L17

Basic Statistics for Economists

Spring 2020

Department of Statistics

Stockholm University

Today

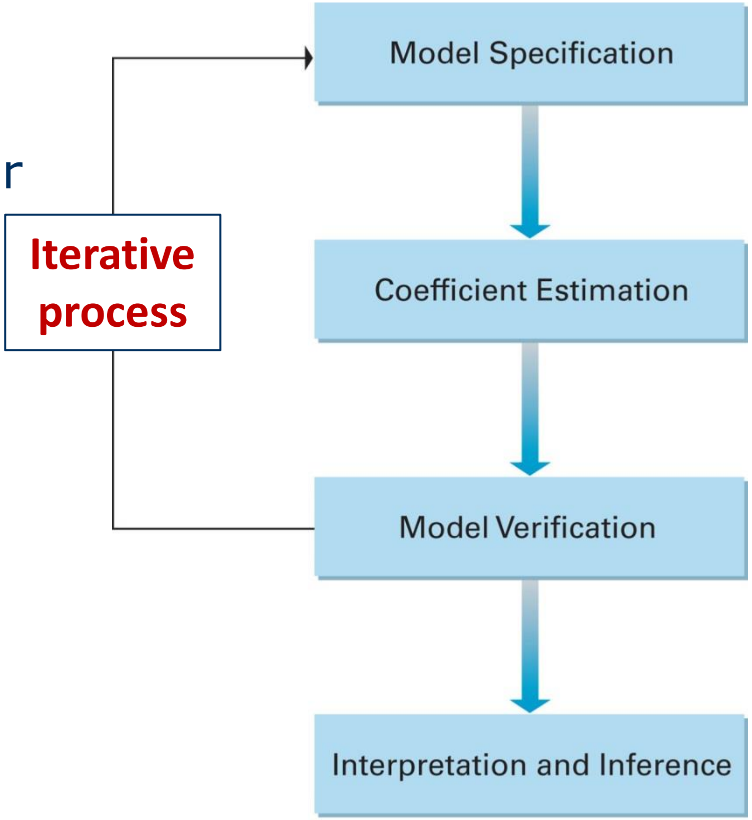
Selected topics from **12.8-9** + **13.1** + **13.4-6 NCT** + review

- Modelling
- Interpretation of t-test in multiple regression
  - correct statement of hypotheses
- Stepwise regression
- Dummy variables
- Specification bias, multicollinearity, heteroscedasticity
- Residual analysis

Maybe in a different order than that of NCT

Analysis and the model process

- This is true any study of relationships, developing explanatory models, models for prognosis, and so on
  - Regression analysis
  - Time series analysis
  - longitudinal models (panel data)
  - ARCH/GARCH-models
  - VAR-models
  - …



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Model building - method

- **Model specification**
  - Choice of variables **grounded in area specific theory**
  - linear – non-linear (we have not talk about this, read NCT 12.7)
- **Estimation of the model (we have covered this)**
  - **Data quality** should be always be considered and assessed
- **Model verification**
  - Does it match the theory: e.g. do the estimates have correct sign?
  - Are the assumptions correct – **residual analysis**
  - A prognosis should be compared to outcomes over time
- **Interpretation, inference, prediction (usage)**
  - Confidence interval, hypothesis test, prognosis

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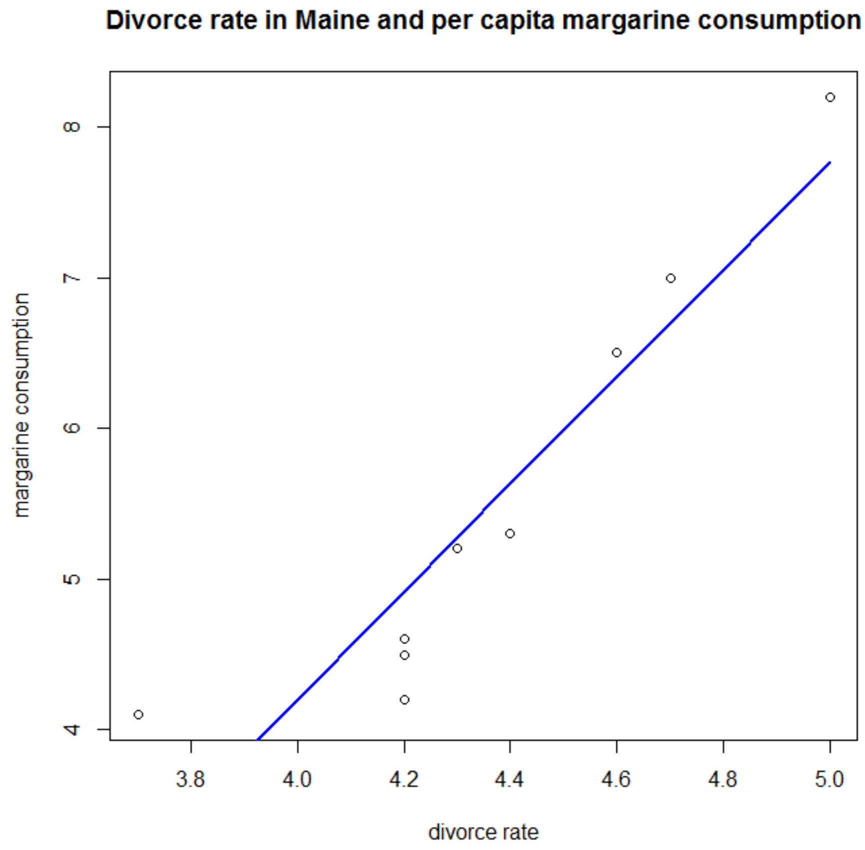
Example: what not to do



source of data: http://www.tylervigen.com/spurious-correlations

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t-test for one of the coefficients, β_j

