

# Bayesian Particle Filter Tracking with CUDA

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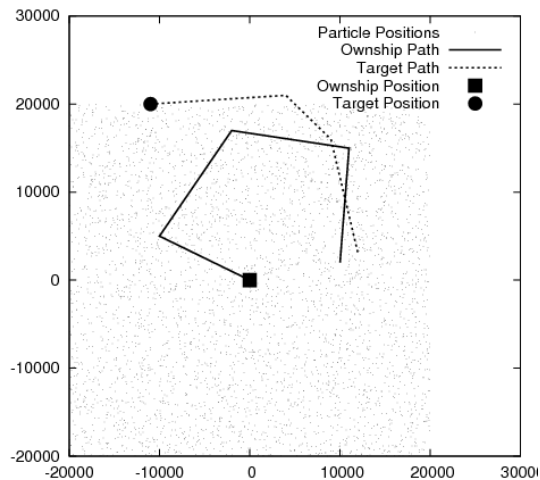
May 6, 2010

# Motivating Example

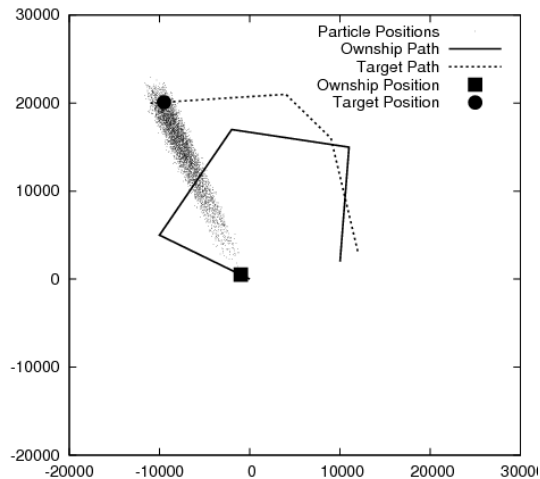
A submarine with a hydrophone (passive sonar) is following another ship using the direction of the sound from the ship's engine.

How can the series of bearing observations from the hydrophone be used to estimate the second ship's position and velocity?

