50.007
Machine Learning

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SINGAPORE UNIVERSITY OF TECHNOLOGY AND DESIGN

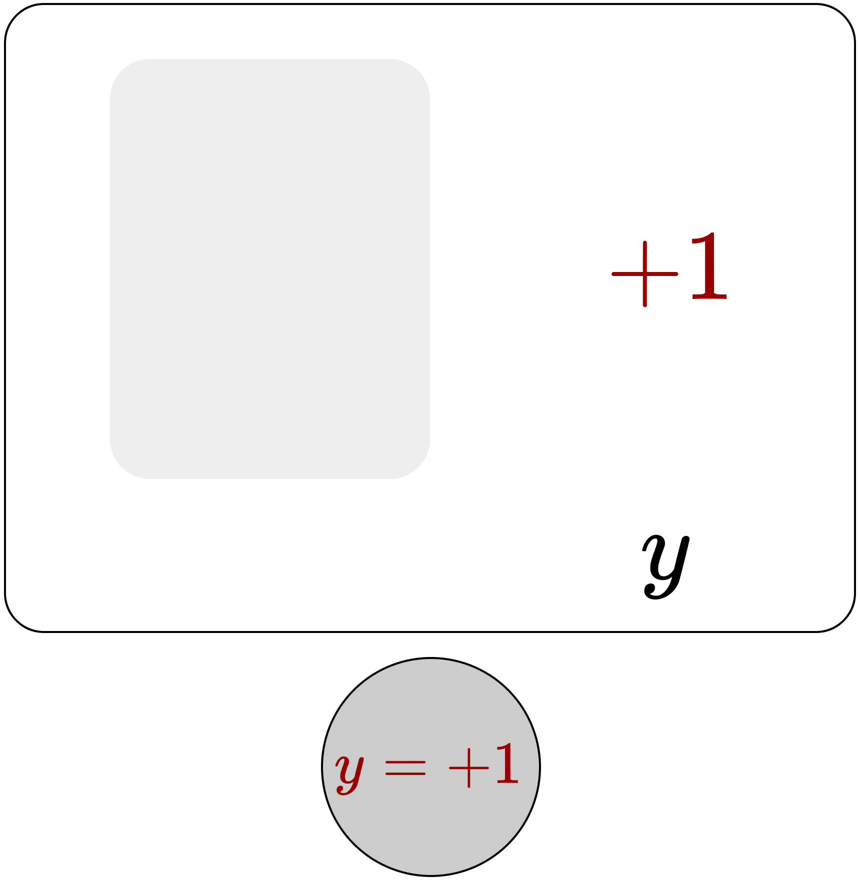
Hidden Markov Model (I)

