Mid-Term Course Work CM1015 / Computational Mathematics

UNIVERSITY OF LONDON

CM1015

COMPUTER SCIENCE

Computational Mathematics

October 2023

INSTRUCTIONS TO STUDENTS:

This paper consists of 5 questions. You should answer **ALL** the questions.

There are 100 marks available on this paper. The marks for each question are indicated at the end of the part in [.] brackets. **Full marks will be awarded for complete answers to a total of 5 questions**.

All answers need to be written clearly

The point of this assessment is to give you the opportunity to consolidate your learning and to assess your understanding of the topics. You do need to submit your answers as a pdf document (probably a single document is best), or photos of your work, or your work properly formatted using the maths mode of your word processor).

The total work is worth 100 marks distributed as follows:

* 15 marks for topic-1 (Number Bases)

* 30 marks for topic-2 (Sequences, Series and Mathematical induction)

* 15 marks for topic-3 (Modular Arithmetic)

* 20 marks for topic-4 (Angles, Triangles and Trigonometry)

* 20 marks for topic-5 (Graph Sketching and Kinematics)

Question 1

(a) State whether the following statements are correct or not. Show your work.

i. $(49)_{10} = (31)_{16} = (110001)_2$, Show your work.

ii. $7B_{16} = (1111011)_2 = (123)_{10}$, show your work. [4]

(b) If you have the binary number 1101.01112, what will be its decimal number equivalent to? Show your work. [2]

(c) Convert 61453_{10} to hexadecimal system, show your work. [4]

(d) A given number in base x can be converted to any other base y. According to the expansion method, if $abc.de$ is any given number in base x, then write its value in base 10. Show your work. [2]

(e) find te value of x if $13_x + 82_x = 17_{10}$. Show your work [3]

Question 2

(a) Use mathematical induction to prove whether that for any positive number n, $n^3 + 2n$ is divisible by 3 or not. Show your work [4]

(b) Suppose that a flu is spreading through a population. Initially, only one person is infected with the flu. It is also known that an infected person will infect two persons after one round of infection (and will not infect more people).Therefore, after first round of infection, there are two newly infected person each of which will infect two persons further and this continues unbounded.Your task is to derive the mathematical form of the rule that tells:

i. Newly infected persons after n rounds of infection, state if this is an arithmetic or a geometric sequence, what is the value of the difference d if it is an arithmetic sequence or the value of the ratio r if it is a geometric sequence.

ii. The total number of infected persons after n rounds of infection

iii. If we let the disease spread at the same rate, how many persons will be infected after 5 rounds?

Show your work in each of the above [9]

(c) Prove that the series whose n^{th} term is $\frac{1}{n(n+1)}$ converges, and find its sum. Show your work [5]

(d) Find the sum of all positive integers, from 5 to 1555 inclusive, that are divisible by 5. Show your work. [4]

(e) The first and eighth terms of an arithmetic series are 51 and 100 respectively.
a) Find the twentieth term of the series. Show your work.
b) Determine the sum of the first twenty terms of the series. Show your work. [8]

Question 3

(a) What is the time 15 hours before 11 p.m.? Show your work to prove that [3]

(b) Find the remainder when 2^{81} is divided by 17. Show your work. [4]

(c) Find the value for each of the following. Show your work

i. What is the smallest non-negative integer for 360 mod 13?

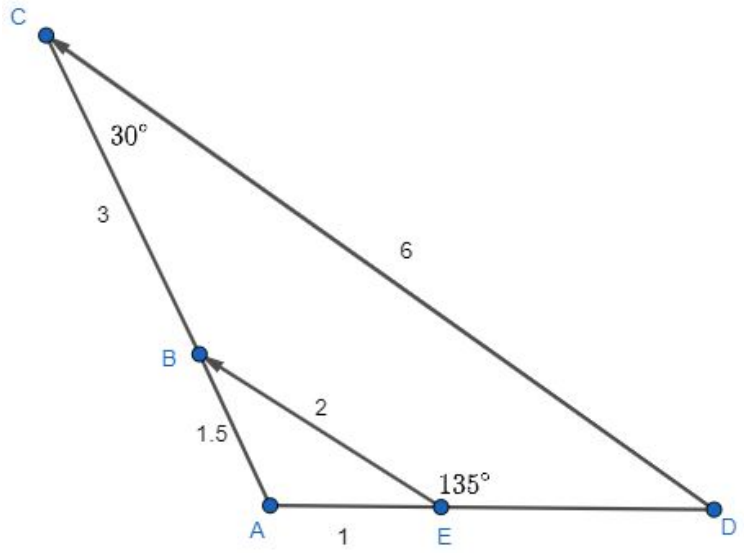
ii. What is the smallest non-negative integer for -86 mod 10? [4]

(d) Compute the remainder of 3^{64} in the division by 67. Show your work [4]

