SOUTHERN FEDERAL UNIVERSITY

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Test

for

“Computer Methods in Discrete Mathematics”

course

Rostov-on-Don

2010

TEST SPECIFICATION

1. Test purpose.

Test tasks could be used for final examinations (basic, advanced and expert levels) on “Computer Methods in Discrete Mathematics” course for students studying for Masters of “EUROPEAN STUDIES ” program and for holding midterm exams in the course of studies. After completion of 1-st unit students supplied with the tasks relevant to it. This kind of tests could be conducted for backward students.

Test tasks could be used by the students for self-control. Teacher may use test as a source for home assignments with further discussion of right and wrong answers.

While using test in midterm exams it should be mandatory to pass the test for all students in the group. Students should know from the very beginning of the semester term that if they fail any midterm exam they wouldn't be admitted to the final exam.

2. Course content incorporated in test.

The purpose of “Computer Methods in Discrete Mathematics” course is to introduce students to the computational discrete mathematics using the Sage mathematics software system, and develop skills of scientific computing in several branches of discrete mathematics like graph theory, group theory and number theory using Python programming language. This will form base for the the Master of Applied Mathematics and Computer Science, so test task include questions from all over the course material, namely:

1. **Unit: Introduction to Python programming.**
    1. Python interpreter, executing scripts.
    2. Defining functions, indentation, if-statements.
    3. Strings, string methods, slices, strings formatting.
    4. Lists, for-statements, range function, list methods, list slices, list comprehensions.
    5. Sorting, key sorting, comparators.
    6. Tuples.
    7. Dictionaries, accessing keys, values and pairs.
    8. Text files.
    9. NumPy arrays.
2. **Unit: Sage Commons.**
    1. Basic worksheet usage.
    2. Defining functions.
    3. Solving equations exactly.
    4. Solving equations numerically.
    5. Differentiation.
    6. Two-dimensional plotting.
3. **Unit : Combinatorics.**
    1. Basics.
    2. Dyck words.
    3. Partitions.

4. Permutations.
5. Posets.

4. Unit: Number Theory.

1. Modular arithmetic.
2. Primality checking.
3. *p*-adic numbers.

5. Unit: Group Theory.

1. Basic group theory.
2. Finite abelian groups.
3. Group presentations.
4. Coset enumeration.

6. Unit: Graph Theory.

1. Creating graphs and digraphs.
2. Displaying graphs.
3. Shortest paths.
4. Canadian traveller problem.
5. Cayley graphs.

3. Control objects list

The test could be used for checking students skills in basic terms, definitions, solviving common problems in respective fields and so on, encountered in the "Computer Methods in Discrete Mathematics" course.

Kind of knowledge and practical skills ckecked by test are as follows

1	Running Python interpreter, use basic language constructs: conditionals, loops, functions, classes. Understanding code semantics.
2	Handling built-in datatypes: strings, lists, tuples and dictionaries. Proccesing text files.
3	Using Sage in interactive mode through worksheets. Defining and calling functions.
4	Solving equations either numerically or symbolycally, differentiating functions in one or several variables, plotting function graphics.
5	Understanding basic Python techniques for handling combinatorial objects. Using Sage facilities to work with Dyck words.
6	Handling partially ordered sets: defining, understanding cover relations, plotting Hasse diagams.
7	Performing modular arithmetic, definnig congruence classes, solving linear congruences. Using multiplicative functions, understanding their role in number theory. Being aquanted with most widely used primality checking methods.
8	Doing basic arithmetics with *p*-adic numbers. Undersastanding common approches and major differences between fields of *p*-adic numbers and real numbers.
9	Familiarity with basic notions of group theory: operations properties,

	homomorphisms, subgroups. Special groups (symmetric, dihedral, cyclic etc.)
10	Understanding structure and properties of finite abelian groups, using these for finding group properties and relationships.
11	Using group presentations for defining and handling (either manually or automatically) various groups. Understanding and solving coset enumeration problem for finitely presented groups.
12	Using various methods for graph construction. Defining basic graph properties. Handling graphs with Sage.
13	Various paths, cycles, circuits etc. on graphs. Using several algorithms for solving paths-related problems.

4. Distribution of tasks by complexity

Test tasks series is divided into three parts by complexity:

- basic level (1–139);
- advanced level (140–178);
- expert level (179–203).

Complexity level	**Task count**	**Maximum raw grade**	**Percentage of maximum raw grade for given complexity level from maximum raw grade for overall work equal to 203**
Базовый	139	139	68,47%
Повышенный	39	39	19,21%
Высокий	25	25	12.32%
Итого	**203**		**100%**

5. Task complexity level

is determined by:

- task substantiveness;
- amount of work to be done to complete the task;
- diversity of the work.

6. Test plan.

#Task	#Contents Item	#Control Object	Course Skills Level
1.	1.1	1	Basic
2.	1.1	1	Basic
3.	1.1	1	Basic
4.	1.1	1	Basic
5.	1.1	1	Basic
6.	1.2	1	Basic
7.	1.2	1	Basic