Generative Models

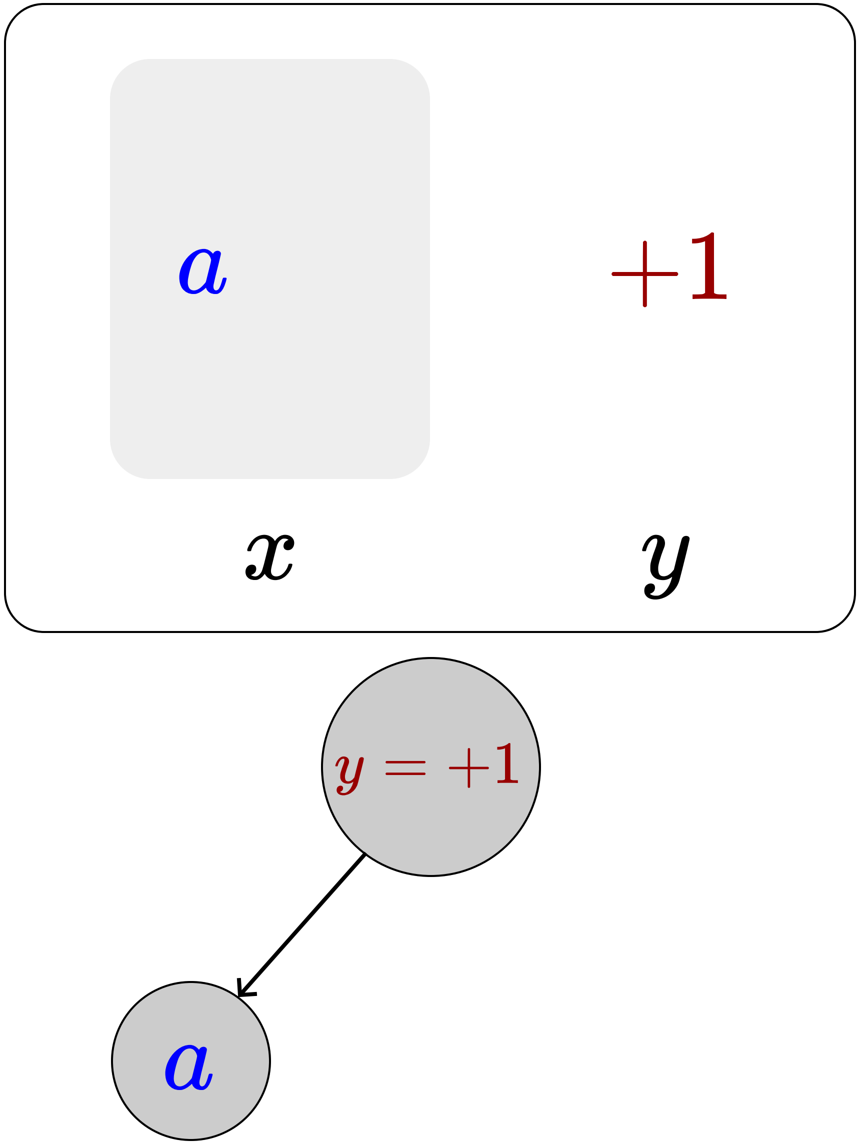
$a\ b\ c$ $\quad +1$

x $\quad y$

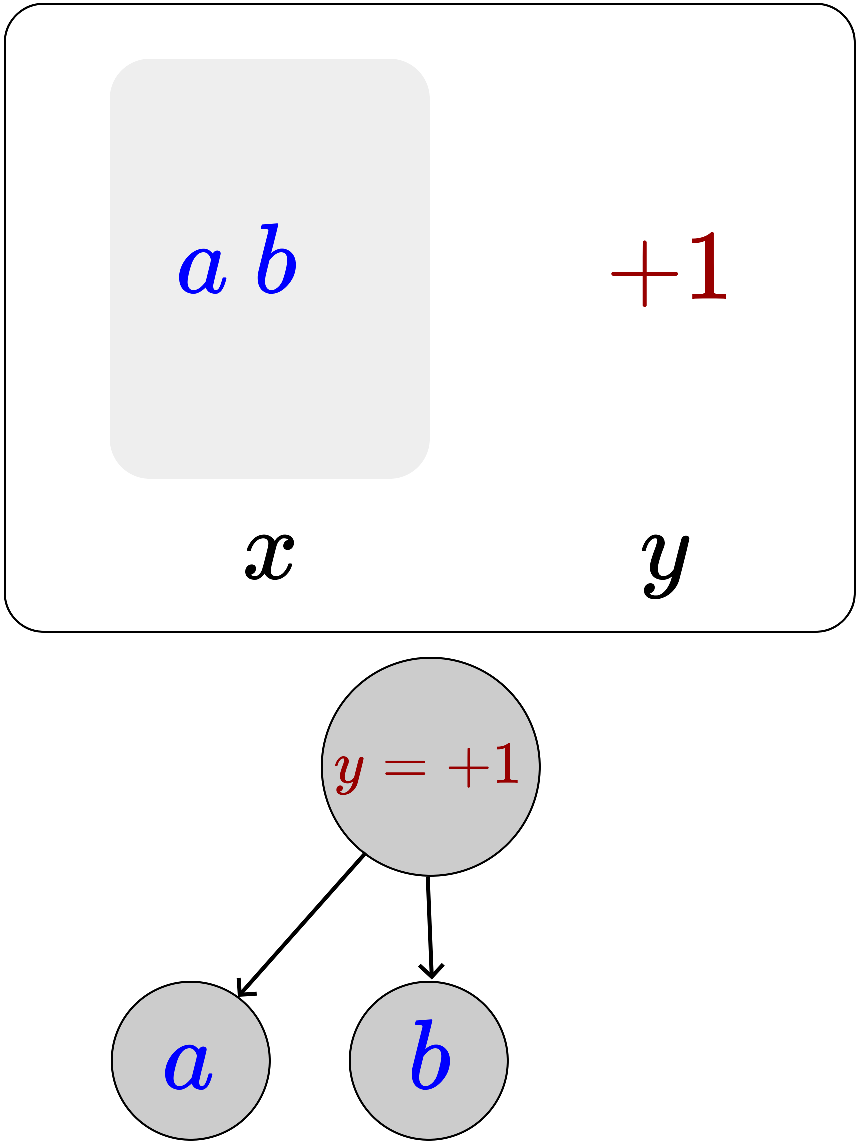
Generative Models



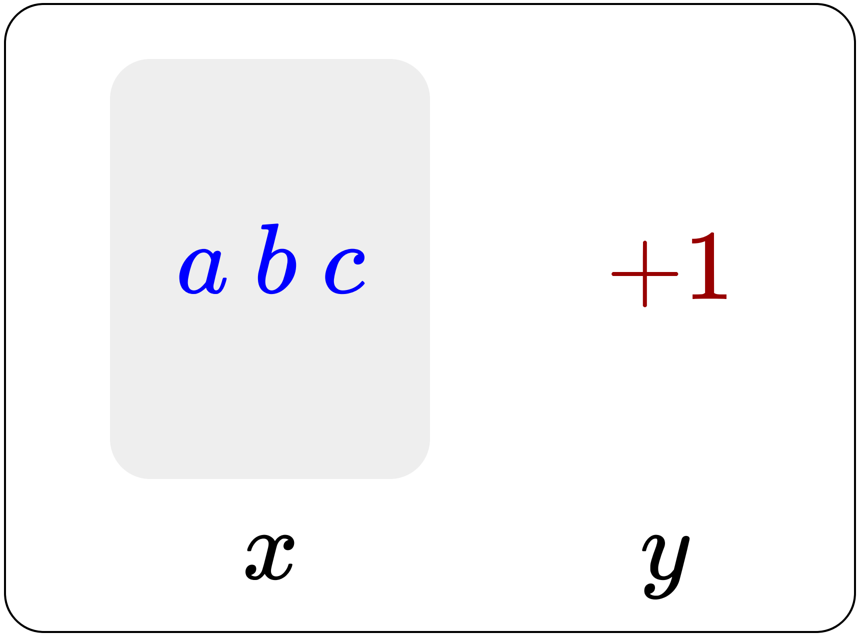
Generative Models

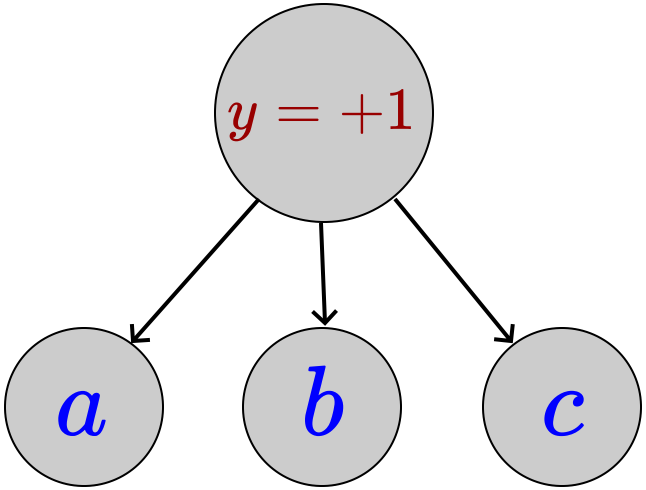


Generative Models



Generative Models





Sequence Labeling

Faith   is   a   fine   invention

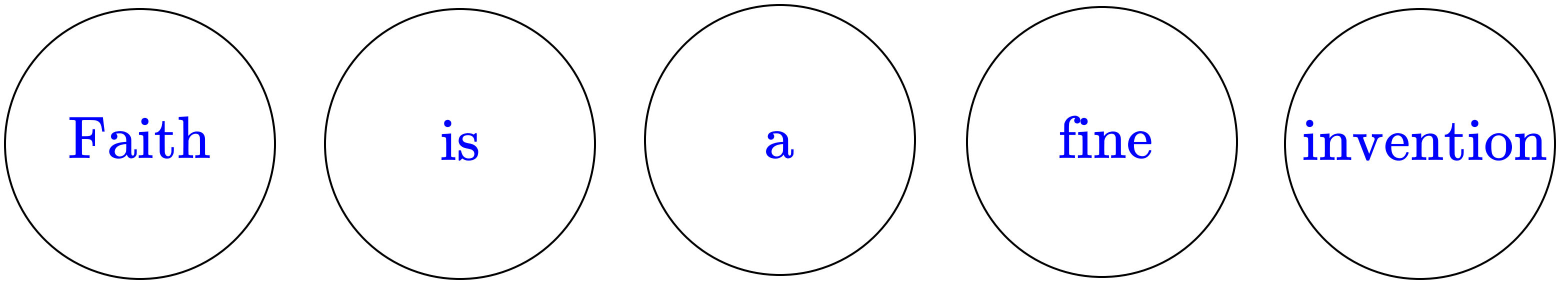
Sequence Labeling

Noun	Verb	Determiner	Adjective	Noun
$\mathbf{N}$	$\mathbf{V}$	$\mathbf{D}$	$\mathbf{A}$	$\mathbf{N}$
Faith	is	a	fine	invention

Sequence Labeling

Sequence Labeling

Sequence Labeling



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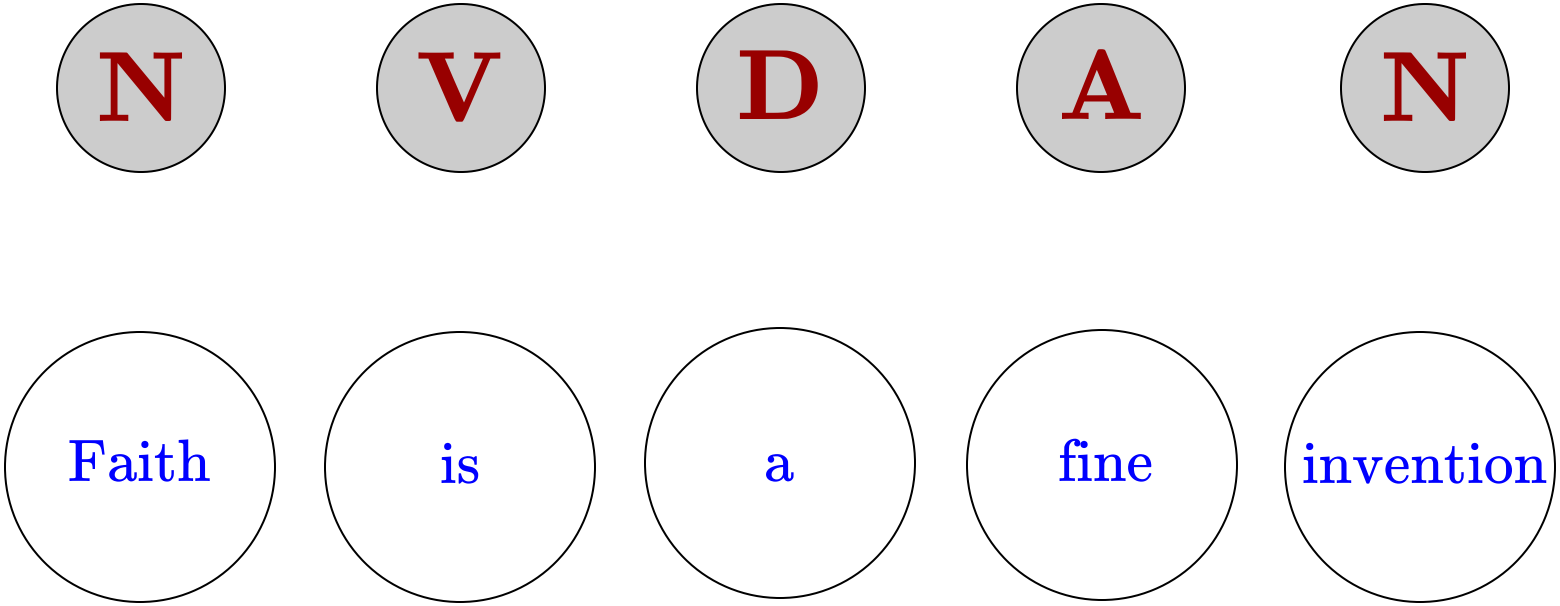

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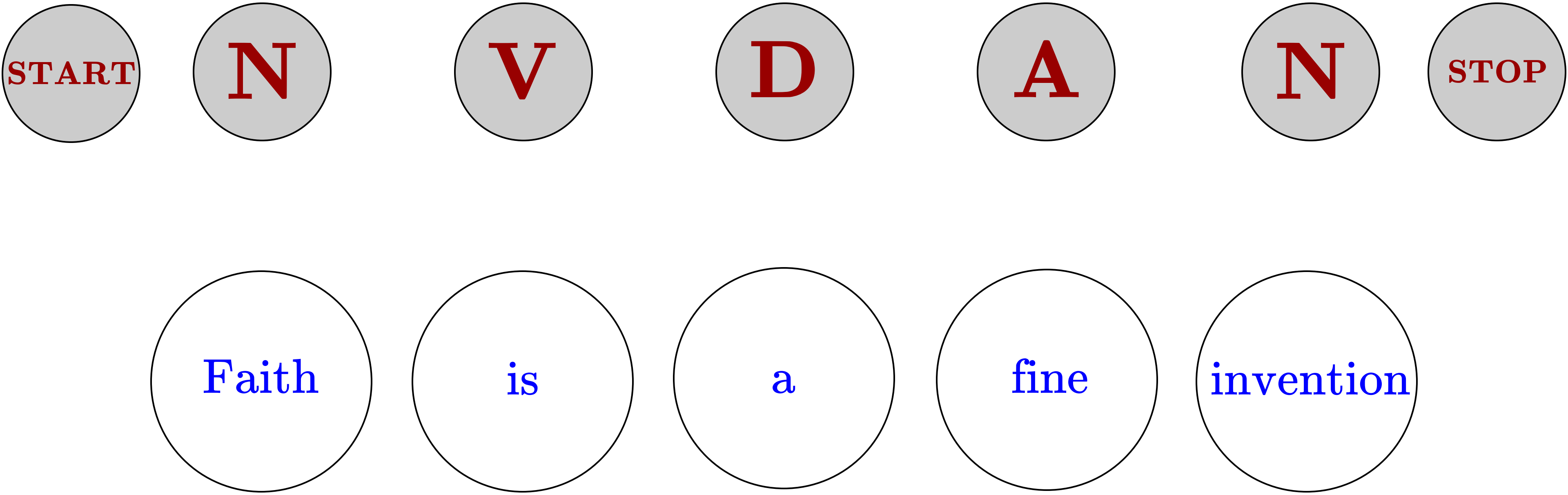
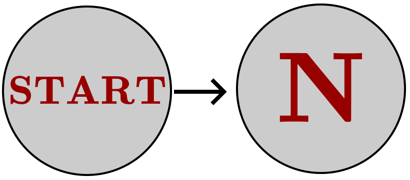


