

Simpson's 1/3 Rule

Theory:

Simpson's 1/3 Rule approximates the integrand by fitting a second-degree polynomial (parabola) over pairs of subintervals.

The formula for Simpson's 1/3 Rule is

$$\int_a^b f(x) dx \approx \frac{h}{3} [y_0 + y_n + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots)]$$

The method requires the number of subintervals to be even. Compared to simpler methods such as the Trapezoidal Rule, Simpson's 1/3 Rule produces significantly smaller errors for the same step size.