

# SVM Handwriting Classification

## Final Exam

Geoffrey Ulman  
CSI747

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### 1 Digit 3 Versus 6

The results obtained by the Nonlinear Rescaling-Augmented Lagrangian Matlab code were identical (to within the sigma specified) to the results obtained for the midterm exam using AMPL. This was the case for both the radial and polynomial kernels. Table 1 summarizes the misclassification error rate achieved.

Table 1: Misclassification Error: Digits 3 vs 6

Data Set	Error	95% Confidence Interval	
		Lower Bound	Upper Bound
Polynomial Training	0.000	0.000	0.000
Radial Training	0.000	0.000	0.000
Polynomial Testing	0.037	-0.004	0.077
Radial Testing	0.024	-0.010	0.058

## 1.1 Polynomial Kernel Output

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 7.017994e+03
* Newton Step: 2 Norm Gradient Phi: 2.110626e+03
* Newton Step: 3 Norm Gradient Phi: 1.583672e+02
* Newton Step: 4 Norm Gradient Phi: 2.820811e+00
* Newton Step: 5 Norm Gradient Phi: 1.207266e+00
* Newton Step: 6 Norm Gradient Phi: 4.798095e-01
* Newton Step: 7 Norm Gradient Phi: 1.613325e-01
Newton Steps: 7
Objective Value: -5.956487e+01
Equality Constraints Infeasibility: 1.026272e-04
Inequality Constraints Infeasibility: 1.461072e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 1.613325e-01
Max Stop Criteria: 1.837724e+00
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 3.095329e+01
* Newton Step: 2 Norm Gradient Phi: 9.970849e-01
* Newton Step: 3 Norm Gradient Phi: 2.440984e-01
* Newton Step: 4 Norm Gradient Phi: 3.200653e-02
* Newton Step: 5 Norm Gradient Phi: 2.926546e-03
Newton Steps: 5
Objective Value: -5.997201e+01
Equality Constraints Infeasibility: 2.554650e-05
Inequality Constraints Infeasibility: -2.915931e-04
Complementarity: 1.067875e-03
Norm of Gradient of Lagrangian: 2.926546e-03
Max Stop Criteria: 3.760220e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 5.925091e-01
* Newton Step: 2 Norm Gradient Phi: 8.074392e-02
* Newton Step: 3 Norm Gradient Phi: 1.736469e-02
* Newton Step: 4 Norm Gradient Phi: 1.350338e-03
* Newton Step: 5 Norm Gradient Phi: 1.591475e-05
Newton Steps: 5
Objective Value: -6.001292e+01
Equality Constraints Infeasibility: 3.850654e-07
Inequality Constraints Infeasibility: -9.212220e-06
Complementarity: 9.993395e-05
Norm of Gradient of Lagrangian: 1.591475e-05
Max Stop Criteria: 4.011331e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 4.298502e-03
* Newton Step: 2 Norm Gradient Phi: 3.332207e-04
* Newton Step: 3 Norm Gradient Phi: 3.305940e-06
Newton Steps: 3
Objective Value: -6.001624e+01
Equality Constraints Infeasibility: 5.967392e-07
Inequality Constraints Infeasibility: 1.017744e-05
Complementarity: 2.162314e-05
Norm of Gradient of Lagrangian: 3.305940e-06
Max Stop Criteria: 5.747959e-03
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 2.343069e-04
* Newton Step: 2 Norm Gradient Phi: 2.512844e-06
Newton Steps: 2
Objective Value: -6.001659e+01
Equality Constraints Infeasibility: 2.448429e-07
Inequality Constraints Infeasibility: 4.805526e-06
Complementarity: 5.361956e-06
Norm of Gradient of Lagrangian: 2.512844e-06
Max Stop Criteria: 1.166411e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 1.617247e-05
* Newton Step: 2 Norm Gradient Phi: 1.649713e-08
Newton Steps: 2
Objective Value: -6.001664e+01
Equality Constraints Infeasibility: 7.590450e-08
Inequality Constraints Infeasibility: 1.619316e-06
Complementarity: 1.403848e-06
Norm of Gradient of Lagrangian: 1.649713e-08
Max Stop Criteria: 2.743425e-04
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 1.157752e-06
* Newton Step: 2 Norm Gradient Phi: 1.990369e-10
Newton Steps: 2
Objective Value: -6.001665e+01
Equality Constraints Infeasibility: 2.186758e-08
Inequality Constraints Infeasibility: 2.134740e-06
Complementarity: 3.670600e-07
Norm of Gradient of Lagrangian: 1.990369e-10
Max Stop Criteria: 6.836852e-05
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 7.788941e-08
Newton Steps: 1
Objective Value: -6.001665e+01
Equality Constraints Infeasibility: 5.995201e-09
Inequality Constraints Infeasibility: 8.251807e-07
Complementarity: 9.578625e-08
Norm of Gradient of Lagrangian: 7.788941e-08
Max Stop Criteria: 1.732893e-05
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 4.772625e-09
Newton Steps: 1
Objective Value: -6.001665e+01
Equality Constraints Infeasibility: 1.594682e-09
Inequality Constraints Infeasibility: 2.539629e-07
Complementarity: 2.572455e-08
Norm of Gradient of Lagrangian: 4.772625e-09
Max Stop Criteria: 4.478936e-06
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 3.376285e-10
Newton Steps: 1
Objective Value: -6.001665e+01
Equality Constraints Infeasibility: 4.293496e-10
Inequality Constraints Infeasibility: 7.194552e-08
Complementarity: 6.787791e-09
Norm of Gradient of Lagrangian: 3.376285e-10
Max Stop Criteria: 1.167655e-06
-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 8.656509e-11
Newton Steps: 1
Objective Value: -6.001665e+01
Equality Constraints Infeasibility: 1.133764e-10
Inequality Constraints Infeasibility: 1.955965e-08
Complementarity: 1.782874e-09
Norm of Gradient of Lagrangian: 8.656509e-11
Max Stop Criteria: 3.032099e-07
-----
Iteration: 12
* Newton Step: 1 Norm Gradient Phi: 3.044651e-11
Newton Steps: 1
Objective Value: -6.001665e+01
Equality Constraints Infeasibility: 2.964669e-11
Inequality Constraints Infeasibility: 5.200108e-09
Complementarity: 4.673983e-10
Norm of Gradient of Lagrangian: 3.044651e-11
Max Stop Criteria: 7.863582e-08

```

## 1.2 Radial Kernel Output

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 6.931331e+03
* Newton Step: 2 Norm Gradient Phi: 2.484910e+03
* Newton Step: 3 Norm Gradient Phi: 1.432481e+03
* Newton Step: 4 Norm Gradient Phi: 4.175104e+02
* Newton Step: 5 Norm Gradient Phi: 1.604049e+02
* Newton Step: 6 Norm Gradient Phi: 1.159400e+02
* Newton Step: 7 Norm Gradient Phi: 1.808279e+00
* Newton Step: 8 Norm Gradient Phi: 7.984806e-01
* Newton Step: 9 Norm Gradient Phi: 3.144736e-01
* Newton Step: 10 Norm Gradient Phi: 9.388754e-02
Newton Steps: 10
Objective Value: -5.325819e+01
Equality Constraints Infeasibility: 8.085655e-05
Inequality Constraints Infeasibility: 5.797742e-04
Complementarity: 9.950263e-03
Norm of Gradient of Lagrangian: 9.388754e-02
Max Stop Criteria: 1.911769e+00
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 4.161087e+01
* Newton Step: 2 Norm Gradient Phi: 1.028650e+00
* Newton Step: 3 Norm Gradient Phi: 2.475090e-01
* Newton Step: 4 Norm Gradient Phi: 3.003328e-02
* Newton Step: 5 Norm Gradient Phi: 1.824418e-03
Newton Steps: 5
Objective Value: -5.379403e+01
Equality Constraints Infeasibility: 3.654177e-05
Inequality Constraints Infeasibility: 1.475506e-03
Complementarity: 9.879219e-04
Norm of Gradient of Lagrangian: 1.824418e-03
Max Stop Criteria: 3.905918e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 2.292880e-01
* Newton Step: 2 Norm Gradient Phi: 5.735189e-02
* Newton Step: 3 Norm Gradient Phi: 1.169970e-02
* Newton Step: 4 Norm Gradient Phi: 9.913998e-04
* Newton Step: 5 Norm Gradient Phi: 1.363580e-05
Newton Steps: 5
Objective Value: -5.383408e+01
Equality Constraints Infeasibility: 2.965729e-06
Inequality Constraints Infeasibility: 6.034579e-04
Complementarity: 1.478299e-04
Norm of Gradient of Lagrangian: 1.363580e-05
Max Stop Criteria: 5.126194e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 4.412096e-03
* Newton Step: 2 Norm Gradient Phi: 2.943626e-04
* Newton Step: 3 Norm Gradient Phi: 2.736765e-06
Newton Steps: 3
Objective Value: -5.383792e+01
Equality Constraints Infeasibility: 1.896586e-06
Inequality Constraints Infeasibility: 2.682708e-04
Complementarity: 3.806888e-05
Norm of Gradient of Lagrangian: 2.736765e-06
Max Stop Criteria: 9.680553e-03
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 4.640537e-04
* Newton Step: 2 Norm Gradient Phi: 6.560533e-06
Newton Steps: 2
Objective Value: -5.383848e+01
Equality Constraints Infeasibility: 7.127341e-07
Inequality Constraints Infeasibility: 7.397461e-05
Complementarity: 1.335408e-05
Norm of Gradient of Lagrangian: 6.560533e-06
Max Stop Criteria: 2.818255e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 7.022949e-05
* Newton Step: 2 Norm Gradient Phi: 1.932518e-07
Newton Steps: 2
Objective Value: -5.383860e+01
Equality Constraints Infeasibility: 2.779462e-07
Inequality Constraints Infeasibility: 2.328036e-05
Complementarity: 6.320571e-06
Norm of Gradient of Lagrangian: 1.932518e-07
Max Stop Criteria: 1.109351e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 1.243716e-05
* Newton Step: 2 Norm Gradient Phi: 7.041632e-09
Newton Steps: 2
Objective Value: -5.383864e+01
Equality Constraints Infeasibility: 1.233954e-07
Inequality Constraints Infeasibility: 9.859381e-06
Complementarity: 3.600585e-06
Norm of Gradient of Lagrangian: 7.041632e-09
Max Stop Criteria: 5.393950e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 2.843813e-06
Newton Steps: 1
Objective Value: -5.383865e+01
Equality Constraints Infeasibility: 6.282017e-08
Inequality Constraints Infeasibility: 5.381736e-06
Complementarity: 2.257747e-06
Norm of Gradient of Lagrangian: 2.843813e-06
Max Stop Criteria: 3.081614e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 9.428880e-07
Newton Steps: 1
Objective Value: -5.383866e+01
Equality Constraints Infeasibility: 3.547164e-08
Inequality Constraints Infeasibility: 3.339605e-06
Complementarity: 1.525655e-06
Norm of Gradient of Lagrangian: 9.428880e-07
Max Stop Criteria: 1.984124e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 4.186526e-07
Newton Steps: 1
Objective Value: -5.383866e+01
Equality Constraints Infeasibility: 2.303498e-08
Inequality Constraints Infeasibility: 2.297566e-06
Complementarity: 1.087752e-06
Norm of Gradient of Lagrangian: 4.186526e-07
Max Stop Criteria: 1.395656e-04
-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 2.045800e-07
Newton Steps: 1
Objective Value: -5.383866e+01
Equality Constraints Infeasibility: 1.612770e-08
