Measuring Correlations Between Spontaneous Parametric Down Converted Photons

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I. INTRODUCTION

Quantum entanglement, or the strong correlation between various degrees of freedom of particles, is a cornerstone in the field of quantum information. The pursuit of strongly correlated, high-repetition, on-demand sources of single photon pairs which are compatible with current day quantum devices is the pursuit of many academic and industry forces alike. Herein this lab report, I will discuss the generation of a photon pair via a Spontaneous parametric down conversion process. Then, I will discuss how via their polarization states which we measure with a single photon detector are in violation of the Clauser-Horne-Shimony-Holt(CHSH) inequality, strongly suggesting that we have observed entangled particles. The equality is meant to place a limit on the possible correlations between two hidden variables, with the intention of proving Bell's Theorem.[1] We will discuss in more detail the theory in section II.

II. THEORY

This experiment is made possible via the creation of photon pairs via spontaneous Parametric Down Conversion(SPDC), a nonlinear process where a photon of higher energy, called the pump photon, is split into two lower energy photons, which we will refer to as the signal and the idler photons. SPDC has three types of phase matching conditions: type 0, type I and type II. In the case of the quED quTools device, which we used to implement this scheme, we have utilized type I phase matching.[2] Type 0 SPDC is characterized by production of photons which share the same polarization as the pump photon. Type I SPDC is referred to when the polarization of the signal and idler are parallel to each other, and orthogonal to the pump photon from which conversion took place. In type II SPDC, the signal and idler photons are orthogonal to each other(See Figure 1)[3]. As we can see, the orientation of the laser pump beam with respect to the optical axis of the crystal can change the size of the emission cones, where the angle is indicated by θ_p. Each cone corresponds to a different frequency of light, It should be noted here that because of the desired entangled state, we must have two crystals whose optical axes are perpendicular. This is required in order to produce the desired entangled state. The idea is to send in a photon with a polarization of 45°, such that the likelihood of two horizontal photons is equally expected as two vertical photons due to the equal superposition between H and V, given by the equation:

$$|\psi\rangle = \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle] \tag{1}$$

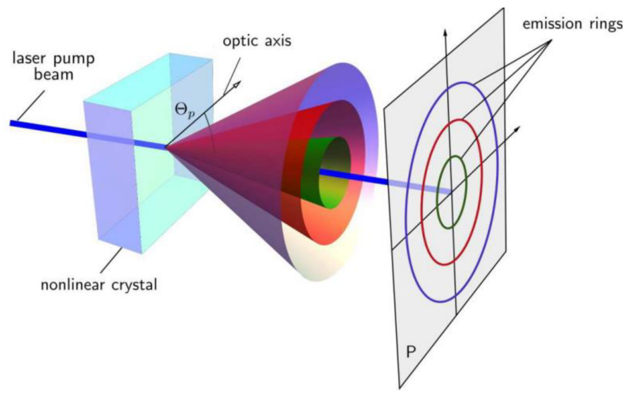

FIG. 1. Type I SPDC source diagram for a single crystal.Credits quTools.

By selecting the photons carefully given the geometry of the process, which we will discuss in more detail in Section III, we can select photons with a polarization state which cannot be factored to separate the wave function of the individual photon polarization states, realizing one of the entangled states described in equation 2 and 3:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}[|H\rangle_1 |H\rangle_2 + e^{i\phi} |V\rangle_1 |V\rangle_2] \tag{2}$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}[|H\rangle_1 |H\rangle_2 - e^{i\phi} |V\rangle_1 |V\rangle_2] \tag{3}$$

We have labeled the photons 1 and 2 in order to describe the spatial modes which is being described. Here the phase ϕ is controlled using the relative phase between the horizontal and vertical photons. If perhaps we have chosen the diagonal/antidiagonal basis, then we can describe the states produced as:

$$|\phi^+\rangle = \frac{1}{\sqrt{2}}[|P\rangle_1 |P\rangle_2 + e^{i\phi} |M\rangle_1 |M\rangle_2] \tag{4}$$

$$|\phi^-\rangle = \frac{1}{\sqrt{2}}[|P\rangle_1 |M\rangle_2 - e^{i\phi} |M\rangle_1 |P\rangle_2] \tag{5}$$

where P and M are:

$$|P\rangle = \frac{1}{\sqrt{2}}[|H\rangle + |V\rangle] \tag{6}$$

$$|M\rangle = \frac{1}{\sqrt{2}}[|H\rangle - |V\rangle] \tag{7}$$

In order to justify that the photons we described above are entangled, we will utilize an experimental implementation of the CHSH inequality, where a parameter S is used to describe the degree of correlation between the polarization states of the photon.

