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ProblemSet2 Answers

Question 1:

a. Experiment and Sample Space: An Experiment is a procedure to observe outcome under specific conditions. Sample space is a set of all the possiple outcomes which are obtained from an experiment.

b. Random Variable: Random variable is a numerical value that matches the possible outcomes of an experiment.

c.Probability Distribution: Probability Distribution is a function with random variable's values as inputs and probability of that value's as output.

d.Cumulative Probability Distribution: The Cumulative Probability Distribution is a function that takes random variable values and numbers less than this value as input and the sum of these probabilities as output.

e. Expected Value and Varience: Expected value is a value that is avarage value for a random variable and it is obtained with repeated trials. Varience is a value that gives how much a random variables value deviate from the avarage.

f. Joint Probability Distribution: Joint Probability Distribution is a funciton gives the probabilities of two or more discrete random variables together.

g. Marginal and conditional probability distributions: Marginal probability distribution is a function which is obtained from a joint probability distrubution by summing the probabilities of all possible outcomes for just one random variable on o a specified value**.** Conditional probability distribution is a function for all the possible outcomes of a random variable under some conditions.

h. Covariance and Correlation: Covariance is a statistical value that measures how two random variable change together. Positive covarience means that variables changes same direction and Negative covariance meant that variables changes opposite direction. Since the covariance is hard to understand covarience we have Corralation that measures how two random variable change together with a value between (-1,1). "-1" means that they have perfect negative relation, "0"means they are independent and "1" means that they have perfect positive relationship.

i. **Skewness and Kurtosis:** Skewness is a statistic to find how the data deviated around the avarage. Kurtosis is a statistics that measure whether tails are fat or not.

Question 2

Let Y denote the number of heads that occur when two coins are tossed. Assume the probability of a head is 0,4 on either coin

a)

$S = \{HH, HT, TH, TT\}$ Y: Number of heads

$P(Y=0) = 0.6 \times 0.6 = 0{,}36$

$P(Y=1) = (0{,}4 \times 0{,}6) 2 = 0{,}48$

$P(Y=2) = 0{,}4 \times 0{,}4 = 0{,}16$

PDF of y =

	$Y=0$	$Y=1$	$Y=2$
$P(Y=y)$	0,36	0,48	0,16

$y_0 = 0$
$y_1 = 1$
$y_2 = 2$

b) $E(Y) = \sum_{i=0}^{2} y_i \times P(Y=y_i) = 0.(0{,}36) + 1.(0{,}48) + 2.(0{,}16)$
$= 0 + 0{,}48 + 0{,}32 = 0{,}8 = E(Y)$

$Var(Y) = \sum (y_i - \mu_y)^2 P(Y=y_i) = (0-0{,}8)^2 (0{,}36) + (1-0{,}8)^2 (0{,}48) +$
$+ (2-0{,}8)^2 (0{,}16)$
$= 0{,}2304 + 0{,}0192 + 0{,}2304 = 0{,}48$
$\Rightarrow Var(Y) = 0{,}48$

Question 3

Marginal Distributions:

	0	1
MDF of Y	0,035	0,965

	0	1
MDF of X	0,602	0,398

a) $E(Y) = \sum y_i \cdot P(Y = y_i) = 0.(0{,}035) + 1.(0{,}965)$
$= 0 + 0{,}965 = E(Y) = 0{,}965$

b) Distribution of $Y|X=0$, Distribution of $Y|X=1$

	0	1
Y	$\frac{0{,}026}{0{,}602}$	$\frac{0{,}576}{0{,}602}$

	0	1
Y	$\frac{0{,}009}{0{,}398}$	$\frac{0{,}389}{0{,}398}$

$\Rightarrow E(Y|X=1) = 0.(0{,}026/0{,}602) + 1.(0{,}576/0{,}602) = 0{,}9568$
$E(Y|X=0) = 0.(0{,}009/0{,}398) + 1.(0{,}389/0{,}398) = 0{,}9773$

c) Unemployment Rate for collage graduates: $\frac{0{,}009}{0{,}398} = 0{,}0226$

" " " non-collage grads: $\frac{0{,}026}{0{,}602} = 0{,}0431$

d) $P(X=1|Y=0) = 0{,}009/0{,}035 = 0{,}2571$
$P(X=0|Y=0) = 0{,}026/0{,}035 = 0{,}7428$

e) Since $P(Y=0|X=0) = 0{,}0431 \neq P(Y=0) = 0{,}035$
it is not independent and there is a strong posit
correlation.

Question 4

$$P(x) = \begin{cases} 1-p \quad , & x = 0 \\ p \quad , & x = 1 \end{cases}$$

a) $E(x^4) = \sum_{i=1}^{2} (x_i)^4 P(X = x_i)$

$= 0^4 \, P(x=0) + 1^4 \, P(x=1) = p \implies E(x^4) = p$

Also $E(x^2) = p$

b) $P(x) = \begin{cases} 0{,}47 & , x = 0 \\ 0{,}53 & , x = 1 \end{cases}$

$E(x) = 0 \cdot (0{,}47) + 1 \cdot (0{,}53) = 0{,}53$

$Var(x) = E(x^2) - (E(x))^2 = 0{,}53 - (0{,}53)^2 = 0{,}2491$

$E((X-0{,}53)^3) = \sum_{i=1}^{2} (x_i - 0{,}53)^3 \cdot P(X = x_i)$

$= (-0{,}53)^3 \cdot (0{,}47) + (0{,}47)^3 (0{,}53) = -0{,}148877 + 0{,}633823$

$= 0{,}4849$

Skewness $= \frac{0{,}4849}{(0{,}2491)^{3/2}} = \frac{0{,}4849}{0{,}1243} = 3{,}9010 \rightarrow$ Long Right Tail

$E((X-0{,}53)^4) = (-0{,}53)^4 \cdot (0{,}47) + (0{,}47)^4 \cdot (0{,}53) = 0{,}0370 + 0{,}0258 = 0{,}0628$

Kurtosis $= \frac{0{,}0628}{(0{,}2491)^2} = \frac{0{,}0628}{0{,}0620} = 1{,}0129 \rightarrow ?$