DM AAA : Automata, Algebra, Applications

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1 Weighted automata

We are placing ourselves in the max-min semiring $S = (\mathbb{N} \cup \{\infty\}, \max, \min, 0, \infty)$.

1.1 Exercise

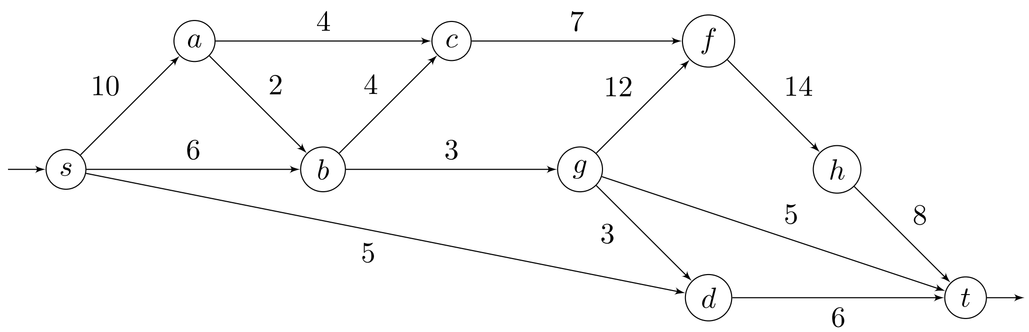
Give a detailed proof that S forms a semiring.

1.2 Exercise

Describe automata weighted over S: What is the value of a path? What is the value of an automaton?

1.3 Exercise

Let A be the following S-automaton:



What is $|A|$?

1.4 Exercise

S-weighted automata are almost the same as maximum-flow problems, but not quite.

1. Seeing the automaton A from above as a maximum-flow problem, what is the maximum flow?
2. What precisely is the difference between maximum-flow problems and S-weighted automata? Is there a semiring S' such that maximum-flow problems can be posed as S'-weighted automata?

1.5 Exercise

Prove that S is star-continuous and compute a^* for all $a \in S$.

1.6 Exercise

Develop the matrix-star formula for a 2-by-2 matrix $\left[\begin{smallmatrix} a & b \\ c & d \end{smallmatrix}\right] \in S^2$.

What is the $(2,3)$-component of the star of the matrix

$$\begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & o & p \end{bmatrix}?$$

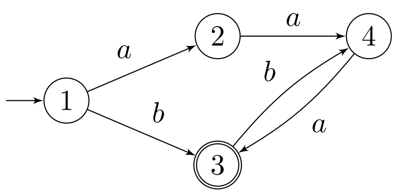
1.7 Exercise

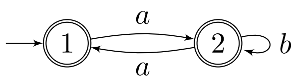
Write a program which implements the recursive matrix-star algorithm to compute M^* for an arbitrary square matrix over S. (Hint: Python, for example, has `math.inf` which should be useful here.) You can test your program on the automaton A from exercise 1.3.

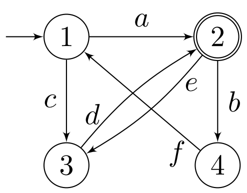
2 ω-Automata

2.1 Exercise

What are the languages of the following Büchi automata?

1. 

2. 

3. 

2.2 Exercise

Let

$$\begin{aligned} L_1 &= \{w \in \{a,b,c\}^\omega \mid w \text{ contains infinitely often the sequence } abc\} \\ &= \{w \in \{a,b,c\}^\omega \mid \{i \in \mathbb{N} \mid w_i = a, w_{i+1} = b, w_{i+2} = c\} \text{ is infinite}\} \end{aligned}$$

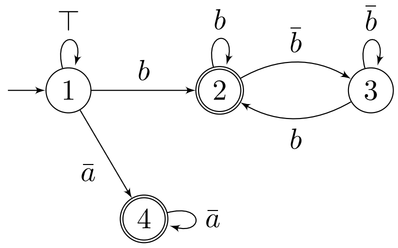
and $L_2 = (c^* a d^* b)^\omega$. Find Büchi automata A_1 and A_2 so that $L_1 = L(A_1)$ and $L_2 = L(A_2)$.

2.3 Exercise

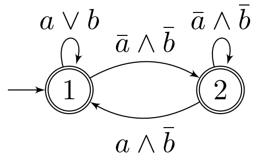
Let $AP = \{a, b\}$, and $\Sigma = 2^{AP}$ (the power set of AP).

For each Büchi automaton $\mathcal{A}$ below, give an LTL formula with $\mathcal{L}(\phi) = \{w \in \Sigma^\omega \mid w \models \phi\} = \mathcal{L}(\mathcal{A})$.

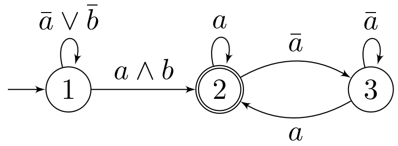
1.



2.



3.



2.4 Exercise

A Büchi automaton $\mathcal{A} = (Q, I, T, F)$ over Σ is called *deterministic* if $|I| \leq 1$, and for each state $q \in Q$ and symbol $a \in \Sigma$, we have $|\{(q, a, q') \in T \mid q' \in Q\}| \leq 1$.

Show that the class of languages recognizable by *deterministic* Büchi automata is closed under

1. intersection,
2. union.

3 Active Learning

3.1 Exercise

Apply the L^* algorithm table to the language $\mathcal{L}(\mathcal{A})$ accepted by the automaton $\mathcal{A}$ of Figure 1. Write the table, the various intermediary models, and the counter-examples to these models you have chosen.

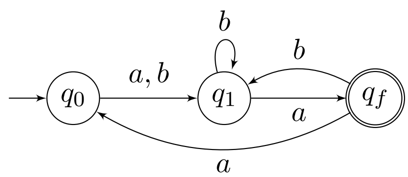


Figure 1: The automaton $\mathcal{A}$.

3.2 Exercise

Using active learning, find the minimal DFA accepting the language $\mathcal{L}_1$ of all words on $\Sigma = \{a, b\}$ with an odd number of a and an even number of b. Write the table, the various intermediary models, and the counter-examples to these models you have chosen.

3.3 Exercise

Using active learning, try to find the minimal DFA accepting the language $\mathcal{L}_2$ of all words on $\Sigma = \{a, b\}$ with the same number of a and b. Is it possible? How and why does the algorithm fail?