ITESO, Universidad Jesuita de Guadalajara

004 – Pairs Trading

MICROSTRUCTURE AND TRADING SYSTEMS

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Table of contents:

Introduction

In this project, a complete pairs trading strategy is developed, forming a low-risk approach that, thanks to formal cointegration tests, error-correction models ($VECM$), and Kalman filters formulated under Powell's Sequential Decision Analytics (SDA) framework, achieves profitable performance across the analyzed periods. The objective was to build a market-neutral strategy capable of identifying temporary imbalances between two cointegrated assets and exploiting them through simultaneous long and short positions, which are balanced and offsetting, adjusted by a dynamically estimated hedge ratio and a trading signal obtained from the dynamically estimated $VECM$ relative to a fixed threshold parameter, theta.

Unlike traditional pairs trading strategies that assume static relationships between assets, this project recognizes that market dynamics are constantly changing; therefore, parameters such as eigenvectors, the hedge ratio, and the VECM must be continuously updated over time. For this reason, a sequential decision approach is implemented, incorporating prediction, updating, and continuous learning through Kalman filters:

1. **Kalman 1:** Used to estimate the daily dynamic hedge ratio (β_t)between the two assets.
2. **Kalman 2:** Used to estimate the $VECM$ eigenvectors daily, generating a stationary signal $(VECM_{hat})$, to produce trading signals, which is essential for capturing mean-reversion opportunities.

These filters were explicitly formulated as a sequential decision process, defining initial states, decisions, exogenous information, a transition function, and an objective function, closely following the mathematical structure of Powell's SDA framework. This allows the strategy to adapt and incorporate new market observations without requiring global recalibration or retraining of regressions, effectively mimicking an online-learning scheme.

The selection of the asset pair used in the strategy was obtained through a rigorous process. A group of 30 assets from 11 different sectors—such as technology, financials, energy, and consumer—was chosen. Historical correlation analysis was then performed, and for those above 60%, a linear regression was estimated whose residuals were analyzed using the Augmented Dickey-Fuller (ADF) statistical test. Finally, cointegration between the two assets was confirmed using the Johansen test. This process ensures the existence of a long-term equilibrium with potential profits through the pairs trading strategy.

Subsequently, a backtesting procedure was carried out under realistic market assumptions:

- **Initial Capital:** $1,000,000 USD
- **Commission:** 0.125% per trade
- **Borrow Rate:** 0.25% annually for short positions
- **Interest:** Daily interest accrued on borrowed amounts
- **Position Size:** Invest 80% of available capital (40% in each asset)

Finally, the strategy was evaluated using performance metrics such as the Sharpe Ratio, Sortino Ratio, Calmar Ratio, and Maximum Drawdown, as well as several trading statistics including the Profit Factor, Win Rate, and the distribution of PnL per trade. This analysis shows that the use of the dynamic VECM and the Kalman filters significantly improves the stability of the spread, and the quality of the signals compared to a static model, resulting in a robust strategy capable of adapting to structural changes in the market.

This project integrates statistical foundations, cointegration theory, and dynamic modeling through a rigorous decision-making process based on sequential decisions, thereby producing a modular, comprehensive solution, resulting in a professional pairs trading system.

Description and Justification of the Strategy

The implemented strategy is based on pairs trading, a form of statistical arbitrage designed to exploit temporary deviations between two assets that maintain a long-term cointegrated relationship. Unlike directional strategies, the main objective is not to predict whether an asset will rise or fall; rather, the strategy seeks to capture the convergence of the spread between two time series that move together. This methodology produces a market-neutral strategy, reducing exposure to broad market movements and focusing instead on relative variations supported by the underlying statistical relationship.

Cointegration is the foundation of this project. Even though each asset in the pair may individually display trends and large fluctuations, they maintain a stable long-term relationship. The linear combination or difference between them—the spread—behaves in a stable manner that tends to revert to its equilibrium level. This feature allows us to identify trading opportunities with a high probability of profit. When the spread moves significantly away from its typical range, we interpret such deviation as a signal to open a position. For example: if the spread widens substantially, we will take a long position in the “cheap” asset and a simultaneous short position in the “expensive” one, assuming both will revert to their historical relationship and converge to the mean. This condition is far more robust than correlation, which only describes short-term co-movements and does not guarantee that the series will return to equilibrium. For this reason, cointegration is treated as the fundamental requirement of the project.

Using Kalman filters is an excellent technique in a dynamic market environment, since neither the hedge ratio (β_t) nor the eigenvectors that define the $VECM$ remain constant over time. Factors such as changing macroeconomic conditions and volatility regimes cause the relationships between assets to evolve. The β estimator obtained through OLS regression is not sufficient for this strategy, as it may become outdated and could distort the spread, making it non-stationary. The Kalman filter updates the hedge ratio and $VECM$ parameters in real time (with each daily iteration), reducing estimation noise and adapting them to new observations. This process improves the stability of the spread and reduces the risk of operating based on delayed signals, fully aligned with a sequential decision-making framework.

Finally, the ideal market conditions for the success of this strategy include persistent economic relationships between assets, sufficiently large deviations from equilibrium to generate trading signals, and the absence of structural breaks that might invalidate cointegration. If the cointegrating relationships remain stable, the cointegrated pairs trading approach combined with Kalman filter updates fulfills the objective of shaping the strategy toward profitable, adaptive, and robust results despite the expected changes in market behavior.

Methodology for Pair Selection

The pair-selection process was conducted through a systematic evaluation of the 30 assets chosen across 11 different sectors. For all possible pair combinations, correlation metrics were computed to identify the initial candidates for cointegration testing. Although the project guidelines suggested using a correlation threshold above 70%, after running multiple tests with different pairs, I observed that the strongest candidates—those that successfully passed the subsequent statistical tests—did not necessarily exhibit such high correlations. Therefore, I decided to adopt a 60% correlation threshold, which yielded pairs with better potential for a functional trading strategy.

