Pumping Lemma

- Lemma: If A is regular language, there is a number p such that if s is in A and of length at least p, s may be divided into s=xyz, satisfying

1) $xy^i z \in A$ for each $i \geq 0$
2) $|y| > 0$
3) $|xy| \leq p$

Application of Pumping Lemma

$L = \{0^i 1^i \mid i \geq 0\}$ is not a regular language

- Proof: Assume it is a regular language.

  There is a automata M to accept it.

  Let p be the number from the pumping lemma

  Consider the string $0^p 1^p$

Proof

Let s= $0^p 1^p$. It is in the language L. By the pumping lemma, s can be divided into s=xyz that satisfies 1), 2) and 3) in the pumping lemma.

- By 3), |xy| is at most p. So, y contains only 0s. By 1) of the pumping, xy^2z is also in the language L. The number of 0s in the string xy^2z is more than the number of 1s. This is a contradiction.

Application of Pumping Lemma

The language L={w | w has an equal number of 0s and 1s} is not a regular language

- Proof: Assume it is a regular language.

  There is a automata M to accept it.

  Let p be the number from the pumping lemma

  Consider the string $0^p 1^p$

Proof

Let s= 0^p1^p . It is in the language L. By the pumping lemma, s can be divided into s=xyz that satisfies 1), 2) and 3) in the pumping lemma.

By 3), |xy| is at most p. y contains only 0s. By 1) of the pumping, the string xy^2z is also in the language L. The number of 0s in the string is more than the number of 1s. This is a contradiction.

Problem

Prove that

$L = \{0^i 1^j \mid j > i \geq 1\}$ is not a regular language.