

### 3SAT

A 3rd conjunctive normal formula (3rd-formula) is a conjunction form with at most 3 literals at each clause

$$(x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_3 \vee \overline{x_5} \vee x_6) \wedge (x_3 \vee \overline{x_6})$$

3SAT={  $\phi$  |  $\phi$  is satisfiable 3rd-formula }

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### 3SAT is NP-complete

$$(a_1 \vee a_2 \vee a_3 \vee a_4)$$

is satisfiable if and only if the following is satisfiable

$$(a_1 \vee a_2 \vee z) \wedge (\overline{z} \vee a_3 \vee a_4)$$

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### 3SAT is NP-complete

$$(a_1 \vee a_2 \vee \dots \vee a_l)$$

is satisfiable if and only if the following is satisfiable

$$(a_1 \vee a_2 \vee z_1) \wedge (\overline{z_1} \vee a_3 \vee z_2) \wedge \\ (\overline{z_2} \vee a_4 \vee z_3) \wedge \dots \wedge (\overline{z_{l-3}} \vee a_{l-1} \vee a_l)$$

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### 3SAT is NP-complete

Convert every clause into

$$(a_1 \vee a_2 \vee \dots \vee a_l)$$

3cnf:

$$(a_1 \vee a_2 \vee z_1) \wedge (\overline{z_1} \vee a_3 \vee z_2) \wedge \\ (\overline{z_2} \vee a_4 \vee z_3) \wedge \dots \wedge (\overline{z_{l-3}} \vee a_{l-1} \vee a_l)$$

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### 3SAT is NP-complete

Conjunctive normal form

$$f_1 \wedge f_2 \wedge \dots \wedge f_k$$

Each clause  $f_i$  is convert into  $g_i (i=1,2,\dots,k)$

$$f_1 \wedge f_2 \wedge \dots \wedge f_k$$

is satisfiable if and only if the following is satisfiable

$$g_1 \wedge g_2 \wedge \dots \wedge g_k$$

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### Problem

Convert the formula F into 3SAT formula F' such that F is satisfiable iff and F' is satisfiable.

$$F = (x_1 \vee \overline{x_2} \vee \overline{x_3} \vee x_4) \wedge (x_3 \vee \overline{x_4} \vee \overline{x_5} \vee x_6)$$

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