2)

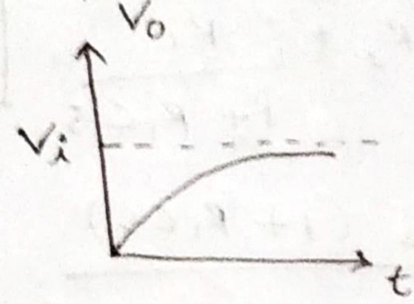
1st order system	2nd order system
→ If the input of the system changes, the output also changes gradually with some delay.	→ If the input of the system changes, the output also changes with a delay & oscillation.

Ans.

1st order system

→ If the input of the system changes, the output also changes gradually with some delay.

→

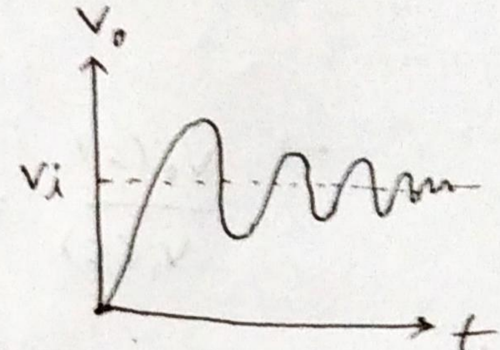


→ Ex: Air conditioner, heater

2nd order system

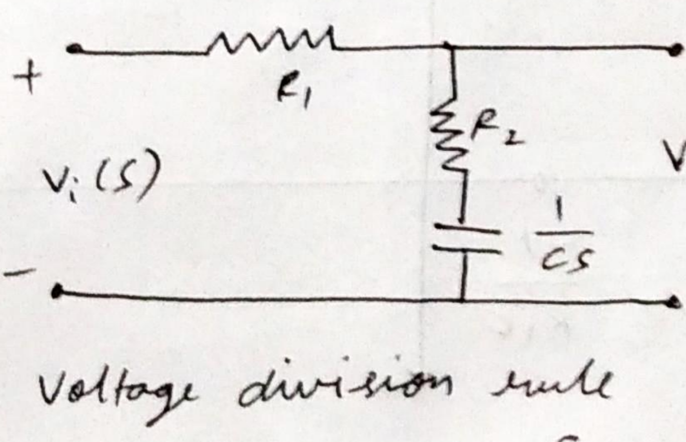
→ If the input of the system changes, the output also changes with a delay & oscillation.

→



Ex: Any analog instrument.