- Hypotheses to test if a coefficient is significant (adds something to the model) ***given that all the other variables are in the model***

- Suppose that $Y = \beta_0^{\circ} + \beta_1^{\circ} X_1 + \varepsilon$ is significant, a "good" model
- Test the extended model with an additional variable X_2

- Re-estimate the model with both X_1 and X_2 and test:

$$H_0: \beta_2 = 0 \mid \beta_1 \neq 0 \quad \text{vs.} \quad H_1: \beta_2 \neq 0 \mid \beta_1 \neq 0$$

- This is a clearer way of stating what is really being tested: does including X_2 significantly improve the model compared to just X_1
  - remember: R^2 and often also R^2_{adj} <u>increase</u> with every extra predictor

Idea: Stepwise regression

- Start with the variable that has the ***strongest correlation*** with $\boldsymbol{Y}$, lets say $\boldsymbol{X_1}$
  - estimate $\boldsymbol{\beta_0}$ and $\boldsymbol{\beta_1}$ and test $\boldsymbol{H_0: \beta_1 = 0}$

- Given that $\boldsymbol{X_1}$ is in the model, choose the one that ***adds the most***, e.g. $\boldsymbol{X_2}$
  - **NOTE**! This is not necessarily the second most one that correlates $\boldsymbol{Y}$ – it will depend on the how the variable correlates with $\boldsymbol{X_1}$
  - estimate $\boldsymbol{\beta_0}$, $\boldsymbol{\beta_1}$ and $\boldsymbol{\beta_2}$ and test $\boldsymbol{H_0: \beta_2 = 0 \mid \beta_1 \neq 0}$

- Continue with more variables $\boldsymbol{X_3}$, $\boldsymbol{X_4}$, …, choose the one that ***adds the most***
  - Estimate $\boldsymbol{\beta_0}$, $\boldsymbol{\beta_1} - \boldsymbol{\beta_{k+1}}$ and test $\boldsymbol{H_0: \beta_{k+1} = 0 \mid \beta_1 \neq 0, \dots, \beta_k \neq 0}$

- Stop when no available variable gives a significant improvement

Stepwise regression

Important to check at every step:

- That the $\boldsymbol{t}$-test is significant
- Changes in coefficients of the $\boldsymbol{X}$ that already are in the model
  - Large changes of the values?
  - Still significant? It happens that something that was significant now is non-significant!
  - Should you exclude a previously included variable?
- Study changes in $\boldsymbol{R^2}$, $\boldsymbol{R^2_{adj}}$, and $\boldsymbol{MSE = s_e^2}$
- Check if the model can be **grounded in area specific theory**
  - You can always find the ”best” model, but is it meaningful?

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Model building

- **Stepwise regression** is often built into statistical software packages. These functions carry out the steps **automatically**

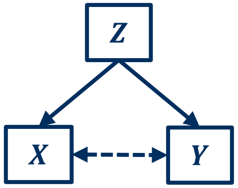
- Different strategies:
  - **Bottom-Up**: start with an ”empty” model and add $\boldsymbol{X}$ one by one.
  - **Top-Down**: start with a ”full” model and reduce it by removing the worst $\boldsymbol{X}$ at every step
  - **Add-Delete strategi**: at every step, the program will do what is “best,” by adding or removing variables

- You have to be careful!

Spurious (fake) relationships

When two variables covary stongly, but there is no explanation

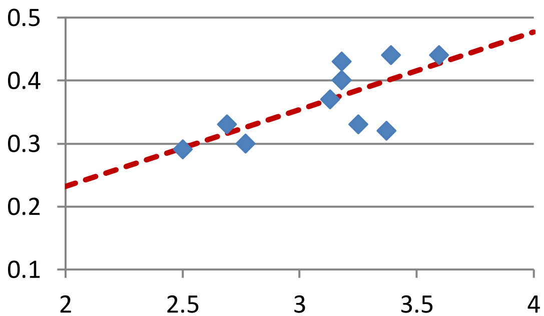
- A third variable influences



... or several...

