Abstract

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1 Introduction

Brief intro about the paper. Past work, theory, references.

2 Theoretical background

Explain the concepts of lattice gauge theory, Wilson and Polyakov loops, center symmetry and Sp(2) group. Mostly from Pepe's work.

Read the paper by Caselle to see if there is something to pure here (SY conjecture? String tension? Universality with spin model? EST?)

3 Simulation and algorithm

Introduce the algorithm used in the simulation. Creutz, Cabibbo Marinari. Explain in details what the code does (without explicitly pasting the code): for example, how to extract SU(2) matrices from Sp(2).

Define the partition function of the system to be

$$\mathcal{Z} = \int \mathcal{D}U e^{-\beta S(U)} \quad \text{where} \quad \int \mathcal{D}U \equiv \prod_{x,\mu} \int \mathrm{d}U_{x,\mu} \,. \tag{3.1}$$

$U_{x,\mu}$ is an element of the group (a link variable starting at the lattice site x, pointing along the direction μ), and S is the action, defined as

$$S = \sum_{\Box} S_{\Box}, \quad S_{\Box} \equiv 1 - \frac{1}{4}\,\mathrm{Tr}\Big(U_{x,\mu} U_{x+\hat{\mu},\nu} U^{\dagger}_{x+\hat{\nu},\mu} U^{\dagger}_{x,\nu}\Big). \tag{3.2}$$

The symbol $\Box$ represents the plaquette: the minimum loop possible on the lattice. Note that a link variable has a direction: the adjoint of a link variable is the link connecting the two sites in the opposite direction. In other words:

$$U_{x,\mu} = U^{\dagger}_{x+\hat{\mu},-\mu}. \tag{3.3}$$

The idea is to follow the *heat-bath* algorithm as designed by Cabibbo and Marinari [1], and generate a new link element with a Boltzmann probability distribution:

$$P(U) = \frac{1}{\mathcal{Z}} e^{-\beta S(U)}\, \mathrm{d}U\,. \tag{3.4}$$

To do this, we consider a set F of SU(2) subgroups of the gauge group Sp(2). Given a Sp(2) element U of the form

$$U_{\mathrm{Sp}(2)} = \begin{pmatrix} W_{11} & W_{12} & X_{11} & X_{12} \\ W_{21} & W_{22} & X_{21} & X_{22} \\ X_{22}^* & -X_{21}^* & W_{22}^* & -W_{21}^* \\ -X_{12}^* & X_{11}^* & -W_{12}^* & W_{11}^* \end{pmatrix}, \tag{3.5}$$

where $W_{ij}, X_{kl} \in \mathbb{C}$ for $i, j, k, l = 1, 2$, we can construct four SU(2) subgroups, by extracting two complex numbers t_1 and t_2 as

- $\begin{cases} t_1 = W_{11} \\ t_2 = X_{12} \end{cases}$
- $\begin{cases} t_1 = W_{22} \\ t_2 = X_{21} \end{cases}$
- $\begin{cases} t_1 = W_{11} + W_{22} \\ t_2 = X_{11} - X_{22} \end{cases}$
- $\begin{cases} t_1 = W_{11} + W_{22}^* \\ t_2 = W_{12} - W_{21}^* \end{cases}$

and building an SU(2) group element as

$$U_{\mathrm{SU}(2)} = \begin{pmatrix} t_1 & t_2 \\ -t_2^* & t_1^* \end{pmatrix}. \tag{3.6}$$

Each choice of t_1 and t_2 above gives a different SU(2) element belonging to a SU(2) subgroup of Sp(2).

4 Results

Put plots and results of the fit and simulations: fit of susceptibility peaks, fit of beta vs nt, etc.

To understand how EST works, let us describe

References

[1] Nicola Cabibbo and Enzo Marinari. "A new method for updating SU(N) matrices in computer simulations of gauge theories". In: *Physics Letters B* 119.4 (1982), pp. 387–390. ISSN: 0370-2693. DOI: `https://doi.org/10.1016/0370-2693(82)90696-7`. URL: `https://www.sciencedirect.com/science/article/pii/0370269382906967`.