Inequality Constraints Infeasibility: 1.674627e-06
Complementarity: 8.088111e-07
Norm of Gradient of Lagrangian: 2.045800e-07
Max Stop Criteria: 1.033786e-04
-----
Iteration: 12
* Newton Step: 1 Norm Gradient Phi: 1.060876e-07
Newton Steps: 1
Objective Value: -5.383866e+01
Equality Constraints Infeasibility: 1.196548e-08
Inequality Constraints Infeasibility: 1.271092e-06
Complementarity: 6.218835e-07
Norm of Gradient of Lagrangian: 1.060876e-07
Max Stop Criteria: 7.953896e-05

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Iteration: 13  
\* Newton Step: 1 Norm Gradient Phi: 5.734159e-08  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 9.255843e-09  
Inequality Constraints Infeasibility: 9.952126e-07  
Complementarity: 4.913232e-07  
Norm of Gradient of Lagrangian: 5.734159e-08  
Max Stop Criteria: 6.299114e-05

Iteration: 14  
\* Newton Step: 1 Norm Gradient Phi: 3.194075e-08  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 7.381562e-09  
Inequality Constraints Infeasibility: 7.987023e-07  
Complementarity: 3.969559e-07  
Norm of Gradient of Lagrangian: 3.194075e-08  
Max Stop Criteria: 5.104040e-05

Iteration: 15  
\* Newton Step: 1 Norm Gradient Phi: 1.822726e-08  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 6.024922e-09  
Inequality Constraints Infeasibility: 6.540616e-07  
Complementarity: 3.267641e-07  
Norm of Gradient of Lagrangian: 1.822726e-08  
Max Stop Criteria: 4.213668e-05

Iteration: 16  
\* Newton Step: 1 Norm Gradient Phi: 1.066175e-08  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 5.008411e-09  
Inequality Constraints Infeasibility: 5.447012e-07  
Complementarity: 2.732712e-07  
Norm of Gradient of Lagrangian: 1.066175e-08  
Max Stop Criteria: 3.533335e-05

Iteration: 17  
\* Newton Step: 1 Norm Gradient Phi: 6.539583e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 4.226518e-09  
Inequality Constraints Infeasibility: 4.601386e-07  
Complementarity: 2.316487e-07  
Norm of Gradient of Lagrangian: 6.539583e-09  
Max Stop Criteria: 3.002438e-05

Iteration: 18  
\* Newton Step: 1 Norm Gradient Phi: 4.412573e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 3.611950e-09  
Inequality Constraints Infeasibility: 3.934905e-07  
Complementarity: 1.986761e-07  
Norm of Gradient of Lagrangian: 4.412573e-09  
Max Stop Criteria: 2.580669e-05

Iteration: 19  
\* Newton Step: 1 Norm Gradient Phi: 3.456332e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 3.120528e-09  
Inequality Constraints Infeasibility: 3.400896e-07  
Complementarity: 1.721440e-07  
Norm of Gradient of Lagrangian: 3.456332e-09  
Max Stop Criteria: 2.240378e-05

Iteration: 20  
\* Newton Step: 1 Norm Gradient Phi: 3.067153e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 2.721469e-09  
Inequality Constraints Infeasibility: 2.966841e-07  
Complementarity: 1.504992e-07  
Norm of Gradient of Lagrangian: 3.067153e-09  
Max Stop Criteria: 1.962085e-05

Iteration: 21  
\* Newton Step: 1 Norm Gradient Phi: 2.883678e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 2.393165e-09  
Inequality Constraints Infeasibility: 2.609539e-07  
Complementarity: 1.326255e-07  
Norm of Gradient of Lagrangian: 2.883678e-09  
Max Stop Criteria: 1.731761e-05

Iteration: 22  
\* Newton Step: 1 Norm Gradient Phi: 2.764939e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 2.120128e-09  
Inequality Constraints Infeasibility: 2.312101e-07  
Complementarity: 1.177052e-07  
Norm of Gradient of Lagrangian: 2.764939e-09  
Max Stop Criteria: 1.539103e-05

Iteration: 23  
\* Newton Step: 1 Norm Gradient Phi: 2.645183e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 1.890505e-09  
Inequality Constraints Infeasibility: 2.062003e-07  
Complementarity: 1.051290e-07  
Norm of Gradient of Lagrangian: 2.645183e-09  
Max Stop Criteria: 1.376410e-05

Iteration: 24  
\* Newton Step: 1 Norm Gradient Phi: 2.523492e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 1.695872e-09  
Inequality Constraints Infeasibility: 1.849808e-07  
Complementarity: 9.443553e-08  
Norm of Gradient of Lagrangian: 2.523492e-09  
Max Stop Criteria: 1.237838e-05

Iteration: 25  
\* Newton Step: 1 Norm Gradient Phi: 2.396222e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 1.529369e-09  
Inequality Constraints Infeasibility: 1.668300e-07  
Complementarity: 8.527081e-08  
Norm of Gradient of Lagrangian: 2.396222e-09  
Max Stop Criteria: 1.118891e-05

Iteration: 26  
\* Newton Step: 1 Norm Gradient Phi: 2.263485e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 1.385853e-09  
Inequality Constraints Infeasibility: 1.511892e-07  
Complementarity: 7.735961e-08  
Norm of Gradient of Lagrangian: 2.263485e-09  
Max Stop Criteria: 1.016067e-05

Iteration: 27  
\* Newton Step: 1 Norm Gradient Phi: 2.133584e-09  
Newton Steps: 1  
Objective Value: -5.383866e+01  
Equality Constraints Infeasibility: 1.261428e-09  
Inequality Constraints Infeasibility: 1.376203e-07  
Complementarity: 7.048552e-08  
Norm of Gradient of Lagrangian: 2.133584e-09  
Max Stop Criteria: 9.266044e-06

Table 2: Misclassification Error: All Digits

Data Set	Error	95% Confidence Interval	
		Lower Bound	Upper Bound
Radial Training C=2	0.0526	0.038	0.067
Radial Training C=100	0.001	-0.001	0.003
Radial Testing C=2	0.222	0.182	0.262
Radial Testing C=100	0.197	0.159	0.236

## 2 All Digits

The radial kernel produced the best results for the 3 versus 6 case, so it was used for the full problem. However, while  $C = 2$  produced the best results for the 3 versus 6 case,  $C = 100$  was used for the full problem. The Nonlinear Rescaling-Augmented Lagrangian method results for  $C = 2$  match the AMPL results from the midterm exactly (to within the sigma specified). However, the Nonlinear Rescaling-Augmented Lagrangian code was also run with  $C = 100$ , which produced a slightly better error rate.

For the full run, epsilon was set to  $1e - 4$  because the calculation ran too slowly using  $1e - 5$ , which was used for the two digit case. This did not appear to affect the overall classifier performance (as compared to the AMPL results from the midterm).

As indicated in Table 2, the overall testing misclassification error achieved by the radial SVM classifier was 0.197 for  $C = 100$  and 0.222 for  $C = 2$ . The  $C = 2$  misclassification error matches the AMPL results from the midterm.

## 2.1 Radial Kernel Digit 0

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 1.851569e+04
* Newton Step: 2 Norm Gradient Phi: 6.630652e+03
* Newton Step: 3 Norm Gradient Phi: 2.648179e+03
* Newton Step: 4 Norm Gradient Phi: 1.015940e+03
* Newton Step: 5 Norm Gradient Phi: 1.039673e+02
* Newton Step: 6 Norm Gradient Phi: 2.419503e+00
* Newton Step: 7 Norm Gradient Phi: 9.412961e-01
* Newton Step: 8 Norm Gradient Phi: 3.392874e-01
Newton Steps: 8
Objective Value: -1.592619e+02
Equality Constraints Infeasibility: 7.534581e-03
Inequality Constraints Infeasibility: 4.155711e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 3.392874e-01
Max Stop Criteria: 2.541708e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 5.387824e+01
* Newton Step: 2 Norm Gradient Phi: 2.102655e+00
* Newton Step: 3 Norm Gradient Phi: 5.155774e-01
* Newton Step: 4 Norm Gradient Phi: 5.998674e-02
* Newton Step: 5 Norm Gradient Phi: 3.150267e-03
Newton Steps: 5
Objective Value: -1.607204e+02
Equality Constraints Infeasibility: 2.141693e-04
Inequality Constraints Infeasibility: -1.388436e-04
Complementarity: 2.393791e-03
Norm of Gradient of Lagrangian: 3.150267e-03
Max Stop Criteria: 1.126405e+00
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 1.214817e+00
* Newton Step: 2 Norm Gradient Phi: 1.103515e-01
* Newton Step: 3 Norm Gradient Phi: 2.526265e-02
* Newton Step: 4 Norm Gradient Phi: 3.087718e-03
Newton Steps: 4
Objective Value: -1.611146e+02
Equality Constraints Infeasibility: 3.238919e-05
Inequality Constraints Infeasibility: 1.187301e-04
Complementarity: 3.133910e-04
Norm of Gradient of Lagrangian: 3.087718e-03
Max Stop Criteria: 2.468780e-01
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 9.183538e-03
* Newton Step: 2 Norm Gradient Phi: 3.237340e-04
Newton Steps: 2
Objective Value: -1.611794e+02
Equality Constraints Infeasibility: 5.834581e-06
Inequality Constraints Infeasibility: 3.154672e-05
Complementarity: 6.370340e-05
Norm of Gradient of Lagrangian: 3.237340e-04
Max Stop Criteria: 4.451503e-02

Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 1.072228e-03
* Newton Step: 2 Norm Gradient Phi: 3.652208e-05
Newton Steps: 2
Objective Value: -1.611900e+02
Equality Constraints Infeasibility: 1.316042e-06
Inequality Constraints Infeasibility: 1.857148e-05
Complementarity: 1.394434e-05
Norm of Gradient of Lagrangian: 3.652208e-05
Max Stop Criteria: 8.983516e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 9.206424e-05
Newton Steps: 1
Objective Value: -1.611919e+02
Equality Constraints Infeasibility: 3.908648e-07
Inequality Constraints Infeasibility: 4.545218e-06
Complementarity: 5.099275e-06
Norm of Gradient of Lagrangian: 9.206424e-05
Max Stop Criteria: 1.944635e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 1.064307e-05
Newton Steps: 1
Objective Value: -1.611923e+02
Equality Constraints Infeasibility: 1.161803e-07
Inequality Constraints Infeasibility: 1.513563e-06
Complementarity: 2.540125e-06
Norm of Gradient of Lagrangian: 1.064307e-05
Max Stop Criteria: 5.095368e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 3.833684e-06
Newton Steps: 1
Objective Value: -1.611924e+02
Equality Constraints Infeasibility: 5.230957e-08
Inequality Constraints Infeasibility: 8.735660e-07
Complementarity: 1.196187e-06
Norm of Gradient of Lagrangian: 3.833684e-06
Max Stop Criteria: 2.158108e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 9.377549e-07
Newton Steps: 1
Objective Value: -1.611924e+02
Equality Constraints Infeasibility: 2.316347e-08
Inequality Constraints Infeasibility: 3.830583e-07
Complementarity: 5.718307e-07
Norm of Gradient of Lagrangian: 9.377549e-07
Max Stop Criteria: 9.939667e-05

