Newton-Raphson Method

Theory:

The Newton-Raphson Method is one of the most powerful numerical techniques for solving non-linear equations. It is based on approximating the function by its tangent line using the first-order Taylor series expansion.

Starting from an initial guess x_0, successive approximations are obtained using the formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

This method converges very rapidly when the initial guess is close to the true root, often requiring fewer iterations than any other root-finding method. However, unlike bracketing methods, it may fail if the derivative becomes zero or if the initial guess is not chosen carefully.

When compared to Bisection, False Position, and Secant methods, Newton-Raphson offers the fastest convergence but at the cost of higher sensitivity to initial conditions.