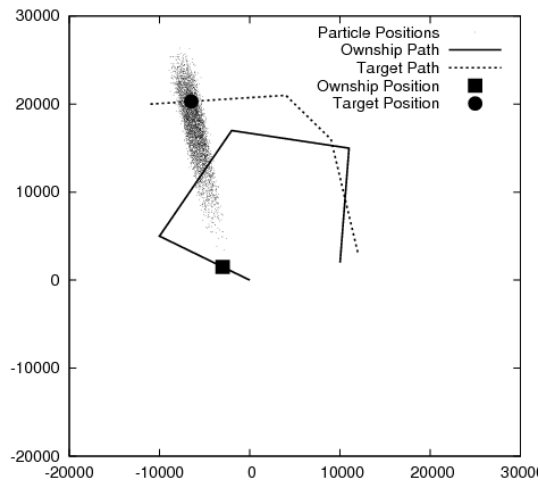
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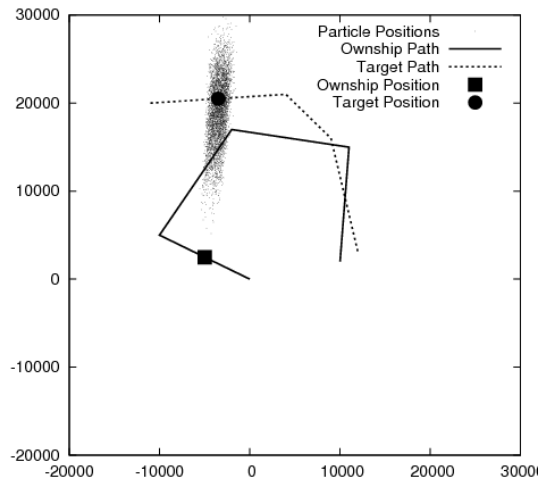
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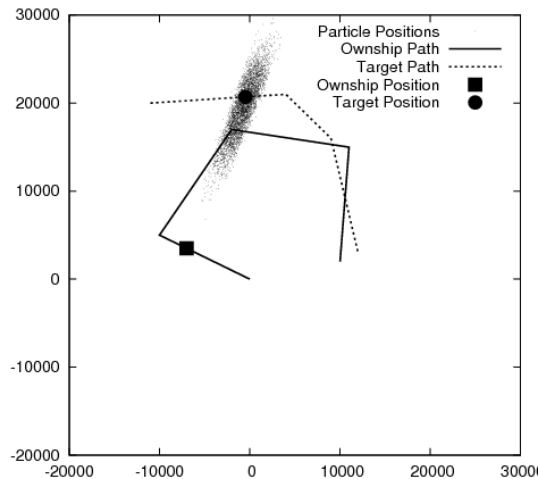
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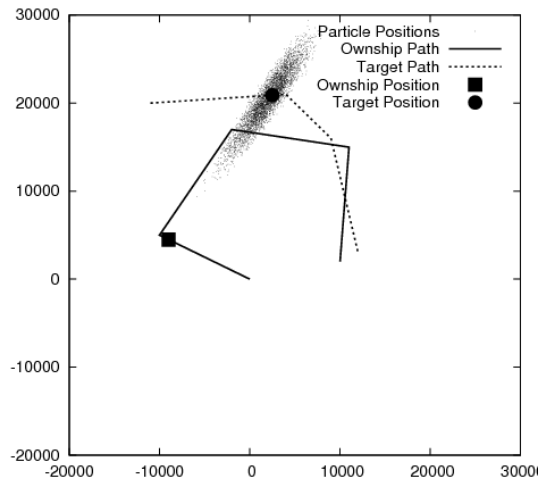
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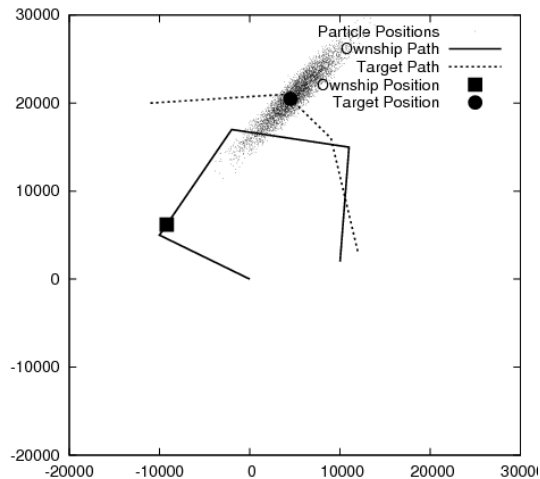


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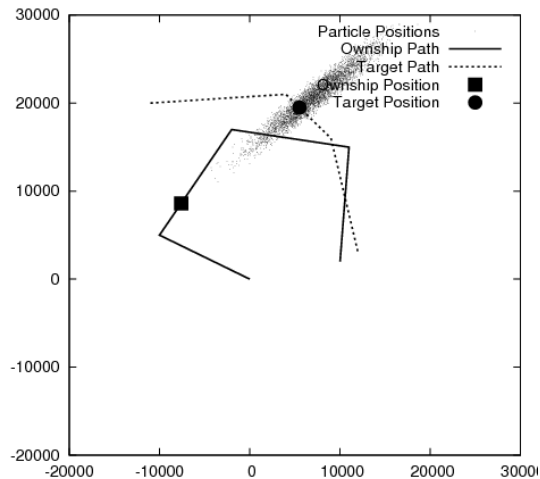




