Parametricity Goes Gradual[1,2]

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1) Ahmed et al. Theorems for Free for Free: Parametricity, With and Without Types. ICFP 2017
2) Xie et al. Consistent Subtyping for All. ESOP 2018

1. Is It Even Possible?

Sealing

(**define** (*create-seal*) (**gensym**))
(**define** (*seal v s1*) (λ (*s2*) (**if** (**eq?** *s1 s2*) *v* (**error**))))
(**define** (*unseal sealed-v s*) (*sealed-v s*))

Syntax 1

Conversion Labels	ϕ	$::=$	$+\alpha \mid -\alpha$
Blame Labels	p, q	$::=$	$+\ell \mid -\ell$
Base Types	ι	$::=$	$\mathsf{int} \mid \mathsf{bool}$
Types	A, B	$::=$	$\iota \mid A \times B \mid A \to B \mid \forall X . A \mid X \mid \alpha \mid \star$
Ground Types	G, H	$::=$	$\iota \mid \star \times \star \mid \star \to \star \mid \alpha$
Operations	$\circledast$	$::=$	$+ \mid - \mid * \mid \ldots$
Expressions	e	$::=$	$n \mid \mathsf{true} \mid \mathsf{false} \mid \mathsf{if}\ e\ \mathsf{then}\ e\ \mathsf{else}\ e \mid e \circledast e \mid x \mid$
			$\lambda(x : A).\, e \mid e\ e \mid \boxed{\Lambda X . v} \mid e\ [B] \mid \langle e, e \rangle \mid \pi_1\, e \mid$
			$\pi_2\, e \mid \boxed{(e : A \overset{\phi}{\Longrightarrow} B) \mid (e : A \overset{p}{\Longrightarrow} B)} \mid \mathsf{blame}\ p$

Syntax 2

$$
\begin{array}{llll}
\textit{Values} & v & ::= & n \mid \mathsf{true} \mid \mathsf{false} \mid \lambda(x:A).\,e \mid \Lambda X.\,v \mid \langle v,v\rangle \mid \\
& & & (v : A \to B \overset{\phi}{\Longrightarrow} A' \to B') \mid (v : \forall X.\,A \overset{\phi}{\Longrightarrow} \forall X.\,B) \mid \\
& & & (v : A \overset{-\alpha}{\Longrightarrow} \alpha) \mid (v : A \to B \overset{p}{\Longrightarrow} A' \to B') \mid \\
& & & \boxed{(v : A \overset{p}{\Longrightarrow} \forall X.\,B)} \mid (v : G \overset{p}{\Longrightarrow} \star) \\
\textit{Type-Name Stores} & \Sigma & ::= & \cdot \mid \Sigma, \alpha := A \\
\textit{Type Environments} & \Delta & ::= & \cdot \mid \Delta, X \\
\textit{Environments} & \Gamma & ::= & \cdot \mid \Gamma, x : A \\
\textit{Evaluation Contexts} & E & ::= & [\cdot] \mid E \circledast e \mid v \circledast E \mid \mathsf{if}\ E\ \mathsf{then}\ e\ \mathsf{else}\ e \mid E\ e \mid v\ E \mid \\
& & & E\ [A] \mid \langle E,e\rangle \mid \langle v,E\rangle \mid (E : A \overset{\phi}{\Longrightarrow} B) \mid (E : A \overset{p}{\Longrightarrow} B)
\end{array}
$$

Typeability

$$\cdots$$

$$\frac{\Sigma ; \Delta , X ; \Gamma \vdash v : A \qquad \Sigma ; \Delta \vdash \Gamma}{\Sigma ; \Delta ; \Gamma \vdash \Lambda X . v : \forall X . A} \qquad \frac{\Sigma ; \Delta ; \Gamma \vdash e : \forall X . A \qquad \Sigma ; \Delta \vdash B}{\Sigma ; \Delta ; \Gamma \vdash e\ [B] : A[B/X]} \qquad \frac{\Sigma ; \Delta ; \Gamma \vdash e : A \qquad \Sigma ; \Delta \vdash A \prec^{\phi} B}{\Sigma ; \Delta ; \Gamma \vdash (e : A \overset{\phi}{\Longrightarrow} B) : B}$$

$$\frac{\Sigma ; \Delta ; \Gamma \vdash e : A \qquad \Sigma ; \Delta \vdash A \prec B}{\Sigma ; \Delta ; \Gamma \vdash (e : A \overset{p}{\Longrightarrow} B) : B} \qquad \frac{\Sigma ; \Delta \vdash \Gamma \qquad \Sigma ; \Delta \vdash A}{\Sigma ; \Delta ; \Gamma \vdash \text{blame}\, p : A}$$