Sequence Labeling

Sequence Labeling



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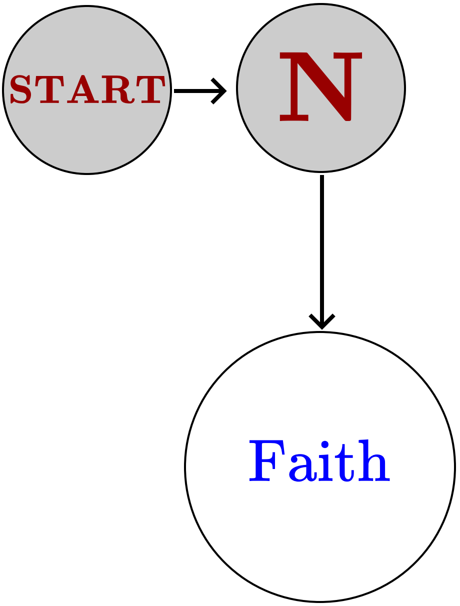

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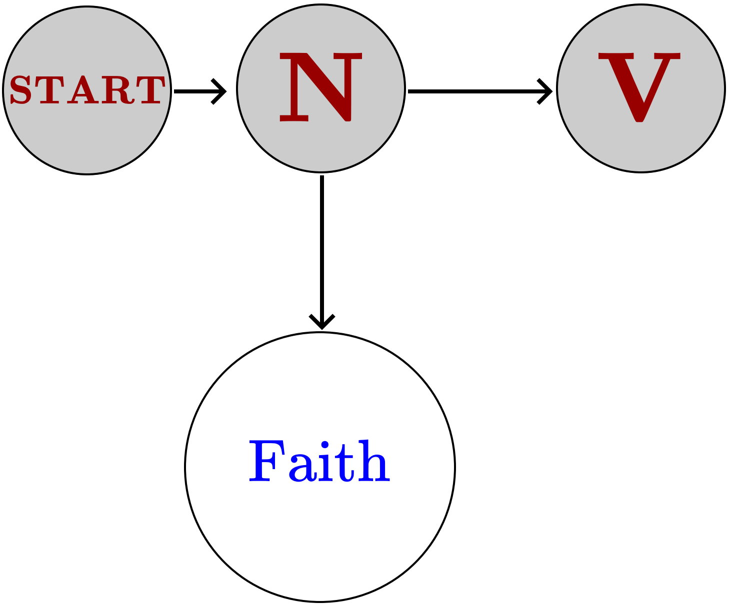

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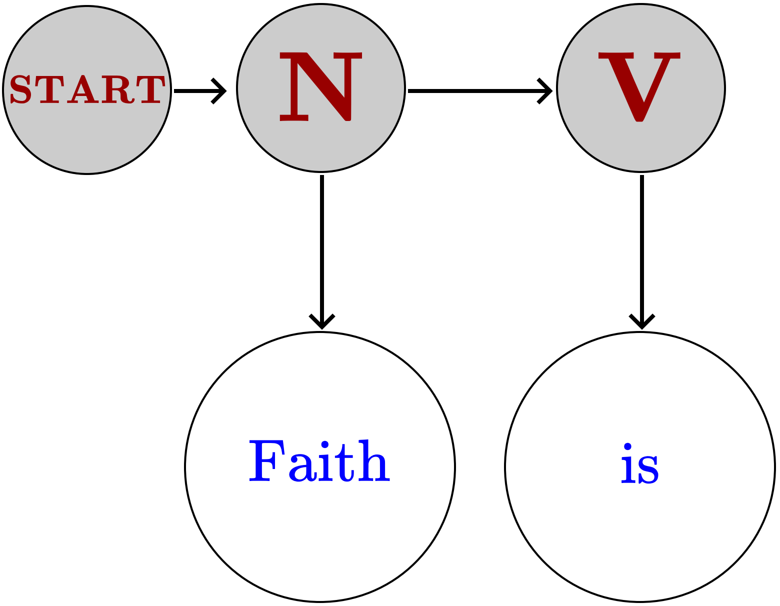

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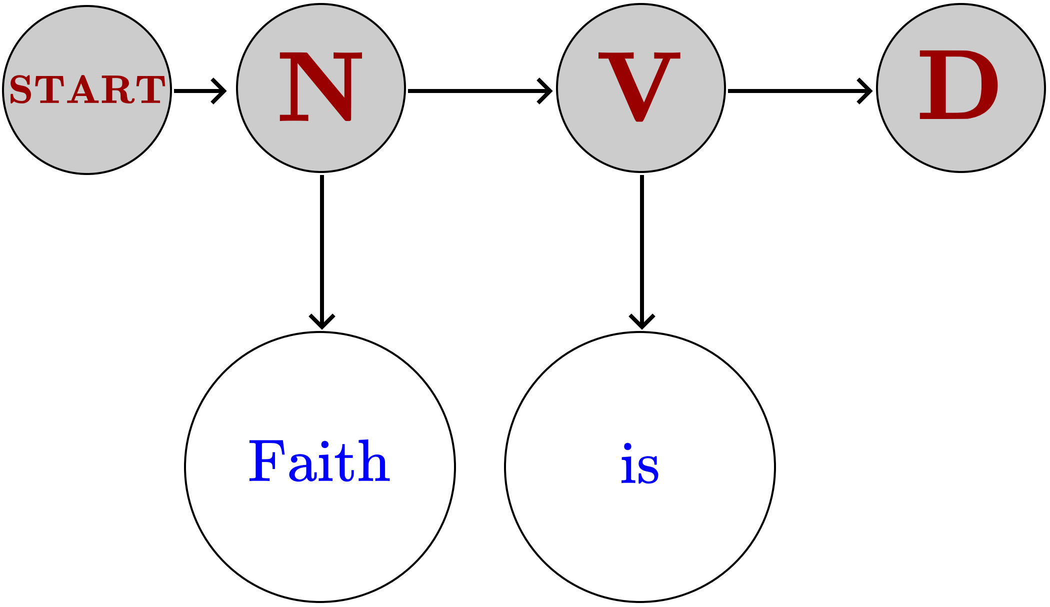

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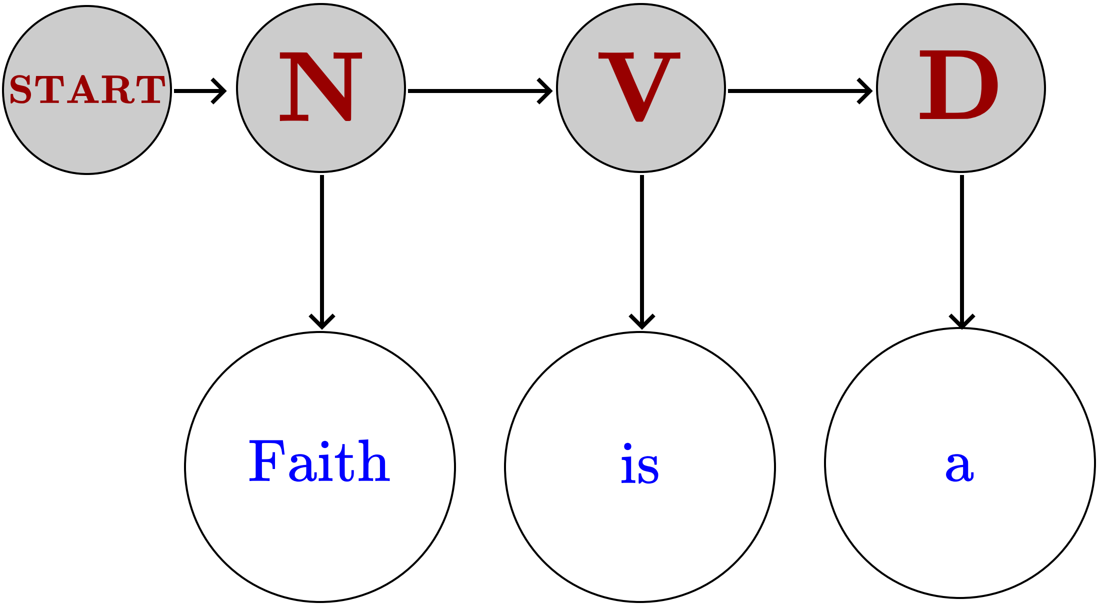

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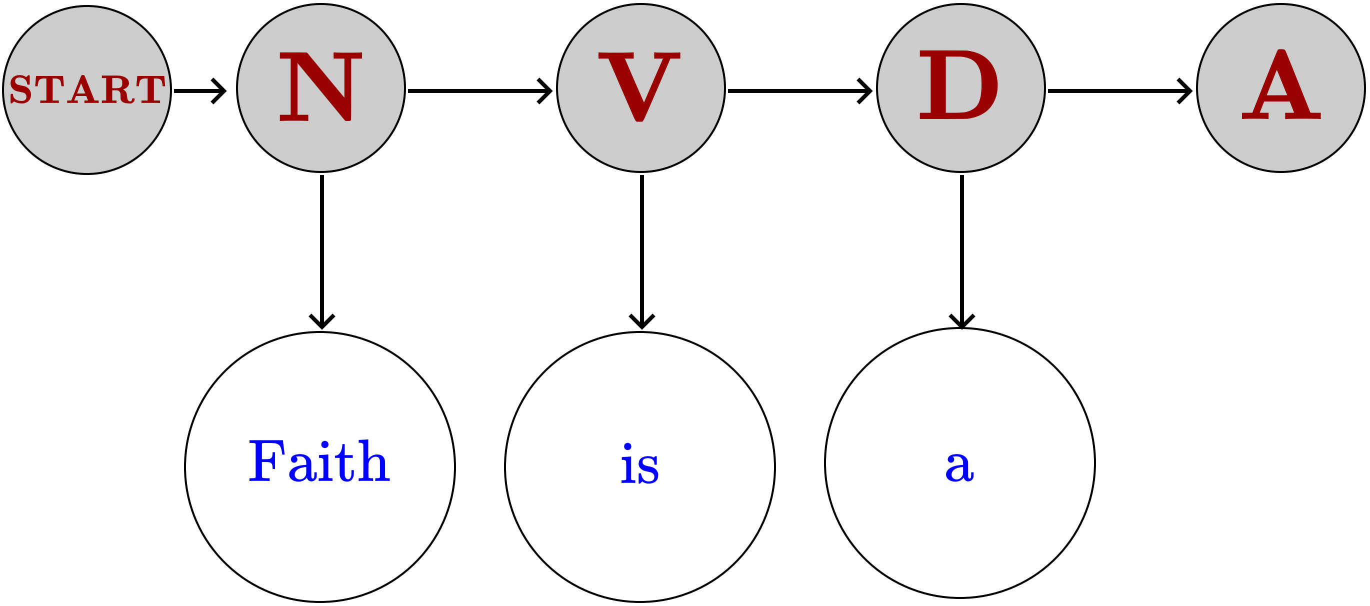

