Homework 1 (CSCI6339, Fall 2020)

Due 09/20/2020(Sunday).

Please type your solution. Handwriting will not be accepted.
Submit your homework to Blackboard account.

Problem 1.

(1) Use inductive method to prove the formula $1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$.

(2) Let $S_k(n) = 1^k + 2^k + \ldots + n^k$. Find a recursion to compute the close formula of $S_k(n)$ via the close formulas of $S_{k-1}(n), S_{k-2}(n), \ldots, S_1(n)$.

Problem 2 .

Give state diagrams of DFA recognizing the following languages. In all cases the alphabet is {0,1}

(1) {w| w contains at least three 1s}

(2) {w| w does not contain substring 110}

Problem 3 A number is a prime if it cannot be a product of two numbers less than it. For examples, 2,3,5,7, 11, 13 are prime numbers, but 6, 8, 10 are not. Prove by contradiction that there are infinite prime numbers.

A twin number is a pair of prime numbers of difference 2. For examples, (3, 5), (11, 13), and (29, 31) are all twin numbers. Search internet about twin number conjecture. Provide its history and current research status.

Hint: Assume that there are only finite prime numbers p1, p2, …, pk. Think about the number p1* p2 *…..* pk+1

Problem 4.

(1) Construct an NFA to accept $L = \{a^{2n}b^{3m} : n \geq 1, m \geq 1\}$.

(2) Construct an NFA to accept $L = \{a^n b^m : n + m$ is odd$\}$.

Problem 5. Use the pumping lemma for the regular language to prove that the following languages are not regular.

(1) $A_1 = \{0^m 1^n 2^n | m, n \geq 0\}$.

(2) $A_2 = \{0^n 1^{n^2} | n \geq 0\}$.