8.	1.2	1	Basic
9.	1.2	1	Basic
10.	1.3	2	Basic
11.	1.3	2	Basic
12.	1.3	2	Basic
13.	1.3	2	Basic
14.	1.3	2	Basic
15.	1.3	2	Basic
16.	1.3	2	Basic
17.	1.3	2	Basic
18.	1.3	2	Basic
19.	1.4	2	Basic
20.	1.4	2	Basic
21.	1.4	1, 2	Basic
22.	1.4	2	Basic
23.	1.4	2	Basic
24.	1.4	2	Basic
25.	1.5	2	Basic
26.	1.5	2	Basic
27.	1.5	2	Basic
28.	1.4	2	Basic
29.	1.6	2	Basic
30.	1.4,6	2	Basic
31.	1.7	2	Basic
32.	1.8	2	Basic
33.	2.1,3	3,4	Basic
34.	2.1,4	3,4	Basic
35.	2.1	3	Basic
36.	2.1	3	Basic
37.	2.2	3	Basic
38.	2.3	4	Basic
39.	2.3,4,5	4	Basic
40.	2.3	4	Basic
41.	2.4	4	Basic
42.	2.5	4	Basic
43.	2.5	4	Basic
44.	2.6	4	Basic
45.	2.6	4	Basic
46.	2.6	4	Basic
47.	2.6	4	Basic
48.	3.1	5	Basic
49.	3.1	5	Basic
50.	3.1	5	Basic

51.	3.2	5	Basic
52.	3.2	5	Basic
53.	3.2	5	Basic
54.	3.2	5	Basic
55.	3.5	6	Basic
56.	3.5	6	Basic
57.	3.5	6	Basic
58.	3.5	6	Basic
59.	3.5	6	Basic
60.	3.5	6	Basic
61.	3.5	6	Basic
62.	3.5	6	Basic
63.	4.1	7	Basic
64.	4.1	7	Basic
65.	4.1	7	Basic
66.	4.1	7	Basic
67.	4.1	7	Basic
68.	4.1,2	7	Basic
69.	4.1,2	7	Basic
70.	4.1,2	7	Basic
71.	4.1,2	7	Basic
72.	4.1,2	7	Basic
73.	4.1,2	7	Basic
74.	4.1,2	7	Basic
75.	4.1,2	7	Basic
76.	4.1,2	7	Basic
77.	4.1,2	7	Basic
78.	4.1,2	7	Basic
79.	4.1,2	7	Basic
80.	4.1,2	7	Basic
81.	4.1,2	7	Basic
82.	4.1,2	7	Basic
83.	4.1,2	7	Basic
84.	4.1	7	Basic
85.	4.2	7	Basic
86.	4.3	8	Basic
87.	4.3	8	Basic
88.	5.1	9	Basic
89.	5.1	9	Basic
90.	5.1	9	Basic
91.	5.1	9	Basic
92.	4.1, 5.1	8,9	Basic
93.	5.1	9	Basic

94.	5.1	9	Basic
95.	5.1	9	Basic
96.	5.1	9	Basic
97.	5.1	9	Basic
98.	3.4, 5.1	5, 9	Basic
99.	5.1	9	Basic
100.	3.4, 5.1	5, 9	Basic
101.	5.1	9	Basic
102.	5.3	11	Basic
103.	5.4	11	Basic
104.	5.1	9	Basic
105.	5.1	9	Basic
106.	5.1	9	Basic
107.	5.1	9	Basic
108.	5.1,2	9,10	Basic
109.	5.1,2	9,10	Basic
110.	5.2	10	Basic
111.	5.3	11	Basic
112.	5.3	11	Basic
113.	6.1	12	Basic
114.	6.1	12	Basic
115.	6.1	12	Basic
116.	6.1	12	Basic
117.	6.1	12	Basic
118.	6.1, 2	12	Basic
119.	6.1	12	Basic
120.	6.1	12, 13	Basic
121.	6.1, 3	13	Basic
122.	6.1, 3	13	Basic
123.	6.1	12	Basic
124.	6.1	12	Basic
125.	6.1	12	Basic
126.	6.1	12	Basic
127.	6.1	12	Basic
128.	6.1	12	Basic
129.	6.1,2	12	Basic
130.	6.1	12, 13	Basic
131.	6.2	12	Basic
132.	6.3	13	Basic
133.	6.3	13	Basic
134.	6.3	13	Basic
135.	6.3	13	Basic
136.	6.1	12	Basic

137.	6.1	12	Basic
138.	6.4	13	Basic
139.	6.1	12	Basic
140.	1.1	1	Advanced
141.	1.1	1	Advanced
142.	1.1	1	Advanced
143.	1.2	1	Advanced
144.	1.2	1	Advanced
145.	1.4	2	Advanced
146.	1.6	2	Advanced
147.	1.4	2	Advanced
148.	1.2, 4	1	Advanced
149.	1.2	1	Advanced
150.	4.1	7	Advanced
151.	4.2	7	Advanced
152.	4.3	8	Advanced
153.	4.3	8	Advanced
154.	4.3	8	Advanced
155.	4.3	8	Advanced
156.	4.3	8	Advanced
157.	4.3	8	Advanced
158.	5.3	11	Advanced
159.	5.1	9	Advanced
160.	4.1, 5.2	7, 10	Advanced
161.	4.1, 5.2	7, 10	Advanced
162.	5.1	9	Advanced
163.	5.1	9	Advanced
164.	5.1	9	Advanced
165.	5.1	9	Advanced
166.	4.1, 5.2	7, 10	Advanced
167.	4.1, 5.2	7, 10	Advanced
168.	5.1	9	Advanced
169.	5.1	9	Advanced
170.	4.1, 5.2	7, 10	Advanced
171.	6.1, 3	12, 13	Advanced
172.	6.3	13	Advanced
173.	6.3	13	Advanced
174.	6.1, 3	13	Advanced
175.	6.1, 3	13	Advanced
176.	6.1	13	Advanced
177.	6.1	13	Advanced
178.	6.1	13	Advanced
179.	4.1, 2	7	Expert

180.	4.2	7	Expert
181.	4.2	7	Expert
182.	4.3	8	Expert
183.	4.3	8	Expert
184.	4.3	8	Expert
185.	3.4, 5.1	9	Expert
186.	5.1	9	Expert
187.	5.1	9	Expert
188.	5.1	9	Expert
189.	5.2	10	Expert
190.	5.2	10	Expert
191.	5.1	9	Expert
192.	5.1	9	Expert
193.	5.1	9	Expert
194.	5.1	9	Expert
195.	5.2	10	Expert
196.	5.1	9	Expert
197.	6.1, 3	13	Expert
198.	6.1, 3	13	Expert
199.	6.1	13	Expert
200.	6.1	13	Expert
201.	6.1	13	Expert
202.	6.1	13	Expert
203.	6.1	13	Expert

7. Test structure by test task forms. Task instruction samples.

Total test task count — 203.

