Homework 8
Part B and C

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1 Problem Statement

Consider the linear system $Ax = b$ where A and b are defined in Equation 1.

$$A = \begin{bmatrix} 1 & 1+\epsilon \\ 1-2\epsilon & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{1}$$

2 Condition Number

Solutions for Equation 1 were calculated for ϵ values from the set $\{2^k \epsilon_{Machine} : k = 0..52\}$. For IEEE Double Precision values, this is approximately from $\epsilon_{Machine}$ to 1. The condition number for the 2 norm of the matrix A is plotted for all tested ϵ values in Figure 1. As expected, the condition numbers calculated by Matlab for other norms were of approximately the same order of magnitude.

3 Solution Error

Although the matrix condition number behaved as expected, the relative error between the exact and Matlab calculated values of x behaved counter-intuitively. The relative error of x as a function of ϵ is plotted in Figure 2. As ϵ approaches approximately 10^{-8}, the relative error increases (which is unexpected because the condition number of A is decreasing on this interval. Further, past 10^{-8} the relative error drops to essentially 0.

This is partially explained by the absolute error of x, shown in Figure 3, and the 2-norm of x, shown in Figure 4. The calculated absolute error is essentially constant from $\epsilon = 10^{-16}$ to $\epsilon = 10^{-8}$, then drops to near 0. The 2-norm of x starts extremely large (which is expected given the near-signular nature of the matrix A) and drops to 0 near $\epsilon = 1$. This means that decreasing relative error of x is driven by the decreasing 2-norm of x on $\epsilon = 10^{-16}$ to $\epsilon = 10^{-8}$.

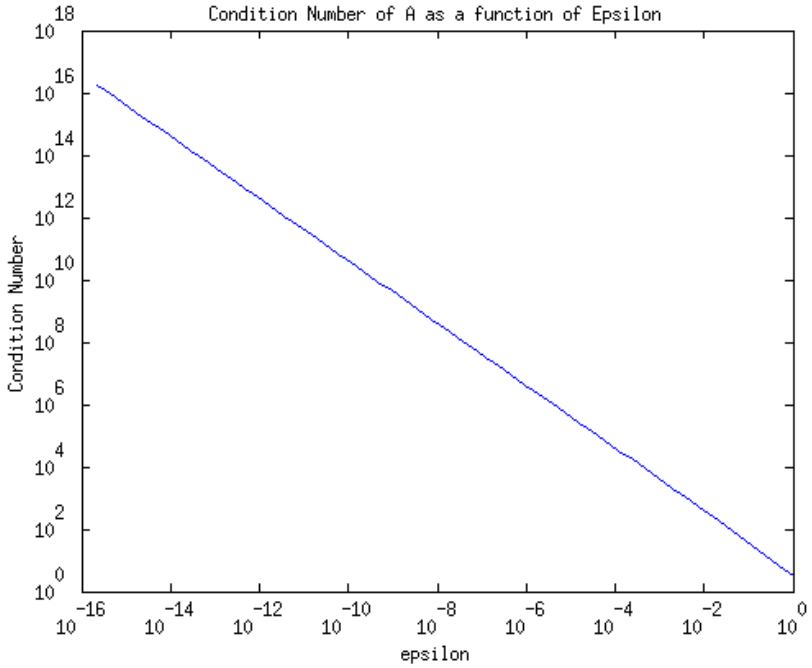

Figure 1: Condition Number of A as a function of Epsilon

It is, however, possible that this result is a result of catestophic cancellation or numeric error by the calculations used to compare they extremely large calculated and exact x values.

4 Other Norms

The only effect using norms besides the 2-norm had on the above calculations was to change the value of the condition number of A (but not the magnitude of the values). Changing norms also changed the height of the plateau observed in the absolute error in Figure 3, but all the plots displayed the same qualitative behavior.

