One- one and onto

- Let A and B be two sets.
- For function f: A→B, if $f(x) \neq f(y)$ whenever $x \neq y$ then f is called one-one.
- For function f: A→ B, say f is onto if f hits every element of B(In other words, for very b in B, there is a in A such that b=f(a))

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Correspondence

- Let A and B be two sets.
- A and B is of the same size if there is a one-one and onto function f: A→B
- For function f: A→B, if it is both one-one and onto, then f is called correspondence.

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Examples

- {1,2,3,…} and {2,4,6,…} are of the same size via f(x)=2x.
- (0,1) and $(-\infty,+\infty)$ are of the same size

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Countable

- A set is countable if it is finite or it has the same size as N={1,2,3,…}
- Theorem: The positive rational numbers set is coutable

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Proof

- Every positive rational number is in the table below

$$\frac{1}{1},\frac{1}{2},\frac{1}{3},\frac{1}{4},\frac{1}{5},\cdots$$
$$\frac{2}{1},\frac{2}{2},\frac{2}{3},\frac{2}{4},\frac{2}{5},\cdots$$
$$\frac{3}{1},\frac{3}{2},\frac{3}{3},\frac{3}{4},\frac{3}{5},\cdots$$
$$\frac{4}{1},\frac{4}{2},\frac{4}{3},\frac{4}{4},\frac{4}{5},\cdots$$
$$\frac{5}{1},\frac{5}{2},\frac{5}{3},\frac{5}{4},\frac{5}{5},\cdots$$

……

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Problem

- Is there any one-one and onto map from the set of integers in [1,20] and the set of odd integers in [1,20]? Why?
- Prove that there is a one-one and onto map from the set of all integers and the set of all odd integers.

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