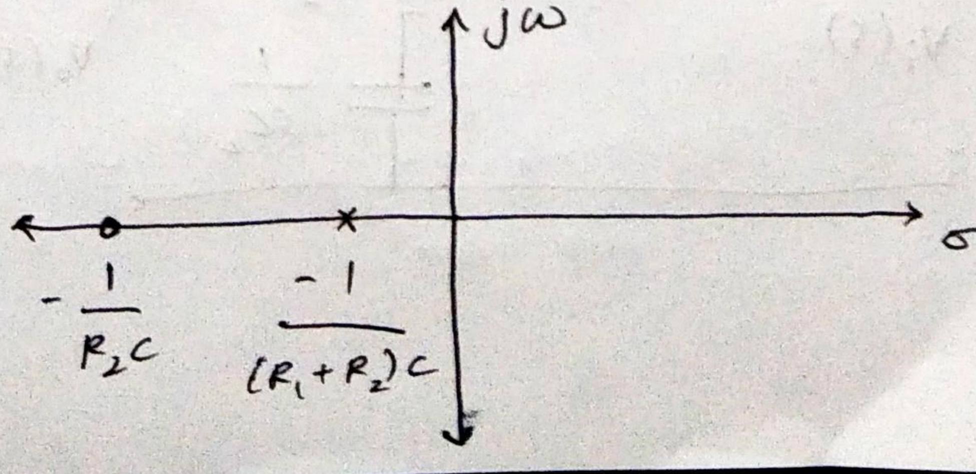
1) a) Lag compensator



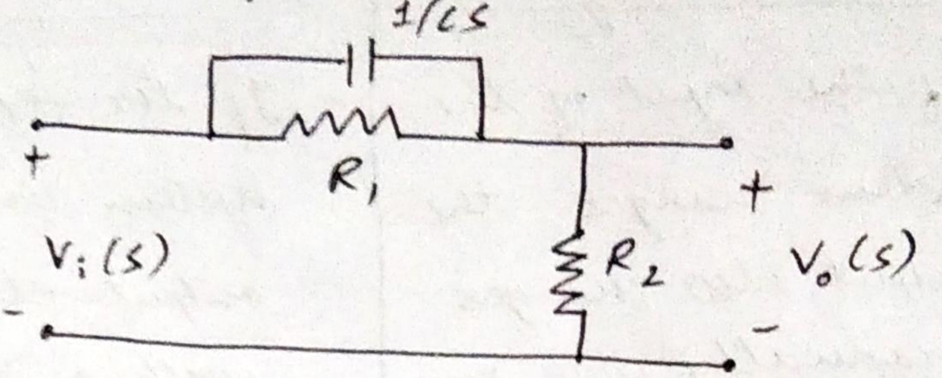
From voltage division rule

$$V_o(s) = V_i(s)\left[\frac{R_2 + \frac{1}{Cs}}{R_2 + \frac{1}{Cs} + R_1}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{1 + sR_2C}{1 + s(R_1+R_2)C}$$



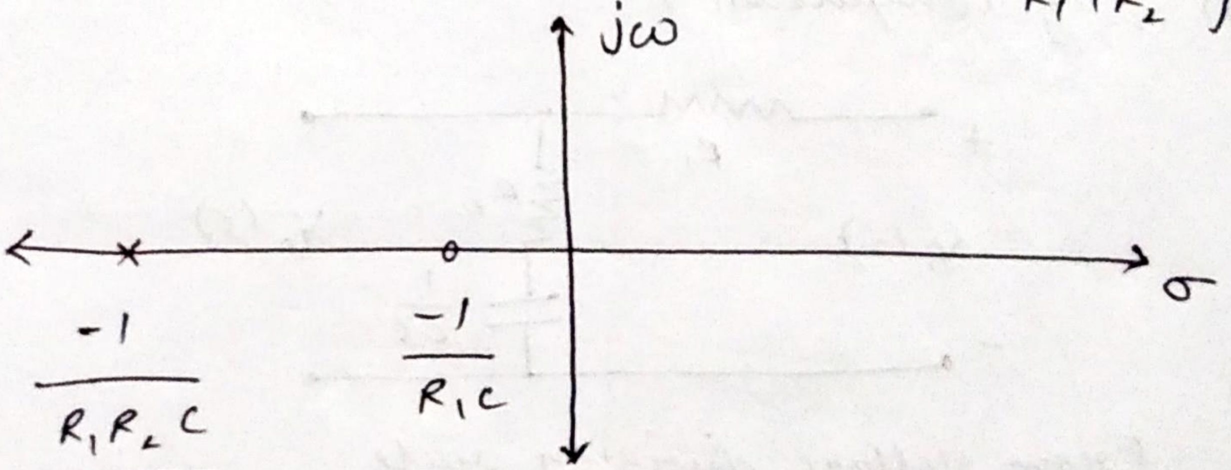
1) b) Lead compensator



$$V_o(s) = V_i(s)\left[\frac{R_2}{R_2 + \dfrac{R_1}{1+R_1CS}}\right]$$

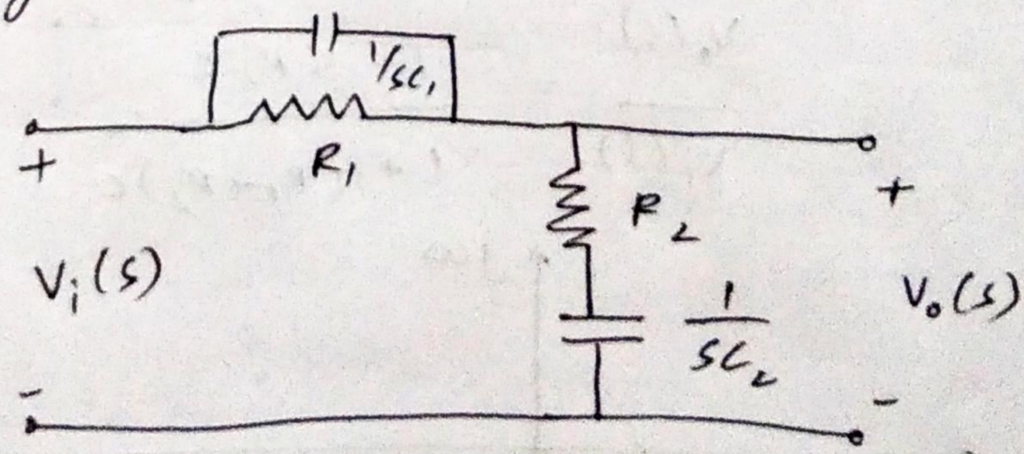
$$\Rightarrow \frac{V_o(s)}{V_i(s)} = \frac{R_2\,(1+R_1CS)}{R_1 + R_1R_2CS + R_2}$$