- E.g. observations are sampled over **time**; both show a **trend**

Y = **Illnesses among public employees,** X = **Price of household electricity, kr**

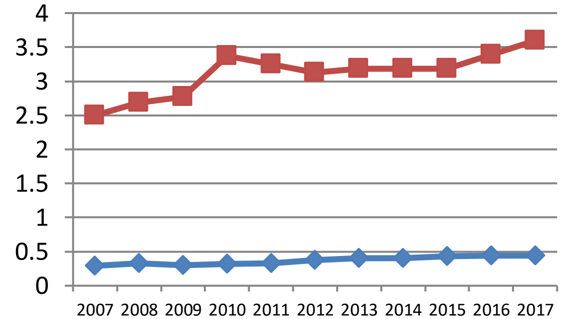


Source: SCB, www.scb.se

$r_{xy} = 0,712$

$R^2 = 0,508$

$p = 0,014$



Hard to see at this scale, but similar trends

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Dummy variables

NCT 12.8

- So far, $\boldsymbol{X}$ has mostly been **continuous** variables, but $\boldsymbol{X}$ can be **discreet**
- No problem if $\boldsymbol{X}$ is **categorical…**
- Special case: $\boldsymbol{X}$ is ***binary*** (***dichotomous***) with $\mathbf{0}$ and $\mathbf{1}$ as possible values
- If you for example want to compare group A to group B

$$\boldsymbol{X} = \begin{cases} \mathbf{0} & \text{if group A} \\ \mathbf{1} & \text{if group B} \end{cases}$$

- $\boldsymbol{X}$ is called ***dummy variable*** or ***indicator variable***

Dummy variable, cont.

- Model with one numerical predictor $\boldsymbol{X_1}$ and one dummy $\boldsymbol{X_2}$:

$$\boldsymbol{\mu_{Y|X} = \beta_0 + \beta_1 X_1 + \beta_2 X_2}$$

Case $\boldsymbol{X_2 = 0}$: $\boldsymbol{\mu_{Y|X} = \beta_0 + \beta_1 X_1 + \beta_2 \cdot 0 = \beta_0 + \beta_1 X_1}$

Case $\boldsymbol{X_2 = 1}$: $\boldsymbol{\mu_{Y|X} = \beta_0 + \beta_1 X_1 + \beta_2 \cdot 1 = (\beta_0 + \beta_2) + \beta_1 X_1}$

- Same slope $\boldsymbol{\beta_1}$ but **different intercept** for the two groups

- Test for different intercept: $\boldsymbol{H_0: \beta_2 = 0 \mid \beta_1 \neq 0}$

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Dummy variable, cont.

- Extend the model with a ”new” variable = $\boldsymbol{X_1 \cdot X_2}$

$$\boldsymbol{\mu_{Y|X} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 (X_1 \cdot X_2)}$$

Case $\boldsymbol{X_2 = 0}$: $\boldsymbol{\mu_{Y|X} = \beta_0 + \beta_1 X_1}$

Case $\boldsymbol{X_2 = 1}$: $\boldsymbol{\mu_{Y|X} = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) X_1}$

- Results in **different intercept <u>and</u> slope** for the two groups

- Test for different slope: $\boldsymbol{H_0 : \beta_3 = 0 \mid \beta_1 \neq 0,\ \beta_2 \neq 0}$

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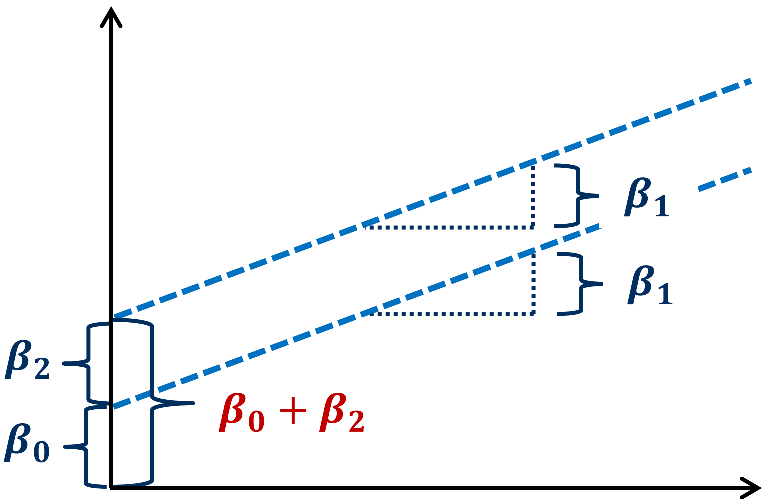
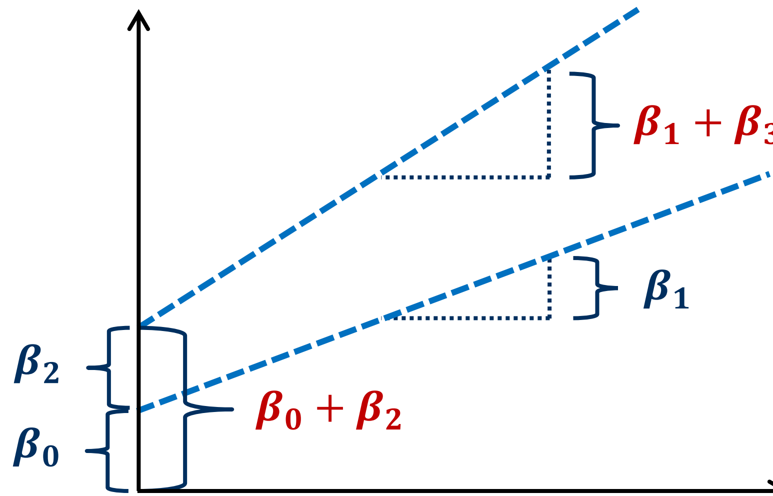
Dummy variable, graphically

- Model with different intercepts, same slope



- Model with different intercepts and slope



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Short discussion of potential problems

NCT 13.4-6

- **Specification bias**
  - Something important is missing from the model
- **Multicollinearity**
  - Predictor variables correlates
- **heteroscedasticity**
  - not constant variance

Specification bias

Section 13.4

Occurs if important (significant) predictor variables are left out of the model.