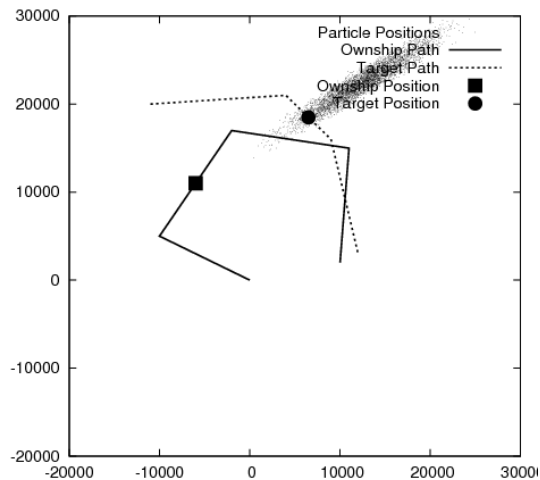
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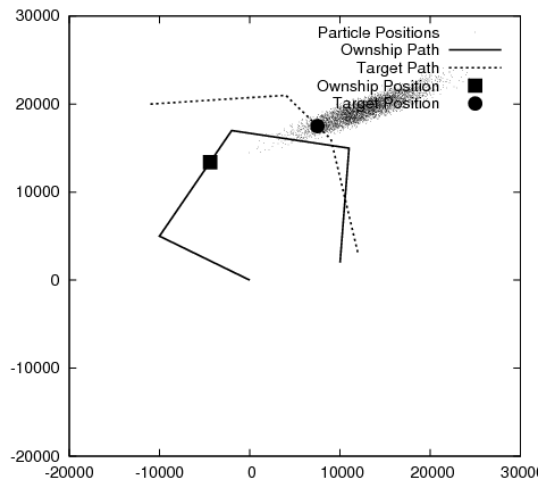
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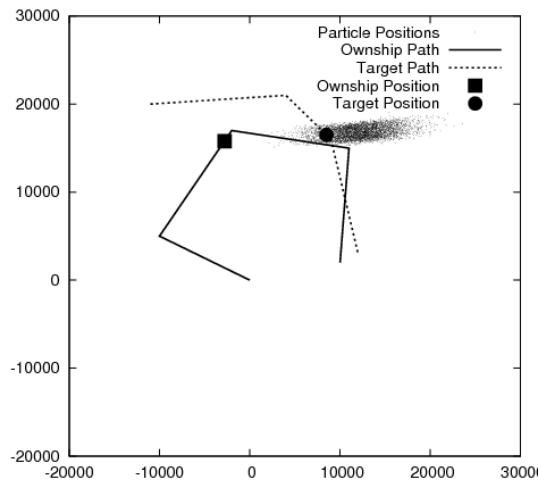
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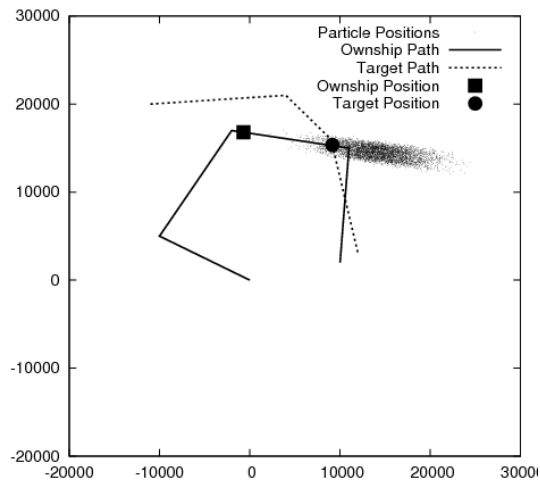
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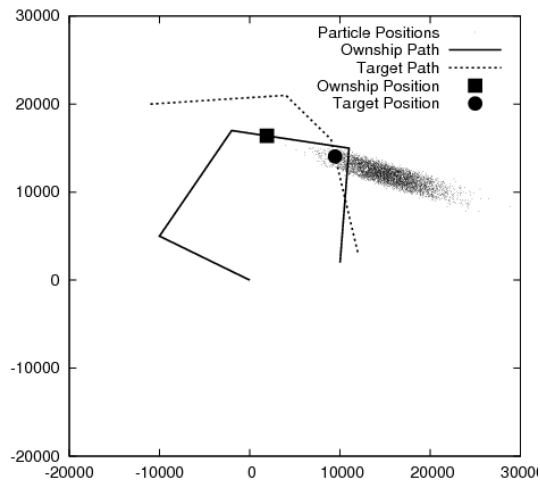
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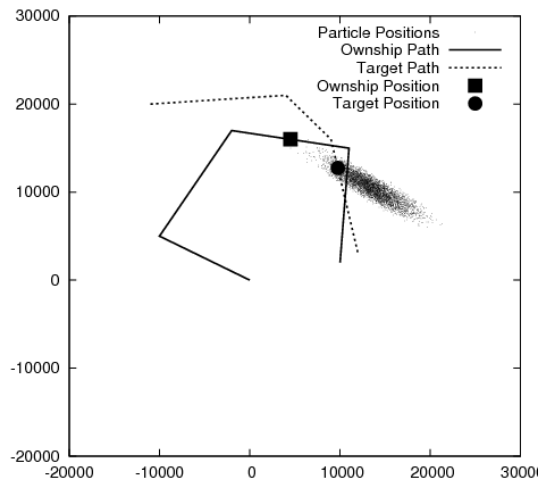
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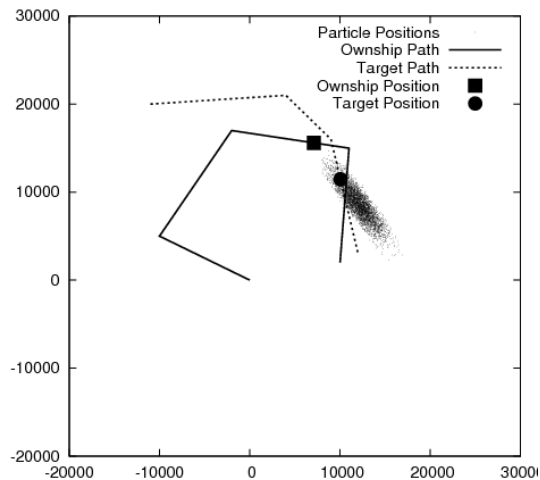