Test is consisted of four test task types. The structure of test tasks is suggested:

Task type "Multiple choice" – task of closed type, where student is offered to choose correct statements from given list.

Task instruction. Each task of this formed is supplied with list of possible answers, among which there is one or more correct ones (2, 3, and more...). Highlight correct answer(s).

Task type "Short answer" – solving this task student should write down a word or a phrase. This kind of task consists of text and blank field for answer.

Task instruction. Answer to this task could be a word, a phrase or a date. Write your answer in block letters down in the blank field.

Task type "Number quiz". It's a variant of "short answer" but the answer should be a number with possible calculating error.

Task instruction. The answer to this question is a nuber. Write your answer down in the blank field.

Task type "Correspondence" – a group of terms is given,the correspondence between them should be established. This kind of task consists of base (text), subquestions and corresponding number of answers. Formatting: compose one list with the questions and answers for them.

Task instruction. In this part of test tasks there are two types of questions:

1. Set up correspondence between terms and their definitions. Write down numbers and letters of answers you chose preserving number order.
2. Point out the sequence. Write it down with letters.

TEST TASKS

BASIC LEVEL

1. Unit: Introduction to Python programming.

1. Python is ___________________ rather than compiled language.

interpreted

2. Python is __________________ rather than statically typed.

dynamically

3. The command for starting Python interpreter is
   - start_python
   - python
   - pythoni
4. Python interpreter prompts for continuation lines with the secondary prompt, by default
   - >>>
   - … (three dots)
   - starting right from the begining of the line
   - ???
5. To get help on using e.g. list types in Python you may type in
   - help(list)
   - help.list
   - list.help
   - list /?
6. Programming block in Python denoted by
   - {}-braces

- begin-end
- identation

7. The keyword for starting function declaration in Python is
    - def
    - decl
    - function
    - no such keyword
8. In ‘if’ statement ‘else’ part is
    - mandatory
    - optional
    - optional in the presence of ‘elif’ parts and mandatory otherwise
9. Usual way for returning value from the function is to use next statement
    - return
    - exit
    - exception throwing
    - sleep
10. String in Python is being indexed from ___ .

0

11. String concatenation operation is
    - +
    - *
    - ++
    - . (dot)
12. Is there implicit conversion from number to string?
    - yes
    - no
    - only in PHP-compatability mode
13. Making string literal raw prevents ________________ in the literal.

escaping

14. Raw string in Python could be obtained using
    - string literal prefixed with ‘r’
    - string literal suffixed with ‘r’
    - calling ‘raw’ function
    - compile-time ‘RAW’ macro

15. Using unicode inside string literal could be allowed by prefixing the literal with ___________________.

u

16. Multi-line string literals in Python are boundedby
    - """ for both sides
    - ''' for both sides
    - " for both sides
    - << and >>
    - `` and "

17. Suppose s was initialized with literal 'Hello'. Match following string slices expressions with their values
    1. s[:]
    2. s[-3:]
    3. s[:-3]
    4. s[1:100]

    1. 'He'
    2. 'llo'
    3. 'Hello'
    4. 'ello'

1 – 3, 2 – 2, 3 – 1, 4 – 4.

18. String formatting operator is
    - *
    - /

- for
- %

19. List declaration is performed with

- []
- {}
- ()
- <>

20. List membership operator is

- in
- out
- contains
- belongs

21. There is only loops over collections in Python. In order to simulate C-like loops on integer parameter you may use __________ function returning desired collection of integer values.

range

22. Appending one list to other is performed with method of list:

- append
- insert
- extend
- index

23. To get index of an element of list you should use method

- pos
- find
- index
- locate

24. Construction of list comprehensions involve

- []

- {}
- ()
- <>

25. Function which returns sorted collection is
    - sort
    - sorted
    - apply_sort
    - perform_sorting

26. To sort list entries on custom criterion you supply a ______ which is a property or method that could be fired against the entries and returns the values which would be used for sorting. Those values in their turn are sorted using default criterion.

key

27. Method of list which perform sorting on the list is
    - sort
    - sorted
    - apply_sort
    - perform_sorting

28. To delete list entry you may use ___________ statement.

del

29. Tuple declaration is performed with
    - []
    - {}
    - ()
    - <>

30. Tuples unlike lists ____________ be changed.

cannot

31. Dictionary declaration is performed with
    - []

- {}
- ()
- <>

32.In order to read text files in custom encodings you should import ____________ module.

codecs

2. Unit: Sage Commons.

33.Result of executing exp(2) command in Sage is

- e^2
- 7.38905609893065 (approx. value of e^2)
- NameError: name 'exp' is not defined

34.In order to get approximate value of some real constant you should use

- n (built-in function)
- n (method against desired constant object)
- numerical_approx (built-in function)
- value (built-in function)
- you don't need to do something special — just type in expression with the constant

35.If you want to use literal integer constant base 8 you should prefix it with

- 0 (zero)
- o (letter)
- you may not use other than 10 bases

36.To get help on using e.g. tan function in Sage ypu may type in

- help(tan)
- help.tan
- tan?
- tan /?

