Part I: Non-Riemannian Hypersquares

Math 256C: From Schemes to Machinations **Fall 2020**

Lecture 1: 26 August

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Tutorial

Here is where lecture content goes, generally a summary or transcription of what is being said or written. Here is a theorem:

Theorem 1.1.1: Kontsevich

The number N_d of rational plane curves of degree d passing through $3d-1$ points in general position is given recursively by

$$N_d = \sum_{d_A+d_B=d} N_{d_A} N_{d_B} d_A^2 d_B \left(d_B \binom{3d-4}{3d_A-2} - d_A \binom{3d-4}{3d_A-1} \right)$$

Here is a margin note: note that the above result was only obtained in the early 1990s, using ideas from theoretical physics.

The above result, is, of course, thoroughly unrelated to the following fact:

Lemma 1.1.2

In a k-free graph on n vertices, there are at most $\binom{k-1}{r}\left(\frac{n}{k-1}\right)^r$ r-cliques.

Setting $r=2$ in the above, we recover the following result:

Corollary 1.1.3: Turan's Theorem

In a k-free graph on n vertices, there are at most $\frac{k-2}{k-1}\frac{n^2}{2}$ edges.

Note that this upper bound can be mildly strengthened into a strict upper bound by considering the different cases for n modulo $k-1$. In particular, if r is the remainder when n is divided by $k-1$, then the upper bound on edges is

$$\frac{k-2}{k-1}\frac{n^2-r^2}{2} + \binom{r}{2}$$

and k-free graphs with precisely that many edges can be straightforwardly constructed.

You can reference any theorem box if you add a reference tag (see the LaTeX code at Corollary 1.1.3 for formatting, and see the style file for the reference prefixes for each theorem style).

Unnnumbered versions of all the theorem boxes exist:

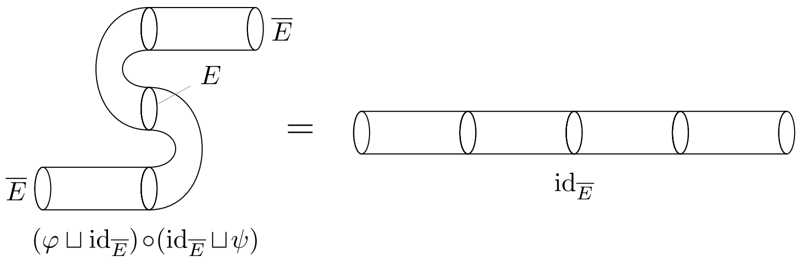
Proposition: Hurwitz

The group of orientation-preserving conformal automorphisms of a compact Riemann surface of genus $g>1$ has order at most $84(g-1)$.

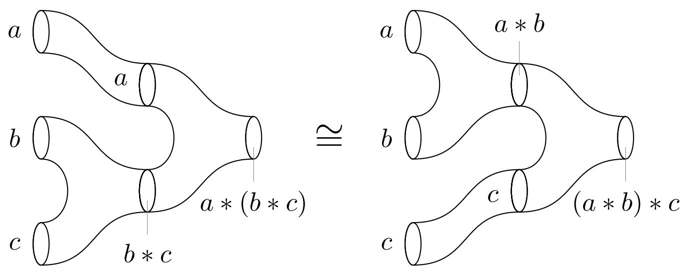
References do not work for unnumbered theorems.

Proof: There is also a proof environment. ■

Some miscellaneous things:



Associativity of the product arising from a 2D TQFT.



Diagrammatic arguments for two-dimensional TQFTs often rely on this strategy of placing vectors at boundary components and using the fact that morphisms in n are only defined up to diffeomorphisms to prove identities in $Z(S^1)$.

Using `https://q.uiver.app/` (whose style file is included in our style file for convenience), we can curve arrows more flexibly in commutative diagrams than tikz-cd normally allows:

$$\begin{tikzcd}
 & \mathbb{P}^n \setminus \{Q\} \arrow[r, "\varphi_0"] & W_0 & \\
X \arrow[ur, hook] \arrow[r, hook] \arrow[rrr, bend right, "\varphi"'] & \mathbb{P}^n \setminus V \arrow[u] \arrow[r, "\varphi_0|_{\mathbb{P}^n \setminus V}"] & W_0 \setminus V' \arrow[u, hook] \arrow[r, two heads, "\varphi'"] & W
\end{tikzcd}$$

Question 1.1.4

Are these diagrams really necessary to include in a sample file?