```

## 2.2 Radial Kernel Digit 1

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 3.732292e+04
* Newton Step: 2 Norm Gradient Phi: 1.484191e+04
* Newton Step: 3 Norm Gradient Phi: 7.327838e+03
* Newton Step: 4 Norm Gradient Phi: 2.687334e+03
* Newton Step: 5 Norm Gradient Phi: 9.600119e+02
* Newton Step: 6 Norm Gradient Phi: 9.586526e+00
* Newton Step: 7 Norm Gradient Phi: 1.501459e+00
* Newton Step: 8 Norm Gradient Phi: 6.155011e-01
* Newton Step: 9 Norm Gradient Phi: 2.316943e-01
Newton Steps: 9
Objective Value: -2.491235e+02
Equality Constraints Infeasibility: 2.387041e-02
Inequality Constraints Infeasibility: 8.619943e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 2.316943e-01
Max Stop Criteria: 6.526028e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 5.641352e+01
* Newton Step: 2 Norm Gradient Phi: 2.333564e+00
* Newton Step: 3 Norm Gradient Phi: 3.267706e-01
* Newton Step: 4 Norm Gradient Phi: 2.463725e-02
* Newton Step: 5 Norm Gradient Phi: 1.405147e-03
Newton Steps: 5
Objective Value: -2.470307e+02
Equality Constraints Infeasibility: 1.159316e-04
Inequality Constraints Infeasibility: 3.183033e-04
Complementarity: 1.878835e-03
Norm of Gradient of Lagrangian: 1.405147e-03
Max Stop Criteria: 5.239052e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 8.803482e-01
* Newton Step: 2 Norm Gradient Phi: 5.920900e-02
* Newton Step: 3 Norm Gradient Phi: 1.226788e-02
* Newton Step: 4 Norm Gradient Phi: 8.593766e-04
* Newton Step: 5 Norm Gradient Phi: 9.019341e-06
Newton Steps: 5
Objective Value: -2.470693e+02
Equality Constraints Infeasibility: 4.133046e-05
Inequality Constraints Infeasibility: 5.956936e-05
Complementarity: 8.504435e-05
Norm of Gradient of Lagrangian: 9.019341e-06
Max Stop Criteria: 8.204599e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 2.060244e-03
* Newton Step: 2 Norm Gradient Phi: 1.627775e-04
Newton Steps: 2
Objective Value: -2.470848e+02
Equality Constraints Infeasibility: 8.287154e-06
Inequality Constraints Infeasibility: 4.748422e-05
Complementarity: 2.124098e-05
Norm of Gradient of Lagrangian: 1.627775e-04
Max Stop Criteria: 1.671530e-02
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 9.968571e-05
Newton Steps: 1
Objective Value: -2.470879e+02
Equality Constraints Infeasibility: 1.760761e-06
Inequality Constraints Infeasibility: 1.826237e-06
Complementarity: 6.345860e-06
Norm of Gradient of Lagrangian: 9.968571e-05
Max Stop Criteria: 3.838994e-03

Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 1.204973e-05
Newton Steps: 1
Objective Value: -2.470886e+02
Equality Constraints Infeasibility: 4.067702e-07
Inequality Constraints Infeasibility: 9.838724e-06
Complementarity: 2.493687e-06
Norm of Gradient of Lagrangian: 1.204973e-05
Max Stop Criteria: 1.063668e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 4.942629e-06
Newton Steps: 1
Objective Value: -2.470887e+02
Equality Constraints Infeasibility: 1.009044e-07
Inequality Constraints Infeasibility: 5.636803e-06
Complementarity: 1.432606e-06
Norm of Gradient of Lagrangian: 4.942629e-06
Max Stop Criteria: 4.600767e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 1.782670e-06
Newton Steps: 1
Objective Value: -2.470888e+02
Equality Constraints Infeasibility: 1.822858e-08
Inequality Constraints Infeasibility: 3.501944e-06
Complementarity: 9.011672e-07
Norm of Gradient of Lagrangian: 1.782670e-06
Max Stop Criteria: 2.689041e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 7.801439e-07
Newton Steps: 1
Objective Value: -2.470888e+02
Equality Constraints Infeasibility: 3.273892e-09
Inequality Constraints Infeasibility: 2.347670e-06
Complementarity: 6.039489e-07
Norm of Gradient of Lagrangian: 7.801439e-07
Max Stop Criteria: 1.832347e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 3.687721e-07
Newton Steps: 1
Objective Value: -2.470888e+02
Equality Constraints Infeasibility: 7.680799e-09
Inequality Constraints Infeasibility: 5.412760e-06
Complementarity: 4.241321e-07
Norm of Gradient of Lagrangian: 3.687721e-07
Max Stop Criteria: 1.315988e-04
-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 1.846258e-07
Newton Steps: 1
Objective Value: -2.470888e+02
Equality Constraints Infeasibility: 7.423726e-09
Inequality Constraints Infeasibility: 5.709365e-06
Complementarity: 3.084846e-07
Norm of Gradient of Lagrangian: 1.846258e-07
Max Stop Criteria: 9.712937e-05

```

## 2.3 Radial Kernel Digit 2

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 5.209840e+06
* Newton Step: 2 Norm Gradient Phi: 1.285635e+04
* Newton Step: 3 Norm Gradient Phi: 5.533639e+03
* Newton Step: 4 Norm Gradient Phi: 2.285356e+03
* Newton Step: 5 Norm Gradient Phi: 3.695606e+02
* Newton Step: 6 Norm Gradient Phi: 2.952644e+00
* Newton Step: 7 Norm Gradient Phi: 1.318872e+00
* Newton Step: 8 Norm Gradient Phi: 6.239989e-01
* Newton Step: 9 Norm Gradient Phi: 3.215332e-01
Newton Steps: 9
Objective Value: -6.347290e+02
Equality Constraints Infeasibility: 1.253713e-02
Inequality Constraints Infeasibility: 5.836365e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 3.215332e-01
Max Stop Criteria: 3.748640e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 6.885732e+01
* Newton Step: 2 Norm Gradient Phi: 2.035773e+00
* Newton Step: 3 Norm Gradient Phi: 4.174253e-01
* Newton Step: 4 Norm Gradient Phi: 4.110007e-02
* Newton Step: 5 Norm Gradient Phi: 1.663760e-03
Newton Steps: 5
Objective Value: -6.354974e+02
Equality Constraints Infeasibility: 3.402519e-04
Inequality Constraints Infeasibility: 4.872119e-03
Complementarity: 1.794207e-03
Norm of Gradient of Lagrangian: 1.663760e-03
Max Stop Criteria: 7.595423e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 1.002789e+00
* Newton Step: 2 Norm Gradient Phi: 6.237712e-02
* Newton Step: 3 Norm Gradient Phi: 1.257798e-02
* Newton Step: 4 Norm Gradient Phi: 1.027925e-03
* Newton Step: 5 Norm Gradient Phi: 1.021925e-05
Newton Steps: 5
Objective Value: -6.356773e+02
Equality Constraints Infeasibility: 3.900675e-05
Inequality Constraints Infeasibility: 1.268359e-04
Complementarity: 1.509164e-04
Norm of Gradient of Lagrangian: 1.021925e-05
Max Stop Criteria: 8.326291e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 4.259643e-03
* Newton Step: 2 Norm Gradient Phi: 1.940024e-04
* Newton Step: 3 Norm Gradient Phi: 2.796757e-06
Newton Steps: 3
Objective Value: -6.356966e+02
Equality Constraints Infeasibility: 4.948742e-06
Inequality Constraints Infeasibility: 5.642939e-05
Complementarity: 4.289966e-05
Norm of Gradient of Lagrangian: 2.796757e-06
Max Stop Criteria: 1.265257e-02