37. The keyword for starting function declaration in Python is

- def
- decl
- function
- no such keyword

38. Sage command for solving equations symbolically is _____________________ .

solve

39. Sage command for declaring variable which would denote equation variable is ________________ .

var

40. To solve system of equation you should pass a ________ of equations to corresponding command.

list

41. Sage command for solving equations numerically is _____________________ .

find_root

42. Sage command for differentiation is _____________________ .

diff

43. To get partial derivative of a function you may call _________ method on the object which represents the function.

diff

44. Basic facility to draw function graphics is ____________ .

plot

45. If you specify variable name, you can create ________________ plots.

parametric

46. To plot a number of plots on one picture you should use operation:

- +
- *

- &
- #

47. In order to create text object to put it on the picture you should use __________ function.

text

3. **Unit: Combinatorics.**

48. In order to create iterator on a list you may pass the list to the __________ function.

iter

49. _________ method of iterator returns currently pointed value and moves iterator one step further.

next

50. There is special class of functions which define iterators. They use ________ keyword instead of return.

yield

51. A __________ word of size n is a word with n ones and n zero's such that in any prefix there are more ones than zero's.

Dyck

52. A collection of Dyck words could be obtained using ____________ function.

DyckWords

53. The number of Dyck words of given order n is equal to _______________ number C_n.

Catalan

54. Viewing the Dyck word as a path from (0, 0) to (2n, 0) in the first quadrant by letting “1” represent steps in the direction (1, 1) and “0” represent steps in the direction (1, −1), the _________ is the maximum y-coordinate reached.

height

55. A partial order on a set P is a binary relation $\leq$ over P which is

- reflexive
- irreflexive
- symmetric
- antisymmetric
- asymmetric
- transitive
- total
- trichotomous
- serial

56. Elements a, b of poset P is said to be ________________ if $a \le b$ or $b \le a$ holds.

comparable

57. In order to create poset object in Sage you should provide list of elements and list of pairs of elements denoting realitions and call ________________ class constructor.

Poset

58. When constructing poset to define relations you may supply two-argument _____________ defining whether two elements relate to each other or not, instead of list of pairs of elements.

function

59. ______________________ of a binary relation R on a set X is a minimal relation R' on X such that the transitive closure of R' is the same as the transitive closure of R.

Transitive reduction

60. ______________________ is a type of diagram used to represent a finite partially ordered set, in the form of a drawing of its transitive reduction.

Hasse diagram

61. In order to obtain all posets of given number of elements in Sage you should call __________________ .

Posets

62. By default 'cardinality' method of posets list obtained from 'Posets' returns pre-computed values obtained from the On-Line Encyclopedia of Integer Sequences , sequence ___________________ .

A000112

4. **Unit: Number Theory.**

63. Two integers a and b are said to be _______________ modulo n, written $a = b$ (mod n) if their difference $a - b$ is an integer multiple of n.

congruent

64. Consider linear congruence $40 = x$ (mod 3); x could be equal to

- 0
- 1
- 2
- 3

65. The least residue of 776^{79} modulo 7 is _____

6

66. Let $a = 3$ (mod 4) and $b = 1$ (mod 4) then the least residue of $a * a + 2 * a * b$ modulo 8 is ____

7

67. Set of congruence classes modulo n for any integer n forms algebraic structure called

- field
- forest
- ring
- ellipse

68. Two integers a and b are said to be __________ if they have no common positive factor other than 1.

coprime

69.________________________________ of two or more non-zero integers, is the largest positive integer that divides the numbers without a remainder.

greatest common divisor

70.gcd(42, 56) = _____

14

71.gcd(3618, 1139) = _____

67

72.For coprime integers a and b it is hold that gcd(a, b) = X, where X is equal to

- 0
- 1
- $a*b$
- max(a, b)

73.For integer a and positive integer m: a has inverse modulo m if and only if gcd(a, m) is equal to

- 1
- m
- a
- min(a, m)

74.Euclidean algorithm for calculating greatest common divisor used it's fundamental property: gcd(a, b) = gcd(b, a @ b), where @ is binary operation:

- * (integer product)
- mod (integer division remainder)
- +
- –

75.$a * x = b$ (mod m) only has solution if gcd(a, m) is equal to

- m
- a
- b

- 1

76.______________ Euclidean algorithm for integer a, b allows to compute Bézout's coefficients.

Extended

77. Let 128 * x = 833 (mod 1001), then x could be equal to

- 812
- 535
- 1
- 1000

78. Inverse of 7 modulo 17 is ___

5

79. Fermat's little theorem states that for any prime p and integer a which is an coprime to p, a^{p-1} is congruent modulo p to

- 0
- 1
- $p - 1$
- p

80._______________ function is a function $f(n)$ of the positive integer n with the property that $f(1) = 1$ and whenever a and b are coprime, then $f(ab)$ = $f(a) * f(b)$.

