



SINGAPORE UNIVERSITY OF
TECHNOLOGY AND DESIGN

Established in collaboration with MIT

Introduction to Deep Learning

Soujanya Poria

50.038 Computational data science

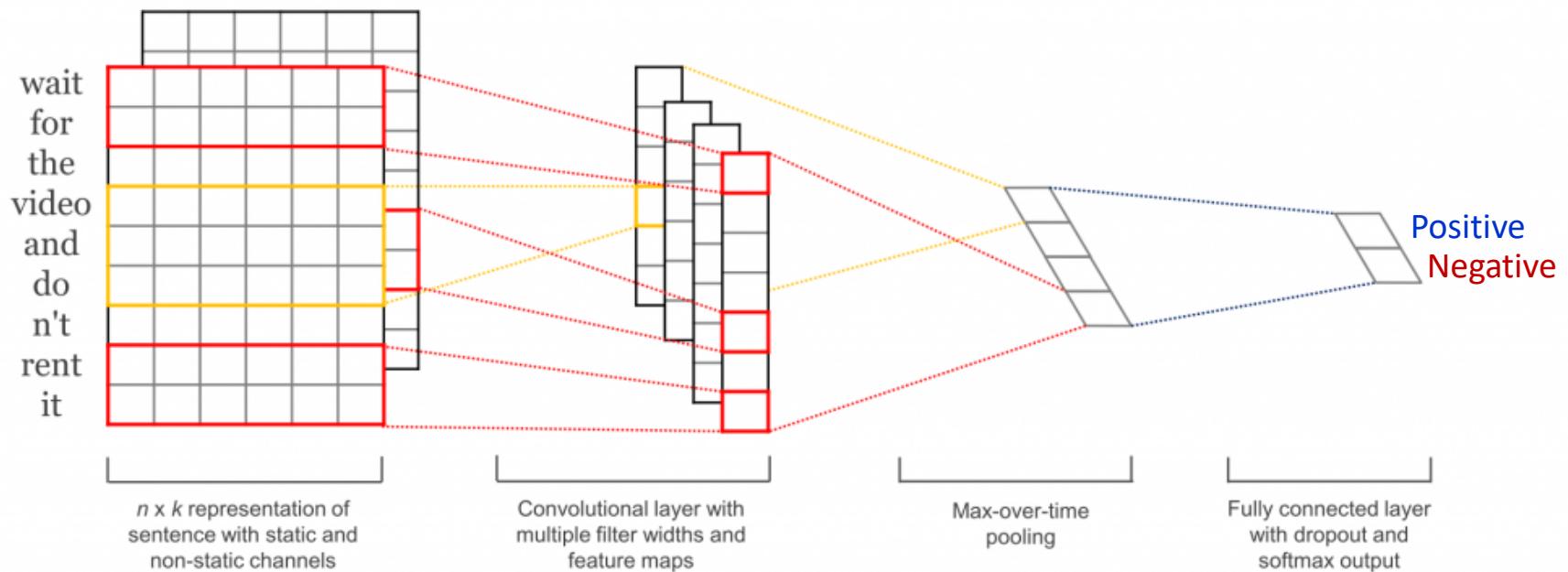
Objectives

- Understand the basic structure of a neural network, particularly a Multi-layer Perceptron
- Understand the various steps in training a neural network

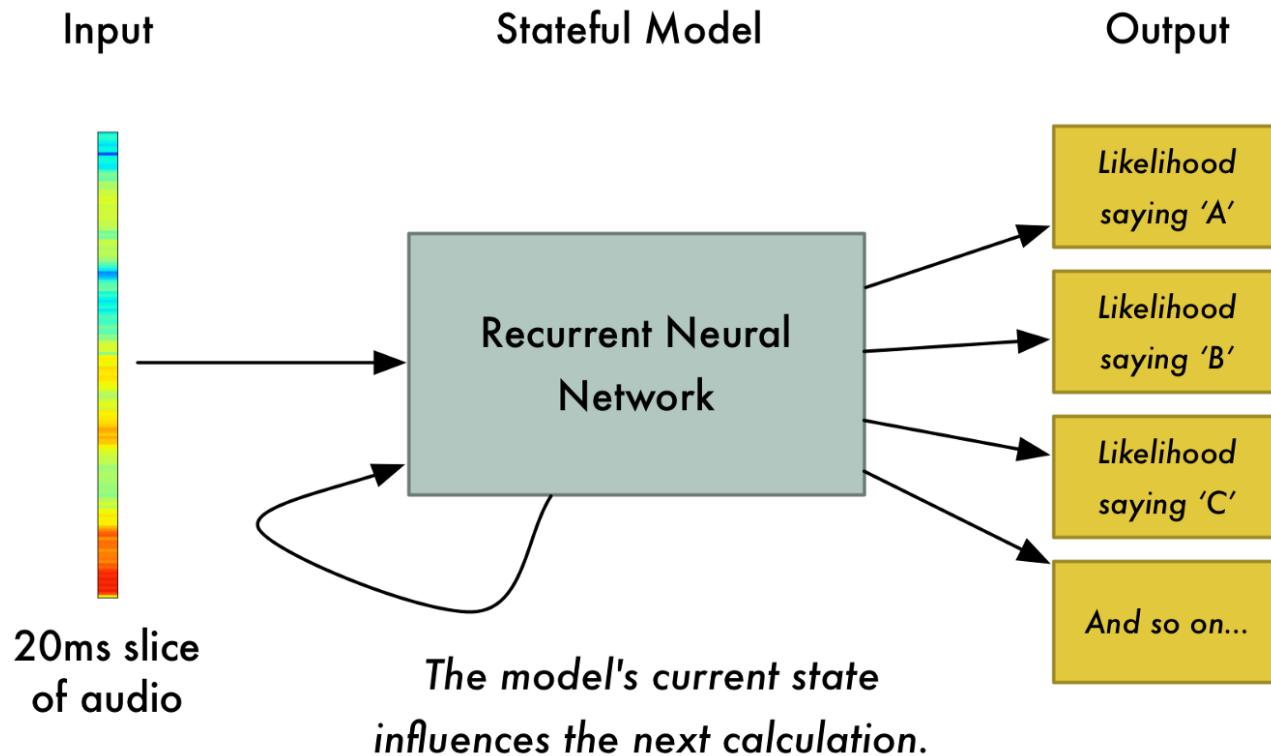
Examples: Images



Examples: Text

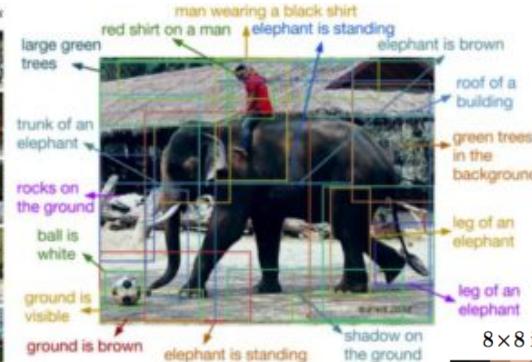


Examples: Audio



<https://medium.com/@ageitgey/machine-learning-is-fun-part-6-how-to-do-speech-recognition-with-deep-learning-28293c162f7a>

Various other examples

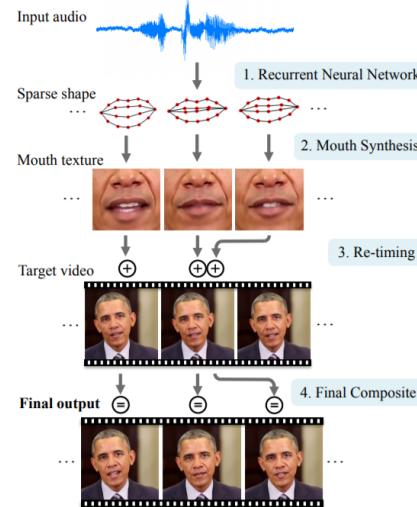


8 × 8 input 32 × 32 samples ground truth

Image Captioning



Colouring B&W Photos



Synthesizing Speech

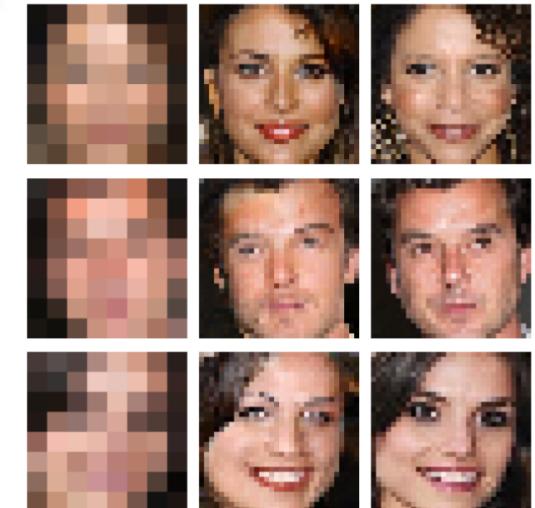
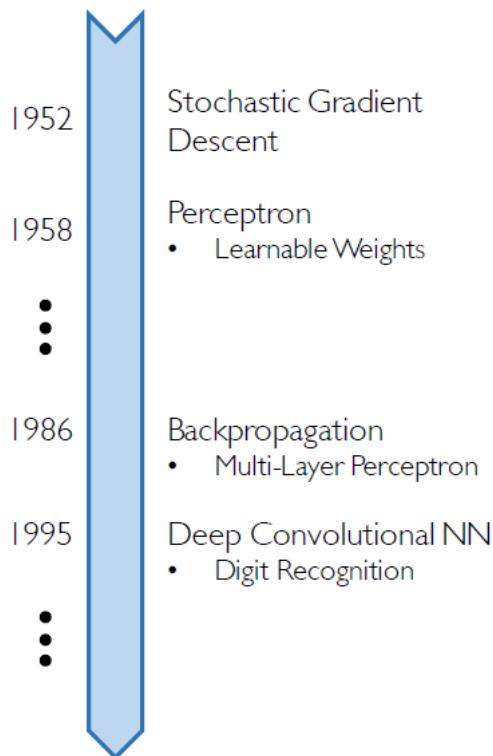


Image Super-resolution

Why Deep Learning?



Neural Networks date back decades, so why the resurgence?

I. Big Data

- Larger Datasets
- Easier Collection & Storage



2. Hardware

- Graphics Processing Units (GPUs)
- Massively Parallelizable

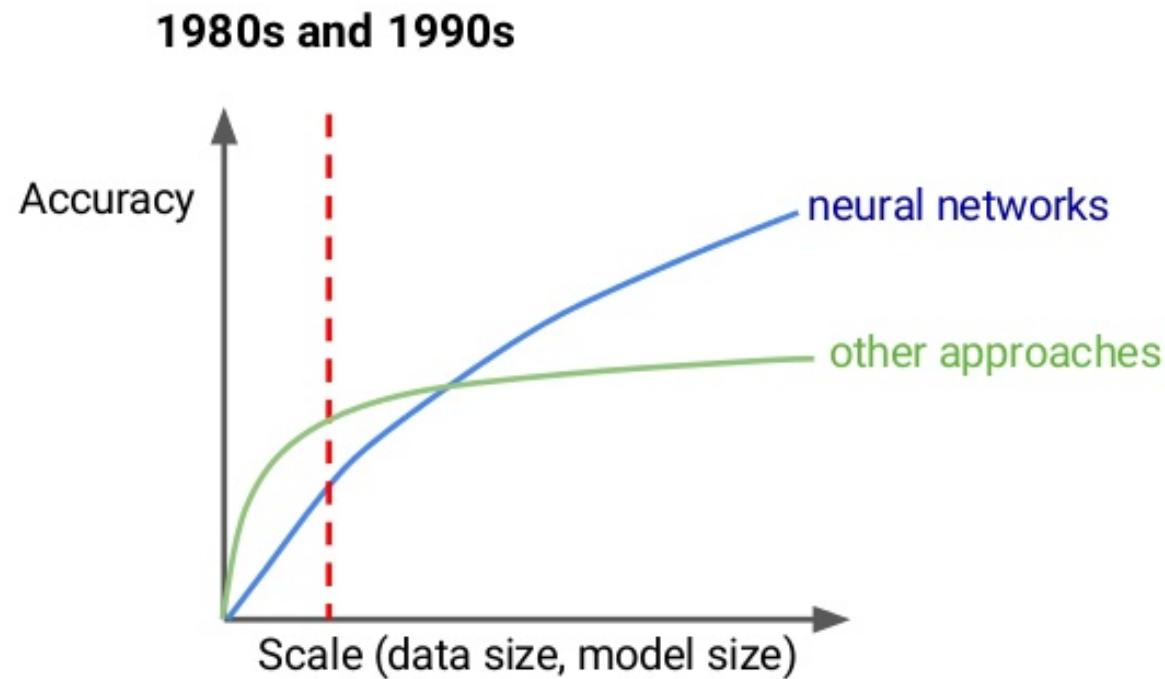


3. Software

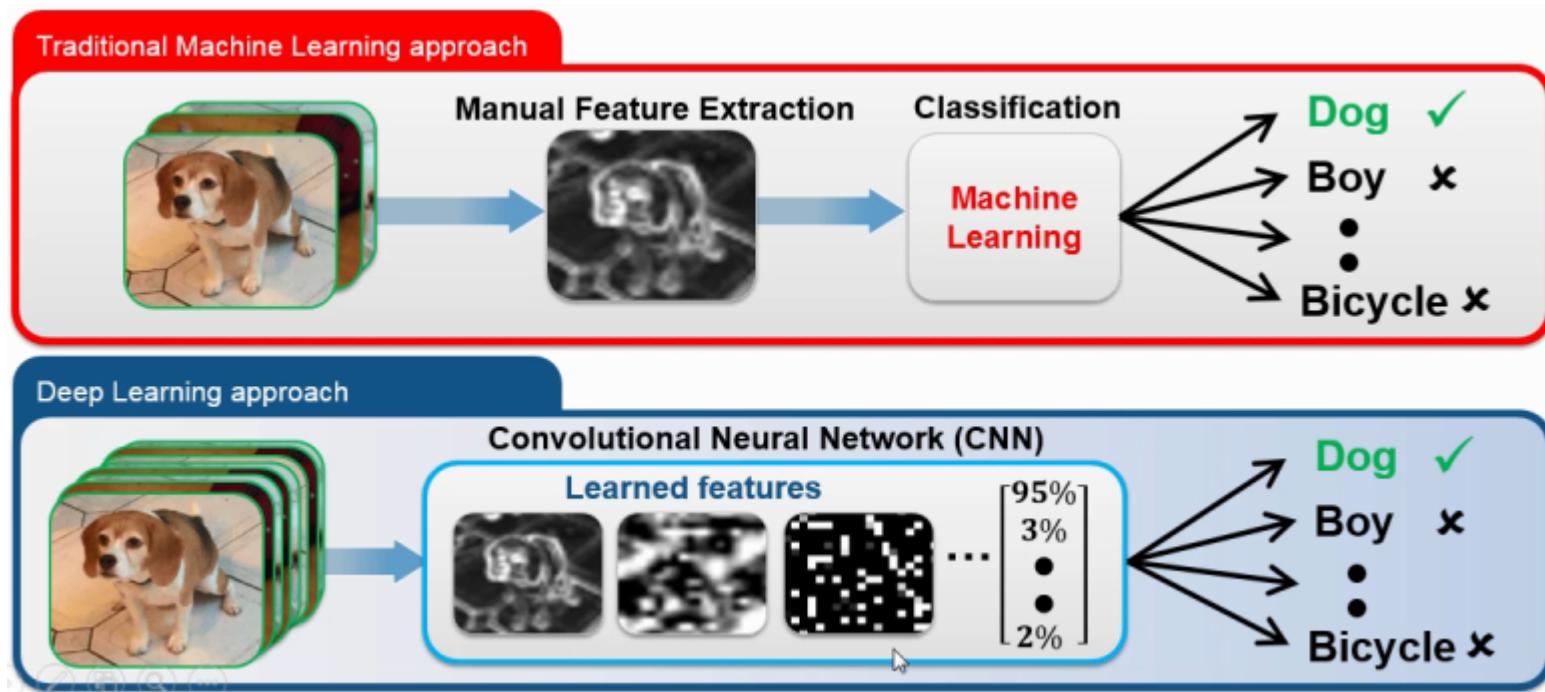
- Improved Techniques
- New Models
- Toolboxes



Why Deep Learning?