$$|S| < 2 \tag{8}$$

Here, S is:

$$S = E(a,b) - E(a,b') + E(a',b) + E(a',b') \tag{9}$$

where E is the normalized expectation value of correlations from the detectors. E is expressed as:

$$E(a,b) = \frac{C(a,b) - C(a,b_\perp) - C(a_\perp,b) + C(a_\perp,b_\perp)}{C(a,b) + C(a,b_\perp) + C(a_\perp,b) + C(a_\perp,b_\perp)} \tag{10}$$

C is the coincidence count rates given the polarizers a and b, which denote the two separate polarizer angles. $a_\perp$ and $b_\perp$ denote the detector parameters perpendicular to a and b, where $a_\perp$ is $\frac{\pi}{2}$ from a.[2]

III. EXPERIMENTAL METHODS

[t] This experiment has several key parameters and features which I will discuss in depth. Figures two and three

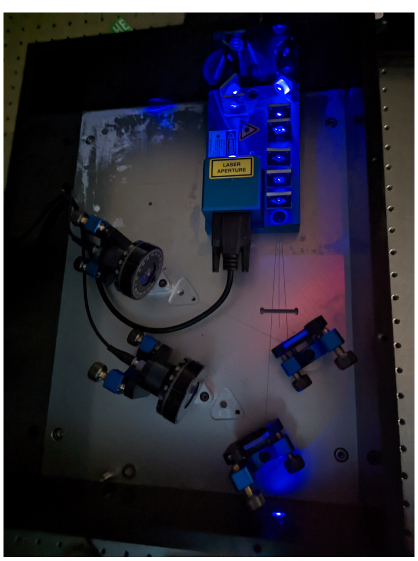

FIG. 2. Picture of uncovered quTools experiment, with an unshielded pump assembly.

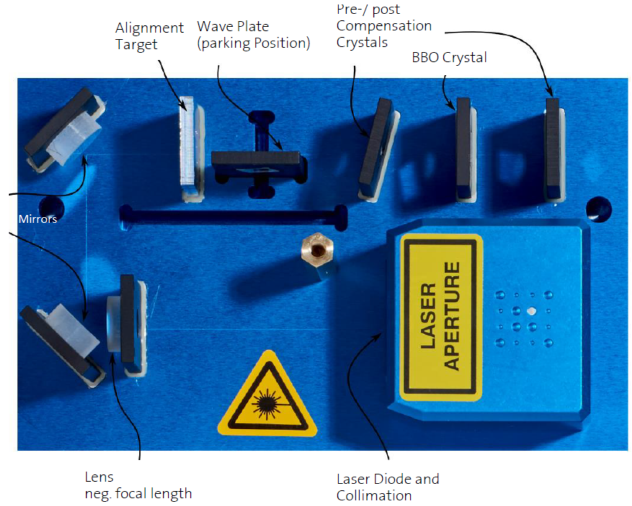

FIG. 3. Diagram of pump assembly and SPDC Source.

will be used as references while we discuss these experimental parameters. in particular, notice the lenses immediately after the polarizers. These lenses lead to single mode fibers, which channel photons to the avalanche photodiodes. These photodiodes are then connected to the quED electronic control unit, which then registers the coincidences of the photons via the avalanche photodetectors.

The pump laser of SPDC source should also be mentioned, and is described as a blue 405nm laser diode in the quED manual. The current of the laser is controllable via the quED touch screen panel. Here, by increasing the lasers current above the lasing threshold, you will observe the count rate of coincidence photons begin to increase as the SPDC source begins to produce signal and idler photons at 810nm. Figure 2 illustrates the red and blue photons being produced in the source, where the red photons are the down-converted photons.