Multiplicative

81. Euler function $\varphi(n)$ of a positive integer n is defined to be the number of positive integers less than or equal to n that are __________ to n.

coprime

82. For any prime integer p Euler function $\varphi(p)$ gives

- 1
- $p * (p - 1) * \ldots * 1$
- $p - 1$
- p

83. $\varphi(36)$ is equal to

- 12
- 5
- 17
- 35

84. Consider system of congruences: $x = 1 \pmod 2$, $x = 2 \pmod 3$, $x = 3 \pmod 5$, $x = 4 \pmod 7$, $x = 5 \pmod{11}$. Then the least residue of x modulo $2 * 3 * 5 * 7 * 11$ is equal to ______

1523

85. The Sieve of Eratosthenes is one of the oldest known algorithms for ___________ testing.

primality

86. The field $\mathbf{Q}_p$ of p-adic numbers can be defined as the _____________ of the field $\mathbf{Q}$ of rational numbers with respect to the p-adic absolute value.

completion

87. The field $\mathbf{Q}_p$ of p-adic numbers can be defined as the _____________ of the ring $\mathbf{Z}_p$ of p-adic integers.

field of fractions

5. Unit: Group Theory.

88. ____________ operation $*$ on a set A is a mapping $A \times A \to A$.

Binary

89. A binary operation $*$ on a set A is called______________ if for all a, b, c from A it holds that $(a * b) * c = a * (b * c)$.

associative

90. A group called __________ if group operation is commutative.

abelian

91. An element e from set A is called _____________ element for the binary operation $*$ on A if for all a from A it holds that $a * e = e * a = a$.

neutral

92. Do non-zero residue classes modulo 4 form a group with respect to multiplication?

- yes
- no

93. Let G and H be groups.__________________ is a mapping $f\colon G \to H$ such that for all a, b from G it holds that $f(a \cdot b) = f(a) \cdot f(b)$.

Homomorphism

94. The __________ of the homomorphism $f\colon G \to H$ (denoted $\operatorname{Ker} f$) is the set of all a from G such that $f(a) = e$, where e denotes neutral element of H.

kernel

95. The _________ of the homomorphism $f\colon G \to H$ ($\operatorname{Im} f$) is the set of all b from H such that $b = f(a)$ for some a from G.

image

96. Bijective homomorphism of a group into itself is called

- monomorphism
- epimorphism
- isomorphism
- automorphism

97. For finite groups $|G|$ is called the __________ of the group G.

order

98. The order of symmetric group S_n is

- n
- $n!$
- $n * (n - 1) / 2$
- 2^n

99. A subset H of a group G is called ____________ if H is also is group under the same operation .

subgroup

100. Cayley's theorem states that every group G is isomorphic to a subgroup of

the______________ group on G.

symmetric

101. The group G is said to be __________ if there exist element a of G such that every element x of G can be written as a power of a.

cyclic

102. Definition of a group by means of set of generators and set of relations is called __________________ of a group.

presentation

103. One of the most widely used algorithm for coset enumeration is the ___________________ algorithm.

Todd–Coxeter

104. Point out which of the following properties correspond to which type of homomorphism $f: G \to H$:

1. $\operatorname{Ker} f = e$.
2. $\operatorname{Im} f = H$.
3. $\operatorname{Ker} f = e$ and $\operatorname{Im} f = H$.

1. f is an epimorphism.
2. f is a monomorphism.
3. f is an isomorphism.

1 – 2, 2 – 1, 3 – 3.

105. If G is finite group and H is a subgroup of G then the order of H is the __________ of the order of G.

divisor

106. Subgroup H of a group G is called ___________ subgroup if $gH = Hg$ for all g from G.

normal

107. The kernel of any homomorphism is a ______________ subgroup.

normal

108. We say that group A is a ____________ of groups B and C if every element a from A can be written uniquely in the form of $a = b + c$ with b from B

and c from C.

direct sum

109. ______________of two groups G_1, G_2 is the set $G_1 \times G_2$ with pairwise operation.

Direct product

110. Finite abelian p-group is said to be of ___________ $(p^{r_1}, \ldots, p^{r_s})$ if it is isomorphic to direct product of cyclic groups of orders p^{r_i}.

type

111. A group G is called free if there is a presentation of if with set of ____________ being empty.

relations

112. There is precisely ______ Tietze transformations.

4

6. Unit: Graph Theory.

113. The order of a graph is the number of its _________________ .

- vertices
- edges
- Euler cycles

114. A graph's size is the number of its __________________ .

- vertices
- edges
- Hamiltonian cycles

115. The ______________________ of a vertex v in a graph G is the maximum distance from v to any other vertex.

Eccentricity

116. The _______________ between two vertices u and v in a graph G is the length of a shortest path between them.

distance

117. A _______________ graph is a graph without loops and multiple edges

where each vertex has the same number of neighbors

- complete
- regular
- planar
- Hamiltonian

118. __________ graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that its edges intersect only at their endpoints.

- complete
- regular
- planar
- Hamiltonian

119. A _________ graph is a graph that consists of a single cycle, or in other words, some number of vertices connected in a closed chain.

cycle

120. __________________________ of two graphs G_1, G_2 is a graph G defined as follows. Vertex set V(G) is equal to Cartesian product V(G_1) × V(G_2) of vertex sets of G_1 and G_2. There is an edge between (u_1, u_2) and (v_1, v_2) in G if and only if $(u_1 = v_1$ and $u_2 \sim v_2)$ or $(u_1 \sim v_1$ and $u_2 = v_2)$ where ~ denotes "is connected by an edge to".

