

$$1) y' + \frac{y}{x+1} = -y^2$$

$$y' + p(x) \cdot y = q(x) \cdot y^n \quad \text{мун 4. } p(x) = \frac{1}{x+1}; q(x) = -1$$

$$\text{Заміна: } \boxed{Z = y^{1-n} = y^{-1}} \quad n=2; \quad 1-n=-1.$$

приводить до лінійного рівн. (мун 3) для функції $Z(x)$

$$Z' + (1-n)p(x) \cdot Z = (1-n)q(x)$$

$$\text{де } \tilde{p}(x) = (1-n)p(x) \quad \text{і} \quad \tilde{q}(x) = (1-n)q(x)$$

$$\tilde{p}(x) = -\frac{1}{x+1}, \quad \tilde{q}(x) = 1. \quad \text{т.з. } 1-n = -1$$

$$Z(x) = R(x) \cdot S(x); \quad R(x) = e^{-\int \tilde{p}(x) dx} = e^{\int \frac{dx}{x+1}} = e^{\ln|x+1|} = x+1$$

$$S(x) = \int \frac{\tilde{q}(x)}{R(x)} dx = \int \frac{1}{x+1} dx = \ln|x+1| + \ln C = \ln C(x+1)$$

$$Z(x) = (x+1) \cdot \ln C(x+1)$$

$$\underline{\underline{y(x) = Z^{-1}(x) = \frac{1}{(x+1) \ln C(x+1)}}}$$

$$2) y' + \frac{2y}{x} = \frac{2\sqrt{y}}{\cos^2 x} \quad \text{мун 4, } p(x) = \frac{2}{x}; q = \frac{2}{\cos^2 x}$$

$$n = \frac{1}{2}, \quad 1-n = \frac{1}{2}$$

$$\text{Заміна: } Z = y^{1-n} = y^{\frac{1}{2}} = \sqrt{y}$$

$$Z' + \underbrace{(1-n)p(x)}_{\tilde{p}(x)} \cdot Z = \underbrace{(1-n)q(x)}_{\tilde{q}(x)} \quad \tilde{p}(x) = \frac{1}{x}; \quad \tilde{q}(x) = \frac{1}{\cos^2 x}$$

$$Z(x) = R(x) \cdot S(x); \quad R(x) = e^{-\int \frac{1}{x} dx} = e^{-\ln|x|} = \frac{1}{x}$$

$$S(x) = \int \frac{1}{\cos^2 x} \cdot x \cdot dx = \left| \begin{array}{l} u = x \quad du = dx \\ dv = \frac{dx}{\cos^2 x} \quad v = \tan x \end{array} \right| = x \cdot \tan x -$$

$$- \int \tan x \cdot dx = x \cdot \tan x + \ln|\cos x| + C$$

$$y(x) = (Z(x))^2 = \frac{1}{x^2} (x \tan x + \ln|\cos x| + C)^2$$