As a first step, a rolling-correlation matrix was generated to identify historically consistent relationships among the assets. This matrix served as an effective initial filter to discard combinations that exhibited no meaningful joint behavior. Through this analysis, only pairs with persistent correlations above 60% were selected, focusing the process on those with a higher likelihood of sharing a common underlying structure over time. The resulting rolling-correlation matrix is generated, stored in a local folder, and partially displayed on screen when running the main script main.py. A segment of this output is shown below:

	AAPL	NVDA	INTC	ORCL
AAPL	0.9999999999998770	0.36200929486061	0.3564795345232680	0.28258598144064900
APD	0.34061417274974500	0.5420147276239370	0.4241665462057170	0.5356239473444540
BA	0.40635309890502600	0.4558955225886220	0.35315515986812700	0.37325299553357500
BAC	0.31038596440877500	0.40419151889326500	0.2740555273123330	0.31584894977459300
C	0.290531934797546	0.3507297618983790	0.153545631149912 0	0.3356871352572990
CAT	0.26107297400515500	0.37197531215147900	0.3632135726825030	0.40652923202357200

Table 1. Rolling Correlation Matrix (First 5 rows, 4 columns)

Asset_1	Asset_2	Mean_Correlation
JPM	BAC	0.8700608859069370
JPM	C	0.8180122444082640
BAC	C	0.7960292222836600
DUK	SO	0.7624541750989190
PEP	JNJ	0.6896484933005400
XOM	CVX	0.6811454556349630
DD	CAT	0.658881144550837
NEE	DUK	0.6546104468138650
KO	PEP	0.6415106880537340
PEP	NEE	0.6265233179100630
KO	DUK	0.6016262721510920

Table 2. Pairs exhibiting high correlation

Subsequently, for each pair that surpassed the threshold, the Engle–Granger method was applied. This procedure evaluates whether the linear combination of two series produces a stationary residual. It consists of running an OLS regression between the prices of both assets to estimate an initial hedge ratio and then applying the Augmented Dickey–Fuller (ADF) test to the residuals. The selected pairs were those whose regression residuals were stationary, meaning that cointegration was confirmed by rejecting the null hypothesis of non-stationarity. This filter also made it possible to distinguish genuine long-term relationships rather than simple short-term correlations.

As a final verification step, the Johansen cointegration test was applied, which allows validating both the presence and dimensionality of cointegration vectors. Through the analysis of the trace and maximum eigenvalue statistics, the existence of a single cointegrating vector was confirmed in two pairs: The Coca-Cola Company and PepsiCo, Inc. (KO–PEP), and The Coca-Cola Company and Duke Energy Corporation (KO–DUK). After analyzing both the historical price behavior and the underlying economic relationship of each pair, I selected the beverage companies (KO–PEP) as the working pair. This analysis not only confirms the cointegration but also provides the eigenvector that will be incorporated into the $VECM$ error-correction model and, once normalized, will serve as the basis for generating trading signals when exceeding a predefined threshold.

The economic relationship between The Coca-Cola Company (KO) and PepsiCo, Inc. (PEP) is understood through their position as a duopoly in the non-alcoholic beverage market, where both firms produce closely substitutable goods. They operate in an imperfectly competitive environment driven by interdependence: when one company raises prices, it can affect the demand for the other, revealing strategic behavior between them.

At the same time, although they compete directly in beverages, their business models show key differences that shape their economic relationship. Coca-Cola focuses almost exclusively on beverages and adopts a pricing strategy based on “matching the competition,” while PepsiCo diversifies across beverages and foods, adjusting prices according to consumer demand. In the soft-drink segment, however, they continue to compete intensely for market share and pricing power. This economic relationship provides a solid foundation for selecting the pair, reinforced by the cointegration tests previously passed.

Asset_1	Asset_2	Mean_Correlation	beta1_OLS	ADF_stat	ADF_pvalue	Stationary	Trace_Stat	Crit_Value_5%	Rank_detected
KO	DUK	0.6016	0.4517	-4.7743	0.0001	True	24.7604	15.4943	1.0
KO	PEP	0.6415	0.2334	-4.3296	0.0004	True	19.5039	15.4943	1.0
BAC	C	0.796	0.5451	-2.9549	0.0393	True	13.6736	15.4943	0.0
JPM	BAC	0.8701	3.4719	-2.9295	0.042	True	9.2965	15.4943	0.0
PEP	NEE	0.6265	1.6917	-2.5958	0.0939	False			

Table 3. Summary cointegration report (First 5 rows)

