PART I:	Non-Riemannia	n Hypersquares

Math 256C: From Schemes to Machinations

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Tutorial

Basics

Here is where lecture content goes, generally a summary or transcription of what is being said or written. Here is a theorem:

Theorem 1.1.1: Kontsevich

The number N_d of rational plane curves of degree d passing through 3d-1 points in general position is given recursively by

$$N_{d} = \sum_{d_{A}+d_{B}=d} N_{d_{A}} N_{d_{B}} d_{A}^{2} d_{B} \left(d_{B} \binom{3d-4}{3d_{A}-2} - d_{A} \binom{3d-4}{3d_{A}-1} \right)$$

The above result, is, of course, thoroughly unrelated to the following fact:

${ m Lemma~1.1.2}$

In a k-free graph on n vertices, there are at most $\binom{k-1}{r}(\frac{n}{k-1})^r$ r-cliques.

Setting r=2 in the above, we recover the following result:

Corollary 1.1.3: Turan's Theorem

In a k-free graph on n vertices, there are at most $\frac{k-2}{k-1} \frac{n^2}{2}$ edges.

You can insert a hyperlinked reference for any theorem box if you add a reference tag (see the LATEX code at Corollary 1.1.3 for formatting, and see the style file for the reference prefixes for each theorem style).

PROOF: There is also a proof environment; the proof heading is configured to live in the left margin.

Citations live in the right margin¹, but will not work correctly if placed

Here is a margin note: I use these generally to annotate my own thoughts or questions during lecture.

You can have multi-paragraph margin notes, which are configured to not have indented paragraphs.

¹ R. Hartshorne. *Algebraic Geometry*. Springer, 1977

inside a theorem box. The available theorem boxes are theorem, lemma, corollary, proposition, definition, example, remark, question, exercise, and conjecture. Unnnumbered versions of all the theorem boxes exist:

Proposition: Hurwitz

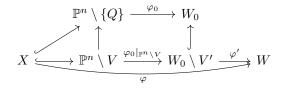
The group of orientation-preserving conformal automorphisms of a compact Riemann surface of genus g > 1 has order at most 84(g-1).

Advanced

Using https://q.uiver.app/ (whose style file is included in our style file for convenience), we can curve arrows more flexibly in commutative diagrams than tikz-cd normally allows:

References do not work for unnumbered theorems.

Theorem box numbering is not affected by subsection; this is set by secnumdepth at the top of the pream-



Finally, there is (in addition to the default environments provided by tufte-book) a two-column environment, which I mostly use to spam examples after a big theorem:

In \mathbb{CP}^2 with its canonical orientation as a complex manifold, we have $H^2 = H_2 = \mathbb{Z}$. Lines L in \mathbb{CP}^2 are embedded copies of $\mathbb{CP}^1 = S^2$, with a single fundamental class $[L] \in H_2$ representing any such line. To compute the self-intersection $[L] \cdot [L]$, choose two representatives L and L' which intersect transversely (in a point, by basic intersection theory); since \mathbb{CP}^1 is canonically oriented as well, and the orientations of L and L'agree with that of \mathbb{CP}^2 (since they are subspaces of \mathbb{CP}^2), we have that $[L] \cdot [L] = 1$.

In $S^2 \times S^2$, the second homology group is \mathbb{Z}^2 , generated by $h = [S^2 \times \{x\}]$ and $v = [\{x\} \times S^2]$. $S^2 \times \{x\}$ and $\{x\} \times S^2$ intersect transversely at (x,x), and, regarding S^2 with its canonical orientation as a complex manifold (\mathbb{CP}^1), the orien-

tations agree to give $h \cdot v = 1$. To compute $h \cdot h$, intersect representatives $S^2 \times \{x\}$ and $S^2 \times \{y\}$ for $x \neq y$, from which it is set-theoretically clear that $h^2 = 0$ (and similarly for v). Thus, the intersection form as a matrix in the basis (h, v)can be written as $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ This intersection form is therefore *even*, meaning that for any k = ah + bv, $k \cdot k = 2ab$ is even.

References

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- M. Kontsevich and Y. Manin. Gromov-witten classes, quantum cohomology, and enumerative geometry. *Communications in Mathematical Physics*, 164:525–562, 1994. DOI: https://doi.org/10.1007/BF02101490.
- E. Witten. Two-dimensional gravity and intersection theory on moduli space. Surveys Diff. Geom., 1:243–310, 1991. DOI: 10.4310/SDG.1990.v1.n1.a5.