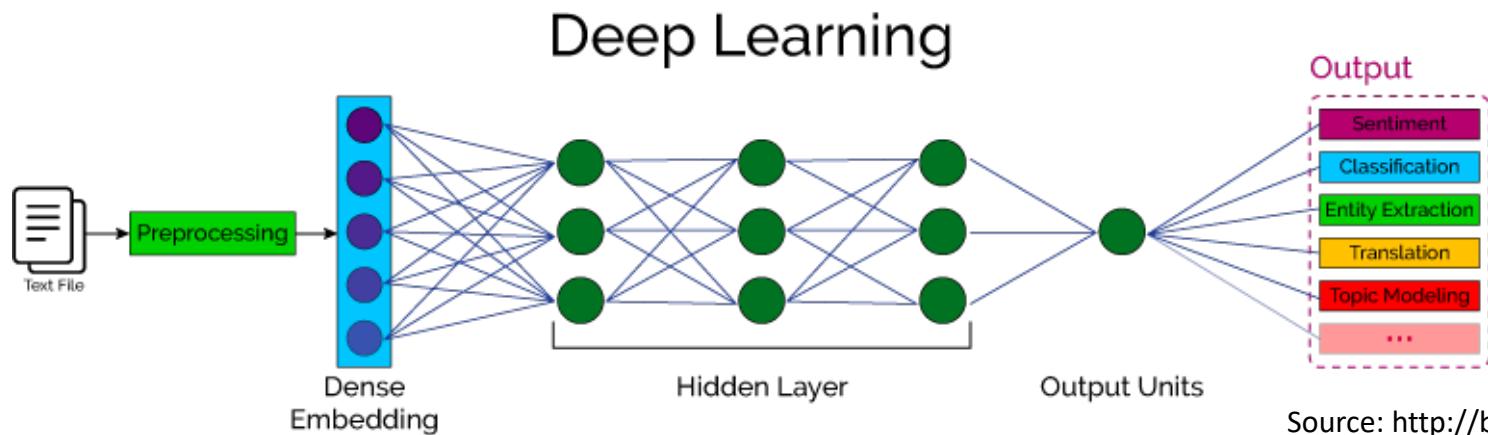
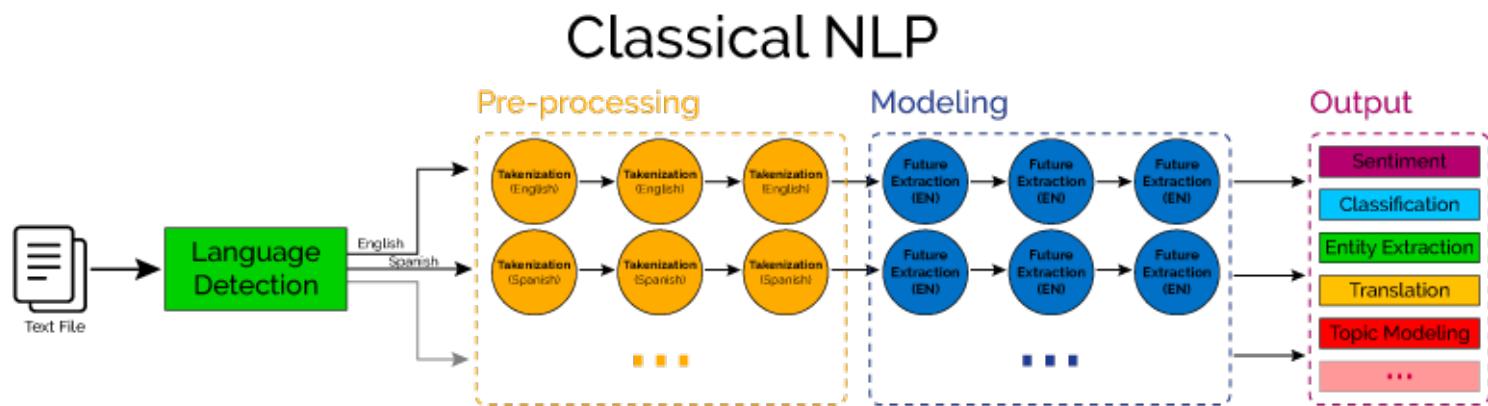


Differences



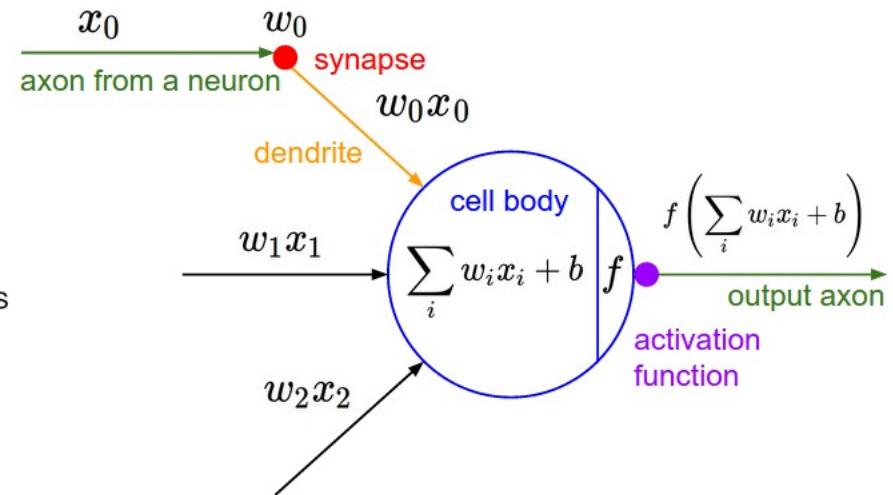
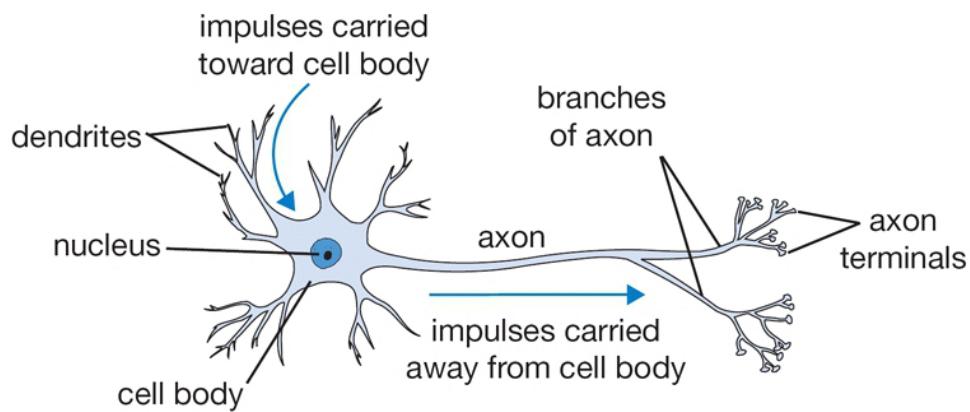
https://leonardoaraujosantos.gitbooks.io/artificial-inteligence/content/deep_learning.html

Differences



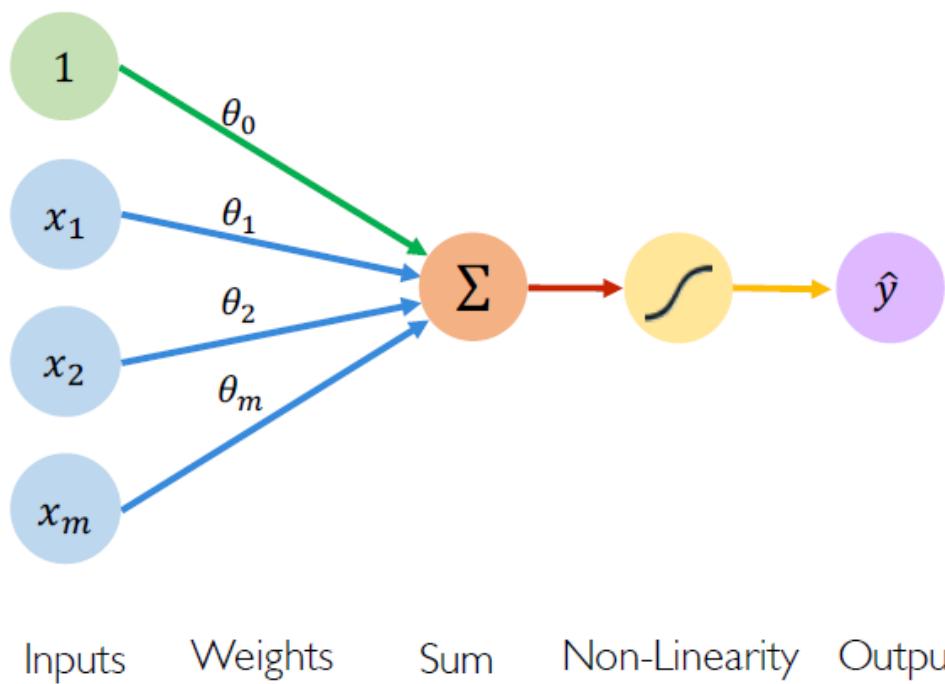
Source: <http://blog.aylien.com>

Inspiration behind Neural Networks



<http://cs231n.github.io/neural-networks-1/>

Perceptron



Linear combination of inputs \downarrow

Output \downarrow

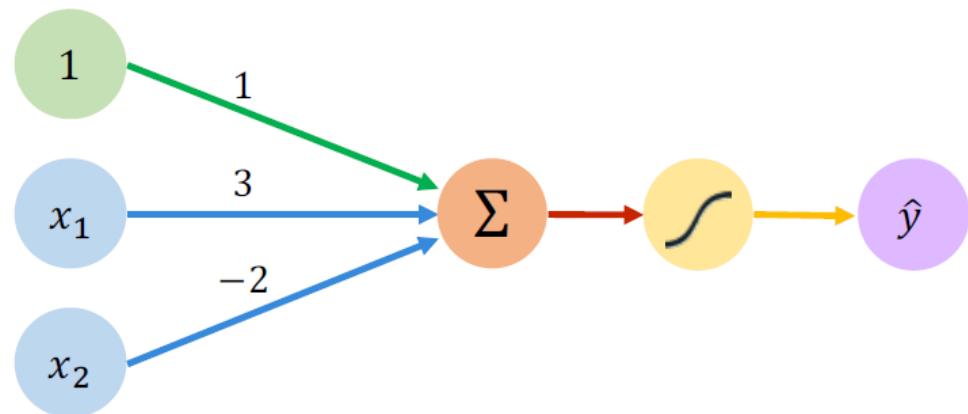
$$\hat{y} = g \left(\theta_0 + \sum_{i=1}^m x_i \theta_i \right)$$

Non-linear activation function \uparrow

Bias \downarrow

Diagram annotations explain the mathematical formula. A red arrow labeled "Linear combination of inputs" points to the term $\sum_{i=1}^m x_i \theta_i$. A purple arrow labeled "Output" points to \hat{y} . A green arrow labeled "Bias" points to θ_0 . A yellow arrow labeled "Non-linear activation function" points to the term g .

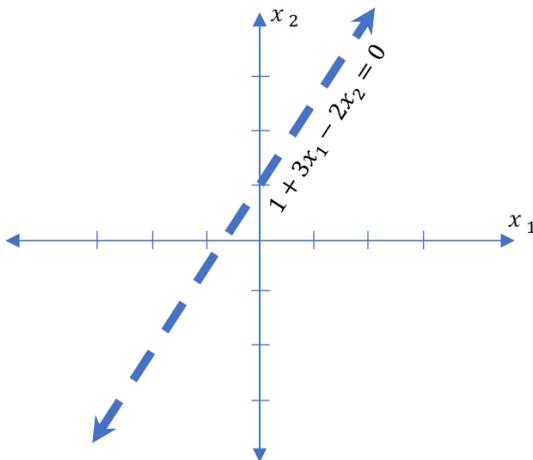
Why Non-linear Activation?



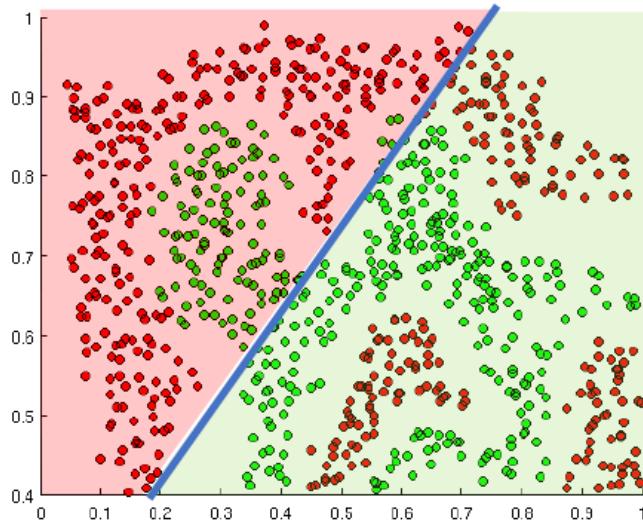
We have: $\theta_0 = 1$ and $\boldsymbol{\theta} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$

$$\begin{aligned}\hat{y} &= g(\theta_0 + \mathbf{X}^T \boldsymbol{\theta}) \\ &= g\left(1 + \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}^T \begin{bmatrix} 3 \\ -2 \end{bmatrix}\right) \\ \hat{y} &= g(1 + 3x_1 - 2x_2)\end{aligned}$$

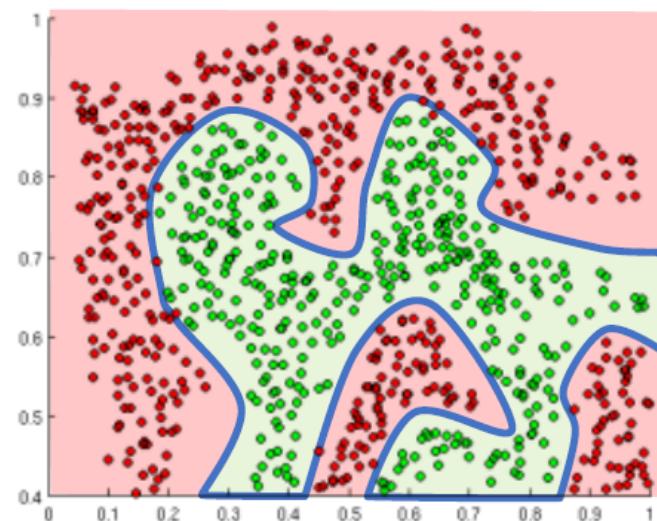
This is just a line in 2D!



Why Non-linear Activation?



Linear Activation functions produce linear decisions no matter the network size

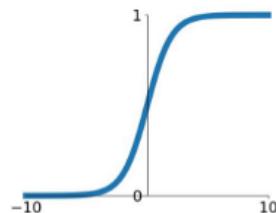


Non-linearities allow us to approximate arbitrarily complex functions

Activation Functions

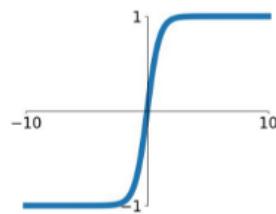
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



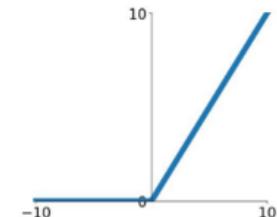
tanh

$$\tanh(x)$$



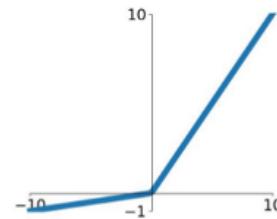
ReLU

$$\max(0, x)$$



Leaky ReLU

$$\max(0.1x, x)$$



Activation Functions

- Main problem with Sigmoid and tanh activation

Vanishing gradient problem

Issue of exceeding small gradients when training neural networks

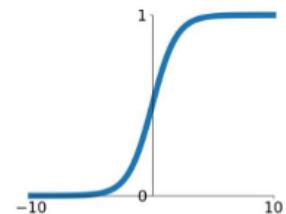
Worse with multiple layers in a NN

Additional problem with Sigmoid

Slower convergence than tanh

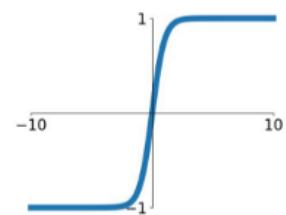
Sigmoid

$$\sigma(x) = \frac{1}{1+e^{-x}}$$



tanh

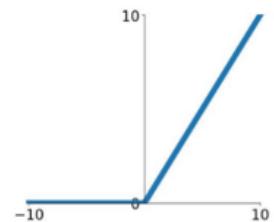
$$\tanh(x)$$



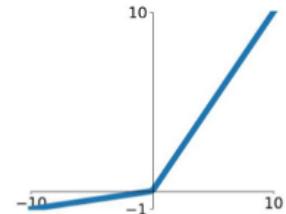
Activation Functions

- Rectified Linear Units (ReLU)
 - Developed to overcome the vanishing gradient problem
 - However, some neuron may “die”
- Leaky ReLU
 - Prevents neurons from “dying” by using a small negative slope

