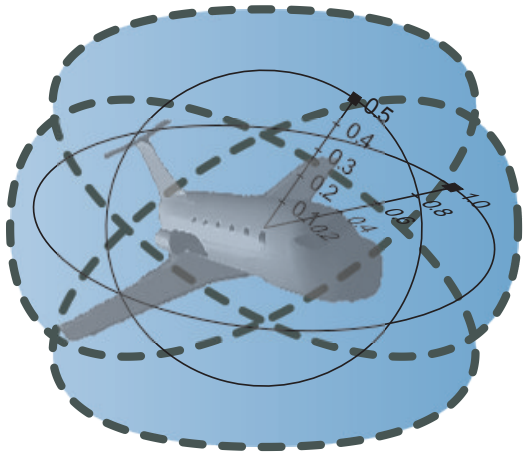
Logic of Distributed Hybrid Systems

André Platzer

Carnegie Mellon University



Outline

Complex Physical Systems:

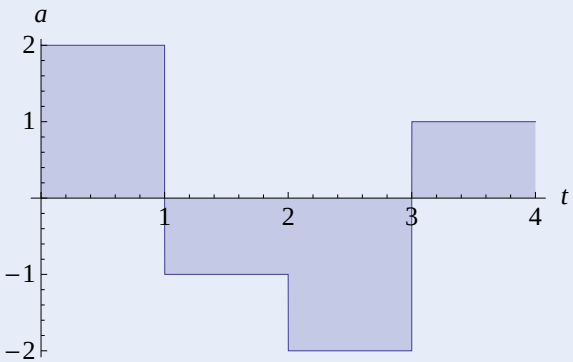
Q: I want to verify my car

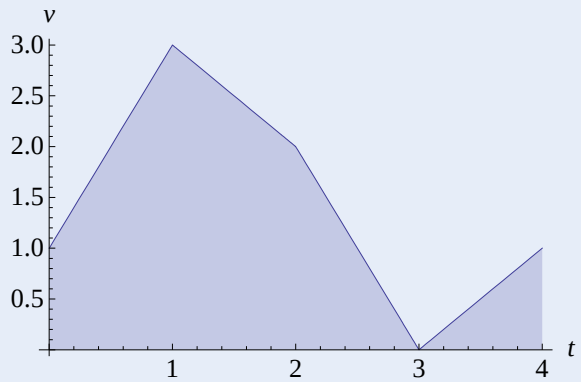
Challenge

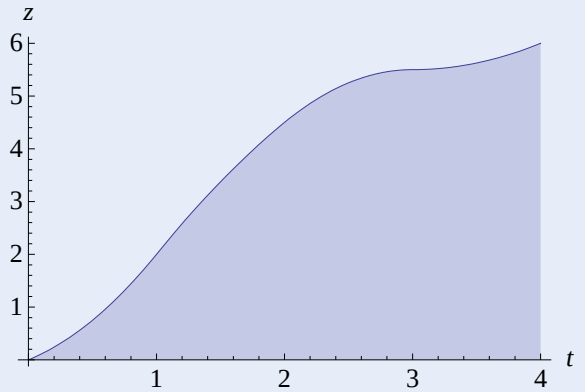
Q: I want to verify my car A: Hybrid systems

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)



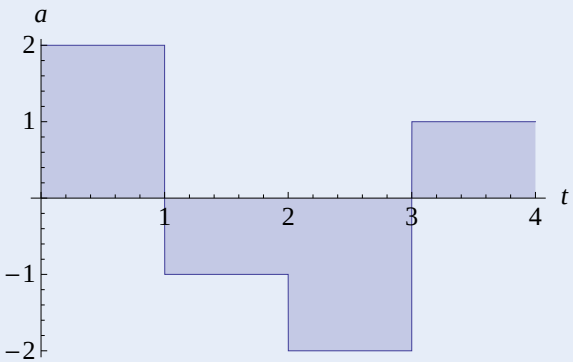


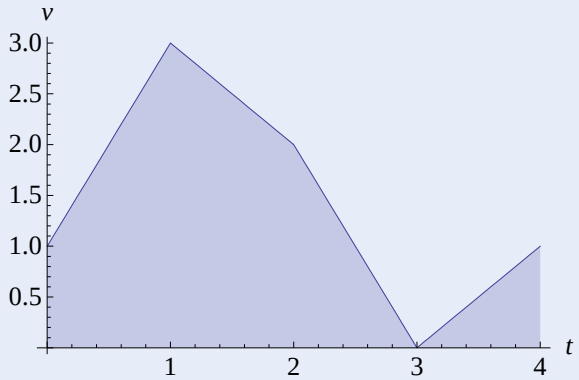


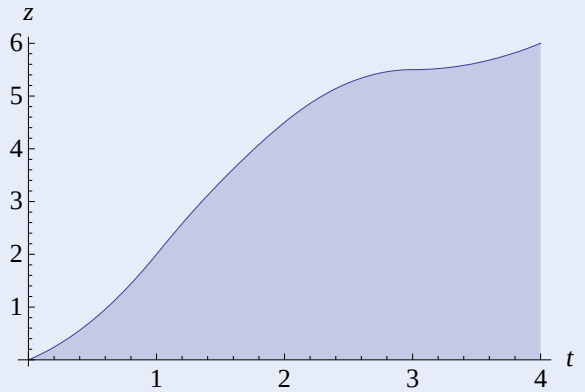
Q: I want to verify my car A: Hybrid systems Q: But there's a lot of cars!

Challenge (Hybrid Systems)

- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)







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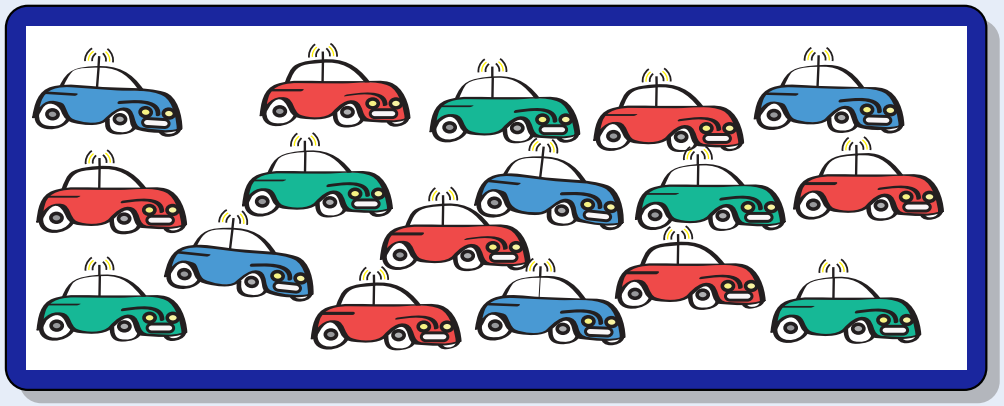
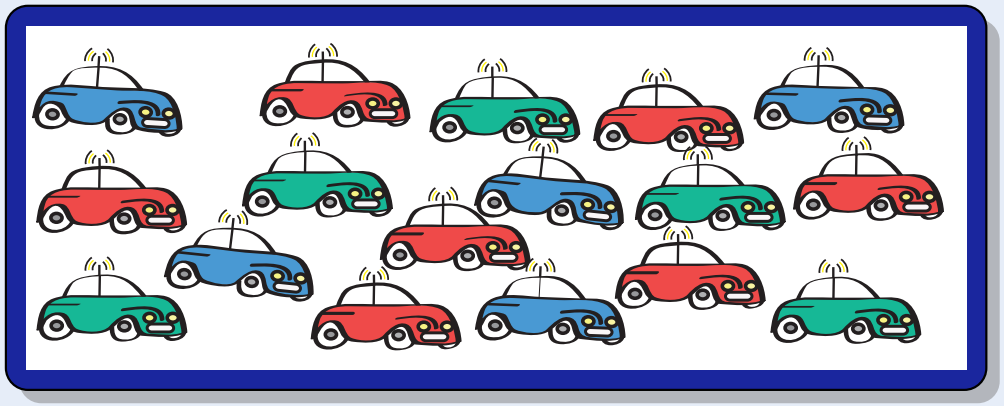
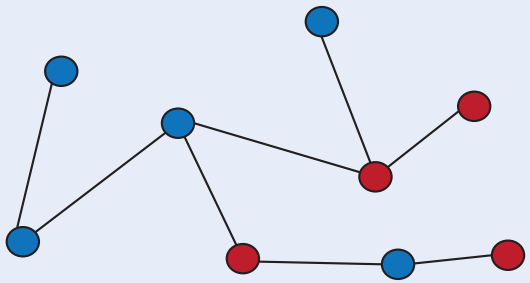


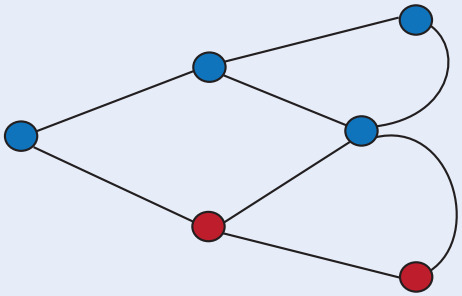
Q: I want to verify a lot of cars A: Distributed systems

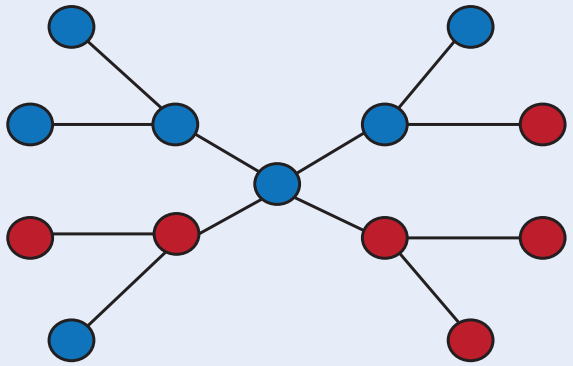
Challenge (Distributed Systems)

- Local computation (finite state automaton)
- Remote communication (network graph)









Q: I want to verify a lot of cars A: Distributed systems Q: But they move!

Challenge (Distributed Systems)

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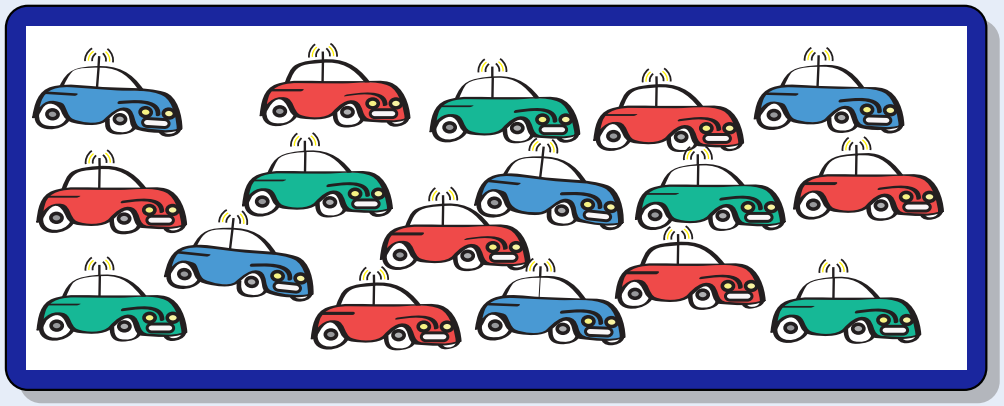
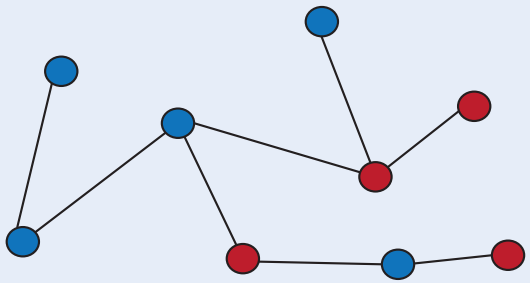
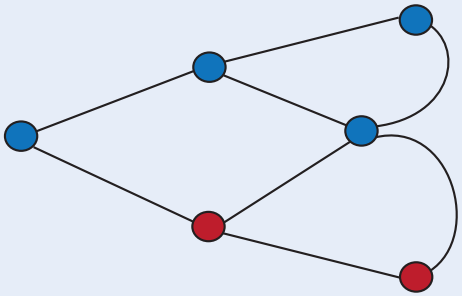
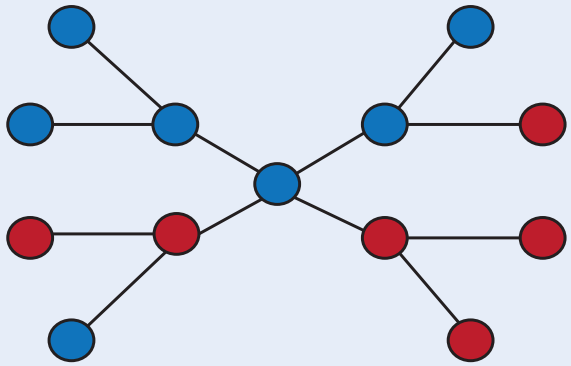
- Remote communication (network graph)









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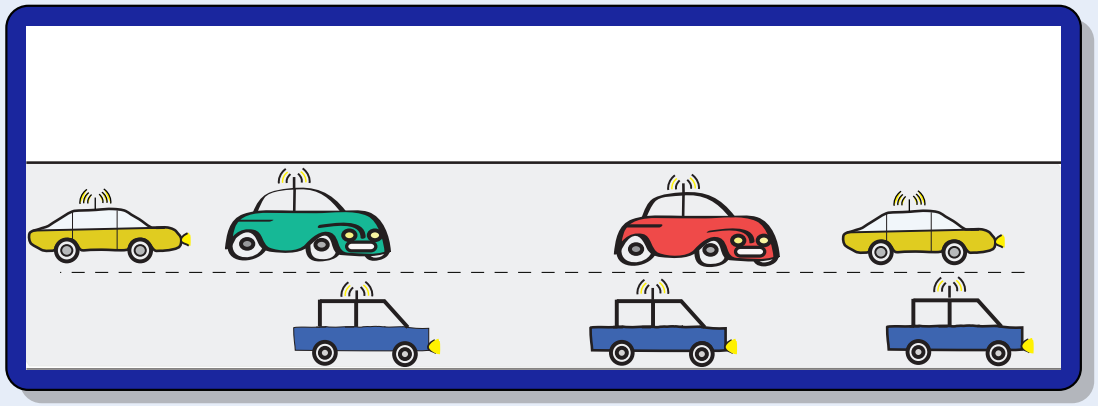

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Challenge (Distributed Hybrid Systems)

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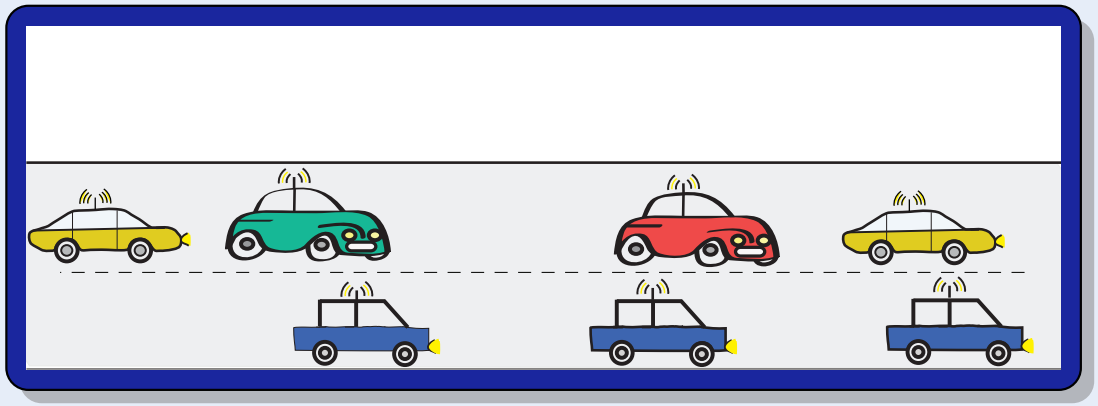
- Discrete dynamics (control decisions)
- Structural dynamics (remote communication)



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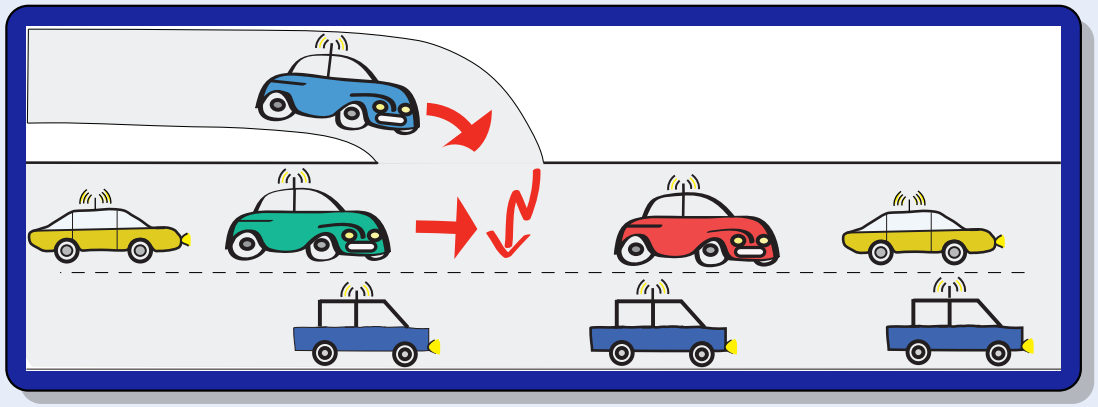

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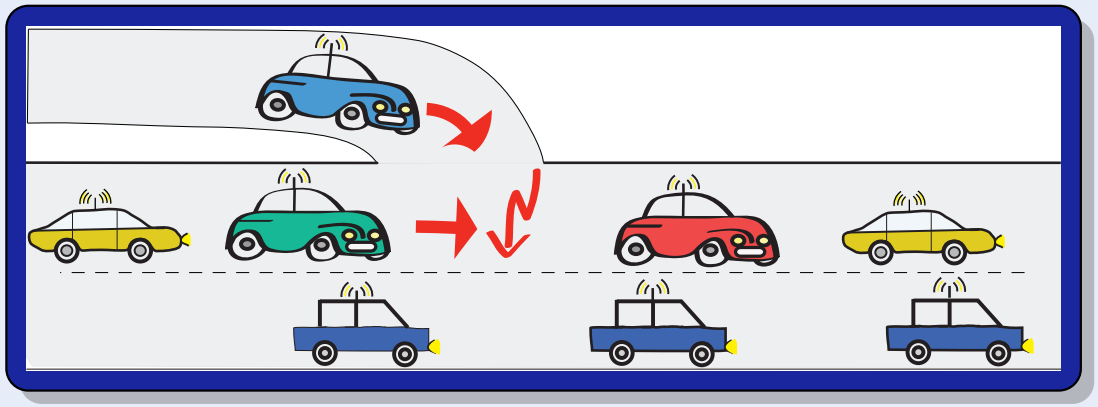

State of the Art: Modeling and Simulation

No formal verification of distributed hybrid systems

Shift [4] The Hybrid System Simulation Programming Language

Hybrid CSP [6] Semantics in Extended Duration Calculus

HyPA [7] Translate fragment into normal form.

