Runge-Kutta Method (Fourth Order)

Theory:

The Runge-Kutta Method is a widely used technique for numerically solving first-order ordinary differential equations of the form

$$\frac{dy}{dx} = f(x, y), y(x_0) = y_0$$

The fourth-order Runge-Kutta method computes the solution using

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

Where,

$$k_1 = hf(x_n, y_n)$$
$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$
$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$
$$k_4 = hf(x_n + h, y_n + k_3)$$

The fourth-order Runge-Kutta method improves calculation by calculating multiple slope estimates within a single step. These slopes are then combined in a weighted manner to produce an accurate approximation of the solution at the next point.