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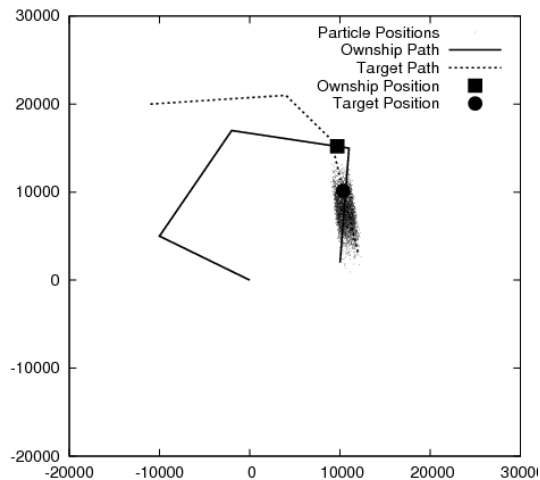




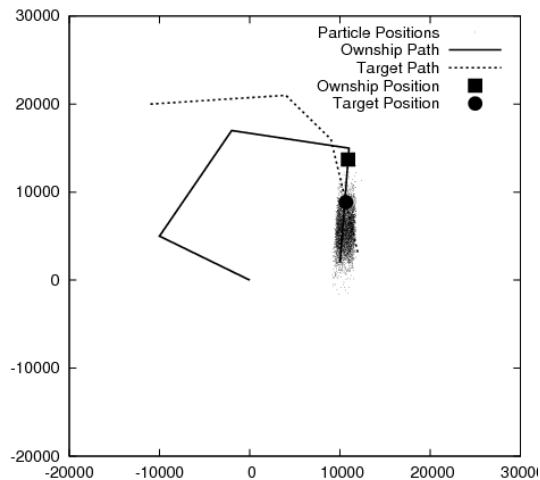
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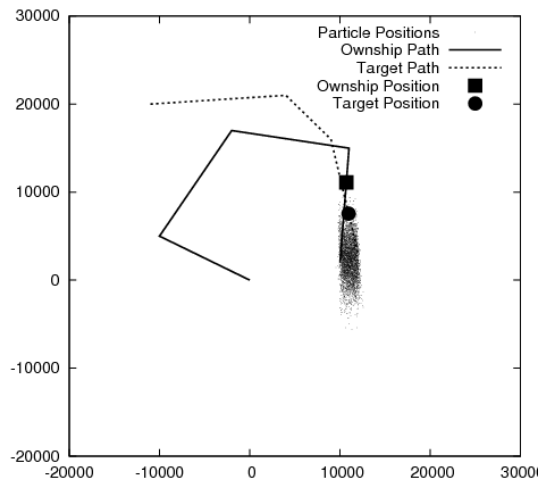
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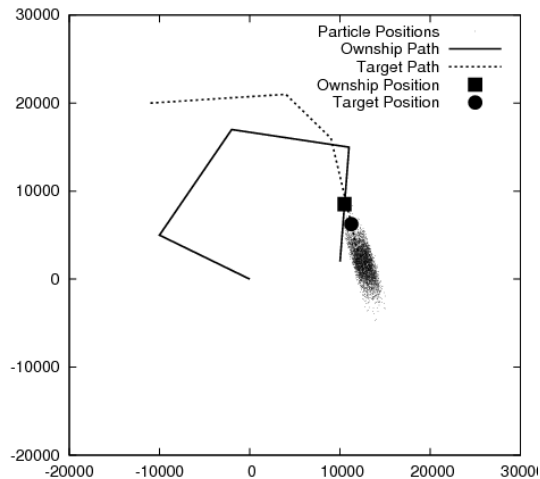
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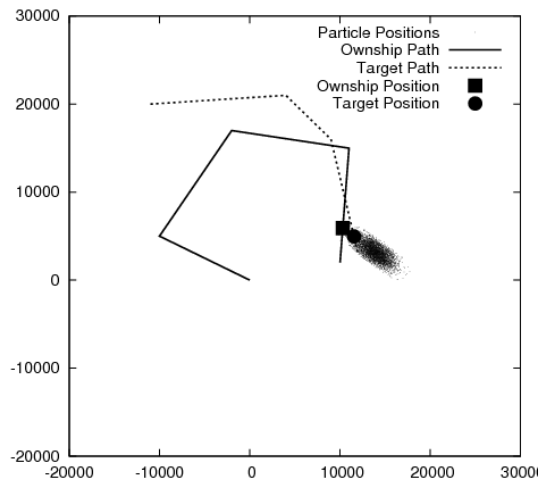
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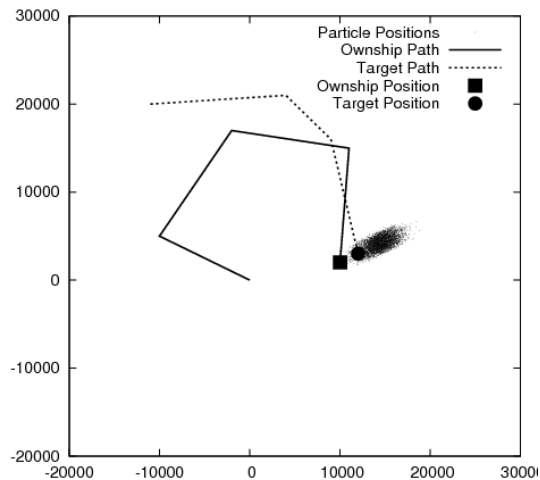
# Motivating Example



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# Motivating Example



# Likelihood Function

$$L(y|x) = P(Y = y|X = x) \quad \text{for } x \in S \quad (1)$$

- ▶  $X$  Random variable on  $S$
- ▶  $Y$  Random variable on measurement space  $H$
- ▶  $P(\cdot|x)$  Probability density function on  $H$
- ▶  $P(y|\cdot)$  Likelihood function relating  $X$  and  $Y$



# Likelihood Function

Why is  $P(y|\cdot)$  called a "Likelihood Function"?

For  $x_1, x_2 \in S$  if  $L(y|x_1) > L(y|x_2)$  then:

The observation  $y$  is more likely to have come from a target with state  $x_1$  than a target with state  $x_2$

# Bayesian Inference

$$P(x|y) = \frac{L(y|x)P(x)}{P(y)} = \frac{L(y|x)P(x)}{\int L(y|x)P(x) dx} \quad (2)$$

- ▶ Observation  $y$  is fixed,  $x$  represents possible target states
- ▶  $P(x)$  Prior probability density function on true target state
- ▶  $P(x|y)$  Posterior distribution given observation  $y$