Convertability — **Almost reflexive!**

$$\boxed{\Sigma ; \Delta \vdash A \prec^{\phi} B} \text{ where } \Sigma ; \Delta \vdash A,\ \Sigma ; \Delta \vdash B, \text{ and } \mathit{FTN}(\phi) \in \Sigma$$

$$\frac{\vdash \Sigma}{\Sigma ; \Delta \vdash \iota \prec^{\phi} \iota} \qquad \frac{\Sigma ; \Delta \vdash A \prec^{\phi} A' \qquad \Sigma ; \Delta \vdash B \prec^{\phi} B'}{\Sigma ; \Delta \vdash A \times B \prec^{\phi} A' \times B'} \qquad \frac{\Sigma ; \Delta \vdash A' \prec^{-\phi} A \qquad \Sigma ; \Delta \vdash B \prec^{\phi} B'}{\Sigma ; \Delta \vdash A \to B \prec^{\phi} A' \to B'}$$

$$\frac{\Sigma ; \Delta , X \vdash A \prec^{\phi} B}{\Sigma ; \Delta \vdash \forall X . A \prec^{\phi} \forall X . B} \qquad \frac{\vdash \Sigma \qquad \alpha := A \in \Sigma}{\Sigma ; \Delta \vdash \alpha \prec^{+\alpha} A} \qquad \frac{\vdash \Sigma \qquad \alpha := A \in \Sigma}{\Sigma ; \Delta \vdash A \prec^{-\alpha} \alpha}$$

$$\frac{\vdash \Sigma \qquad \alpha := A \in \Sigma \qquad \alpha \notin \phi}{\Sigma ; \Delta \vdash \alpha \prec^{\phi} \alpha} \qquad \frac{\vdash \Sigma \qquad X \in \Delta}{\Sigma ; \Delta \vdash X \prec^{\phi} X} \qquad \frac{\vdash \Sigma}{\Sigma ; \Delta \vdash \star \prec^{\phi} \star}$$

Compatibility

$\boxed{\Sigma;\Delta \vdash A \prec B}$ where $\Sigma;\Delta \vdash A$ and $\Sigma;\Delta \vdash B$

$$\frac{\vdash \Sigma}{\Sigma;\Delta \vdash \iota \prec \iota} \qquad \frac{\Sigma;\Delta \vdash A \prec A' \qquad \Sigma;\Delta \vdash B \prec B'}{\Sigma;\Delta \vdash A \times B \prec A' \times B'} \qquad \frac{\Sigma;\Delta \vdash A' \prec A \qquad \Sigma;\Delta \vdash B \prec B'}{\Sigma;\Delta \vdash A \to B \prec A' \to B'}$$

$$\boxed{\frac{\Sigma;\Delta, X \vdash A \prec B \qquad X \notin A}{\Sigma;\Delta \vdash A \prec \forall X\,.\,B}} \qquad \boxed{\frac{\Sigma;\Delta \vdash A[\star/X] \prec B}{\Sigma;\Delta \vdash \forall X\,.\,A \prec B}} \qquad \frac{\vdash \Sigma \qquad \alpha \in \Sigma}{\Sigma;\Delta \vdash \alpha \prec \alpha} \qquad \frac{\vdash \Sigma \qquad X \in \Delta}{\Sigma;\Delta \vdash X \prec X}$$

$$\boxed{\frac{\Sigma;\Delta \vdash A}{\Sigma;\Delta \vdash A \prec \star} \qquad \frac{\Sigma;\Delta \vdash A}{\Sigma;\Delta \vdash \star \prec A}}$$