Cartesian product

121. A Hamiltonian cycle in a Hamiltonian graph of order 24 has _____ edges.

- 12
- 24
- 23
- none of the above

122. A spanning tree for a simple graph of order 24 has ____ edges.

- 12
- 6

- 23
- none of the above

123. The name of the command to create graph in Sage is

- graph
- Graph
- make_graph
- create_graph

124. In order to list edges of the graph g in Sage you should type in

- g.edges()
- g.Edges()
- g.get_edges()
- g.getEdges()

125. In order to draw graph g in Sage you should type in:

- g.plot()
- g.draw()
- g.show()
- g.print()

126. The ____________ matrix of a finite graph G on n vertices is the $n \times n$ matrix where the non-diagonal entry a_{ij} is the number of edges from vertex i to vertex j.

adjacency

127. The ____________ matrix of G is a $p \times q$ matrix (b_{ij}), where p and q are the numbers of vertices and edges respectively, such that $b_{ij} = 1$ if the vertex v_i and edge x_j are incident and 0 otherwise.

incidence

128. In order to create graph from the matrix of desired type in Sage you should use graph constructor with two parameters, the matrix and the named parameter ______________.

format

129. Plotting using the Circular Layout, which places all the vertices on a circle could be performed by passing plot() method of a graph named parameter layout with value _______________ .

circular

130. Expression that yields cycle graph of order 9 in Sage is ______________________________

graphs.CycleGraph(9)

131. In order to get LaTeX code for your gaph g in Sage you should do:

- latex(g)
- g.latex()
- g.getLatex()
- latexCode(g)

132. The graph method for finding Hamiltonian cycle on given graph is called ________________________________

hamiltonian_cycle

133. All paths from vertex 0 to vertex 5 on undirected graph g in Sage could be obtained by performing:

- all_paths(g, 0, 6)
- g.all_paths(0, 6)
- g.get_all_paths(0, 6)

134. Distance between 1-st and 6-th vertices of the graph g in Sage could be obtained by performing ________________________ command.

g.distance(1, 6)

135. The name of the graph method yielding all distances on graph in Sage is ____________________________ .

distance_all_pairs

136. The name of the graph method in Sage to determine whether current graph isomorphic to other one is ________________________ .

is_isomorphic

137. The name of the graph method in Sage to construct product of graphs is ____________________________ .

cartesian_product

138. The Canadian Traveller Problem is a generalization of the shortest path problem to graphs that are ________________ observable.

partially

139. Kuratowski's theorem states that a finite graph is planar if and only if it does not contain a subgraph that is homeomorphic to _____ or $K_{3,3}$.

K_5

ADVANCED LEVEL

1. Unit: Introduction to Python programming.

140. Python programming language was named after

- a group of snakes found in Africa and Asia
- serpent, the earth-dragon of Delphi
- orator, diplomat of Philip II of Macedon
- television series “Monty Python's Flying Circus”

141. The way of exiting Python interpreter in a platform independent way is to type in

- Ctrl+Z
- Ctrl+D
- quit()

142. Source code files for Python should be encoded only with

- ASCII
- UTF-8
- UTF-16
- any supported by your implementation encoding with pointing it in first line of the file with special comment: # -*- coding: *encoding* -*-

143. docstring is a

- special API for proccessing MS Word documents in Python

- documentation chapter for string type in Python
- string literal on the first line of function definition in Python
- tool for automatic indentation of strings in source code

144. Parameters passing strategy in Python is the
    - call by value
    - call by name
    - call by need

145. You may use list as ____________ through append and pop methods. (Here append acts as conventional push operation.)

stack

146. Tuples could be involved in group _________________ .

assignment

147. In order to avoid KeyError you may use ____________ method of dictionary instead of []-operation.

get

148. ‘else’ clause could be part of
    - if statement
    - for statement
    - while statement
    - return statement

149. Constructor in Python class is a special function denoted by
    - ‘constructor’ keyword
    - the name equal to class name
    - __init__

2. Unit: Sage Commons.

3. Unit: Combinatorics.

4. Unit: Number Theory.

150. If $a * s + b * t = 1$ then gcd(a, b) is equal to

- 0
- 1
- $a*b$
- $\max(a, b)$
- the answer could not be determined for all a, b uniformly

151. In computational complexity theory, the formal language corresponding to the prime numbers is denoted as

- P
- NP
- PRIMES
- co-NP

152. Dramatic difference of p-adic absolute value $|\cdot|_p$ and usual Euclidian absolute value $|\cdot|$ on rational numbers is that $|\cdot|_p$ is ______________________ absolute value.

non-archimedean

153. The Theorem by ________________ states that every non-trivial absolute value on $\mathbf{Q}$ is equivalent to either the Euclidian absolute value $|\cdot|$ or to the p-adic absolute value $|\cdot|_p$ for some prime number p.

Ostrowski

154. If a_i are integers from 0 to $p-1$ for all i from 0 to infinity and $-1 = a_0 + a_1 p + a_2 p^2 + \ldots$ then all a_i are equal to

- 0
- 1
- $(p-1)/2$
- $p-1$

155. What rational number has the 5-adic expansion $1 + 3 * 5 + 1 * 5^2 + 3 * 5^3 + \ldots$? ________

$-2/3$

156. Let $x = a_0 + a_1p + a_2p^2 + \ldots$ be integer p-adic number. Point out which of the following properties correspond to which type of integer p-adic number.

1. All but finitely many a_i are equal to $p - 1$.
2. $\{a_i\}$ is periodic sequence.

1. x is a negative number.
2. x is a rational number.

1 – 1; 2 – 2.

157. 3 have square root in $\mathbf{Q}_p$ for $p > 3$.