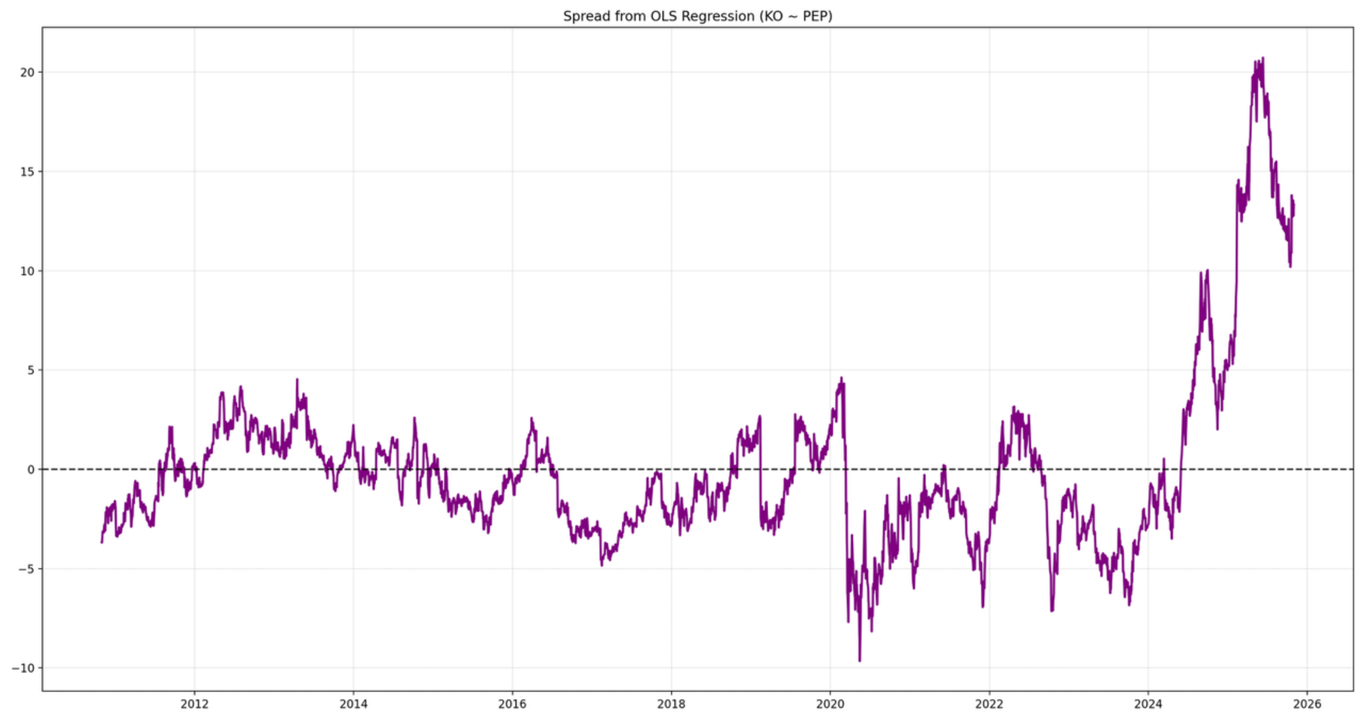

Figure 1. Spread of pair KO-PEP from OLS Regression

Complementing this analysis, a price chart was generated to observe the joint behavior of the assets and the historical evolution of the resulting spread. These visualizations help verify the relative stability between both assets and confirm that the spread fluctuates around its long-term equilibrium level, which further supports the suitability of the selected pair. This chart illustrates both the simultaneous historical behavior of the two assets and the individual trend each one has followed over time. The spread (shown in its real scale on the right-hand axis) exhibits a stable, mean-reverting pattern, reinforcing that the statistically obtained relationship is functional and appropriate for the pairs trading strategy.

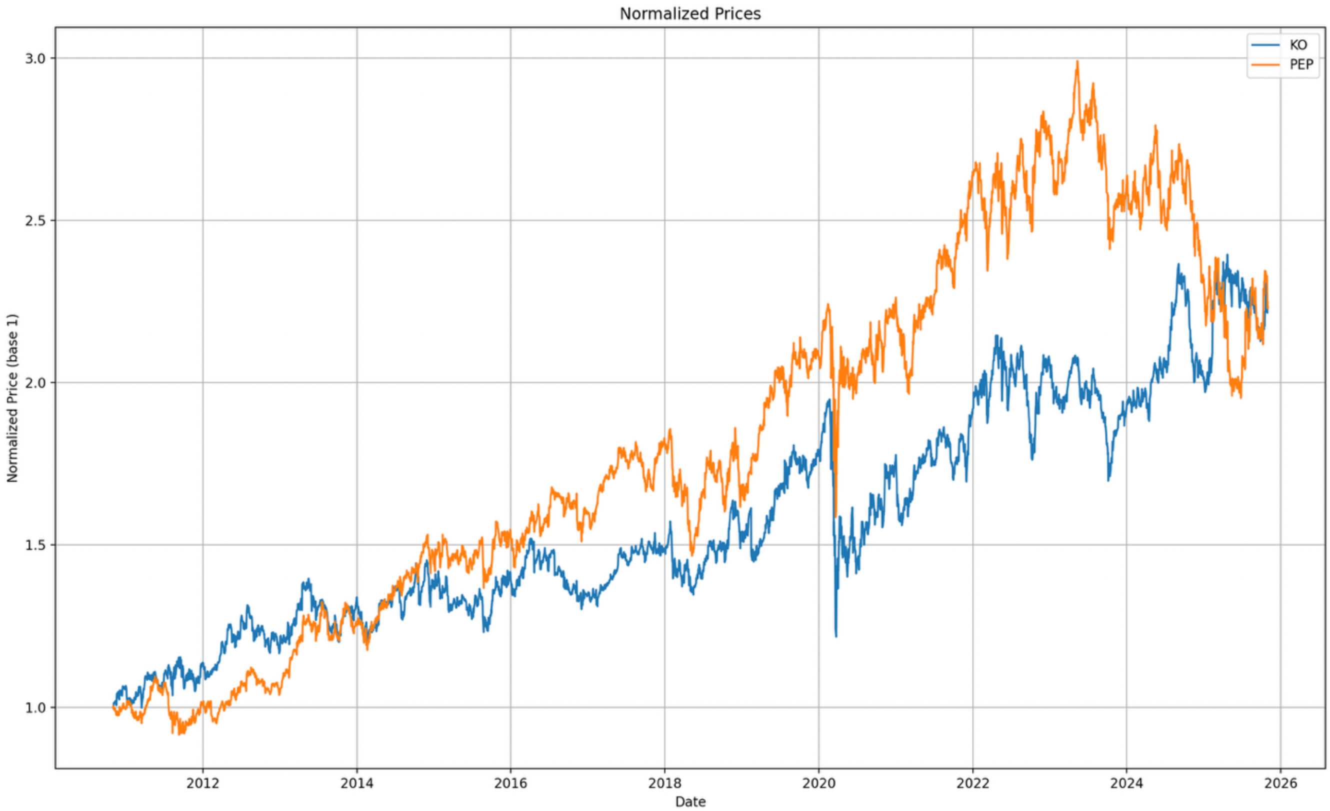

Figure 2. Normalized price series of pair KO-PEP

Kalman Filters Formulated as Sequential Decision Models (SDA)

Model 1: Kalman Filter 1

Kalman Filter 1 – Dynamic Hedge Ratio Estimation

1. **Narrative**

The objective of this filter is to dynamically estimate the parameter β_t (the Hedge Ratio) to obtain the pair's dynamic hedge ratio. New prices arrive each day, and the filter updates its beliefs based on the predefined parameters describing the linear relationship between the two assets.

2. **Core Elements**

- **Metric to impact:** Stability of the hedge ratio and its predictive capability.
- **Decision:** Applying the filter's update through the dynamic parameter estimators.
- **Uncertainity:** Variation in the hedge ratio, market noise, and observation error.
- **Exogenous information:** Daily prices observed at each iteration (x_t, y_t).
- **Prior uncertainity:** There is no certainty regarding how the hedge ratio will change over time.

3. **Mathematical Model (5 Components of the SDA Framework)**

State

The state contains the dynamic vector containing α_t and β_t

$$S_t = (w_t, P_t) \qquad\qquad w_t = \begin{bmatrix}\alpha_t \\ \boldsymbol{\beta_t}\end{bmatrix}$$

Decision

$$x_t = \pi(S_t) = Predict - Update(S_t, W_{t+1})$$

Exogenous Information

$$W_{t+1} = (x_{t+1}, \qquad y_{t+1})$$

Where W_{t+1} represents the new prices arriving to the filter at each iteration.

Transition Function

Prediction:

$$w_{t|t-1} = Fw_{t-1} \qquad\qquad P_{t|t-1} = FP_{t-1}F + Q$$

Update:

$$K_t = P_{t|t-1}\, C_t^T \left(C_t\, P_{t|t-1} C_t^T + R\right)^{-1}$$

Where K_t is the Kalman gain

$$w_t = w_{t|t-1} + K_t \left(y_t - C_t\, w_{t|t-1}\right)$$

Here, $w_t = (\alpha_t, \beta_t)$ represents the vector of dynamic parameters.

$$P_t = (I - K_t C_t)\, P_{t|t-1}$$

Objective
To estimate the dynamic hedge ratio, which determines the proportion of units of each asset that should be traded in order to maintain a market-neutral position.

