

1 ultrasphere and ultrasphere-harmonics: Python 2 packages for Vilenkin–Kuznetsov–Smorodinsky 3 polyspherical coordinates and hyperspherical 4 harmonics techniques in array API

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10 Summary

11 Spherical harmonics, which are the solutions to the angular part of the laplace equation,
12 have been widely used in various fields of science and engineering. Especially, hyperspherical
13 harmonics, which are spherical harmonics in higher dimensions, have been applied to many-body
14 problems in quantum mechanics and nuclear physics

15 Statement of need

16 ultrasphere and ultrasphere-harmonics are Python packages for hyperspherical coordinates
17 and hyperspherical harmonics techniques. Our packages is that they support any type of
18 Vilenkin–Kuznetsov–Smorodinsky polyspherical coordinate systems ([Vilenkin & Klimyk, 1993](#)).
19 This allows to write codes that work in any type of polyspherical coordinates and thus in any
20 number of dimensions. To demonstrate this, we implemented acoustic scattering from a single
21 sphere for any type of polyspherical coordinates, which could be verified by command-line
22 interface.

23 Our api is compatible with the array API standard ([Meurer et al., 2023](#)). This enables writing
24 code which runs on multiple array libraries (e.g., NumPy([Harris et al., 2020](#)), PyTorch([Paszke
25 et al., 2019](#))) and multiple hardware (e.g., CPU, GPU). Our packages fully support vectorization
26 for high performance computing.

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References

- Harris, C. R., Millman, K. J., Walt, S. J. van der, Gommers, R., Virtanen, P., Cournapeau, D., Wieser, E., Taylor, J., Berg, S., Smith, N. J., Kern, R., Picus, M., Hoyer, S., Kerkwijk, M. H. van, Brett, M., Haldane, A., Río, J. F. del, Wiebe, M., Peterson, P., ... Oliphant, T. E. (2020). Array programming with NumPy. *Nature*, 585(7825), 357–362. <https://doi.org/10.1038/s41586-020-2649-2>
- Meurer, A., Reines, A., Gommers, R., Fang, Y.-L. L., Kirkham, J., Barber, M., Hoyer, S., Müller, A., Zha, S., Shanabrook, S., Gacha, S. J., Lezcano-Casado, M., Fan, T. J., Reddy, T., Passos, A., Kwon, H., Oliphant, T., & Standards, C. for P. D. A. (2023). Python array API standard: Toward array interoperability in the scientific python ecosystem. *Scipy*. <https://doi.org/10.25080/gerudo-f2bc6f59-001>
- Paszke, A., Gross, S., Massa, F., Lerer, A., Bradbury, J., Chanan, G., Killeen, T., Lin, Z., Gimelshein, N., Antiga, L., Desmaison, A., Köpf, A., Yang, E., DeVito, Z., Raison, M., Tejani, A., Chilamkurthy, S., Steiner, B., Fang, L., ... Chintala, S. (2019). PyTorch: An imperative style, high-performance deep learning library. In *Proceedings of the 33rd international conference on neural information processing systems* (pp. 8026–8037). Curran Associates Inc.
- Vilenkin, N. Ja., & Klimyk, A. U. (1993). *Representation of lie groups and special functions* (Vol. 74). Springer Netherlands. <https://doi.org/10.1007/978-94-017-2883-6>