- This will make the **OLS estimates** of the coefficients of the included variables **biased**.

Shown on 572, but not part of the course!

- The **statistical inference** (hypothesis test and CI) can in this case be **misleading**.
- The effect of the missing variables are instead captured by the residual variance (s_e^2) which typically becomes much greater, i.e. **worse precision**.

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Multicollinearity

Section 13.5

- When two or more of the explanatory variables are strongly correlated with each other ⇒ serious problems for the entire model!
- Illustration (constructed example):

Variables

- Y: **Sales (mkr)**
- X_1: **Print advertising (100 tkr)**
- X_2: **Other advertising (100 tkr)**
- $n = 7$

Y	X_1	X_2	X_3
7	4	1	5
9	7	2	9
16	9	5	14
19	12	8	20
25	15	10	25
26	17	14	31
33	20	17	37

Multicollinearity, cont.

```
Regression Analysis: Y versus X1
The regression equation is
Y = - 0,31 + 1,63 X1

Predictor     Coef   SE Coef      T      P
Constant    -0,306     1,388  -0,22  0,834
X1          1,6327    0,1058  15,42  0,000

S = 1,48186   R-Sq = 97,9%   R-Sq(adj) = 97,5%
```

$r_{x_1,y} = 0,990$

NOTE: R^2 close to 1 and significant coefficients ($p = 0$) for both models

```
Regression Analysis: Y versus X2
The regression equation is
Y = 6,68 + 1,55 X2

Predictor     Coef   SE Coef      T      P
Constant     6,676     1,282   5,21  0,003
X2          1,5485    0,1302  11,89  0,000

S = 1,90830   R-Sq = 96,6%   R-Sq(adj) = 95,9%
```

$r_{x_2,y} = 0,983$

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Multicollinearity, cont.

```
Regression Analysis: Y versus X1; X2
The regression equation is
Y = 0,89 + 1,34 X1 + 0,279 X2

Predictor     Coef  SE Coef     T      P
Constant     0,890    3,594  0,25  0,817
X1          1,3436   0,7948  1,69  0,166
X2          0,2791   0,7592  0,37  0,732

S = 1,62948   R-Sq = 98,0%   R-Sq(adj) = 97,0%

Analysis of Variance
Source          DF      SS      MS      F      P
Regression       2  522,81  261,40  98,45  0,000
Residual Error   4   10,62    2,66
Total            6  533,43
```

Comments: see next page

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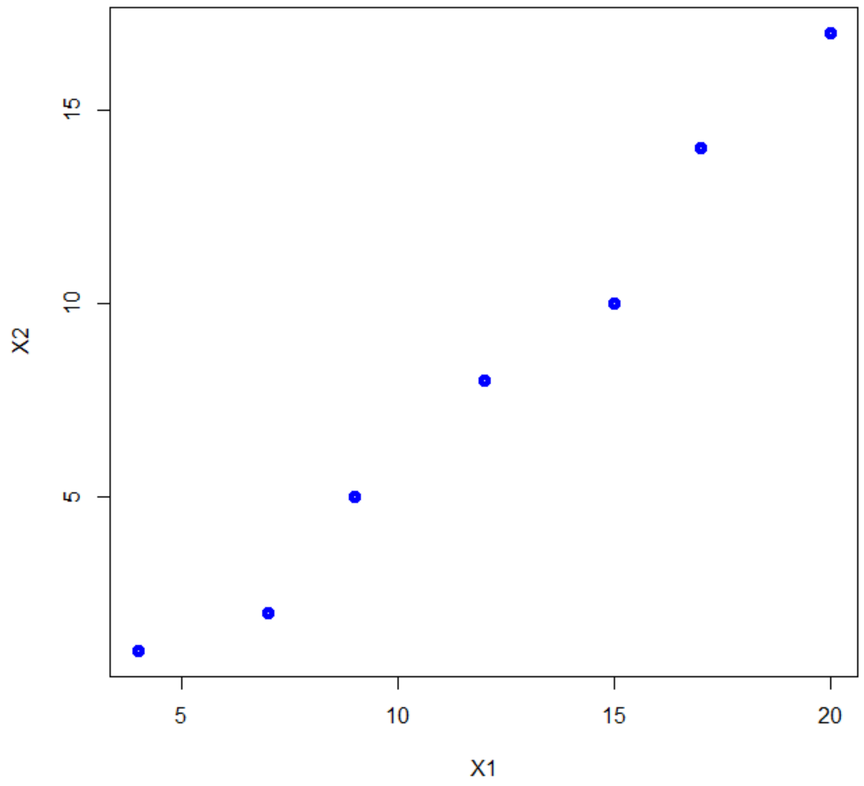
Plot: Print ads vs. Other ads



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Multicollinearity, cont.