$$= \frac{R_2\,(1+sR_1C)}{R_1+R_2\left(1+s\dfrac{R_1R_2C}{R_1+R_2}\right)}$$



$$\frac{-1}{\dfrac{R_1R_2C}{R_1+R_2}} \qquad \frac{-1}{R_1C}$$

1) c) Lag-Lead compensator



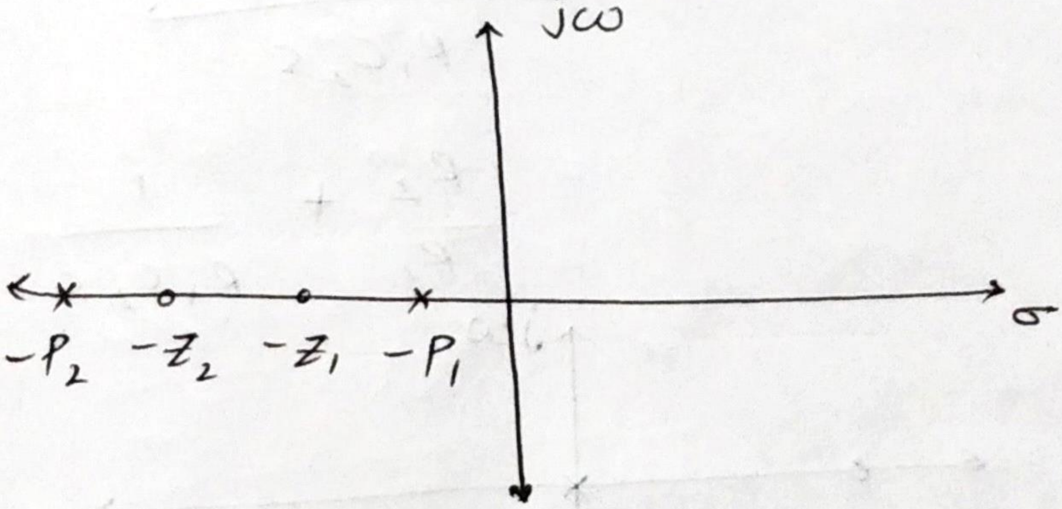
$$V_o(s) = V_i(s)\left[\frac{R_2 + \frac{1}{sC_2}}{R_2 + \frac{1}{C_2 s} + \frac{R_1}{1 + R_1 C_1 s}}\right]$$

$$\frac{V_o(s)}{V_i(s)} = \frac{(1 + C_2 R_2 s)(1 + R_1 C_1 s)}{(1 + R_2 C_2 s + R_2 C_2 R_1 C_1 s^2 + R_1 C_1 s + R_1 C_2 s)}$$

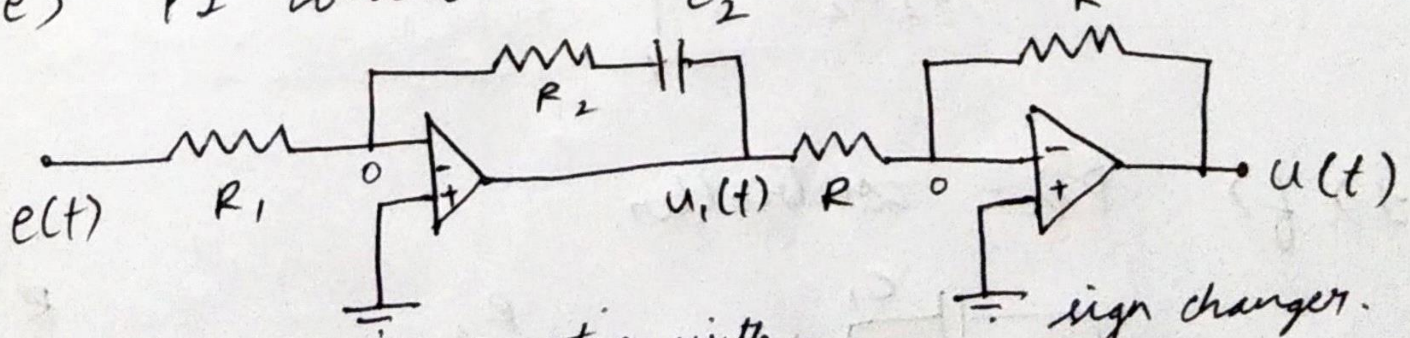
$$= \frac{(R_1 C_1 s + 1)(s R_2 C_2 + 1)}{R_2 C_2 R_1 C_1 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_2) s + 1}$$