χ process algebra [8] Simulation, translation of fragments to PHAVER, UPPAAL

R-Charon [5] Modeling Language for Reconfigurable Hybrid Systems

Φ-calculus [9] Semantics in rich set theory

ACP_{hs}^{srt} [10] Modeling language proposal

OBSHS [11] Partial random simulation of objects

Contributions

1. System model and semantics for distributed hybrid systems: QHP
2. Specification and verification logic: Qd$\mathcal{L}$
3. Proof calculus for Qd$\mathcal{L}$
4. First verification approach for distributed hybrid systems
5. Sound and complete axiomatization relative to differential equations
6. Prove collision freedom in a (simple) distributed car control system, where new cars may appear dynamically on the road
7. Logical foundation for analysis of distributed hybrid systems
8. Fundamental extension: first-order $x(i)$ versus primitive x

CSL'10,LMCS'12

Outline

Outline (Conceptual Approach)

Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems

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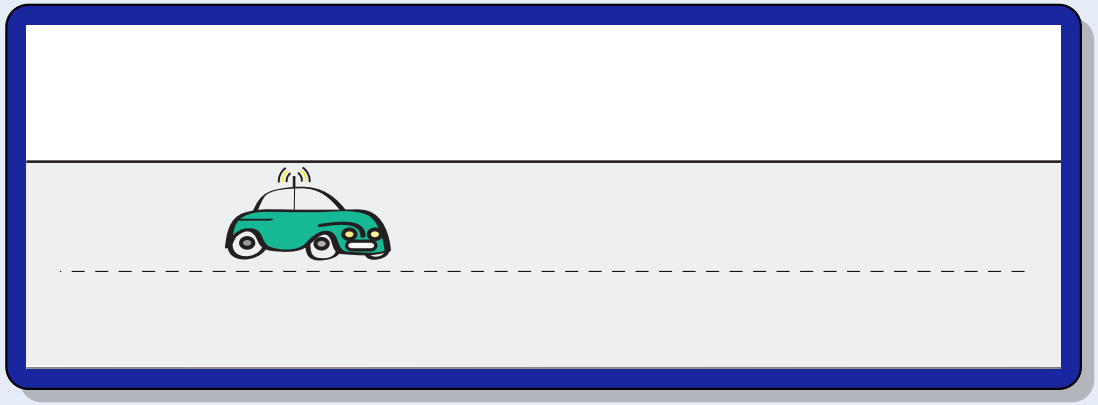
- Continuous dynamics (differential equations)
- Discrete dynamics (control decisions)
- Structural dynamics (communication/coupling)



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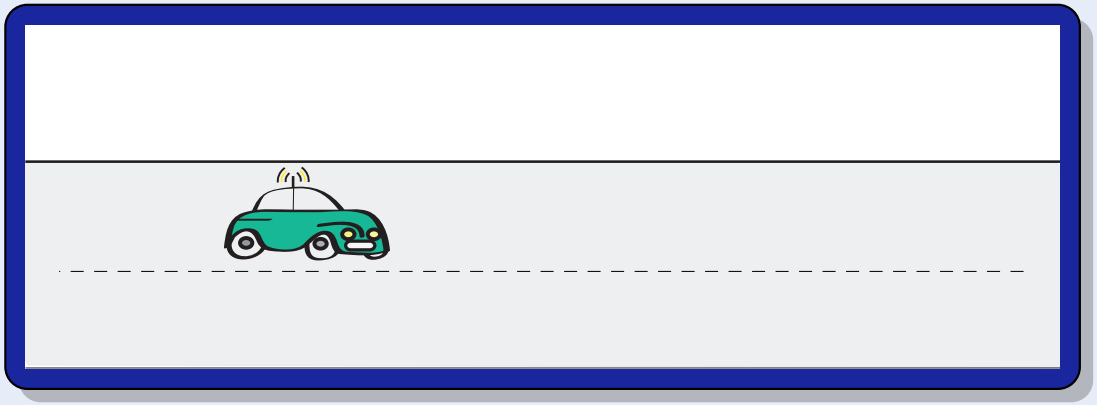

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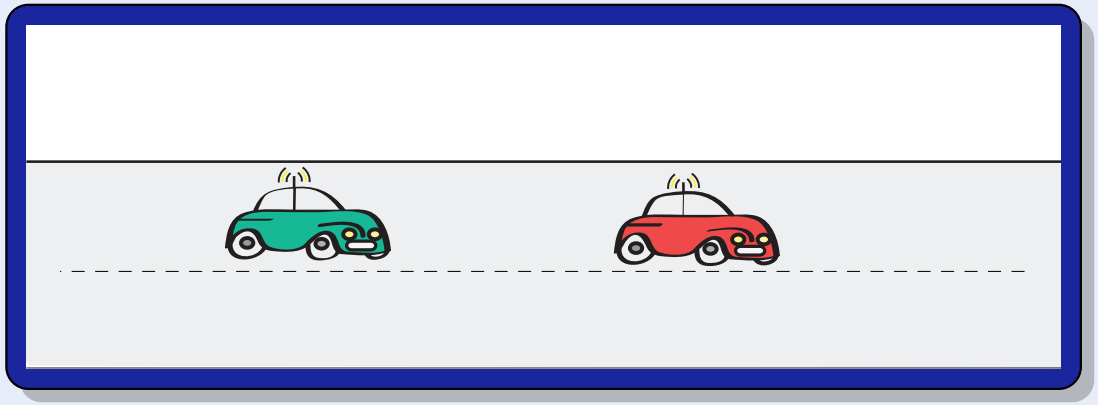

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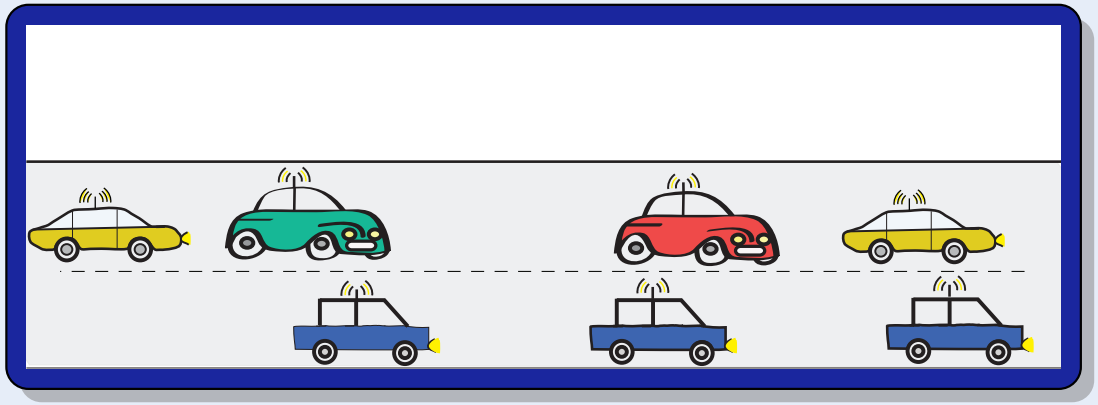

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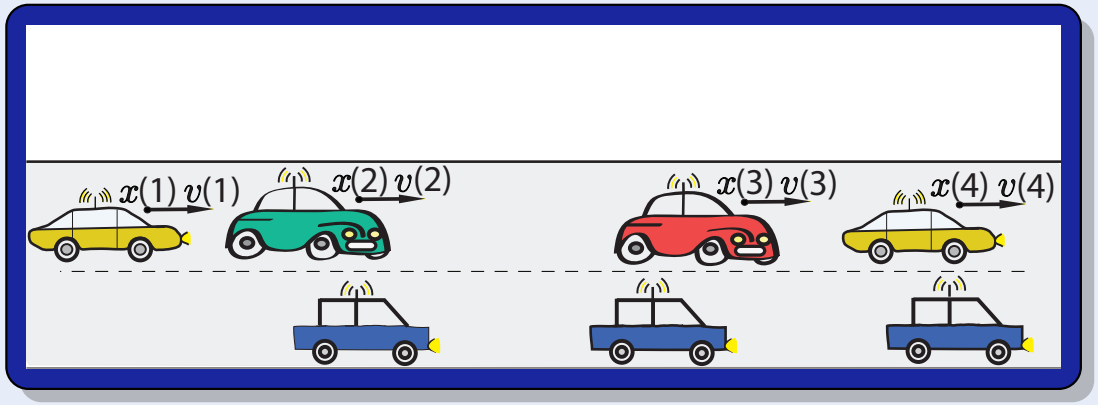

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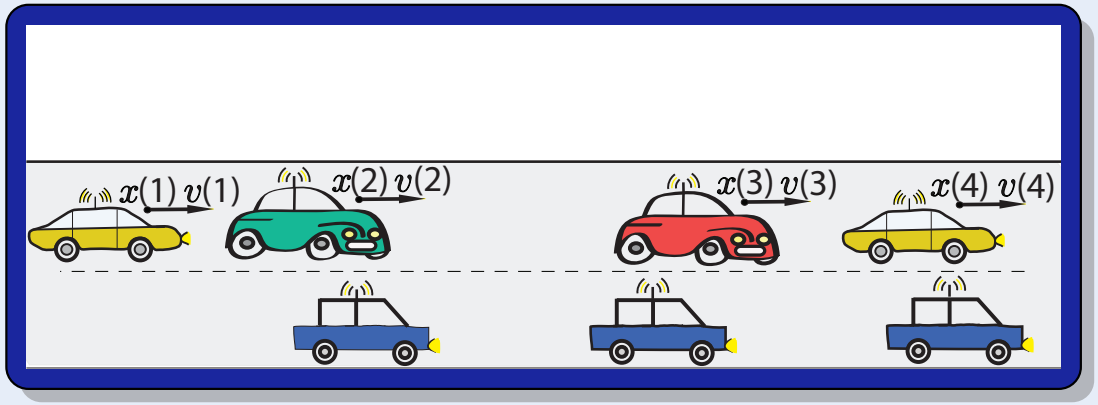

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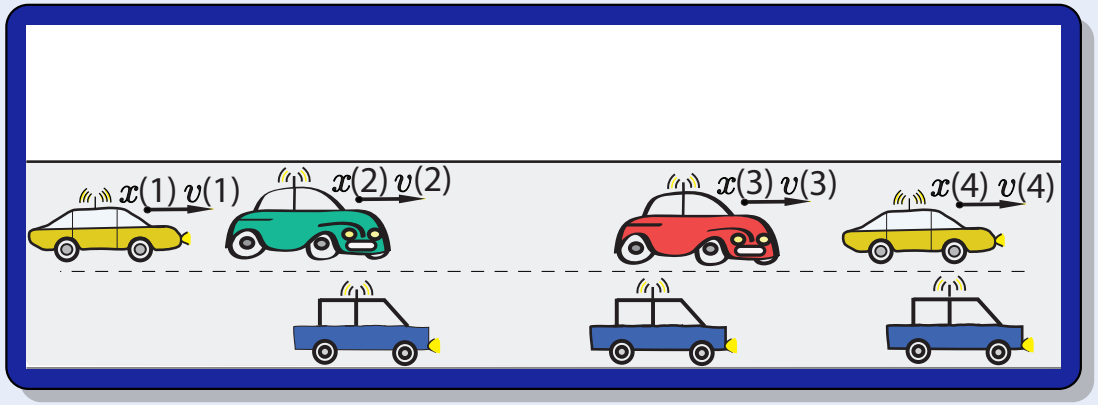

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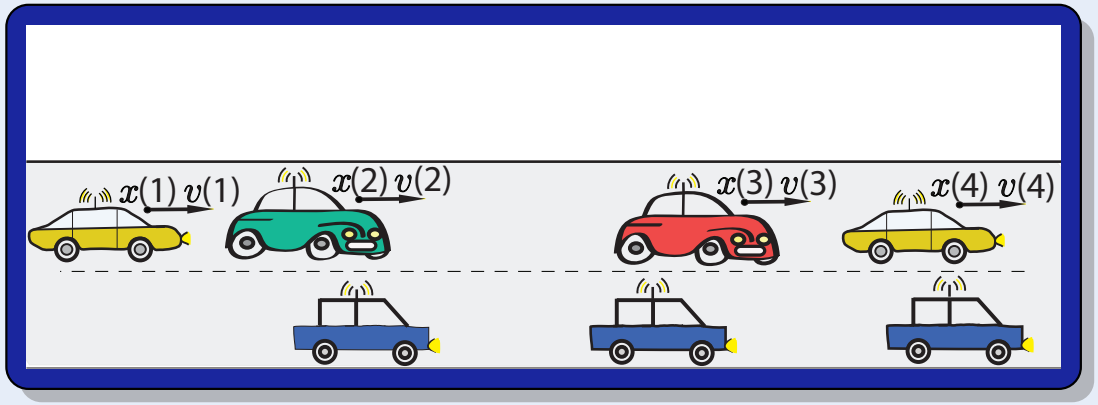
- Structural dynamics (communication/coupling)
  $\ell(i) := \mathit{carInFrontOf}(i)$



Model for Distributed Hybrid Systems

Q: How to model distributed hybrid systems A: Quantified Hybrid Programs

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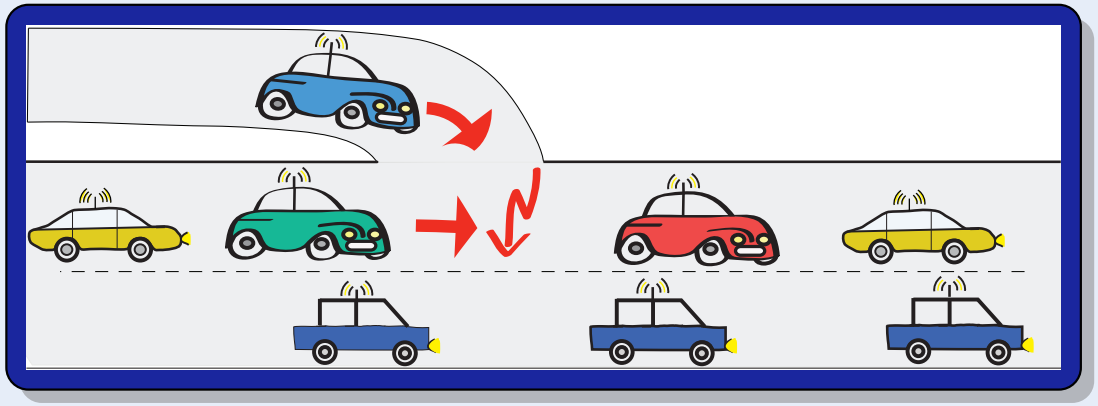

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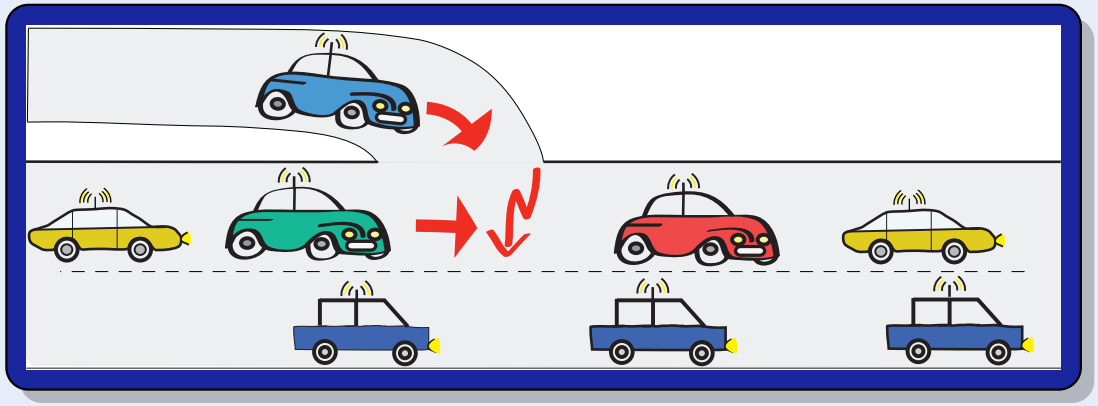
  $n := \texttt{new}\, Car$



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

Definition (Quantified hybrid program α)

$\forall i : C\ x(i)' = \theta$	(quantified ODE)	
$\forall i : C\ x(i) := \theta$	(quantified assignment)	jump & test
$?Q$	(conditional execution)	jump & test
$\alpha; \beta$	(seq. composition)	Kleene algebra
$\alpha \cup \beta$	(nondet. choice)	Kleene algebra
α^*	(nondet. repetition)	Kleene algebra

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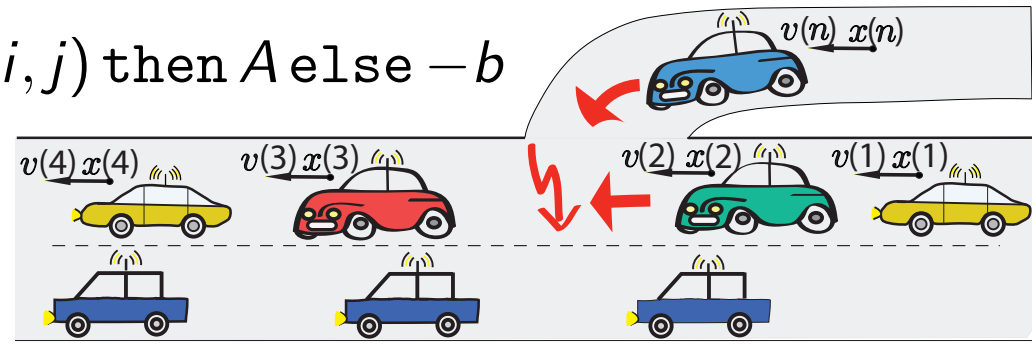
$$DCCS \equiv (ctrl; drive)^*$$

$$ctrl \equiv \forall i : C\ a(i) := \texttt{if}\ \forall j : C\ far(i,j)\ \texttt{then}\ A\ \texttt{else}\ -b$$

$$drive \equiv \forall i : C\ x(i)'' = a(i)$$



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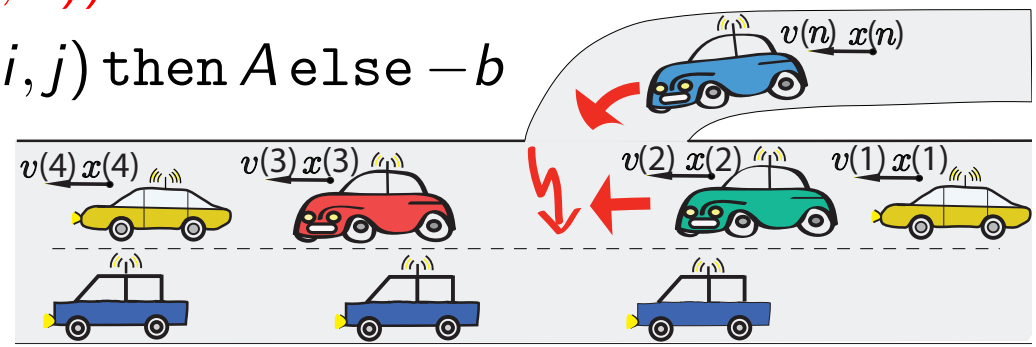

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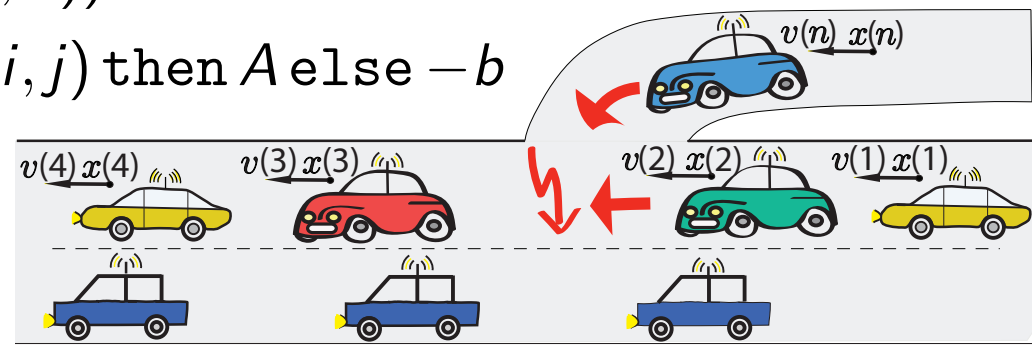
$\texttt{new}\ C$ is definable!