- Large $\boldsymbol{R^2}$ but marginally greater than the previous models
  - but $\boldsymbol{R^2_{adj}}$ has decreased compared to model with just $\boldsymbol{X_1}$
- Test $\boldsymbol{H_0: \beta_1 = \beta_2 = 0}$ gives significant results ($\boldsymbol{F}$-test, $\boldsymbol{p = 0}$)
  - But neither $\boldsymbol{b_1}$ or $\boldsymbol{b_1}$ are significantly different from zero with $\boldsymbol{p = 0,166}$ and $\boldsymbol{0,732}$ respectively
- Large change in the estimated $\boldsymbol{X_2}$ coefficient, slightly smaller change of the $\boldsymbol{X_1}$ coefficient
- Standard errors $\boldsymbol{s_{b_1}}$ and $\boldsymbol{s_{b_2}}$ are approximately 7,5 and 6 times greater compared to the model with only $\boldsymbol{X_1}$, the residual standard error $\boldsymbol{s_e}$ has also increased
  - We have also lost a degree of freedom

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Multicollinearity, cont.

- X_1 and X_2 covary so strongly that we cannot discern their respective effects on Y
  - Their correlation is $r_{x_1,x_2} = 0,989$

- Do try a model with all three explanatory variables X_1, X_2, and X_3 and see what happens!
  - The problems will be even more serious. Do you see why?

What can we do?

- Specify a different model!

Absolute multicollinearity

- Anta $\boldsymbol{Y}$ = shipping cost, $\boldsymbol{X_1}$ = distance in kilometers, $\boldsymbol{X_2}$ = distance in Swedish mil (10 km = 1 mil).
- Model including both:

$$\boldsymbol{\mu_{Y|X} = \beta_0 + \beta_1 X_1 + \beta_2 X_2 \ = \ \beta_0 + \beta_1 X_1 + \beta_2 \cdot 10 X_1}$$
$$\boldsymbol{= \ \beta_0 + (\beta_1 + 10\beta_2) X_1 \ = \ \beta_0 + \alpha_1 X_1}$$

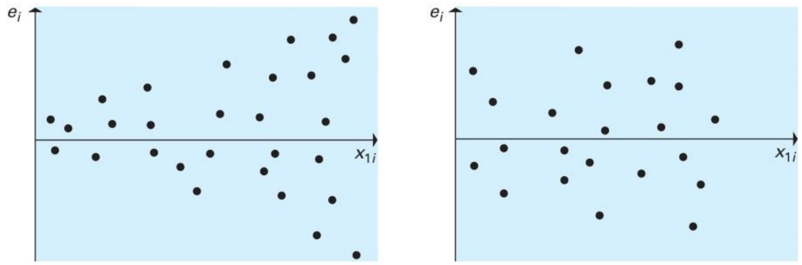
- This is really two variables, $\boldsymbol{X_1}$ and $\boldsymbol{Y}$, but we are trying to estimate this using 3 coefficients
- Mathematically, this will not work!

The relationships between the independent variables do not need to this obvious. Strong correlations can also occur in more complex situations. Be careful!

heteroscedasticity

Section 13.6

- <u>Model assumptions</u>: the error terms are normally distributed with expected value = 0 and **constant variance σ_ε^2**
- This is often not a good assumption



(a) Heterosecedasticity (b) No Apparent Heterosecedasticity

Plot the residuals (e_i) against each of the explanatory variables (x_{ki})

- E.g. Larger companies are subject to more factors with may affect variation of $\boldsymbol{Y}$, compared to smaller companies.

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heteroscedasticity, cont.

Problem:

- OLS estimates are **not efficient** (not best precision)
- Hypothesis test and confidence interval for the coefficients are **not correctly defined** – e.g. you can not be sure that your CI or p-values are even approximately correct.
- **NCT** describes a test for homoscedasticity – **skip it!**
- Do plot the residuals $\boldsymbol{e_i}$ against the explanatory variables $\boldsymbol{x_{ki}}$ and also against the predicted values $\hat{\boldsymbol{y}}_{\boldsymbol{i}}$!

Verification of the model: Residual analysis

NCT 12.9 p. 534-537

- The model assumption must be verified!

1. $\boldsymbol{Y}$ is a ***linjear*** function of $\boldsymbol{X_1, \dots, X_K}$ + error term $\boldsymbol{\varepsilon_i}$
2. $\boldsymbol{X_1, \dots, X_K}$ are all ***independent*** of the error terms $\boldsymbol{\varepsilon_i}$.
3. The error terms are $\boldsymbol{\varepsilon_i}$ ***normally distributed*** r.v. with expected value $\boldsymbol{E(\varepsilon_i) = 0}$ and ***constant variance*** $\boldsymbol{\sigma_\varepsilon^2}$
4. The error terms are pairwise ***independent*** of each other

- The residuals $\boldsymbol{e_i}$ are proxies of $\boldsymbol{\varepsilon_i}$ $\Rightarrow$ **analyze the residuals**
  - We have just talked about the assumption of constant variance and how we can check this