Let P_1 & P_2 be poles &

Z_1 Z_2 be zeroes



1) e) PI controller



Sol.

$$\frac{0 - e(s)}{R_1} + \frac{0 - u_1(s)}{R_2 + \frac{1}{C_2 s}} = 0$$

$$\frac{e_1(s)}{R_1} = -u_1(s) \cdot \frac{1}{R_2 + \frac{1}{C_2 s}} \quad \text{—— (1)}$$

$$\frac{0 - u_1(s)}{R} + \frac{0 - u(s)}{R} = 0$$

$$\rightarrow u_1(s) = -u(s) \quad \text{——} \quad ②$$

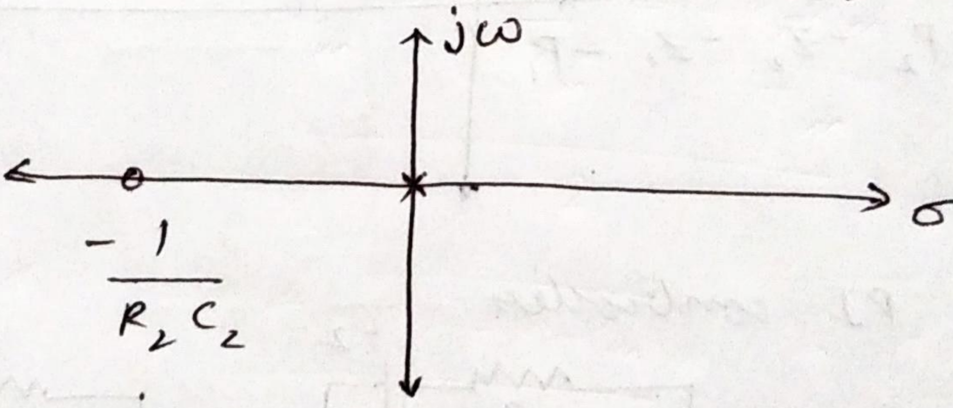
From ① & ②

$$\frac{e_1(s)}{R_1} = u(s) \cdot \frac{1}{R_2 + \frac{1}{C_2 s}}$$

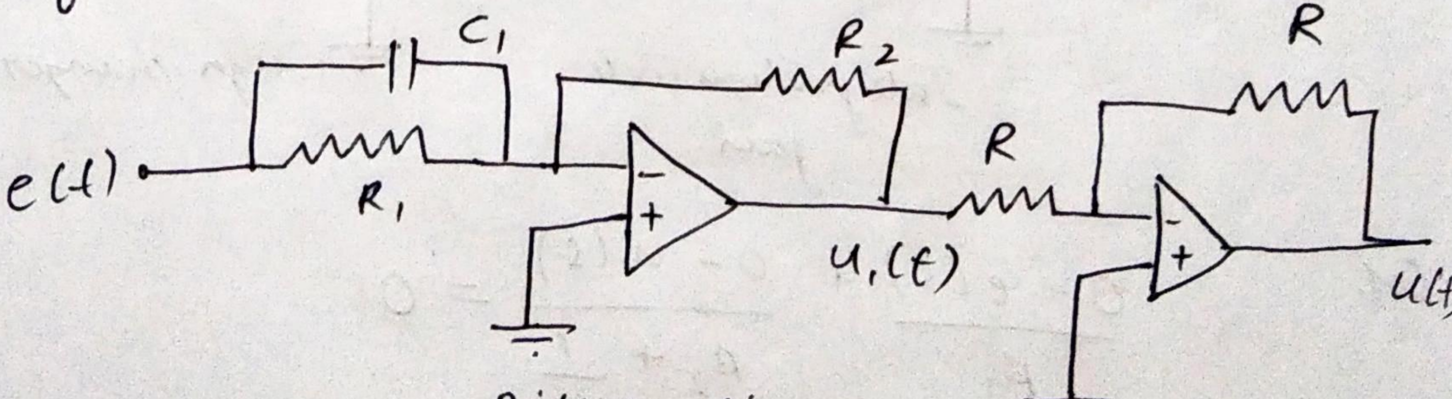
$$T(s) = \frac{u(s)}{e_1(s)} = \frac{R_2 + \frac{1}{C_2 s}}{R_1}$$