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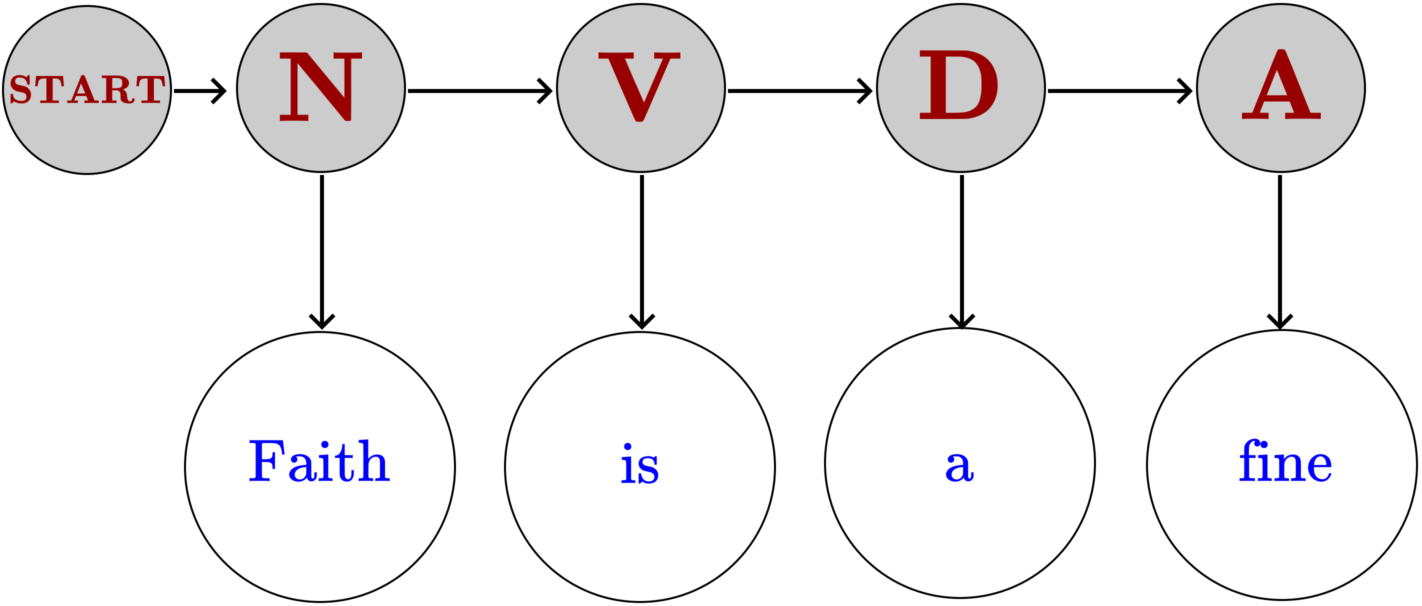

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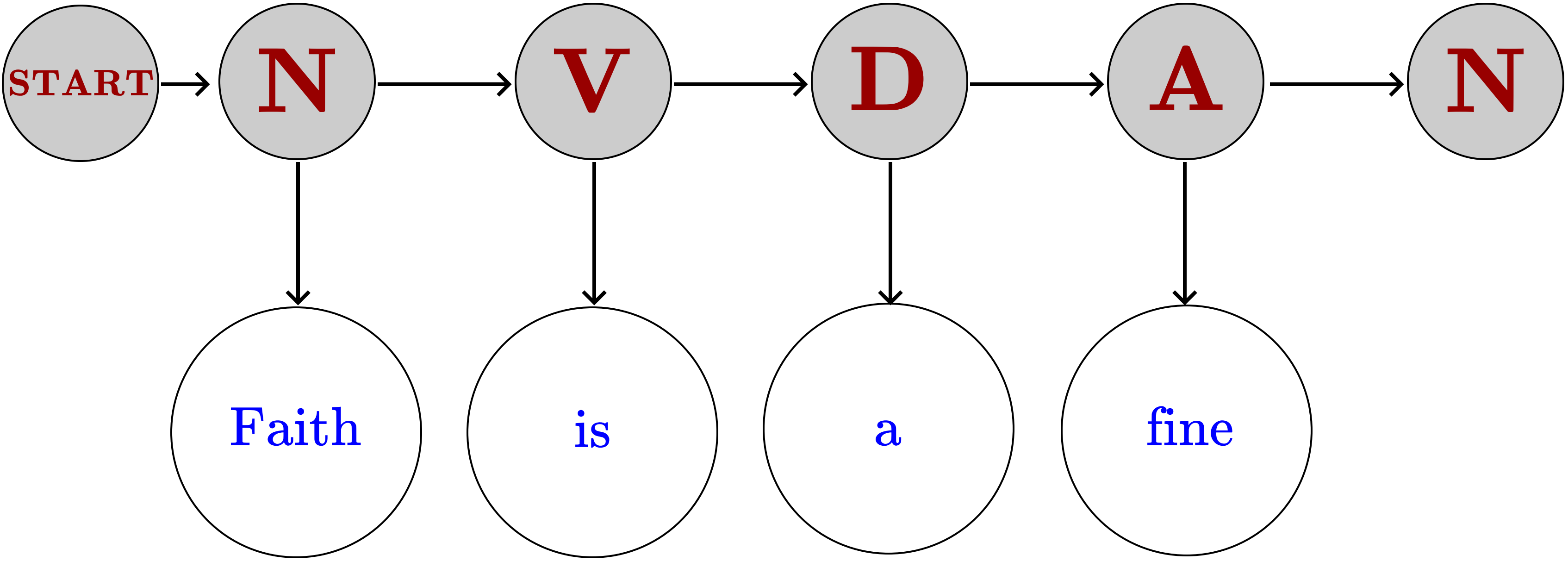

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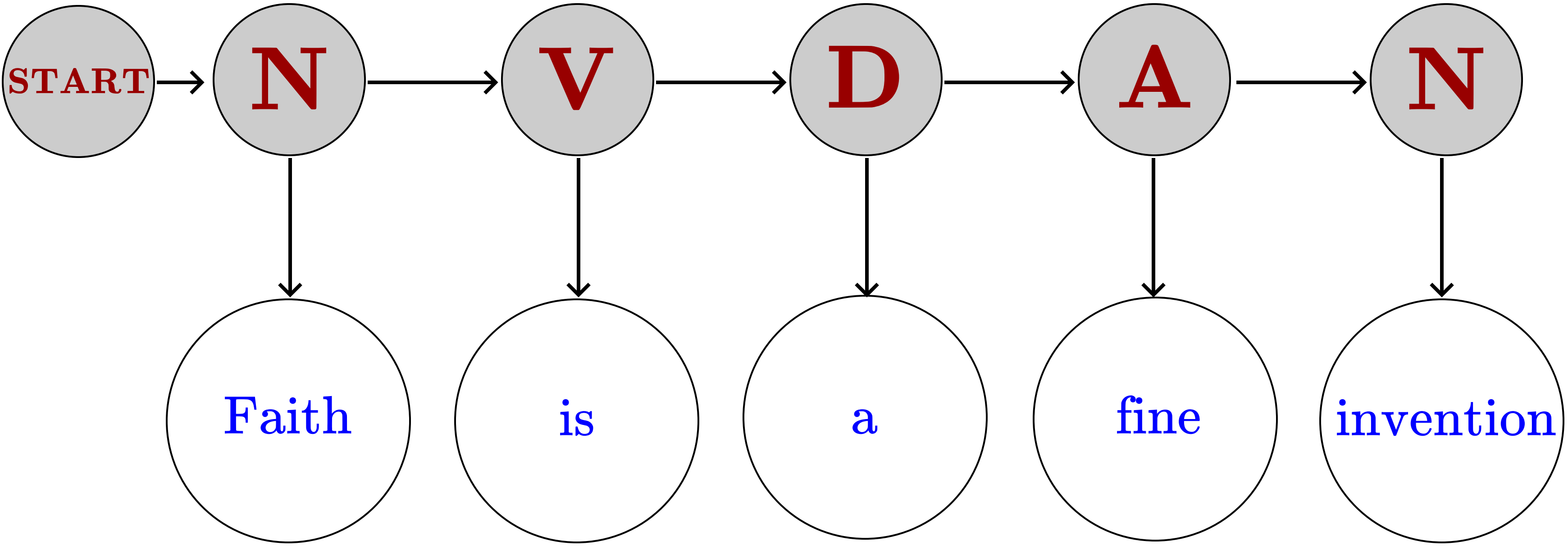