```

```

Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 1.432980e-04
* Newton Step: 2 Norm Gradient Phi: 8.789280e-07
Newton Steps: 2
Objective Value: -6.356986e+02
Equality Constraints Infeasibility: 9.058686e-07
Inequality Constraints Infeasibility: 2.027835e-05
Complementarity: 1.591626e-05
Norm of Gradient of Lagrangian: 8.789280e-07
Max Stop Criteria: 3.140923e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 8.727036e-06
Newton Steps: 1
Objective Value: -6.356988e+02
Equality Constraints Infeasibility: 2.576446e-07
Inequality Constraints Infeasibility: 9.686387e-06
Complementarity: 6.821732e-06
Norm of Gradient of Lagrangian: 8.727036e-06
Max Stop Criteria: 1.199273e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 1.280149e-06
Newton Steps: 1
Objective Value: -6.356989e+02
Equality Constraints Infeasibility: 9.926605e-08
Inequality Constraints Infeasibility: 5.081421e-06
Complementarity: 3.323358e-06
Norm of Gradient of Lagrangian: 1.280149e-06
Max Stop Criteria: 5.623771e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 2.968621e-07
Newton Steps: 1
Objective Value: -6.356989e+02
Equality Constraints Infeasibility: 4.513808e-08
Inequality Constraints Infeasibility: 2.625121e-06
Complementarity: 1.968064e-06
Norm of Gradient of Lagrangian: 2.968621e-07
Max Stop Criteria: 2.935791e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 6.806404e-08
Newton Steps: 1
Objective Value: -6.356989e+02
Equality Constraints Infeasibility: 2.144513e-08
Inequality Constraints Infeasibility: 1.385736e-06
Complementarity: 1.222695e-06
Norm of Gradient of Lagrangian: 6.806404e-08
Max Stop Criteria: 1.630774e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 1.680559e-08
Newton Steps: 1
Objective Value: -6.356989e+02
Equality Constraints Infeasibility: 1.034971e-08
Inequality Constraints Infeasibility: 7.513223e-07
Complementarity: 7.841448e-07
Norm of Gradient of Lagrangian: 1.680559e-08
Max Stop Criteria: 9.549244e-05

```



## 2.4 Radial Kernel Digit 3

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 4.297456e+04
* Newton Step: 2 Norm Gradient Phi: 1.289545e+04
* Newton Step: 3 Norm Gradient Phi: 5.231594e+03
* Newton Step: 4 Norm Gradient Phi: 1.582732e+03
* Newton Step: 5 Norm Gradient Phi: 2.264873e+01
* Newton Step: 6 Norm Gradient Phi: 2.484300e+00
* Newton Step: 7 Norm Gradient Phi: 1.120879e+00
* Newton Step: 8 Norm Gradient Phi: 4.745314e-01
* Newton Step: 9 Norm Gradient Phi: 2.013696e-01
Newton Steps: 9
Objective Value: -5.949923e+02
Equality Constraints Infeasibility: 1.590373e-02
Inequality Constraints Infeasibility: 7.059411e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 2.013696e-01
Max Stop Criteria: 4.698938e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 6.921989e+01
* Newton Step: 2 Norm Gradient Phi: 2.261957e+00
* Newton Step: 3 Norm Gradient Phi: 4.174322e-01
* Newton Step: 4 Norm Gradient Phi: 3.877653e-02
* Newton Step: 5 Norm Gradient Phi: 4.797372e-03
Newton Steps: 5
Objective Value: -5.951737e+02
Equality Constraints Infeasibility: 3.625927e-04
Inequality Constraints Infeasibility: 4.732402e-04
Complementarity: 7.832286e-04
Norm of Gradient of Lagrangian: 4.797372e-03
Max Stop Criteria: 7.769935e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 2.461743e-01
* Newton Step: 2 Norm Gradient Phi: 4.218244e-02
* Newton Step: 3 Norm Gradient Phi: 8.646029e-03
* Newton Step: 4 Norm Gradient Phi: 9.386652e-04
* Newton Step: 5 Norm Gradient Phi: 2.583588e-05
Newton Steps: 5
Objective Value: -5.953357e+02
Equality Constraints Infeasibility: 4.498559e-05
Inequality Constraints Infeasibility: 2.702953e-05
Complementarity: 1.224544e-04
Norm of Gradient of Lagrangian: 2.583588e-05
Max Stop Criteria: 1.086829e-01
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 5.908811e-03
* Newton Step: 2 Norm Gradient Phi: 7.880056e-04
* Newton Step: 3 Norm Gradient Phi: 4.758147e-05
Newton Steps: 3
Objective Value: -5.953485e+02
Equality Constraints Infeasibility: 6.059668e-06
Inequality Constraints Infeasibility: 2.419104e-05
Complementarity: 2.917893e-05
Norm of Gradient of Lagrangian: 4.758147e-05
Max Stop Criteria: 1.627899e-02

```

```

Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 3.644832e-04
* Newton Step: 2 Norm Gradient Phi: 3.414080e-06
Newton Steps: 2
Objective Value: -5.953499e+02
Equality Constraints Infeasibility: 1.137056e-06
Inequality Constraints Infeasibility: 8.407905e-06
Complementarity: 8.052752e-06
Norm of Gradient of Lagrangian: 3.414080e-06
Max Stop Criteria: 3.329665e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 3.274605e-05
Newton Steps: 1
Objective Value: -5.953501e+02
Equality Constraints Infeasibility: 2.895986e-07
Inequality Constraints Infeasibility: 4.651022e-06
Complementarity: 3.753307e-06
Norm of Gradient of Lagrangian: 3.274605e-05
Max Stop Criteria: 9.089859e-04
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 3.901473e-06
Newton Steps: 1
Objective Value: -5.953502e+02
Equality Constraints Infeasibility: 8.091747e-08
Inequality Constraints Infeasibility: 3.748121e-06
Complementarity: 2.197046e-06
Norm of Gradient of Lagrangian: 3.901473e-06
Max Stop Criteria: 3.240841e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 1.427375e-06
Newton Steps: 1
Objective Value: -5.953502e+02
Equality Constraints Infeasibility: 3.588929e-08
Inequality Constraints Infeasibility: 2.606425e-06
Complementarity: 1.407688e-06
Norm of Gradient of Lagrangian: 1.427375e-06
Max Stop Criteria: 1.809314e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 6.077789e-07
Newton Steps: 1
Objective Value: -5.953502e+02
Equality Constraints Infeasibility: 1.757209e-08
Inequality Constraints Infeasibility: 4.586650e-06
Complementarity: 9.657749e-07
Norm of Gradient of Lagrangian: 6.077789e-07
Max Stop Criteria: 1.175712e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 2.874463e-07
Newton Steps: 1
Objective Value: -5.953502e+02
Equality Constraints Infeasibility: 1.035641e-08
Inequality Constraints Infeasibility: 6.447303e-06
Complementarity: 6.972626e-07
Norm of Gradient of Lagrangian: 2.874463e-07
Max Stop Criteria: 8.430197e-05

```

## 2.5 Radial Kernel Digit 4

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 4.705215e+04
* Newton Step: 2 Norm Gradient Phi: 1.571905e+04
* Newton Step: 3 Norm Gradient Phi: 6.460616e+03
* Newton Step: 4 Norm Gradient Phi: 2.345630e+03
* Newton Step: 5 Norm Gradient Phi: 2.141179e+02
* Newton Step: 6 Norm Gradient Phi: 1.909383e+02
* Newton Step: 7 Norm Gradient Phi: 2.013132e+00
* Newton Step: 8 Norm Gradient Phi: 9.269356e-01
* Newton Step: 9 Norm Gradient Phi: 3.860099e-01
* Newton Step: 10 Norm Gradient Phi: 1.190911e-01
Newton Steps: 10
Objective Value: -6.520045e+02
Equality Constraints Infeasibility: 8.912807e-03
Inequality Constraints Infeasibility: 6.385065e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 1.190911e-01
Max Stop Criteria: 2.755928e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 6.857702e+01
* Newton Step: 2 Norm Gradient Phi: 2.000148e+00
* Newton Step: 3 Norm Gradient Phi: 3.734989e-01
* Newton Step: 4 Norm Gradient Phi: 3.556324e-02
* Newton Step: 5 Norm Gradient Phi: 2.583244e-03
Newton Steps: 5
Objective Value: -6.517194e+02
Equality Constraints Infeasibility: 2.334661e-04
Inequality Constraints Infeasibility: 1.581451e-03
Complementarity: 1.062306e-03
Norm of Gradient of Lagrangian: 2.583244e-03
Max Stop Criteria: 5.284254e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 5.284724e-01
* Newton Step: 2 Norm Gradient Phi: 4.465176e-02
* Newton Step: 3 Norm Gradient Phi: 8.988851e-03
* Newton Step: 4 Norm Gradient Phi: 6.881584e-04
Newton Steps: 4
Objective Value: -6.518977e+02
Equality Constraints Infeasibility: 3.418451e-05
Inequality Constraints Infeasibility: 8.995204e-05
Complementarity: 1.270415e-04
Norm of Gradient of Lagrangian: 6.881584e-04
Max Stop Criteria: 8.334295e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 3.823012e-03
* Newton Step: 2 Norm Gradient Phi: 2.446538e-04
* Newton Step: 3 Norm Gradient Phi: 2.029240e-06
Newton Steps: 3
Objective Value: -6.519206e+02
Equality Constraints Infeasibility: 4.558850e-06
Inequality Constraints Infeasibility: 2.712002e-05
Complementarity: 2.722803e-05
Norm of Gradient of Lagrangian: 2.029240e-06
Max Stop Criteria: 1.374056e-02
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 2.679267e-04
* Newton Step: 2 Norm Gradient Phi: 2.079151e-06
Newton Steps: 2
Objective Value: -6.519237e+02
Equality Constraints Infeasibility: 7.075670e-07
Inequality Constraints Infeasibility: 1.282519e-05
Complementarity: 1.017659e-05
Norm of Gradient of Lagrangian: 2.079151e-06
Max Stop Criteria: 3.373550e-03