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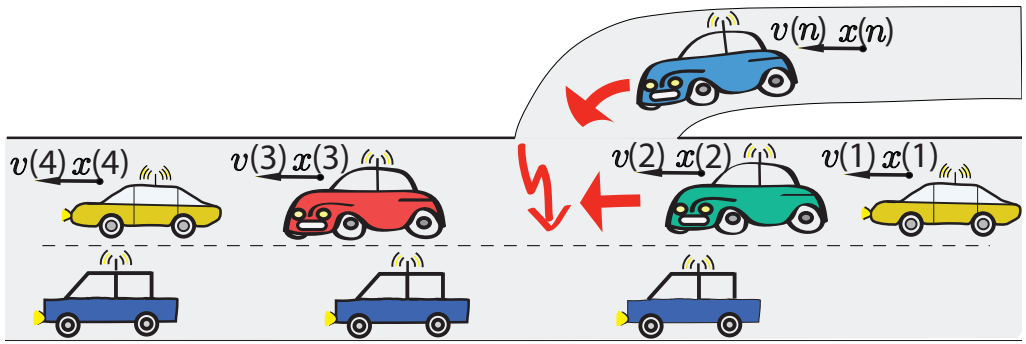
Definition (Qd$\mathcal{L}$ Formula ϕ)

$\neg, \wedge, \vee, \rightarrow, \ \forall x\,, \exists x\,, \ \ =, \geq, \ +, \cdot$ ($\mathbb{R}$-first-order part)
$[\alpha]\phi, \quad \langle\alpha\rangle\phi$ (dynamic part)

$$[(\mathit{appear}\,;\mathit{ctrl}\,;\mathit{drive})^*]\ \forall i{\neq}j : C\ x(i) \neq x(j)$$



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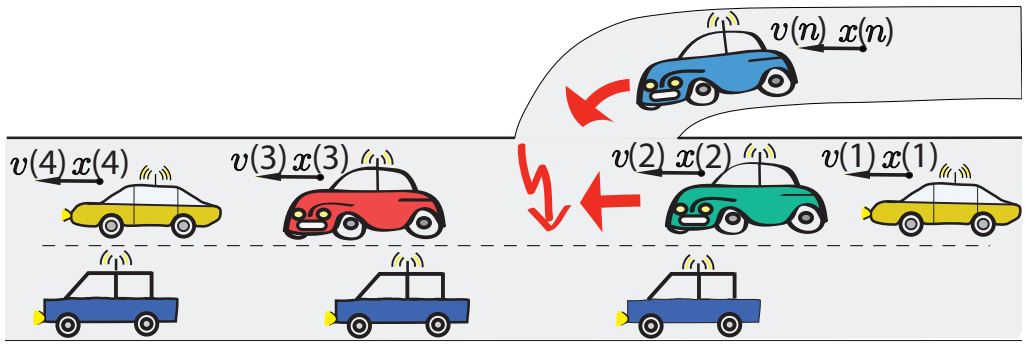
Definition (Qd$\mathcal{L}$ Formula ϕ)

$\neg, \wedge, \vee, \rightarrow,\ \forall x\,, \exists x\,,\ \ =, \geq,\ +, \cdot$ ($\mathbb{R}$-first-order part)
$[\alpha]\phi,\quad \langle\alpha\rangle\phi$ (dynamic part)

$$\forall i, j : C\ \ far(i,j) \rightarrow [(appear\,;ctrl\,;drive)^*]\ \forall i{\neq}j : C\ \ x(i) \neq x(j)$$



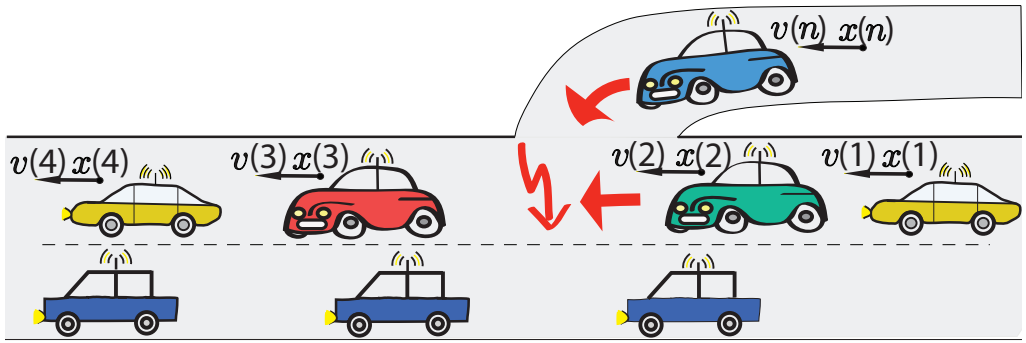
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Syntax

Definition (Qd$\mathcal{L}$ Formula ϕ)

$\neg, \wedge, \vee, \rightarrow,\ \forall x\,, \exists x\,,\ \ =, \geq,\ +, \cdot$ ($\mathbb{R}$-first-order part)
$[\alpha]\phi,\quad \langle\alpha\rangle\phi$ (dynamic part)

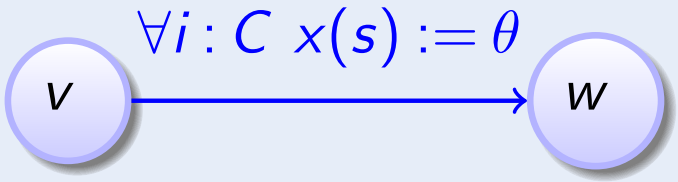
$$\forall i, j : C\ \ far(i,j) \rightarrow [(appear\,;ctrl\,;drive)^*]\ \forall i{\neq}j : C\ \ x(i) \neq x(j)$$

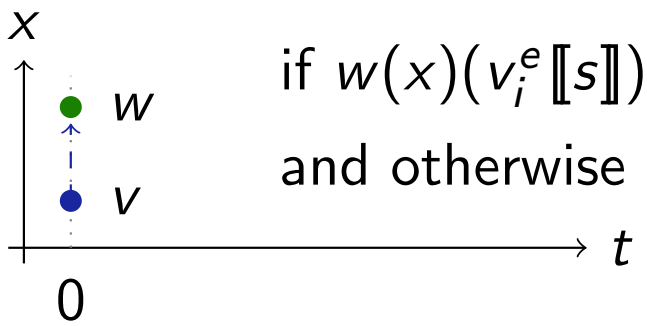
$$far(i,j)\ \equiv\ i \neq j\ \rightarrow\ x(i) < x(j) \wedge v(i) \leq v(j) \wedge a(i) \leq a(j)$$
$$\vee\ x(i) > x(j) \wedge v(i) \geq v(j) \wedge a(i) \geq a(j) \ldots$$



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



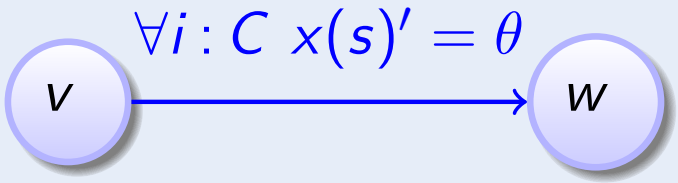


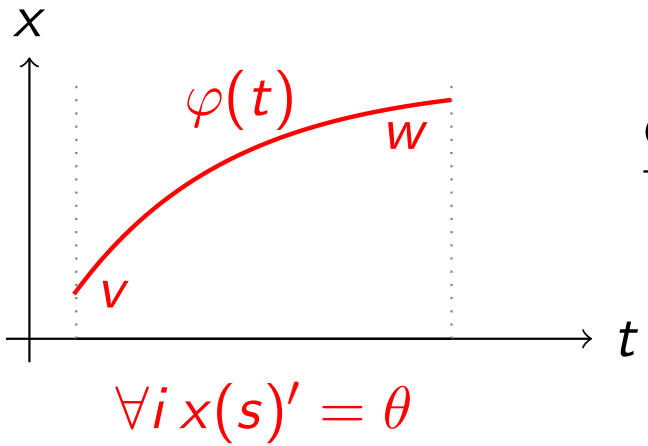
if $w(x)(v_i^e[\![s]\!]) = v_i^e[\![\theta]\!]$ (for all e)

and otherwise unchanged

Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)

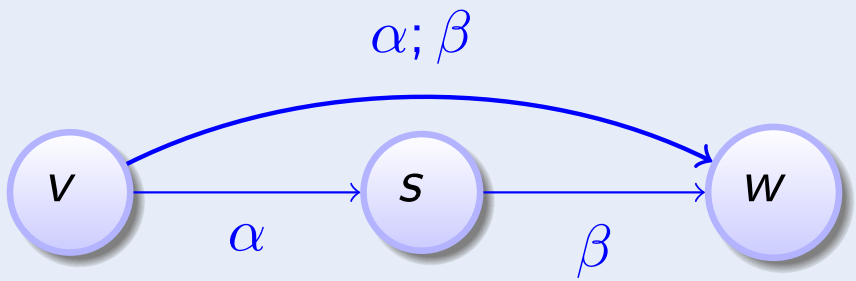




$$\frac{\mathrm{d}\,\varphi(t)_i^e[\![x(s)]\!]}{\mathrm{d}t}(\zeta) = \varphi(\zeta)_i^e[\![\theta]\!] \quad \text{(for all } e\text{)}$$

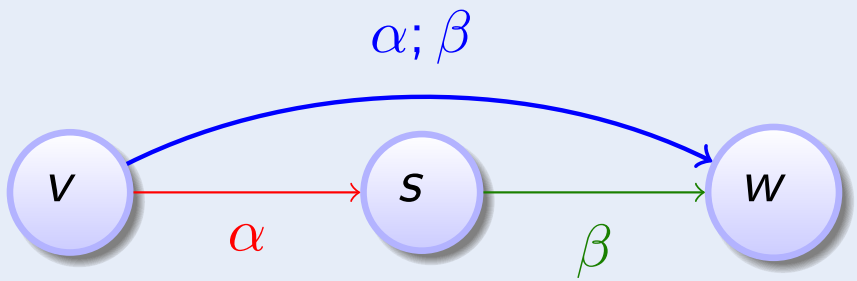
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

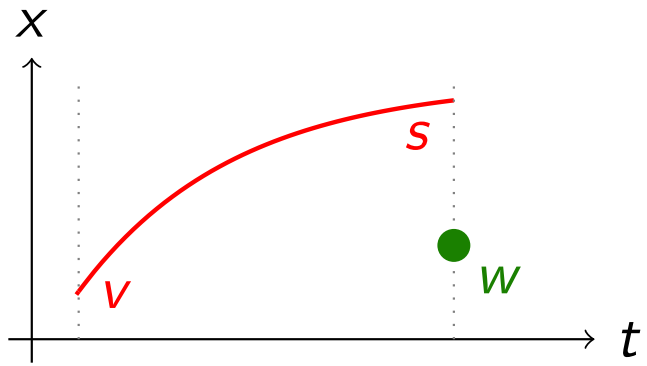
Definition (Quantified hybrid program α: transition semantics)



Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

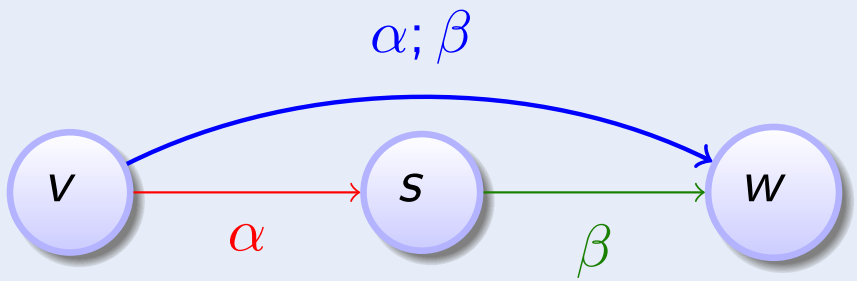
Definition (Quantified hybrid program α: transition semantics)

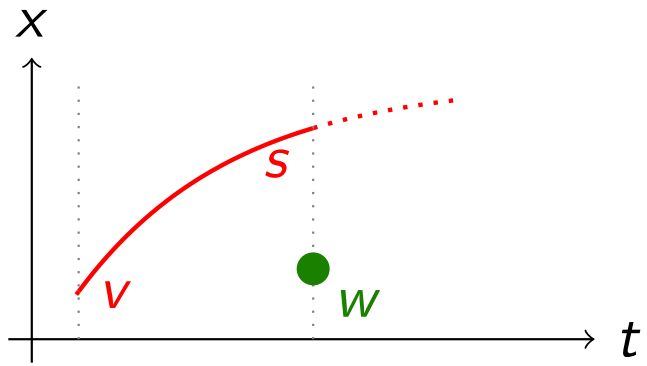




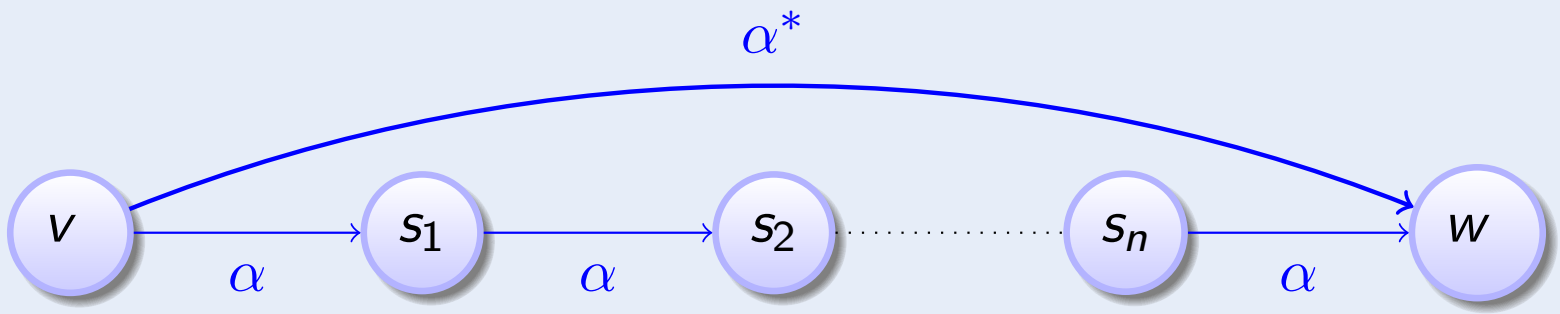
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)

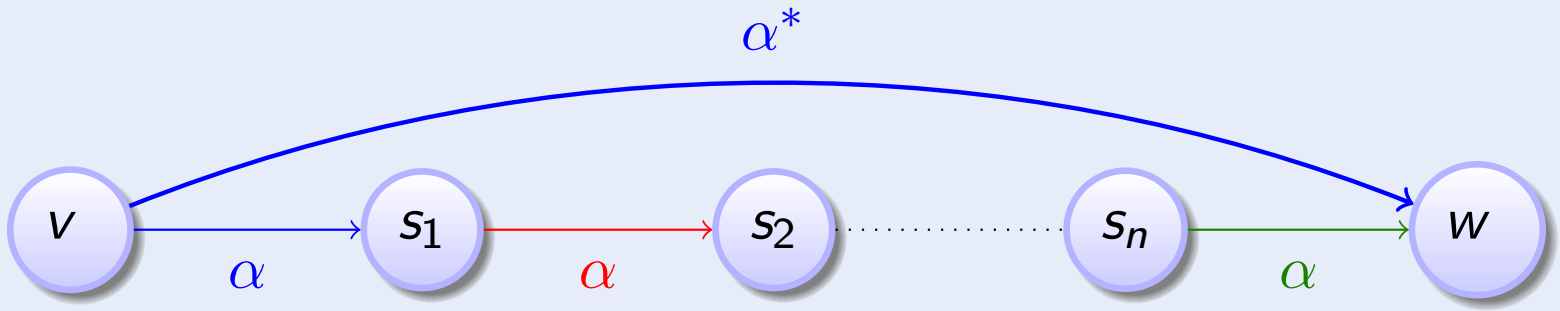


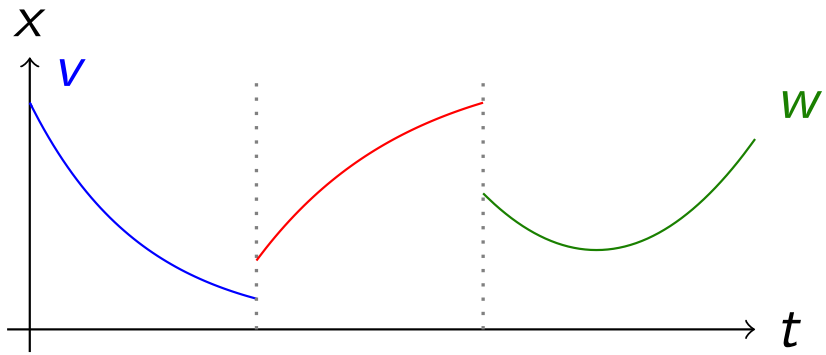


Definition (Quantified hybrid program α: transition semantics)



