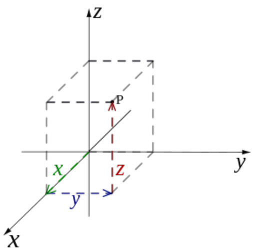
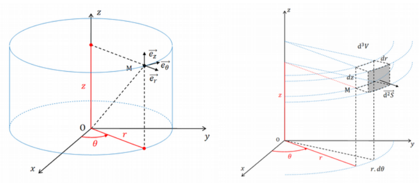
FORMULAIRE D'ANALYSE VECTORIELLE

Coordonnées cartésiennes



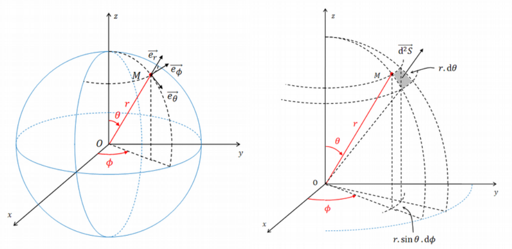
$$\overrightarrow{OM}=x\vec{e}_x+y\vec{e}_y+z\vec{e}_z$$
$$d\overrightarrow{OM}=dx\,\vec{e}_x+dy\,\vec{e}_y+dz\,\vec{e}_z$$
$$dS=dx\,dy$$
$$dV=dx\,dy\,dz$$
$$\overrightarrow{grad}\,U=\frac{\partial U}{\partial x}\vec{e}_x+\frac{\partial U}{\partial y}\vec{e}_y+\frac{\partial U}{\partial z}\vec{e}_z$$
$$\mathrm{div}\,\vec{V}=\frac{\partial V_x}{\partial x}+\frac{\partial V_y}{\partial y}+\frac{\partial V_z}{\partial z}$$
$$\overrightarrow{rot}\,\vec{V}=\left(\frac{\partial V_z}{\partial y}-\frac{\partial V_y}{\partial z}\right)\vec{e}_x+\left(\frac{\partial V_x}{\partial z}-\frac{\partial V_z}{\partial x}\right)\vec{e}_y+\left(\frac{\partial V_y}{\partial x}-\frac{\partial V_x}{\partial y}\right)\vec{e}_x$$
$$\Delta U=\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}$$
$$\Delta\vec{V}=\Delta V_x\vec{e}_x+\Delta V_y\vec{e}_y+\Delta V_z\vec{e}_z$$

Coordonnées cylindriques



$$\overrightarrow{OM}=r\,\vec{e}_r+z\,\vec{e}_z$$
$$d\overrightarrow{OM}=dr\,\vec{e}_r+r\,d\theta\,\vec{e}_\theta+dz\,\vec{e}_z$$
$$dS=r\,d\theta\,dz$$
$$dV=r\,dr\,d\theta\,dz$$
$$\overrightarrow{grad}\,U=\frac{\partial U}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial U}{\partial\theta}\vec{e}_\theta+\frac{\partial U}{\partial z}\vec{e}_z$$
$$\mathrm{div}\,\vec{V}=\frac{1}{r}\frac{\partial(r V_r)}{\partial r}+\frac{1}{r}\frac{\partial V_\theta}{\partial\theta}+\frac{\partial V_z}{\partial z}$$
$$\overrightarrow{rot}\,\vec{V}=\left(\frac{1}{r}\frac{\partial V_z}{\partial\theta}-\frac{\partial V_\theta}{\partial z}\right)\vec{e}_r+\left(\frac{\partial V_r}{\partial z}-\frac{\partial V_z}{\partial r}\right)\vec{e}_\theta+\left(\frac{\partial(r V_\theta)}{\partial r}-\frac{\partial V_r}{\partial\theta}\right)\vec{e}_z$$
$$\Delta U=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial U}{\partial r}\right)+\frac{1}{r^2}\frac{\partial^2 U}{\partial\theta^2}+\frac{\partial^2 U}{\partial z^2}$$
$$\Delta\vec{V}=\left[\Delta V_r-\frac{1}{r^2}\left(V_r+2\frac{\partial V_\theta}{\partial\theta}\right)\right]\vec{e}_r+\left[\Delta V_\theta-\frac{1}{r^2}\left(V_\theta-2\frac{\partial V_r}{\partial r}\right)\right]\vec{e}_\theta+\Delta V_z\vec{e}_z$$