4. **Uncertainty Model**

Not applicable.
(External uncertainty is already incorporated through the state noise Q and observation noise R.

5. **Policy Design**

Kalman Filter 1 applies a policy that dynamically estimates the hedge ratio from the system's state. At each period, the policy takes the previous state and the new observations and performs the predict and update steps, producing:

$$X^{\pi}(S_t) = (\widehat{\alpha_t}, \widehat{\beta}_t)$$

6. **Policy Evaluation**

Not applicable.

Model 2: Kalman Filter 2

Kalman Filter 2 – Dynamic Hedge Ratio Estimation

1. **Narrative**

This second Kalman Filter dynamically estimates the cointegrating vector (e_{1_t}, e_{2_t}) that characterizes the long-run equilibrium relationship between the two assets. Unlike the hedge-ratio filter, this model focuses on tracking the evolution of the $VECM$ residual in real time, allowing the strategy to detect deviations from equilibrium under structural changes in the market. Each day, new prices arrive, and the filter updates its belief about the cointegration vector, producing a dynamic $VECM$ signal used to generate long/short positions, to hold active positions, and to close them when the $VECM$ reverts toward zero.

2. **Core Elements**

- **Metric to impact:** Stability and mean-reversion properties of the $VECM$ residual.
- **Decision:** Applying the estimation policy (predict-update) to obtain the dynamic cointegration vector $(\widehat{e_{1_t}}, \widehat{e_{2_t}})$
- **Uncertainty:** Changes or variability in the long-run equilibrium structure, market noise, and observation noise of the $VECM_t$ residual
- **Exogenous information:** Daily prices observed at each iteration (x_t, y_t) and $VECM_t$
- **Prior uncertainty:** True cointegration vector may drift over time due to regime shifts, structural breaks, or changes in market conditions.

3. **Mathematical Model (5 Components of the SDA Framework)**

State
The state contains the dynamic cointegrating vector and is uncertainty

$$S_t = (w_t, P_t) \qquad w_t = \begin{bmatrix} e_{1_t} \\ e_{2_t} \end{bmatrix}$$

This vector defines the instantaneous $VECM_t$residual:

$$VECM_t = e_1 * p_{1_t} + e_{2_t} * p_{2_t}$$

Decision

$$x_t = \pi(S_t) = Predict - Update(S_t, W_{t+1})$$

The policy computes the new estimates $(\widehat{e_{1_t}}, \widehat{e_{2_t}})$

Exogenous Information

$$W_{t+1} = (p_{1_{t+1}}, p_{2_{t+1}}, VECM_{t+1})$$

Where $p_{1_{t+1}}, p_{2_{t+1}}$ are the new price observations
And $VECM_{t+1}$ the new estimated correction model

Transition Function

Prediction:

$$W_{t|t-1} = Aw_{t-1} \qquad\qquad P_{t|t-1} = AP_{t-1}A^T + Q$$

Where $A = I_2$, the model assumes the cointegration vector follows a random walk.

Update:

Observation:

$$\epsilon_t = C_t w_t + v_t \qquad\qquad C_t = [p_{1_t}, p_{2_t}]$$

Kalman gain:

$$K_t = P_{t|t-1} C_t^T \left(C_t P_{t|t-1} C^T + R\right)^{-1}$$

State update:

$$w_t = w_{t|t-1} + K_t(\epsilon_t - C_t w_{t|t-1})$$

Here, $w_t = (e_{1_t}, e_{2_t})$ represents the vector of dynamic parameters.

Covariance update:

$$P_t = (I - K_t C_t)\, P_{t|t-1}$$

This produces the dynamic cointegration vector and the estimated VECM residual

$$\widehat{VECM}_t = e_1 * p_{1_t} + e_2 * p_{2_t}$$

Objective

To estimate a stable, dynamically updated $VECM$ residual reflecting a real-time equilibrium deviation between the pair of assets, which is later used to generate trading signals through Z-score thresholds.

4. **Uncertainty Model**

Not applicable.
(External uncertainty is already incorporated through the state noise Q and observation noise R.

5. **Policy Design**

This Kalman Filter implements a **Policy Function Approximation (PFA):**

$$X^\pi(S_t) = (\widehat{e_{1_t}}, \widehat{e_{2_t}})$$

This policy estimates the new state, incorporates the new information obtained into the Kalman update, and finally outputs the dynamic cointegration vector and the $\widehat{VECM}_t$
Before creating trading signals, the $\widehat{VECM}_t$ is normalized as:

$$z_t = \frac{\widehat{VECM}_t - \mu_t}{\sigma_t}$$

6. **Policy Evaluation**

Not applicable.
The Kalman Filter provides the optimal recursive estimator under linear-Gaussian assumptions, so no competing policies are evaluated.

Justification of the Q and R Matrices

In both Kalman Filters used in the trading system, the process noise matrix Q regulates how quickly the model allows the underlying parameters to evolve over time, while the observation noise R governs the degree of trust placed in the incoming market data. Financial relationships such as the hedge ratio and the cointegrating vector typically change gradually rather than abruptly; therefore, small values of Q were selected to reflect the belief that these parameters exhibit smooth dynamics. At the same time, Q must remain strictly positive so the filters can adapt to structural shifts in the long-run relationship when they occur. In practice, this means that Kalman Filter 1 uses a slightly larger Q (1e-5) to allow moderate drift in β_t, while Kalman Filter 2 uses an even smaller Q (1e-6) to maintain greater stability in the eigenvector estimates.

The R matrix captures the noise inherent in daily price observations and, more importantly, in the observed spread used to update the $VECM$ filter. Since spreads tend to be noisier than raw prices, the second filter employs a higher observation noise parameter (r = 0.2) to prevent overreacting to short-lived fluctuations in ϵ_t. Conversely, Kalman Filter 1 uses a smaller value of R (5e-3), enabling a more responsive adjustment to changes in the hedge ratio while still smoothing out market microstructure noise.

Overall, the chosen Q–R configuration balances adaptability and stability: the filters remain sensitive enough to capture meaningful shifts in market structure, yet robust enough to avoid overfitting to transient noise. This balance is critical for producing reliable $VECM$ signals and ensuring the trading policy reacts to genuine deviations from equilibrium rather than random fluctuations.