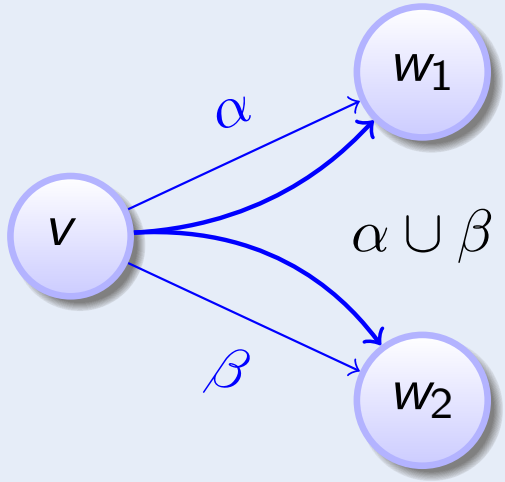
Definition (Quantified hybrid program α: transition semantics)



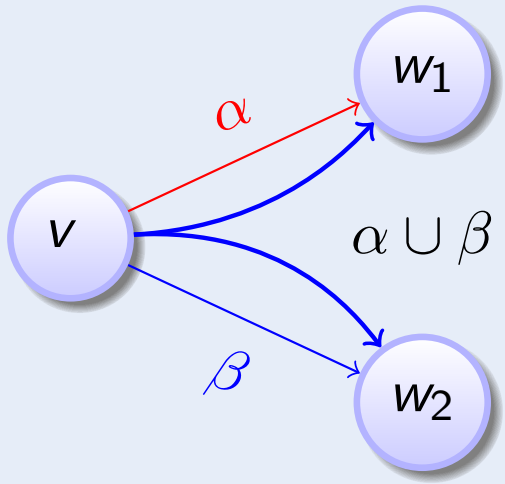


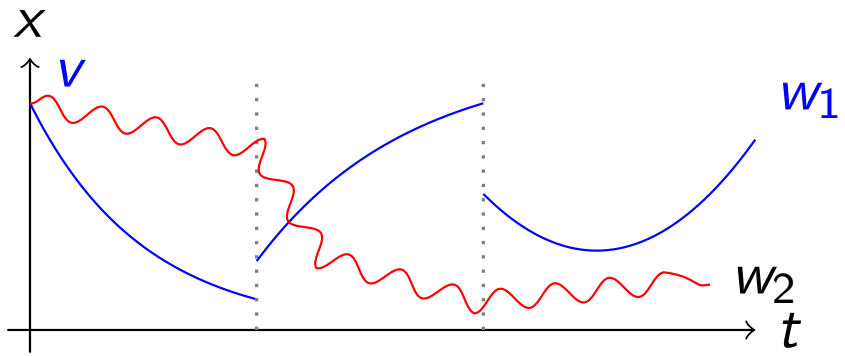
Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



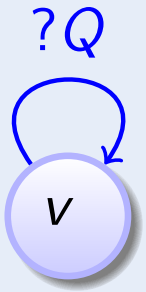
Definition (Quantified hybrid program α: transition semantics)



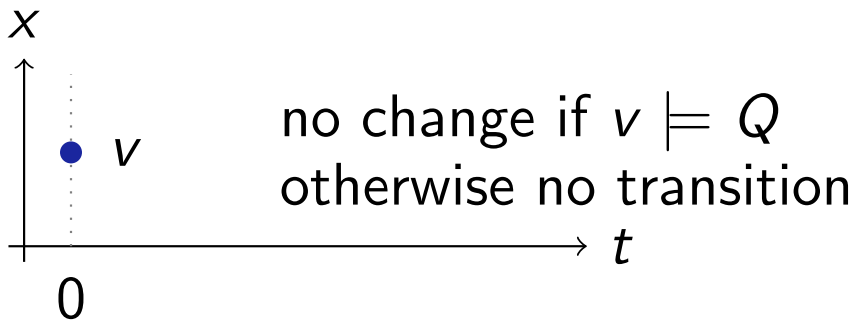


Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

Definition (Quantified hybrid program α: transition semantics)



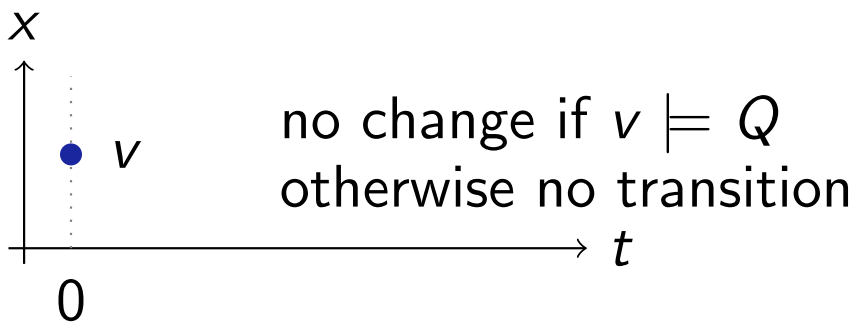
if $v \models Q$



no change if $v \models Q$
otherwise no transition

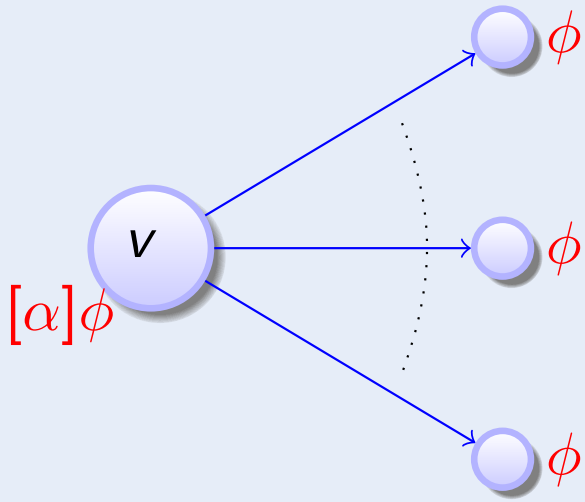
Definition (Quantified hybrid program α: transition semantics)

if $v \not\models Q$



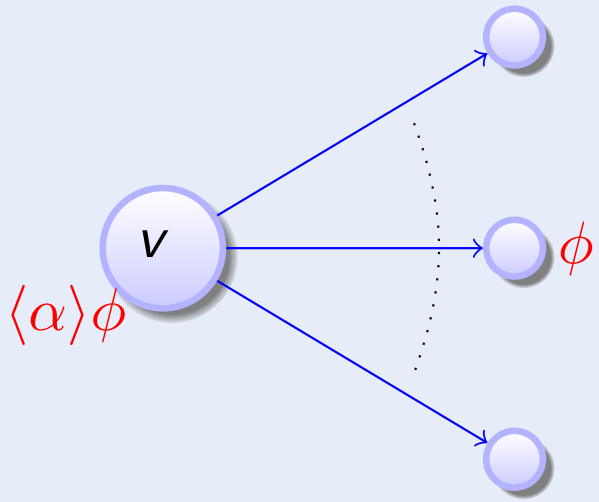
no change if $v \models Q$
otherwise no transition

Definition (Qd$\mathcal{L}$ Formula ϕ)

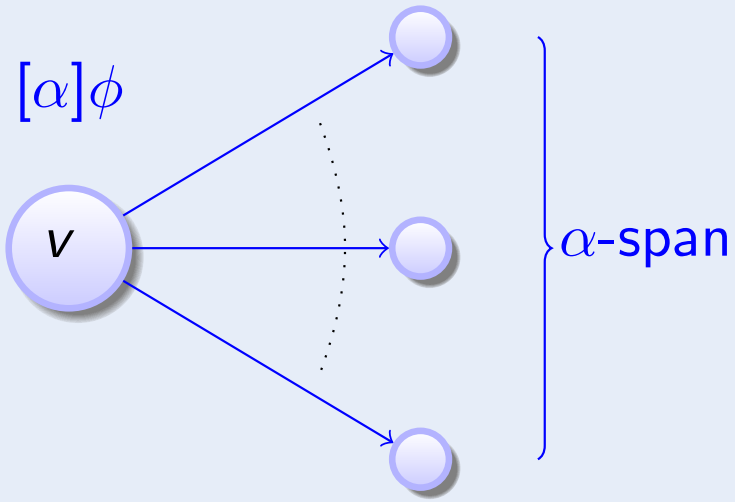


Quantified Differential Dynamic Logic Qd$\mathcal{L}$: Semantics

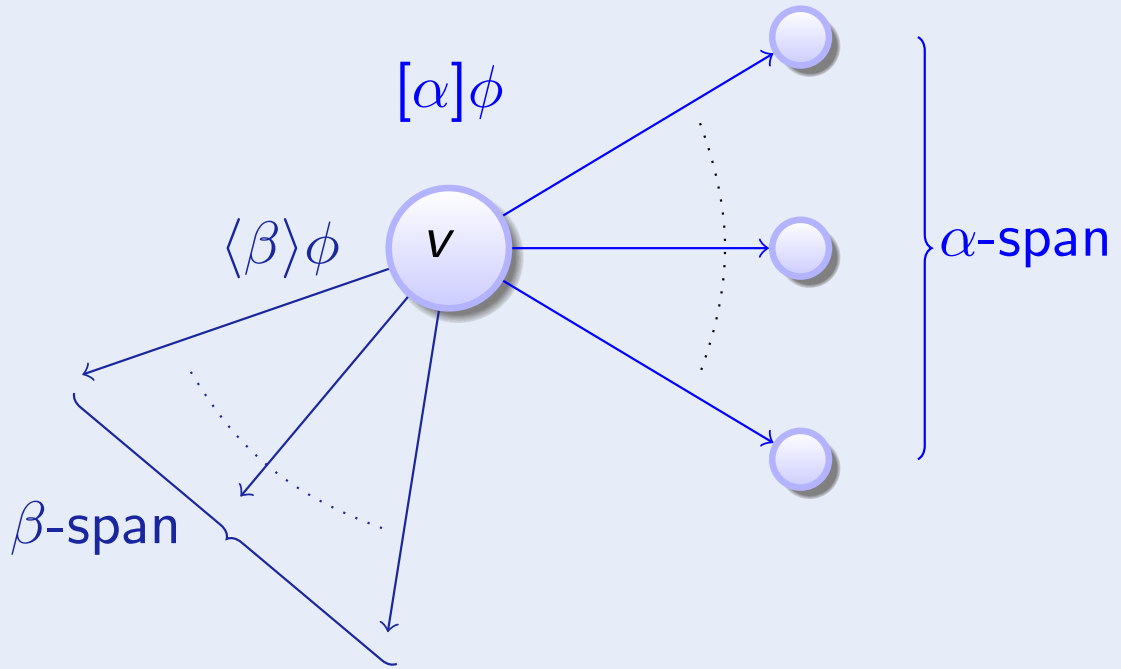
Definition (Qd$\mathcal{L}$ Formula ϕ)



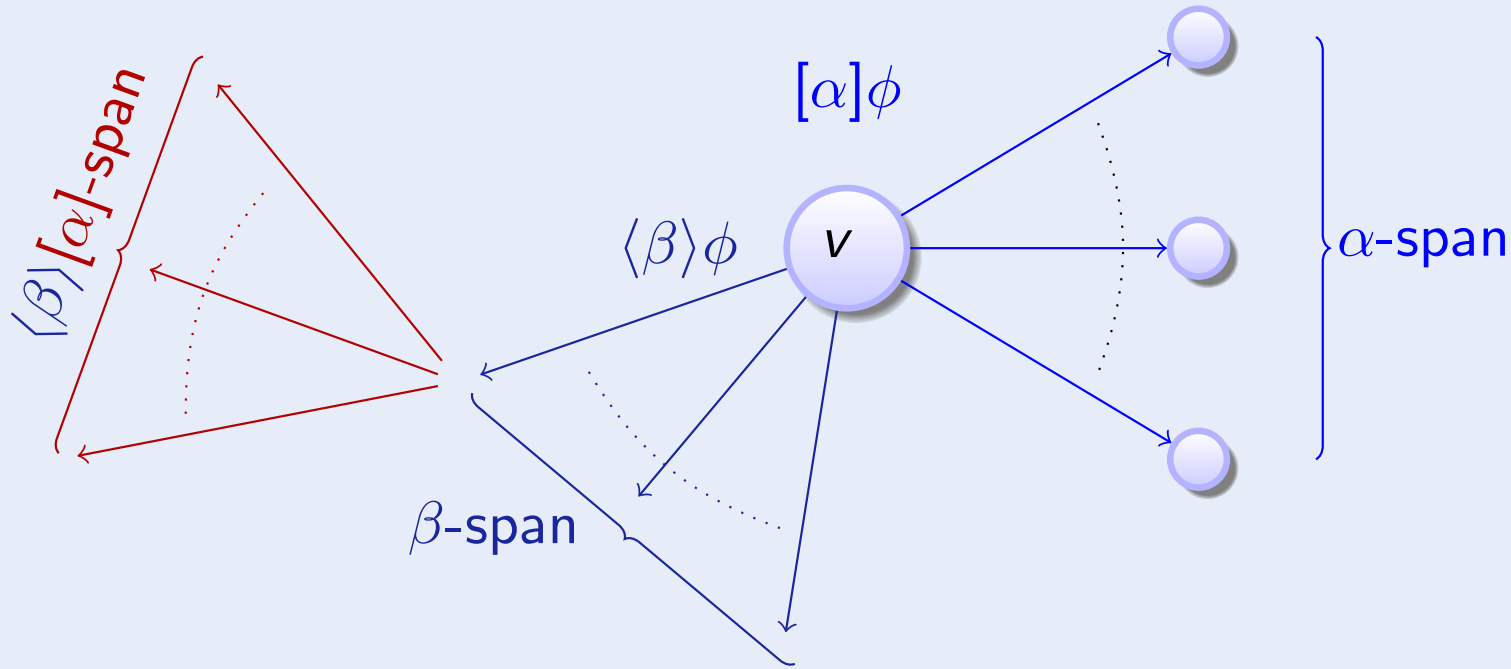
Definition (Qd$\mathcal{L}$ Formula ϕ)



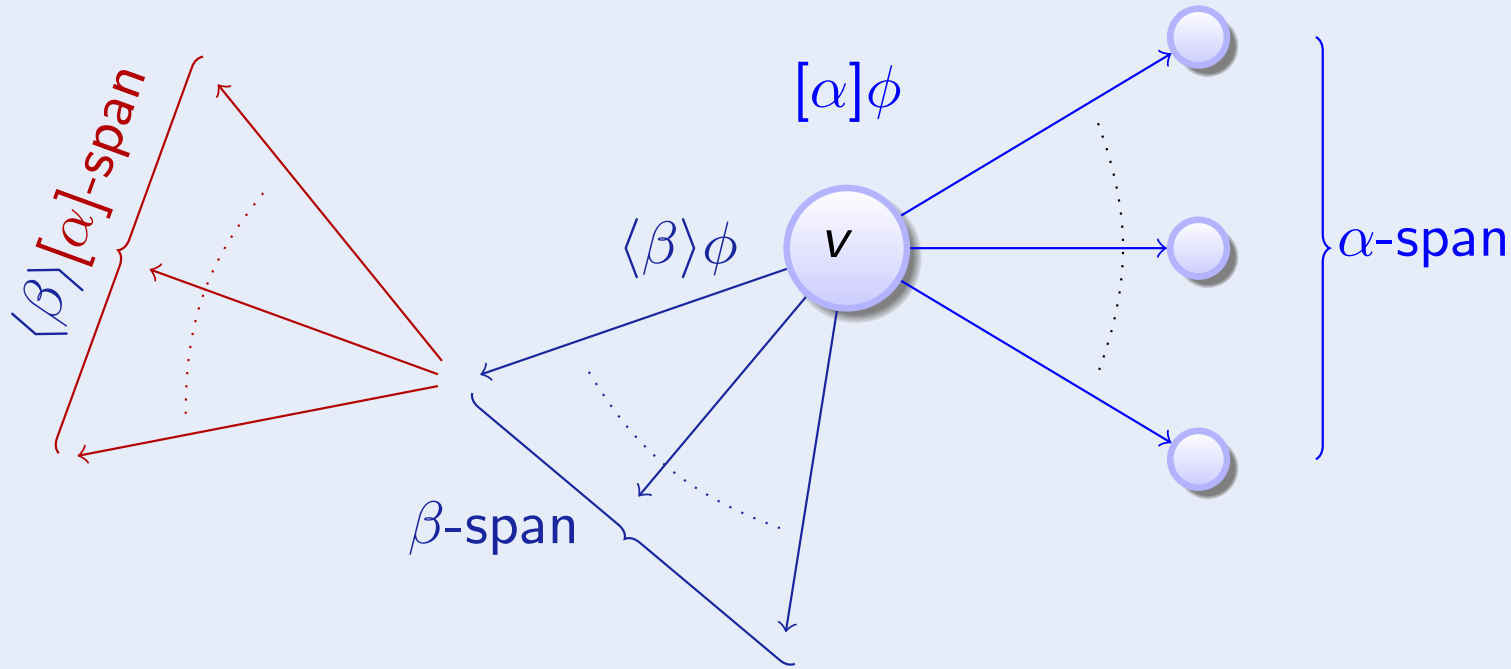
Definition (QdℒL Formula ϕ)



Definition (Qd$\mathcal{L}$ Formula ϕ)



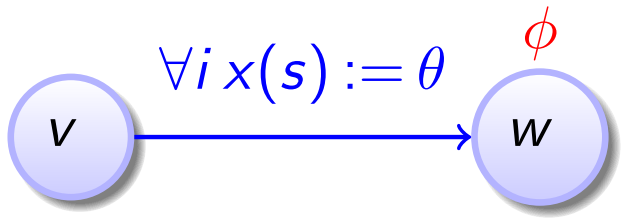
Definition (Qd$\mathcal{L}$ Formula ϕ)



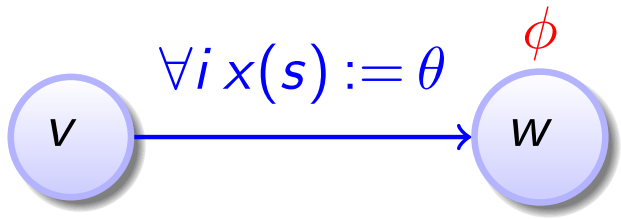
compositional semantics $\Rightarrow$ compositional calculus!