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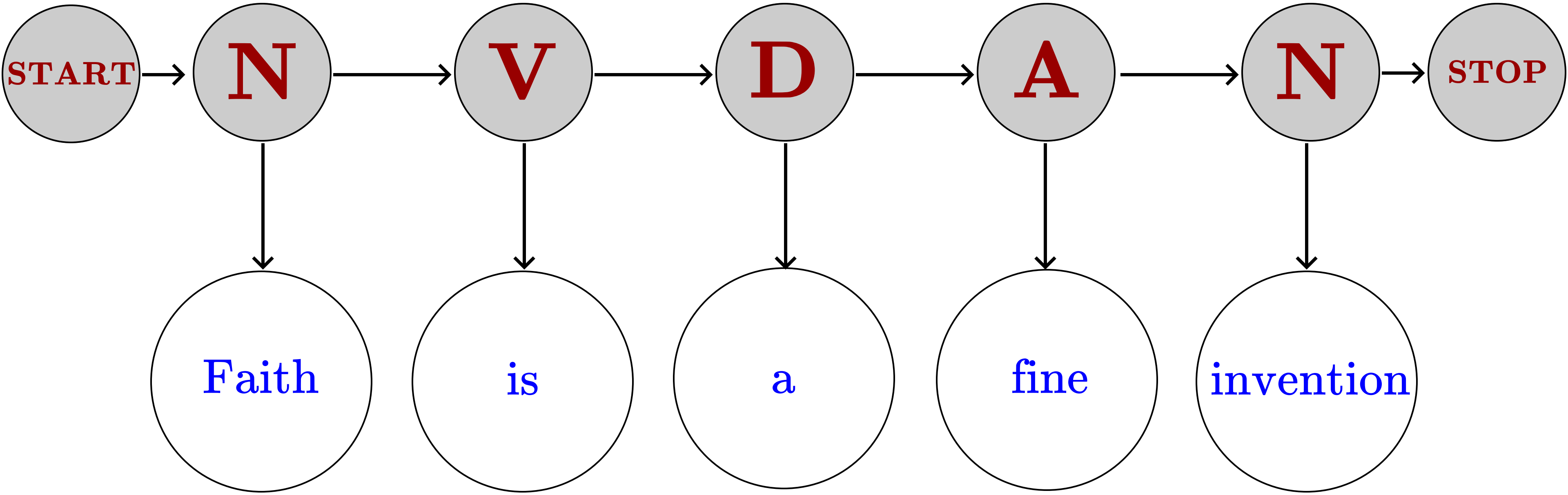

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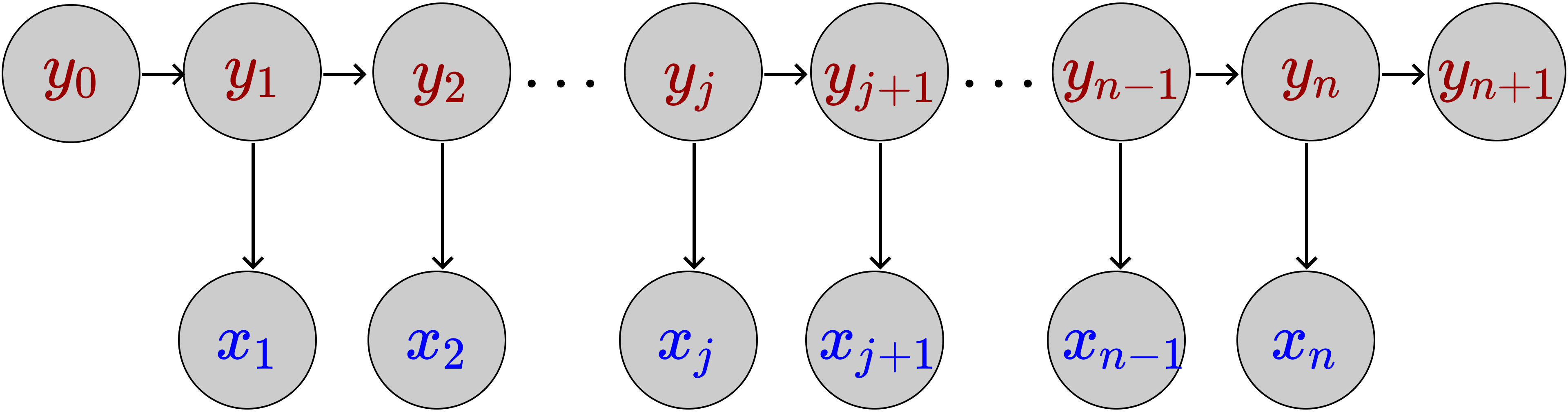

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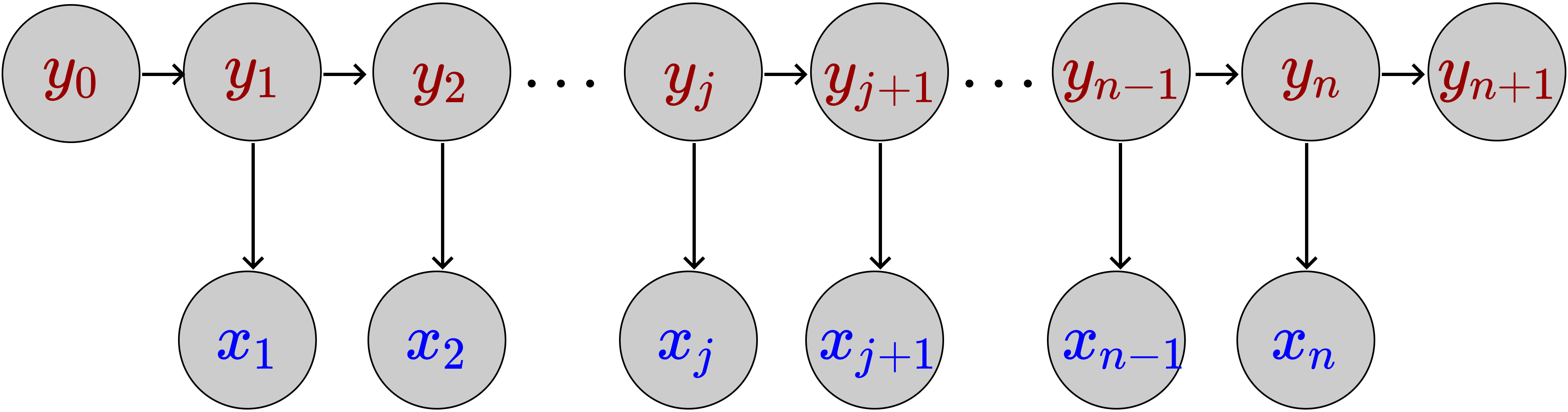
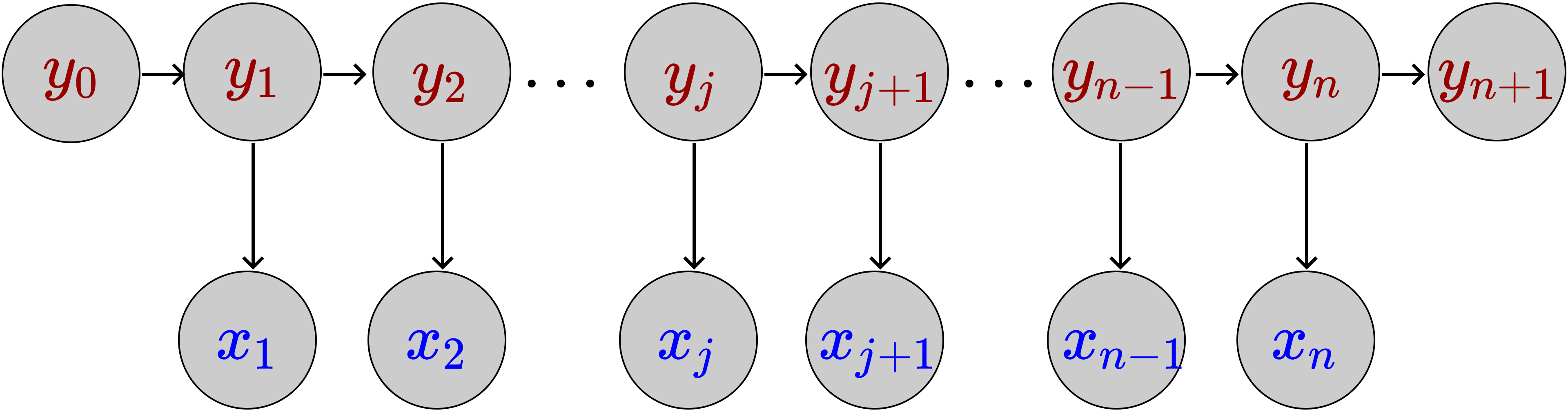


$$p(x_1, x_2, \ldots, x_{n-1}, x_n, y_0, y_1, y_2, \ldots, y_{n-1}, y_n, y_{n+1})$$

START

STOP

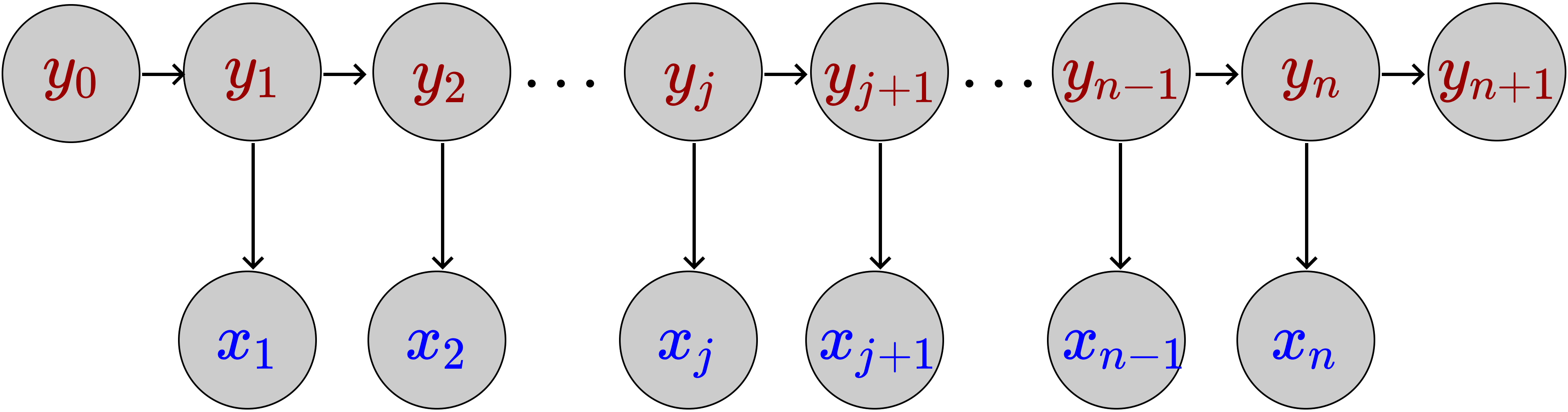
Sequence Labeling



$$p(x_1, x_2, \ldots, x_{n-1}, x_n, y_0, y_1, y_2, \ldots, y_{n-1}, y_n, y_{n+1})$$

$$\prod_{j=0}^{n} p(y_{j+1} | y_j) \times \prod_{j=1}^{n} p(x_j | y_j)$$

Hidden Markov Model



$$p(x_1, x_2, \ldots, x_{n-1}, x_n, y_0, y_1, y_2, \ldots, y_{n-1}, y_n, y_{n+1})$$

$$\underbrace{\prod_{j=0}^{n} a_{y_j, y_{j+1}}}_{\text{Transition probabilities}} \times \underbrace{\prod_{j=1}^{n} b_{y_j}(x_j)}_{\text{Emission probabilities}}$$

Hidden Markov Model

An HMM is defined by a tuple $\langle \mathcal{T}, \mathcal{O}, \theta \rangle$, where

$\mathcal{T}$

a set of states including `START` and `STOP` states.

