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Problem Set 3 Answers

Question 1

- a. Normal distribution and standard normal distribution:** A bell shaped and symmetric around the mean distribution with mean of 0 and variance of 1 in standard version.
- b. Chi-squared distribution:** A type of distribution used in hypotheses tests and sum of squared independent normal random variables.
- c. t distribution:** t distribution is like normal distribution but with fatter tails.
- d. F distribution:** The F distribution results from the ratio of two scaled chi-square distributions.
- e. Independently and identically distributed (i.i.d):** i.i.d refers to independent random variables with same probability distribution.
- f. Sampling distribution:** Sampling distribution is the probability distribution of the statistics of many samples drawn from a population.
- g. Law of large number:** Law of Large number says that as size of sample increase, average of sample gets closer to the mean of population
- h. Central limit theorem:** Central limit theorem states that as size of sample increase, sample average distribution gets closer to the normal distribution.
- i. Consistency:** Consistency means that the closeness of an estimator to the truth increases as the sample size increases.
- j. Asymptotic distribution:** The asymptotic distribution describes the limiting behaviour of a succession of distributions as the sample size approaches infinity.

Question 2

```
> ?pnorm
> pnorm(3, mean = 1, sd = 2, lower.tail = TRUE)
[1] 0.8413447
> pnorm(0, mean = 3, sd = 3, lower.tail = FALSE)
[1] 0.8413447
> pnorm(52, mean = 50, sd = 5, lower.tail = TRUE) -pnorm
(40, mean = 50, sd = 5, lower.tail = TRUE)
[1] 0.6326716
```

Question 3

```
> ?pchisq
> pchisq(7.78, df = 4, lower.tail = T)
[1] 0.9000223
> pchisq(18.31, df = 10, lower.tail = F)
[1] 0.04995417
> ?pf
> pf(q = 1.831, df1 = 10, df2 = Inf, lower.tail = F)
[1] 0.04995417
> pchisq(1.0, df = 1, lower.tail = F)
```

Question 3 d. Since $F(df1, Inf) \approx \text{ChiSq}(df1)/df1$ and $1.831 = 18.31/10$

Question 4

a.

```
> pnorm(10.4, mean = 10, sd = sqrt(4/20), lower.tail = T)
-pnorm(9.6, mean = 10, sd = sqrt(4/20), lower.tail = T)
[1] 0.6289066
> pnorm(10.4, mean = 10, sd = sqrt(4/100), lower.tail = T)
-pnorm(9.6, mean = 10, sd = sqrt(4/100), lower.tail = T)
[1] 0.9544997
> pnorm(10.4, mean = 10, sd = sqrt(4/1000), lower.tail = T)
-pnorm(9.6, mean = 10, sd = sqrt(4/1000), lower.tail = T)
[1] 1
> |
```

b. and c.

$$\begin{aligned} b) P(10-c \leq \bar{Y} \leq 10+c) &= P(M-c \leq \bar{Y} \leq M+c) \\ &= P(-c \leq \bar{Y} - M \leq c) = P\left(\frac{-c}{\frac{\sigma}{\sqrt{n}}} \leq Z \leq \frac{c}{\frac{\sigma}{\sqrt{n}}}\right) \\ n \rightarrow \infty &\Rightarrow \frac{\pm c}{\frac{\sigma}{\sqrt{n}}} \rightarrow \pm \infty \Rightarrow P(-\infty \leq Z \leq \infty) = 1 // \end{aligned}$$

c) By the Law of Large Numbers \bar{Y} converges true mean.
For any $c > 0$ $P(10-c \leq \bar{Y} \leq 10+c) \rightarrow 1$ as $n \rightarrow \infty$ implies that $\bar{Y} \xrightarrow{P} 10 //$