- yes
- no
- the answer could not be determined for all $p > 3$ uniformly

5. Unit: Group Theory.

158. If we try to give presentation of cyclic group of order n with the least number of generators, the number would be

- 1
- $n * (n - 1) / 2$
- $n - 1$
- n

159. Which of the following operations has neutral element:

- composition of functions on the set of all functions from set X to X
- composition of functions on the set of all bijective functions from set X to X
- max(a; b) on the set of real numbers **R**
- max(a; b) on the set of nonnegative real numbers
- vector product of vectors in 3-dimensional real vector space
- gcd(a; b) on the set of natural numbers **N**
- gcd(a; b) on the set of natural numbers with zero **N** U {0}

160. Let p be a prime number. Sufficient condition for group G to be isomorphic to $\mathbf{Z}_p$ is that

- $|G| = p$
- G has no proper subgroups
- G is a subgroup of symmetric group S_n

161. Let n be a natural number. Sufficient condition for cyclic group G to be isomorphic to $\mathbf{Z}_n$ is that ______

- $|G| = n$
- G has no proper subgroups
- G is a subgroup of symmetric group S_n

162. Every cyclic group with infinite number of elements is isomorphic to the additive group of _____________ numbers.

integer

163. Let A be an abelian group, B and C – its subgroups. Let A be a direct product of B and C. Sufficient condition for A being direct sum of B and C is

- $B \cap C = \{0\}$
- B is a subset of C or vice versa
- $B = \{0\}$ or $C = \{0\}$

164. For every $n > N$ there is at least two non-isomorphic groups of order n . N is equal to

- 1 000 000
- 1024
- 42
- no such number

165. Is intersection of two subgroup of a group is the subgroup of the group?

- yes
- no
- if and only if those two subgroups are normal

166. $\mathbf{Z}_{64}$ has exactly ____ subgroups.

7

167. Suppose that besides e group Z_n contains only elements of orders 3, 5, and

n. What is n? ______

15

168. Let G be a cyclic group with exactly two non-trivial proper subgroups, H_1 and H_2 , and let $|H_1| = 5$ and $|H_2| = 25$. What is $|G|$? ______

125

169. Let a be a group element that has an infinite order. Let $i \neq j$ and $<a^i>=$ $<a^j>$, What is j? _______

$-i$

170. $(\mathbf{Z}_7)^*$ with multiplication modulo 7 is isomorphic to $\mathbf{Z}_6$. How many isomorphisms do they have? _____

2

6. Unit: Graph Theory.

171. Suppose G is a graph of order n and deg(v) greater or equal than $(n-1)/2$. Then G is

- connected
- disconnected
- complete

172. One of basic algorithms for solving shortest path problem is the _____________ algorithm.

Dijkstra's

173. One of the basic heuristic algorithms for solving shortest path problem is the _____________ algorithm.

A* search

174. The number of spanning trees in the complete graph K_8 is

- 48
- 6^8
- 8^6
- none of the above

175. A complete bipartite graph $K_{3,3}$ has ________ spanning trees.

- 27
- 81
- 6
- none of the above

176. The order of a forest with 17 vertices and 4 components is

- 17
- 4
- 16
- none of the above

177. The number of different labelled trees of order n is

- n^n
- $(n-2)^n$
- n^{n-2}
- none of the above

178. If G is a connected plane graph of order v, size e and with f faces, then

- $v-e+f=2$
- $e-v+f=2$
- $v+e-f=2$
- none of the above

EXPERT LEVEL

1. **Unit: Introduction to Python programming.**
2. **Unit: Sage Commons.**
3. **Unit: Combinatorics.**
4. **Unit: Number Theory.**

179. Wilson's theorem state that for any prime integer p the least residue (by absolute value) of $(p-1)!$ modulo p is equal to ______

-1

180. The Miller–Rabin and Solovay–Strassen tests are most widely used ____________________ primality test rather than deterministic ones.

probabilistic

181. It is known that PRIMES is in _________ complexity class.

P

182. For any non-zero rational x it is hold that $|x| * \Pi_p|x|_p$ = _____ (where $\Pi_p|x|_p$ denotes the product of p-adic absolute values $|x|_p$ for each prime p and $|x|$ is the usual Euclidian absolute value).

1

183. Topological space of integer 2-adic numbers $\mathbf{Z}_2$ is homeomorphic to the

- Cantor set
- Mandelbrot set
- set of complex numbers

184. The field of p-adic numbers $\mathbf{Q}_p$ and the field of real numbers $\mathbf{R}$ have following algebraic connections:

- they are isomorphic
- they are not isomorphic
- $\mathbf{Q}_p$ could be monomorphically embeded into $\mathbf{R}$
- $\mathbf{R}$ could be monomorphically embeded into $\mathbf{Q}_p$

5. Unit: Group Theory.

185. If G is a non-abelian group of order 6 then it is isomorphic to the group of all __________________ of three elements.

permutations

186. Dihedral group D_n can be generated by _____________ and reflection s.

rotations

187. The center of dihedral group D_n has at most _____ elements.

2

188. How many normal subgroups has alternating group A_n, $n > 4$, besides

itself and (e)?

- 0
- 1
- $\varphi(n)$, where φ denotes Euler's function
- $n\,(n-1)\,/\,2$

189. Let G be the group of all $n \times n$ diagonal matrices with ± 1 in the diagonal entries. Them G isomorphic to

- $(\mathbf{Z}_2)^n$
- $\mathbf{Z}_{(2^n)}$
- none of the above

190. Let G be a finite abelian group . Sufficient condition for $f(a) = a^n$ $(n > 0)$ to be G-automorphism is that

- n divides $|G|$
- $|G|$ divides n
- $\gcd(n, |G|) = 1$

191. Let G be a group. All its subgroups of index _____ are normal.

2

192. The number of non-isomorphic groups of order 8 is

- 5
- 6
- 7
- 8

193. All groups of order $2p$ are either __________ or dihedral.

cyclic

194. Consider dihedral group D_{12}. Let R_i, $i = 0, 1, \ldots, 11$ be a rotation by a $i\pi\,/\,6$ and F_i be a reflection with i and $i + 6$ fixed. Whether following element of D_{12} rotation or reflection: $R_2\,R_7\,F_2\,F_7\,F_6\,R_2\,F_5\,R_9\,F_6$

