$, and

$K_a = \frac{1}{T^2} \lim_{z \rightarrow 1} (z-1)^2 G(z) = 0$. Checking for stability, we find that the system is

stable for $T = 0.1$ second, since $T(z) = \frac{G(z)}{1 + G(z)} = \frac{1.5z - 1.109}{z^2 + 0.222z - 0.703}$ has poles

inside the unit circle at -0.957 and $+0.735$.

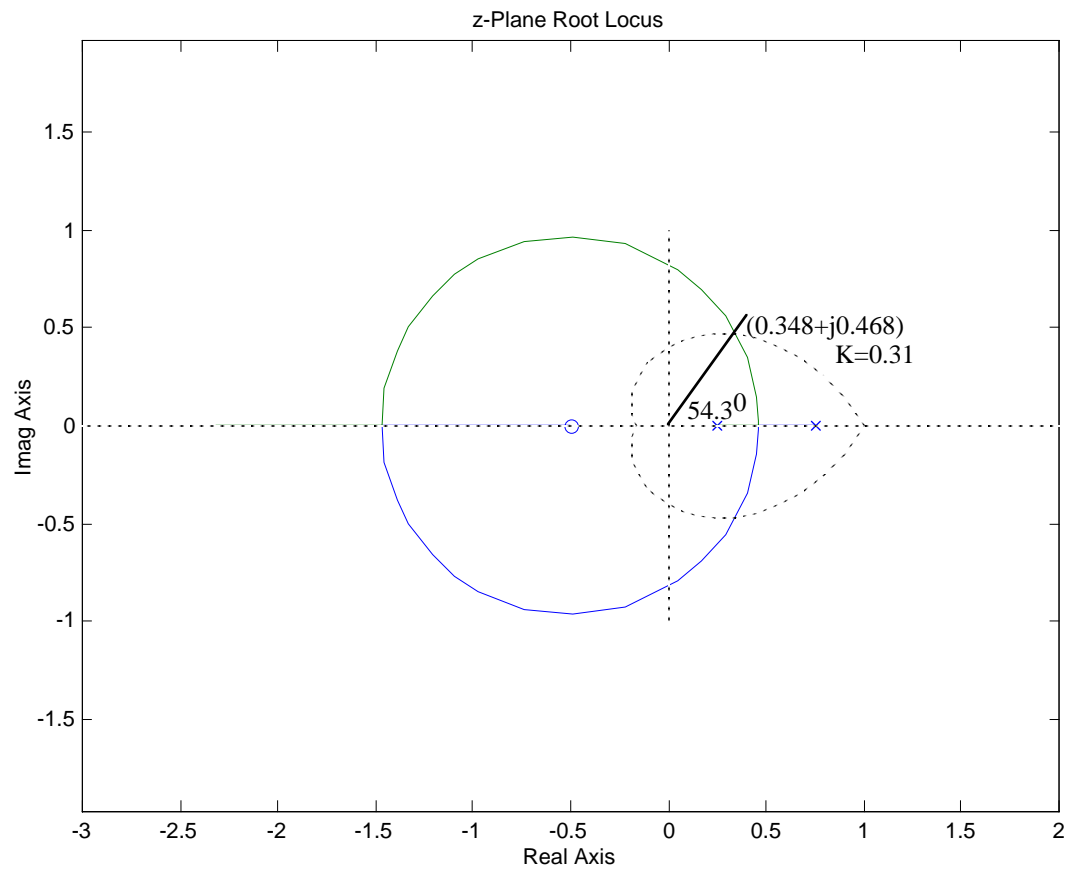
Again, checking for stability, we find that the system is unstable for $T = 0.5$

second, since $T(z) = \frac{G(z)}{1 + G(z)} = \frac{3.02z - 0.6383}{z^2 + 2.802z - 0.6272}$ has poles inside and outside

the unit circle at $+0.208$ and -3.01 , respectively.

13.8.

Draw the root locus superimposed over the $\zeta = 0.5$ curve shown below. Searching along a 54.3° line, which intersects the root locus and the $\zeta = 0.5$ curve, we find the point $0.587 \angle 54.3^\circ = (0.348 + j0.468)$ and $K = 0.31$.