```

```

Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 2.554797e-05
Newton Steps: 1
Objective Value: -6.519242e+02
Equality Constraints Infeasibility: 1.488310e-07
Inequality Constraints Infeasibility: 6.479363e-06
Complementarity: 4.650661e-06
Norm of Gradient of Lagrangian: 2.554797e-05
Max Stop Criteria: 1.276733e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 4.908463e-06
Newton Steps: 1
Objective Value: -6.519244e+02
Equality Constraints Infeasibility: 4.605376e-08
Inequality Constraints Infeasibility: 3.106685e-06
Complementarity: 2.397970e-06
Norm of Gradient of Lagrangian: 4.908463e-06
Max Stop Criteria: 5.919030e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 1.957067e-06
Newton Steps: 1
Objective Value: -6.519244e+02
Equality Constraints Infeasibility: 2.124574e-08
Inequality Constraints Infeasibility: 1.707053e-06
Complementarity: 1.325332e-06
Norm of Gradient of Lagrangian: 1.957067e-06
Max Stop Criteria: 3.290717e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 7.183747e-07
Newton Steps: 1
Objective Value: -6.519244e+02
Equality Constraints Infeasibility: 1.153805e-08
Inequality Constraints Infeasibility: 9.893613e-07
Complementarity: 7.659304e-07
Norm of Gradient of Lagrangian: 7.183747e-07
Max Stop Criteria: 1.908430e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 2.730915e-07
Newton Steps: 1
Objective Value: -6.519244e+02
Equality Constraints Infeasibility: 6.734104e-09
Inequality Constraints Infeasibility: 5.929204e-07
Complementarity: 4.575402e-07
Norm of Gradient of Lagrangian: 2.730915e-07
Max Stop Criteria: 1.142069e-04
-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 1.045520e-07
Newton Steps: 1
Objective Value: -6.519244e+02
Equality Constraints Infeasibility: 4.068361e-09
Inequality Constraints Infeasibility: 3.611900e-07
Complementarity: 2.773786e-07
Norm of Gradient of Lagrangian: 1.045520e-07
Max Stop Criteria: 6.951168e-05

```

## 2.6 Radial Kernel Digit 5

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 3.177861e+04
* Newton Step: 2 Norm Gradient Phi: 9.974503e+03
* Newton Step: 3 Norm Gradient Phi: 2.872137e+03
* Newton Step: 4 Norm Gradient Phi: 3.968737e+02
* Newton Step: 5 Norm Gradient Phi: 4.469698e+00
* Newton Step: 6 Norm Gradient Phi: 1.886190e+00
* Newton Step: 7 Norm Gradient Phi: 7.291223e-01
* Newton Step: 8 Norm Gradient Phi: 2.454204e-01
Newton Steps: 8
Objective Value: -3.914340e+02
Equality Constraints Infeasibility: 1.200865e-02
Inequality Constraints Infeasibility: 4.498182e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 2.454204e-01
Max Stop Criteria: 3.937285e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 8.820601e+01
* Newton Step: 2 Norm Gradient Phi: 2.356458e+00
* Newton Step: 3 Norm Gradient Phi: 5.390633e-01
* Newton Step: 4 Norm Gradient Phi: 6.434345e-02
* Newton Step: 5 Norm Gradient Phi: 1.069343e-02
Newton Steps: 5
Objective Value: -3.935885e+02
Equality Constraints Infeasibility: 2.945021e-04
Inequality Constraints Infeasibility: 2.206061e-03
Complementarity: 1.258278e-03
Norm of Gradient of Lagrangian: 1.069343e-02
Max Stop Criteria: 6.298825e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 1.312494e+00
* Newton Step: 2 Norm Gradient Phi: 1.119279e-01
* Newton Step: 3 Norm Gradient Phi: 2.243456e-02
* Newton Step: 4 Norm Gradient Phi: 1.565652e-03
* Newton Step: 5 Norm Gradient Phi: 1.353173e-05
Newton Steps: 5
Objective Value: -3.938212e+02
Equality Constraints Infeasibility: 2.991688e-05
Inequality Constraints Infeasibility: 1.042958e-04
Complementarity: 1.083993e-04
Norm of Gradient of Lagrangian: 1.353173e-05
Max Stop Criteria: 7.339688e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 9.900454e-03
* Newton Step: 2 Norm Gradient Phi: 1.627480e-03
* Newton Step: 3 Norm Gradient Phi: 9.738931e-05
Newton Steps: 3
Objective Value: -3.938403e+02
Equality Constraints Infeasibility: 3.323246e-06
Inequality Constraints Infeasibility: 7.780799e-05
Complementarity: 1.962797e-05
Norm of Gradient of Lagrangian: 9.738931e-05
Max Stop Criteria: 1.034544e-02

```

```

Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 4.059047e-04
* Newton Step: 2 Norm Gradient Phi: 6.810572e-06
Newton Steps: 2
Objective Value: -3.938422e+02
Equality Constraints Infeasibility: 3.699642e-07
Inequality Constraints Infeasibility: 2.562605e-05
Complementarity: 4.872429e-06
Norm of Gradient of Lagrangian: 6.810572e-06
Max Stop Criteria: 2.020850e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 3.196839e-05
Newton Steps: 1
Objective Value: -3.938425e+02
Equality Constraints Infeasibility: 1.529236e-08
Inequality Constraints Infeasibility: 7.933976e-06
Complementarity: 1.972671e-06
Norm of Gradient of Lagrangian: 3.196839e-05
Max Stop Criteria: 5.824617e-04
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 2.713099e-06
Newton Steps: 1
Objective Value: -3.938425e+02
Equality Constraints Infeasibility: 1.728958e-08
Inequality Constraints Infeasibility: 2.710300e-06
Complementarity: 1.122546e-06
Norm of Gradient of Lagrangian: 2.713099e-06
Max Stop Criteria: 2.422907e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 9.628024e-07
Newton Steps: 1
Objective Value: -3.938425e+02
Equality Constraints Infeasibility: 1.576384e-08
Inequality Constraints Infeasibility: 2.115381e-06
Complementarity: 7.008054e-07
Norm of Gradient of Lagrangian: 9.628024e-07
Max Stop Criteria: 1.433209e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 3.739023e-07
Newton Steps: 1
Objective Value: -3.938425e+02
Equality Constraints Infeasibility: 1.106462e-08
Inequality Constraints Infeasibility: 1.425552e-06
Complementarity: 4.698021e-07
Norm of Gradient of Lagrangian: 3.739023e-07
Max Stop Criteria: 9.228617e-05

```

## 2.7 Radial Kernel Digit 6

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 3.250235e+04
* Newton Step: 2 Norm Gradient Phi: 1.195457e+04
* Newton Step: 3 Norm Gradient Phi: 5.298462e+03
* Newton Step: 4 Norm Gradient Phi: 3.151725e+03
* Newton Step: 5 Norm Gradient Phi: 2.817518e+02
* Newton Step: 6 Norm Gradient Phi: 3.813862e+00
* Newton Step: 7 Norm Gradient Phi: 1.474092e+00
* Newton Step: 8 Norm Gradient Phi: 5.498910e-01
* Newton Step: 9 Norm Gradient Phi: 1.766787e-01
Newton Steps: 9
Objective Value: -3.663020e+02
Equality Constraints Infeasibility: 1.280051e-02
Inequality Constraints Infeasibility: 8.290002e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 1.766787e-01
Max Stop Criteria: 4.330545e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 5.724300e+01
* Newton Step: 2 Norm Gradient Phi: 2.333306e+00
* Newton Step: 3 Norm Gradient Phi: 3.921453e-01
* Newton Step: 4 Norm Gradient Phi: 4.265869e-02
* Newton Step: 5 Norm Gradient Phi: 2.301060e-03
Newton Steps: 5
Objective Value: -3.653497e+02
Equality Constraints Infeasibility: 5.318937e-04
Inequality Constraints Infeasibility: 3.518503e-06
Complementarity: 1.378997e-03
Norm of Gradient of Lagrangian: 2.301060e-03
Max Stop Criteria: 1.136557e+00
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 3.661755e-01
* Newton Step: 2 Norm Gradient Phi: 6.183224e-02
* Newton Step: 3 Norm Gradient Phi: 1.148524e-02
* Newton Step: 4 Norm Gradient Phi: 8.541579e-04
Newton Steps: 4
Objective Value: -3.655355e+02
Equality Constraints Infeasibility: 8.782728e-05
Inequality Constraints Infeasibility: 9.474220e-05
Complementarity: 2.180017e-04
Norm of Gradient of Lagrangian: 8.541579e-04
Max Stop Criteria: 1.842478e-01
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 5.790418e-03
* Newton Step: 2 Norm Gradient Phi: 3.629340e-04
* Newton Step: 3 Norm Gradient Phi: 5.745380e-06
Newton Steps: 3
Objective Value: -3.655676e+02
Equality Constraints Infeasibility: 1.655450e-05
Inequality Constraints Infeasibility: 4.863241e-05
Complementarity: 5.610863e-05
Norm of Gradient of Lagrangian: 5.745380e-06
Max Stop Criteria: 3.663367e-02

```

```

Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 5.312925e-04
* Newton Step: 2 Norm Gradient Phi: 9.711987e-06
Newton Steps: 2
Objective Value: -3.655729e+02
Equality Constraints Infeasibility: 4.082783e-06
Inequality Constraints Infeasibility: 1.744522e-05
Complementarity: 2.023606e-05
Norm of Gradient of Lagrangian: 9.711987e-06
Max Stop Criteria: 9.729003e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 6.098738e-05
Newton Steps: 1
Objective Value: -3.655740e+02
Equality Constraints Infeasibility: 1.323126e-06
Inequality Constraints Infeasibility: 1.745828e-05
Complementarity: 9.797352e-06
Norm of Gradient of Lagrangian: 6.098738e-05
Max Stop Criteria: 3.445685e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 1.096106e-05
Newton Steps: 1
Objective Value: -3.655742e+02
Equality Constraints Infeasibility: 5.157215e-07
Inequality Constraints Infeasibility: 8.806262e-06
Complementarity: 5.558462e-06
Norm of Gradient of Lagrangian: 1.096106e-05
Max Stop Criteria: 1.504169e-03
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 4.128358e-06
Newton Steps: 1
Objective Value: -3.655743e+02
Equality Constraints Infeasibility: 2.458999e-07
Inequality Constraints Infeasibility: 5.153806e-06
Complementarity: 3.470900e-06
Norm of Gradient of Lagrangian: 4.128358e-06
Max Stop Criteria: 8.078809e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 1.680598e-06
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 1.338055e-07
Inequality Constraints Infeasibility: 2.966042e-06
Complementarity: 2.331087e-06
Norm of Gradient of Lagrangian: 1.680598e-06
Max Stop Criteria: 4.955162e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 7.659872e-07
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 8.153759e-08
Inequality Constraints Infeasibility: 1.798389e-06
Complementarity: 1.654466e-06
Norm of Gradient of Lagrangian: 7.659872e-07
Max Stop Criteria: 3.346852e-04