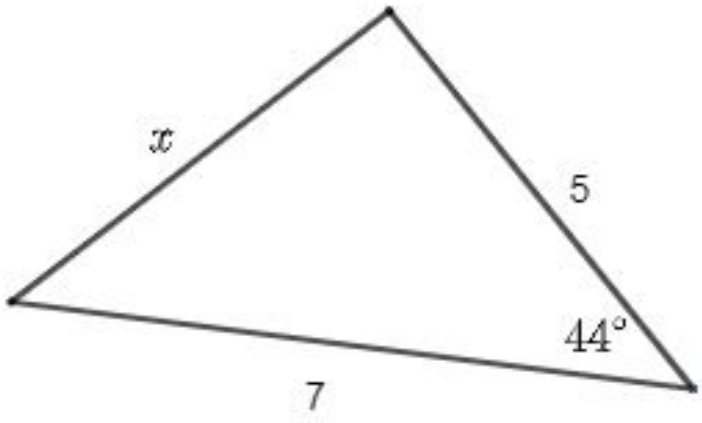
Question 4

(a) If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$; $0^\circ < A+B \leq 90^\circ$; $A > B$, find A and B. Show your work [5]

(b) Find the measure of the angle A and the length of AD. Show your work [4]



(c) Find the side-length marked x to 2 dp. Show your work [5]



(d) If $\tan(\theta)=\frac{a}{b}$, show that $\frac{a\sin\theta - b\cos\theta}{a\sin\theta + b\cos\theta} = \frac{a^2-b^2}{a^2+b^2}$. Show your work. [6]

Question 5

(a) Discuss each of the following statements, showing your work: [6]

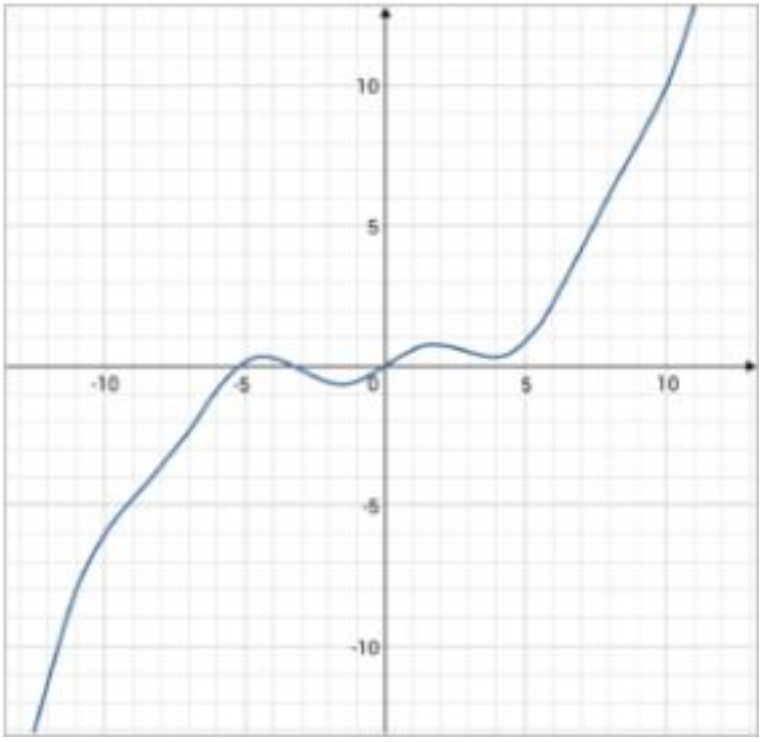
i. An object is moving at constant speed. Which statement MUST be true? Show your work
A. The acceleration of the object must be zero
B. The direction of the object is not changing
C. The velocity of the object is constant
D. All of the above
E. None of the above

ii. A tennis player tosses a tennis ball straight up in the air If a is the acceleration of the ball, and v is its velocity, which statement is true for when the ball reaches the highest point of its trajectory? Show your work
A. Both v and a are zero
B. Only v is zero and a is not
C. Only a is zero and v is not
D. Both v and a are non-zero
E. Impossible to answer without knowing the initial speed of the ball

iii. When a stone is thrown directly upwards with initial velocity of $30.0m/s$, what will be the maximum height it will reach and when will it be? Acceleration due to gravity is $10m/s^2$. Show your work
A. 45 m in 3 s
B. 90 m in 6 s
C. 1.5 m in 3 s
D. 90 m in 3 s
E. 45 m in 6 s

(b) Consider the function given by $f(x) = \frac{x-3}{x^2+9x-22}$. Show your work
(i) state the domain of the function
(ii) state the range of the function
(iii) plot the graph of the function. [6]

(c) From the following plot, say whether the function $f(x)$ is one-to-one or not. Is it onto or not? Show your work [4]



(d) IF $f : \Re \to \Re$, defined as $f(x) = x^2 - 2$,

i. Is the function one to one? Why?

ii. Show if it onto or not, if not, where can it be onto? [4]

END OF PAPER

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