Here's a lemma that I think clarifies the key step in the proof.

Lemma 1. *Let $(A, \mathfrak{m})$ be a Noetherian local ring.*

By Lemma 1, we may assume

REFERENCES