$$\rightarrow T(s) = \frac{R_2 C_2 s + 1}{R_1 C_2 s}$$

$$= \frac{R_2}{R_1} + \frac{1}{R_1 C_2 s}$$



1) f) PD - controller



Differentiator with gain

sign changer

Differentiator with gain

$$\frac{0 - e(s)}{\dfrac{R_1}{1 + R_1 C_1 s}} + \frac{0 - u_1(s)}{R_2} = 0$$

$$\Rightarrow e(s) = -u_1(s) \cdot \frac{R_1}{(1 + R_1 C_1 s) R_2} \quad \text{— ①}$$

Sign changer

$$0 - u_1(s) + 0 - u(s) = 0$$

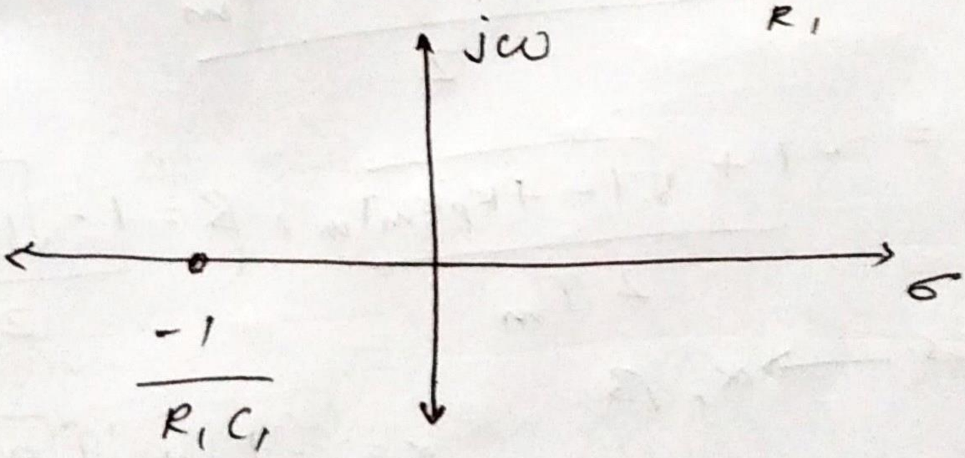
$$u_1(s) = -u(s) \quad \text{— ②}$$

From ① & ②

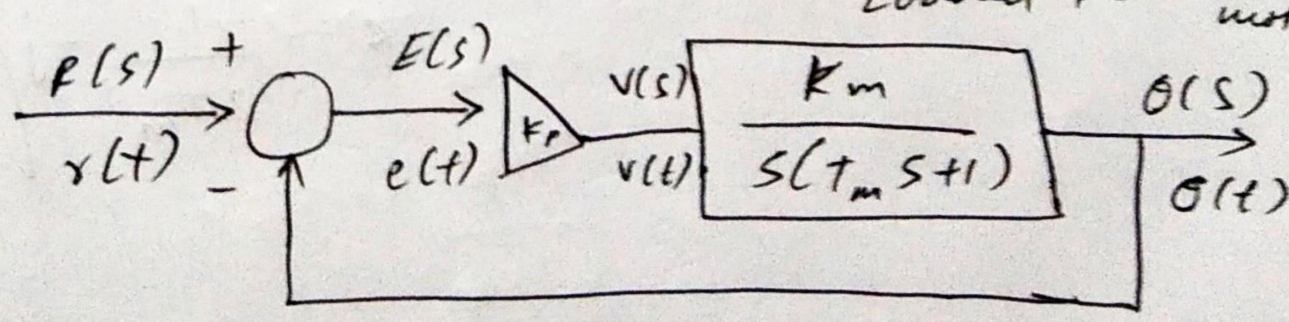
$$e(s) = u(s) \cdot \frac{R_1}{(1 + R_1 C_1 s) R_2}$$

