

# Final Exam (CSCI6339, Fall 2020)

Due 11:59pm Monday Dec 7, 2020

Instruction: Dr. Bin Fu

Name:

ID Number:

Please submit exam in MS word format.

1. a) Design an automata to accept  $\{(10^2)^i \mid i \geq 0\}$ .  
b) Prove that the language  $\{0^i 1^{2i} \mid i \geq 0\}$  is not regular.

2. Let  $L$  be the set of undirected graphs. Prove that  $L$  is in NL.

3. For a Turing machine  $M(\cdot)$  with an input  $w$ , let  $M(w)$  be the output on the output tape if it stops in finite steps. Let  $L = \{(M1, M2, w) \mid M1 \text{ and } M2 \text{ are Turing machines, } w \text{ is an input, and } M1(w) \text{ is not equal to } M2(w)\}$ . Prove that  $L$  is undecidable by finding a reduction from  $A_{TM}$  to it, where  $A_{TM} = \{\langle M, w \rangle \mid \text{Turing machine } M \text{ accepts } w\}$ .

4. The subgraph-isomorphism problem takes two graphs  $G_1$  and  $G_2$  and asks whether  $G_1$  is isomorphic to a subgraph of  $G_2$ . Show that
- the subgraph-isomorphism problem is in NP; and
  - it is NP-complete by giving a polynomial time reduction from SAT problem to it.

Note: Two graphs  $G_1=(V_1, E_1)$  and  $G_2=(V_2, E_2)$  are isomorphic if there exists a one-one and onto function  $f(\cdot)$  from  $V_1$  to  $V_2$  such that for every two nodes  $u$  and  $v$  in  $V_1$ ,  $(u,v)$  is in  $E_1$  if and only if  $(f(u), f(v))$  is in  $E_2$ . For examples,  $G_1$  is isomorphic to a subgraph with vertices  $\{1,2,5,4\}$  of  $G_2$  below.