Outline (Verification Approach)

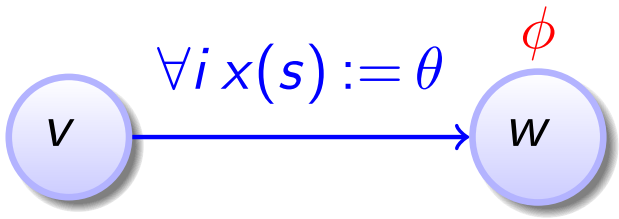
$$\frac{}{\phi([\forall i\, x(i) := \theta]x(u))}$$



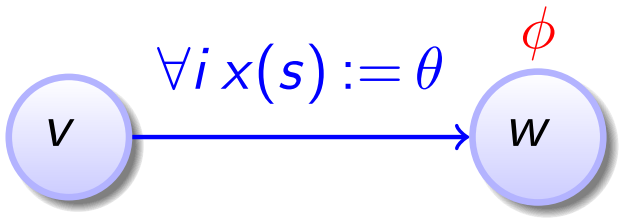
$$\frac{\forall i\,(i = u \to \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$

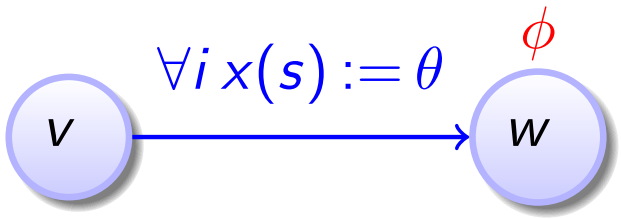


$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



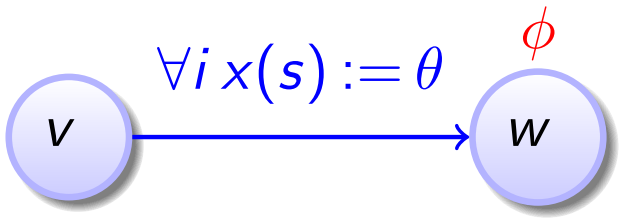
$$\frac{}{\phi([\underbrace{\forall i\, x(s) := \theta}]x(u))}$$

$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



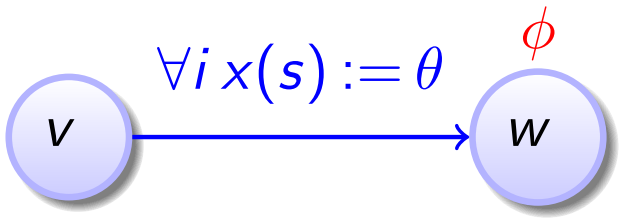
$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\,(s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi(\underbrace{[\forall i\, x(s) := \theta]}x(u))}$$

$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



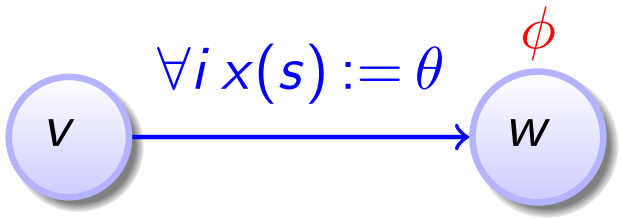
$$\frac{\texttt{if}\ \exists i\ s = u\ \texttt{then}\ \forall i\,(s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi(\underbrace{[\forall i\, x(s) := \theta]}x(u))}$$

$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



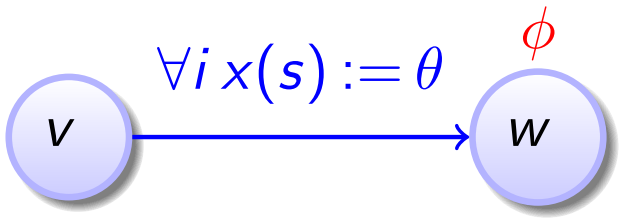
$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\,(s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi(\underbrace{[\forall i\, x(s) := \theta]}x(u))}$$

$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



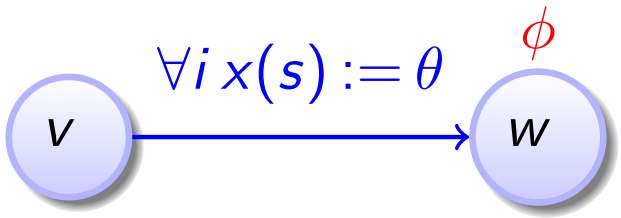
$$\frac{\texttt{if}\ \exists i\, s = u\ \texttt{then}\ \forall i\,(s = u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x(u))}{\phi(\underbrace{[\forall i\, x(s) := \theta]}x(u))}$$

$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \rightarrow \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



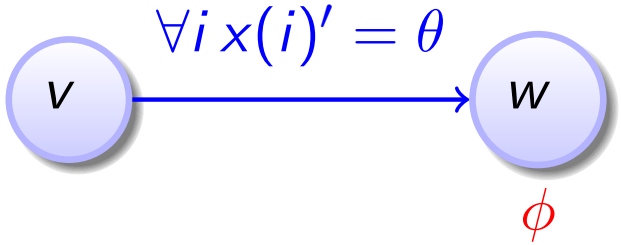
$$\frac{\texttt{if}\ \exists i\, s = [\mathcal{A}]u\ \texttt{then}\ \forall i\,(s = [\mathcal{A}]u \rightarrow \phi(\theta))\ \texttt{else}\ \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$

$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \to \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$

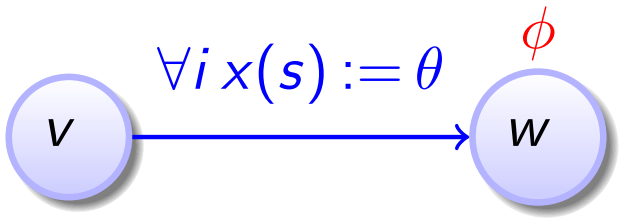


$$\frac{\texttt{if}\ \exists i\, s = [\mathcal{A}]u\ \texttt{then}\ \forall i\,(s = [\mathcal{A}]u \to \phi(\theta))\ \texttt{else}\ \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$

$$\frac{\forall t{\geq}0\,[\forall i\, x(i) := x_i(t)]\phi}{[\forall i\, x(i)' = \theta]\phi}$$

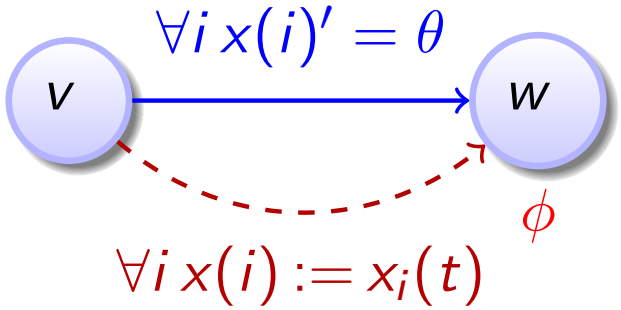


$$\frac{\forall i\,(i = [\forall i\, x(i) := \theta]u \to \phi(\theta))}{\phi([\forall i\, x(i) := \theta]x(u))}$$



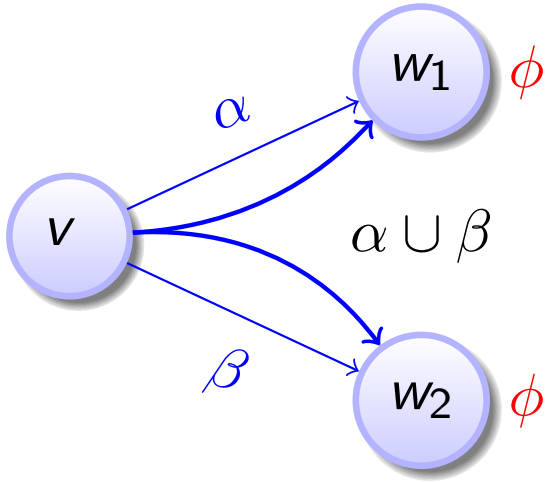
$$\frac{\texttt{if}\ \exists i\, s = [\mathcal{A}]u\ \texttt{then}\ \forall i\,(s = [\mathcal{A}]u \to \phi(\theta))\ \texttt{else}\ \phi(x([\mathcal{A}]u))}{\phi([\underbrace{\forall i\, x(s) := \theta}_{\mathcal{A}}]x(u))}$$

$$\frac{\forall t{\geq}0\,[\forall i\, x(i) := x_i(t)]\phi}{[\forall i\, x(i)' = \theta]\phi}$$

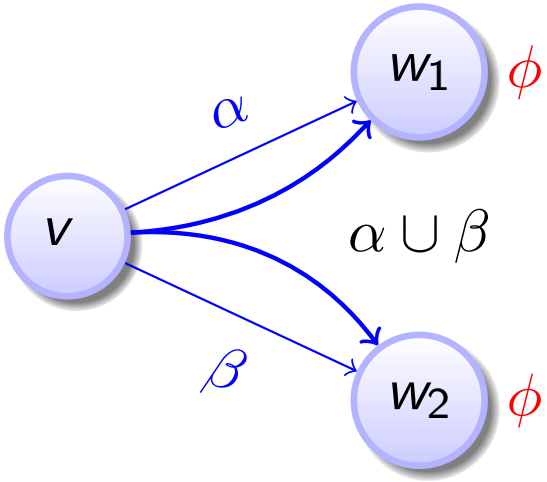


compositional semantics $\Rightarrow$ compositional rules!

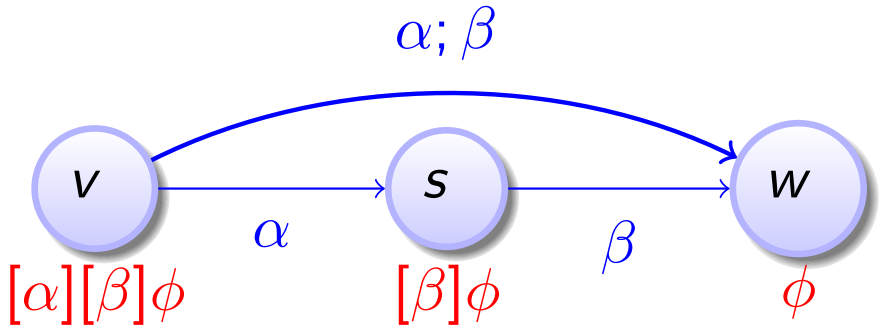
$$\frac{[\alpha]\phi \wedge [\beta]\phi}{[\alpha \cup \beta]\phi}$$



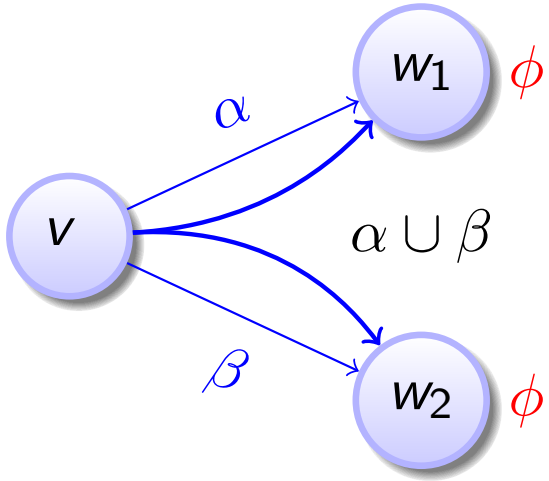
$$\frac{[\alpha]\phi \wedge [\beta]\phi}{[\alpha \cup \beta]\phi}$$



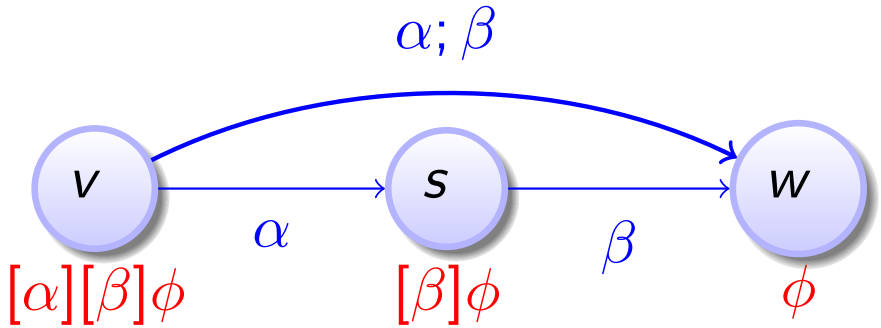
$$\frac{[\alpha][\beta]\phi}{[\alpha;\beta]\phi}$$



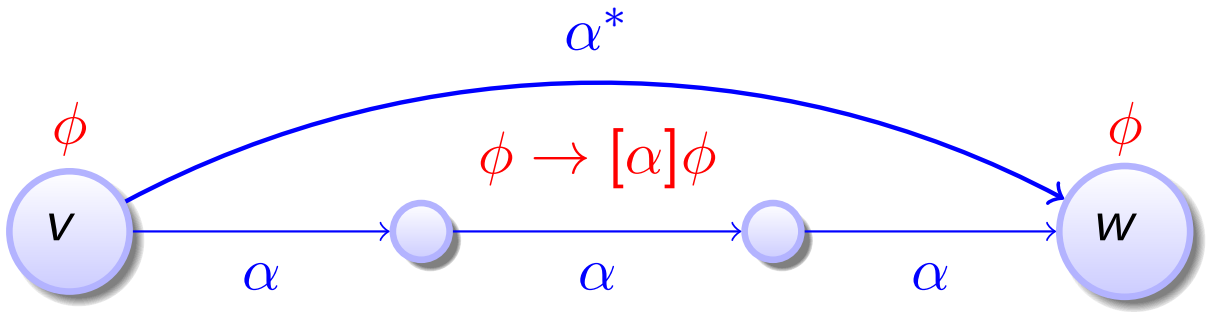
$$\frac{[\alpha]\phi \wedge [\beta]\phi}{[\alpha \cup \beta]\phi}$$



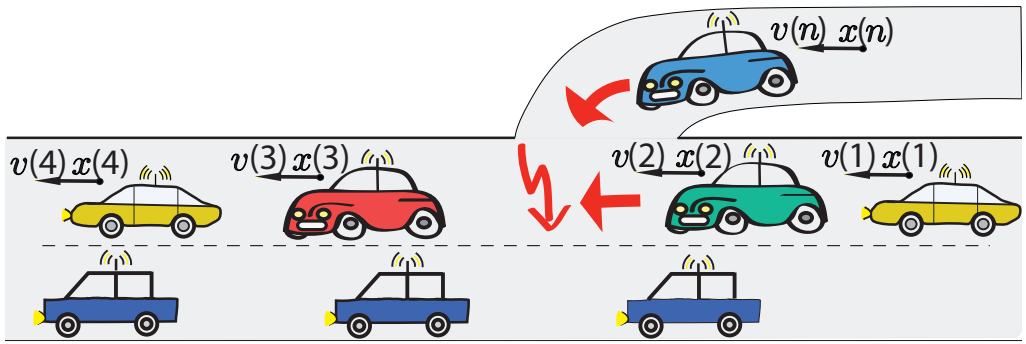
$$\frac{[\alpha][\beta]\phi}{[\alpha;\beta]\phi}$$



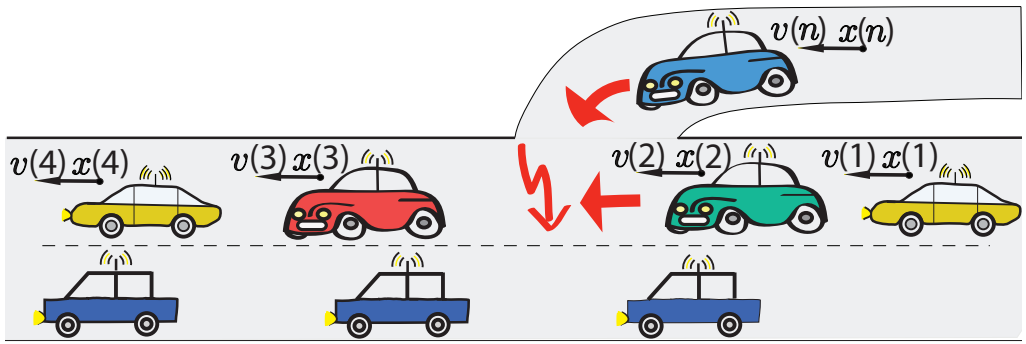
$$\frac{\phi \quad (\phi \rightarrow [\alpha]\phi)}{[\alpha^*]\phi}$$



$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

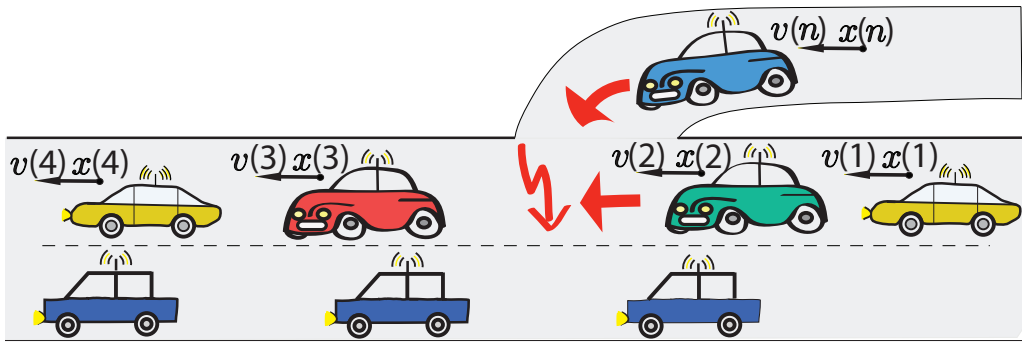


$$\frac{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}$$



$$\frac{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)]\forall j{\neq}k\, x(j){\neq}x(k)}{\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)}$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



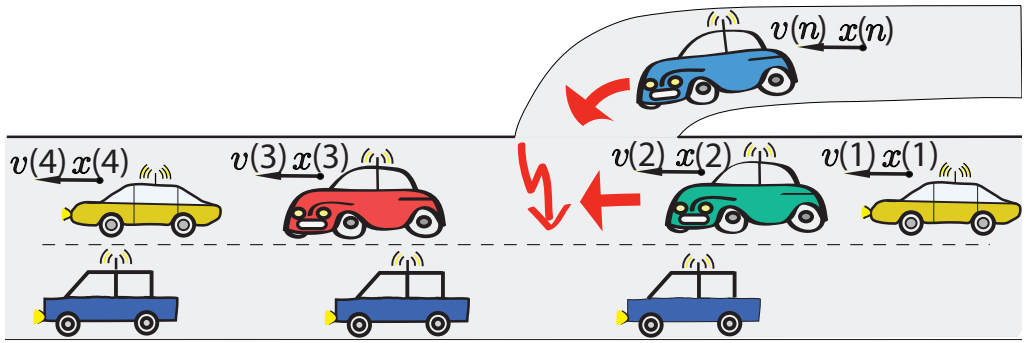
Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\ x(i){\neq}x(j) \to s{\geq}0 \to [\forall i\ x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\ \forall j{\neq}k\ x(j){\neq}x(k)$$

$$\forall i{\neq}j\ x(i){\neq}x(j) \to \forall t{\geq}0\ [\forall i\ x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)]\forall j{\neq}k\ x(j){\neq}x(k)$$

$$\forall i{\neq}j\ x(i){\neq}x(j) \to [\forall i\ x(i)' = v(i), v(i)' = -b]\ \forall j{\neq}k\ x(j){\neq}x(k)$$

$$\forall i{\neq}j\ x(i){\neq}x(j) \to [\forall i\ x(i)'' = -b]\ \forall j{\neq}k\ x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