# Prior Distribution

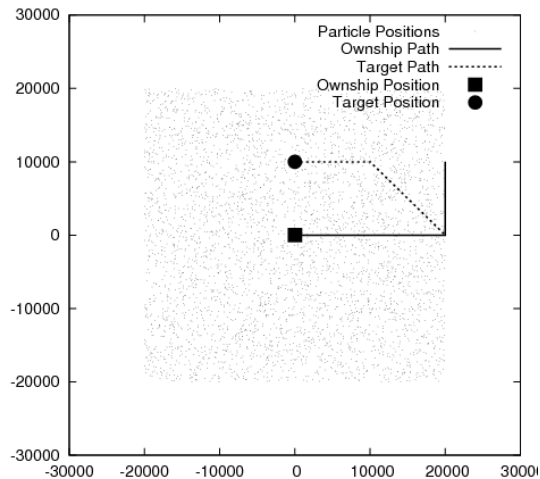


Figure: Prior Particle Position Distribution

# Information Update

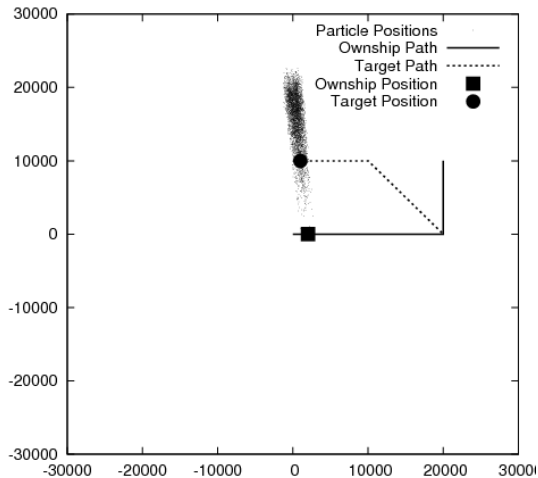


Figure: Posterior Particle Position Distribution after Azimuth Observation

# Resampling

$$C = \frac{n}{\sum_{i=0}^n w_i} \quad (3)$$

- ▶  $C$  Normalizing constant
- ▶  $w_i$  Particle weights
- ▶  $n$  Number of particles

Example  $w_i$  Array:

0.8	0.4	6.6	0.6	1.6
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$$C = \frac{5}{10} = 0.5$$

# Resampling

$$\overline{w}_i = Cw_i \quad (4)$$

Example  $\overline{w}_i$  Array:

0.4	0.2	3.3	0.3	0.8
-----	-----	-----	-----	-----

# Resampling

$$\hat{w}_i = \text{floor}\left(\sum_{j=0}^i \bar{w}_j\right) \quad (5)$$

Example  $\hat{w}_i$  Array:

Cumulative Sums:

0.4	0.6	3.9	4.2	5.0
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Floor:

0	0	3	4	5
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# Resampling Results