The optical path of the source is quite simple, and figure 3 has been included to illustrate the elements. After the blue laser diode there are two lenses to focus the LED light: an aspheric lens and a lens with a negative focal length. An alignment target has been placed in front of two mirrors which redirect the light 180°, where then an alignment target is situated to assist in optimal conversion and coupling. Once choice alignment has been obtained, the half wave plate directly in front of the target can be inserted without effect on the alignment of the system to manipulate the polarization of the pump beam. This element is extremely important, as the angle of polarization along the Poincare sphere will determine the basis of polarization, as mentioned in the section II. The pump beam is now passes through the two BBO nonlinear crystals. Herein, the SPDC process takes place; for the crystal with optical axis parallel to the vertical plane, two horizontal photons can be created. In the other crystal which is parallel to the horizontal plane, two vertical photons are probabilistically created. This process of photons spontaneously being created orthogonal to the pump photons is due to the type I phase matching, and is the heart of the entanglement generation. It is important to note that the orientation of the polarization is key to creating the probabilistic interaction, where it is unknown which crystal a photon interacts with due to its equal superposition.[4]

The insertion of the two compensation crystals is another vital element in this setup. The inclusion of the two YVO crystals, labeled pre- and post- compensation crystals in figure 3, allows for for compensation of any temporal shifts or dispersion effects due to the generation of slightly different polarization. Temporal dispersion can be described as the separation in time of the two photons, Δt. This keeps the photons equal in time approximately, and allows them to arrive at the detectors in a small enough interval to consider them coincident.

Figure 4 describes the process of the difference in arrival times for the photons due to the geometry of the experiment. In general, there is a negative effect in terms of arrive time due to the two-crystal photon generation. Because the photons are non-degenerate in frequency, the photon pairs experience dispersion and crystal birefringence dependent on their frequency. Labeled in the figure 5, we can see the differences in arrival time for photons from crystal 1 and 2. τ_- corresponds to the difference in the arrival of the green photon and the red photon from the corresponding crystal. τ_+ corresponds to the median time of arrival for each photon. the superscripts 1 and 2 correspond to the crystal which produces the photons in question. Values closer to the origin of the x-axis arrive sooner. In general, $\tau_+^{(1)} < \tau_+^{(2)}$, which says that the mean photon arrival time for crystal 1 photons is sooner than the arrival time of crystal 2 photons, due to birefringence effects via the polarization and orientation of the optical axis.[5]

Due to dispersion, the photons from crystal 1 are more dispersed than the photons which only pass through crystal 2, $\tau_-^{(1)} < \tau_-^{(2)}$.

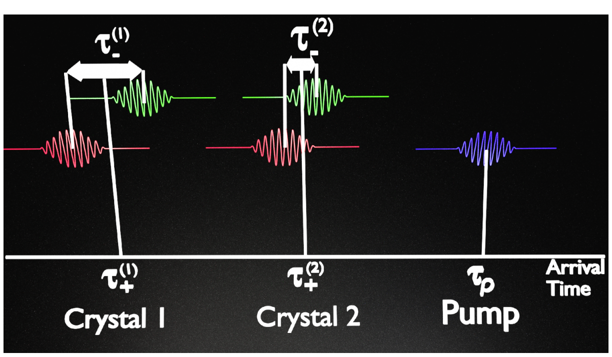

FIG. 4. Diagram detailing photon delay due to dispersion and birefrigence from crystal 1 and 2.

A. Measurement of Correlation Curves

In order to verify that entangled photons are being created, we will attempt to find a basis in which we can separate the wave function of photon 1 and photon 2. This is done via the polarizers which are seen in figure 2, right before the collimating lenses on the left. The procedure is as follows: fix polarizer a to some chosen basis, while incrementing polarizer b. Then, change your basis, and repeat the experiment. It is traditionally done in the basis of $a = 0°$ or $a = 45°$, corresponding to horizontal/vertical or antidiagonal/diagonal basis, respectively. For my data, I have done $a = 0°$, $a = 45°$,$a = 90°$, and $a = 135°$. We expect the correlation curves to be proportional to Malus' Law, where the intensity of our electromagnetic radiation is described by $sin^2(b-a)$, and thus we should also expect that each of the bases will correspond to a Bell state of its own:

$$|\Psi\rangle = A\,|\psi^+\rangle + B\,|\psi^-\rangle + C\,|\phi^-\rangle + D\,|\phi^+\rangle \qquad (11)$$

We refer to this as the **global fitting function**, which describes the state of any superposition of entangled photons. We will show in section IV that a function can be defined to fit each of these states with an analog to Malus' Law. The expectation of this is a high coefficient for one of the states, and proportionally low for all others. In particular, the expectation is that we will see either ϕ^+ or ϕ^-, since in my experimentation I did not modify the pre-compensation crystal to achieve both of them. To obtain ψ^+ and ψ^-, we can insert a half wave plate, just after the post-compensation crystal.

With the data we will take for each of the corresponding bases, it is then possible to extract several key parameters. Namely the visibility of the correlation curves, as well as the parameter S which describes the quantum correlation which we expect to violate the CHSH Inequality.

