Theory

MATRIX INVERSION METHOD

OVERVIEW:

Matrix inversion finds the inverse of a square matrix A, denoted as A^(-1), such that A * A^(-1) = I, where I is the identity matrix. For solving Ax = b, if A^(-1) exists, then x = A^(-1) * b.

MATHEMATICAL FOUNDATION:

For an n×n matrix A, the inverse A^(-1) exists if and only if:

- det(A) ≠ 0 (A is non-singular)
- A is a square matrix
- A has full rank

Property: A * A^(-1) = A^(-1) * A = I

GAUSS-JORDAN METHOD FOR INVERSION:

The most common method uses Gauss-Jordan elimination on [A|I]:

1. Form augmented matrix [A|I] where I is the identity matrix
2. Apply row operations to transform A to I
3. The result is [I|A^(-1)]

Algorithm Steps: For each column k from 1 to n: a. Make diagonal element 1 (divide row k by a[k][k]) b. Eliminate all other elements in column k c. Apply same operations to the right side (identity part)

COFACTOR METHOD:

Alternative approach: A^(-1) = (1/det(A)) * adj(A)

Where:

- adj(A) is the adjugate (transpose of cofactor matrix)
- Cofactor C[i][j] = (-1)^(i+j) * M[i][j]
- M[i][j] is the minor (determinant of submatrix)

VERIFICATION:

After finding A^(-1):

- Multiply A * A^(-1)
- Result should be identity matrix I
- Check if diagonal elements = 1, off-diagonal = 0

TIME COMPLEXITY:

- Gauss-Jordan method: O(n^3)
- Cofactor method: O(n^4) - less efficient

NUMERICAL CONSIDERATIONS:

- Avoid division by very small numbers
- Use partial pivoting for stability
- Check condition number of matrix
- Ill-conditioned matrices have unstable inverses

ADVANTAGES:

- Direct method for solving Ax = b
- Useful when solving multiple systems
- Gives complete inverse matrix

DISADVANTAGES:

- Computationally expensive (O(n^3))
- Numerically unstable for ill-conditioned matrices
- Not recommended for sparse matrices
- Unnecessary for solving single linear system