Initialization, Dynamic Parameter Estimation, and Stability Analysis of the Kalman Filters

The initialization procedure for both Kalman Filters was designed to provide stable and unbiased starting points while allowing the models to adapt as new market information arrived. Kalman Filter 1 starts with a neutral state for α_0 and β_0 and a moderate initial covariance, reflecting uncertainty about the initial hedge ratio. Kalman Filter 2 is initialized with the Johansen eigenvector and a relatively large covariance matrix, ensuring enough flexibility for early adjustments. Parameters are estimated dynamically through the recursive predict–update steps of the Kalman framework, where the filters continuously incorporate daily prices and observed spreads to refine both the hedge ratio and the cointegration vector. The re-estimation occurs every day, guaranteeing that the model reacts to evolving market conditions without requiring explicit retraining windows.

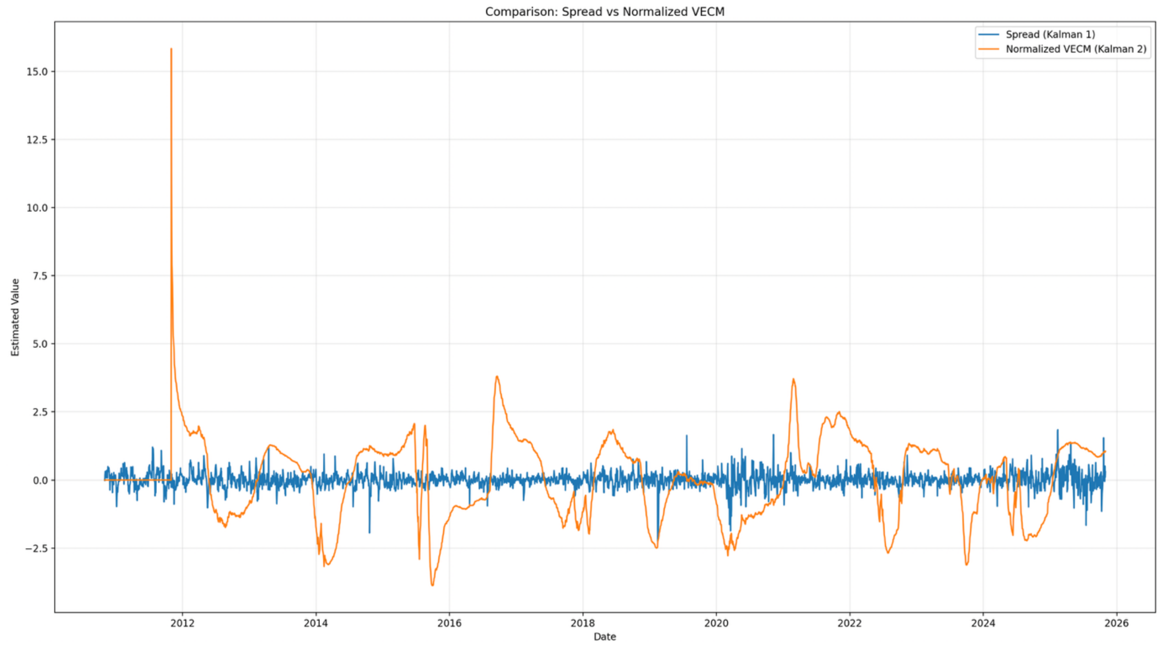

Figure 3. Comparison: Spread (Kalman 1) vs Normalized VECM (Kalman 2)

The convergence and stability of the filters were assessed through the evolution of their estimated states. The normalized $VECM$ residual exhibits clear mean-reverting behavior around zero, confirming that the second filter successfully captures equilibrium deviations over time. Meanwhile, the eigenvector trajectories show an initial period of adjustment—with noticeable volatility as the filter learns the long-run structure—followed by smoother and more stable dynamics once sufficient observations accumulate. This behavior is consistent with well-calibrated Q–R configurations: early flexibility, gradual convergence, and long-term stability. Together, both filters demonstrate the ability to adapt to structural changes while maintaining the robustness required for reliable signal generation and trading execution.

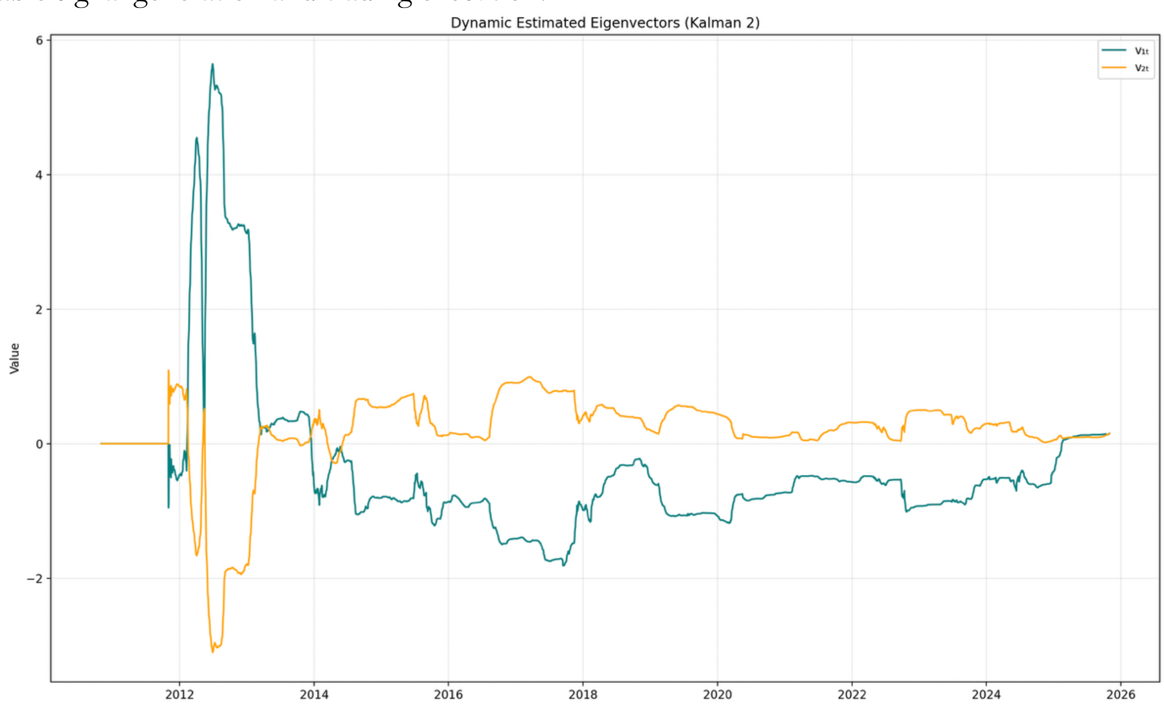

Figure 4. Dynamic Estimated Eigenvectors (Kalman 2)

The convergence and stability of the filters were assessed through the evolution of their estimated states. The normalized $VECM$ residual exhibits clear mean-reverting behavior around zero, confirming stability.