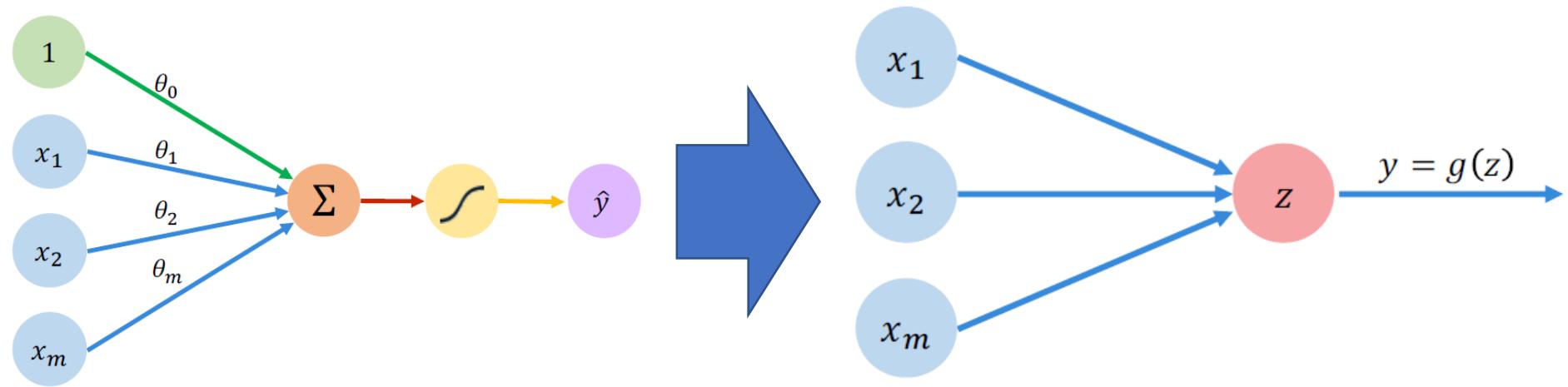
ReLU
 $\max(0, x)$



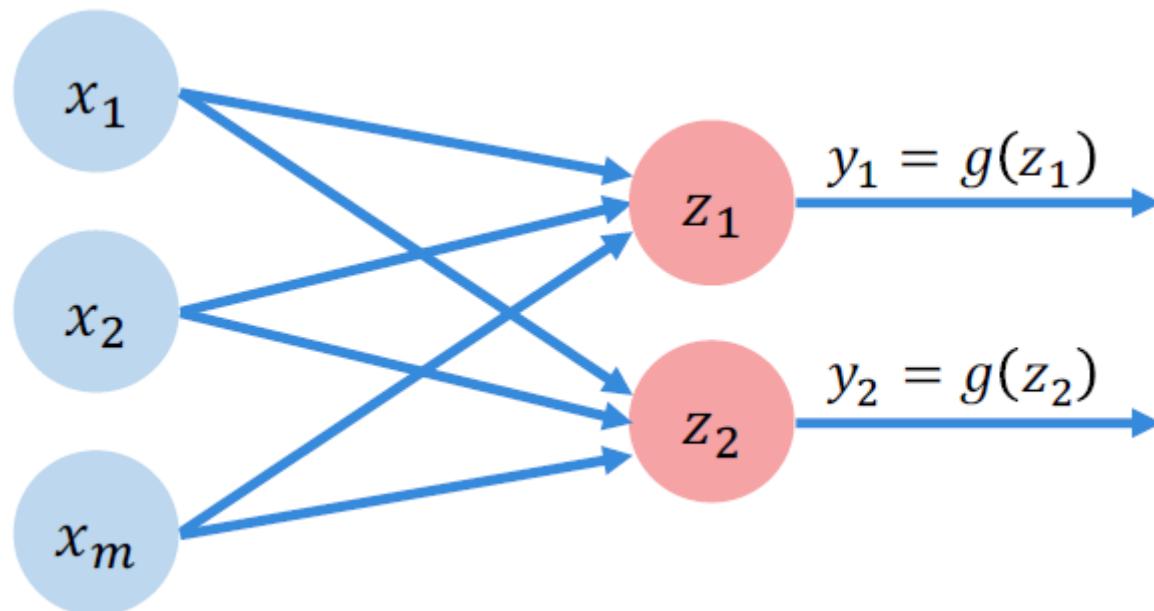
Leaky ReLU
 $\max(0.1x, x)$



Simplifying the Perceptron

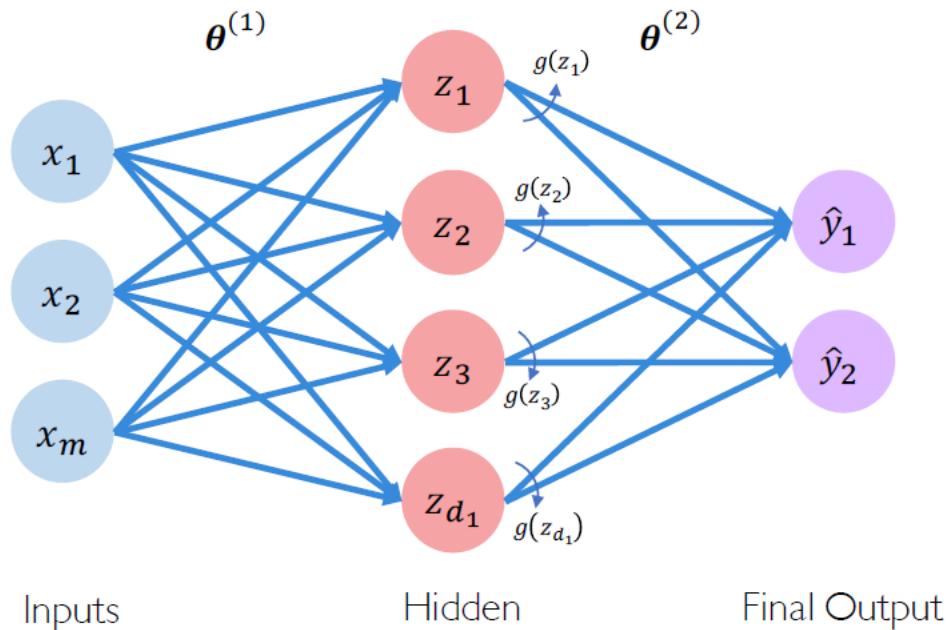


Multiple Perceptron



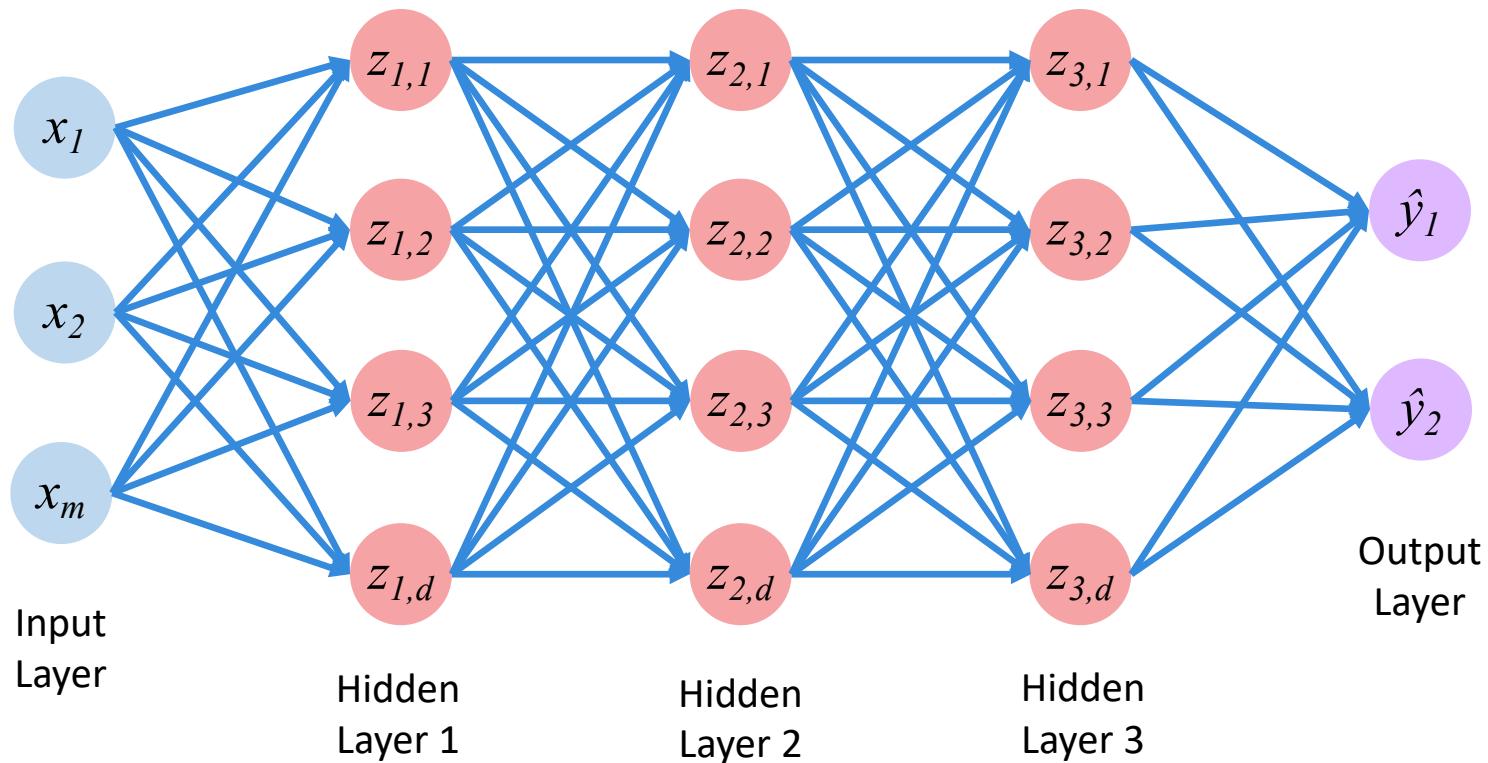
Neural Network

- Multiple perceptrons, aka a Multi-layer Perceptron (MLP)

