Gauss Jordan Elimination

Theory

GAUSS-JORDAN ELIMINATION METHOD

OVERVIEW:

Gauss-Jordan Elimination is an extension of Gauss Elimination that transforms the coefficient matrix into a diagonal form (reduced row echelon form). Unlike Gauss Elimination, it eliminates both above and below the pivot element, eliminating the need for back substitution.

MATHEMATICAL FOUNDATION:

For a system AX = B, the method transforms the augmented matrix [A|B] into the form [I|X], where I is the identity matrix and X is the solution vector.

ALGORITHM STEPS:

1. Forward Elimination (Similar to Gauss):
    - Transform matrix to upper triangular form
    - For each pivot position (k,k):
        - Make the pivot element 1 by dividing the entire row
        - Eliminate all elements below the pivot

```
for (int i = 0; i < n - 1; i++)
        {

            for (int j = i + 1; j < n; j++)
            {
                double r = v[j][i] / v[i][i];
                for (int k = 0; k < var + 1; k++)
                {
                    v[j][k] -= v[i][k] * r;
                }
            }
```

13. Backward Elimination:
    - Starting from the last row, moving upward:
    - Eliminate all elements above each pivot

- Result: Diagonal matrix with 1's on diagonal

```
for (int i = 1; i < n; i++)
       {
           for (int j = i - 1; j >= 0; j--)
           {
               double ratio = v[j][i] / v[i][i];
               for (int k = 1; k <= var; k++)
               {
                   v[j][k] -= ratio * v[i][k];
               }
           }
       }
```

25. Solution:
    - Solution is directly available in the augmented column
    - No back substitution needed

PIVOT NORMALIZATION:

For each pivot row k:

- Divide entire row by v[k][k]
- This makes the diagonal element equal to 1

TIME COMPLEXITY:

O(n^3) - slightly more operations than Gauss Elimination due to backward elimination

ADVANTAGES:

- No back substitution required
- Solution obtained directly
- Can compute matrix inverse simultaneously
- Better for finding matrix inverse

DISADVANTAGES:

- More computational steps than Gauss Elimination
- Still susceptible to round-off errors
- Requires non-zero pivot elements