```

```

-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 3.756609e-07
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 5.444058e-08
Inequality Constraints Infeasibility: 1.158868e-06
Complementarity: 1.225644e-06
Norm of Gradient of Lagrangian: 3.756609e-07
Max Stop Criteria: 2.417176e-04
-----
Iteration: 12
* Newton Step: 1 Norm Gradient Phi: 1.946534e-07
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 3.897795e-08
Inequality Constraints Infeasibility: 7.951791e-07
Complementarity: 9.393080e-07
Norm of Gradient of Lagrangian: 1.946534e-07
Max Stop Criteria: 1.829736e-04
-----
Iteration: 13
* Newton Step: 1 Norm Gradient Phi: 1.054299e-07
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 2.939826e-08
Inequality Constraints Infeasibility: 5.775864e-07
Complementarity: 7.398535e-07
Norm of Gradient of Lagrangian: 1.054299e-07
Max Stop Criteria: 1.433401e-04
-----
Iteration: 14
* Newton Step: 1 Norm Gradient Phi: 5.896601e-08
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 2.304192e-08
Inequality Constraints Infeasibility: 4.399576e-07
Complementarity: 5.959974e-07
Norm of Gradient of Lagrangian: 5.896601e-08
Max Stop Criteria: 1.152576e-04
-----
Iteration: 15
* Newton Step: 1 Norm Gradient Phi: 3.387059e-08
Newton Steps: 1
Objective Value: -3.655744e+02
Equality Constraints Infeasibility: 1.859029e-08
Inequality Constraints Infeasibility: 3.480063e-07
Complementarity: 4.891837e-07
Norm of Gradient of Lagrangian: 3.387059e-08
Max Stop Criteria: 9.460906e-05

```

## 2.8 Radial Kernel Digit 7

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 3.410043e+04
* Newton Step: 2 Norm Gradient Phi: 1.179148e+04
* Newton Step: 3 Norm Gradient Phi: 5.879060e+03
* Newton Step: 4 Norm Gradient Phi: 2.962630e+03
* Newton Step: 5 Norm Gradient Phi: 2.781926e+02
* Newton Step: 6 Norm Gradient Phi: 3.100677e+00
* Newton Step: 7 Norm Gradient Phi: 1.225527e+00
* Newton Step: 8 Norm Gradient Phi: 4.646988e-01
* Newton Step: 9 Norm Gradient Phi: 1.582370e-01
Newton Steps: 9
Objective Value: -2.941389e+02
Equality Constraints Infeasibility: 1.631274e-02
Inequality Constraints Infeasibility: 8.417762e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 1.582370e-01
Max Stop Criteria: 4.933057e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 6.845477e+01
* Newton Step: 2 Norm Gradient Phi: 3.799705e+00
* Newton Step: 3 Norm Gradient Phi: 4.232281e-01
* Newton Step: 4 Norm Gradient Phi: 3.848513e-02
* Newton Step: 5 Norm Gradient Phi: 3.195887e-03
Newton Steps: 5
Objective Value: -2.930688e+02
Equality Constraints Infeasibility: 3.488188e-04
Inequality Constraints Infeasibility: 6.594380e-05
Complementarity: 1.252308e-03
Norm of Gradient of Lagrangian: 3.195887e-03
Max Stop Criteria: 6.995190e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 2.929649e-01
* Newton Step: 2 Norm Gradient Phi: 5.793933e-02
* Newton Step: 3 Norm Gradient Phi: 1.216575e-02
* Newton Step: 4 Norm Gradient Phi: 9.562483e-04
Newton Steps: 4
Objective Value: -2.932350e+02
Equality Constraints Infeasibility: 5.534231e-05
Inequality Constraints Infeasibility: 6.768244e-05
Complementarity: 1.447408e-04
Norm of Gradient of Lagrangian: 9.562483e-04
Max Stop Criteria: 1.159512e-01
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 3.201625e-03
* Newton Step: 2 Norm Gradient Phi: 1.367691e-04
Newton Steps: 2
Objective Value: -2.932574e+02
Equality Constraints Infeasibility: 9.100699e-06
Inequality Constraints Infeasibility: 1.021039e-05
Complementarity: 3.276161e-05
Norm of Gradient of Lagrangian: 1.367691e-04
Max Stop Criteria: 2.095788e-02
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 1.927366e-04
* Newton Step: 2 Norm Gradient Phi: 1.235856e-06
Newton Steps: 2
Objective Value: -2.932603e+02
Equality Constraints Infeasibility: 1.871423e-06
Inequality Constraints Infeasibility: 4.903463e-06
Complementarity: 1.048991e-05
Norm of Gradient of Lagrangian: 1.235856e-06
Max Stop Criteria: 4.671207e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 2.369612e-05
Newton Steps: 1
Objective Value: -2.932608e+02
Equality Constraints Infeasibility: 4.825457e-07
Inequality Constraints Infeasibility: 5.306549e-06
Complementarity: 3.629645e-06
Norm of Gradient of Lagrangian: 2.369612e-05
Max Stop Criteria: 1.351897e-03
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 3.026343e-06
Newton Steps: 1
Objective Value: -2.932609e+02
Equality Constraints Infeasibility: 1.358493e-07
Inequality Constraints Infeasibility: 3.239988e-06
Complementarity: 1.685020e-06
Norm of Gradient of Lagrangian: 3.026343e-06
Max Stop Criteria: 4.965253e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 1.047845e-06
Newton Steps: 1
Objective Value: -2.932609e+02
Equality Constraints Infeasibility: 4.656808e-08
Inequality Constraints Infeasibility: 3.235840e-06
Complementarity: 1.068356e-06
Norm of Gradient of Lagrangian: 1.047845e-06
Max Stop Criteria: 2.580971e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 4.078128e-07
Newton Steps: 1
Objective Value: -2.932610e+02
Equality Constraints Infeasibility: 1.775677e-08
Inequality Constraints Infeasibility: 3.091435e-06
Complementarity: 7.281371e-07
Norm of Gradient of Lagrangian: 4.078128e-07
Max Stop Criteria: 1.586539e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 1.819545e-07
Newton Steps: 1
Objective Value: -2.932610e+02
Equality Constraints Infeasibility: 8.036061e-09
Inequality Constraints Infeasibility: 2.522831e-06
Complementarity: 5.237934e-07
Norm of Gradient of Lagrangian: 1.819545e-07
Max Stop Criteria: 1.067946e-04
-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 8.755241e-08
Newton Steps: 1
Objective Value: -2.932610e+02
Equality Constraints Infeasibility: 4.401271e-09
Inequality Constraints Infeasibility: 2.041674e-06
Complementarity: 3.927793e-07
Norm of Gradient of Lagrangian: 8.755241e-08
Max Stop Criteria: 7.589202e-05

```

## 2.9 Radial Kernel Digit 8

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 5.209830e+06
* Newton Step: 2 Norm Gradient Phi: 1.304886e+04
* Newton Step: 3 Norm Gradient Phi: 5.906741e+03
* Newton Step: 4 Norm Gradient Phi: 2.390849e+03
* Newton Step: 5 Norm Gradient Phi: 2.152047e+02
* Newton Step: 6 Norm Gradient Phi: 1.308916e+02
* Newton Step: 7 Norm Gradient Phi: 1.404523e+00
* Newton Step: 8 Norm Gradient Phi: 5.603643e-01
* Newton Step: 9 Norm Gradient Phi: 1.989465e-01
Newton Steps: 9
Objective Value: -7.222127e+02
Equality Constraints Infeasibility: 1.031602e-02
Inequality Constraints Infeasibility: 6.160873e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 1.989465e-01
Max Stop Criteria: 3.257213e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 7.075971e+01
* Newton Step: 2 Norm Gradient Phi: 2.215356e+00
* Newton Step: 3 Norm Gradient Phi: 4.522316e-01
* Newton Step: 4 Norm Gradient Phi: 3.840371e-02
* Newton Step: 5 Norm Gradient Phi: 1.578215e-03
Newton Steps: 5
Objective Value: -7.229749e+02
Equality Constraints Infeasibility: 3.669519e-04
Inequality Constraints Infeasibility: 9.554068e-05
Complementarity: 1.824849e-03
Norm of Gradient of Lagrangian: 1.578215e-03
Max Stop Criteria: 7.510938e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 8.721971e-01
* Newton Step: 2 Norm Gradient Phi: 8.300803e-02
* Newton Step: 3 Norm Gradient Phi: 1.755636e-02
* Newton Step: 4 Norm Gradient Phi: 1.319860e-03
* Newton Step: 5 Norm Gradient Phi: 1.123496e-05
Newton Steps: 5
Objective Value: -7.231686e+02
Equality Constraints Infeasibility: 4.015986e-05
Inequality Constraints Infeasibility: 3.714269e-04
Complementarity: 1.183469e-04
Norm of Gradient of Lagrangian: 1.123496e-05
Max Stop Criteria: 9.030533e-02
-----
Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 3.890715e-03
* Newton Step: 2 Norm Gradient Phi: 2.527109e-04
* Newton Step: 3 Norm Gradient Phi: 8.833841e-06
Newton Steps: 3
Objective Value: -7.231861e+02
Equality Constraints Infeasibility: 4.413471e-06
Inequality Constraints Infeasibility: 2.038065e-04
Complementarity: 2.160372e-05
Norm of Gradient of Lagrangian: 8.833841e-06
Max Stop Criteria: 1.113933e-02
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 2.542586e-04
* Newton Step: 2 Norm Gradient Phi: 2.811193e-06
Newton Steps: 2
Objective Value: -7.231880e+02
Equality Constraints Infeasibility: 4.471567e-07
Inequality Constraints Infeasibility: 9.088977e-05
Complementarity: 8.348665e-06
Norm of Gradient of Lagrangian: 2.811193e-06
Max Stop Criteria: 2.098549e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 4.028132e-05
Newton Steps: 1
Objective Value: -7.231882e+02
Equality Constraints Infeasibility: 1.035744e-09
Inequality Constraints Infeasibility: 3.550212e-05
Complementarity: 3.916026e-06
Norm of Gradient of Lagrangian: 4.028132e-05
Max Stop Criteria: 8.581893e-04
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 7.475947e-06
Newton Steps: 1
Objective Value: -7.231883e+02
Equality Constraints Infeasibility: 2.925192e-08
Inequality Constraints Infeasibility: 1.376400e-05
Complementarity: 2.196187e-06
Norm of Gradient of Lagrangian: 7.475947e-06
Max Stop Criteria: 4.330321e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 2.928006e-06
Newton Steps: 1
Objective Value: -7.231883e+02
Equality Constraints Infeasibility: 2.316298e-08
Inequality Constraints Infeasibility: 6.961229e-06
Complementarity: 1.312805e-06
Norm of Gradient of Lagrangian: 2.928006e-06
Max Stop Criteria: 2.612629e-04
-----
Iteration: 9
* Newton Step: 1 Norm Gradient Phi: 1.153650e-06
Newton Steps: 1
Objective Value: -7.231883e+02
Equality Constraints Infeasibility: 1.544166e-08
Inequality Constraints Infeasibility: 3.780625e-06
Complementarity: 8.259662e-07
Norm of Gradient of Lagrangian: 1.153650e-06
Max Stop Criteria: 1.617667e-04
-----
Iteration: 10
* Newton Step: 1 Norm Gradient Phi: 4.945636e-07
Newton Steps: 1
Objective Value: -7.231883e+02
Equality Constraints Infeasibility: 1.014217e-08
Inequality Constraints Infeasibility: 2.163397e-06
Complementarity: 5.375860e-07
Norm of Gradient of Lagrangian: 4.945636e-07
Max Stop Criteria: 1.041199e-04
-----
Iteration: 11
* Newton Step: 1 Norm Gradient Phi: 2.198802e-07
Newton Steps: 1
Objective Value: -7.231883e+02
Equality Constraints Infeasibility: 6.743528e-09
Inequality Constraints Infeasibility: 1.459445e-06
Complementarity: 3.581086e-07
Norm of Gradient of Lagrangian: 2.198802e-07
Max Stop Criteria: 6.883592e-05