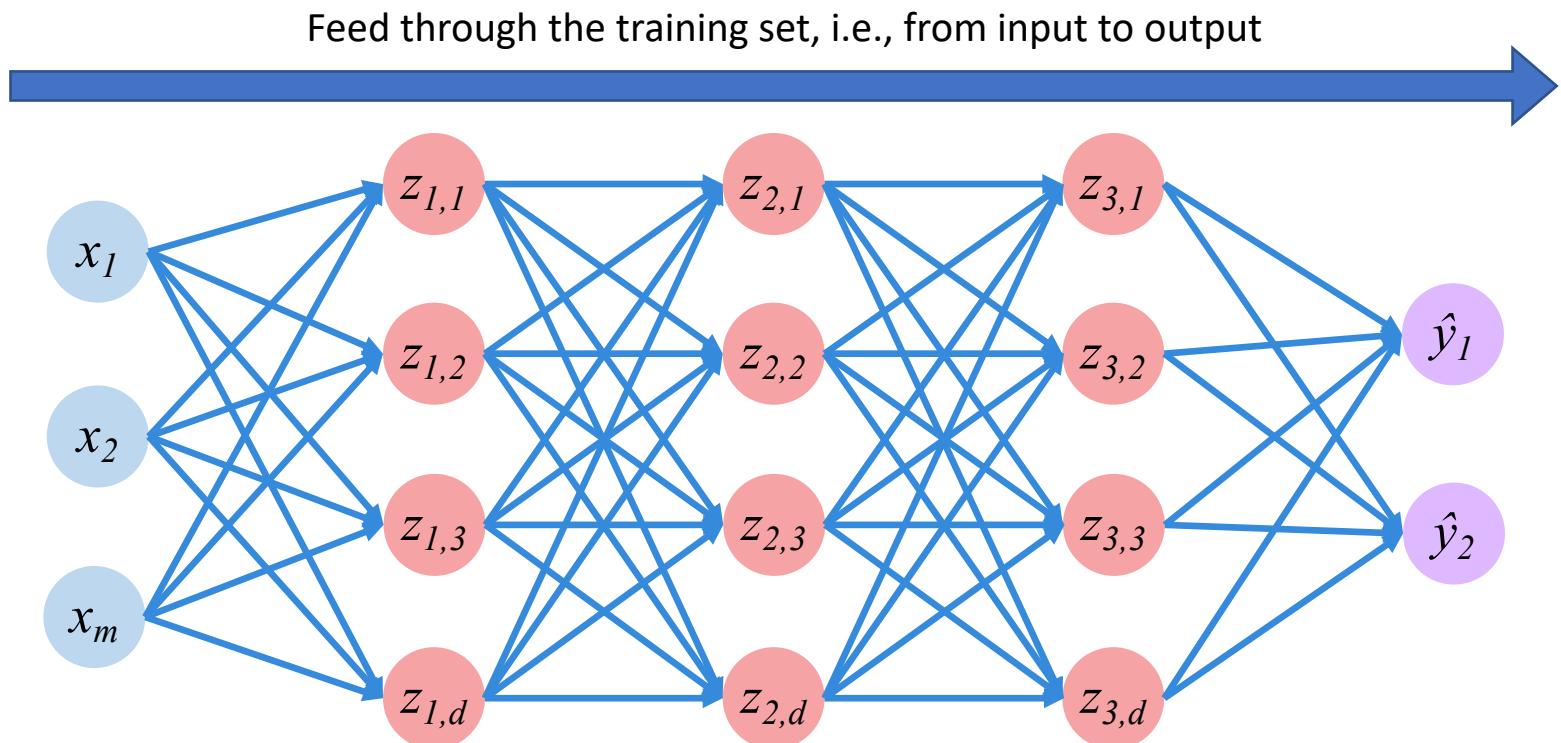


Deep Neural Network

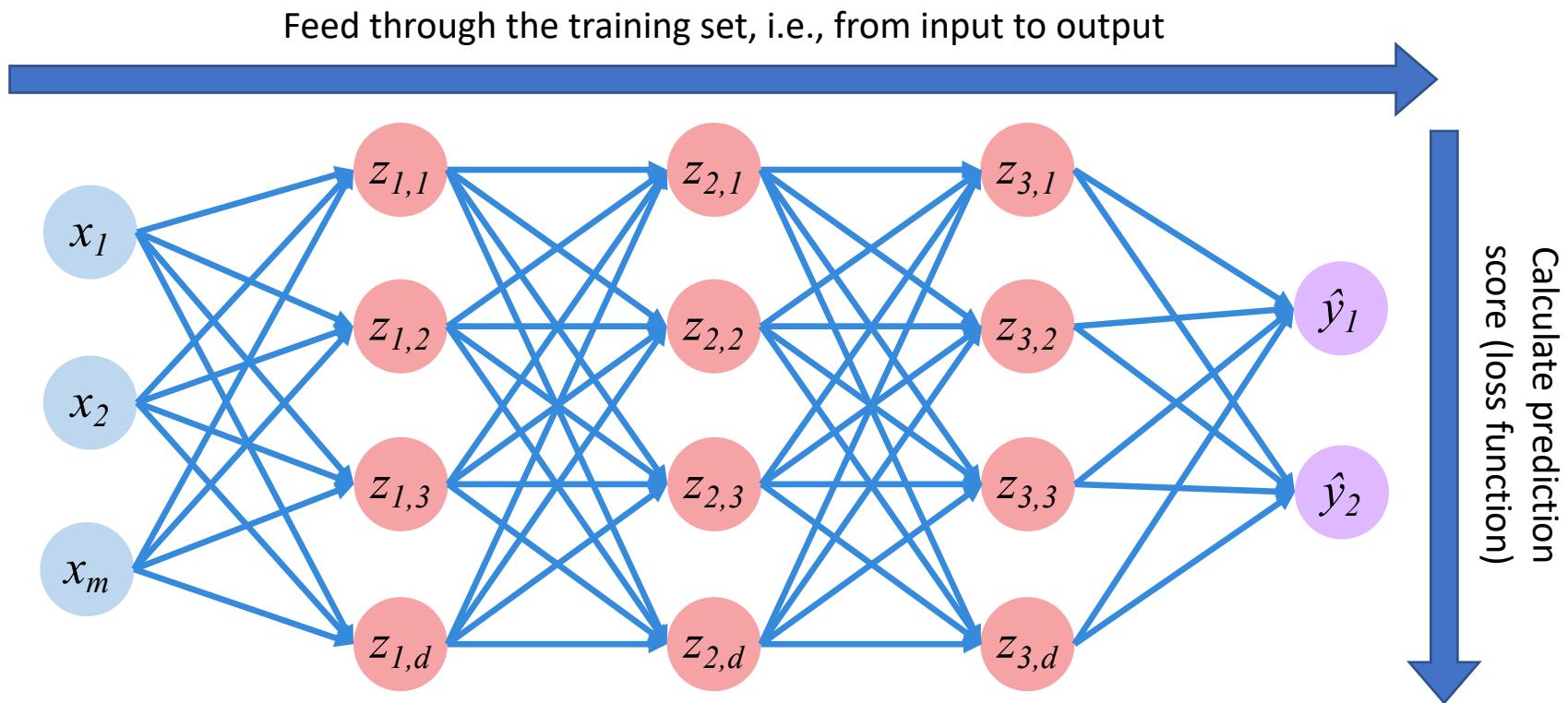
- Adding on multiple hidden layers



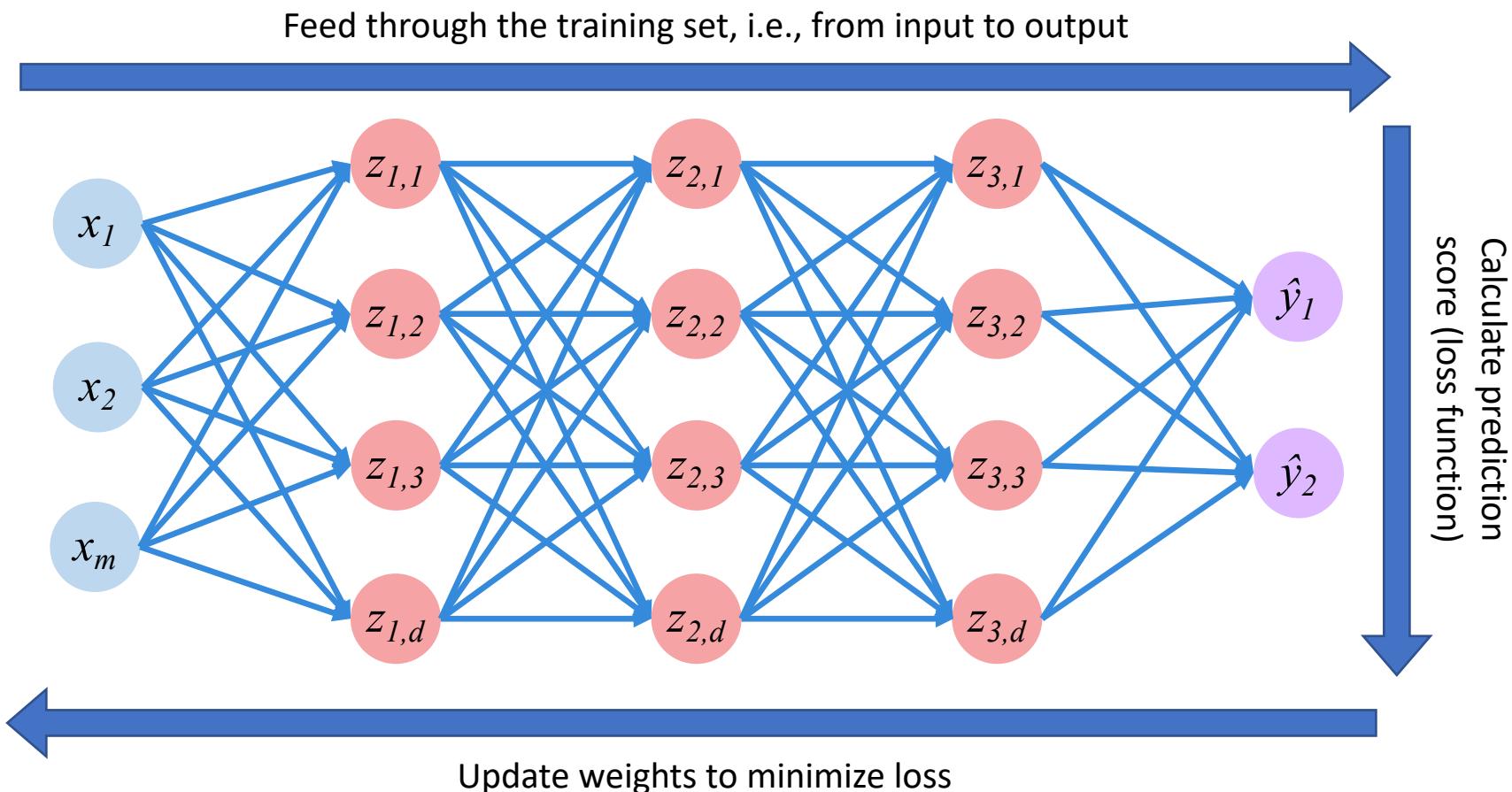
Training a Neural Network



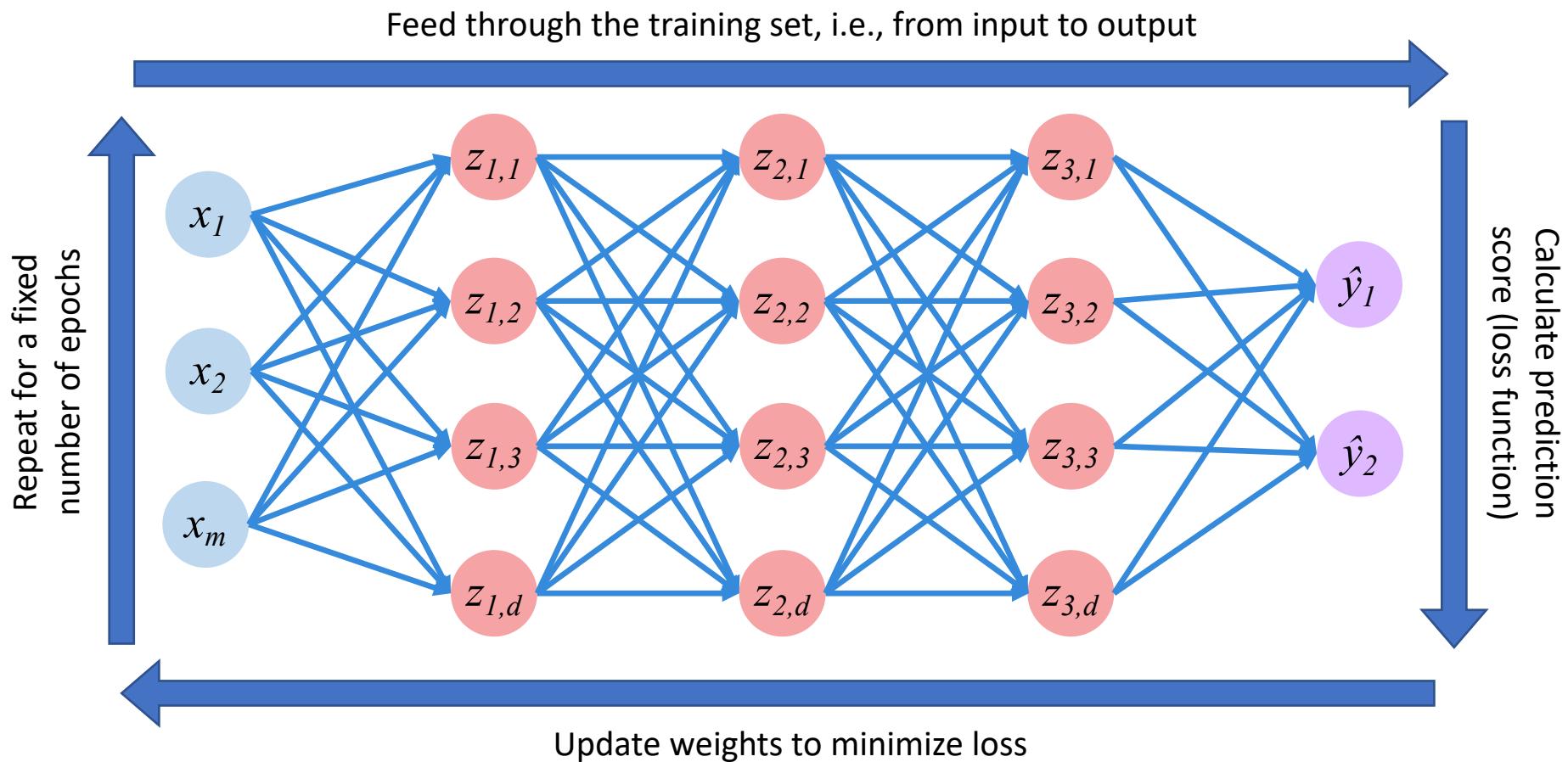
Training a Neural Network



Training a Neural Network



Training a Neural Network



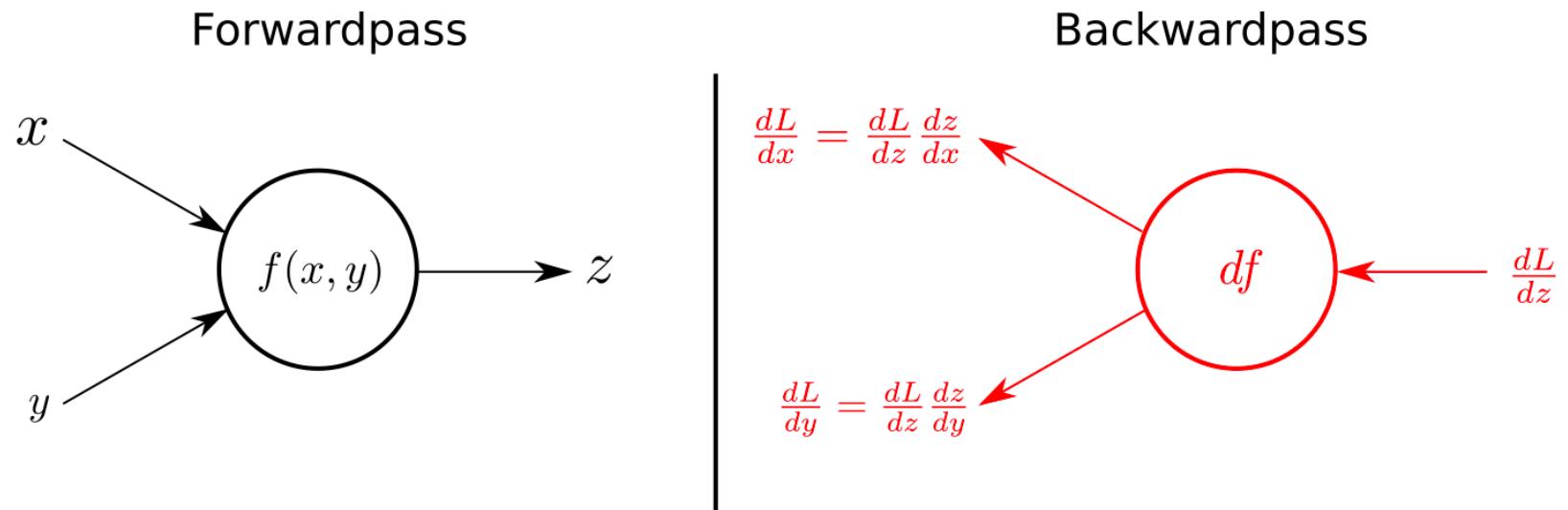
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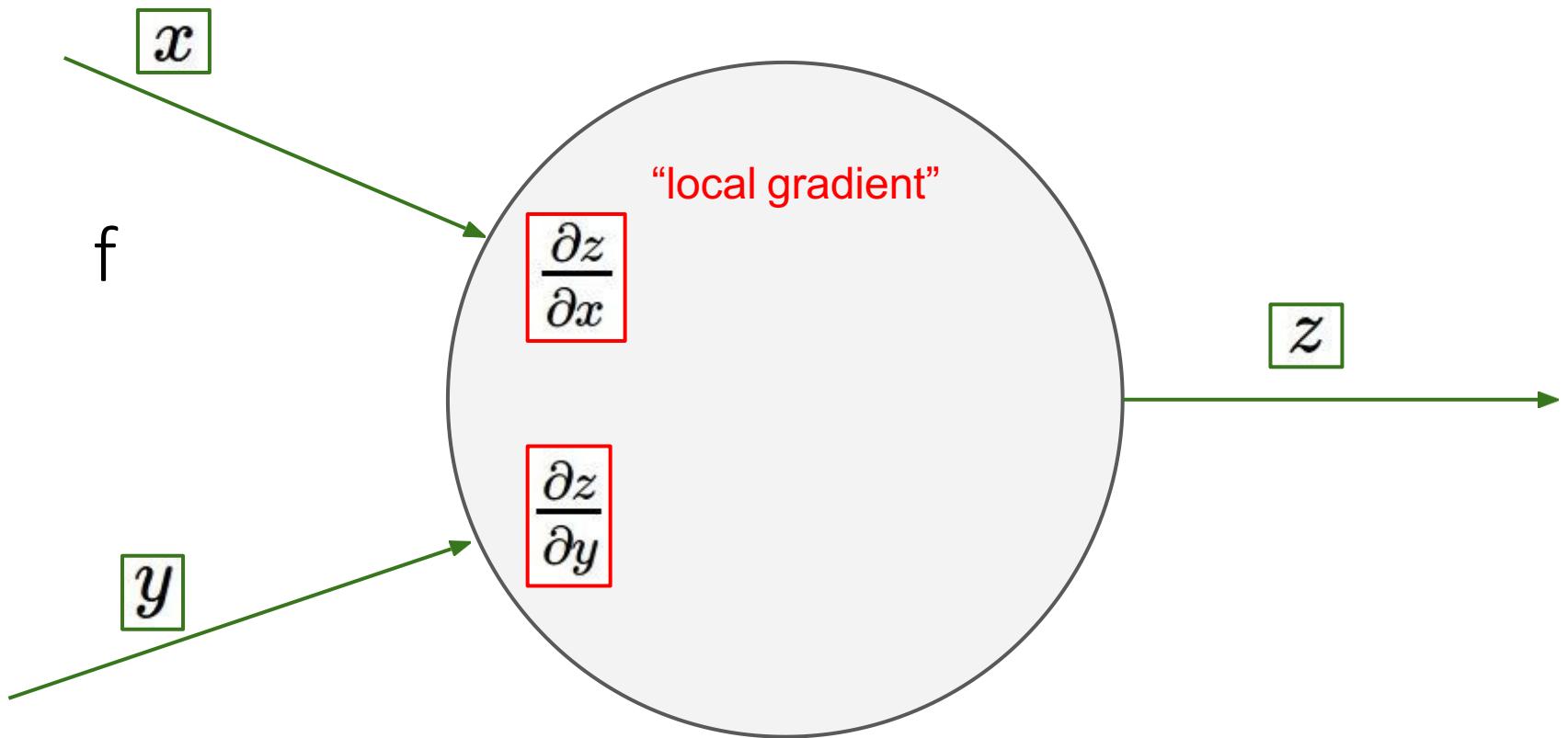
- What type of loss function?
- How to learn and update weights?
- How much training data to use?

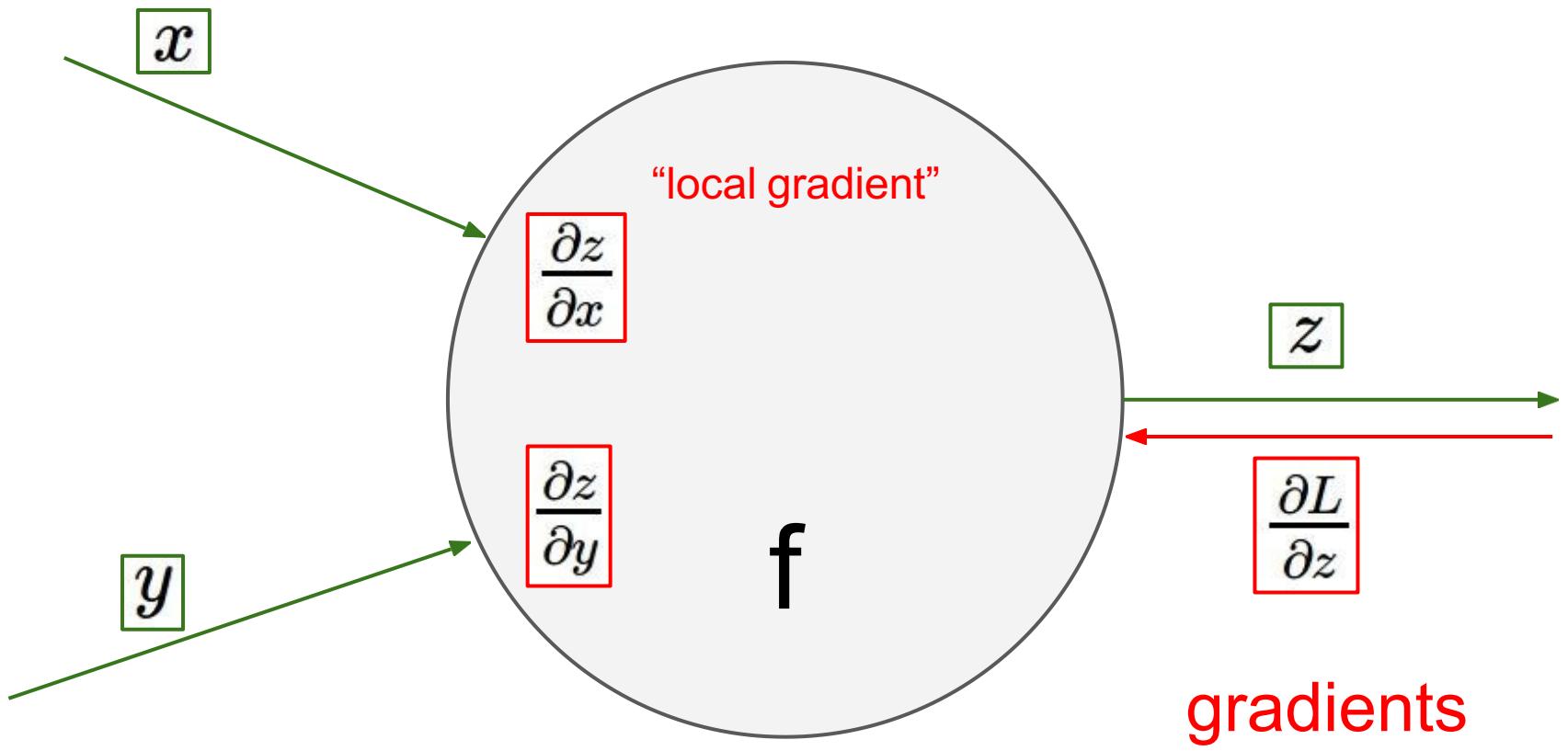
Forward propagation and Backpropagation

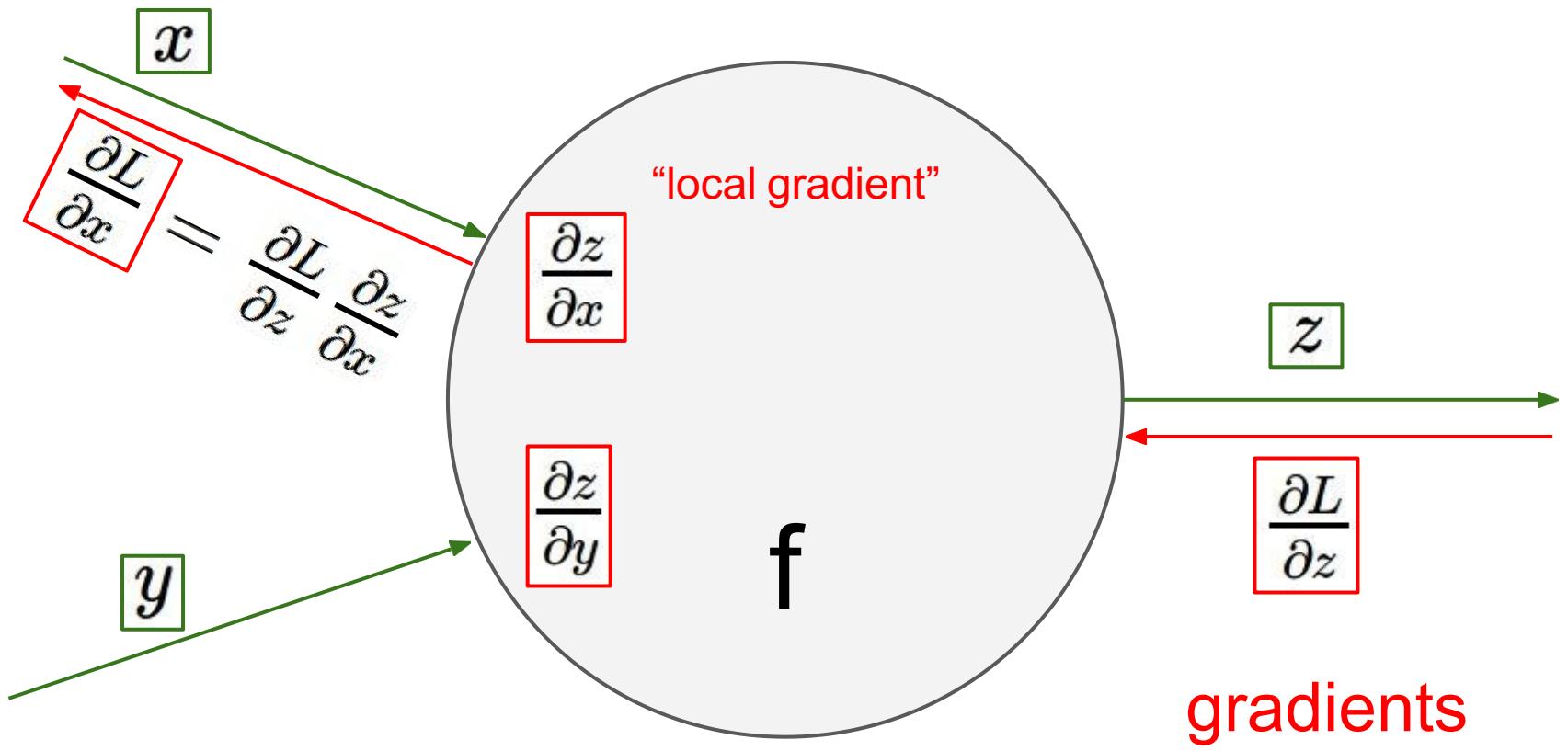
1. Given inputs and the labels first do forward propagation through the network. It is done by feeding the input to the network. Finally, we will get an output from the last layer. This process is called forward propagation.
2. Calculate the error/loss and gradients of the loss with respect to each of the parameters in the network. The error/loss is the difference between the network generated output and the actual output in the training dataset.
3. Backpropagate the gradients and update/tune the weights using gradient descent technique. This process is called backpropagation.