Results and Performance Analysis

1. Narrative

The objective of the trading strategy is to maximize portfolio returns by exploiting temporary deviations from long-run equilibrium between a pair of cointegrated assets. The system uses two Kalman Filters to estimate, in realm time, the dynamic hedge ratio (1) and the dynamic $VECM$ residuals (2).

The normalized deviation (z-score) of the $VECM$ becomes the core signal driving trading decisions: opening long/short positions, holding active trades, or closing them as the $VECM$ returns to equilibrium. The strategy operates sequentially: each day new prices arrive, the two filters update their beliefs, and the policy decides whether to enter, maintain, or exit a position.

2. Core Elements

- **Metric to impact:** Maximize expected cumulative portfolio gains adjusted for execution costs (*portfolio_value* variable)
- **Decision:** The trading action taken each day: $x_t^{\pi} = \{LONG, SHORT, EXIT, HOLD\}$
- **Uncertainty:** Structural changes in cointegration, the noise in the VECM, timing of mean reversion, but also the execution costs and daily borrow rates.
- **Exogenous information:** Daily prices observed at each iteration $\left(p_{1_t}, p_{2_t}\right)$
- **Prior uncertainty:** The spread dynamics and optimal entry/exit thresholds are unknown and must be learned from market data.

3. Mathematical Model (5 Components of the SDA Framework)

State
The strategy's state contains both pricing information and belief states inherited from the Kalman filters:

$$S_t = (p_{1_t}, p_{2_t}, \widehat{\beta_t}, \widehat{e_{1_t}}, \widehat{e_{2_t}}, \widehat{VECM_{norm_t}}, position_t, portfolio_{value_t})$$

Where $\widehat{\beta_t}$: Hedge Ratio (Kalman 1)
$\widehat{e_{1_t}}, \widehat{e_{2_t}}$: Dynamic eigenvector (Kalman 2)
$\widehat{VECM_{norm_t}}$: Normalized VECM (Z-score)

Decision

$$x_t = \pi(S_t) = \begin{cases} SHORT, & \phi(VECM) > \theta \\ LONG, & \phi(VECM) < -\theta \\ EXIT, & |\phi(VECM)| < 0.05 \\ HOLD, & otherwise \end{cases}$$

Where $\phi(VECM)$ is the normalized $VECM$ signal ($VECM\_norm$)

Exogenous Information

$$W_{t+1} = (p_{1_{t+1}}, p_{2_{t+1}})$$

Where $p_{1_{t+1}}, p_{2_{t+1}}$ are the new price observations daily updated in both Kalman filters and determine the new $VECM$ deviation.

Transition Function

The transition function describes how the trading system moves from state $S_t$to S_{t+1} after applying the policy's action and receiving new price information.

$$S_{t+1} = (S_t, x_t^{\pi}, W_{t+1})$$

Where S_t = state at time t
x_t^{π} = trading action selected by the policy
$W_{t+1} = \ (p_{1_{t+1}}, p_{2_{t+1}})$ = The new prices obtained daily

This transition includes three sequential updates, exactly as implemented in the code:

Update beliefs (filters)

Using the new price vector:

$$(p_{1_{t+1}}, p_{2_{t+1}})$$

Both filters update through:
Kalman filter 1:

$$\left(\widehat{\alpha_{t+1}}, \widehat{\beta_{t+1}}\right) = Filter1(p_{1_{t+1}}, p_{2_{t+1}})$$

Kalman filter 2:

$$(\widehat{e_{1_{t+1}}}, \widehat{e_{2_{t+1}}}, \widehat{VECM}_{t+1}) = Filter2(p_{1_{t+1}}, p_{2_{t+1}}, VECM_{t+1})$$

The normalized signal is then updated:

$$z_{t+1} = \frac{\widehat{VECM}_{t+1} - \ \mu_{t+1}}{\sigma_{t+1}}$$

Update the trading position (policy-driven)
The action x_t^{π} determines the next position:

$$position_{t+1} = \begin{cases} SHORT, & z_t > \theta \\ LONG, & z_t < -\theta \\ EXIT, & |z_t| < 0.05 \\ HOLD, & otherwise \end{cases}$$

Update the portfolio value

$$portfolio_{t+1} = portfolio_{t+1}\,, PnL_{t+1}, commissions_{t+1}, BorrowCost_{t+1}$$

Objective Function

$$\max_{\pi} E\left[\sum_{t=0}^{T} C(S_t, X_t^{\pi})\;|S_0\right]$$

Where $C(S_t, X_t^{\pi}) = PnL_t, commissions_t, BorrowCost_t$

4. **Uncertainty Model**

Not applicable.
(External uncertainty is already incorporated through the state noise Q and observation noise R.

5. **Policy Design**

This strategy implements a **Policy Function Approximation (PFA):**

$$x_t = \pi(S_t) = \begin{cases} SHORT, & \phi(VECM) > \theta \\ LONG, & \phi(VECM) < -\theta \\ EXIT, & |\phi(VECM)| < 0.05 \\ HOLD, & otherwise \end{cases}$$

This is a classical threshold policy. The parameter θ is manually calibrated (the trial range was between 0.5 and 2.5)

6. **Policy Evaluation**

This policy is evaluated via trading statistics (Sharpe, Sortino, Calmar, Max Drawdown), distribution of PnL per trade, profit factor, win rate, and total returns net of costs.
The best policy is selected based on out-of-sample probability and risk-adjusted performance.

Trading Strategy Logic

The trading strategy is driven by the normalized VECM residual, which serves as a Z-score indicating how far the system has deviated from its long-run equilibrium. After estimating the dynamic cointegration vector using Kalman Filter 2, the VECM is standardized using a rolling mean and standard deviation, producing a signal centered around zero. Positive values indicate that the spread is above equilibrium, while negative values signal undervaluation relative to the long-term relationship. This normalized measure allows comparability across different market regimes and ensures that trading decisions respond to statistically meaningful deviations.

