Math 256C: From Schemes to Conspiracies **Fall 2020**

Chapter I: Points with Endomorphisms

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Tutorial

Basics

Lecture 1: September 3rd, 1752

I tend to use chapter headings for larger sections of a course (as opposed to individual lectures); to keep track of where lectures start you can use the `bmn` (boxed margin note) command, which writes to the margin.

Here is where lecture content goes, generally a summary or transcription of what is being said or written. Here is a theorem:

Theorem 1.1.1: Kontsevich[1]

The number N_d of rational plane curves of degree d passing through $3d-1$ points in general position is given recursively by

$$N_d = \sum_{d_A+d_B=d} N_{d_A} N_{d_B} d_A^2 d_B \left(d_B \binom{3d-4}{3d_A-2} - d_A \binom{3d-4}{3d_A-1} \right)$$

[1] Kontsevich and Manin, "Gromov-Witten classes, quantum cohomology, and enumerative geometry"

The above result, is, of course, thoroughly unrelated to the following fact:

Lemma 1.1.2

In a k-free graph on n vertices, there are at most $\binom{k-1}{r}(\frac{n}{k-1})^r$ r-cliques.

Here is a margin note: I use these generally to annotate my own thoughts or questions during lecture.

You can have multi-paragraph margin notes, which are configured to not have indented paragraphs.

Setting $r=2$ in the above, we recover the following result:[2]

[2] There are also numbered sidenotes.

Corollary 1.1.3: Turan's Theorem

In a k-free graph on n vertices, there are at most $\frac{k-2}{k-1}\frac{n^2}{2}$ edges.

You can insert a hyperlinked reference for any theorem box if you add a reference tag (see formatting below), e.g, `Corollary~\ref{cor:turan}` becomes Corollary 1.1.3 (see the style file for the reference prefixes for each theorem style).

PROOF : There is also a proof environment; the proof heading is configured to live in the left margin. ■

Citations live in the right margin.[3] Repeated citations appear as Ibid.[4] The available theorem boxes are `theorem`, `lemma`, `corollary`, `proposition`, `definition`, `example`, `remark`, `question`, `exercise`, `counterexample`, and `conjecture`.

[3] Fantechi, "Stacks for Everybody", p. 232.
[4] Ibid.

Unnnumbered versions of all the theorem boxes exist:

Proposition: Hurwitz[5]

The group of orientation-preserving conformal automorphisms of a compact Riemann surface of genus $g > 1$ has order at most $84(g-1)$.

[5] Fantechi, "Stacks for Everybody"; Hartshorne, *Algebraic Geometry*

For obvious reasons, internal references do not work for unnumbered theorems.

Formatting for a theorem box is as follows:

```
\begin{theorem}[<theorem name/author>]%
    [<optional tag for references>]%
    [<optional citation keys/tags (comma separated) for this theorem>]

    ... <theorem statement goes here> ...
\end{theorem}
```

The above is the *only* way to use citations inside one of the supplied theorem environments due to some incompatbilities between the `tcolorbox` package and the `tufte-book` class. Numbered sidenotes do not work inside theorem boxes, unnumbered marginnotes often work fine.

References

Fantechi, Barbara. "Stacks for Everybody". In: *European Congress of Mathematics*. Ed. by Carles Casacuberta et al. Basel: Birkhäuser Basel, 2001, pp. 349–359. ISBN: 978-3-0348-8268-2.

Hartshorne, Robin. *Algebraic Geometry*. Springer, 1977.

Kontsevich, Maxim and Yuri Manin. "Gromov-Witten classes, quantum cohomology, and enumerative geometry". In: *Communications in Mathematical Physics* 164 (1994), pp. 525–562. DOI: https://doi.org/10.1007/BF02101490.

Witten, Edward. "Two-Dimensional Gravity and Intersection Theory on Moduli Space". In: *Surveys Diff. Geom.* 1 (1991), pp. 243–310. DOI: 10.4310/SDG.1990.v1.n1.a5.