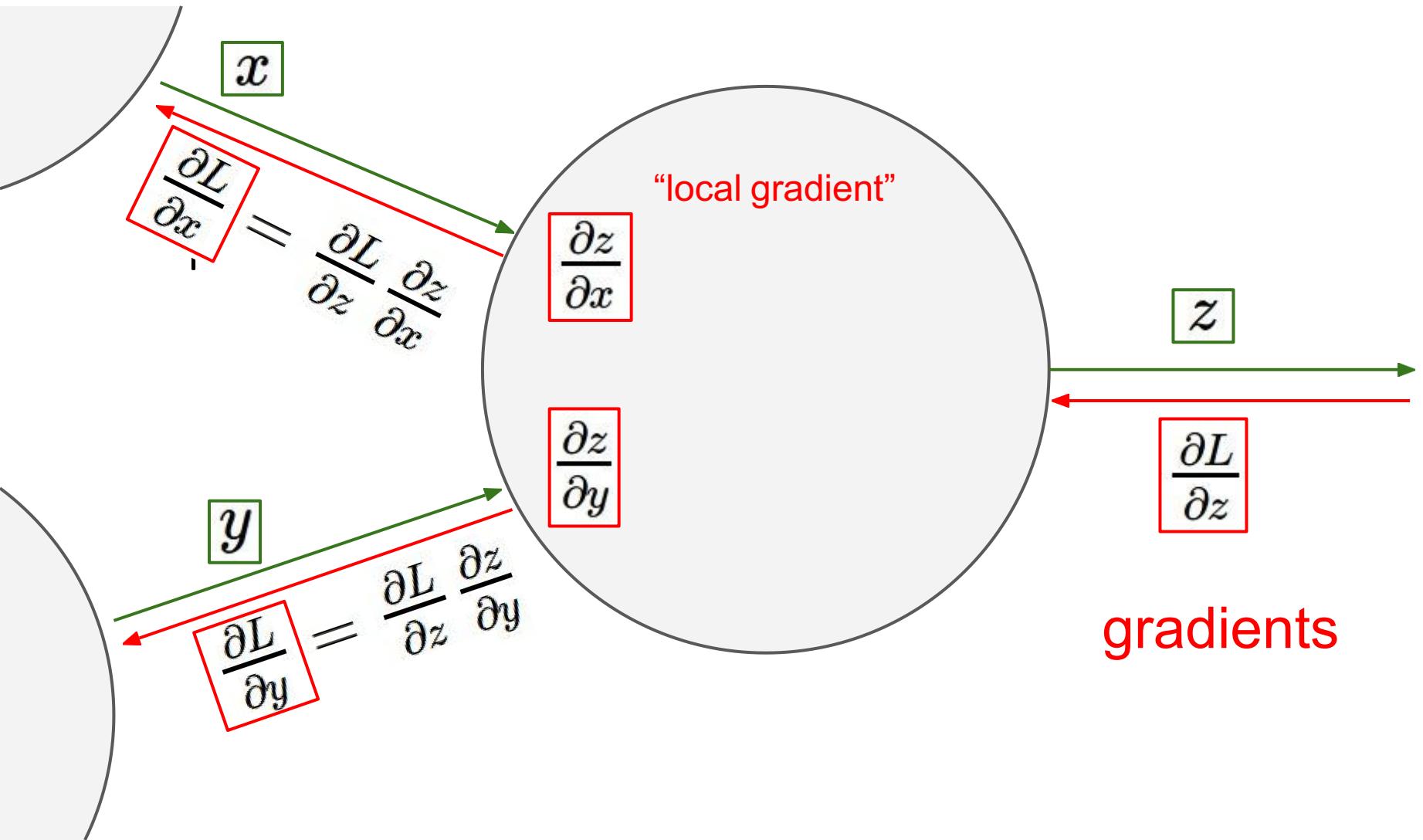
Forward propagation and Backpropagation





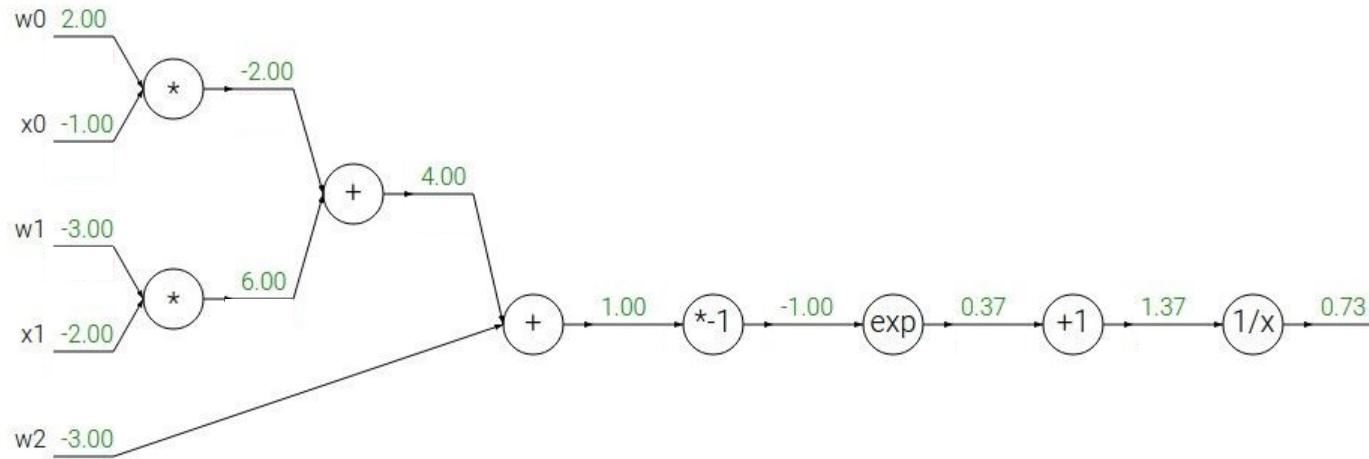






Another example:

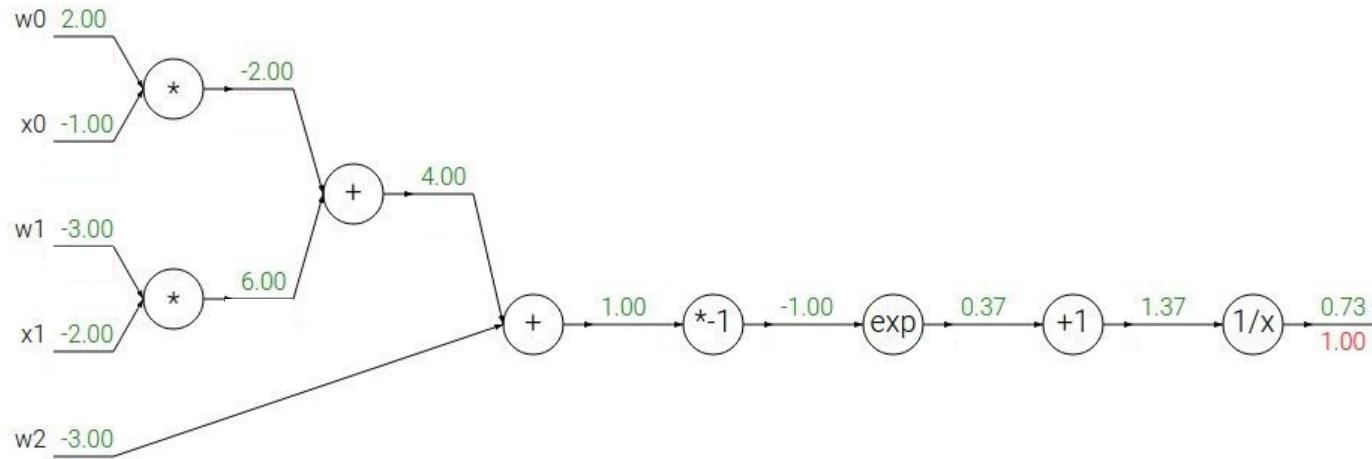
$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



Calculate the gradient of the LOSS with respect to w_0, x_0, w_1, x_1 and w_2 .

Another example:

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$



$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

$$f_c(x) = c + x$$

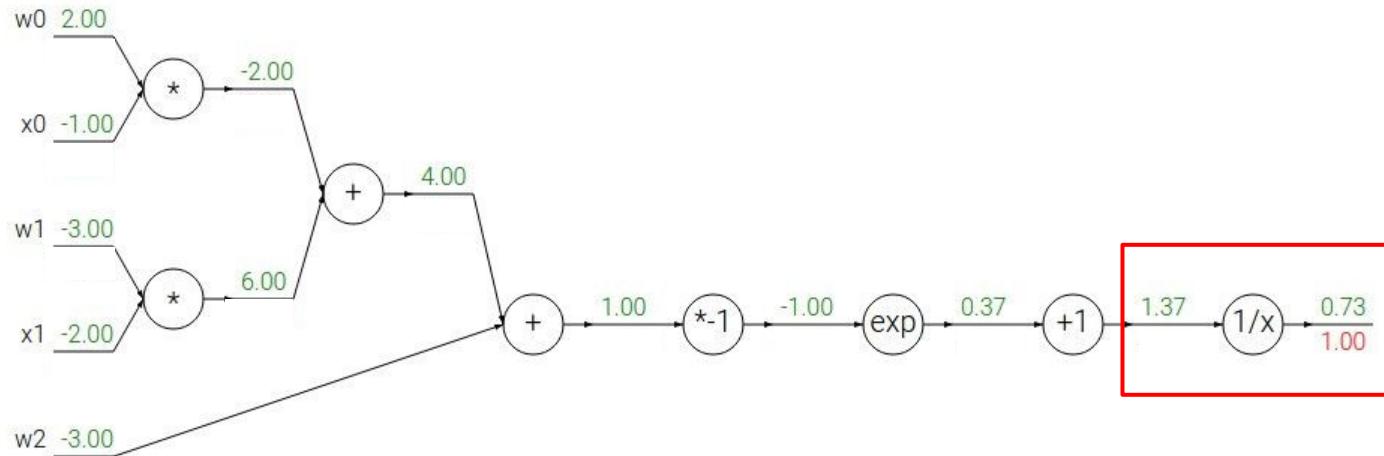
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$$\frac{df}{dx} = -1/x^2$$

