PART I:	Non-Riemannia	n Hypersquares

Math 256C: From Schemes to Conspiracies

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# **Tutorial**

# Basics

Here is where lecture content goes, generally a summary or transcription of what is being said or written. Here is a theorem:

#### Theorem 1.1.1: Kontsevich

The number  $N_d$  of rational plane curves of degree d passing through 3d-1 points in general position is given recursively by

$$N_d = \sum_{d_A + d_B = d} N_{d_A} N_{d_B} d_A^2 d_B \left( d_B \left( \frac{3d - 4}{3d_A - 2} \right) - d_A \left( \frac{3d - 4}{3d_A - 1} \right) \right)$$

The above result, is, of course, thoroughly unrelated to the following fact:

#### Lemma 1.1.2

In a k-free graph on n vertices, there are at most  $\binom{k-1}{r}(\frac{n}{k-1})^r$  r-cliques.

Setting r=2 in the above, we recover the following result:

### Corollary 1.1.3: Turan's Theorem

In a k-free graph on n vertices, there are at most  $\frac{k-2}{k-1} \frac{n^2}{2}$  edges.

You can insert a hyperlinked reference for any theorem box if you add a reference tag (see the LATEX code at Corollary 1.1.3 for formatting, and see the style file for the reference prefixes for each theorem style).

PROOF: There is also a proof environment; the proof heading is configured to live in the left margin.

Citations live in the right margin<sup>1</sup>, but will not work correctly if placed inside a theorem box. The available theorem boxes are theorem, lemma, corollary, proposition, definition, example, remark, question, exercise, and conjecture. Unnnumbered versions of all the theorem boxes exist:

Here is a margin note: I use these generally to annotate my own thoughts or questions during lecture.

You can have multi-paragraph margin notes, which are configured to not have indented paragraphs.

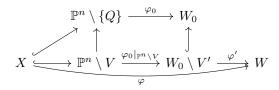
 $<sup>^{1}\,\</sup>mathrm{R.}$  Hartshorne. Algebraic Geometry. Springer, 1977

## Proposition: Hurwitz

The group of orientation-preserving conformal automorphisms of a compact Riemann surface of genus g > 1 has order at most 84(g-1).

#### Advanced

Using https://q.uiver.app/ (whose style file is included in our style file for convenience), we can curve arrows more flexibly in commutative diagrams than tikz-cd normally allows:



Finally, there is (in addition to the default environments provided by tufte-book) a two-column environment, which I mostly use to spam examples after a big theorem:

In  $\mathbb{CP}^2$  with its canonical orientation as a complex manifold, we have  $H^2 = H_2 = \mathbb{Z}$ . Lines L in  $\mathbb{CP}^2$  are embedded copies of  $\mathbb{CP}^1 = S^2$ , with a single fundamental class  $[L] \in H_2$  representing any such line. To compute the self-intersection  $[L] \cdot [L]$ , choose two representatives L and L' which intersect transversely (in a point, by basic intersection theory); since  $\mathbb{CP}^1$  is canonically oriented as well, and the orientations of L and L'agree with that of  $\mathbb{CP}^2$  (since they are subspaces of  $\mathbb{CP}^2$ ), we have that  $[L] \cdot [L] = 1$ .

#### References do not work for unnumbered theorems.

Theorem box numbering is not affected by subsection; this is set by secnumdepth at the top of the pream-

## Example 1.1.5

In  $S^2 \times S^2$ , the second homology group is  $\mathbb{Z}^2$ , generated by  $h = [S^2 \times \{x\}]$  and  $v = [\{x\} \times S^2]$ .  $S^2 \times \{x\}$  and  $\{x\} \times S^2$  intersect transversely at (x,x), and, regarding  $S^2$  with its canonical orientation as a complex manifold ( $\mathbb{CP}^1$ ), the orientations agree to give  $h \cdot v = 1$ . To compute  $h \cdot h$ , intersect representatives  $S^2 \times \{x\}$  and  $S^2 \times \{y\}$ for  $x \neq y$ , from which it is set-theoretically clear that  $h^2 = 0$  (and similarly for v). Thus, the

intersection form as a matrix in the basis (h, v)can be written as  $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$  This intersection form is therefore *even*, meaning that for any k = ah + bv,  $k \cdot k = 2ab$  is even.

# References

B. Fantechi. Stacks for everybody. In C. Casacuberta, R. M. Miró-Roig, J. Verdera, and S. Xambó-Descamps, editors, European Congress of Mathematics, pages 349–359, Basel, 2001. Birkhäuser Basel. ISBN 978-3-0348-8268-2.

- R. Hartshorne. Algebraic Geometry. Springer, 1977.
- M. Kontsevich and Y. Manin. Gromov-witten classes, quantum cohomology, and enumerative geometry. *Communications in Mathematical Physics*, 164:525–562, 1994. DOI: https://doi.org/10.1007/BF02101490.
- E. Witten. Two-dimensional gravity and intersection theory on moduli space. Surveys Diff. Geom., 1:243–310, 1991. DOI: 10.4310/SDG.1990.v1.n1.a5.