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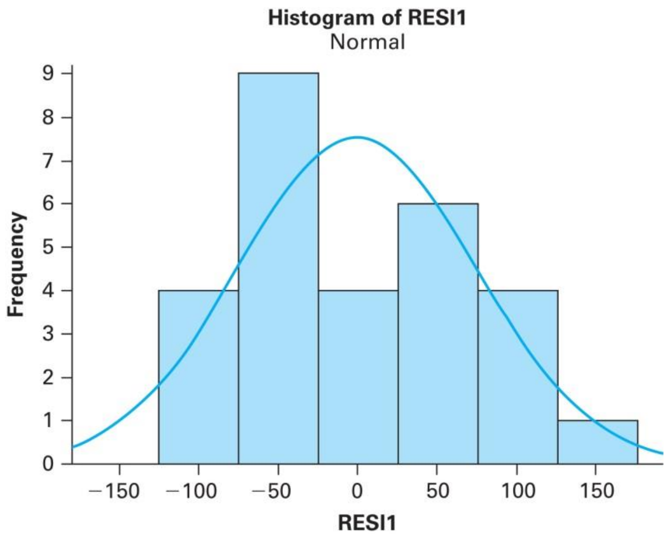
Analysis of residuals, cont.

Normally distributed error terms $\boldsymbol{\varepsilon_i}$

- There are test, e.g. Jarque-Bera test. (not covered)
- **Histogram** of $\boldsymbol{e_i}$ or ”Probability plots” (not covered)



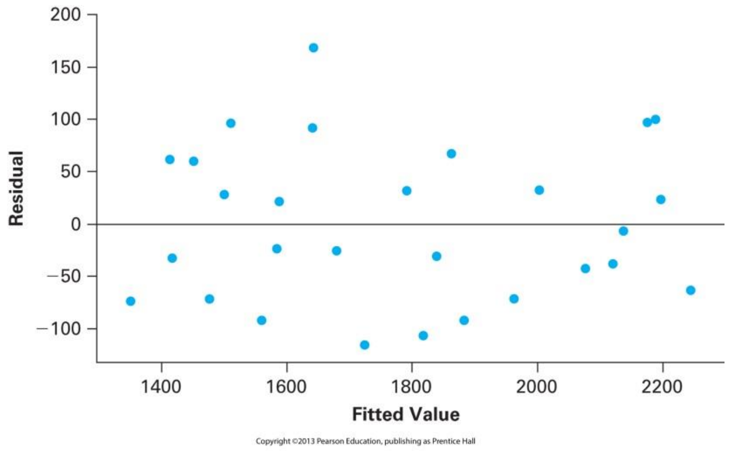
Does not have to be very normal like. If it looks ”almost” normal, then that is ok, too.

The other assumptions are more critical: independence, constant variance, linearity

Analysis of residuals, cont.

$X_1, \dots, X_K$ **independent** of error terms ε_i

- Plot residuals e_i against each of $X_1, \dots, X_K$
- Also plot against the predicted values $\hat{y}$ (= linj.komb. av $X_1, \dots, X_K$)
- Should be void of clear patterns and should look random

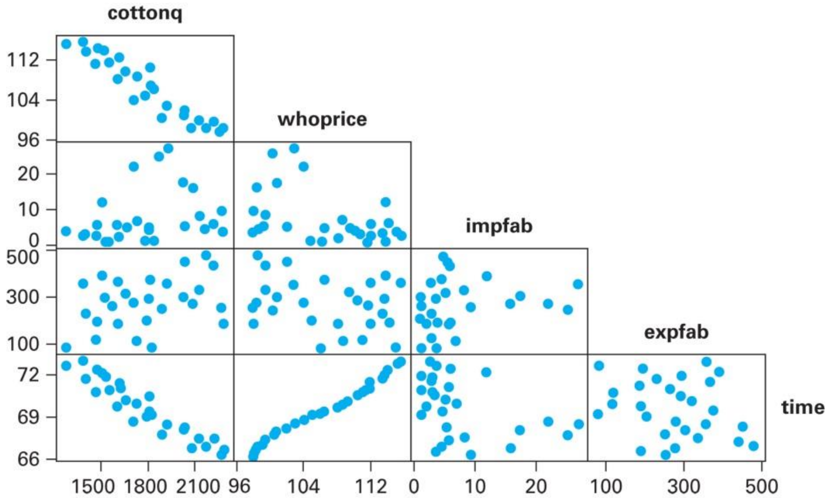


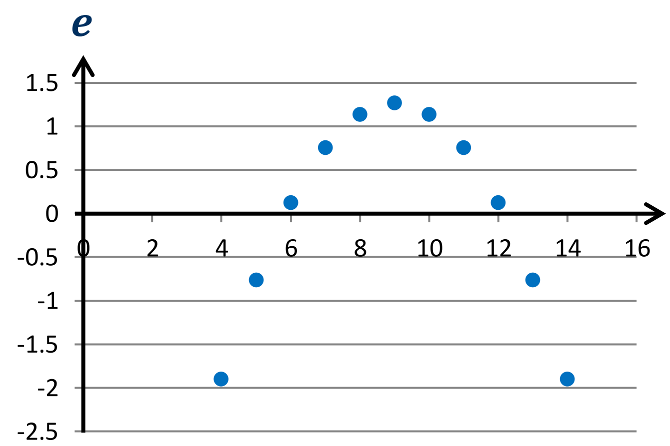
A ”clear pattern” could be any kind of pattern. To understand what causes the pattern requires more thinking and discussion. But dependence, non-linearity causes typical patterns (see next page).