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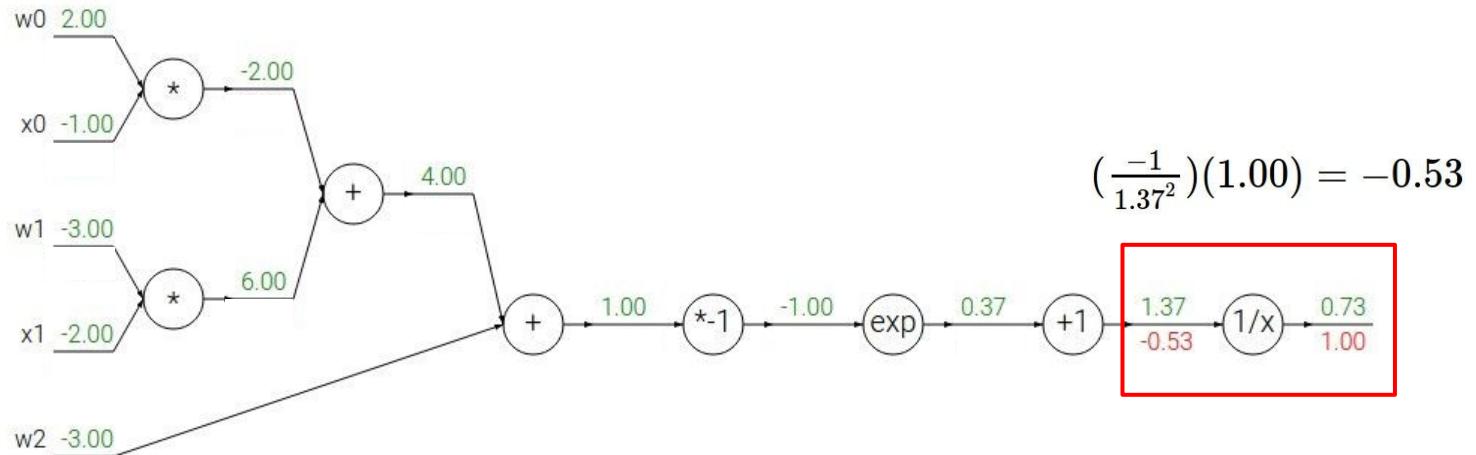
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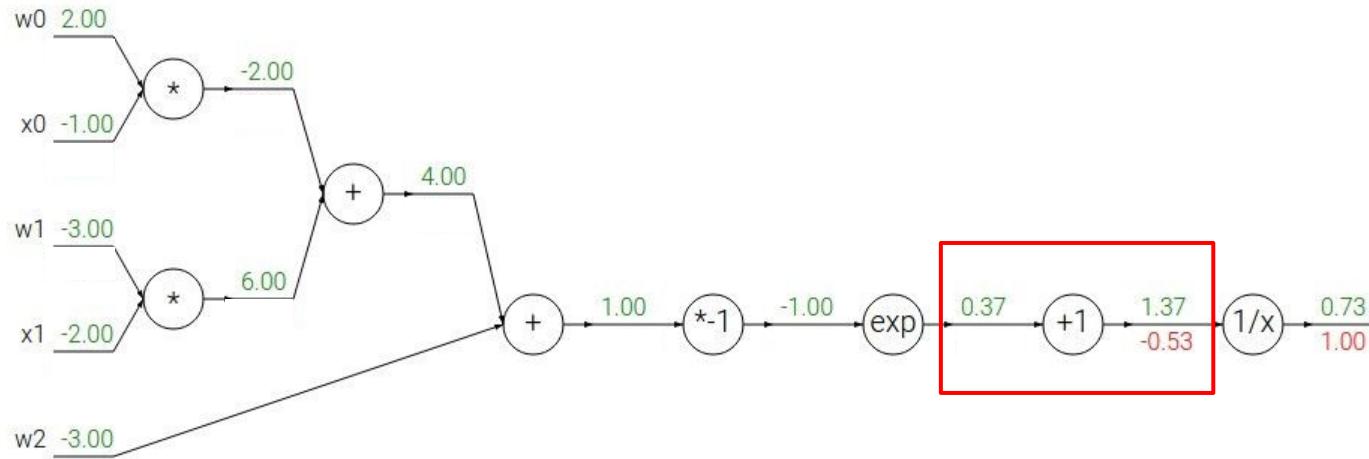
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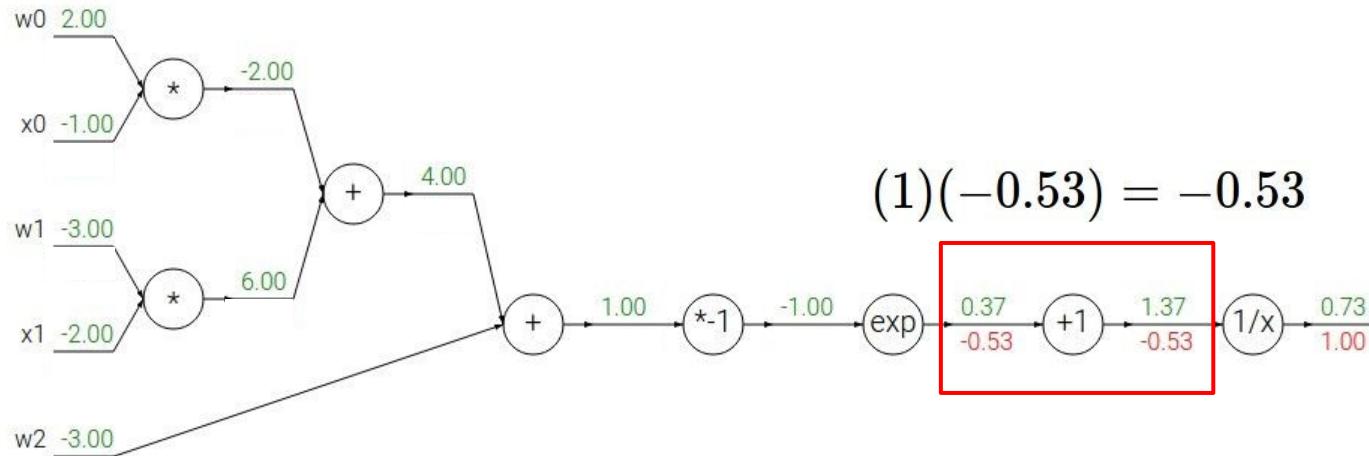
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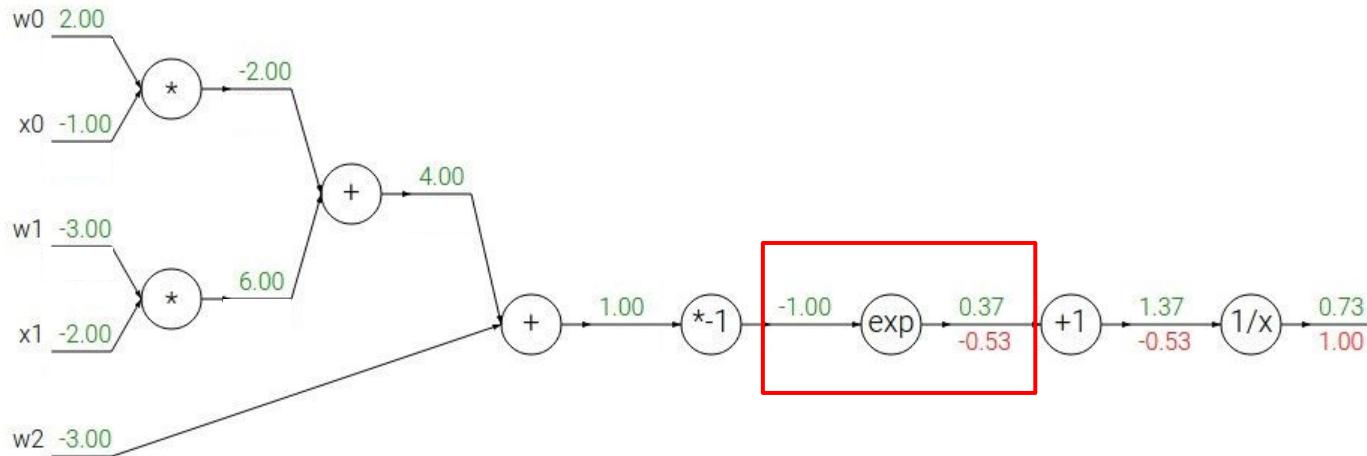
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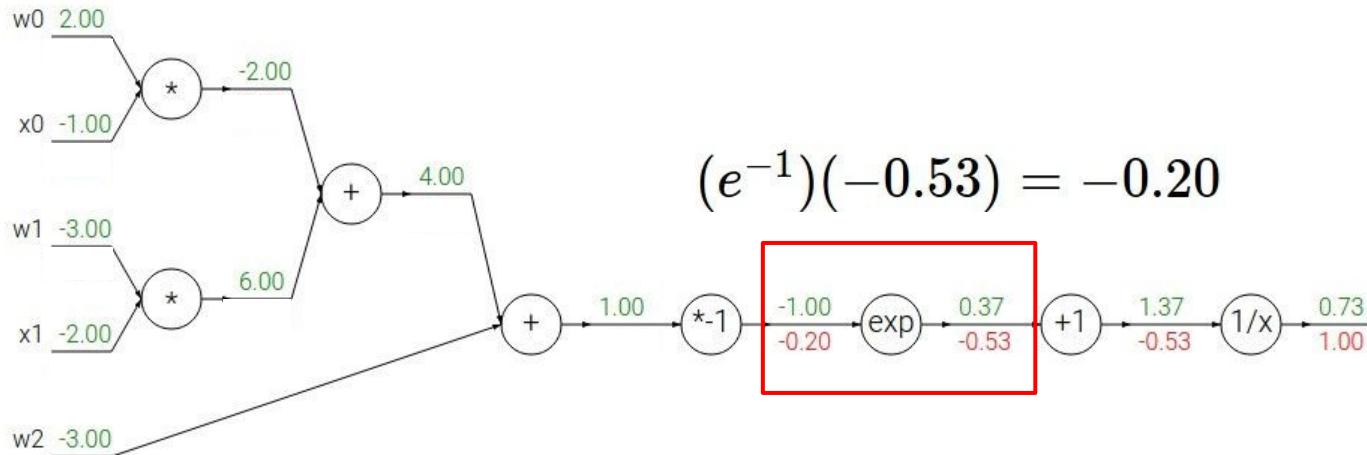
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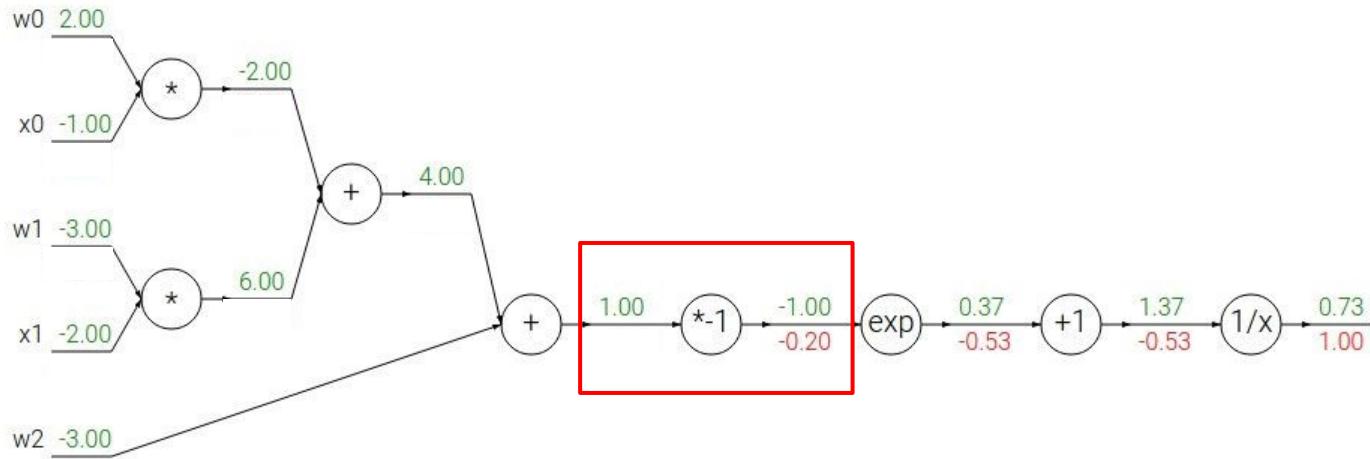
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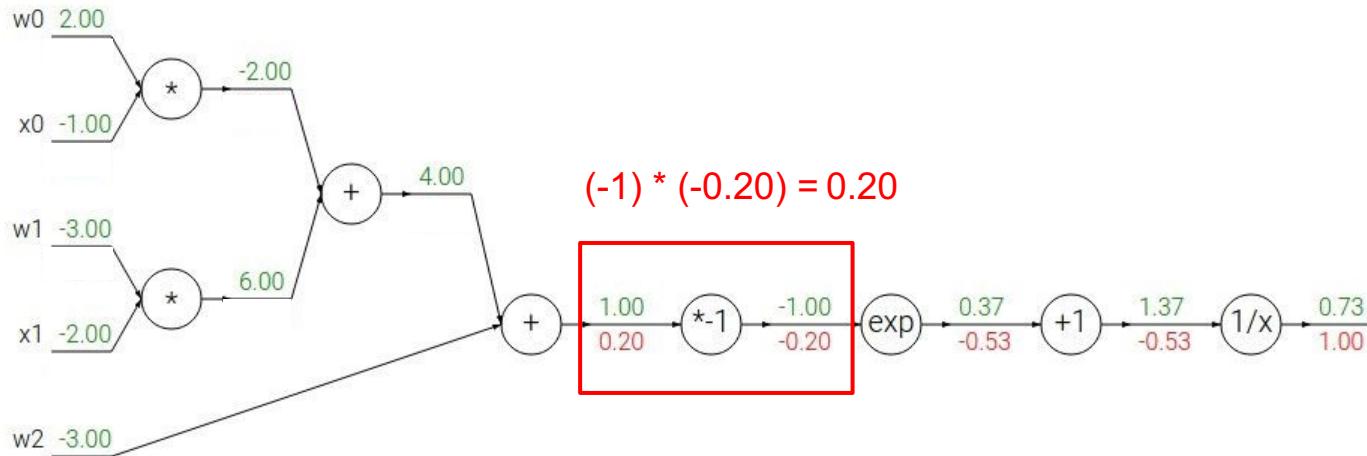
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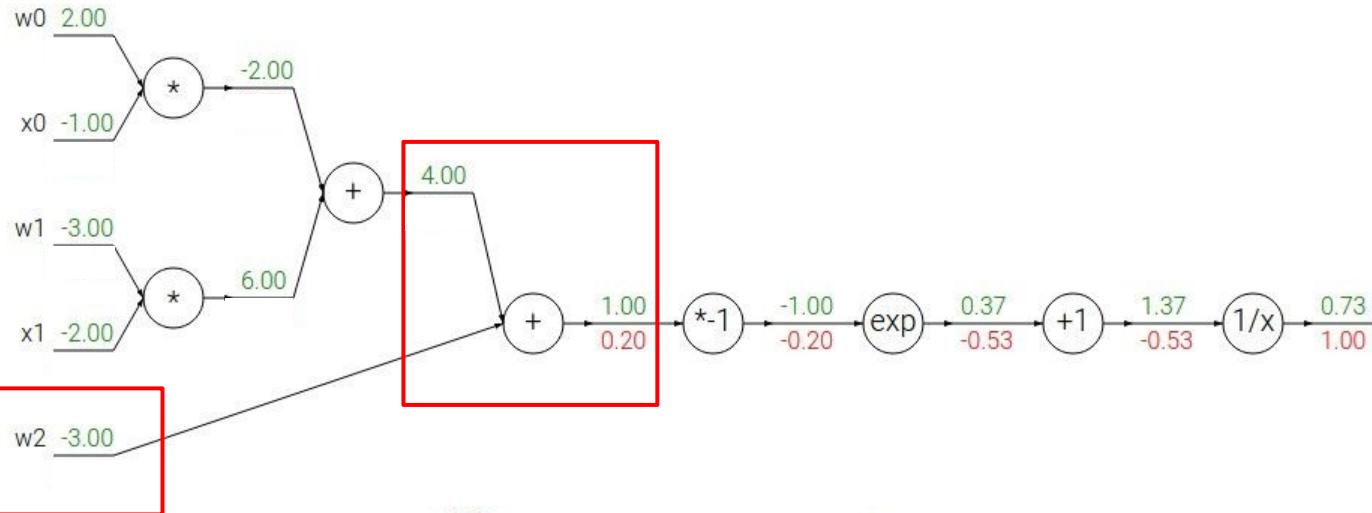
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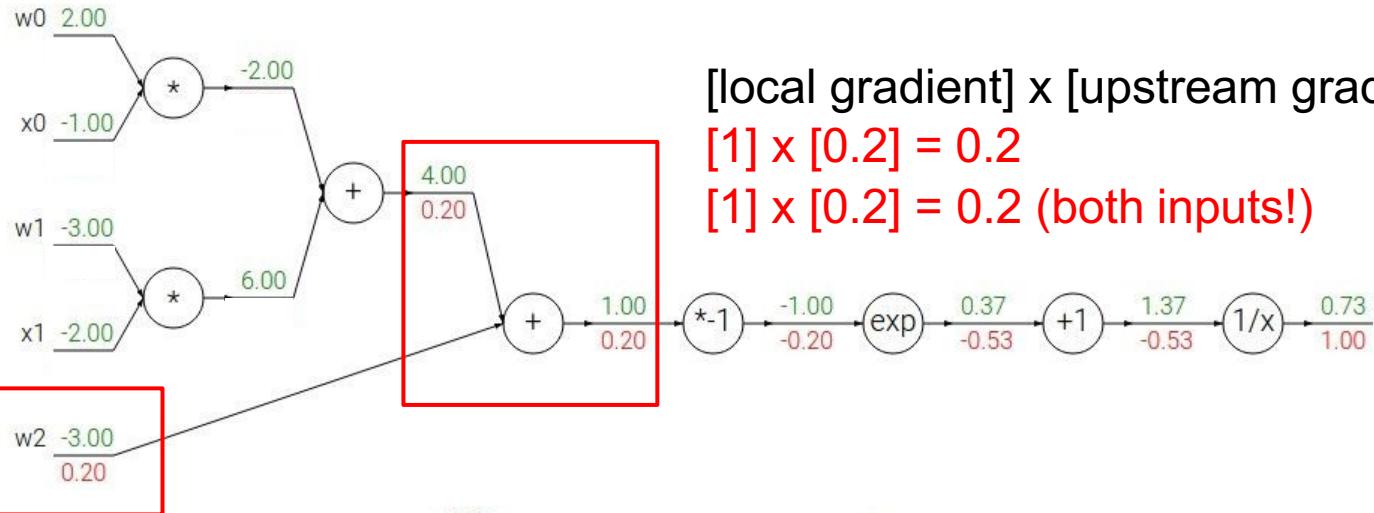
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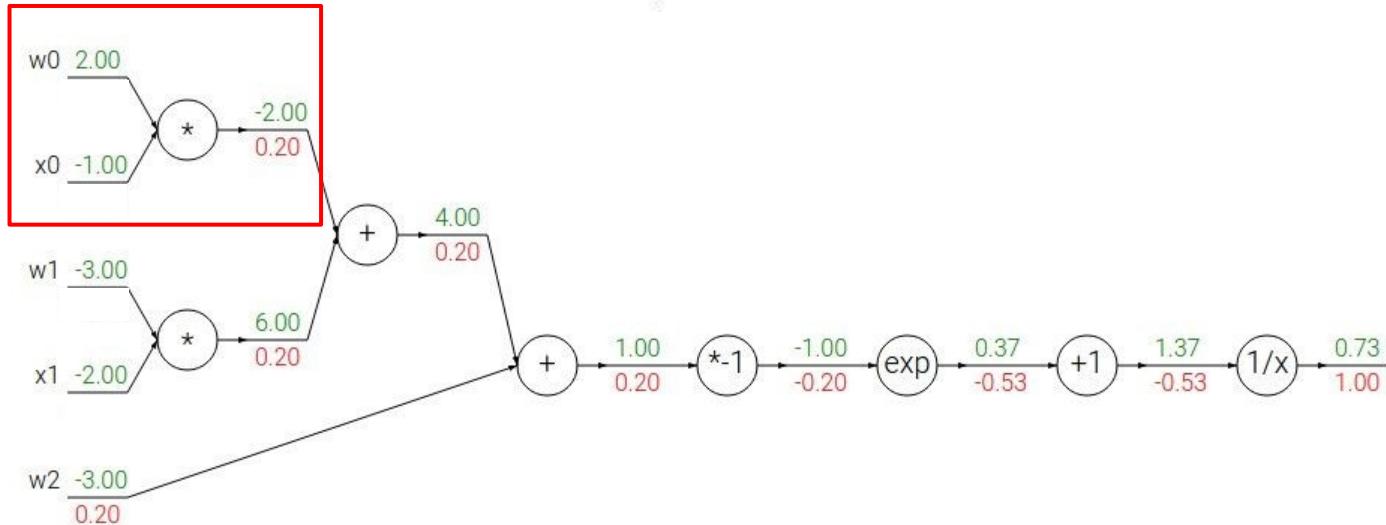
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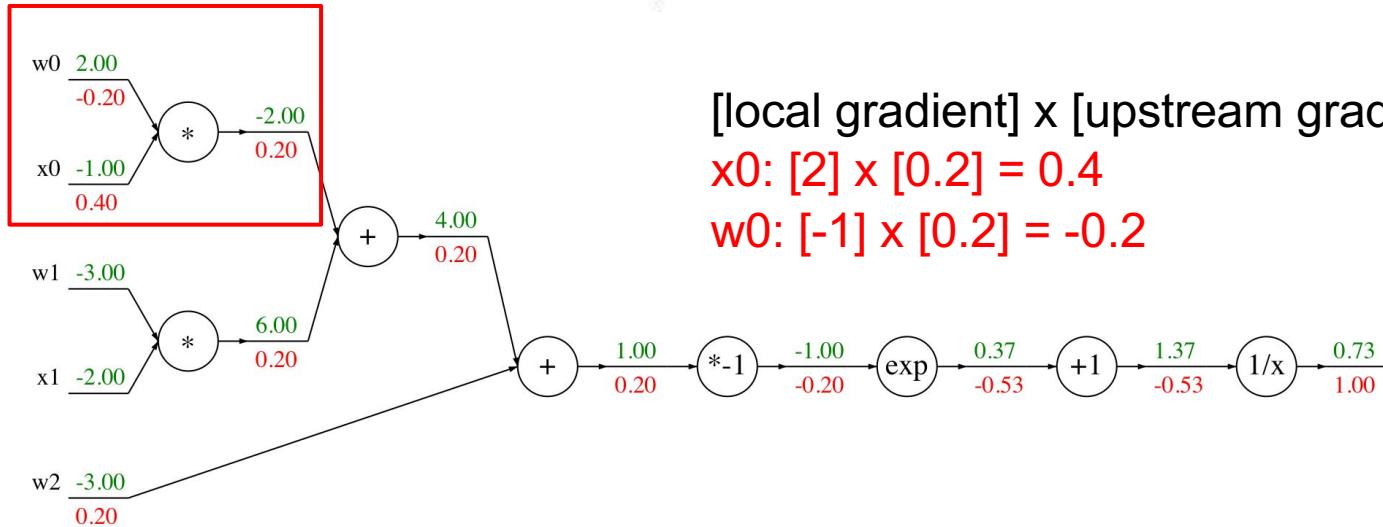
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Another example:

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[local gradient] x [upstream gradient]
 $x_0: [2] \times [0.2] = 0.4$
 $w_0: [-1] \times [0.2] = -0.2$

$$f(x) = e^x$$

→

$$\frac{df}{dx} = e^x$$

$$f_a(x) = ax$$

→

$$\frac{df}{dx} = a$$

$$f(x) = \frac{1}{x}$$

$$f_c(x) = c + x$$

$$\frac{df}{dx} = -1/x^2$$

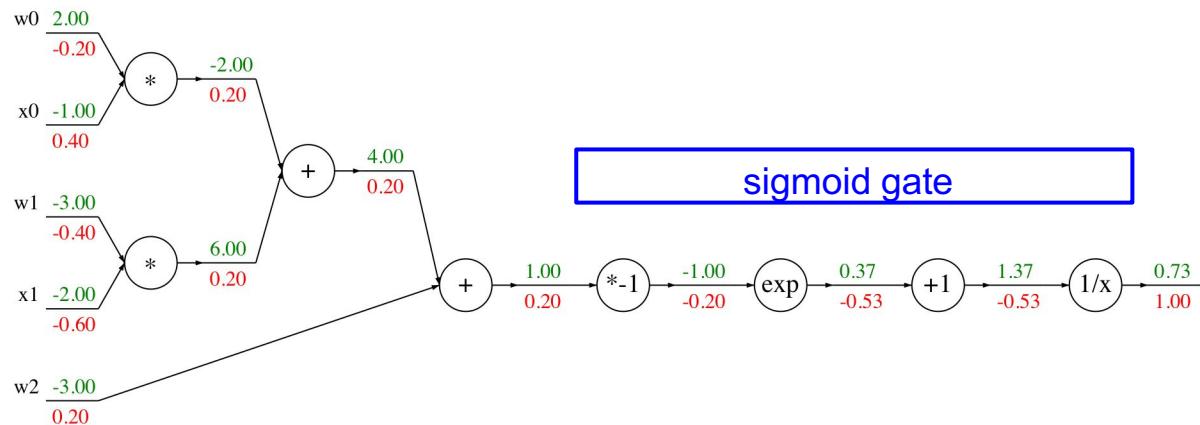
$$\frac{df}{dx} = 1$$

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left(\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left(\frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$

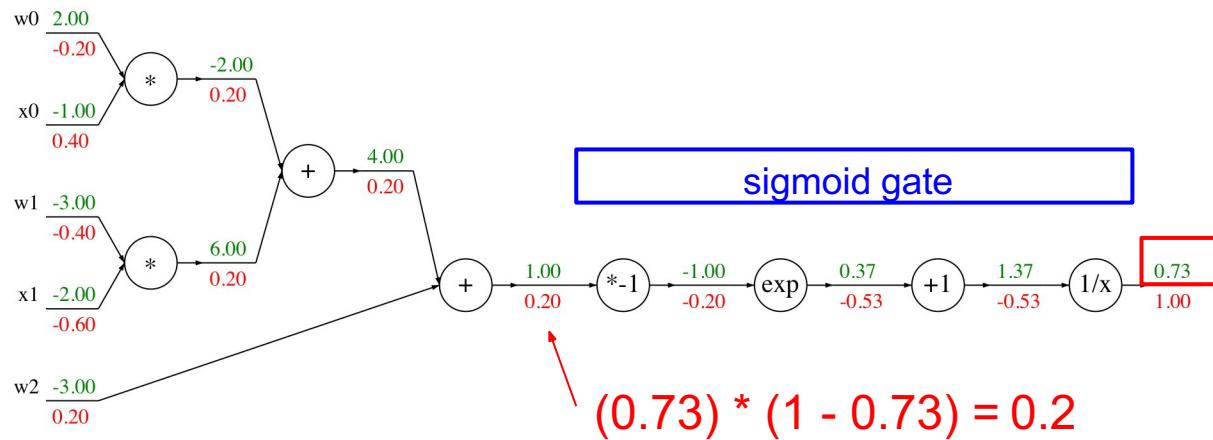


$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2)}}$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

sigmoid function

$$\frac{d\sigma(x)}{dx} = \frac{e^{-x}}{(1 + e^{-x})^2} = \left(\frac{1 + e^{-x} - 1}{1 + e^{-x}} \right) \left(\frac{1}{1 + e^{-x}} \right) = (1 - \sigma(x))\sigma(x)$$



Logistic regression: binary classification

The loss function is -

$$: L(w) = -\frac{1}{m} \sum_{i=1}^m [y^{(i)} \log h(x^{(i)}) + (1 - y^{(i)}) \log(1 - h(x^{(i)}))]$$

$x^{(i)}$ is a vector for all x_j ($j=0,1, \dots, n$), and $y^{(i)}$ is the target value for this example.

$$h(x) = \frac{1}{1 + e^{-w^T x}}$$

Softmax

- Use softmax for multi-class classification
 - K is the number classes

$$P(y = j \mid z^{(i)}) = \phi_{\text{softmax}}(z^{(i)}) = \frac{e^{z^{(i)}}}{\sum_{j=0}^k e^{z_j^{(i)}}},$$

where we define the net input z as

$$z = w_0x_0 + w_1x_1 + \dots + w_mx_m = \sum_{l=0}^m w_lx_l = \mathbf{w}^T \mathbf{x}.$$

The loss function is $H(y, p) = - \sum_i y_i \log(p_i)$

Softmax vs Sigmoid function in Logistic classifier?

