CSCI 6333/6315 Database Systems

Spring 2020

ASSIGNMENT 3: Functional Dependencies and Normalization

Sample Answers

For this homework assignment, consider the schema $R = (A, B, C, D, E)$ and the following set F of functional dependencies holds on R:

$$\begin{aligned} &A \rightarrow BC \\ &CD \rightarrow E \\ &B \rightarrow D \\ &E \rightarrow A \end{aligned}$$

Problem 1. Suppose that we decompose the schema R into $R_1 = (A, B, C)$ and $R_2 = (A, D, E)$. Show that this decomposition is a lossless-join decomposition with respect to F.

Answer: We only to show that the intersection of R_1 and R_2 is a superkey for either R_1 or R_2. Since $R_1 \cap R_2 = A$, we first find the closure of A.

$$\begin{aligned} A &\Longrightarrow A, \;\; A \longrightarrow A \\ &\Longrightarrow ABC, \;\; A \longrightarrow BC \\ &\Longrightarrow ABCD, \;\; B \longrightarrow D \\ &\Longrightarrow ABCDE, \;\; CD \longrightarrow E \end{aligned}$$
$$\text{Hence, } A^+ = ABCDE, A^+ \longrightarrow R.$$

The above proves that A is a superkey for R, so is for R_1 and R_2. Therefore, the decomposition is lossless.

Problem 2. Suppose that we decompose the relation schema R into $R_3 = (A, B, C)$ and $R_4 = (C, D, E)$. Show that this decomposition is not a lossless-join decomposition.

Hint: Give an example of a relation r on schema R such that

$$r \neq \Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r)$$

Answer: We need to construct a concrete relation r so that r satisfies the given functional dependencies but the decomposition of R_3 and R_4 is not lossless. Below is the relation r:

r =

A	B	C	D	E
2	4	5	6	7
20	40	5	60	70

$r(R_3) = \Pi_{A,B,C}(r) =$

A	B	C
2	4	5
20	40	5

$r(R_4) = \Pi_{C,D,E}(r) =$

C	D	E
5	6	7
5	60	70

However,

$r \neq \Pi_{A,B,C}(r) \bowtie \Pi_{C,D,E}(r) =$

A	B	C	D	E
2	4	5	6	7
2	4	5	60	70
20	40	5	6	7
20	40	5	60	70

Problem 3. Compute $(BC)^+$.

Answer:

$$\begin{aligned} BC &\Longrightarrow BC, \;\; BC \longrightarrow BC \\ &\Longrightarrow BCD, \;\; B \longrightarrow D \\ &\Longrightarrow BCDE, \;\; CD \longrightarrow E \\ &\Longrightarrow ABCDE, \;\; E \longrightarrow A \end{aligned}$$

Hence, $(BC)^+ = ABCDE$.

Problem 4. Compute the canonical cover F_c.

Answer: We only need to consider two functional dependencies $A \longrightarrow BC$ and $CD \longrightarrow E$ to see whether there are any extraneous attributes in them.

First, consider $A \longrightarrow BC$. Let us check whether B or C is extraneous. For B, we need to compute A^+ from

$$F' = \{A \longrightarrow C, CD \longrightarrow E, B \longrightarrow D, E \longrightarrow A\}$$

$$\begin{aligned} A &\Longrightarrow A, \;\; A \longrightarrow A \\ &\Longrightarrow AC, \;\; A \longrightarrow C \end{aligned}$$

We have $A^+ = AC \not\rightarrow$ BC, so B is not extraneous. For C, we need to compute A^+ from

$$F' = \{A \longrightarrow B, CD \longrightarrow E, B \longrightarrow D, E \longrightarrow A\}$$

$$\begin{aligned} A &\Longrightarrow A, \quad A \longrightarrow A \\ &\Longrightarrow AB, \quad A \longrightarrow B \\ &\Longrightarrow ABD, \quad B \longrightarrow D \end{aligned}$$

We have $A^+ = ABD \not\rightarrow$ BC, so C is not extraneous.

Second, consider $CD \longrightarrow E$. Let us check whether C or D is extraneous. For C, we need to compute D^+ from the original collection F of functional dependencies.

$$D \Longrightarrow D, \quad D \longrightarrow D$$

We have $D^+ = D \not\rightarrow E$, so C is not extraneous. For D, we need to compute C^+ from the original collection F of functional dependencies.

$$C \Longrightarrow C, \quad C \longrightarrow C$$

We have $C^+ = C \not\rightarrow E$, so D is not extraneous.

From the above analysis, none of the attributes in F is extraneous. Hence, $F_c = F$.

Problem 5. Show that the decomposition of R into $R_1 = (A, B, C)$ and $R_2 = (A, D, E)$ is not a dependency-preserving decomposition.

Answer: We project F into R_1 and R_2 to obtain $F_1 = \{A \longrightarrow BC\}$ and $F_2 = \{CD \longrightarrow E\}$, respectively. Now, we need to show that $(F_1 \cup F_2)^+ \neq F^+$. Since F has $B \longrightarrow D$, but B⟶D cannot be derived from $F_1 \cup F_2$, we have $(F_1 \cup F_2)^+ \neq F^+$, hereby the decomposition is not dependency-preserving.

Problem 6. Give a lossless-join decomposition into BCNF of R.

Answer: We start with R and shall check for every *nontrivial* functional dependency $\alpha \longrightarrow \beta$ in F to see α is a superkey or not. If not, we'll then decompose R.

For $A \longrightarrow BC$, since $A^+ = ABCDE$, A is superkey for R.

For $CD \longrightarrow E$, since $(CD)^+ = ABCDE$, CD is superkey for R.

For $B \longrightarrow D$, since $B^+ = BD$, B is not superkey for R. Hence, with this functional dependency, we decompose R into $R_1 = (B, D)$ and $R_2 = (A, B, C, E)$. This decomposition is a BCNF decomposition.

Problem 7. Give a lossless-join, dependency preserving decomposition into 3NF of R.

Answer: Following the 3NF decomposition algorithm, we first find the canonical cover F_c of F. By answer to Problem 4, we have $F_c = F = \{A \longrightarrow BC,\ CD \longrightarrow E,\ B \longrightarrow D,\ E \longrightarrow A\}$. Second, for functional dependency in F_c, we add a new but smaller relational schema:

$$R_1 = (A, B, C), \text{ for } A \longrightarrow BC$$
$$R_2 = (C, D, E), \text{ for } CD \longrightarrow E$$
$$R_3 = (B, D), \text{ for } B \longrightarrow D$$
$$R_4 = (A, E), \text{ for } E \longrightarrow A$$

We notice that A is a candidate key for R and A is contained in R_1. We also notice that none of the four decomposed schema is a subset of another decomposed schema. Hence, R_1, R_2, R_3 and R_4 form a 3NF decomposition for R.