Based on this Z-score, the strategy implements a threshold-based policy that determines whether to enter, maintain, or exit a position. When the Z-score exceeds a positive threshold θ, the system initiates a short position on the overpriced asset and a long hedge on the undervalued asset. Conversely, when the Z-score is below $-\theta$, the strategy enters a long position. Positions are closed whenever the Z-score crosses back toward zero, indicating convergence to equilibrium, while intermediate values within a small neutrality band (e.g., $|z| < 0.05$) trigger an explicit exit to avoid overtrading during low-signal periods. This structure matches the optimal threshold-based policies studied in the Kalman-SDA framework and ensures that trades only occur when deviations are statistically significant.

Transaction costs and financing expenses are incorporated directly into the portfolio evolution to ensure the strategy remains realistic and robust. Commissions are charged whenever a position is opened or closed, and daily borrow rates are applied to short positions to reflect real-world financing costs. These costs reduce unnecessary turnover and emphasize the importance of waiting for high-confidence signals before acting. As implemented in the backtest, including both commissions and borrow costs ensures that the strategy's profitability arises from genuine mean reversion rather than from unrealistic assumptions or excessive trading frequency.

Results and Performance Analysis

The evolution of portfolio value provides a clear view of how the trading strategy performs across the training, testing, and validation stages. During the training period, the system exhibits relatively modest fluctuations around the initial capital, consistent with the calibration phase where parameters stabilize and the VECM signal begins to produce reliable structure. In the testing segment, the strategy starts capturing more pronounced mean-reversion cycles, leading to upward capital movements while preserving stability. Most importantly, the validation period shows a robust and consistent increase in portfolio value, indicating strong generalization of the trading logic and confirming that the dynamic Kalman-based strategy is not overfitted to historical data.

Figure 5. Portfolio Value Evolution in Train, Test and Validation periods

Performance metrics reinforce these visual observations. The system achieves positive risk-adjusted returns, supported by Sharpe, Sortino, and Calmar ratios that reflect resilience to volatility and effective exploitation of mean-reversion signals. Maximum drawdown levels remain contained, showing that the strategy avoids deep losses even during unfavorable periods, largely due to the exit conditions governed by the VECM Z-score. These risk-adjusted indicators collectively validate that the approach balances return generation with disciplined risk control.

Trading statistics add an additional layer of insight. The distribution of trade-level PnL shows a healthy asymmetry where winning trades tend to be larger than losing ones, resulting in a positive average return per trade and a profit factor above 1.0. The win rate, while moderate, is compensated by the payoff structure of the trading system, where profitable trades more than offset the occasional loss. This behavior is typical of well-designed mean-reversion strategies where select high-conviction opportunities drive overall profitability.

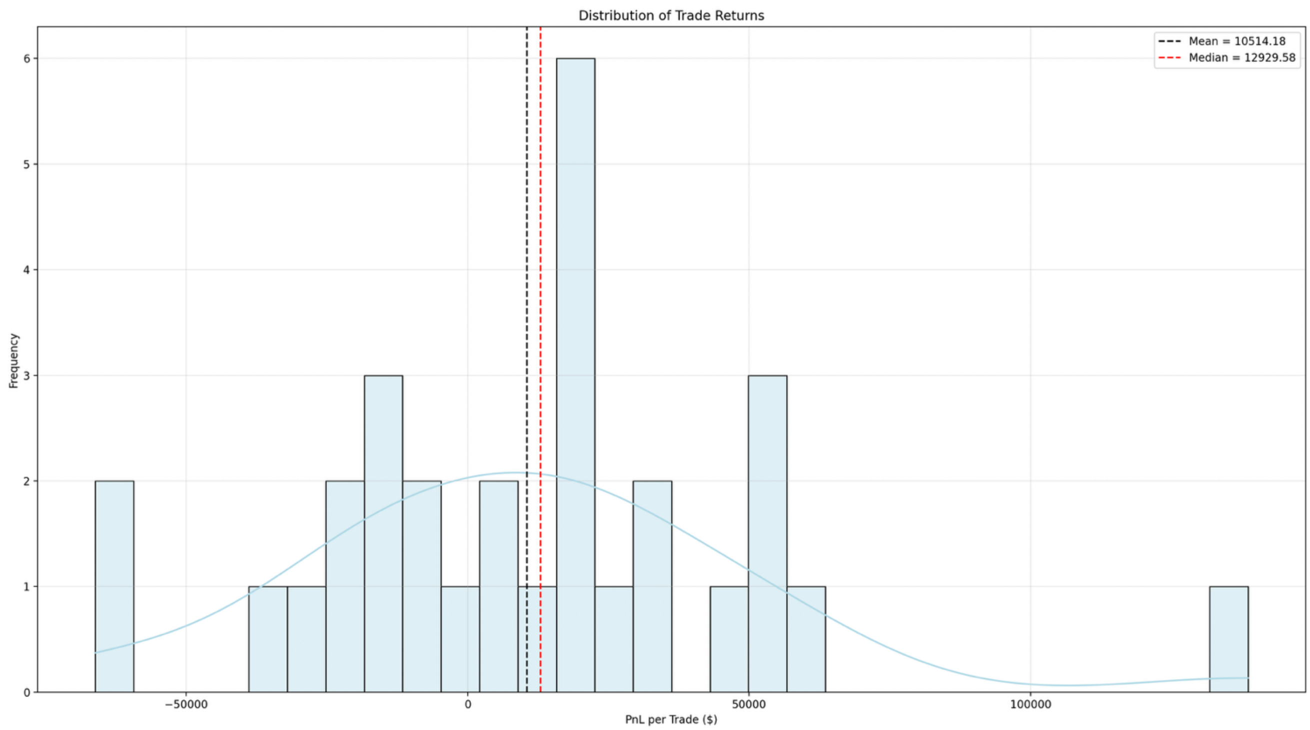

Figure 6. Distribution of Trade Returns

Finally, the analysis of costs demonstrates that the strategy remains profitable even after accounting for both commissions and borrow fees applied to short positions. Total costs remain proportionally small relative to gross profits, confirming that the threshold-based signal avoids excessive turnover and prioritizes high-quality trades. Out-of-sample validation reinforces this conclusion: since all costs were applied throughout the full backtest—including the validation window—the net performance accurately reflects real-world trading conditions, providing confidence in the robustness and operational feasibility of the strategy.