- In the two-class logistic regression, the predicted probabilities are as follows, using the sigmoid function:

$$\Pr(Y_i = 0) = \frac{e^{-\beta \cdot \mathbf{X}_i}}{1 + e^{-\beta_0 \cdot \mathbf{X}_i}}$$

$$\Pr(Y_i = 1) = 1 - \Pr(Y_i = 0) = \frac{1}{1 + e^{-\beta \cdot \mathbf{X}_i}}$$

In the multiclass logistic regression, with K classes, the predicted probabilities are as follows, using the softmax function:

$$\Pr(Y_i = k) = \frac{e^{\beta_k \cdot \mathbf{X}_i}}{\sum_{0 \leq c \leq K} e^{\beta_c \cdot \mathbf{X}_i}}$$

Softmax vs Sigmoid function in Logistic classifier?

- One can observe that the softmax function is an extension of the sigmoid function to the multiclass case, as explained below. Let's look at the multiclass logistic regression, with K=2 classes:

$$\Pr(Y_i = 0) = \frac{e^{\beta_0 \cdot \mathbf{X}_i}}{\sum_{0 \leq c \leq K} e^{\beta_c \cdot \mathbf{X}_i}} = \frac{e^{\beta_0 \cdot \mathbf{X}_i}}{e^{\beta_0 \cdot \mathbf{X}_i} + e^{\beta_1 \cdot \mathbf{X}_i}} = \frac{e^{(\beta_0 - \beta_1) \cdot \mathbf{X}_i}}{e^{(\beta_0 - \beta_1) \cdot \mathbf{X}_i} + 1} = \frac{e^{-\beta \cdot \mathbf{X}_i}}{1 + e^{-\beta \cdot \mathbf{X}_i}}$$

$$\Pr(Y_i = 1) = \frac{e^{\beta_1 \cdot \mathbf{X}_i}}{\sum_{0 \leq c \leq K} e^{\beta_c \cdot \mathbf{X}_i}} = \frac{e^{\beta_1 \cdot \mathbf{X}_i}}{e^{\beta_0 \cdot \mathbf{X}_i} + e^{\beta_1 \cdot \mathbf{X}_i}} = \frac{1}{e^{(\beta_0 - \beta_1) \cdot \mathbf{X}_i} + 1} = \frac{1}{1 + e^{-\beta \cdot \mathbf{X}_i}}$$

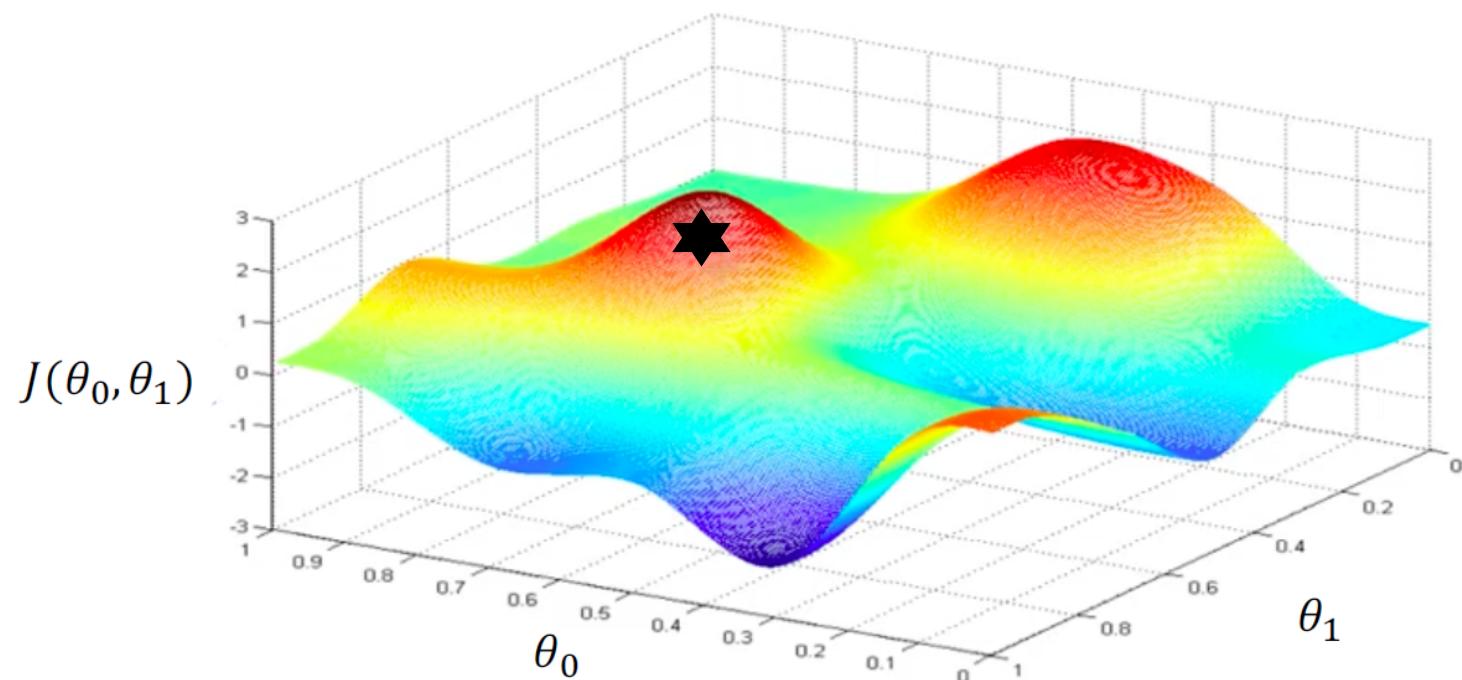
where $\beta = -(\beta_0 - \beta_1)$

Types of Loss Functions

- For Regression tasks (continuous values)
 - Mean Absolute Error
 - Mean Squared Error
- For Classification tasks (discrete categories)
 - Categorical Cross Entropy
 - Binary Cross Entropy
- And various others in Keras

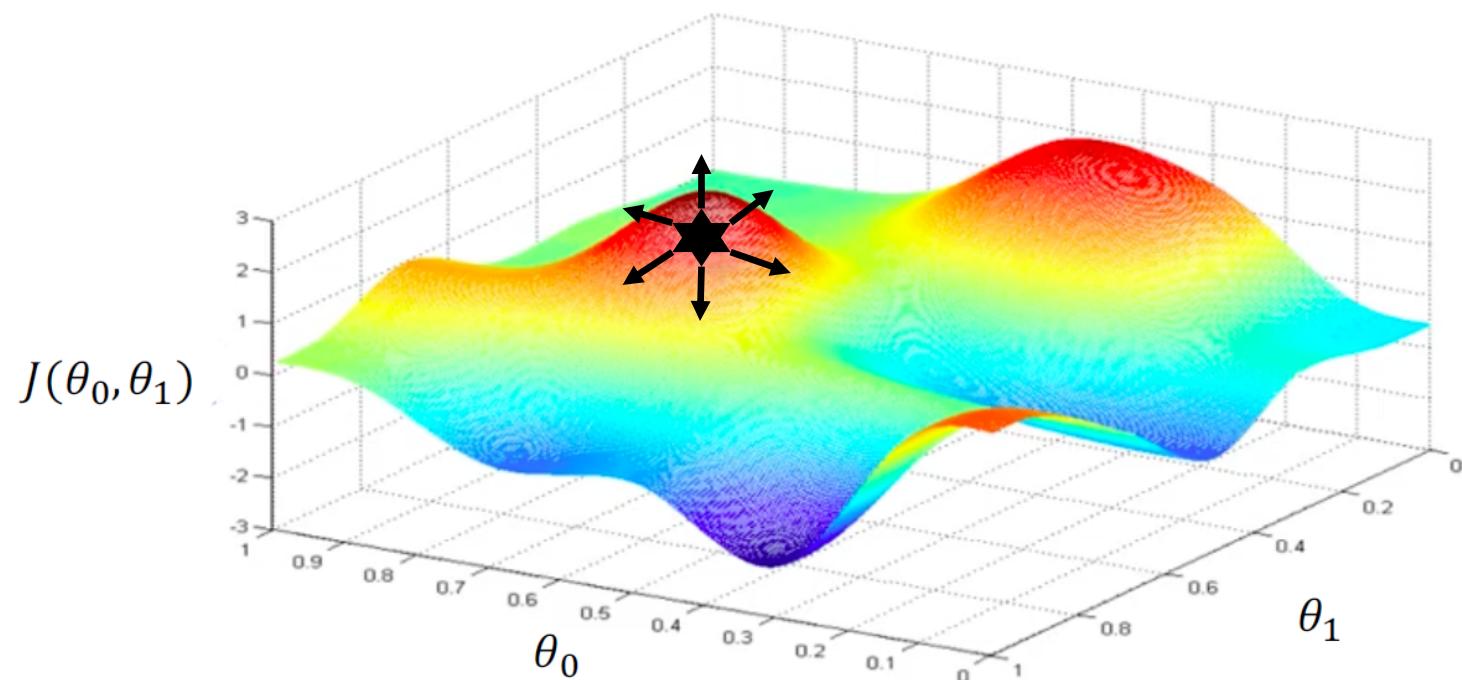
Gradient Descent

- Standard approach for learning weights



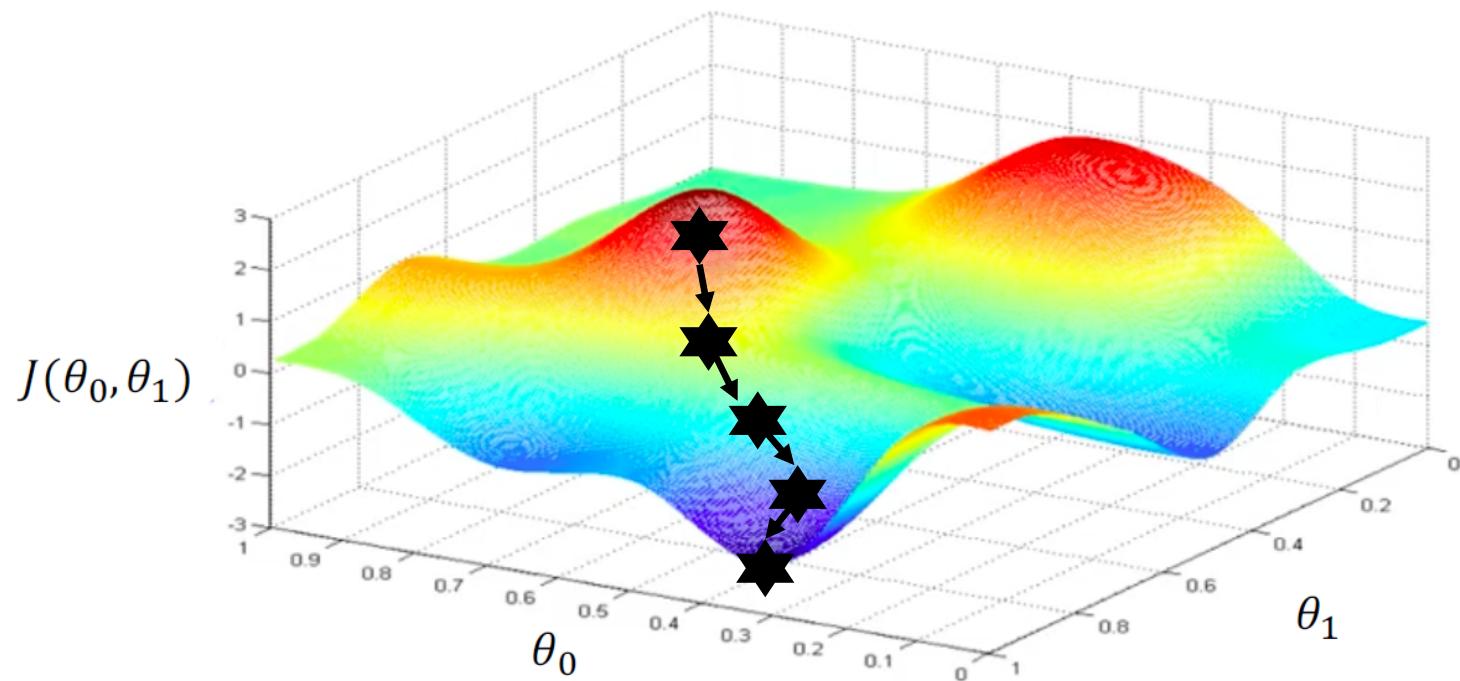
Gradient Descent

- Standard approach for learning weights



Gradient Descent

- Standard approach for learning weights
 - What about the learning rate?



Exercise 1: Gradient Descent

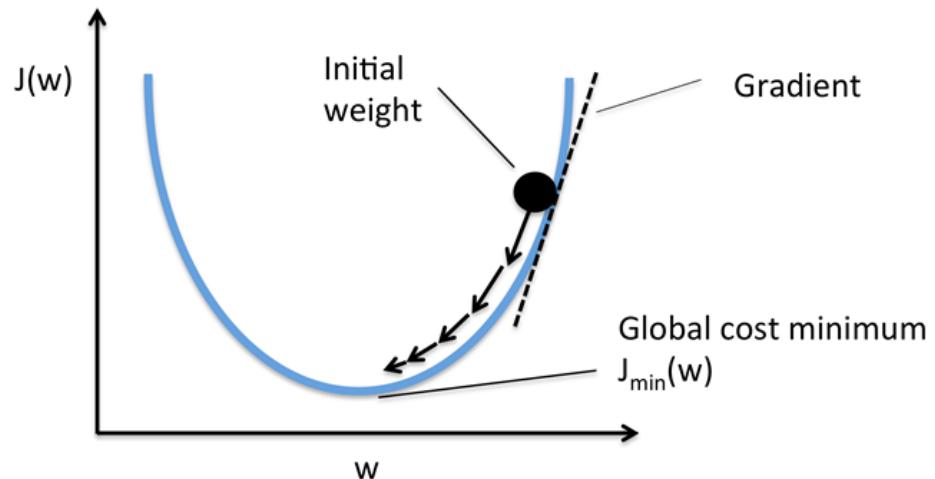
- What are the effects of learning rates on gradient descent?

Mini-batch Training

- Batch gradient descent (BGD)
 - Compute weights using entire training set
- Stochastic gradient descent (SGD)
 - Compute weights using an instance at a time
- Mini-batch gradient descent
 - Compute weights using small batches (e.g., 32) from training set
- Typically, mini-batch is used in training neural networks
 - More robust convergence (avoid local minima) than BGD
 - Computationally more efficient than SGD
 - Parallelization to speed up computation

Adaptive Learning Rates

- Instead of fixed learning rates, use learning rates that change based on:
 - Value of gradient
 - Speed of learning
 - Sizes of weights

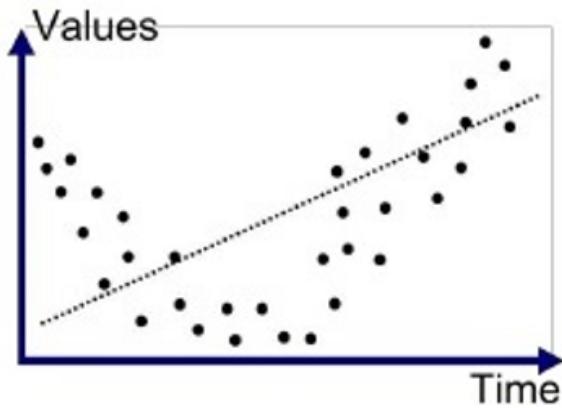


<https://wiki.tum.de/display/Ifdv/Adaptive+Learning+Rate+Method>

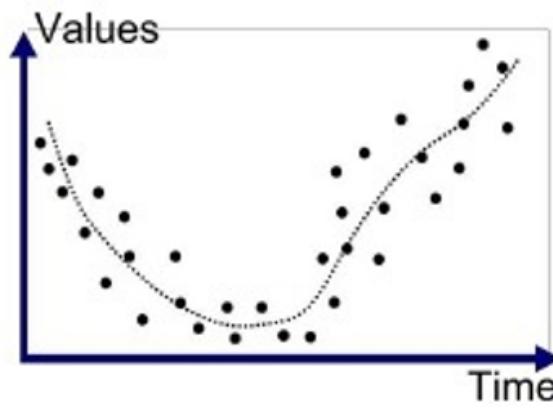
Adaptive Learning Rates

- Various approaches available in Keras
 - Adagrad
 - Adadelta
 - Adam
 - Nadam
 - RMSprop

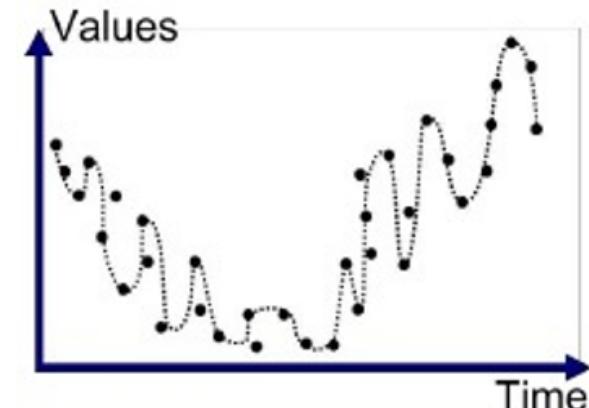
Under/Over-fitting Problem



Underfitted



Good Fit/R robust



Overfitted

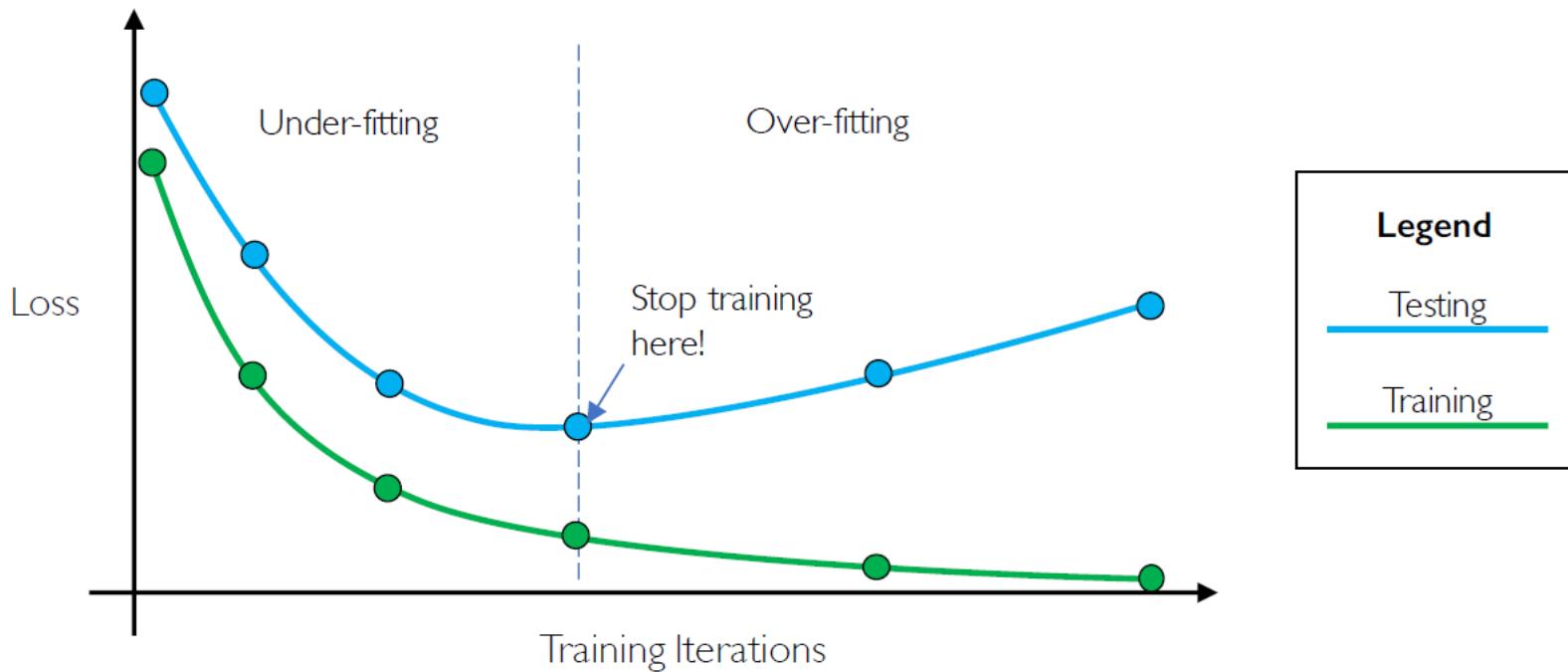
Source: <https://medium.com/greyatom/what-is-underfitting-and-overfitting-in-machine-learning-and-how-to-deal-with-it-6803a989c76>

