

Language recognized by TM

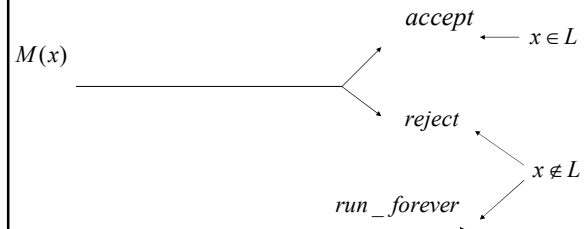
For a Turing machine M , $L(M)$ denotes the set of all strings accepted by M .

A language is Turing recognizable if some Turing machine recognizes it.

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Turing Recognizable

- Turing machine M recognizes language L



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EQ_{TM}

$EQ_{TM} = \{ \langle A, B \rangle \mid A \text{ and } B \text{ are Turing machines and } L(A) = L(B) \}$

Theorem: Neither EQ_{TM} nor its complement is Turing recognizable

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Proof

We first prove $A_{TM} \leq_m \overline{EQ_{TM}}$

For input $\langle M, w \rangle$ for A_{TM}

TM M_1 : rejects any input

TM M_2 : accepts any input if M accepts w .

It is easy to see $L(M_1) \neq L(M_2)$ iff M accepts w

So, $\langle M, w \rangle \in A_{TM} \Leftrightarrow \langle M_1, M_2 \rangle \in \overline{EQ_{TM}}$

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Proof

We prove $A_{TM} \leq_m EQ_{TM}$

For input $\langle M, w \rangle$ for A_{TM}

TM M_1 : accepts any input

TM M_2 : accepts any input if M accepts w .

It is easy to see $L(M_1) = L(M_2)$ iff M accepts w

So, $\langle M, w \rangle \in A_{TM} \Leftrightarrow \langle M_1, M_2 \rangle \in EQ_{TM}$

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Problem

Show that the language $F = \{ \langle M \rangle \mid M \text{ is a Turing machine, and } L(M) \text{ contains infinite elements} \}$ is not Turing recognizable.

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