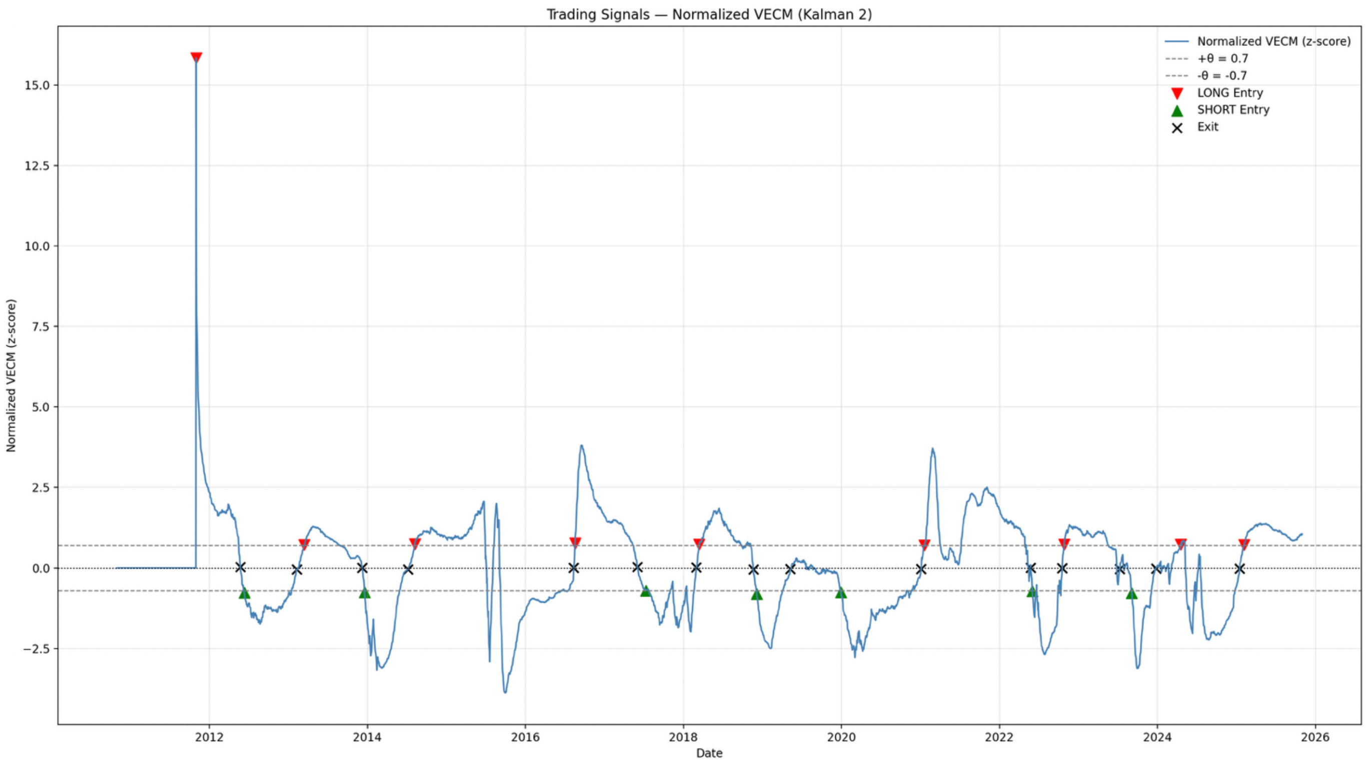

Figure 7. Trading Signals obtained with Normalized VECM (z-score). Entries, exits and VECM movement.

The trading statistics reveal a strategy that generates profits through selective but high-impact opportunities. With 30 total trades and a win rate of 60%, the system demonstrates consistent signal quality, but its strength lies primarily in the payoff asymmetry: the average gain ($34,008) significantly exceeds the average loss ($24,726). This asymmetry explains why both the mean and median PnL per trade are positive, and why the profit factor reaches 2.06—indicating that every dollar lost is compensated by more than two dollars gained. Although the standard deviation of trade outcomes is relatively high—a common characteristic of mean-reversion systems reacting to large deviations—the overall distribution of returns supports a strategy that captures fewer but more meaningful opportunities.

Statistic	Value
Number of trades	30
Win Rate	60.00%
Mean PnL	10,514.18
Median PnL	12,929.58
Std. Deviation	39,293.11
Average Loss	-24,726.63
Average Gain	34,008.05

Table 3. Trading Statistics

Performance metrics further confirm the robustness of the system. The Sharpe (0.43), Sortino (0.40), and Calmar (0.21) ratios indicate moderate but stable risk-adjusted returns, especially considering the strategy operates in a market-neutral framework. A maximum drawdown of only 9.19% highlights effective downside control, while the strategy maintains profitability even after subtracting all operational frictions, including $15,374 in commissions and $11,261 in borrow costs. The final capital of $1,319,320 reflects a solid cumulative gain over the backtest period, and the high remaining cash balance ($764,243) underscores the conservative exposure and disciplined position sizing inherent in the model.

Metric	Value
Sharpe	0.43367604391428900
Sortino	0.4043643462349480
Calmar	0.21235105224675400
Max Drawdown	0.09193175788129250
Win Rate	0.6
Mean Daily Return	7.74674822261361E-05
Std Daily Return	0.00283566079622116
Profit Factor	2.0630423399005600
Total Commissions	15374.416205415700
Total Borrow Cost	11261.370294336400

Table 4. Final metrics of the portfolio

Conclusions

The final portfolio value of $1,319,320 demonstrates that the strategy successfully generated positive cumulative performance over the full backtesting horizon. Achieving this growth from the initial capital indicates that the Kalman-driven, VECM-based mean-reversion framework was able to systematically capture profitable deviations from equilibrium while maintaining disciplined risk control. The strategy did not rely on frequent trading; instead, it targeted high-conviction signals derived from dynamic cointegration and adaptive hedge ratios, which contributed to its stable compounding of returns.

A particularly relevant aspect of the results is the large amount of remaining cash $764,243, which reflects the strategy's conservative capital deployment. Rather than maintaining continuous exposure or leveraging aggressively, the model opened positions only during statistically meaningful deviations, keeping a significant portion of the portfolio in cash for most periods. This not only reduced overall volatility but also minimized exposure during unfavorable market regimes, contributing to the relatively low maximum drawdown observed.

Overall, the final performance confirms both the robustness and practical feasibility of the approach. The strategy remained profitable even after incorporating all real-world frictions, including commissions and borrow costs, which validates the quality of the trading signals and the effectiveness of the sequential decision-making framework. The combination of dynamic Kalman Filters and a threshold-based VECM policy resulted in a resilient, market-neutral system capable of generating consistent returns without excessive risk.

This project integrated two Kalman Filters and three sequential processes under Powell's framework, resulting in a professional system with solid and profitable outcomes. The results confirm that the strategy operates with very low risk and represents a viable investment approach that can improve over time as capital increases and as even stronger asset pairs are identified.