$\mathcal{O}$

a set of observation symbols

θ

Transition and emission parameters $a_{u,v}$ and $b_u(o)$.

Hidden Markov Model
An Example

$$\mathcal{T} = \{\texttt{START}, A, B, \texttt{STOP}\}$$

$$\mathcal{O} = \{\text{``the''}, \text{``dog''}\}$$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9

$$a_{u,v}$$

$$b_u(o)$$

Hidden Markov Model

An Example

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9

$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

What is $p(\mathbf{x}, \mathbf{y})$?

Hidden Markov Model

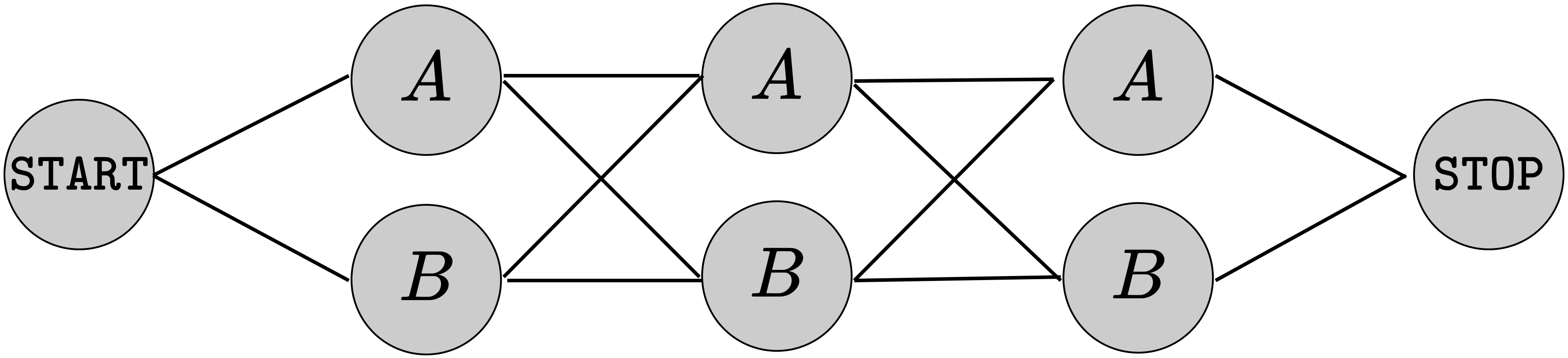
$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9



One path corresponds to one label sequence.

Hidden Markov Model

$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
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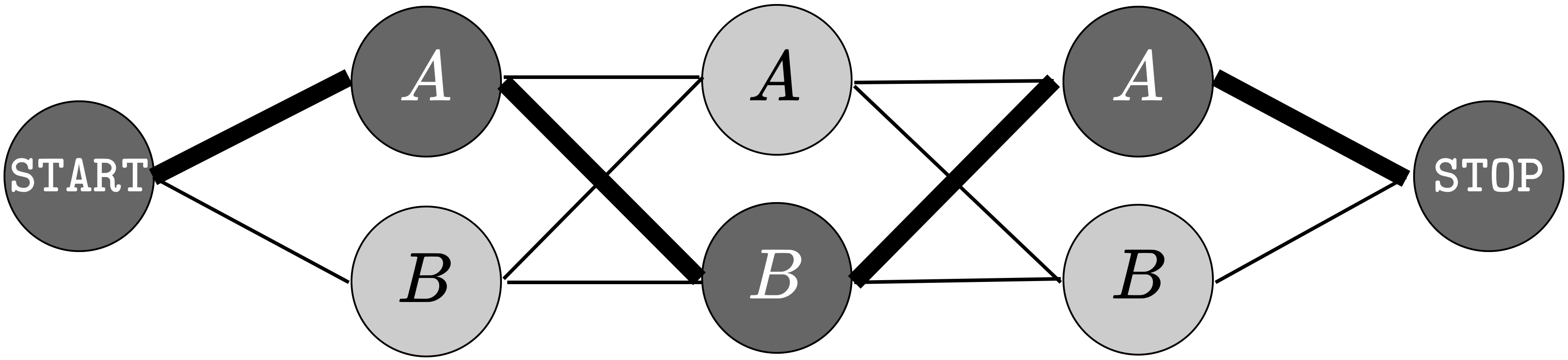


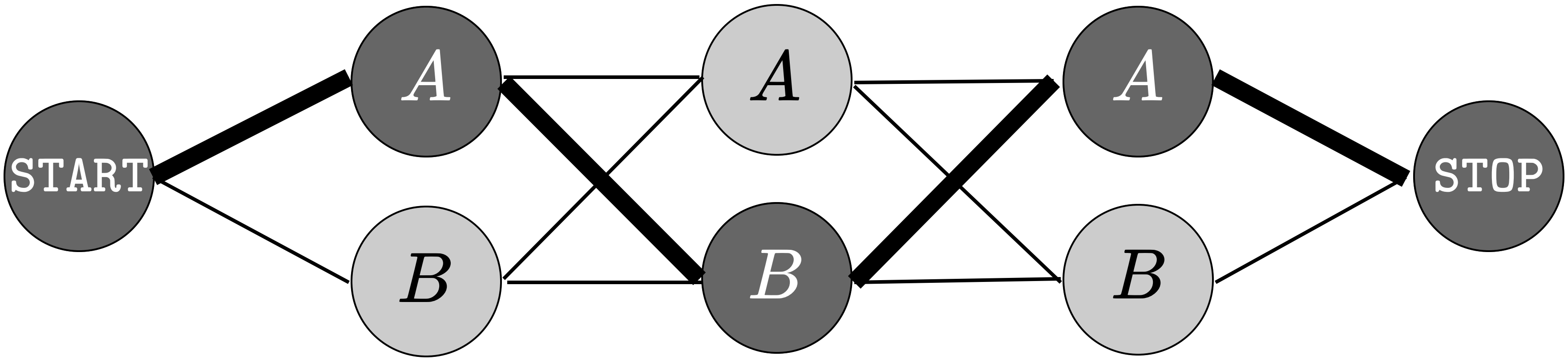
$a_{\texttt{START},A}$

Hidden Markov Model

$$a_{u,v} \qquad (\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A \qquad b_u(o)$$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9



$$a_{\texttt{START},A} \times b_A(\text{``The''})$$

Hidden Markov Model

$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
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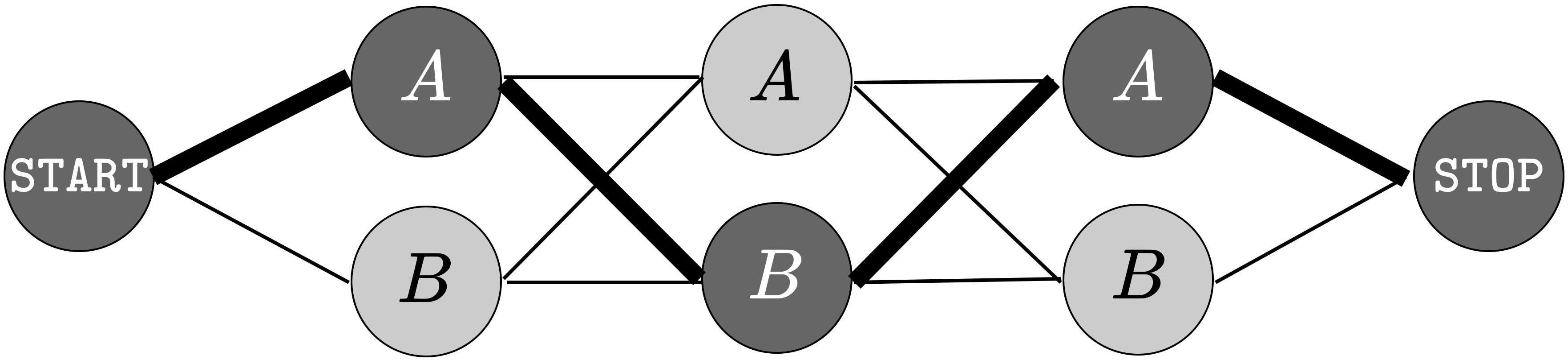

$a_{\texttt{START},A} \times b_A(\text{“The”}) \times a_{A,B}$

Hidden Markov Model

$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
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$b_u(o)$

