Homework #5

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December 11, 2016

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Problem 1

$$p(y = c_1|x) = \frac{p(x|y = c_1)p(y = c_1)}{p(x, y = c_1) + p(x, y = c_2)}$$

$$p(y = c_1|x) = \frac{p(x|y = c_1)p(y = c_1)}{p(x|y = c_1)p(y = c_1) + p(x|y = c_2)p(y = c_2)}$$

$$p(y = c_1|x) = \frac{1}{1 + \frac{p(x|y=c_2)}{p(x|y=c_1)}}$$

Assuming $p(y = c_1) = p(y = c_2)$, now plugging in the Gaussians where the prefactors cancel out due to equal covariance matrices,

$$\frac{p(x|y = c_2)}{p(x|y = c_1)} = \prod_{j=1}^{d} \frac{\exp(-1/2(x_j^2 - 2x_j\mu_{2,j} + \mu_{2,j}^2))}{\exp(-1/2(x_j^2 - 2x_j\mu_{1,j} + \mu_{1,j}^2))}$$

$$\frac{p(x|y = c_2)}{p(x|y = c_1)} = \prod_{j=1}^{d} \exp(\frac{1}{2\sigma_j^2}((2x_j\mu_{2,j} - \mu_{2,j}^2) - (2x_j\mu_{1,j} - \mu_{1,j}^2)))$$

$$\frac{p(x|y = c_2)}{p(x|y = c_1)} = \prod_{j=1}^{d} \exp(\frac{1}{2\sigma_j^2}(2x_j(\mu_{2,j} - \mu_{1,j}) + \mu_{1,j}^2 - \mu_{2,j}^2))$$

$$\frac{p(x|y = c_2)}{p(x|y = c_1)} = \exp(\sum_{j=1}^{d} \frac{1}{2\sigma_j^2}(2x_j(\mu_{2,j} - \mu_{1,j}) + \mu_{1,j}^2 - \mu_{2,j}^2))$$

$$\frac{p(x|y = c_2)}{p(x|y = c_1)} = \exp(w \cdot x + w_0)$$

where, $w = \frac{1}{\sigma_j^2}(\mu_{2,j} - \mu_{1,j})$ and $w_0 = \sum_{j=1}^{d} \frac{1}{2\sigma_j^2}(\mu_{1,j}^2 - \mu_{2,j}^2)$ and therefore,

$$p(y = c_1|x) = \frac{1}{1 + \exp(w \cdot x + w_0)}$$

Problem 2

Logistic regression we classify if the sigmoid is > 0.5 or < 0.5 while the linear discriminant analysis picks the class that maximizes the probability of x given y. They will not have the same value. We maximize log likelihood, so the values will be negative as probability is bounded between 0 and 1.

Problem 3

See attached notebook.