Analysis of residuals, cont.

Linear correlation

- Plot Y against each of $X_1, \dots, X_K$; matrix plots
- Also plots with e_i against each of $X_1, \dots, X_K$ and $\hat{y}$ can indicate non-linearity





Clear pattern because of non-linearity!

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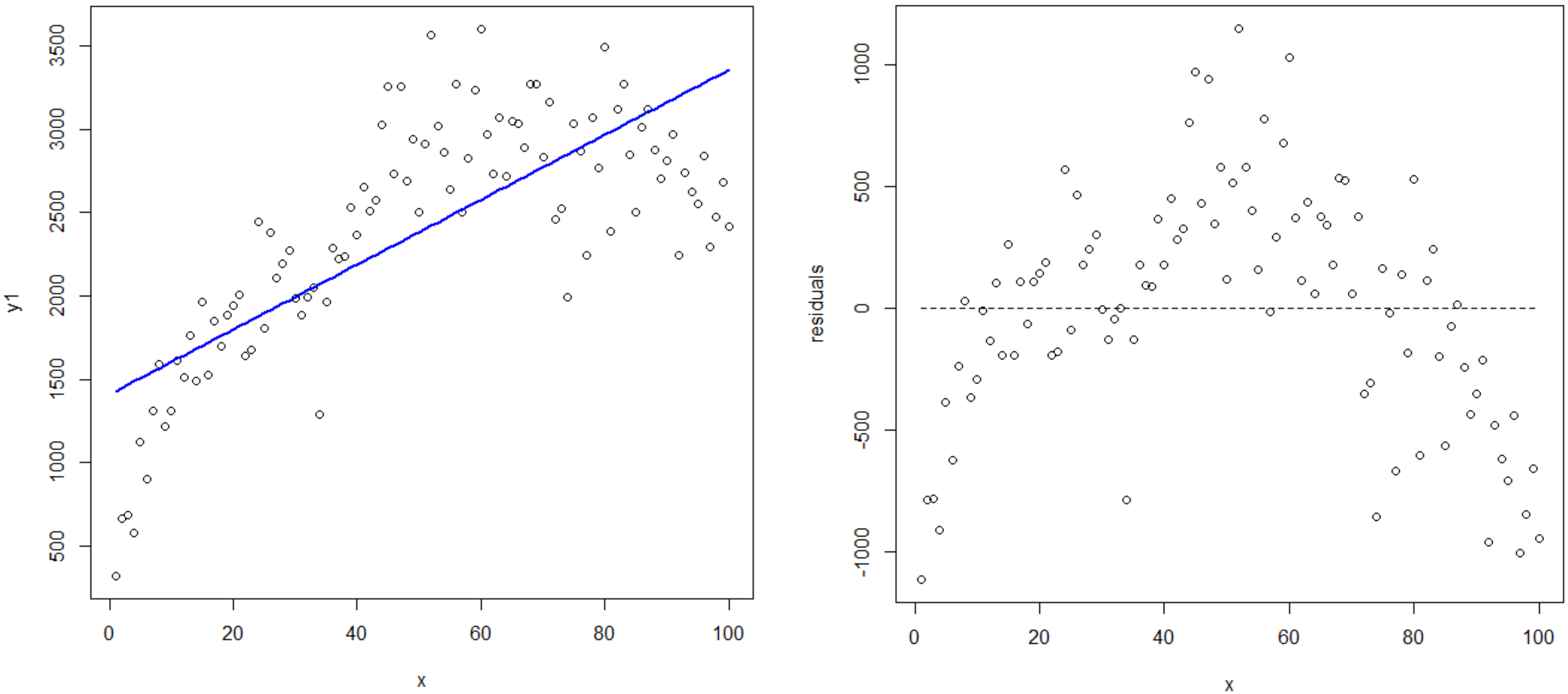
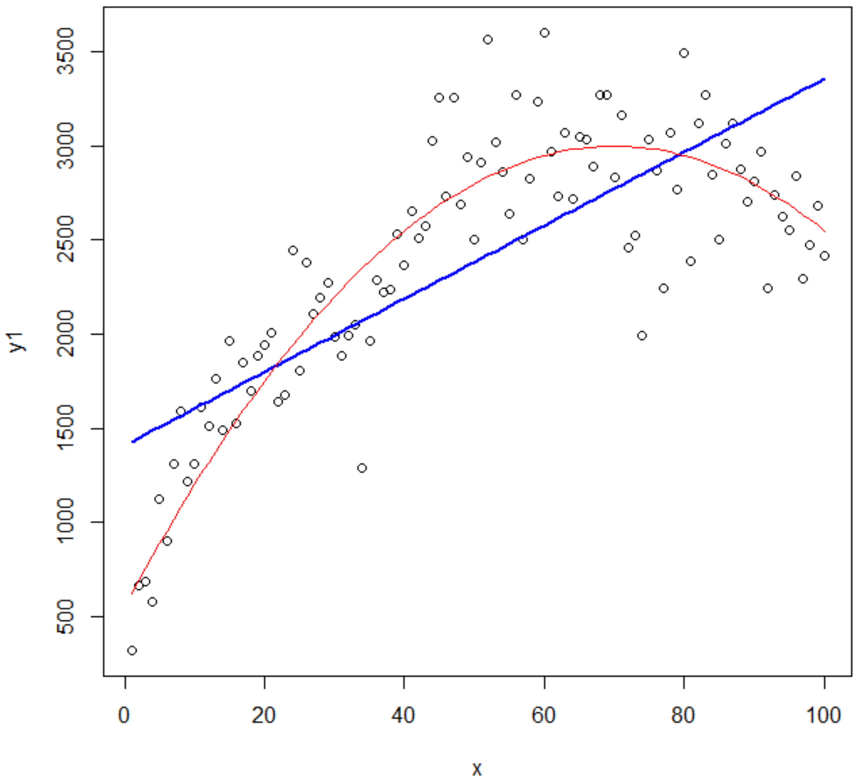
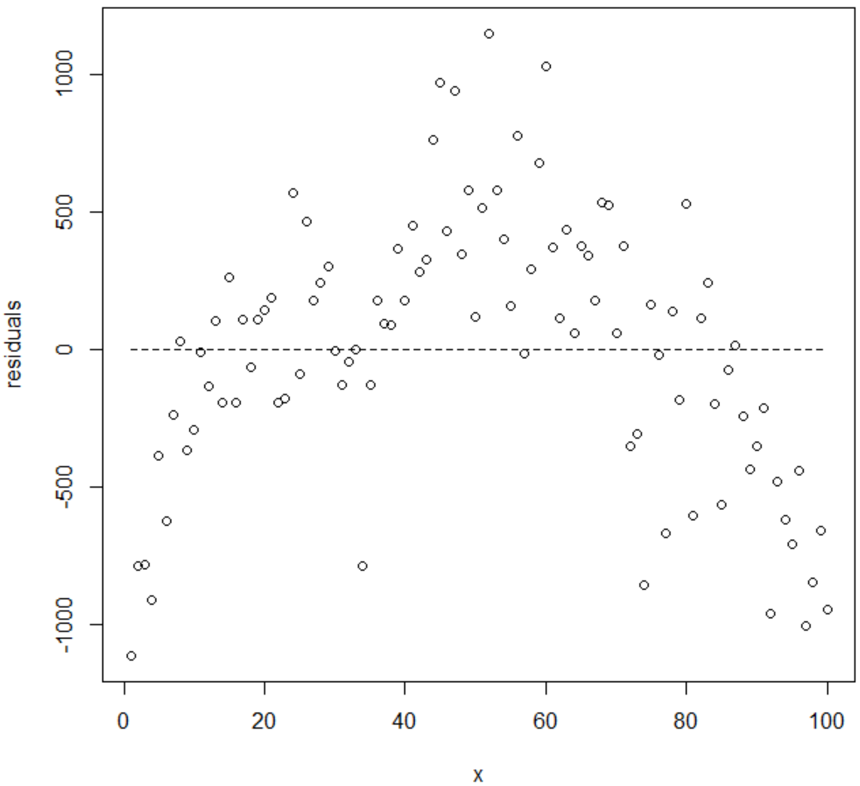
Example: non-linear relationship