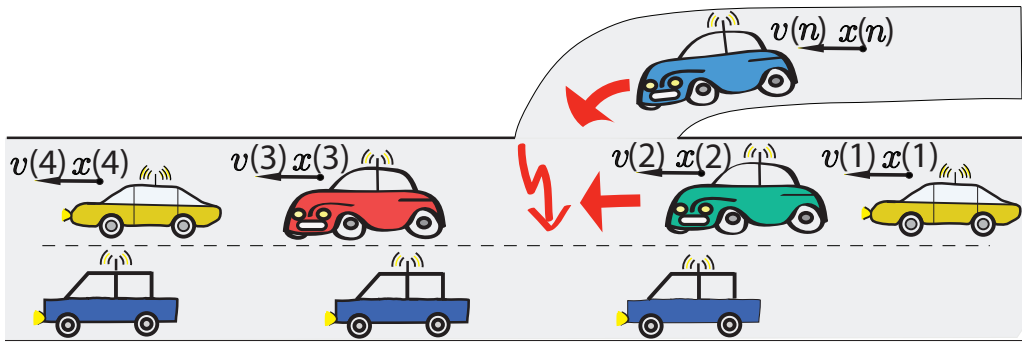
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, \left(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k)\right)$$

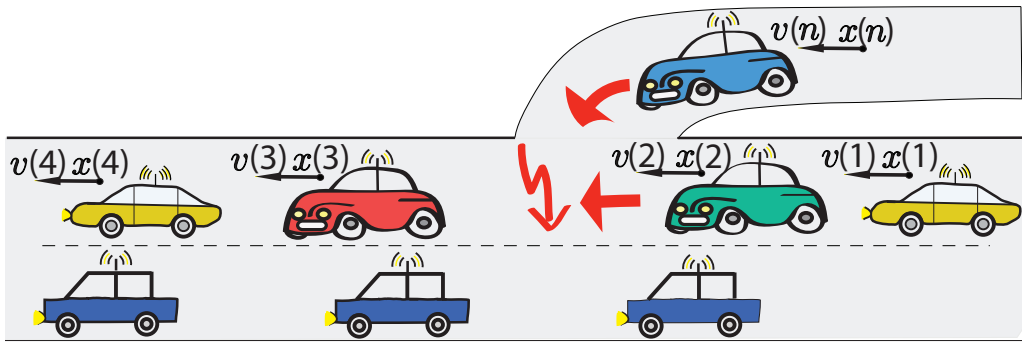
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall j{\neq}k \qquad \forall s{\geq}0\left(-\tfrac{b}{2}s^2+v(j)s+x(j) \neq -\tfrac{b}{2}s^2+v(k)s+x(k)\right)$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, \left(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k)\right)$$

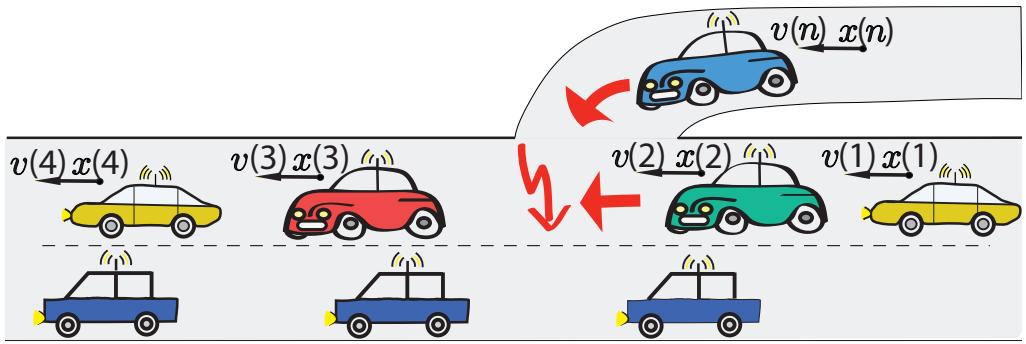
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall j{\neq}k\, \mathrm{QE}\forall s{\geq}0\left(-\tfrac{b}{2}s^2+v(j)s+x(j) \neq -\tfrac{b}{2}s^2+v(k)s+x(k)\right)$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, \left(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k)\right)$$

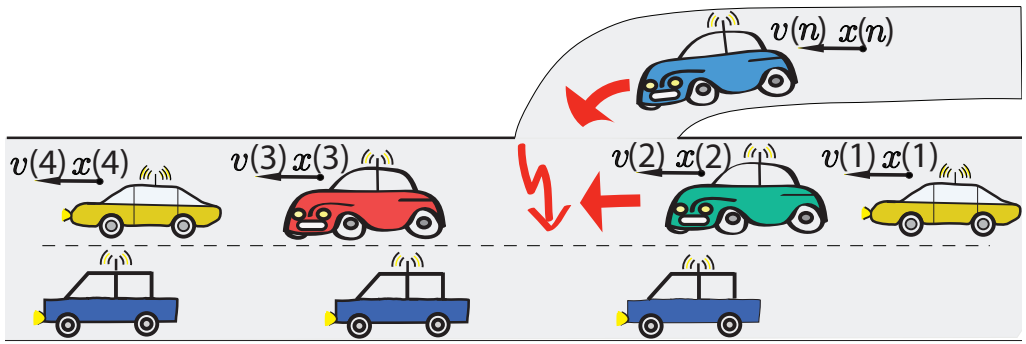
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall j{\neq}k\, (x(j){\leq}x(k){\wedge}v(j){\leq}v(k) \vee x(j){\geq}x(k){\wedge}v(j){\geq}v(k))$$

$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow \forall j{\neq}k\, (-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

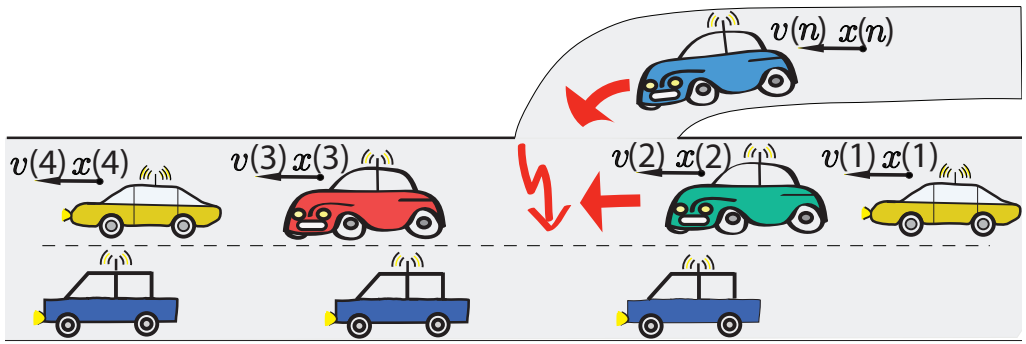
$$\forall i{\neq}j\, x(i){\neq}x(j), s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow s{\geq}0 \rightarrow [\forall i\, x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow \forall t{\geq}0\, [\forall i\, x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)' = v(i), v(i)' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$

$$\forall i{\neq}j\, x(i){\neq}x(j) \rightarrow [\forall i\, x(i)'' = -b]\, \forall j{\neq}k\, x(j){\neq}x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall X, Y, V, W\,(X \neq Y \rightarrow X \leq Y \wedge V \leq W \vee X \geq Y \wedge V \geq W)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow \forall j \neq k\,(x(j) \leq x(k) \wedge v(j) \leq v(k) \vee x(j) \geq x(k) \wedge v(j) \geq v(k))$$

$$\forall i \neq j\; x(i) \neq x(j), s \geq 0 \rightarrow \forall j \neq k\,(-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

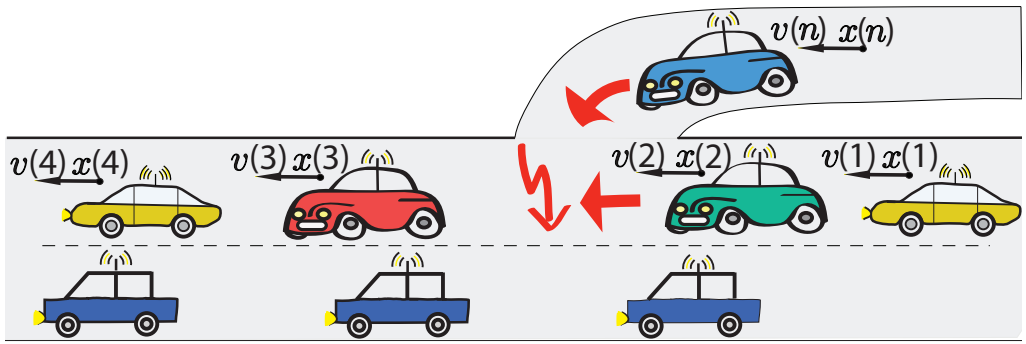
$$\forall i \neq j\; x(i) \neq x(j), s \geq 0 \rightarrow [\forall i\; x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow s \geq 0 \rightarrow [\forall i\; x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\, \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow \forall t \geq 0\, [\forall i\; x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow [\forall i\; x(i)' = v(i), v(i)' = -b]\, \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow [\forall i\; x(i)'' = -b]\, \forall j \neq k\; x(j) \neq x(k)$$



Deduction Modulo with Free Variables & Skolemization

$$\forall X, Y, V, W\,(X \neq Y \rightarrow X \leq Y \wedge V \leq W \vee X \geq Y \wedge V \geq W)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow \forall j \neq k\; (x(j) \leq x(k) \wedge v(j) \leq v(k) \vee x(j) \geq x(k) \wedge v(j) \geq v(k))$$

$$\forall i \neq j\; x(i) \neq x(j), s \geq 0 \rightarrow \forall j \neq k\; (-\tfrac{b}{2}s^2 + v(j)s + x(j) \neq -\tfrac{b}{2}s^2 + v(k)s + x(k))$$

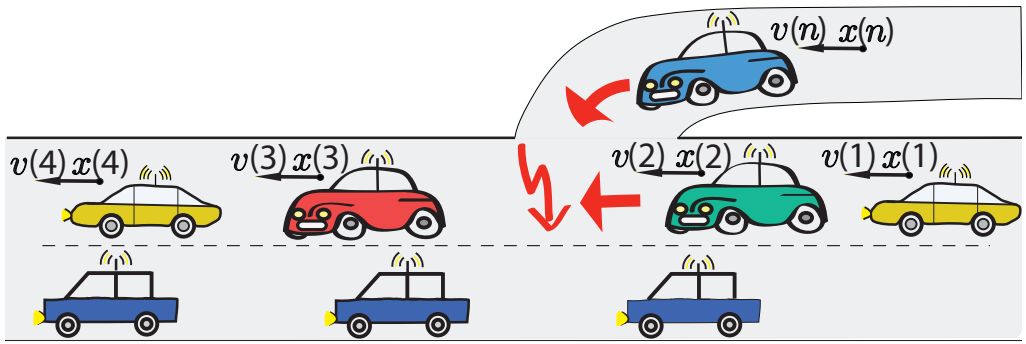
$$\forall i \neq j\; x(i) \neq x(j), s \geq 0 \rightarrow [\forall i\; x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\; \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow s \geq 0 \rightarrow [\forall i\; x(i) := -\tfrac{b}{2}s^2 + v(i)s + x(i)]\; \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow \forall t \geq 0\; [\forall i\; x(i) := -\tfrac{b}{2}t^2 + v(i)t + x(i)] \forall j \neq k\; x(j) \neq x(k)$$

$$\forall i \neq j\; x(i) \neq x(j) \rightarrow [\forall i\; x(i)' = v(i), v(i)' = -b]\; \forall j \neq k\; x(j) \neq x(k)$$

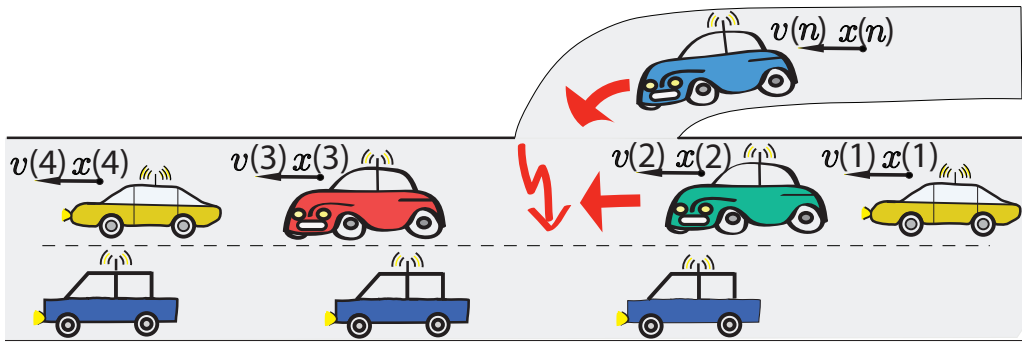
$$\forall i \neq j\; x(i) \neq x(j) \rightarrow [\forall i\; x(i)'' = -b]\; \forall j \neq k\; x(j) \neq x(k)$$



Actual Existence and Creation

Actual Existence Function $\mathbb{E}(\cdot)$

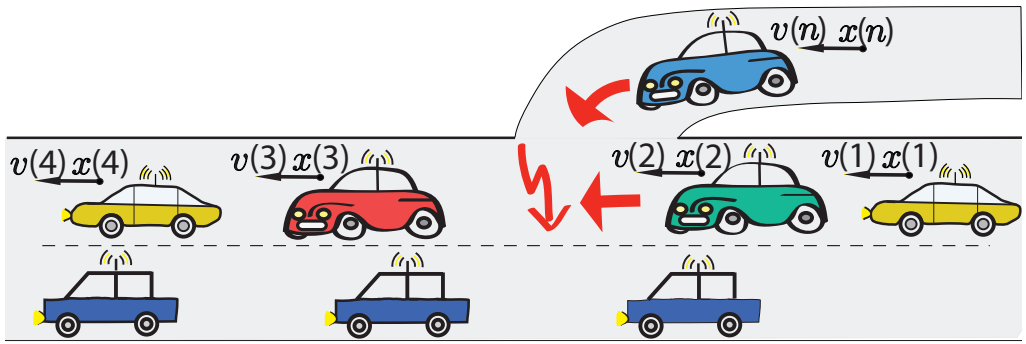
$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$



Actual Existence Function $\mathbb{E}(\cdot)$

$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$

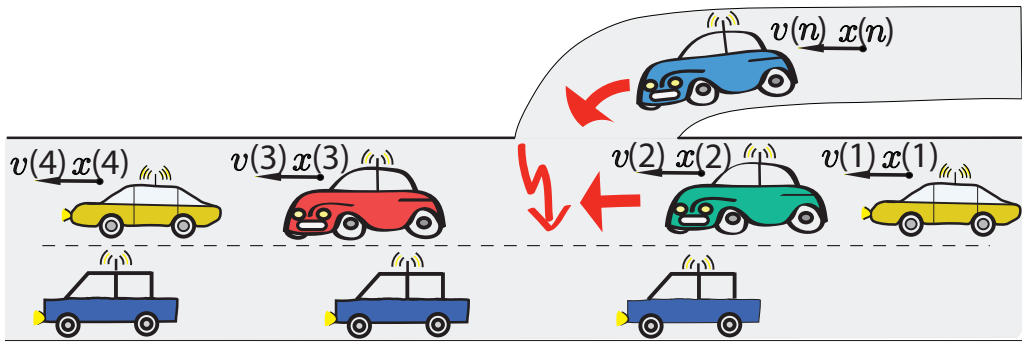
$$\frac{}{[n := \texttt{new } C]\phi}$$



Actual Existence Function $\mathbb{E}(\cdot)$

$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$

$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathbb{E}(n) = 0);\;\; \mathbb{E}(n) := 1]\phi}{[n := \texttt{new}\; C]\phi}$$



Actual Existence Function $\mathbb{E}(\cdot)$

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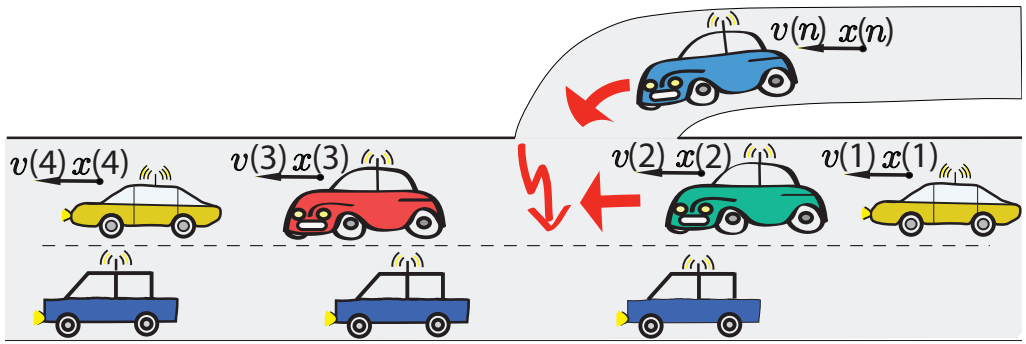
$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathbb{E}(n) = 0);\;\; \mathbb{E}(n) := 1]\phi}{[n := \texttt{new}\; C]\phi}$$



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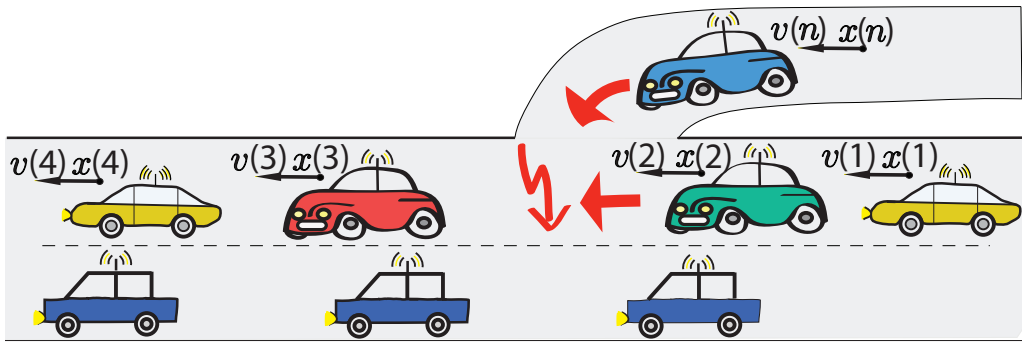
$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathbb{E}(n) = 0);\;\; \mathbb{E}(n) := 1]\phi}{[n := \texttt{new}\; C]\phi}$$

