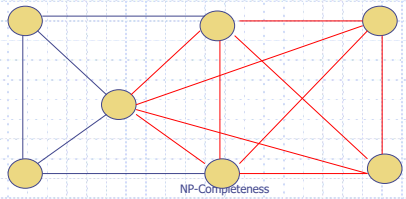
Clique Problem

Given undirected graph G, a clique is a set of nodes of G such that every two nodes are connected by an edge.

A k-clique is a clique with k nodes



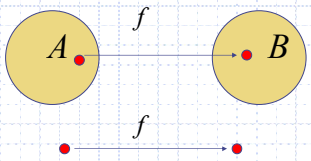
1

Polynomial Time Reduction

Assume that A and B are two languages on Σ

A is polynomial time mapping reducible to A if a polynomial time computable function $f : \Sigma^* \to \Sigma^*$ exists such that

$$w \in A \Leftrightarrow f(w) \in B$$



$A \leq_P B$

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Boolean Formula

A literal is either a boolean variable or its negation:

$$x \qquad \overline{x}$$

A clause is the disjunction of several literals

$$x_1 \vee \overline{x_2} \vee \overline{x_3} \vee x_4$$

Conjunctive normal form is the conjunction of several clauses

$$(x_1 \vee \overline{x_2} \vee \overline{x_3} \vee x_4) \wedge (x_3 \vee \overline{x_5} \vee x_6) \wedge (x_3 \vee \overline{x_6})$$

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3SAT

A 3nd conjunctive normal formula (3nd-formula) is a conjunction form with at most 3 literals at each clause

$$(x_1 \vee \overline{x_2} \vee \overline{x_3}) \wedge (x_3 \vee \overline{x_5} \vee x_6) \wedge (x_3 \vee \overline{x_6})$$

3SAT={ ϕ | ϕ is satisfiable 3nd-formula}

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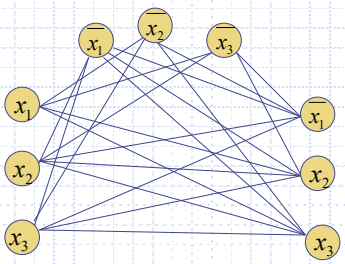
SAT to CLIQUE

Example: $(x_1 \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3}) \wedge (\overline{x_1} \vee x_2 \vee x_3)$



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Problem

- Given the formula f=

$$(x_1 \vee x_2) \wedge (\overline{x_1} \vee \overline{x_2}) \wedge (\overline{x_1} \vee x_3 \vee x_4)$$

- construct a graph G such that f is satisfiable iff G has a clique of size 3.

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