1) $y' = \frac{x+y}{x-y} \begin{matrix} |:x \\ |:x \end{matrix}$; тип 2

$y' = \frac{1+\frac{y}{x}}{1-\frac{y}{x}}$ $\quad t = \frac{y}{x}$; $y = t\cdot x$; $y' = t'x + t$

$t = t(x)$

$t'x + t = \frac{1+t}{1-t}$; $t'\cdot x = \frac{1+\cancel{t}-\cancel{t}+t^2}{1-t} = \frac{1+t^2}{1-t}$: тип 1

$\frac{dt}{dx}x = \frac{1+t^2}{1-t}$; $\frac{(1-t)dt}{1+t^2} = \frac{dx}{x}$

$\int\frac{dt}{1+t^2} - \frac{1}{2}\int\frac{d(t^2+1)}{1+t^2} = \ln|x| + \ln C$

$\operatorname{arctg} t - \frac{1}{2}\ln(1+t^2) = \ln Cx$; $\underline{\underline{\operatorname{arctg}\frac{y}{x} = \ln\left(Cx\sqrt{1+\frac{y^2}{x^2}}\right)}}$

2) $y^2 + x^2y' = xy\cdot y'$; $y'(xy - x^2) = y^2$

$y' = \frac{y^2}{xy-x^2} \begin{matrix} |:x^2 \\ |:x^2 \end{matrix}$ тип 2. $\quad y' = \frac{y^2/x^2}{y/x - 1}$; $t = \frac{y}{x}$...

$t'x + t = \frac{t^2}{t-1}$; $t'\cdot x = \frac{\cancel{t^2}-\cancel{t^2}+t}{t-1}$

$\frac{dt}{dx}\cdot x = \frac{t}{t-1}$; $\frac{(t-1)\,dt}{t} = \frac{dx}{x}$

$\int\left(1-\frac{1}{t}\right)dt = \ln|x| + \ln C$

$t - \ln|t| = \ln Cx$

$t = \ln(Cx\cdot t)$; $\underline{\underline{\frac{y}{x} = \ln Cy}}$

3) $2y' = \frac{y^2}{x^2} + 8\frac{y}{x} + 10$ тип 2. $t = \frac{y}{x}$; $y' = t'x + t$

$2(t'x + t) = t^2 + 8t + 10$

$2t'x = t^2 + 8t + 10 - 2t$; $2t'x = t^2 + 6t + 1$

$2x \cdot \frac{dt}{dx} = t^2 + 6t + 10$; $\underbrace{\frac{dt}{t^2+6t+10}}_{10=9+1} = \frac{1}{2} \cdot \frac{dx}{x}$;

$\int \frac{dt}{(t+3)^2+1} = \frac{1}{2}\int \frac{dx}{x}$; $arctg(t+3) = \frac{1}{2}\ln|x| + \frac{1}{2}\ln C$

$arctg(\frac{y}{x}+3) = \frac{1}{2}\ln Cx$

$\underline{\underline{2arctg(\frac{y}{x}+3) = \ln Cx}}$

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4) $(xy' - y) \cdot arctg\frac{y}{x} = x$; $y|_{x=1} = 0$ З-ча Коші.

поділимо ДР на x; $(x \neq 0)$

$(y' - \frac{y}{x}) \cdot arctg\frac{y}{x} = 1$; тип 2 ; $t = \frac{y}{x}$; $y' = t'x + t$

$(t' \cdot x + \cancel{t} - \cancel{t}) \cdot arctg t = 1$.

$\frac{dt}{dx} \cdot x \cdot arctg t = 1$; $\int arctg t \cdot dt = \int \frac{dx}{x}$

$\int arctg t \cdot dt = \begin{vmatrix} u = arctg t & du = \frac{1}{1+t^2}dt \\ dV = dt & V = t \end{vmatrix} = t \cdot arctg t -$

$-\frac{1}{2}\int \frac{2t\,dt}{1+t^2} = t \cdot arctg t - \frac{1}{2}\ln(1+t^2)$

$\frac{y}{x} arctg\frac{y}{x} - \frac{1}{2}\ln(1+\frac{y^2}{x^2}) = \ln Cx$

$\frac{y}{x} arctg\frac{y}{x} = \ln\left(\frac{\sqrt{x^2+y^2}}{\cancel{x}} \cdot C\cancel{x}\right)$

$$\frac{y}{x}\operatorname{arctg}\frac{y}{x}=\ln\left(C\sqrt{x^2+y^2}\right)$$ загальний інтеграл ДР

Знайдемо C із початкових умов: $y|_{x=1}=0$

Підставимо $y=0$, $x=1$ у загальний інтеграл:

$$0=\ln C \Rightarrow C=1$$

Частинний інтеграл ДР: $\frac{y}{x}\operatorname{arctg}\frac{y}{x}=\ln\sqrt{x^2+y^2}$

5) $y'=\frac{3y^3+6x^2y}{2xy^2+3x^3}\begin{array}{l}|:x^3\\|:x^3\end{array}$ min 2 $\quad t=\frac{y}{x};\ x\cdot t'+t=y'$

$$x\cdot t'+t=\frac{3t^3+6t}{2t^2+3};\quad x\cdot t'=\frac{3t^3+6t-2t^3-3t}{2t^2+3}=\frac{t^3+3t}{2t^2+3}$$

$$x\cdot\frac{dt}{dx}=\frac{t^3+3t}{2t^2+3};\quad \int\frac{2t^2+3}{t^3+3t}\,dt=\int\frac{dx}{x}$$

$$\int\frac{2t^2+3+t^2-t^2}{t^3+3t}\,dt=\int\frac{(3t^2+3)-t^2}{t^3+3t}\,dt=\int\frac{d(t^3+3t)}{t^3+3t}-\int\frac{t^{\cancel{2}}\,dt}{\cancel{t}(t^2+3)}=$$

$$d(t^3+3t)=(3t^2+3)dt$$

$$=\ln|t^3+3t|-\frac{1}{2}\int\frac{2t\,dt}{t^2+3}=\ln|t^3+3t|-\frac{1}{2}\ln(t^2+3)$$

$$\ln|t^3+3t|-\frac{1}{2}\ln(t^2+3)=\ln Cx$$

$$\ln\frac{t(t^2+3)}{\sqrt{t^2+3}}=\ln Cx;\quad \ln\left(t\cdot\sqrt{t^2+3}\right)=\ln Cx$$

$$\frac{y}{x}\cdot\sqrt{\frac{y^2}{x^2}+3}=Cx;\quad y\sqrt{3x^2+y^2}=Cx^3$$