IV. RESULTS

By the methods described above, several plots have been produced, represented below. By setting the angle a, data was taken in increments of 10° for b, where each data point is over a minute. Using a fitting function:

$$\begin{aligned}\Psi \equiv N(ASin^2(P(a+b)) + BSin^2(P(a-b))\\ +CCos^2(P(a+b)) + DCos(^2P(a-b))),\end{aligned} \qquad (12)$$

we can define a method to fit each correlation curve. The results are illustrated in figure 5.

TABLE I. Table of fitting parameters for correlation curves.

angle(a)	A	B	C	D	P	N
$a = 0^\circ$	5.34e-03	5.34e-03	1.00e+00	-3.71e-04	3.41e+03	1.74e-02
$a = 45^\circ$	3.40e-02	5.59e-03	9.94e-01	1.50e-02	3.31e+03	1.69e-02
$a = 90^\circ$	1.33e-02	2.88e-04	9.99e-01	9.95e-04	3.56e+03	1.70e-02
$a = 135^\circ$	5.11e-02	9.52e-03	9.97e-01	5.73e-03	3.12e+03	1.69e-02

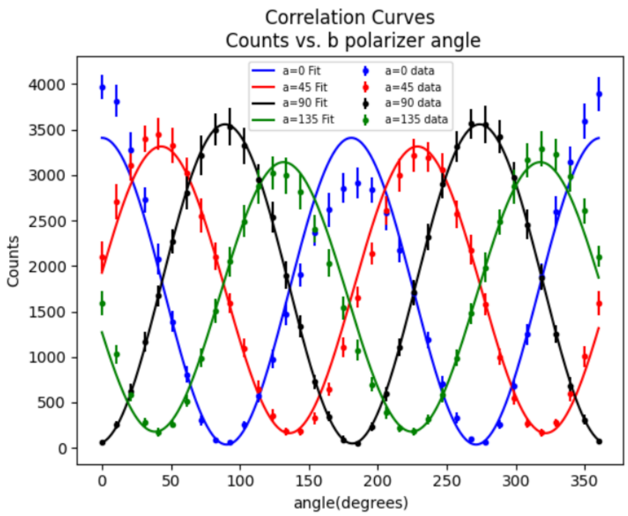

FIG. 5. Theoretical and experimental data for Correlation Curve.

As we can see from the fitting results in figure 5, we see an agreement of the fitting function with the raw data taken. For If we observe the a=0 term, we see that there is some discrepancy in the curve fitting compared to the experimental data. This is most likely due to the adjustment of the polarizer having some effect on the alignment of the light into the collimating lens which leads to the photodiode. We can predict that the entanglement source was most likely aligned with the polarizers aligned at the a,b=0 mark. From table 1, we can infer that have an entanglement state corresponding approximately to the state $|\phi^-\rangle$. This is expected, as our setup had no half waveplate installed during data acquisition, and the pre-compensation crystal was left unmodified.

Also of note is the case where the polarizers are varied such that a=b. In this case it is assumed that if the photons are in the state described by the parameter fitting, that there is an extremely low probability that at some angle of polarizers, no photons will pass through, since the state of the polarization is orthogonal to the polarizer.

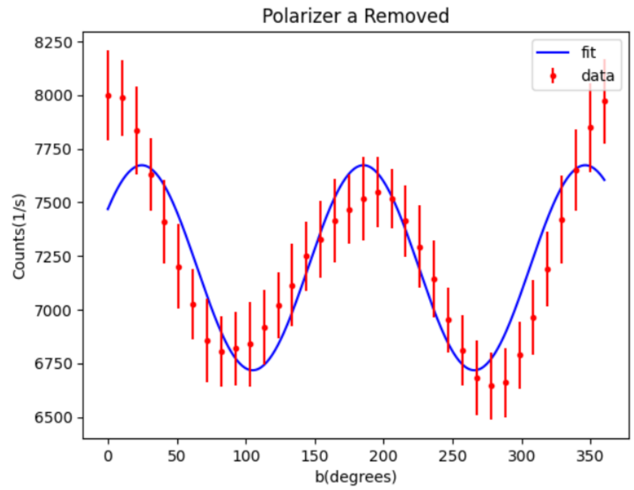

FIG. 7. Polarizer a removed.