Reductions of conversion expressions

$$
\begin{array}{rcl}
(v : \iota \overset{\phi}{\Longrightarrow} \iota) & \longrightarrow & v \\
(\langle v_1, v_2 \rangle : A \times B \overset{\phi}{\Longrightarrow} A' \times B') & \longrightarrow & \langle (v_1 : A \overset{\phi}{\Longrightarrow} B), (v_2 : A' \overset{\phi}{\Longrightarrow} B') \rangle \\
(v : A \to B \overset{\phi}{\Longrightarrow} A' \to B')\ v' & \longrightarrow & (v\ (v' : A' \overset{-\phi}{\Longrightarrow} A) : B \overset{\phi}{\Longrightarrow} B') \\
(v : \alpha \overset{\phi}{\Longrightarrow} \alpha) & \longrightarrow & v \quad \text{if } \alpha \notin \phi \\
((v : A \overset{-\alpha}{\Longrightarrow} \alpha) : \alpha \overset{+\alpha}{\Longrightarrow} A) & \longrightarrow & v \\
(v : \star \overset{\phi}{\Longrightarrow} \star) & \longrightarrow & v
\end{array}
$$

Reductions of cast expressions

$$
\begin{array}{rcll}
(\langle v_1, v_2\rangle : A \times B \overset{p}{\Longrightarrow} A' \times B') & \longrightarrow & \langle (v_1 : A \overset{p}{\Longrightarrow} B), (v_2 : A' \overset{p}{\Longrightarrow} B')\rangle & \\
(v : A \to B \overset{p}{\Longrightarrow} A' \to B')\ v' & \longrightarrow & (v\ (v' : A' \overset{-p}{\Longrightarrow} A) : B \overset{p}{\Longrightarrow} B') & \\
(v : \forall X.\, A \overset{p}{\Longrightarrow} B) & \longrightarrow & (v\ [\star] : A[\star/X] \overset{p}{\Longrightarrow} B) & \text{if } B \neq \forall Y.\, B' \text{ for any } Y, B' \\
(v : \alpha \overset{p}{\Longrightarrow} \alpha) & \longrightarrow & v & \\
(v : \star \overset{p}{\Longrightarrow} \star) & \longrightarrow & v & \\
((v : G \overset{p}{\Longrightarrow} \star) : \star \overset{q}{\Longrightarrow} G) & \longrightarrow & v & \\
((v : G \overset{p}{\Longrightarrow} \star) : \star \overset{q}{\Longrightarrow} H) & \longrightarrow & \text{blame } q & \text{if } G \neq H \\
(v : A \overset{p}{\Longrightarrow} \star) & \longrightarrow & ((v : A \overset{p}{\Longrightarrow} G) : G \overset{p}{\Longrightarrow} \star) & \text{if } A \sim G, A \neq G, A \neq \star \\
(v : \star \overset{p}{\Longrightarrow} A) & \longrightarrow & ((v : \star \overset{p}{\Longrightarrow} G) : G \overset{p}{\Longrightarrow} A) & \text{if } A \sim G, A \neq G, A \neq \star
\end{array}
$$

Reductions of configurations

$$\boxed{\Sigma \triangleright e \longmapsto \Sigma' \triangleright e'}$$

$$\frac{e \longrightarrow e'}{\Sigma \triangleright E[e] \longmapsto \Sigma \triangleright E[e']}$$

$$\frac{\Sigma \triangleright e \longmapsto \Sigma' \triangleright e'}{\Sigma \triangleright E[e] \longmapsto \Sigma' \triangleright E[e']}$$

$$\frac{}{\Sigma \triangleright E[\mathrm{blame}\ p\ ] \longmapsto \Sigma \triangleright \mathrm{blame}\ p}$$

$$\frac{\Sigma; (\cdot, X); \cdot \vdash v : A \qquad \alpha \notin \mathrm{dom}(\Sigma)}{\Sigma \triangleright (\Lambda X\,.\,v)\ [B] \longmapsto \Sigma, \alpha{:=}B \triangleright (v[\alpha/X] : A[\alpha/X] \overset{+\alpha}{\Longrightarrow} A[B/X])}$$

$$\frac{\alpha \notin \mathrm{dom}(\Sigma)}{\Sigma \triangleright (v : A \overset{p}{\Longrightarrow} \forall X.\, A')\ [B] \longmapsto \Sigma, \alpha{:=}B \triangleright ((v : A \overset{p}{\Longrightarrow} A'[\alpha/X]) : A'[\alpha/X] \overset{+\alpha}{\Longrightarrow} A'[B/X])}$$

$$\frac{\alpha \notin \mathrm{dom}(\Sigma)}{\Sigma \triangleright (v : \forall X.\, A \overset{\phi}{\Longrightarrow} \forall X.\, A')\ [B] \longmapsto \Sigma, \alpha{:=}B \triangleright ((v\ [\alpha] : A[\alpha/X] \overset{\phi}{\Longrightarrow} A'[\alpha/X]) : A'[\alpha/X] \overset{+\alpha}{\Longrightarrow} A'[B/X])}$$