$$\therefore T(s) = \frac{u(s)}{e(s)} = \frac{(1 + R_1 C_1 s) * R_2}{R_1}$$

$$= \frac{R_2}{R_1} + R_2 C_1 s$$



1) g) servo motor



Gain (open loop) $G = \frac{K_p K_m}{s(T_m s+1)}$; $H = 1$

Transfer fn : $\frac{\theta(s)}{R(s)} = \frac{G}{1+G}$

$$= \frac{K_p K_m}{s(T_m s+1) + K_p K_m}$$

$$= \frac{K_p K_m}{s^2 T_m + K_p K_m + s}$$

$$= \frac{K_p K_m}{T_m} \left[\frac{1}{s^2 + \frac{s}{T_m} + \frac{K_p K_m}{T_m}} \right]$$

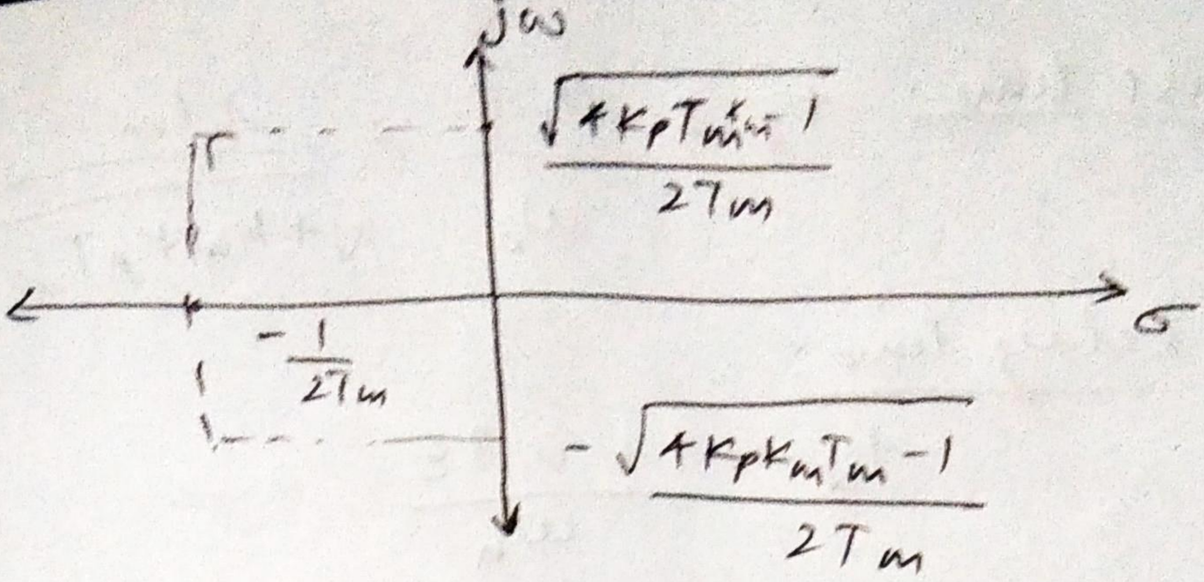
$$s = \frac{-\frac{1}{T_m} \pm \sqrt{\frac{1}{T_m^2} - \frac{4 K_p K_m}{T_m}}}{2}$$

Let $\alpha = \frac{-1 + \sqrt{1 - 4K_p K_m T_m}}{2T_m}$ & $\beta = \frac{1 - \sqrt{1 - 4K_p K_m T_m}}{2T_m}$

poles $\longrightarrow \alpha, \beta$

$$\alpha = \frac{-1}{2T_m} + j\left[\frac{\sqrt{4K_p K_m T_m - 1}}{2T_m}\right]$$

$$\beta = \frac{-1}{2T_m} - j\left[\frac{\sqrt{4K_p K_m T_m - 1}}{2T_m}\right]$$



1) h) Time domain specifications:

$$\frac{\theta(s)}{R(s)} = \frac{K_pK_m}{T_m}\left[\frac{1}{s^2+\frac{1}{T_m}\cdot s+\frac{K_pK_m}{T_m}}\right] \quad -①$$