Regularization

- Technique to help prevent overfitting on training data
 - Helps to generalize our model on unseen (testing) data
- Use
 - Early Stopping

Early Stopping

- Stop training of model before overfitting



Exercise 2: Application to NLP

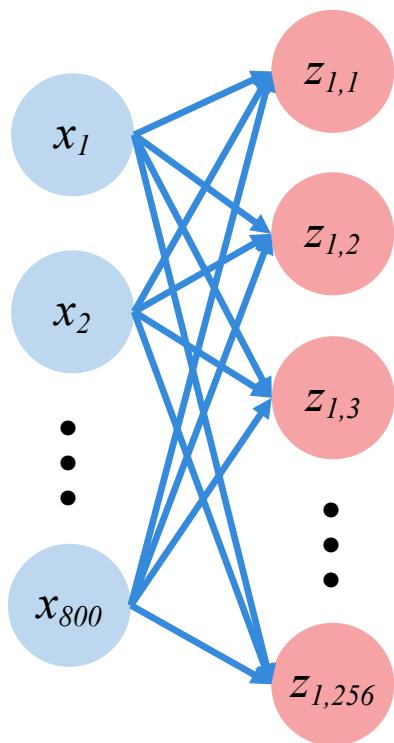
- What would your neural network architecture look like for the following sentiment analysis task?



Vocabulary = 800 Words

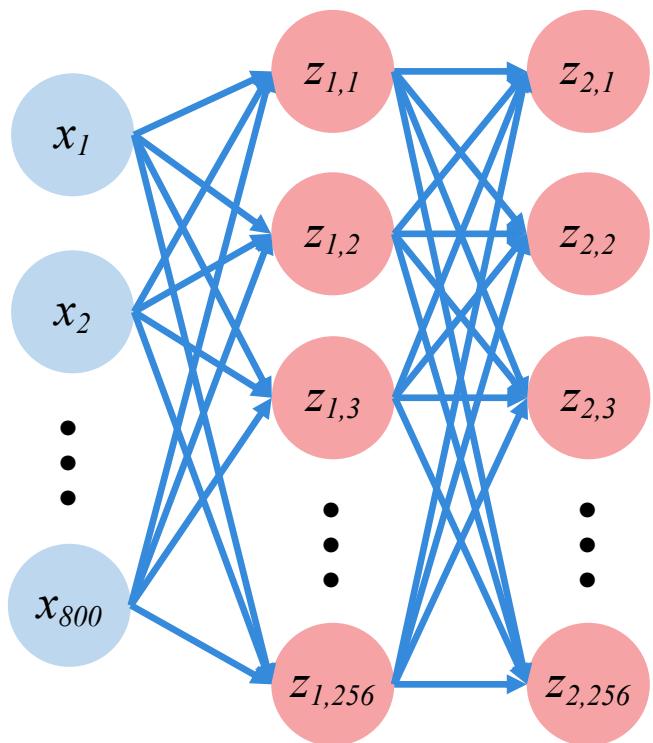
Classes =
{V.Good, Good,
Neutral, Bad,
V.Bad}

Keras Example: NLP Application



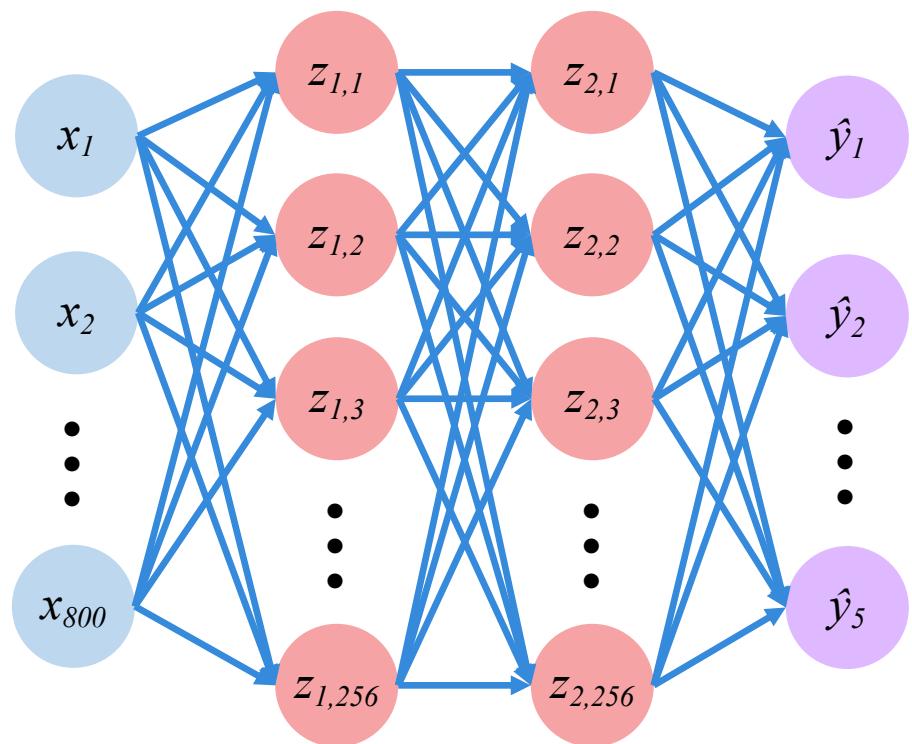
```
1 model = Sequential()  
2  
3 # input layer and hidden layer 1  
4 model.add(Dense(256, activation='relu', input_dim=800))  
5 model.add(Dropout(0.5))
```

Keras Example: NLP Application



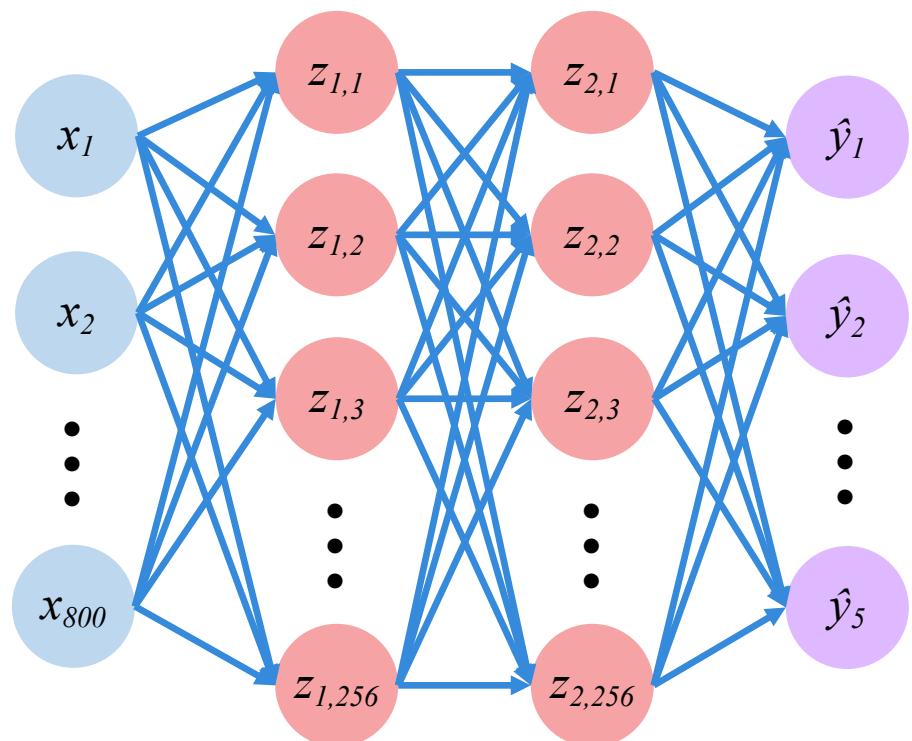
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3 # input layer and hidden layer 1
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5 model.add(Dropout(0.5))
6
7 # hidden layer 2
8 model.add(Dense(256, activation='relu'))
9 model.add(Dropout(0.5))
```

Keras Example: NLP Application



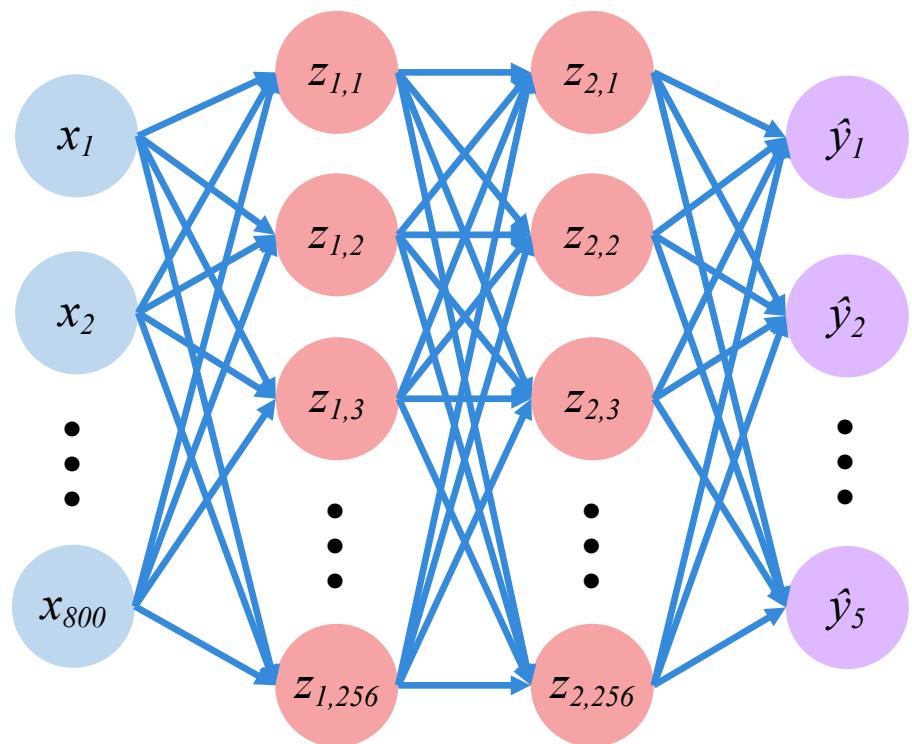
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8 model.add(Dense(256, activation='relu'))
9 model.add(Dropout(0.5))
10
11 # output layer
12 model.add(Dense(5, activation='softmax'))
```

Keras Example: NLP Application



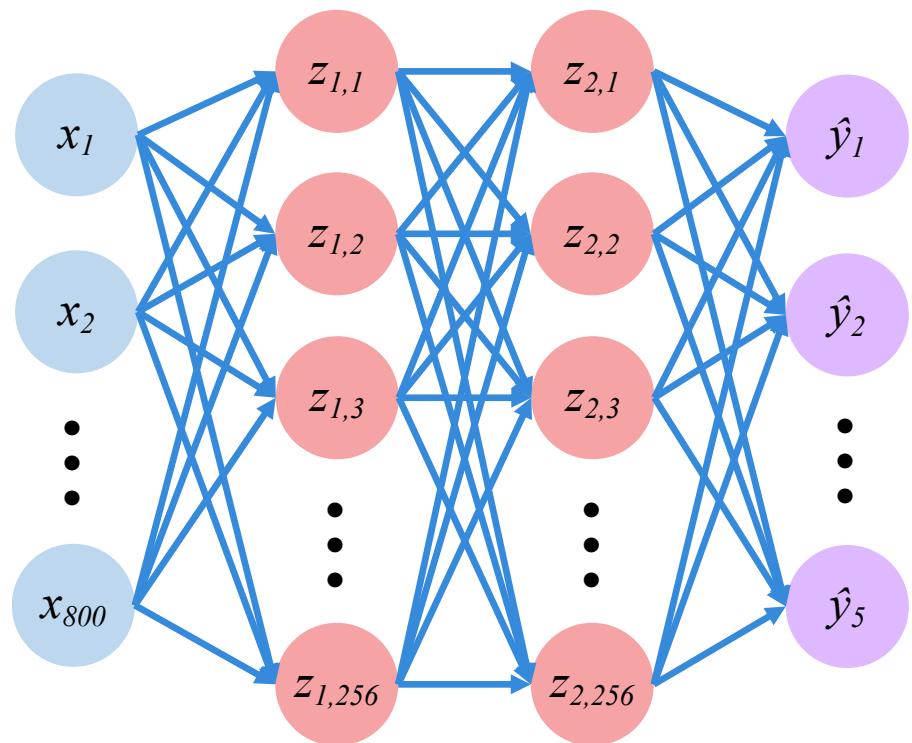
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12 model.add(Dense(5, activation='softmax'))
13
14 # configure Learning process
15 model.compile(loss='categorical_crossentropy',
16                 optimizer='adam',
17                 metrics=['accuracy'])
```

Keras Example: NLP Application



```
1 model = Sequential()
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16                 optimizer='adam',
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18
19 # train our model
20 model.fit(x_train, y_train,
21             epochs=50,
22             batch_size=128)
```

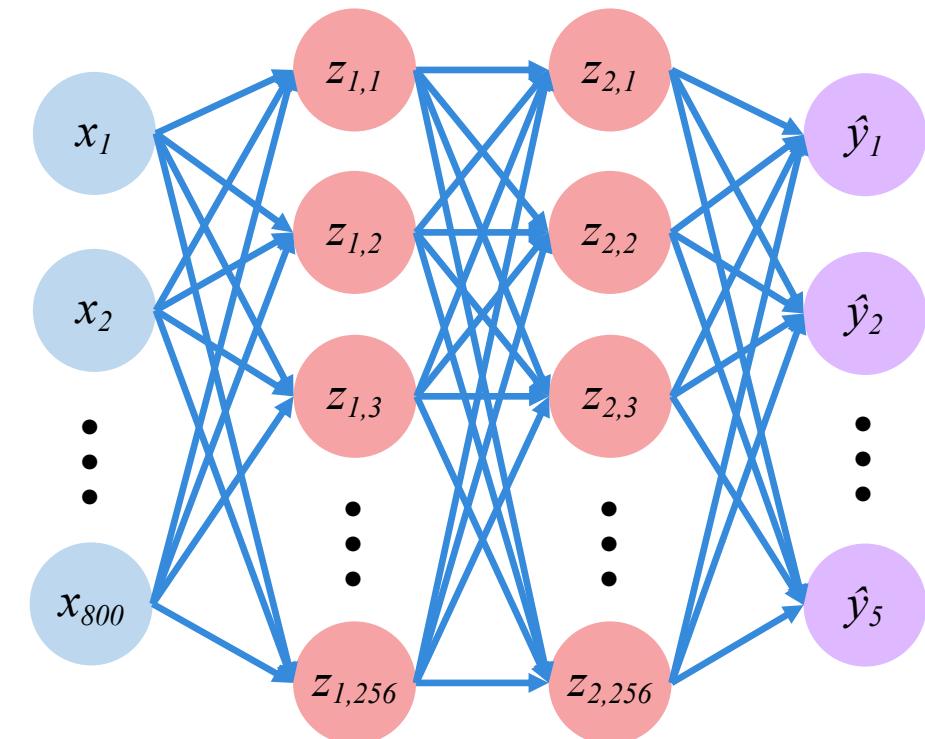
Keras Example: NLP Application



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17                 metrics=['accuracy'])
18
19 # train our model
20 model.fit(x_train, y_train,
21             epochs=50,
22             batch_size=128)
23
24 # test our model
25 score = model.evaluate(x_test, y_test, batch_size=128)
```

Exercise 3: Discussion of Shortcomings

- What are the possible shortcomings of our previous Naïve Bayes and this MLP approach to sentiment classification?



How deep learning helps

- Suppose we have L input sentences each having M words. Each of these words have N number of features. Hence, the input matrix X has dimension of $L \times M \times N$.
- Now say, we want to classify sentiment of each sentence.
- Can we automatically learn a function that learns and tells us the role of each word in the classification?
- We call this attention mechanism.
- The idea is simple.
- We apply a dense layer or FC network on the input matrix X. FC of dimension $N \times 1$. We get X' of dimension $L \times M \times 1$.
- We add some non-linearity to X' so we get $X'' = \tanh(X')$
- Now, apply softmax to the last dimension to get the importance probability of each word. Say, we store this to $X''' = \text{softmax}(X'', \text{axis}=-1)$ of dimension $L \times M \times 1$

How deep learning helps

- We multiply X''' with the input X to find an abstract representation of our inputs that encode the role of each word along with the original semantic of the words.
- Hence we have $X'''' = X''' * X$ to get the abstract features of dimension $L * N$
- These features can be sent to a softmax layer for classification
- Using the backpropagation method, the deep network can learn the weights and function required to leverage the role of each word in the classification.

Things to do

- For the upcoming lab, please install Keras
 - See <https://keras.io/#installation>

References

- http://introtodeeplearning.com/materials/2018_6S191_Lecture1.pdf
- <http://www.yaronhadad.com/deep-learning-most-amazing-applications/>
- <https://medium.com/@shrutijadon10104776/survey-on-activation-functions-for-deep-learning-9689331ba092>
- <https://towardsdatascience.com/activation-functions-and-its-types-which-is-better-a9a5310cc8f>