Contextual Approximation and Equivalence

$$
\begin{aligned}
\Sigma;\Delta;\Gamma \vdash e_1 \preceq^{ctx} e_2 : A \quad &\stackrel{\text{def}}{=} \quad \boxed{\Sigma;\Delta;\Gamma \vdash e_1 : A \ \wedge\ \Sigma;\Delta;\Gamma \vdash e_2 : A} \wedge \\
&\boxed{\forall C, \Sigma', B.\ \vdash C : (\Sigma;\Delta;\Gamma \vdash A) \rightsquigarrow (\Sigma';\ \cdot;\ \cdot \vdash B)} \implies \\
&\quad \boxed{(\Sigma' \triangleright C[e_1]\Downarrow \implies \Sigma' \triangleright C[e_2]\Downarrow)} \wedge \\
&\quad \boxed{\begin{aligned}&(\exists \Sigma_1.\ \Sigma' \triangleright C[e_1] \longmapsto^* \Sigma_1 \triangleright \mathrm{blame}\ p \implies \\ &\quad \exists \Sigma_2.\ \Sigma' \triangleright C[e_2] \longmapsto^* \Sigma_2 \triangleright \mathrm{blame}\ p\ )\end{aligned}} \\
\Sigma;\Delta;\Gamma \vdash e_1 \approx^{ctx} e_2 : A \quad &\stackrel{\text{def}}{=} \quad \Sigma;\Delta;\Gamma \vdash e_1 \preceq^{ctx} e_2 : A \ \wedge\ \Sigma;\Delta;\Gamma \vdash e_2 \preceq^{ctx} e_1 : A
\end{aligned}
$$

Theorems For Free Are Back!

Theorem 5.1 (Free Theorem: K-Combinator).
If $\Sigma \vdash v : \forall X.\, \forall Y.\, X \to Y \to X$, $\Sigma \vdash v_1 : A$, *and* $\Sigma \vdash v_2 : B$, *then either*

(1) $\Sigma \triangleright v\ [A]\ [B]\ v_1\ v_2 \longmapsto^* \Sigma' \triangleright v_1'$ *and* $v_1' \approx^{ctx} v_1$, *for some* Σ', v_1', *or*

(2) $\Sigma \triangleright v\ [A]\ [B]\ v_1\ v_2 \Uparrow$, *or*

(3) $\Sigma \triangleright v\ [A]\ [B]\ v_1\ v_2 \longmapsto^* \Sigma' \triangleright \mathrm{blame}\, p$, *for some* Σ', p.

Kripke Logical Relations Track The State of the World

$$\begin{aligned}
\mathrm{World}_n \quad &= \quad \{(j, \Sigma_1, \Sigma_2, \kappa) \in \mathrm{Nat} \times \mathrm{TNStore} \times \mathrm{TNStore} \times (\mathrm{TName} \xrightarrow{\mathrm{fin}} \mathrm{Rel}_j) \mid \\
&\qquad j < n \ \wedge\ \vdash \Sigma_1 \ \wedge\ \vdash \Sigma_2 \ \wedge\ \forall \alpha \in \mathrm{dom}(\kappa).\ \kappa(\alpha) \in \mathrm{Rel}_j\,[\Sigma_1(\alpha), \Sigma_2(\alpha)]\,\}
\end{aligned}$$

$$\begin{aligned}
\mathcal{V}\,[\![\mathrm{int}]\!]\,\rho \quad &= \quad \{(W, n, n) \in \mathrm{Atom}\,[\mathrm{int}]\,\rho\} \\
\mathcal{V}\,[\![\mathrm{bool}]\!]\,\rho \quad &= \quad \{(W, b, b) \in \mathrm{Atom}\,[\mathrm{bool}]\,\rho\}
\end{aligned}$$

$$\mathrm{Atom}\,[A]\,\rho \quad = \quad \bigcup_{n \geq 0} \{(W, e_1, e_2) \in \mathrm{Atom}_n\,[\rho(A), \rho(A)]\,\}$$

$$\begin{aligned}
\mathrm{Atom}_n\,[A_1, A_2] \quad &= \quad \{(W, e_1, e_2) \mid W.j < n \ \wedge\ W \in \mathrm{World}_n \ \wedge \\
&\qquad W.\Sigma_1; \cdot; \cdot \vdash e_1 : A_1 \ \wedge\ W.\Sigma_2; \cdot; \cdot \vdash e_2 : A_2\}
\end{aligned}$$