- rotation
- reflection

195. How many automorphisms does $\mathbf{Z}_3 \oplus \mathbf{Z}_5$ have? ________

8

196. Let G be group and H and K its subgroups of orders 55 and 25, respectively. Let $H \cap K$ is not trivial. What is $|H \cap K|$? _____

5

6. Unit: Graph Theory.

197. How many Hamilton circuits does the K_5 (complete graph with 5 vertices) have ? ______

24

198. Consider the Prüfer sequence, $S = (1, 7, 1, 5, 2, 5)$. Let T be the labelled tree corresponding to S. The size of T is

- 2
- 6
- 5
- 7
- none of the above

199. The hypercube graph Q_5 is planar.

- true
- false

200. The complete bipartite graph graph $K_{4,3}$ is non-planar.

- true
- false

201. If G is a simple connected 3-regular planar graph where every face is bounded by exactly 3 edges, then the size of G is

- 3
- 4
- 6
- 5

- none of the above

202. The chromatic number of the cyclic graph C_{15} is
    - 3
    - 2
    - 6
    - 15
    - none of the above

203. The chromatic number of the cyclic graph K_{15} is
    - 3
    - 2
    - 6
    - 15
    - none of the above

TEST ANSWERS (KEYS)

1	**2**	**3**	**4**	**5**
interpreted	dynamically	2	2	1
6	**7**	**8**	**9**	**10**
3	1	2	1	0
11	**12**	**13**	**14**	**15**
1	2	escaping	1	u
16	**17**	**18**	**19**	**20**
1	1 – 3, 2 – 2, 3 – 1, 4 – 4.	4	1	1
21	**22**	**23**	**24**	**25**
range	3	3	1	2
26	**27**	**28**	**29**	**30**
key	1	del	3	cannot
31	**32**	**33**	**34**	**35**
2	codecs	1	1, 2, 3	1
36	**37**	**38**	**39**	**40**
3	1	solve	var	list
41	**42**	**43**	**44**	**45**
find_root	diff	diff	plot	parametric
46	**47**	**48**	**49**	**50**
1	text	iter	next	yield
51	**52**	**53**	**54**	**55**
Dyck	DyckWords	Catalan	height	1, 4, 6
56	**57**	**58**	**59**	**60**
comparable	Poset	function	Transitive reduction	Hasse diagram
61	**62**	**63**	**64**	**65**
Posets	A000112	congruent	2	6
66	**67**	**68**	**69**	**70**
7	3	coprime	greatest common divisor	14
71	**72**	**73**	**74**	**75**
67	2	1	2,4	4
76	**77**	**78**	**79**	**80**
Extended	1	5	2	Multiplicative
81	**82**	**83**	**84**	**85**
coprime	3	1	1523	primality
86	**87**	**88**	**89**	**90**
completion	field of fractions	Binary	associative	abelian
91	**92**	**93**	**94**	**95**
neutral	2	Homomorphism	kernel	image
96	**97**	**98**	**99**	**100**
4	order	2	subgroup	symmetric
101	**102**	**103**	**104**	**105**
cyclic	presentation	Todd–Coxeter	1 – 2, 2 – 1 3 – 3	divisor

106	**107**	**108**	**109**	**110**
nornal	nornal	direct sum	Direct product	type
111	**112**	**113**	**114**	**115**
relations	4	1	2	Eccentricity
116	**117**	**118**	**119**	**120**
distance	2	3	cycle	Cartesian product
121	**122**	**123**	**124**	**125**
2	3	2	1	1,3
126	**127**	**128**	**129**	**130**
adjacency	incidence	format	circular	graphs.CycleGraph(9)
131	**132**	**133**	**134**	**135**
1	hamiltonian_cycle	2	g.distance(1, 6)	distance_all_pairs
136	**137**	**138**	**139**	**140**
is_isomorphic	cartesian_product	partially	K_5	4
141	**142**	**143**	**144**	**145**
3	4	3	1	stack
146	**147**	**148**	**149**	**150**
assignment	get	1, 2, 3	3	2
151	**152**	**153**	**154**	**155**
3	non-archimedean	Ostrowsky	4	– 2 / 3
156	**157**	**158**	**159**	**160**
1 – 1, 2 – 2.	3	1	1, 2, 4, 7	1
161	**162**	**163**	**164**	**165**
1	integer	1	4	1
166	**167**	**168**	**169**	**170**
7	15	125	$-i$	2
171	**172**	**173**	**174**	**175**
1	Dijkstra's	A* search	3	2
176	**177**	**178**	**179**	**180**
1	3	1	– 1	prbabilistic
181	**182**	**183**	**184**	**185**
P	1	1	2	permutations
186	**187**	**188**	**189**	**190**
rotations	2	1	1	3
191	**192**	**193**	**194**	**195**
2	1	cyclic	2	8
196	**197**	**198**	**199**	**200**
5	24	4	2	1
201	**202**	**203**		
3	1	4		

GRADING POLICY
(SCALING TEST GRADES TO TREDITIONAL ONES)

Test grading could be evaluated on 1 correct answer – 1 point basis. So 202 points is the maximum.

50% correct answers — C grade.

70% correct answers — B grade.

More than 80% correct answers — A grade.

TEST HOLDING RECOMENDATIONS

Timing should be computed from the 1 task – 3.5 minutes base.

If testing is being hold in computer class then tests are to check by computers. Otherwise test are to check by teacher with the kyes.

If testing holds without assistance then checking should be performed only after test completion.