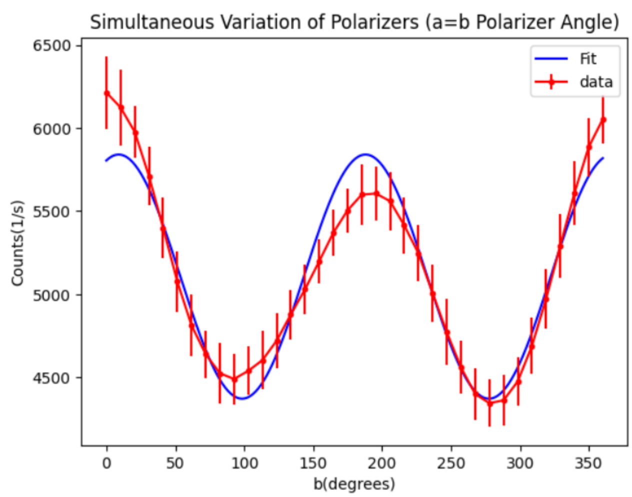

FIG. 6. Polarizers varied simultaneously.

Figure 6 illustrates that we do not have a perfect entangled state of $|\phi^-\rangle$, and perhaps have some superposition with the other states described in equation 11. In confirmation, This is precisely what table I describes with coefficients A,B, and D. These other states occur due to several factors. Some amount of birefringence of the crystal is inevitably left uncorrected by the pre- and post-compensation crystal. There is also the factor of the dispersion which each photon experiences and is unique to the frequency of the photon and the crystal material.

These factors, in combination with the inevitably imperfect alignment of the pump to the optical axes of our crystal, as well as an imperfect polarizer causing amplitude variation as it precesses, will cause this drop in the amplitude of the counts versus the angle of our polarizers.

A. Correlation Parameter

Finally, we should discuss the calculation of the parameter S, or the degree of correlation, using our experimental data. Using the best fit functions of the correlation plots, we are able to derive a value of S which works out to be

$$S = 2.638 \pm 0.018 \tag{13}$$

Well within agreement with the theoretical value of $2\sqrt{2}$, discussed below along with the error calculation. To calcuate S, we have taken advantage of equations 9 and 10. The significance of the parameter S is that it has violated locality. This parameter, if governed by two classical, local variables, will never exceed a value of 2, which is a limit called Tsirelson's Bound. The correlation of two classical particles will never exceed 2, according to the CHSH inequality. Quantum mechanics, however predicts that two particles may be correlated to a factor of 2 $\sqrt{2} \approx 2.828$. While we have shown something marginally off in terms of error, this parameter is still very much over the classical limit.

TABLE II. A table with coincidence counts with polarizer angles a and b. Counts are from fits to correlation curves.

a°	b°	Coincidence Counts
0	22.5	2918
0	67.5	542
0	112.5	510
0	157.5	2886
90	22.5	672
90	67.5	3099
90	112.5	3034
90	157.5	602
45	22.5	2953
45	67.5	2799
45	112.5	626
45	157.5	568
135	22.5	393
135	67.5	828
135	112.5	2841
135	157.5	2612

To estimate the error in our value of S, we utilize an equation determined by gaussian error correction listed here:

$$\Delta E(a,b) = 2\frac{(C(a,b)+C(a_\perp,b_\perp))(C(a,b_\perp)+C(a_\perp,b))}{(C(a,b)+C(a_\perp,b)+C(a,b_\perp)+C(a_\perp,b_\perp))^2} \times\sqrt{\frac{1}{C(a,b)+C(a_\perp,b_\perp)}+\frac{1}{C(a_\perp,b)+C(a,b_\perp)}}$$

V. CONCLUSIONS

While an excellent demonstration, we cannot trully say with certainty that these photons are entangled. Without prodding all possible loopholes, it is impossible to make the claim that quantum mechanics is complete. This is simply one of the many experiments devised to prod a particular loophole. We have investigated the coincidence loophole, though there exist several more which also perhaps have not been thoroughly explored. Even our methodology of researching the detection loophole is not without its flaws. We cannot assume that we have detected all photons, and therefore we cannot assume that the statistical analysis which we have performed can be perfectly accurate. This poses an issue, in particular with regard to the concept of hidden variable theories which operate on data rejection.

Other loopholes must be addressed before we can claim that quantum mechanics is complete. There are many others, including the locality, detection, and memory loophole for example.[6]

Assuming that no other loopholes are causing some degree of error, we can say with some degree that we have violated to some degree classical mechanics, and have seen the coincidence loophole of quantum mechanics, with a violation of magnitude $n_\Delta = \frac{S-2}{\Delta S} = 0.638$. This yields an high degree of violation.

ACKNOWLEDGMENTS

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