KLR For Functions:

In a future world, related values go to related expressions

$$
\begin{aligned}
\mathcal{V}\,[\![A \rightarrow B]\!]\,\rho \quad = \quad & \{(W, v_{f1}, v_{f2}) \in \mathrm{Atom}\,[A \rightarrow B]\,\rho \mid \\
& \forall W' \sqsupseteq W.\ \forall v_1, v_2.\ (W', v_1, v_2) \in \mathcal{V}\,[\![A]\!]\,\rho \implies \\
& (W', v_{f1}\ v_1, v_{f2}\ v_2) \in \mathcal{E}\,[\![B]\!]\,\rho\}
\end{aligned}
$$

$$
\begin{aligned}
W' \sqsupseteq W \quad & \stackrel{\text{def}}{=} \quad W'.j \le W.j \ \wedge\ W'.\Sigma_1 \supseteq W.\Sigma_1 \ \wedge\ W'.\Sigma_2 \supseteq W.\Sigma_2 \ \wedge \\
& \qquad W'.\kappa \sqsupseteq \lfloor W.\kappa \rfloor_{W'.j} \ \wedge\ W, W' \in \mathrm{World} \\
\kappa' \sqsupseteq \kappa \quad & \stackrel{\text{def}}{=} \quad \forall \alpha \in \mathrm{dom}(\kappa).\ \kappa'(\alpha) = \kappa(\alpha)
\end{aligned}
$$

The Problem With $\lessdot$

$$\frac{\Sigma ; \Delta \vdash A[\star/X] \lessdot B}{\Sigma ; \Delta \vdash \forall X.\, A \lessdot B} \quad \Rightarrow \quad \forall a.a \rightarrow a \ \lessdot \ \mathrm{Int} \rightarrow \mathrm{Bool}$$

Consistency And Subtyping Decoupled

$$\boxed{A \sim B}$$

$$A \sim A \qquad A \sim \star \qquad \star \sim A \qquad \frac{A_1 \sim B_1 \qquad A_2 \sim B_2}{A_1 \to A_2 \sim B_1 \to B_2} \qquad \frac{A \sim B}{\forall a.A \sim \forall a.B}$$

$$\boxed{\Psi \vdash A <: B}$$

$$\frac{\Psi, a \vdash A <: B}{\Psi \vdash A <: \forall a.B}\ \textsc{S-ForallR} \qquad \boxed{\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] <: B}{\Psi \vdash \forall a.A <: B}\ \textsc{S-ForallL}} \qquad \frac{a \in \Psi}{\Psi \vdash a <: a}\ \textsc{S-TVar}$$

$$\frac{}{\Psi \vdash \mathsf{Int} <: \mathsf{Int}}\ \textsc{S-Int} \qquad \frac{\Psi \vdash B_1 <: A_1 \qquad \Psi \vdash A_2 <: B_2}{\Psi \vdash A_1 \to A_2 <: B_1 \to B_2}\ \textsc{S-Fun} \qquad \frac{}{\Psi \vdash \star <: \star}\ \textsc{S-Unknown}$$

An Option: No Subtyping, Stricter Consistency [Igarashi et al., 2017]

$$\frac{A \sim B}{\forall a.A \sim \forall a.B} \qquad \frac{A \sim B \qquad B \neq \forall a.B' \qquad \star \in \mathsf{Types}(B)}{\forall a.A \sim B}$$

Precludes both:

- ∀a.a → a ~ Int → Bool
- ∀a.a → a ~ Int → Int

Even: ∀a.Int → Int ~/~ Int → Int

The Road To Consistent Subtyping Through [2008]

Proposition 2 (Properties of Consistent-Subtyping). *The following are equivalent:*

FAIL!