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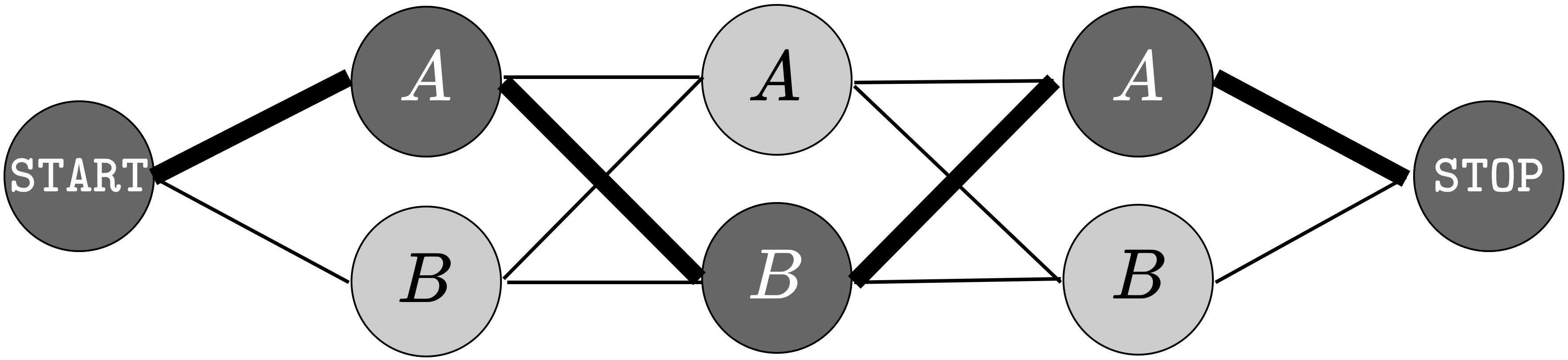


$$a_{\texttt{START},A} \times b_A(\text{“The”}) \times a_{A,B} \times b_B(\text{“dog”})$$

Hidden Markov Model

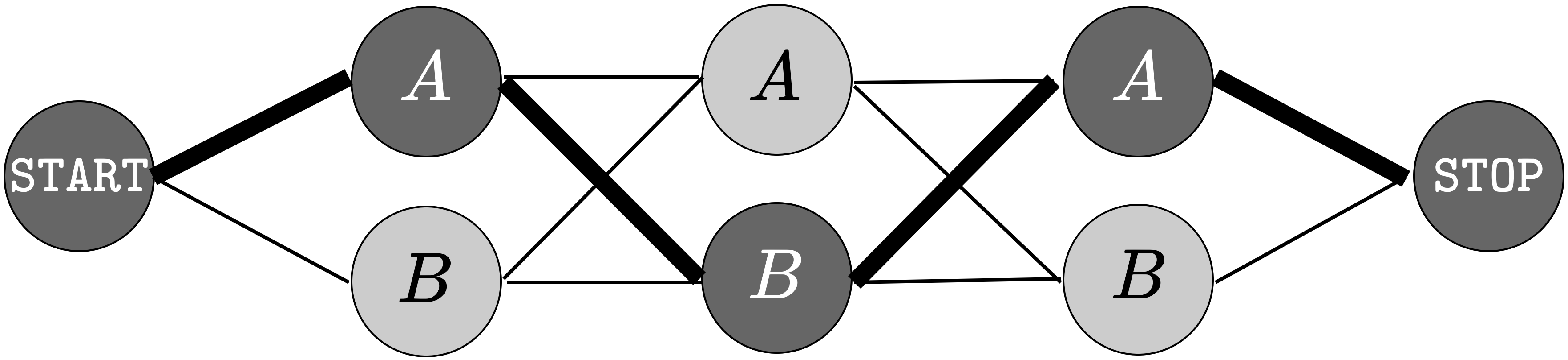
$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9



$$a_{\texttt{START},A} \times b_A(\text{"The"}) \times a_{A,B} \times b_B(\text{"dog"}) \times a_{B,A}$$

Hidden Markov Model

$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
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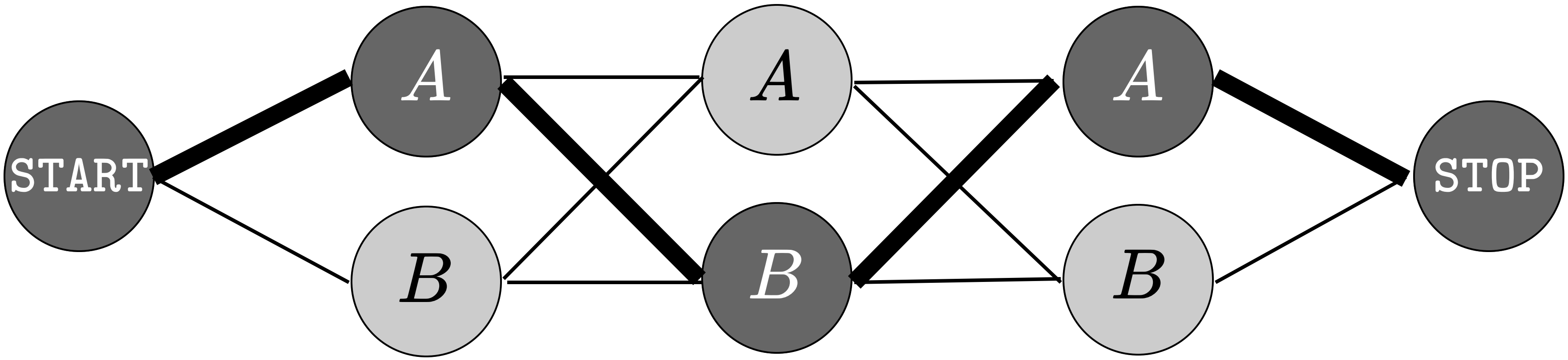

$$a_{\texttt{START},A} \times b_A(\text{“The”}) \times a_{A,B} \times b_B(\text{“dog”}) \times a_{B,A} \times b_A(\text{“the”})$$

Hidden Markov Model

$$(\mathbf{x}, \mathbf{y}) = \text{the}/A, \text{dog}/B, \text{the}/A$$

$a_{u,v}$

$u \backslash v$	A	B	$\texttt{STOP}$
$\texttt{START}$	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
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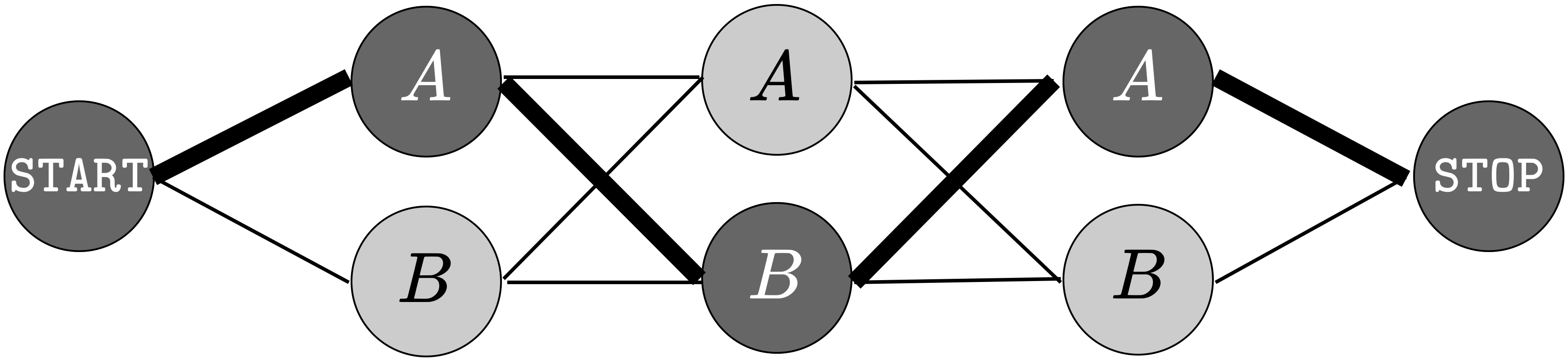
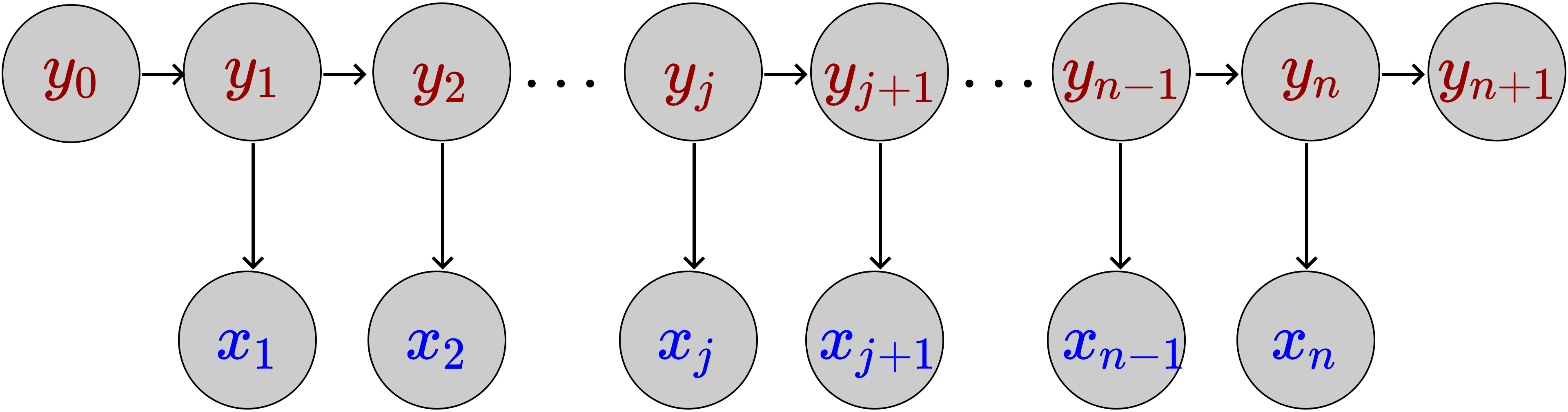


$$a_{\texttt{START},A} \times b_A(\text{“The”}) \times a_{A,B} \times b_B(\text{“dog”}) \times a_{B,A} \times b_A(\text{“the”}) \times a_{A,\texttt{STOP}}$$

