

Abstract

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1 Introduction

Brief intro about the paper. Past work, theory, references.

2 Theoretical background

Explain the concepts of lattice gauge theory, Wilson and Polyakov loops, center symmetry and $\text{Sp}(2)$ group. Mostly from Pepe's work.

Read the paper by Caselle to see if there is something to pure here (SY conjecture? String tension? Universality with spin model? EST?)

3 Simulation and algorithm

Introduce the algorithm used in the simulation. Creutz, Cabibbo Marinari. Explain in details what the code does (without explicitly pasting the code): for example, how to extract $SU(2)$ matrices from $Sp(2)$.

Define the partition function of the system to be

$$\mathcal{Z} = \int \mathcal{D}U e^{-\beta S(U)} \quad \text{where} \quad \int \mathcal{D}U \equiv \prod_{x,\mu} \int dU_{x,\mu}. \quad (3.1)$$

$U_{x,\mu}$ is an element of the group (a link variable starting at the lattice site x , pointing along the direction μ), and S is the action, defined as

$$S = \sum_{\square} S_{\square}, \quad S_{\square} \equiv 1 - \frac{1}{4} \text{Tr} \left(U_{x,\mu} U_{x+\hat{\mu},\nu} U_{x+\hat{\nu},\mu}^{\dagger} U_{x,\nu}^{\dagger} \right). \quad (3.2)$$

The symbol \square represents the plaquette: the minimum loop possible on the lattice. Note that a link variable has a direction: the adjoint of a link variable is the link connecting the two sites in the opposite direction. In other words:

$$U_{x,\mu} = U_{x+\hat{\mu},-\mu}^{\dagger}. \quad (3.3)$$

The idea is to follow the *heat-bath* algorithm as designed by Cabibbo and Marinari [1], and generate a new link element with a Boltzmann probability distribution:

$$P(U) = \frac{1}{\mathcal{Z}} e^{-\beta S(U)} dU. \quad (3.4)$$

To do this, we consider a set F of $SU(2)$ subgroups of the gauge group $Sp(2)$. Given a $Sp(2)$ element U of the form

$$U_{Sp(2)} = \begin{pmatrix} W_{11} & W_{12} & X_{11} & X_{12} \\ W_{21} & W_{22} & X_{21} & X_{22} \\ X_{22}^* & -X_{21}^* & W_{22}^* & -W_{21}^* \\ -X_{12}^* & X_{11}^* & -W_{12}^* & W_{11}^* \end{pmatrix}, \quad (3.5)$$

where $W_{ij}, X_{kl} \in \mathbb{C}$ for $i, j, k, l = 1, 2$, we can construct four $SU(2)$ subgroups, by extracting two complex numbers t_1 and t_2 as

- $\begin{cases} t_1 = W_{11} \\ t_2 = X_{12} \end{cases}$
- $\begin{cases} t_1 = W_{22} \\ t_2 = X_{21} \end{cases}$
- $\begin{cases} t_1 = W_{11} + W_{22} \\ t_2 = X_{11} - X_{22} \end{cases}$
- $\begin{cases} t_1 = W_{11} + W_{22}^* \\ t_2 = W_{12} - W_{21}^* \end{cases}$

and building an $SU(2)$ group element as

$$U_{SU(2)} = \begin{pmatrix} t_1 & t_2 \\ -t_2^* & t_1^* \end{pmatrix}. \quad (3.6)$$

Each choice of t_1 and t_2 above gives a different $SU(2)$ element belonging to a $SU(2)$ subgroup of $Sp(2)$.

4 Results

Put plots and results of the fit and simulations: fit of susceptibility peaks, fit of beta vs nt, etc.

To understand how EST works, let us describe

References

- [1] Nicola Cabibbo and Enzo Marinari. “A new method for updating $SU(N)$ matrices in computer simulations of gauge theories”. In: *Physics Letters B* 119.4 (1982), pp. 387–390. ISSN: 0370-2693. DOI: [https://doi.org/10.1016/0370-2693\(82\)90696-7](https://doi.org/10.1016/0370-2693(82)90696-7). URL: <https://www.sciencedirect.com/science/article/pii/0370269382906967>.