For 2nd order system T.F is

$$\frac{\theta(s)}{R(s)} = \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2} \quad -②$$

comp. ① & ②

$$\omega_n^2 = \frac{K_pK_m}{T_m} \Rightarrow \omega_n = \sqrt{\frac{K_pK_m}{T_m}}$$

$$\xi\omega_n = \frac{1}{2T_m} \Rightarrow \xi = \frac{\frac{1}{T_m}}{2\sqrt{\frac{K_pK_m}{T_m}}} = \frac{1}{2\sqrt{K_pK_mT_m}}$$

$$\phi^\circ = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = \cos^{-1}(\xi)$$

$$\phi^c = \frac{\pi}{180}\cos^{-1}(\xi)$$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = \frac{1}{2T_m}\sqrt{4K_pK_mT_m-1}$$

i) rise time; $t_r = \frac{\pi - \phi}{\omega_d} = \frac{2T_m}{\sqrt{4K_mK_pT_m - 1}}(\pi - \phi^c)$

ii) Delay time;

$$t_d = \frac{1 + 0.7\varepsilon}{\omega_n}$$

$$= \frac{1 + 0.7\left(\frac{1}{2\sqrt{K_pK_mT_m}}\right)}{\sqrt{\frac{K_mK_p}{T_m}}}$$

$$= \sqrt{\frac{T_m}{K_mK_p}} + 0.35 = \frac{0.35 + \sqrt{K_mK_pT_m}}{K_mK_p} \text{ sec}$$

iii) Peak time;-

$$T_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\varepsilon^2}} = \frac{2\pi T_m}{\sqrt{4K_pK_mT_m - 1}} \text{ sec}$$

iv) Peak overshoot;-

$$MP = e^{-\frac{\varepsilon\pi}{\sqrt{1-\varepsilon^2}}} = e^{-\frac{1}{2\sqrt{K_pK_mT_m}} \cdot \frac{\pi}{\sqrt{1-\frac{1}{4K_pK_mT_m}}}}$$

$$= e^{-\frac{\pi}{\sqrt{4K_pK_mT_m - 1}}}$$

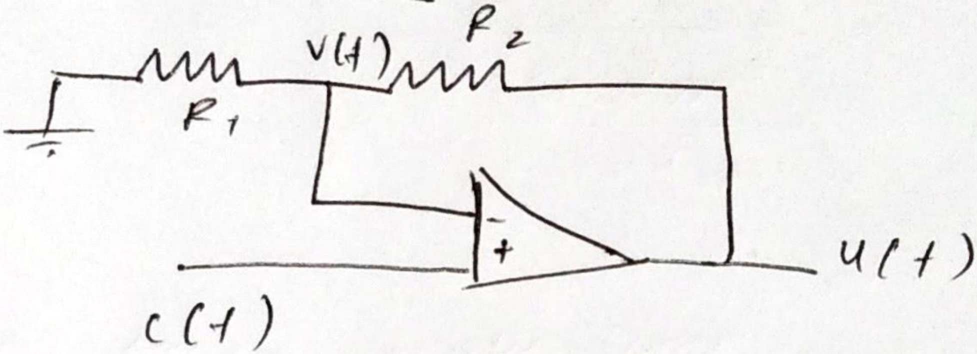
v) Settling time;-

$$\frac{4}{\varepsilon\omega_n} \rightarrow 2\%$$

$$\frac{3}{\varepsilon\omega_n} = 5\%$$

$$T_s = 8T_m \rightarrow 2\% \quad = 6T_m \rightarrow 5\%.$$

1) d) <u>P - controller:</u>



$$V(t) = c(t) \Rightarrow V(s) = C(s)$$

$$\Rightarrow \frac{c(s)}{R_1} + \frac{c(s) - u(s)}{R_2} = 0$$

$$\Rightarrow c(s)\left[\frac{R_1 + R_2}{R_1 \cancel{R_2}}\right] = \frac{u(s)}{\cancel{R_2}}$$

$$\Rightarrow \frac{u(s)}{\cancel{Ra}\, c(s)} = \frac{R_1 + R_2}{R_1} \text{ (constant).}$$