Example: non-linear relationship, cont.





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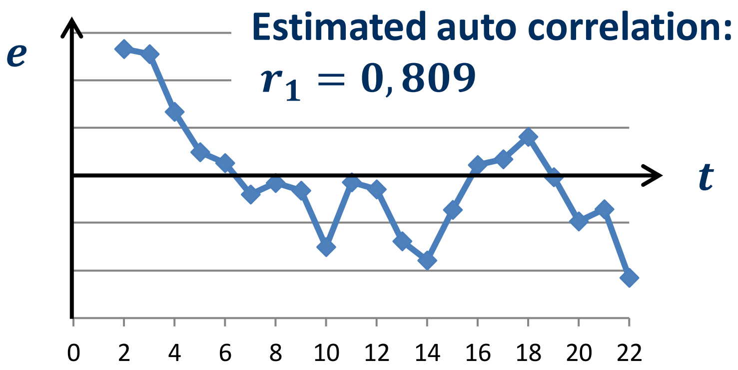
Residual analysis, cont.

The error terms $\boldsymbol{\varepsilon_i}$ are **dependent** of each other

- Common vid repeated measures over time
  - The value of today influences the value of tomorrow
- Let $\boldsymbol{\varepsilon_t}$ denote the error term at time $\boldsymbol{t}$
- **Auto correlation**, serial correlation: $\boldsymbol{\rho_1 = Corr(\varepsilon_t, \varepsilon_{t-1})}$



Note! Described in section 12.9, but not part of reading instructions!

Can be difficult to notice if the error terms are dependent in some other way.

Area specific knowledge required!

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Next time

- Another kind of test: **χ^2-test**
- Used for **categorical data**
  - Goodness-of-fit test
  - Independence test/ Homogeneity test

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