```

## 2.10 Radial Kernel Digit 9

```

Iteration: 1
* Newton Step: 1 Norm Gradient Phi: 5.006381e+04
* Newton Step: 2 Norm Gradient Phi: 1.876691e+04
* Newton Step: 3 Norm Gradient Phi: 1.088150e+04
* Newton Step: 4 Norm Gradient Phi: 3.274279e+03
* Newton Step: 5 Norm Gradient Phi: 4.473892e+02
* Newton Step: 6 Norm Gradient Phi: 5.360784e+02
* Newton Step: 7 Norm Gradient Phi: 2.058079e+00
* Newton Step: 8 Norm Gradient Phi: 8.749511e-01
* Newton Step: 9 Norm Gradient Phi: 4.095136e-01
* Newton Step: 10 Norm Gradient Phi: 1.731676e-01
Newton Steps: 10
Objective Value: -9.237480e+02
Equality Constraints Infeasibility: 2.464143e-02
Inequality Constraints Infeasibility: 9.554396e-03
Complementarity: 9.999000e-03
Norm of Gradient of Lagrangian: 1.731676e-01
Max Stop Criteria: 7.051693e+01
-----
Iteration: 2
* Newton Step: 1 Norm Gradient Phi: 6.487336e+01
* Newton Step: 2 Norm Gradient Phi: 2.576436e+00
* Newton Step: 3 Norm Gradient Phi: 3.227255e-01
* Newton Step: 4 Norm Gradient Phi: 2.311970e-02
* Newton Step: 5 Norm Gradient Phi: 7.301489e-04
Newton Steps: 5
Objective Value: -9.213781e+02
Equality Constraints Infeasibility: 1.292835e-04
Inequality Constraints Infeasibility: 1.693638e-04
Complementarity: 7.561376e-04
Norm of Gradient of Lagrangian: 7.301489e-04
Max Stop Criteria: 4.626902e-01
-----
Iteration: 3
* Newton Step: 1 Norm Gradient Phi: 1.213999e-01
* Newton Step: 2 Norm Gradient Phi: 3.090568e-02
* Newton Step: 3 Norm Gradient Phi: 5.256942e-03
* Newton Step: 4 Norm Gradient Phi: 3.003113e-04
Newton Steps: 4
Objective Value: -9.214256e+02
Equality Constraints Infeasibility: 3.131447e-05
Inequality Constraints Infeasibility: 2.458840e-05
Complementarity: 6.416766e-05
Norm of Gradient of Lagrangian: 3.003113e-04
Max Stop Criteria: 7.329791e-02

```

```

Iteration: 4
* Newton Step: 1 Norm Gradient Phi: 6.024430e-04
* Newton Step: 2 Norm Gradient Phi: 2.009011e-05
Newton Steps: 2
Objective Value: -9.214343e+02
Equality Constraints Infeasibility: 3.994033e-06
Inequality Constraints Infeasibility: 4.854216e-06
Complementarity: 1.047910e-05
Norm of Gradient of Lagrangian: 2.009011e-05
Max Stop Criteria: 9.478215e-03
-----
Iteration: 5
* Newton Step: 1 Norm Gradient Phi: 7.022253e-05
Newton Steps: 1
Objective Value: -9.214353e+02
Equality Constraints Infeasibility: 5.072431e-07
Inequality Constraints Infeasibility: 1.298983e-06
Complementarity: 3.041750e-06
Norm of Gradient of Lagrangian: 7.022253e-05
Max Stop Criteria: 1.271033e-03
-----
Iteration: 6
* Newton Step: 1 Norm Gradient Phi: 6.526771e-06
Newton Steps: 1
Objective Value: -9.214355e+02
Equality Constraints Infeasibility: 6.735584e-08
Inequality Constraints Infeasibility: 2.997654e-07
Complementarity: 1.545691e-06
Norm of Gradient of Lagrangian: 6.526771e-06
Max Stop Criteria: 2.451241e-04
-----
Iteration: 7
* Newton Step: 1 Norm Gradient Phi: 3.388013e-06
Newton Steps: 1
Objective Value: -9.214355e+02
Equality Constraints Infeasibility: 1.160526e-08
Inequality Constraints Infeasibility: 2.123180e-07
Complementarity: 6.557002e-07
Norm of Gradient of Lagrangian: 3.388013e-06
Max Stop Criteria: 1.244068e-04
-----
Iteration: 8
* Newton Step: 1 Norm Gradient Phi: 6.559602e-07
Newton Steps: 1
Objective Value: -9.214355e+02
Equality Constraints Infeasibility: 2.799243e-09
Inequality Constraints Infeasibility: 1.316630e-07
Complementarity: 2.944887e-07
Norm of Gradient of Lagrangian: 6.559602e-07
Max Stop Criteria: 5.512888e-05

```

## 2.11 Radial Kernel Alphas Digit 9

Below, optimal alpha values are provided for one of the ten cases in the full problem (digit 9) with  $C = 2$ . Results for  $C = 2$  were chosen because they can be compared with the AMPL results in the midterm (which also provided optimal alpha values for digit 9,  $C = 2$ ). The results match to within the  $1e - 4$  prevision used by the NRAL code.