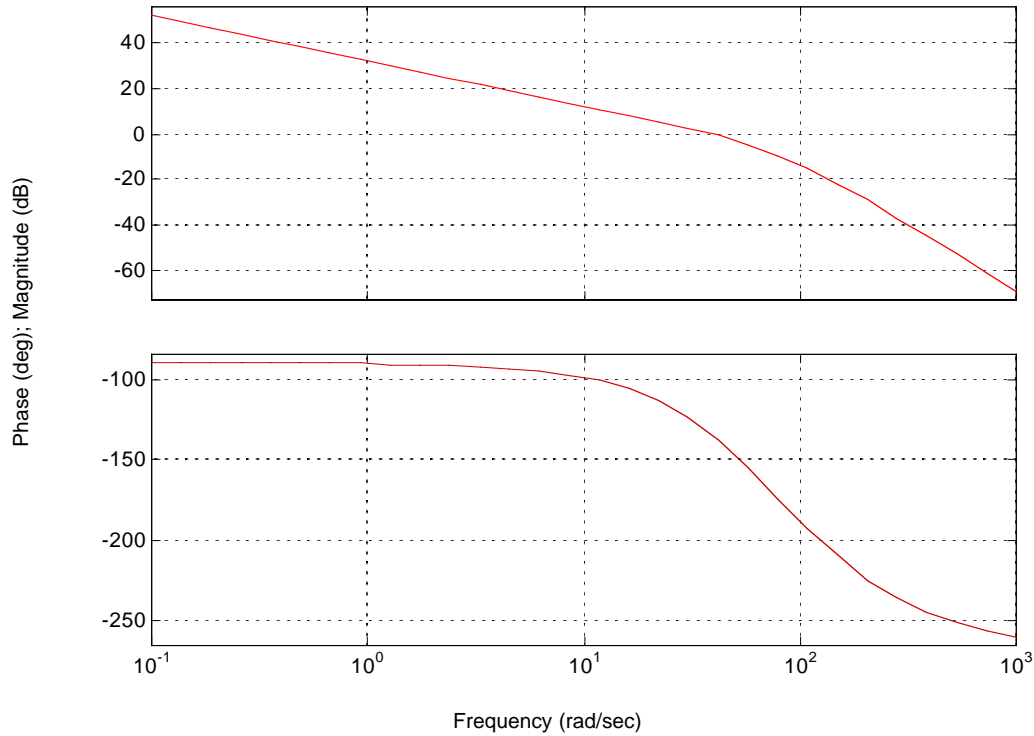
**13.9.**

Let

$$G_e(s) = G(s)G_c(s) = \frac{100K}{s(s+36)(s+100)} \frac{2.38(s+25.3)}{(s+60.2)} = \frac{342720(s+25.3)}{s(s+36)(s+100)(s+60.2)}.$$

The following shows the frequency response of $G_e(j\omega)$.

Bode Diagrams



We find that the zero dB frequency, ω_{Φ_M} , for $G_e(j\omega)$ is 39 rad/s. Using Astrom's guideline the value of T should be in the range, $0.15 / \omega_{\Phi_M} = 0.0038$ second to $0.5 / \omega_{\Phi_M} = 0.0128$ second. Let us use T = 0.001 second.

Now find the Tustin transformation for the compensator. Substituting $s = \frac{2(z-1)}{T(z+1)}$

into $G_c(s) = \frac{2.38(s+25.3)}{(s+60.2)}$ with T = 0.001 second yields

$$G_c(z) = 2.34 \frac{(z-0.975)}{(z-0.9416)}.$$

13.10.

$$G_c(z) = \frac{X(z)}{E(z)} = \frac{1899z^2 - 3761z + 1861}{z^2 - 1.908z + 0.9075}. \text{ Cross-multiply and obtain}$$

$(z^2 - 1.908z + 0.9075)X(z) = (1899z^2 - 3761z + 1861)E(z)$. Solve for the highest power of z operating on the output, X(z), and obtain

$$z^2 X(z) = (1899z^2 - 3761z + 1861)E(z) - (-1.908z + 0.9075)X(z). \text{ Solving for}$$

$X(z)$ on the left-hand side yields

$X(z) = (1899 - 3761z^{-1} + 1861z^{-2})E(z) - (-1.908z^{-1} + 0.9075z^{-2})X(z)$. Finally, we implement this last equation with the following flow chart:

