

Bayesian Particle Filter Tracking with CUDA

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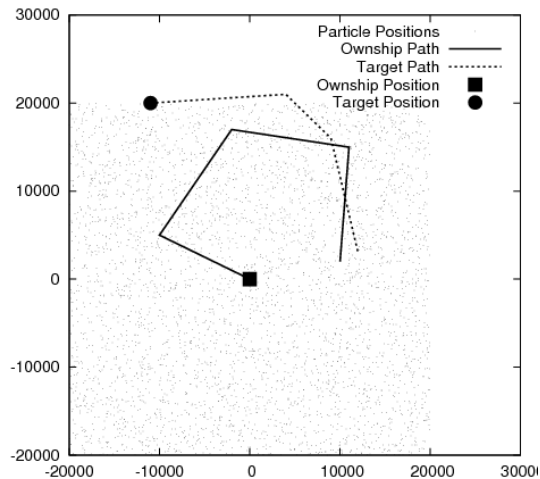
May 6, 2010

Motivating Example

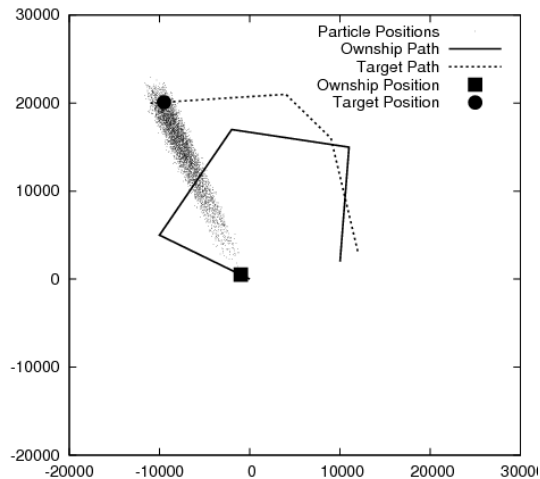
A submarine with a hydrophone (passive sonar) is following another ship using the direction of the sound from the ship's engine.

How can the series of bearing observations from the hydrophone be used to estimate the second ship's position and velocity?

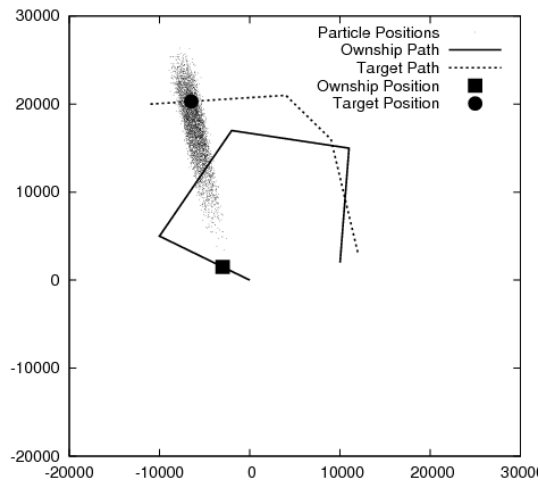
Motivating Example



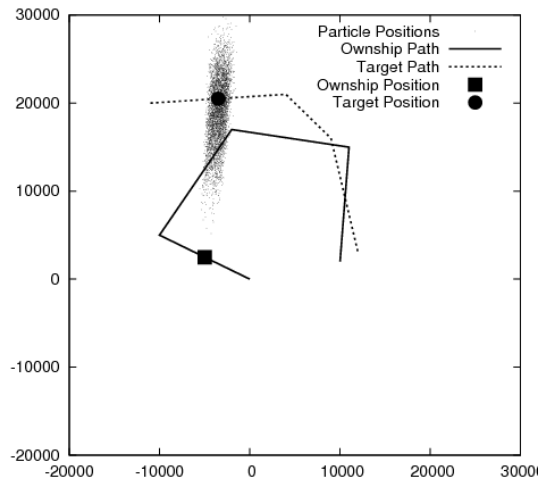
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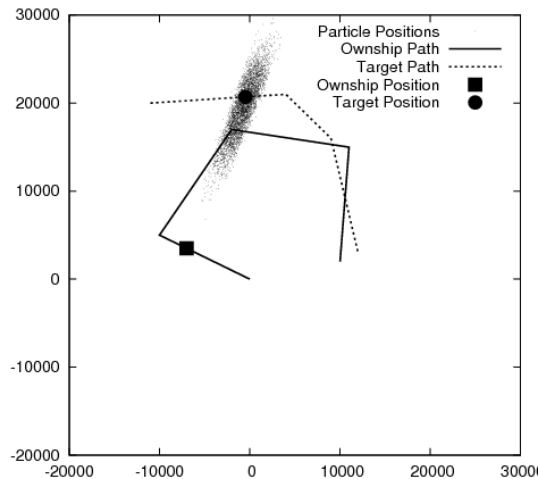
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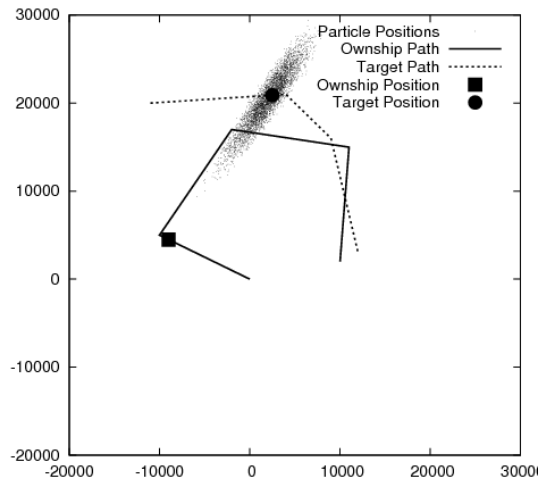
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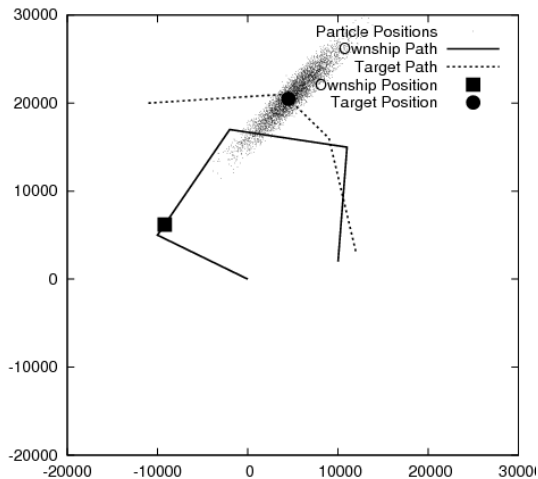
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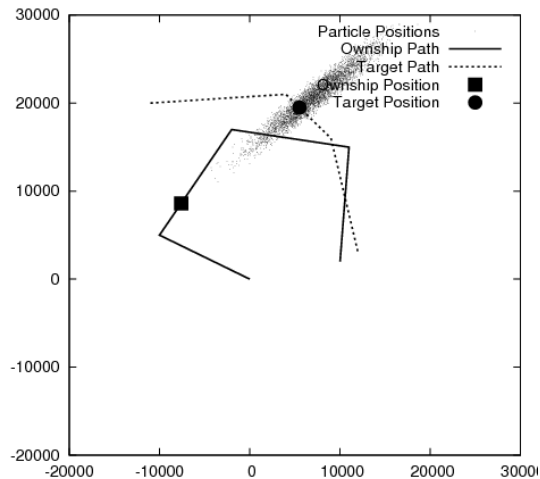
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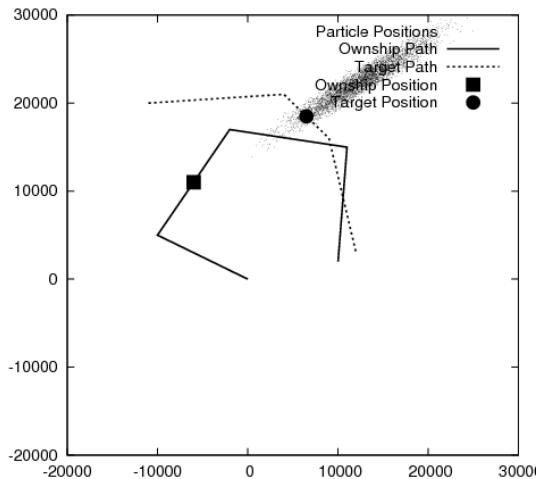
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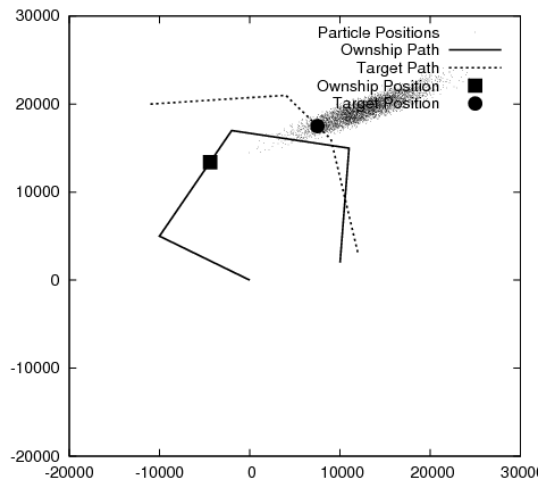
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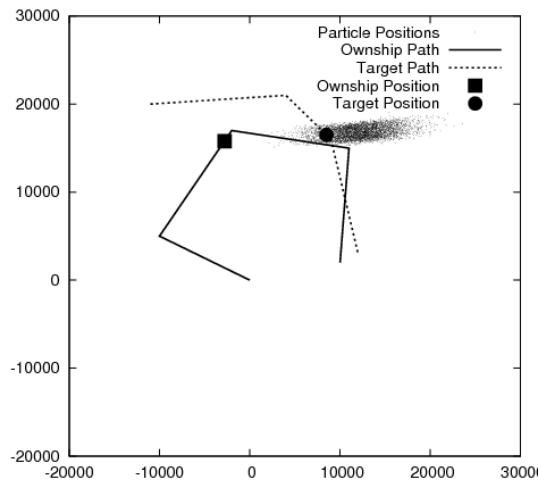
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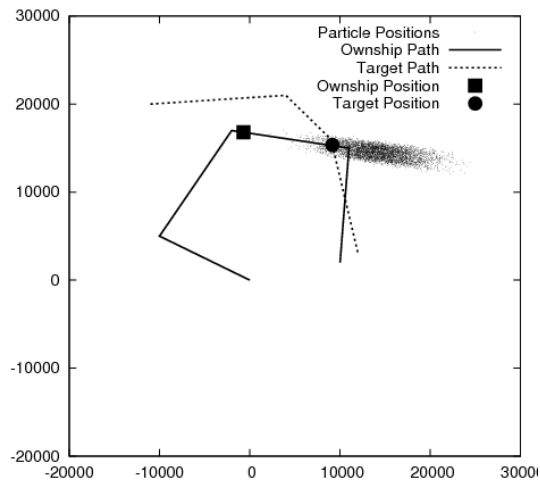
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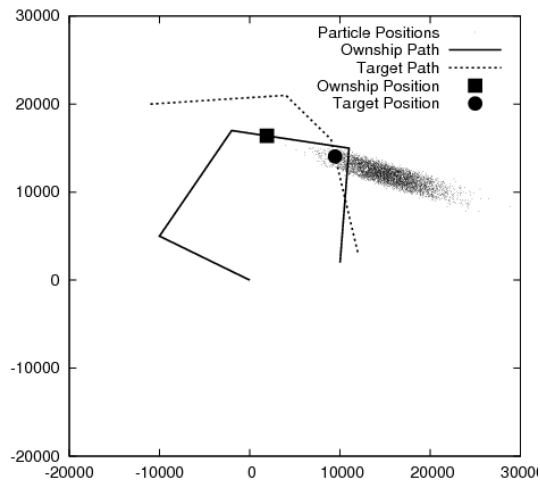
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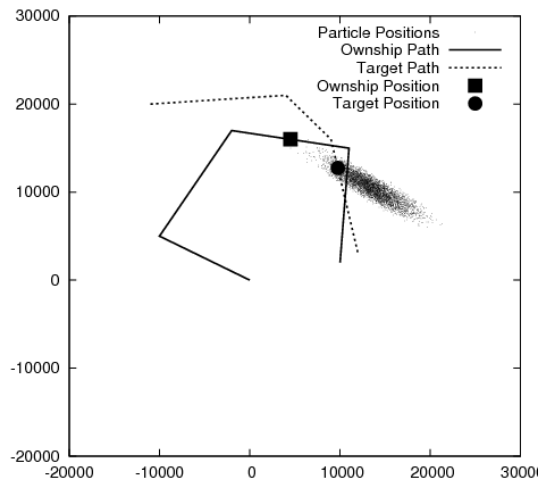
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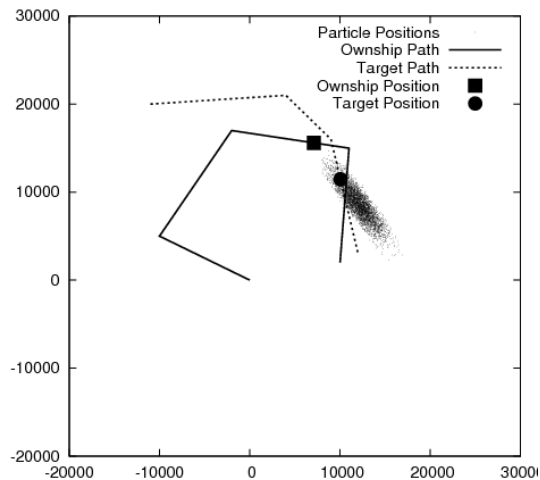
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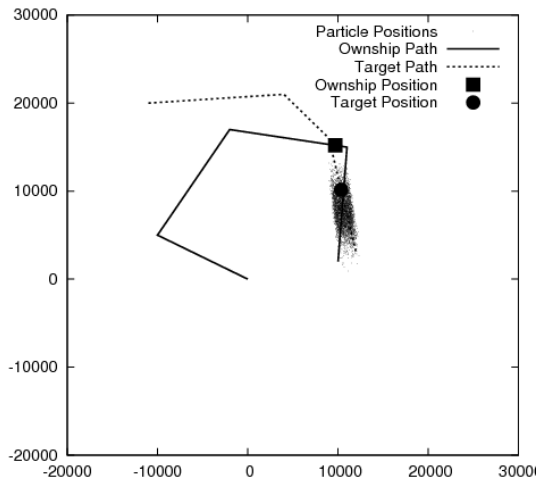
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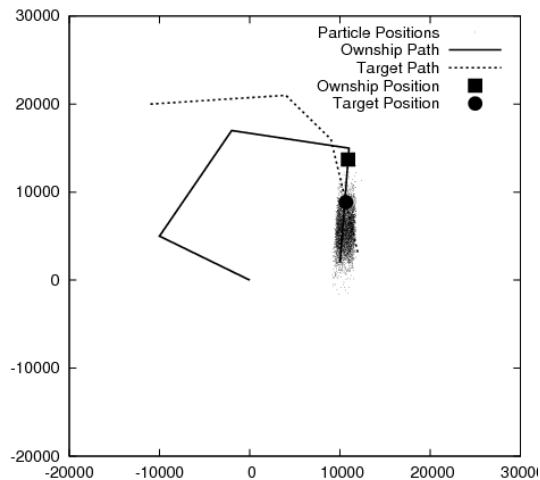
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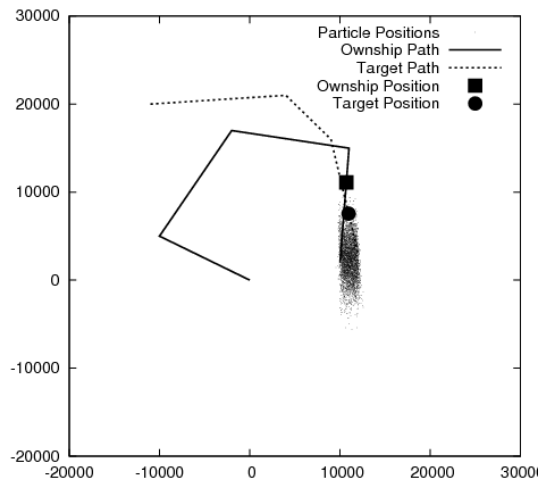
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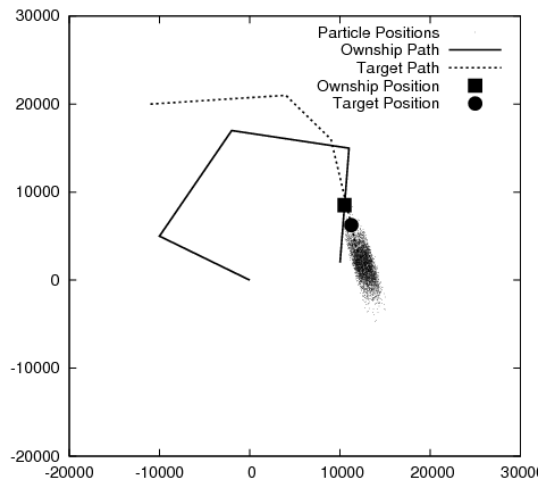
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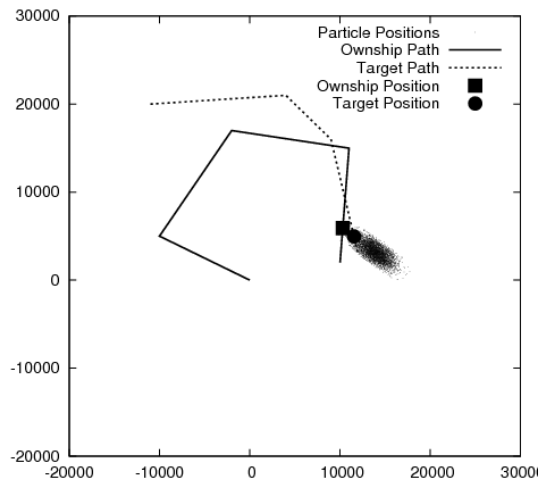
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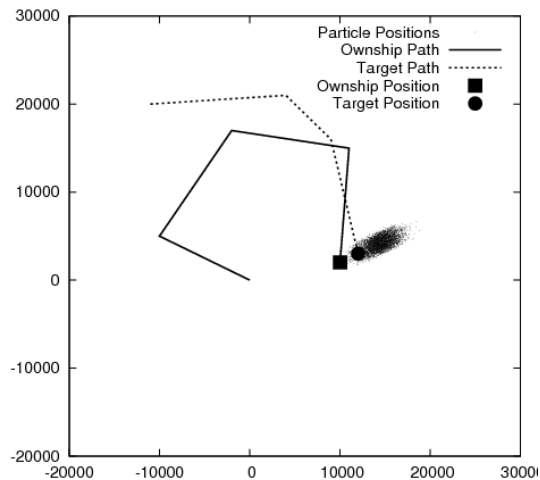
Motivating Example



Motivating Example



Motivating Example



Likelihood Function

$$L(y|x) = P(Y = y|X = x) \quad \text{for } x \in S \quad (1)$$

- ▶ X Random variable on S
- ▶ Y Random variable on measurement space H
- ▶ $P(\cdot|x)$ Probability density function on H
- ▶ $P(y|\cdot)$ Likelihood function relating X and Y

Likelihood Function

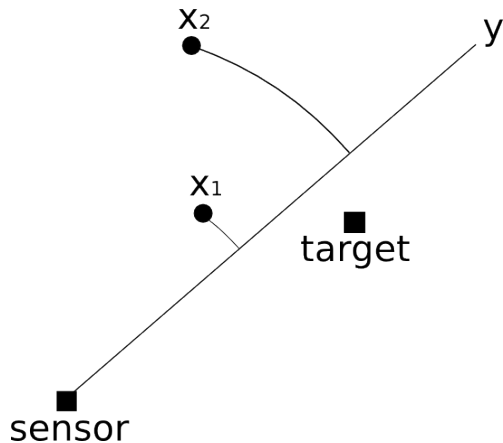
Why is $P(y|\cdot)$ called a "Likelihood Function"?

For $x_1, x_2 \in S$ if $L(y|x_1) > L(y|x_2)$ then:

The observation y is more likely to have come from a target with state x_1 than a target with state x_2

Likelihood Function

$$L(y|x_1) > L(y|x_2)$$



Bayes' Rule

Reverend Thomas Bayes (1702-1761):

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)} \quad (2)$$

Bayesian Inference

$$P(x|y) = \frac{L(y|x)P(x)}{P(y)} = \frac{L(y|x)P(x)}{\int L(y|x)P(x) dx} \quad (3)$$

- ▶ Observation y is fixed, x represents possible target states
- ▶ $P(x)$ Prior probability density function on true target state
- ▶ $P(x|y)$ Posterior distribution given observation y

"Multiply each particle's weight by the likelihood function evaluated at the particle then normalize by the sum over all particle weights."

Prior Distribution

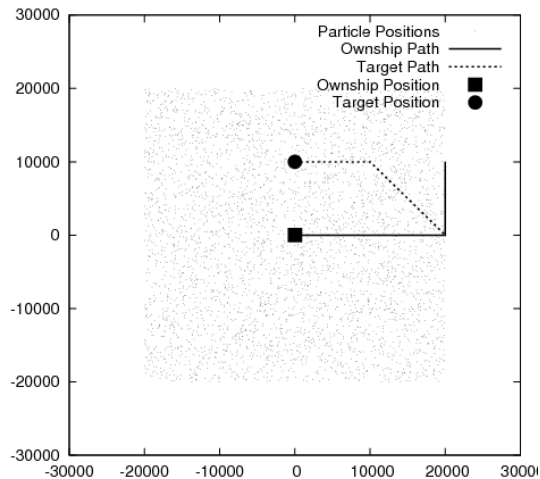


Figure: Prior Particle Position Distribution

Information Update

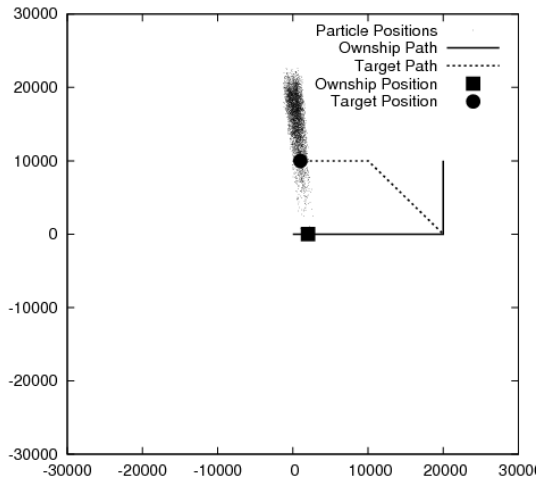


Figure: Posterior Particle Position Distribution after Azimuth Observation

Resampling

$$C = \frac{n}{\sum_{i=0}^n w_i} \quad (4)$$

- ▶ C Normalizing constant
- ▶ w_i Particle weights
- ▶ n Number of particles

Example w_i Array:

0.8	0.4	6.6	0.6	1.6
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$$C = \frac{5}{10} = 0.5$$

Resampling

$$\overline{w}_i = Cw_i \quad (5)$$

Example \overline{w}_i Array:

0.4	0.2	3.3	0.3	0.8
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Resampling

$$\hat{w}_i = \text{floor}\left(\sum_{j=0}^i \bar{w}_j\right) \quad (6)$$

Example \hat{w}_i Array:

Cumulative Sums:

0.4	0.6	3.9	4.2	5.0
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Floor:

0	0	3	4	5
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Resampling Results