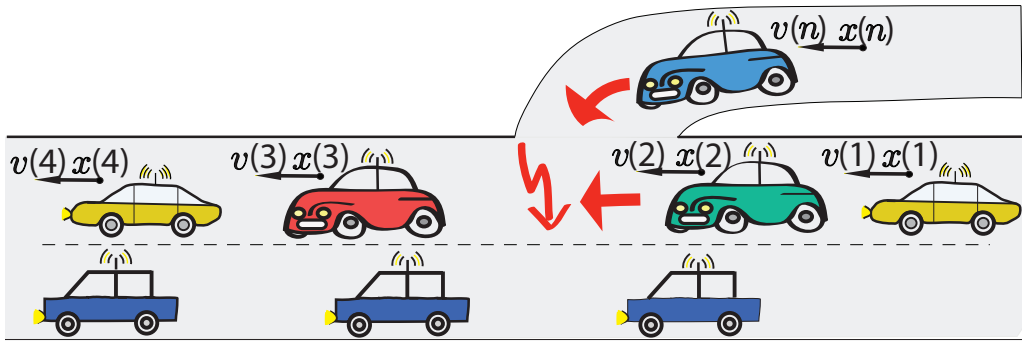


Actual Existence Function $\mathbb{E}(\cdot)$

$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$

$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathbb{E}(n) = 0);\;\; \mathbb{E}(n) := 1]\phi}{[n := \texttt{new}\; C]\phi}$$

$$\begin{aligned} \forall i : C!\;\; \phi &\equiv \\ \forall i : C!\;\; f(s) := \theta &\equiv \\ \forall i : C!\;\; f(s)' = \theta &\equiv \end{aligned}$$

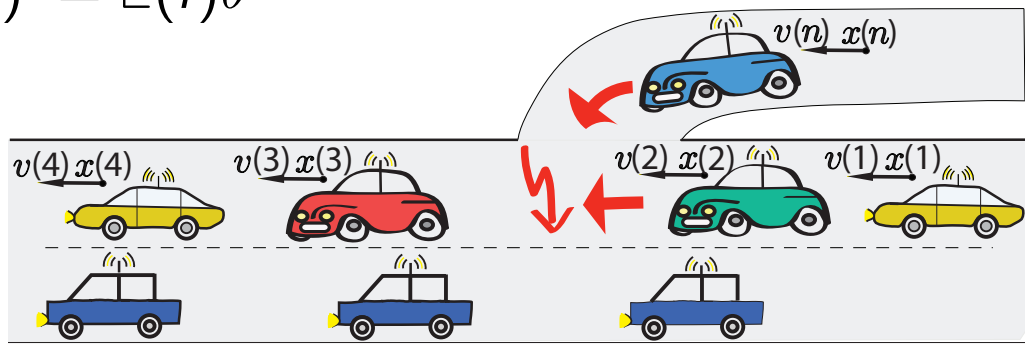


Actual Existence Function $\mathbb{E}(\cdot)$

$$\mathbb{E}(i) = \begin{cases} 0 & \text{if } i \text{ denotes a possible object} \\ 1 & \text{if } i \text{ denotes an actively existing objects} \end{cases}$$

$$\frac{[(\forall j : C \;\; n := j);\;\; ?(\mathbb{E}(n) = 0);\;\; \mathbb{E}(n) := 1]\phi}{[n := \texttt{new}\; C]\phi}$$

$$\begin{aligned} \forall i : C!\; \phi &\equiv \forall i : C\; (\mathbb{E}(i) = 1 \to \phi) \\ \forall i : C!\; f(s) := \theta &\equiv \forall i : C\; f(s) := (\texttt{if}\; \mathbb{E}(i) = 1\; \texttt{then}\; \theta\; \texttt{else}\; f(s)) \\ \forall i : C!\; f(s)' = \theta &\equiv \forall i : C\; f(s)' = \mathbb{E}(i)\theta \end{aligned}$$



Soundness and Completeness

Theorem (Relative Completeness) (LMCS'12)

Qd$\mathcal{L}$ calculus is a sound & complete axiomatisation of distributed hybrid systems relative to quantified differential equations.

▸ Proof 16p.

Soundness and Completeness

Theorem (Relative Completeness) (LMCS'12)

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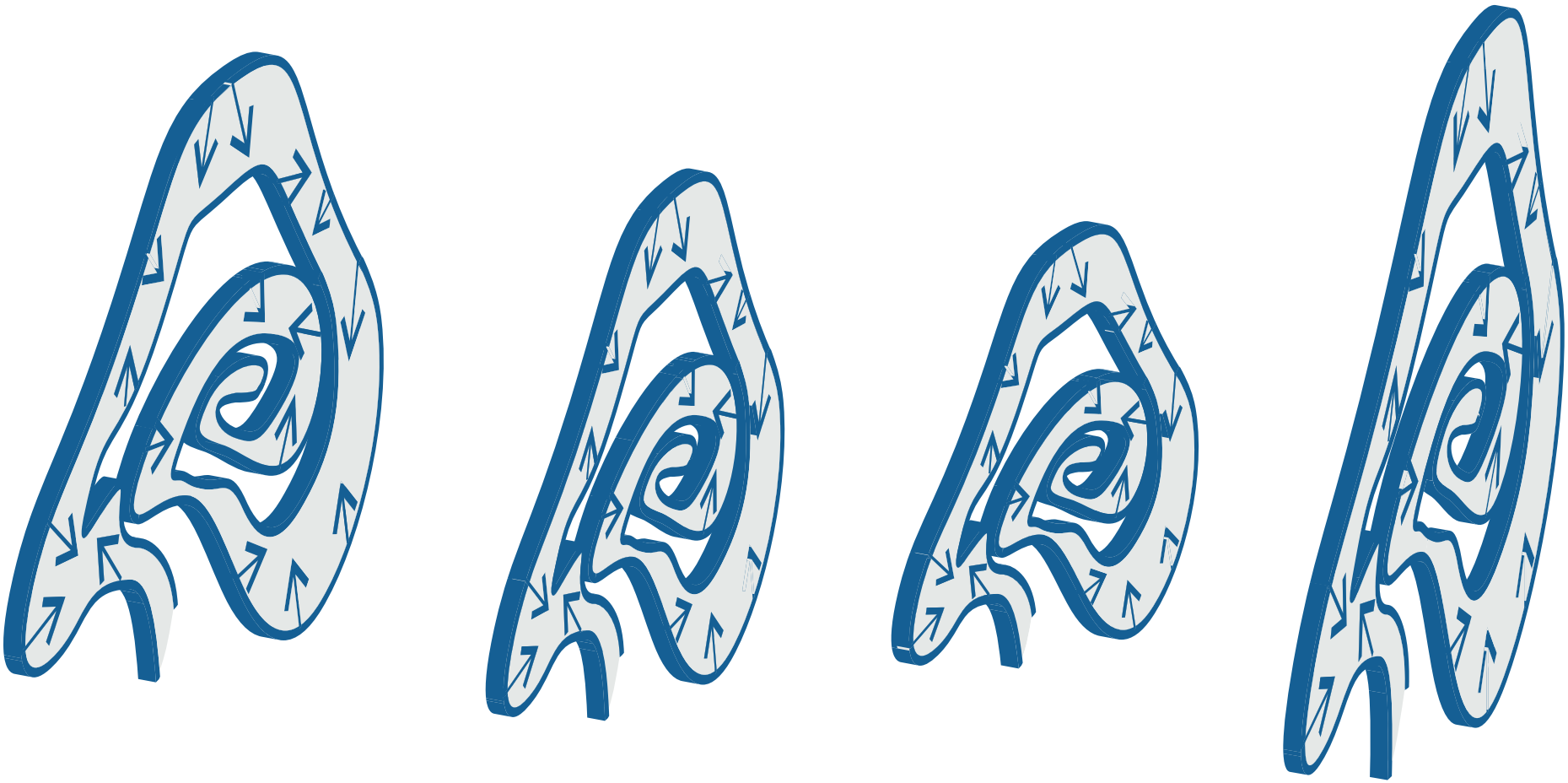
Corollary (Proof-theoretical Alignment)

proving distributed hybrid systems = proving dynamical systems!

Quantified Differential Invariants

Theorem (Quantified Differential Invariant) (HSCC'11)

$$\text{(QdI)}\quad \frac{Q\rightarrow[\forall i : C\ f(i)' := \theta]F'}{F\rightarrow[\forall i : C\ f(i)' = \theta \& Q]F} \quad \textit{is sound}$$



A Simple Proof with Quantified Differential Invariants

$$\forall i : C\ 2x(i)^3 \geq 1 \rightarrow [\forall i : C\ x(i)' = x(i)^2 + x(i)^4 + 2] \forall i : C\ 2x(i)^3 \geq 1$$

A Simple Proof with Quantified Differential Invariants

$$\frac{}{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2](\forall i : C\ 2x(i)^3 \geq 0)'}$$
$$\forall i : C\ 2x(i)^3 \geq 1 \rightarrow [\forall i : C\ x(i)' = x(i)^2 + x(i)^4 + 2]\forall i : C\ 2x(i)^3 \geq 1$$

A Simple Proof with Quantified Differential Invariants

$$\frac{}{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2]\forall i : C\ (2x(i)^3)' \geq 0}$$

$$\frac{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2](\forall i : C\ 2x(i)^3 \geq 0)'}{\forall i : C\ 2x(i)^3 \geq 1 \rightarrow [\forall i : C\ x(i)' = x(i)^2 + x(i)^4 + 2]\forall i : C\ 2x(i)^3 \geq 1}$$

A Simple Proof with Quantified Differential Invariants

$$\frac{}{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2]\forall i : C\ 6x(i)^2x(i)' \geq 0}$$

$$\frac{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2]\forall i : C\ (2x(i)^3)' \geq 0}{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2](\forall i : C\ 2x(i)^3 \geq 0)'}$$

$$\frac{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2](\forall i : C\ 2x(i)^3 \geq 0)'}{\forall i : C\ 2x(i)^3 \geq 1 \rightarrow [\forall i : C\ x(i)' = x(i)^2 + x(i)^4 + 2]\forall i : C\ 2x(i)^3 \geq 1}$$

A Simple Proof with Quantified Differential Invariants

$$\frac{}{\forall i : C\ 6x(i)^2(x(i)^2 + x(i)^4 + 2) \geq 0}$$

$$\frac{\forall i : C\ 6x(i)^2(x(i)^2 + x(i)^4 + 2) \geq 0}{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2]\forall i : C\ 6x(i)^2x(i)' \geq 0}$$

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A Simple Proof with Quantified Differential Invariants

$$\dfrac{}{\textit{true}}$$

$$\dfrac{}{\forall i : C\ 6x(i)^2(x(i)^2 + x(i)^4 + 2) \geq 0}$$

$$\dfrac{}{[\forall i : C\ x(i)' := x(i)^2 + x(i)^4 + 2]\forall i : C\ 6x(i)^2 x(i)' \geq 0}$$

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$$\forall i : C\ 2x(i)^3 \geq 1 \rightarrow [\forall i : C\ x(i)' = x(i)^2 + x(i)^4 + 2]\forall i : C\ 2x(i)^3 \geq 1$$

Outline

Driver's License Test for Robotic Cars?

Driver's License Test for Robotic Cars?

Driver's License Test for Robotic Cars? Proof!

Car Control: Local Lane Control Challenge

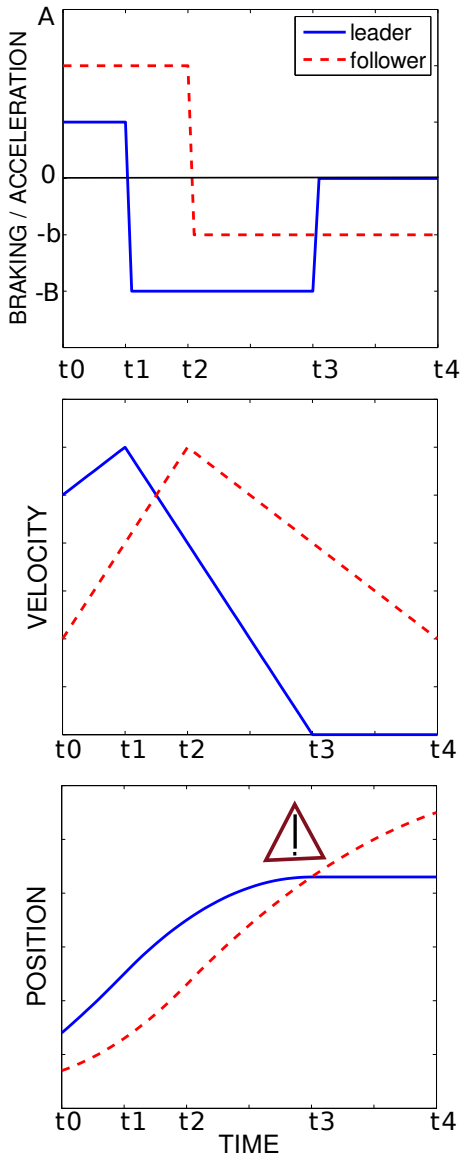
Challenge: Local lane dynamics

- A car controller for a differential equation respects separation of local lane.

Car Control: Local Lane Control Challenge

Challenge: Local lane dynamics

- A car controller for a differential equation respects separation of local lane.
- Follower car maintains safe distance to leader:



Car Control: Local Lane Control Challenge

Challenge: Local lane dynamics

- A car controller for a differential equation respects separation of local lane.
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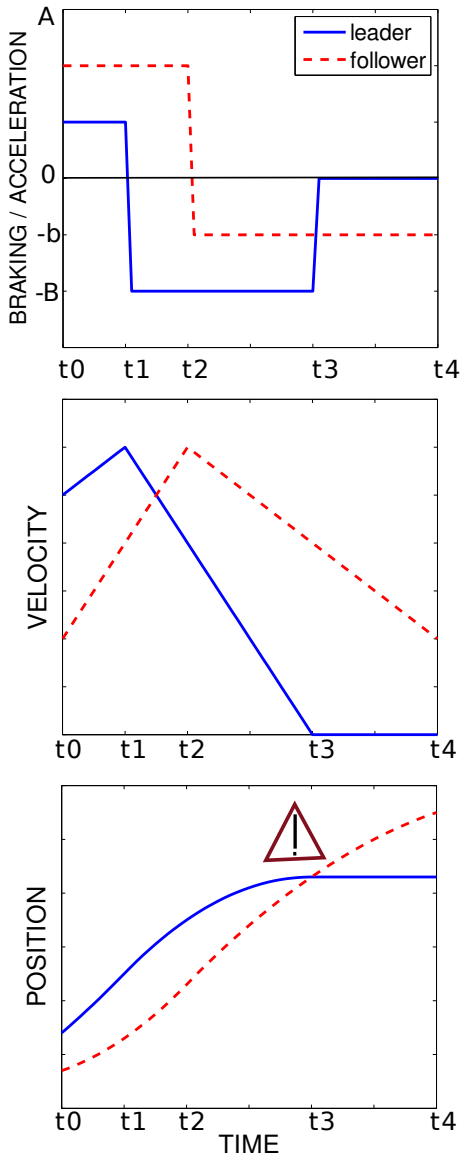
$$f \ll \ell \rightarrow [(a_i := ctrl;\ x_i'' = a_i)^*]\, f \ll \ell$$

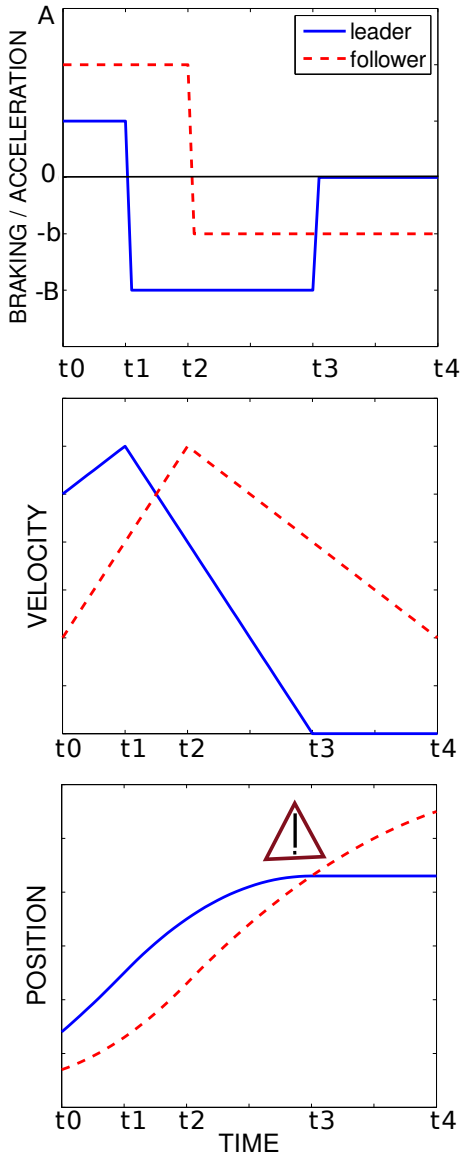


Challenge: Local lane dynamics

- A car controller for a differential equation respects separation of local lane.
- Follower car maintains safe distance to leader:

$$f \ll \ell \rightarrow [(a_i := ctrl;\ x_i'' = a_i)^*]\, f \ll \ell$$

$$f \ll \ell \equiv (x_f \leq x_\ell) \wedge (f \neq \ell) \rightarrow (x_\ell > x_f + \frac{v_f^2}{2b} - \frac{v_\ell^2}{2B} \wedge x_\ell > x_f \wedge v_f \geq 0 \wedge v_\ell \geq 0)$$



Car Control: Global Lane Control Challenge

Challenge: Global lane dynamics

- All controllers for arbitrarily many differential equations respect separation globally on lane.

Car Control: Global Lane Control Challenge

Challenge: Global lane dynamics

- All controllers for arbitrarily many differential equations respect separation globally on lane.
- Each car safe behind all others

Car Control: Global Lane Control Challenge

Challenge: Global lane dynamics

- All controllers for arbitrarily many differential equations respect separation globally on lane.
- Each car safe behind all others

$$[(\forall i\, a(i) := ctrl;\ \forall i\, x(i)'' = a(i))^*]\forall i,j\, i \ll j$$

Car Control: Local Highway Control Challenge

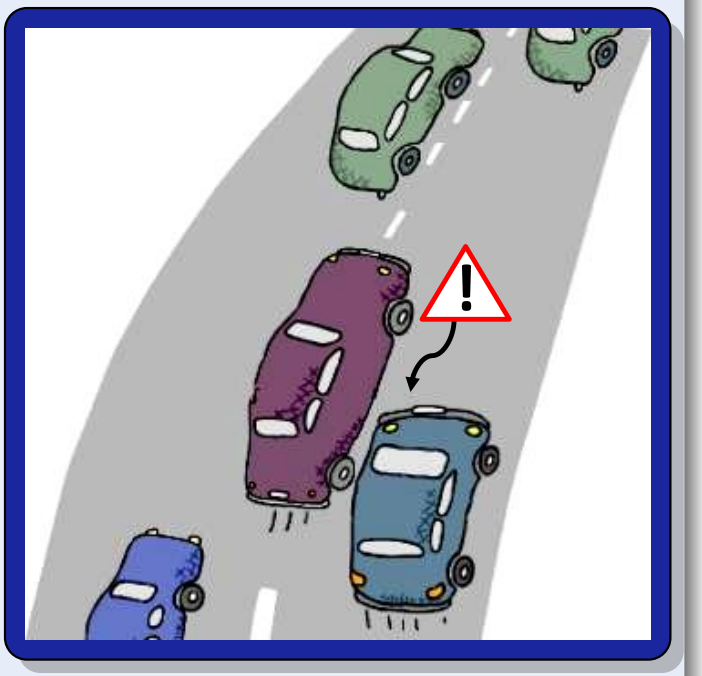
Challenge: Local highway dynamics

- All controllers for arbitrarily many differential equations respect separation locally on highway.

