

Math 256C: From Schemes to Conspiracies

Fall 2020

Chapter I: Points with Endomorphisms

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Tutorial

Basics

Lecture 1: September 3rd, 1752

I tend to use chapter headings for larger sections of a course (as opposed to individual lectures); to keep track of where lectures start you can use the `\bmn` (boxed margin note) command, which writes to the margin.

Here is where lecture content goes, generally a summary or transcription of what is being said or written. Here is a theorem:

Theorem 1.1: Kontsevich

The number N_d of rational plane curves of degree d passing through $3d - 1$ points in general position is given recursively by

$$N_d = \sum_{d_A + d_B = d} N_{d_A} N_{d_B} d_A^2 d_B \left(d_B \binom{3d-4}{3d_A-2} - d_A \binom{3d-4}{3d_A-1} \right)$$

The above result, is, of course, thoroughly unrelated to the following fact:

Lemma 1.2

In a k -free graph on n vertices, there are at most $\binom{k-1}{r} \left(\frac{n}{k-1}\right)^r$ r -cliques.

Setting $r = 2$ in the above, we recover the following result:

Corollary 1.3: Turan's Theorem

In a k -free graph on n vertices, there are at most $\frac{k-2}{k-1} \frac{n^2}{2}$ edges.

You can insert a hyperlinked reference for any theorem box if you add a reference tag (see the `\LaTeX` code at Corollary 1.3 for formatting, and see the style file for the reference prefixes for each theorem style).

PROOF : There is also a proof environment; the proof heading is configured to live in the left margin. ■

Here is a margin note: I use these generally to annotate my own thoughts or questions during lecture.

You can have multi-paragraph margin notes, which are configured to not have indented paragraphs.

Citations live in the right margin,¹ but will not work correctly if placed inside a theorem box. Repeated citations appear as *ibid.*² The available theorem boxes are `theorem`, `lemma`, `corollary`, `proposition`, `definition`, `example`, `remark`, `question`, `exercise`, `counterexample`, and `conjecture`. Unnumbered versions of all the theorem boxes exist:

Proposition: Hurwitz

The group of orientation-preserving conformal automorphisms of a compact Riemann surface of genus $g > 1$ has order at most $84(g-1)$.

¹ Fantechi, “Stacks for Everybody”, p. 232.

² *Ibid.*

References do not work for unnumbered theorems.

References

- Fantechi, Barbara. “Stacks for Everybody”. In: *European Congress of Mathematics*. Ed. by Carles Casacuberta et al. Basel: Birkhäuser Basel, 2001, pp. 349–359. ISBN: 978-3-0348-8268-2.
- Hartshorne, Robin. *Algebraic Geometry*. Springer, 1977.
- Kontsevich, Maxim and Yuri Manin. “Gromov-Witten classes, quantum cohomology, and enumerative geometry”. In: *Communications in Mathematical Physics* 164 (1994), pp. 525–562. DOI: <https://doi.org/10.1007/BF02101490>.
- Witten, Edward. “Two-Dimensional Gravity and Intersection Theory on Moduli Space”. In: *Surveys Diff. Geom.* 1 (1991), pp. 243–310. DOI: [10.4310/SDG.1990.v1.n1.a5](https://doi.org/10.4310/SDG.1990.v1.n1.a5).