1	-1.4537e-08	101	1.0438e-08	201	9.2658e-09	301	3.2110e-08	401	2.7270e-09
2	1.9897e-08	102	-4.0290e-09	202	2.0524e-08	302	1.4024e-09	402	1.1490e+00
3	1.2926e-08	103	3.6985e-08	203	1.7176e-08	303	4.5505e-08	403	2.0000e+00
4	1.0958e-01	104	-4.7998e-07	204	3.4917e-08	304	1.0913e-08	404	4.7079e-01
5	-1.3697e-08	105	-6.6537e-09	205	1.5871e-08	305	9.0716e-09	405	4.7079e-01
6	2.5008e-01	106	-8.5294e-07	206	-4.9151e-09	306	-4.1537e-07	406	2.0000e+00
7	1.6348e-08	107	-6.8158e-09	207	1.2809e-07	307	3.8604e-08	407	2.0000e+00
8	1.4777e-09	108	-8.6290e-09	208	7.0755e-09	308	-7.3719e-09	408	4.6392e-07
9	-1.0397e-08	109	-6.0811e-09	209	2.5731e-08	309	1.1851e-08	409	2.9402e-01
10	-5.0000e-08	110	-6.6537e-09	210	1.0593e-08	310	1.4308e-08	410	1.7725e+00
11	-8.2480e-08	111	-1.3676e-08	211	6.4152e-08	311	2.4242e-08	411	2.0000e+00
12	9.1618e-09	112	-1.4777e-07	212	1.4721e-08	312	-3.4228e-08	412	-1.1616e-07
13	1.4533e-08	113	-1.2641e-07	213	7.7678e-08	313	5.8346e-01	413	2.6726e-09
14	6.1841e-09	114	-1.2641e-07	214	2.8009e-08	314	-5.5440e-09	414	2.0000e+00
15	4.7366e-08	115	-5.0443e-08	215	2.7270e-08	315	2.9188e-01	415	1.3485e+00
16	6.8323e-01	116	2.3669e-08	216	2.5698e-08	316	6.9442e-09	416	2.0000e+00
17	-2.5362e-08	117	-1.1066e-07	217	8.4243e-08	317	1.5572e+00	417	2.0000e+00
18	9.9011e-08	118	-1.7403e-08	218	-1.1871e-09	318	6.0714e-09	418	2.0000e+00
19	-3.4207e-08	119	-1.7403e-08	219	3.3551e-09	319	-7.2794e-09	419	2.0000e+00
20	2.3519e-08	120	-1.1505e-09	220	2.0360e-08	320	-7.3004e-09	420	8.3866e-01
21	5.0526e-08	121	-1.3676e-08	221	1.2888e-09	321	8.3031e-09	421	2.0000e+00
22	-1.1937e-08	122	-1.3676e-08	222	3.1066e-08	322	1.7658e-08	422	1.4466e-08
23	1.7064e-08	123	-1.1505e-09	223	3.2706e-08	323	1.2487e-08	423	3.2751e-08
24	4.5492e-09	124	-6.6537e-09	224	2.4315e-08	324	3.0624e-01	424	-1.4101e-08
25	3.8127e-01	125	-3.7797e-08	225	1.7061e-08	325	-1.2221e-08	425	8.0350e-01
26	1.0280e+00	126	-1.7308e-08	226	-1.8188e-11	326	1.2479e-06	426	2.0000e+00
27	-1.0365e-08	127	-1.7458e-07	227	9.3004e-09	327	8.2227e-09	427	7.2236e-01
28	2.5000e-08	128	-1.3489e-07	228	1.0647e-08	328	-3.4263e-09	428	1.4075e-09
29	2.9072e-08	129	2.2510e-08	229	2.9104e-09	329	7.1002e-09	429	4.9850e-09
30	2.9072e-08	130	-2.6066e-09	230	2.5591e-08	330	1.4958e-01	430	-1.4303e-07
31	-1.5435e-08	131	3.3118e-07	231	1.2415e-08	331	-9.9232e-08	431	-6.7821e-07
32	2.0460e-08	132	-1.5933e-08	232	1.3324e-08	332	1.8794e-06	432	1.1097e-07
33	4.4651e-08	133	-6.6537e-09	233	2.8310e-08	333	-1.7880e-09	433	2.5341e-08
34	2.1262e-11	134	-8.2719e-08	234	1.0700e-10	334	4.3017e-01	434	-9.0735e-09
35	1.0769e-08	135	-7.3560e-09	235	-2.6931e-08	335	-5.1801e-09	435	1.6608e-08
36	1.5094e-08	136	-1.8148e-09	236	2.8099e-08	336	5.9742e-09	436	7.2247e-09
37	8.0243e-09	137	1.8392e+00	237	-1.1849e-07	337	-6.0326e-09	437	1.4536e-08
38	1.4693e-08	138	-3.6997e-08	238	5.2822e-09	338	1.5392e+00	438	2.9475e-07
39	-1.8816e-08	139	-4.4575e-08	239	1.8679e-08	339	1.6407e-08	439	5.0409e-01
40	-1.6452e-08	140	-1.5614e-08	240	3.3678e-08	340	7.3612e-09	440	-1.4523e-08
41	-3.0812e-08	141	9.4619e-01	241	2.4066e-08	341	2.0000e+00	441	2.7909e-08
42	-4.1648e-08	142	-3.0208e-09	242	6.7097e-09	342	4.0293e-08	442	2.0000e+00
43	-1.1616e-08	143	-6.6537e-09	243	1.0692e-08	343	-6.1807e-10	443	5.3105e-08
44	-1.9012e-08	144	-2.6724e-08	244	2.1693e-08	344	3.2671e-09	444	2.0000e+00
45	-1.0823e-08	145	-5.6542e-08	245	4.3869e-02	345	4.9912e-09	445	2.0000e+00
46	-3.9909e-08	146	-5.6542e-08	246	-1.5865e-08	346	-2.3640e-08	446	2.0000e+00
47	-2.2879e-08	147	1.7705e-01	247	1.9067e-08	347	2.0000e+00	447	2.0000e+00
48	-1.0344e-08	148	9.7331e-09	248	8.0530e-08	348	-1.6783e-08	448	1.8435e-08
49	-1.4416e-08	149	5.1656e-01	249	8.5985e-09	349	1.5692e-08	449	-5.8675e-08
50	1.6888e-08	150	5.9630e-08	250	-2.7577e-08	350	-7.5273e-09	450	6.4350e-09
51	-3.8618e-09	151	1.4236e-07	251	-2.6963e-08	351	6.3897e-09	451	2.0774e-08
52	4.8273e-08	152	1.5734e-07	252	1.0124e-08	352	7.8601e-09	452	2.0000e+00
53	1.9124e+00	153	-1.7403e-08	253	9.0701e-09	353	-3.7991e-09	453	-3.8227e-08
54	-4.8402e-08	154	-2.8817e-08	254	2.2988e-08	354	3.1469e-08	454	2.6624e-08
55	-1.1302e-09	155	2.5730e-08	255	1.6583e-08	355	-1.7650e-08	455	3.0279e-08
56	-1.1661e-08	156	-2.1439e-08	256	3.0064e-09	356	-8.2904e-08	456	7.0255e-08
57	1.3145e-09	157	-1.1505e-09	257	-3.5012e-09	357	1.9907e-09	457	2.0000e+00
58	1.9954e-08	158	-1.1505e-09	258	8.5368e-09	358	-5.8304e-09	458	-1.8603e-08
59	-1.8770e-09	159	-9.0993e-08	259	1.5453e-08	359	-1.1567e-08	459	1.0890e-08
60	8.1970e-09	160	-8.8153e-08	260	-1.5177e-08	360	8.1643e-08	460	1.6457e+00
61	4.6336e-07	161	1.2839e+00	261	-3.7597e-09	361	1.0629e-01	461	2.0000e+00
62	-1.1028e-08	162	5.0578e-08	262	1.9472e-01	362	2.6973e-02	462	4.2341e-08
63	-1.7032e-08	163	2.0000e+00	263	-1.2559e-07	363	2.0000e+00	463	2.1624e-07
64	2.2741e-08	164	-2.1439e-08	264	1.7976e-08	364	1.6003e-08	464	3.8928e-08
65	2.3233e-08	165	3.4149e-08	265	8.9977e-09	365	1.0932e-07	465	4.2973e-09
66	1.2249e-09	166	3.6068e-09	266	-2.5699e-08	366	2.0000e+00	466	-6.6627e-08
67	1.7890e-08	167	-7.7678e-10	267	1.4513e-08	367	2.7346e-08	467	2.8477e-08
68	2.9277e-08	168	1.3272e-08	268	3.3434e-08	368	2.2819e-01	468	5.5718e-08
69	1.6212e-08	169	1.4835e+00	269	1.5349e-08	369	-2.1287e-09	469	2.9434e-08
70	-1.2587e-08	170	-2.7239e-08	270	2.0642e-08	370	3.8888e-09	470	6.3213e-08
71	-3.2567e-09	171	3.5861e-08	271	1.5378e-07	371	-7.4088e-09	471	-4.1329e-08
72	4.1232e-02	172	-2.8672e-08	272	1.8429e-08	372	-1.6927e-08	472	1.0077e-08
73	1.8409e-08	173	-6.6537e-09	273	6.1578e-09	373	1.3825e+00	473	-1.0488e-08
74	-7.0110e-09	174	-6.6537e-09	274	-3.9041e-08	374	1.7398e-07	474	-3.2552e-08
75	1.6337e-08	175	-2.1439e-08	275	-9.1389e-08	375	3.7036e-09	475	2.3952e-01
76	1.4252e-08	176	-1.7366e-08	276	2.0110e-09	376	8.3987e-09	476	7.9065e-09
77	-6.3864e-09	177	1.2923e-08	277	9.0739e-01	377	1.6497e-08	477	3.1596e-09
78	2.9822e-08	178	1.3700e-08	278	-3.2980e-09	378	2.0000e+00	478	3.2315e-01
79	1.0688e-08	179	-9.9879e-09	279	4.4978e-09	379	1.1370e-08	479	-6.9053e-09
80	-5.8399e-08	180	-2.0820e-08	280	1.9324e-01	380	1.5584e-01	480	-9.0668e-07
81	-4.9076e-08	181	-2.1439e-08	281	-1.3047e-09	381	-4.2430e-08	481	-9.5059e-09
82	-7.9131e-09	182	-1.3676e-08	282	2.6803e-08	382	7.6520e-09	482	-1.2359e-09
83	9.3370e-09	183	8.5468e-08	283	1.1540e+00	383	7.4573e-01	483	4.4584e-09
84	9.7819e-09	184	-1.3676e-08	284	8.9554e-09	384	4.8229e-09	484	-5.4035e-08
85	1.9221e-08	185	-6.6537e-09	285	-3.0884e-11	385	3.2705e-08	485	3.7950e-01
86	2.4396e-08	186	-4.7998e-07	286	-5.1291e-08	386	2.0000e+00	486	2.0684e-08
87	-1.5381e-08	187	8.9692e-09	287	1.3474e-08	387	1.7429e+00	487	-2.4548e-09
88	-1.0879e-08	188	3.4347e-09	288	-1.3547e-08	388	2.0000e+00	488	-1.8749e-08
89	-2.0736e-09	189	9.7305e-09	289	3.5100e-01	389	2.0000e+00	489	7.7232e-01
90	4.8506e-09	190	6.4259e-09	290	-1.5875e-08	390	2.0000e+00	490	-2.9347e-08
91	-4.5377e-11	191	3.1555e-08	291	-6.3616e-09	391	2.0000e+00	491	6.3213e-08
92	-3.0184e-08	192	3.1977e-09	292	7.8260e-09	392	1.7723e+00	492	-1.8749e-08
93	-4.6582e-08	193	2.3108e-08	293	2.5456e-08	393	2.0000e+00	493	1.1690e-08
94	-1.3676e-08	194	1.4357e-08	294	2.5732e-08	394	2.0000e+00	494	5.9392e-09
95	-3.2413e-07	195	1.0752e-08	295	-6.0104e-09	395	-1.5599e-08	495	7.3842e-09
96	1.2663e-07	196	-5.5654e-08	296	-1.1842e-08	396	-1.0644e-08	496	-1.6758e-08
97	2.0000e+00	197	9.4224e-09	297	-2.9908e-09	397	3.7226e-08	497	4.9524e-08
98	2.0000e+00	198	9.0359e-08	298	2.1752e-08	398	1.5854e-08	498	-1.8378e-08
99	2.7254e-08	199	4.1381e-08	299	-3.4003e-08	399	2.5854e-08	499	9.0953e-09
100	1.7857e-08	200	1.4864e-08	300	1.0710e-08	400	2.2177e-08	500	-2.5756e-08
501	7.5171e-09	601	-5.5818e-08	701	1.2646e-08	801	-6.7521e-09	901	2.0000e+00
502	-3.2976e-08	602	1.0172e-10	702	-1.8673e-08	802	1.0650e+00	902	2.0000e+00
503	8.5339e-09	603	3.3393e-09	703	-2.9196e-08	803	-5.0981e-08	903	2.0000e+00
504	-1.0650e-08	604	-8.4243e-09	704	8.2491e-01	804	-1.2359e-08	904	