Assignment 3: CSCI 4310/6323

Instructor: Dr. Bin Fu. Due April 29, 2021 (Thursday).

Please type your solution in MS word format. Submit your homework solution to Blackboard with file name firstname-lastname-hw3.

Problem 1. Problem 24.5-7. Let $G = (V, E)$ be a weighted, directed graph that contains no negative-weight cycles. Let S in V be the source vertex, and let G be initialized by Initialize-Single-Source(G, s). Prove that there exists a sequence of $|V| - 1$ relaxation steps that produces $d[v]$ for the length of the shortest path from s to v for all v in V.

Problem 2. 34.5-1 to prove the subgraph-isomorphism problem is NP-complete.

Problem 3. In graph theory, a dominating set for a graph $G = (V, E)$ is a subset D of V such that every vertex not in D is adjacent to at least one member of D. The domination number $\upsilon(G)$ is the number of vertices in a smallest dominating set for G.

The dominating set problem concerns testing whether $\upsilon(G) \leq K$ for a given graph G and input K. Prove that the dominating set problem is NP-complete via a polynomial reduction from vertex cover problem to it.

Problem 4. The half-clique problem is to determine if a graph of n vertices has a clique of at least $n/2$ vertices. Prove that half-clique problem is NP-complete.

Problem 5. Show that for every problem A in NP, there is an algorithm which solves A in time $O(2^{p(n)})$, where n is the size of the input instance and $p(n)$ is a polynomial (which may depend on A).

Problem 6. (Problem 35-3 in the textbook) Suppose that we generalize the set-covering problem so that each set S_i in the family F has an associated weight w_i and the weight of a cover L is $\sum_{S_i \in L} w_i$. We wish to determine a minimum-weight cover.

Show how to generalize the greedy set-covering heuristic in a natural manner to provide an approximate solution for any instance of the weighted set-covering problem. Show that your heuristic has an approximation ratio $H(d)$, where d is the maximum size of any set S_i.