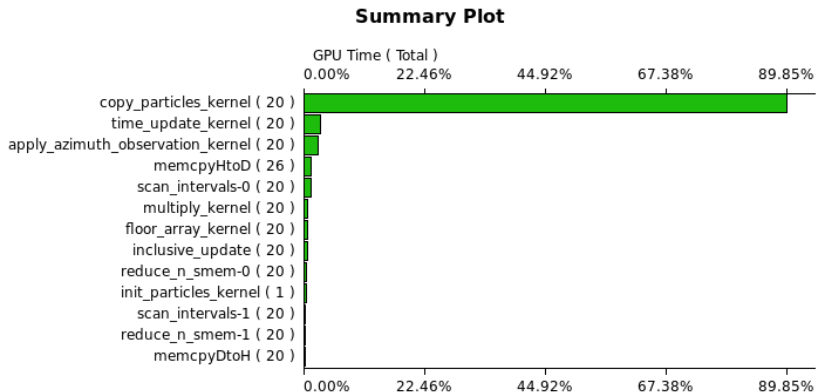


Figure: CUDA Visual Profiler Version 1 Results

Improved Resampling Results

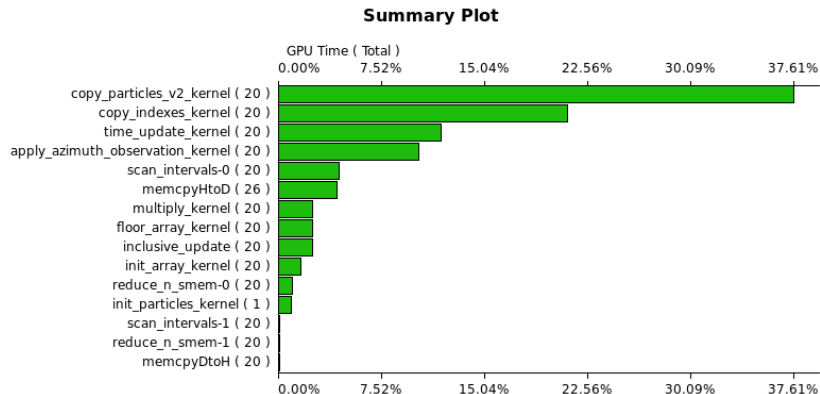
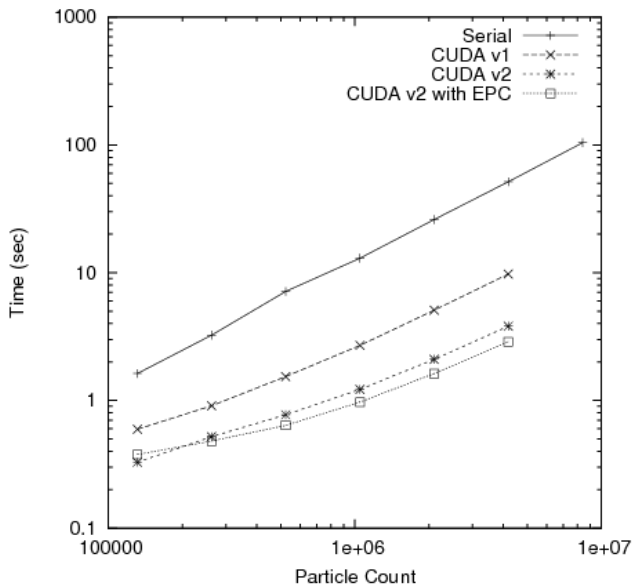
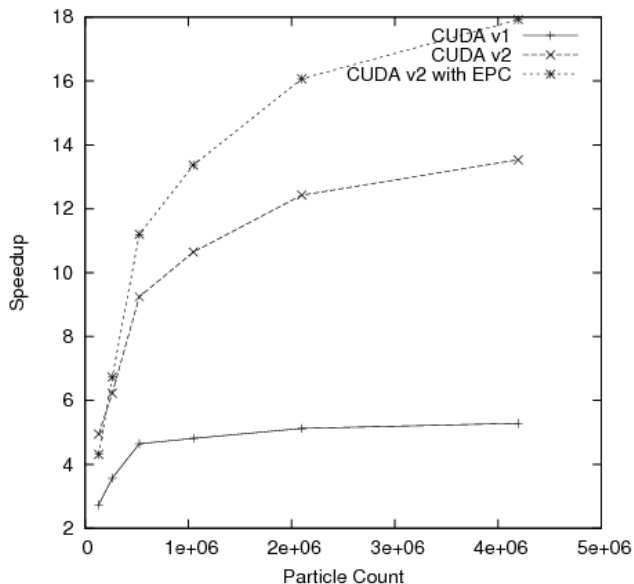


Figure: CUDA Visual Profiler Version 2 Results

Performance



Performance



Effective Particle Count

$$\overline{w}_i = \frac{w_i}{\sum_{i=1}^n w_i} \quad (7)$$

$$N_{eff} = \frac{1}{\sum_{i=1}^n \overline{w}_i^2} \quad (8)$$

$$\text{For } \overline{w}_i = \frac{1}{n} \quad N_{eff} = \frac{1}{n \frac{1}{n^2}} = n$$

$$\text{For } \overline{w}_i = \begin{cases} 1 & \text{for } i = 0 \\ 0 & \text{otherwise} \end{cases} \quad N_{eff} = \frac{1}{1} = 1$$