Car Control: Local Highway Control Challenge

Challenge: Local highway dynamics

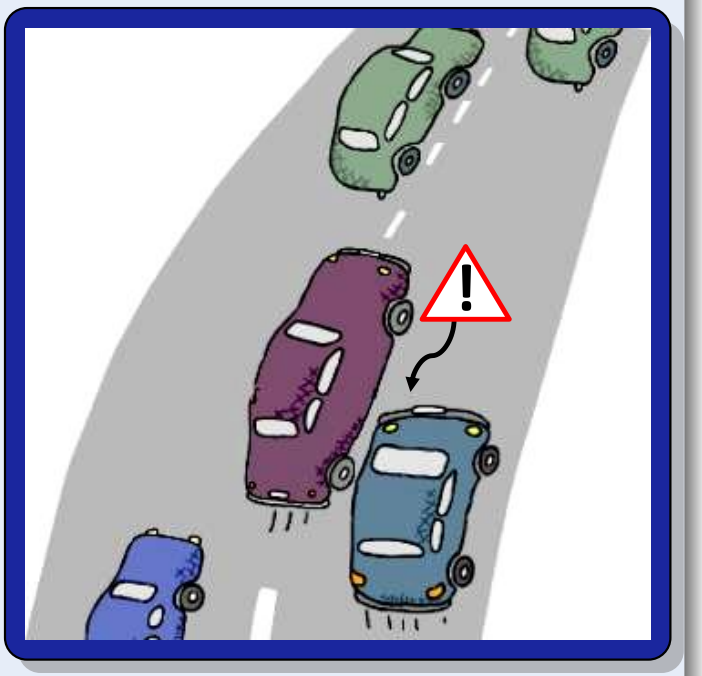
- All controllers for arbitrarily many differential equations respect separation locally on highway.
- For each lane: all controllers for the differential equations respect separation even if cars appear or disappear.



Car Control: Local Highway Control Challenge

Challenge: Local highway dynamics

- All controllers for arbitrarily many differential equations respect separation locally on highway.
- For each lane: all controllers for the differential equations respect separation even if cars appear or disappear.
- Each car safe behind all others, even if new cars appear or disappear.



Challenge: Local highway dynamics

- All controllers for arbitrarily many differential equations respect separation locally on highway.
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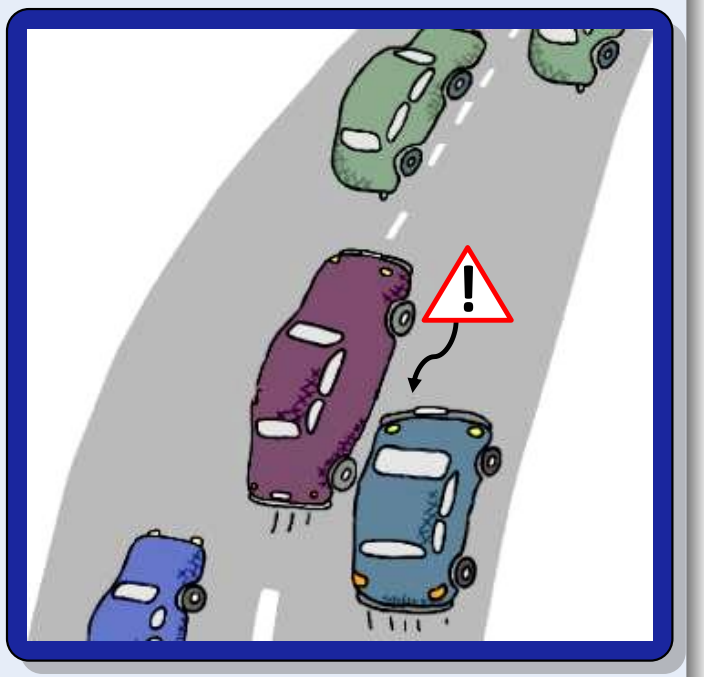


$$[(n := \texttt{new}\ C;\ \forall i\ a(i) := ctrl;\ \forall i\ x(i)'' = a(i))^*]\forall i,j\ i \ll j$$

Car Control: Global Highway Control Challenge

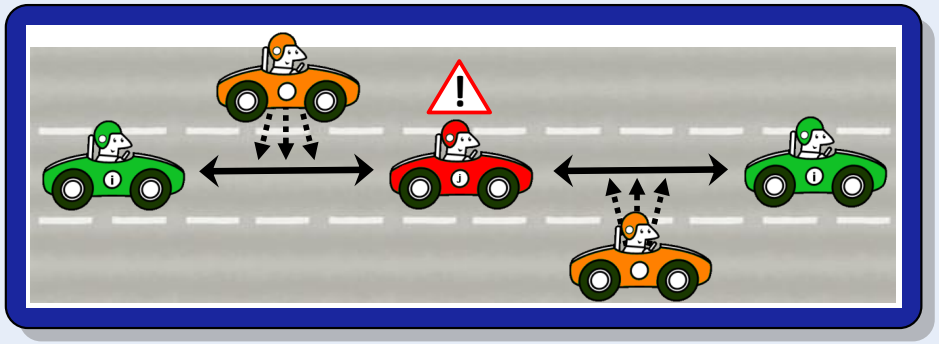
Challenge: Global highway dynamics

- All controllers for arbitrarily many differential equations respect separation globally on highway.

Car Control: Global Highway Control Challenge

Challenge: Global highway dynamics

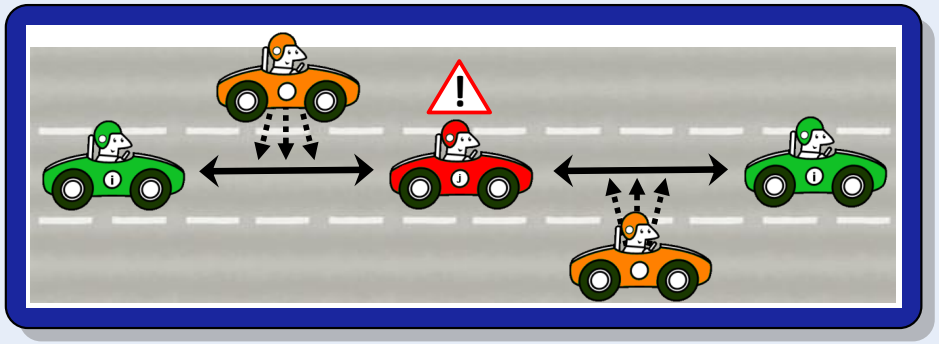
- All controllers for arbitrarily many differential equations respect separation globally on highway.
- All controllers for the differential equations respect separation even if cars switch lanes.



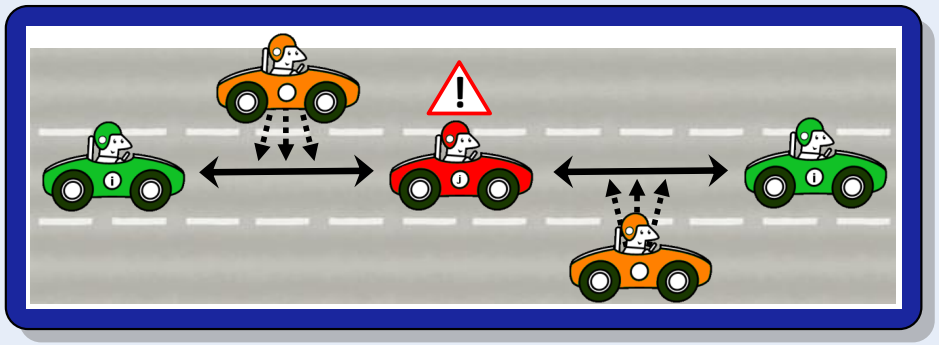
Car Control: Global Highway Control Challenge

Challenge: Global highway dynamics

- All controllers for arbitrarily many differential equations respect separation globally on highway.
- All controllers for the differential equations respect separation even if cars switch lanes.
- On all lanes, all car safe behind all others on their lanes, even if cars switch lanes.



Challenge: Global highway dynamics



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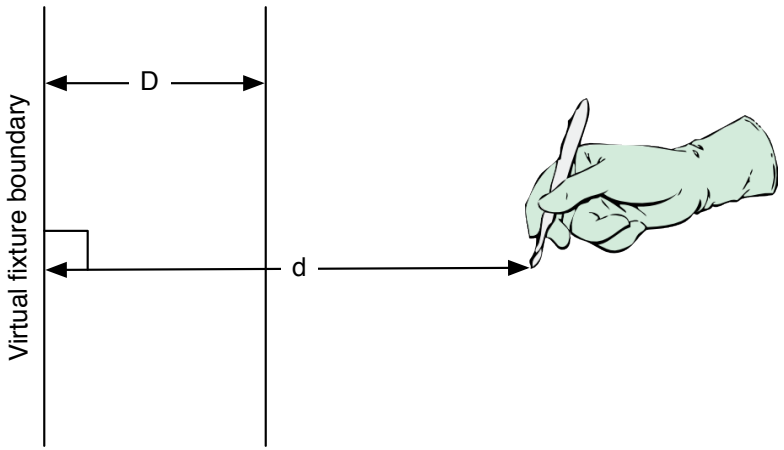
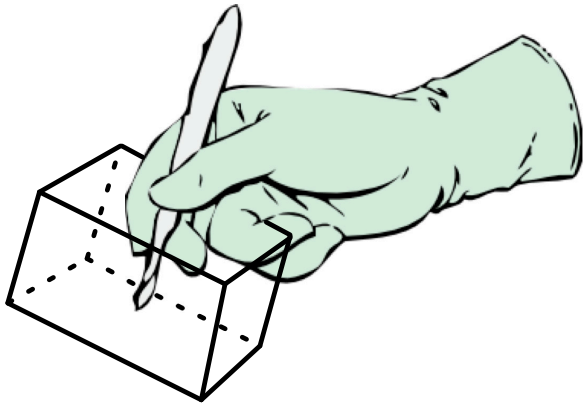
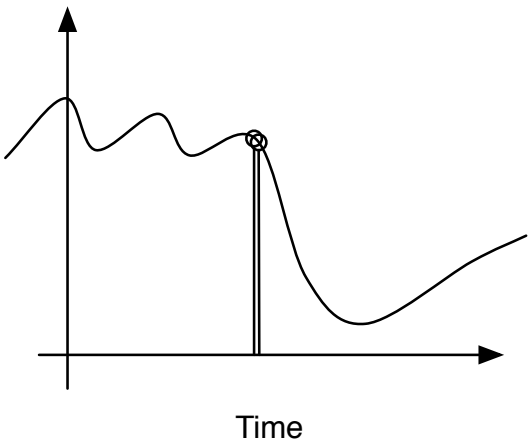
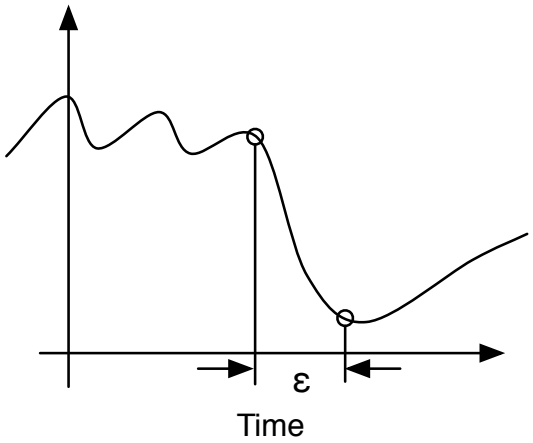
$$[\forall l\,(n := \texttt{new}\ C;\ \forall i\ a(i) := ctrl;\ \forall i\ x(i)'' = a(i))^*]\forall l\,\forall i,j\ i \ll j$$

Surgical Robot Verification: Skull-base Surgery

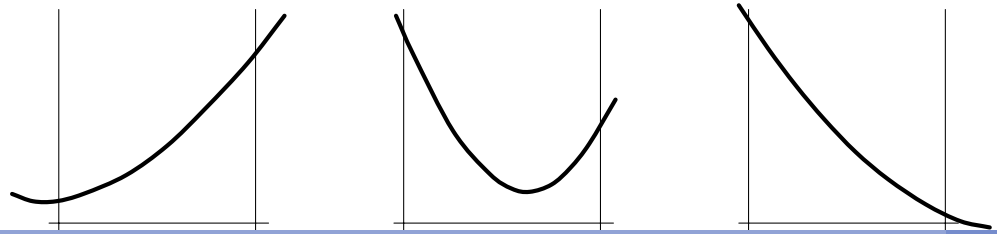








Redesign to predictive control



- Negligible lag?

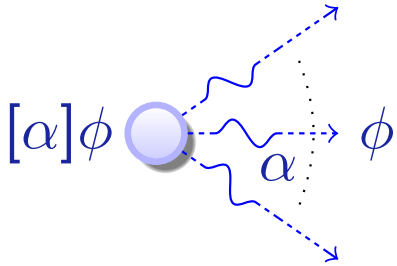
HSCC'13

Outline

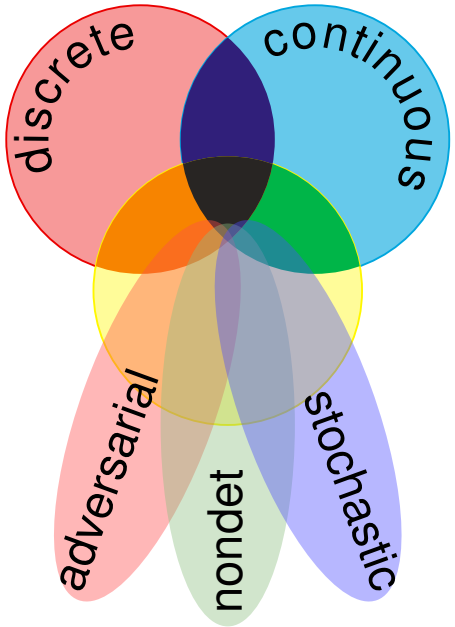
Conclusions

quantified differential dynamic logic

$$\text{Qd}\mathcal{L} = \text{FOL} + \text{DL} + \text{QHP}$$



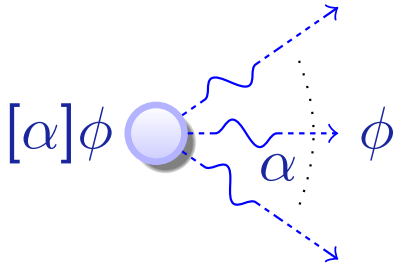
- Distributed hybrid systems everywhere
- System model and semantics
- Logic for distributed hybrid systems
- Compositional proof calculus
- First verification approach
- Sound & complete / diff. eqn.
- Quantified differential invariants
- Distributed car control verified
- Distributed aircraft control verified
- Robot verified for many obstacles



Conclusions

quantified differential dynamic logic

$$\text{Qd}\mathcal{L} = \text{FOL} + \text{DL} + \text{QHP}$$



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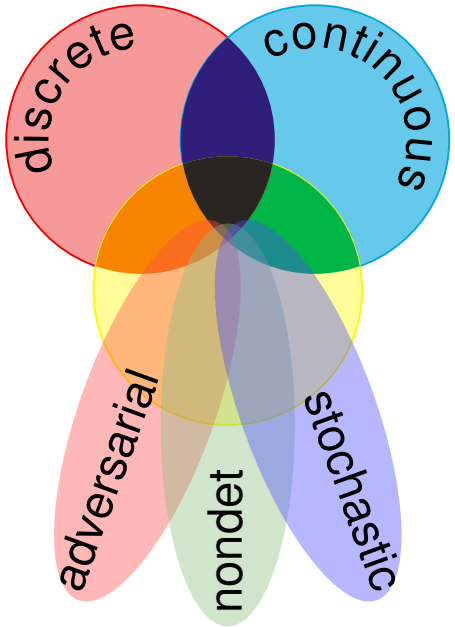

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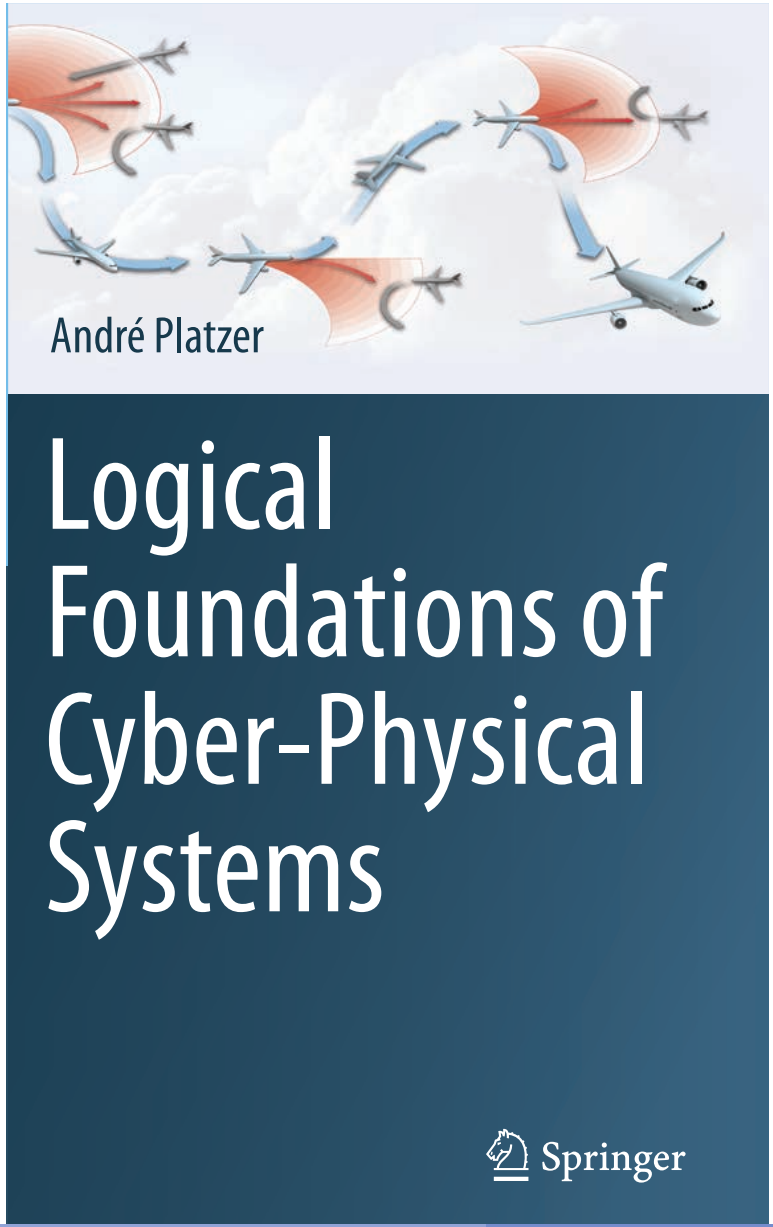


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