Prof. Dr. J. Giesl M. Hark

Notes:

- Please solve these exercises in **groups of four**!
- The solutions must be handed in **directly before (very latest: at the beginning of)** the exercise course on Wednesday, 03.07.2019, 14:30, in lecture hall **AH I**. Alternatively you can drop your solutions into a box which is located right next to Prof. Giesl's office (until 30 minutes before the exercise course starts).
- Please write the **names** and **immatriculation numbers** of all students on your solution. Also please staple the individual sheets!

Exercise 1 ($\beta\delta$-Reduction): (3 points)

Recall the notion of "δ-reduction", which is defined as follows:
A set δ of rules of the form $c\,t_1 \dots t_n \;\to\; r$ with $c \in \mathcal{C}$, $t_1, \dots, t_n, r \in \Lambda$ is called a delta-rule set if

(1) $t_1, \dots, t_n, r$ are closed lambda terms

(2) all t_i are in $\to_\beta$-normal form

(3) the t_i do not contain any left-hand side of a rule from δ

(4) in δ there exist no two different rules $c\,t_1 \dots t_n \;\to\; r$ and $c\,t_1 \dots t_m \;\to\; r'$ with $m \geq n$

For such a set δ we define the relation $\to_\delta$ as the smallest relation with

- $l \;\to_\delta\; r$ for all $l \to r \;\in\; \delta$
- if $t_1 \;\to_\delta\; t_2$, then also $(t_1\,r) \;\to_\delta\; (t_2\,r)$, $(r\,t_1) \;\to_\delta\; (r\,t_2)$ and $\lambda y.t_1 \;\to_\delta\; \lambda y.t_2$ for all $r \in \Lambda$, $y \in \mathcal{V}$.

We denote the combination of β- and δ-reduction by $\to_{\beta\delta}$, i.e., $\to_{\beta\delta} = \to_\beta \cup \to_\delta$.

The four conditions (1), (2), (3), (4) are required in order to ensure that $\to_{\beta\delta}$ is confluent. Now assume that we replace condition (4) by the following condition:

in δ there exist no two different rules $c\,t_1 \dots t_n \;\to\; r$ and $c\,t_1 \dots t_m \;\to\; r'$
with $m \geq n$ $\underline{\text{ and } r \neq r'}$

Would $\to_{\beta\delta}$ still be confluent? Please explain your answer.

Exercise 2 (Weak Head Normal Order Reduction): (2 + 2 + 2 = 6 points)

For each of the following terms please show the reduction steps of the WHNO-reduction with the $\to_{\beta\delta}$-relation up to weak head normal form. In each step, please indicate whether it is a $\to_\beta$– or $\to_\delta$–step. Also indicate if the reduction stops because no more $\beta\delta$-reduction is possible or if it stops only because the term is already in WHNO. Note that Nil, Cons, and False are constructors. Here the set δ contains the following rules:

$\texttt{if}\ \mathsf{False} \to \lambda x\ y.y$

$\texttt{gt}\ 22\ 23 \to \mathsf{False}$

$\texttt{mult}\ 4\ 2 \to 8$

$\texttt{isa}_{\mathsf{Cons}}\ \texttt{Nil} \to \mathsf{False}$

a) $(\lambda x.\texttt{mult}\ x\ 2)\ (\texttt{if}\ (\texttt{gt}\ 22\ 23)\ 1\ 4)$

b) $(\lambda x\ y.x\ y)((\lambda z.\texttt{mult}\ 4\ z)\ 2)$

c) $(\lambda z.\texttt{if}\ (\texttt{isa}_{\mathsf{Cons}}\ z)\ \texttt{Nil}\ (\texttt{Cons}\ ((\lambda x.\texttt{mult}\ 4\ x)\ 2)\ \texttt{Nil}))\ \texttt{Nil}$

Exercise 3 (Simple Haskell to Lambda Calculus): (5 points)

Please translate the following Haskell-expression into a lambda term using $\mathcal{L}am$:

```
let length = \zs -> if isa_Nil zs then
                      0
                      else
                      1 + length (sel_2,2 (argof_Cons zs))
in length Nil
```

Hint: If you want, you can write `(plus x y)` instead of `(x + y)`, `i` instead of `isa_Nil` etc.