1. $\sigma \lesssim \tau$,
2. $\sigma <: \sigma'$ *and* $\sigma' \sim \tau$ *for some* σ', *and*
3. $\sigma \sim \sigma''$ *and* $\sigma'' <: \tau$ *for some* σ''.

$$\begin{tikzcd}
\mathsf{Int} \to \mathsf{Int} \arrow[r, dash, "\sim"] & \mathsf{Int} \to \star \\
\forall a.a \arrow[u, "<:"] \arrow[r, dash, "\sim"'] & \bot \arrow[u, "<:"']
\end{tikzcd}$$

$$\begin{tikzcd}
\bot \arrow[r, dash, "\sim"] & (((\star \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \star) & \\
(((\forall a.a \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\forall a.a) \arrow[u, "<:"] \arrow[r, dash, "\sim"'] & \bot \arrow[u, "<:"] \arrow[r, dash] & (\star \to \mathsf{Int}) \to \mathsf{Int} \\
& (\forall a.a \to \mathsf{Int}) \to \mathsf{Int} \arrow[u, "<:"] \arrow[r, dash, "\sim"'] & (\forall a.\star \to \mathsf{Int}) \to \mathsf{Int} \arrow[u, "<:"]
\end{tikzcd}$$

Reasonable Property To Fix What Failed

$$\frac{\Psi \vdash A <: C \qquad C \sim D \qquad \Psi \vdash D <: B}{\Psi \vdash A \lesssim B}$$

$$\begin{tikzcd}
A_2 \arrow[r, dash, "\sim"] & A_3 \arrow[d, "<:"] \\
A_1 \arrow[u, "<:"] \arrow[r, dashed, bend right, "\lesssim"] & A_4
\end{tikzcd}$$

$$A_1 = (((\forall a.a \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\forall a.a)$$
$$A_2 = ((\forall a.a \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \mathsf{Int})$$
$$A_3 = ((\forall a.\star \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \star)$$
$$A_4 = (((\star \to \mathsf{Int}) \to \mathsf{Int}) \to \mathsf{Bool}) \to (\mathsf{Int} \to \star)$$

Maybe Ahmed et al. [2017] Were Close Enough!

$$\boxed{\Psi \vdash A \lesssim B}$$

$$\frac{\Psi, a \vdash A \lesssim B}{\Psi \vdash A \lesssim \forall a.B}\ \textsc{CS-ForallR} \qquad \boxed{\frac{\Psi \vdash \tau \qquad \Psi \vdash A[a \mapsto \tau] \lesssim B}{\Psi \vdash \forall a.A \lesssim B}\ \textsc{CS-ForallL}}$$

$$\frac{\Psi \vdash B_1 \lesssim A_1 \qquad \Psi \vdash A_2 \lesssim B_2}{\Psi \vdash A_1 \to A_2 \lesssim B_1 \to B_2}\ \textsc{CS-Fun} \qquad \frac{a \in \Psi}{\Psi \vdash a \lesssim a}\ \textsc{CS-TVar} \qquad \frac{}{\Psi \vdash \mathsf{Int} \lesssim \mathsf{Int}}\ \textsc{CS-Int}$$

$$\frac{}{\Psi \vdash \star \lesssim A}\ \textsc{CS-UnknownL} \qquad \frac{}{\Psi \vdash A \lesssim \star}\ \textsc{CS-UnknownR}$$

Theorem 1. $\boxed{\Psi \vdash A \lesssim B} \Leftrightarrow \boxed{\Psi \vdash A <: C,\ C \sim D,\ \Psi \vdash D <: B}$ *for some* C, D.

Discussion

What is *good* and *bad* about global store of seals?

Even regular big-lambda reduction generates stuff!

$$\frac{\Sigma; (\cdot, X);\ \cdot\ \vdash v : A \qquad \alpha \notin \mathrm{dom}(\Sigma)}{\Sigma \triangleright (\Lambda X\,.\,v)\ [B] \ \longmapsto\ \Sigma\,, \alpha{:=}B \triangleright (v\,[\alpha/X] : A[\alpha/X] \stackrel{+\alpha}{\Longrightarrow} A[B/X])}$$

Is $\Lambda a.a \to a < \mathsf{Int} \to \mathsf{Bool}$ a road blocker?

► **Theorem 1** (Equivalence to the STLC for fully annotated terms).
Suppose e *is fully annotated and* T *is static.*
1. $\vdash_S e : T$ *if and only if* $\vdash e : T$. *(Siek and Taha [49]).*
2. $e \Downarrow_S v$ *if and only if* $e \Downarrow v$.

What’s With Graduality?

Fin