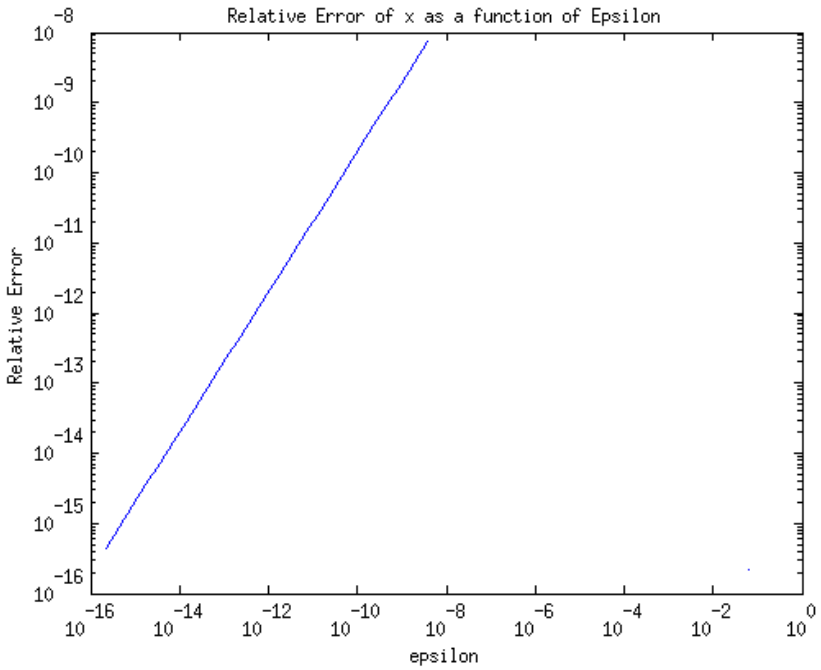

Figure 2: Relative Error of x as a function of Epsilon

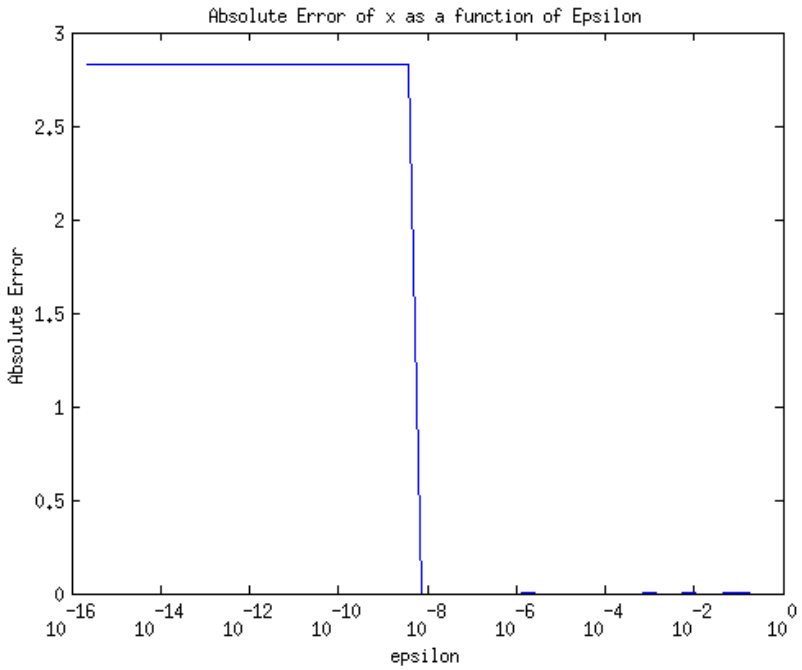

Figure 3: Absolute Error of x as a function of Epsilon

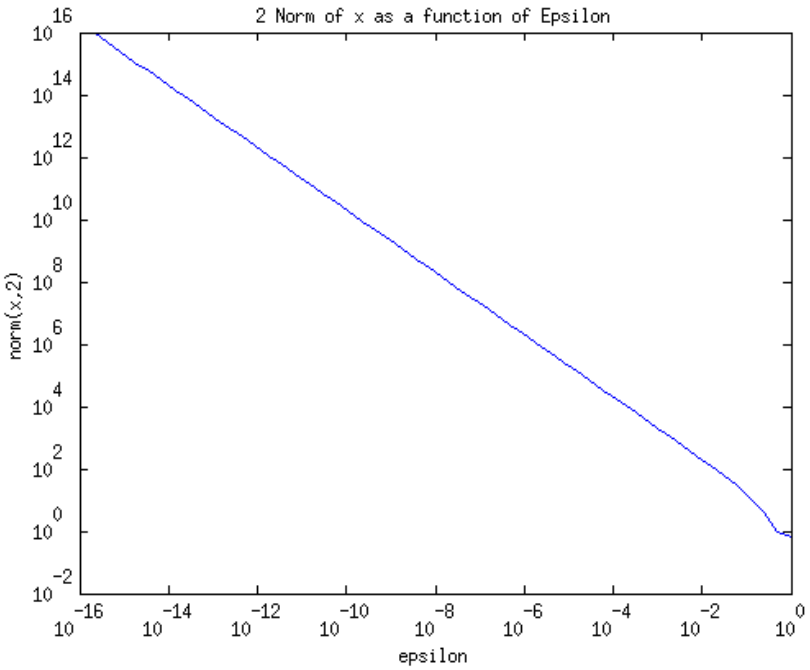

Figure 4: 2-Norm of x as a function of Epsilon