Coordonnées sphériques



$$\overrightarrow{OM}=r\,\vec{e}_r$$
$$d\overrightarrow{OM}=dr\,\vec{e}_r+r\,d\theta\,\vec{e}_\theta+r\sin\theta\,d\phi\,\vec{e}_\phi$$
$$dS=r^2\sin\theta\,d\theta\,d\phi$$
$$dV=r^2\,dr\sin\theta\,d\theta\,d\phi$$
$$\overrightarrow{grad}\,U=\frac{\partial U}{\partial r}\vec{e}_r+\frac{1}{r}\frac{\partial U}{\partial\theta}\vec{e}_\theta+\frac{1}{r\sin\theta}\frac{\partial U}{\partial\phi}\vec{e}_\phi$$
$$\mathrm{div}\,\vec{V}=\frac{1}{r^2}\frac{\partial(r^2 V_r)}{\partial r}+\frac{1}{r\sin\theta}\frac{\partial(\sin\theta V_\theta)}{\partial\theta}+\frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial\phi}$$
$$\overrightarrow{rot}\,\vec{V}=\frac{1}{r\sin\theta}\left(\frac{\partial(\sin\theta V_\phi)}{\partial\theta}-\frac{\partial V_\theta}{\partial\phi}\right)\vec{e}_r+\left(\frac{1}{r\sin\theta}\frac{\partial V_r}{\partial\phi}-\frac{\partial(r V_\phi)}{\partial r}\right)\vec{e}_\theta+\frac{1}{r}\left(\frac{\partial(r V_\theta)}{\partial r}-\frac{\partial V_r}{\partial\theta}\right)\vec{e}_\phi$$
$$\Delta U=\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial U}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial U}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2 U}{\partial\phi^2}$$
$$\Delta\vec{V}=\left(\Delta V_r-\frac{2V_r}{r^2}-\frac{2V_\theta}{r^2}\frac{\cos\theta}{\sin\theta}-\frac{2}{r^2}\frac{\partial V_\theta}{\partial\theta}-\frac{2}{r^2\sin^2\theta}\frac{\partial V_\phi}{\partial\phi}\right)\vec{e}_r+\left(\Delta V_\theta-\frac{V_\theta}{r^2\sin^2\theta}+\frac{2}{r^2}\frac{\partial V_r}{\partial\theta}-\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial V_\phi}{\partial\phi}\right)\vec{e}_\theta+\left(\Delta V_\phi-\frac{V_\phi}{r^2\sin^2\theta}+\frac{2}{r^2\sin\theta}\frac{\partial V_r}{\partial\phi}+\frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial V_\theta}{\partial\phi}\right)\vec{e}_\phi$$

Composition des opérateurs

$$\mathrm{div}\left(\overrightarrow{rot}\,\vec{V}\right)=0$$
$$\overrightarrow{rot}\left(\overrightarrow{rot}\,\vec{V}\right)=\overrightarrow{grad}\left(\mathrm{div}\,\vec{V}\right)-\Delta\vec{V}$$
$$\mathrm{div}\left(\overrightarrow{grad}\,U\right)=\Delta U$$
$$\overrightarrow{rot}\left(\overrightarrow{grad}\,U\right)=\vec{0}$$

Produits scalaires et vectoriels :

$$\vec{A}\cdot\vec{B}=|\vec{A}|\cdot|\vec{B}|\cdot\cos\theta \quad \text{et} \quad \vec{A}\cdot\vec{B}=x_A\cdot x_B+y_A\cdot y_B+z_A\cdot z_B$$
$$|\vec{C}|=|\vec{A}\wedge\vec{B}|=|\vec{A}|\cdot|\vec{B}|\cdot\sin\theta \quad \text{et} \quad \vec{A}\wedge\vec{B}=(y_A\cdot z_B-y_B\cdot z_A)\vec{u}_x+(z_A\cdot x_B-z_B\cdot x_A)\vec{u}_y+(x_A\cdot y_B-x_B\cdot y_A)\vec{u}_z$$

Constantes :

permittivité diélectrique du vide : $\epsilon_0=8{,}854\cdot10^{-12}\,\mathrm{A^2\cdot s^4\cdot kg^{-1}\cdot m^{-3}}$
perméabilité magnétique du vide : $\mu_0=4\pi\cdot10^{-7}\,\mathrm{kg\cdot m\cdot A^{-2}\cdot s^{-2}}$
vitesse de la lumière : $c\approx3\cdot10^8\,\mathrm{m\cdot s^{-1}}$
masse de l'électron : $m_e=9{,}109\cdot10^{-31}\,\mathrm{kg}$
charge de élémentaire : $e=1{,}6\cdot10^{-19}\,\mathrm{C}$