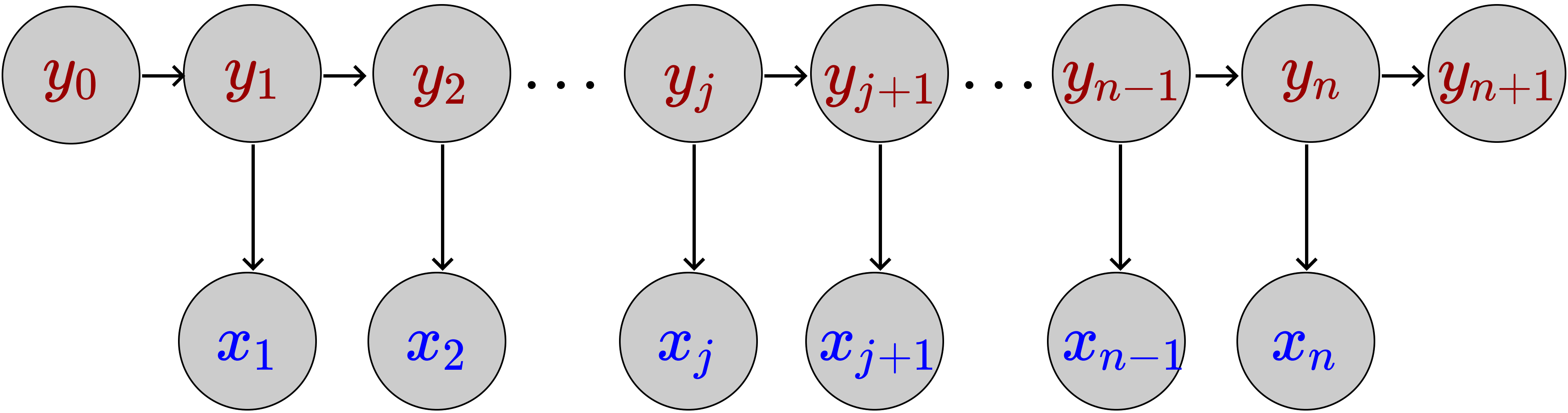
Hidden Markov Model



$$p(\mathbf{x},\mathbf{y}) = \underbrace{\prod_{j=0}^{n} a_{y_j,y_{j+1}}}_{\text{Transition probabilities}} \times \underbrace{\prod_{j=1}^{n} b_{y_j}(x_j)}_{\text{Emission probabilities}}$$

Now that we know what are the model parameters, how do we estimate them? In other words, how to do learning?

Hidden Markov Model

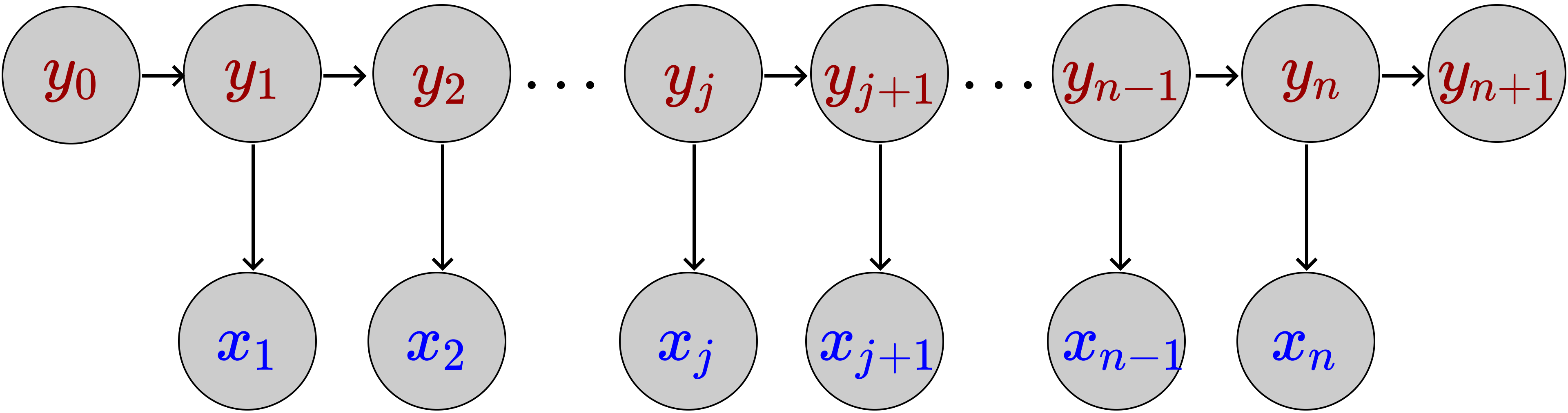


Number of times we see a transition from u to v

$$a_{u,v} = \frac{\text{count}(u,v)}{\text{count}(u)} \qquad b_u(o) = \frac{\text{count}(u \rightarrow o)}{\text{count}(u)}$$

Number of times we see the state u in the training set

Hidden Markov Model



Number of times we see observation o generated from u

$$a_{u,v} = \frac{\text{count}(u,v)}{\text{count}(u)} \qquad b_u(o) = \frac{\text{count}(u \to o)}{\text{count}(u)}$$

Number of times we see the state u in the training set

Hidden Markov Model

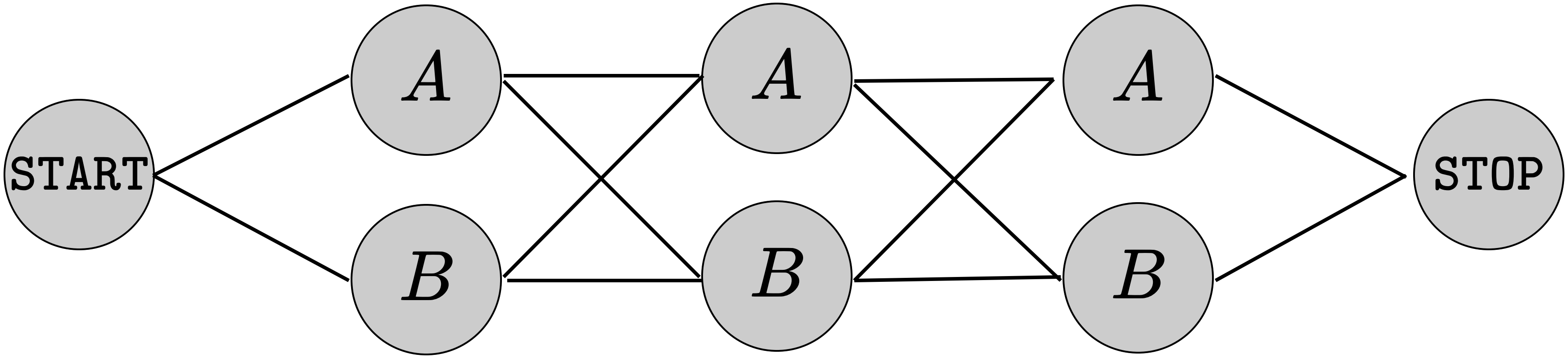
$a_{u,v}$

$u \backslash v$	A	B	STOP
START	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	0.9

$$\mathbf{x} = \text{the dog the}$$



Which label sequence $\mathbf{y}$ is the most probable given the word sequence $\mathbf{x}$?

Hidden Markov Model

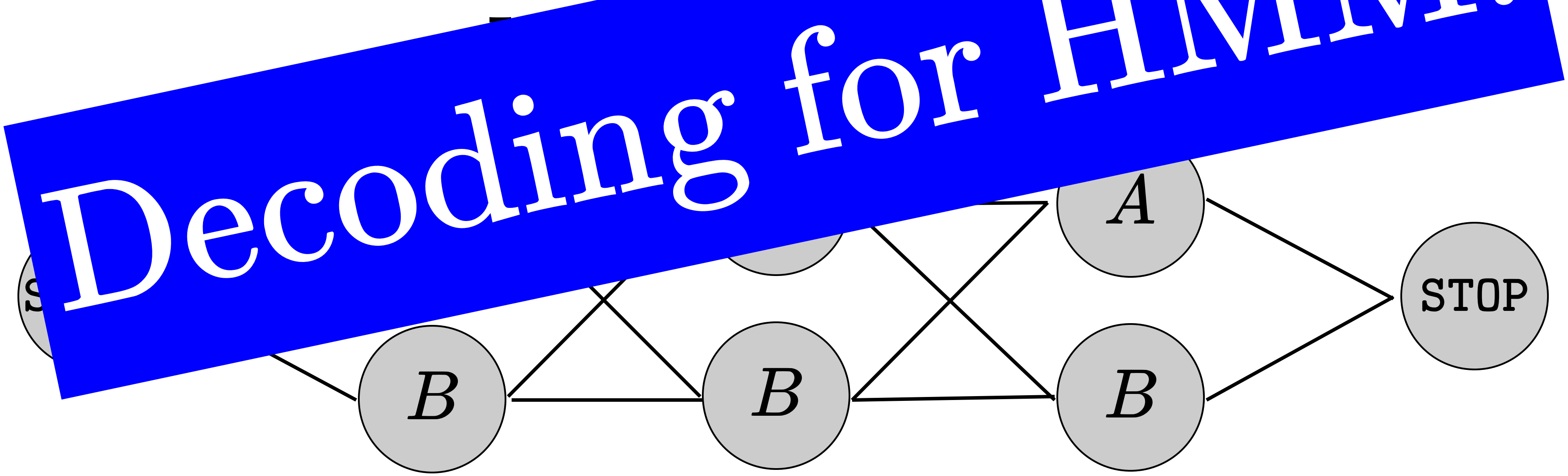
$a_{u,v}$

$u \backslash v$	A	B	STOP
START	1.0	0.0	0.0
A	0.5	0.5	0.0
B	0.0	0.8	0.2

$b_u(o)$

$u \backslash o$	"the"	"dog"
A	0.9	0.1
B	0.1	[illegible]

Decoding for HMM!



Which label sequence $\mathbf{y}$ is the most probable given the word sequence $\mathbf{x}$?