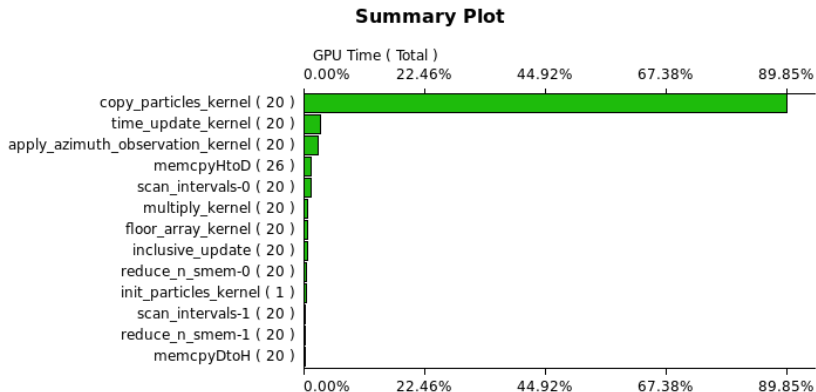


Figure: CUDA Visual Profiler Version 1 Results



# Improved Resampling Results

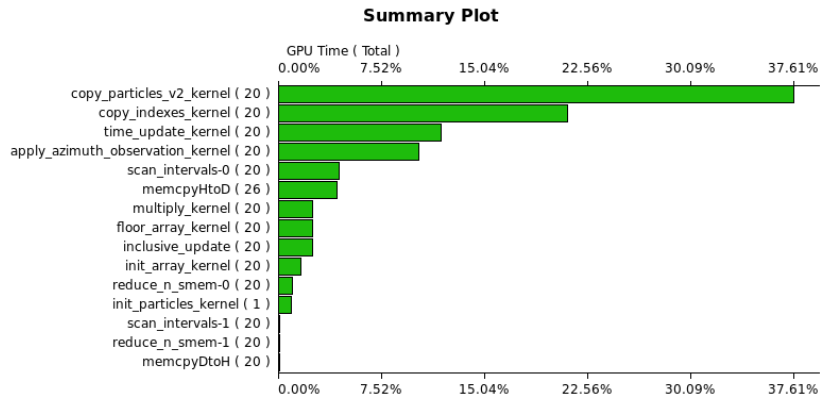
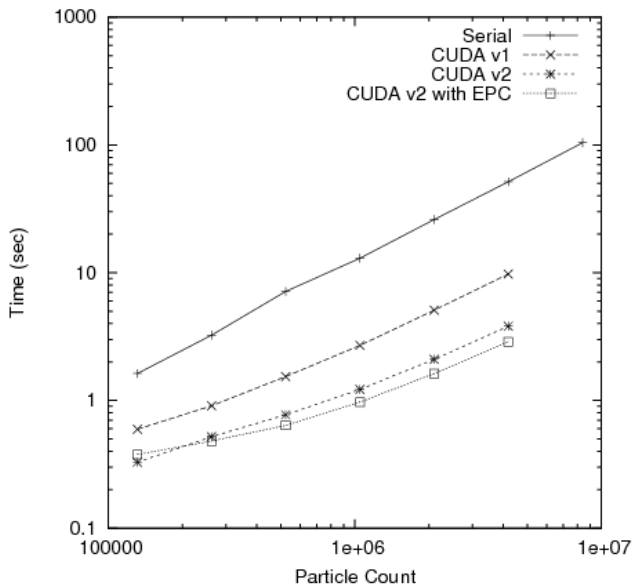
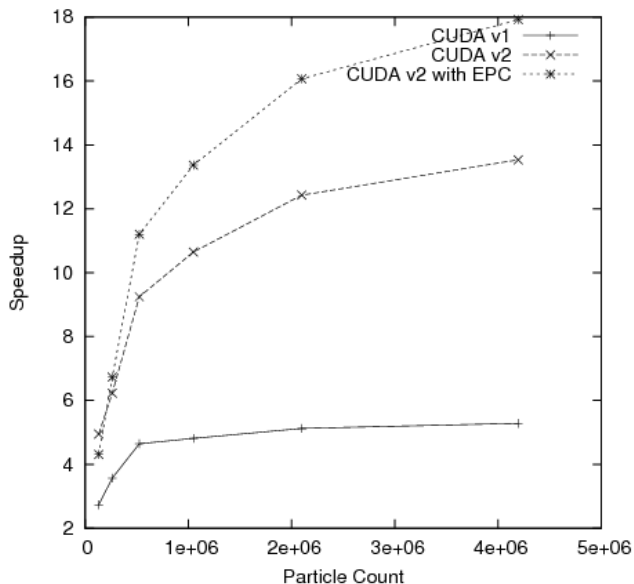


Figure: CUDA Visual Profiler Version 2 Results

# Performance



# Performance



# Effective Particle Count

$$\overline{w}_i = \frac{w_i}{\sum_{i=1}^n w_i} \quad (6)$$

$$N_{eff} = \frac{1}{\sum_{i=1}^n \overline{w}_i^2} \quad (7)$$

$$\text{For } \overline{w}_i = \frac{1}{n} \quad N_{eff} = \frac{1}{n \frac{1}{n^2}} = n$$

$$\text{For } \overline{w}_i = \begin{cases} 1 & \text{for } i = 0 \\ 0 & \text{otherwise} \end{cases} \quad N_{eff} = \frac{1}{1} = 1$$