

Versatile Robust Clustering of Ad Hoc Cognitive Radio Network

Di Li, *Member, IEEE*, and James Gross, *Member, IEEE*

Abstract—In this paper, we will see the application of congestion game in solving the channel allocation problem in the context of TV white space. The channel allocation problem we will address is a general problem, as the transmission power is not identical for every transmitter and on each channel, actually, the transmission power could be unique for each transmitter-channel combination. With the suitable utility function designed for transmitters, the behaviours of the transmitters can be described by a congestion game. The algorithm of channel allocation is derived from the dynamics of the transmitter in the game, which reaches Nash equilibrium quickly.

Furthermore, we provide a complete solution to fully exploit TV white space complying with IEEE 802.22 standard. We propose a centralized methods to regulate the upper bound of transmission power, so that to strictly protect the primary users. The the distributed channel allocation and power control are conducted sequentially.

Index Terms—TV white space, IEEE 802.22, channel allocation

1 INTRODUCTION

Wireless data communication needs to access radio waves, known as spectrum. Spectrum is divided into different frequency bands, known as channels, and then allocated for different types of use. The majority of the spectrum is allocated with licences, for instance licenses to military units or mobile operators. A small portion is made licence exempt, such as the ISM band where Wi-Fi works.

The allocated spectrum is not used in all locations or at all times. Such temporally and spatially unused spectrum is known as white space, and it can be used by other devices and services. The white space devices can change their spectrum use according to the spectrum availability and don't make harm to the licensed users' operation. Because white space devices are cognitive to the radio environment, they are also called cognitive radio users.

Among the white space, the UHF TV band attracts a lot of attention from research, industry and regulatory bodies. Allowing license exempted users to work with TV white space is promising to cope with the scarcity of spectrum resources [1]. Firstly, more unused TV white frequencies become vacant than ever with the ongoing transition from analogue to digital broadcasts. Secondly, the frequencies of TV bands enable broadband access over larger geographic ranges compared to higher frequency bands. WSDs can potentially provide a wide range of services such as hot spot coverage, in home broadband, rural broadband and machine to

machine communications [fcom]. Figure 1 shows the available TV white space in some parts of the east coast of United States.

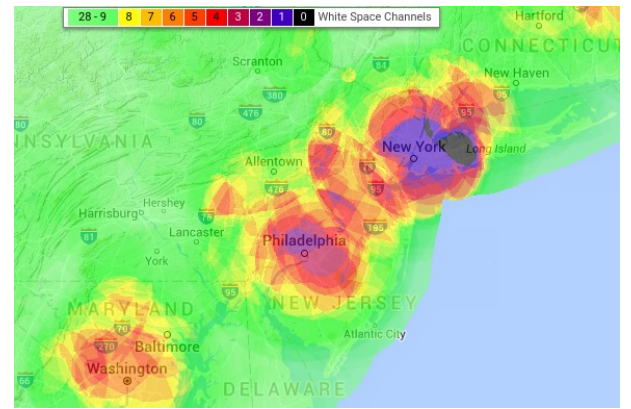


Fig. 1. Variability of available channels in a densely populated area. This figure is obtained from [5]

The devices working with TV white space is call white space device (WSD). Together with academy and industry, the regulatory bodies in different regions have issued regulations or guidelines on the operation of WBSs. Figure 1 shows a high level summary of the regulations decided by Federal Communications Commission (FCC) in United States, Electronic Communications Committee (ECC) in Europe and Office of Communications (Ofcom) in UK respectively. The regulations are different in many aspects due to the special realities in different regions, but they have consensus on two important aspects.

- Firstly, the harmful interference from the white space devices to the TV receivers should be prevented. In this regard, a WBS should work on proper channels and with suitable transmission power according to its location and the locations of the digital terrestrial television (DTT) receivers.
- Secondly, white space databases (WSDBs) should be available. Some WSDs should access WSDB to obtain a list of channels (and the corresponding maximum transmission power according to ECC and ofcom) which are allowed to use at their location. According to the regulations, WSDs are categorised as master and slave devices.¹ The master WSDs communicate their geographical location and certain technical characteristics about

1. FCC adopts different terminologies, fixed and portable mode II are equivalent to master devices, and portable mode I acts as slave device

TABLE 1
Summery of rules on using TV white space by regulatory bodies

| Specification | FCC | ECC | ofcom |
|--|--|---|---|
| target spectrum range (certain channels in between are excluded) | 54-698 MHz | 470-790 MHz | 470 - 790 MHz |
| categorise of WSDs | Fixed/portable, master/slave | master/slave | master/slave |
| maximum permissible power of WSDs | fixed: 30dBm; portable: 16dBm-20dBm | According to location and WSD characteristics | According to location and WSD characteristics |
| criterion for the transmission power as to protect the DTT receiver | power level at the protected contour | maximal degradation of location probability (e.g. 01.%) | maximal likelihood that the rise in the noise plus interference floor exceeds 1 dB at the edge of DTT coverage (e.g. 10%) |
| database availability | Yes | Yes | Yes |
| Translation from WSD's location to frequency availability happens in | regulatory body | database | database |
| sensing ability required | No | No, but recommended | No |
| Frequency for WSD to update available channel and power | 24 hours | 2 hours | 2 hours |
| information provided by the database to WSD | list of channels | list of channels and corresponding transmission power | list of channels and corresponding transmission power |
| information provided to the database before a WSD operates | location | location and equipment types | location |
| separation from the protected contour | Yes | No | No |
| Consideration of out of band emissions | Yes | Yes | Yes |

the devices such as device model or antenna height to the WSDB, which in turn inform the master WSDs which channel to use and at what powers². Then the master WSDs inform their slave devices the operation parameters. With this structure, the database takes over the task of spectrum sensing for the WSDs, as a result, the design of WSDs becomes simpler and the devices are more affordable.

IEEE 802.22 is proposed in response to the rules issued by the regulatory bodies. It adopts a master/slave structure for the WSDs, except for the geolocation capability, it requires the WSDs to be capable of detecting the incumbent services on working channel. According to the rules of ECC and ofcom, In the database, the transformation of the reported information from WSDs and the information on incumbent services, i.e., location, devices types, into the list of allowed frequencies and associated transmit powers for WSDs is conduct. As spectrum sensing is not mandatory to be deployed on WSDs by the regulatory bodies, this translation is critical to the co-existing issue. The co-existing issue includes two aspects. The first, the operation of WSDs should not be harmful to the incumbent services. The second, WSDs should choose proper channel and power, which are obtained from the WSDB, to mitigate interferences among them. The regulatory bodies regulate prudent rules on the transmission power and working channel, so as to prevent the co-channel interference and adjacent channel leakage to cause harmful interference to incumbent services. But they don't regulate the co-existing problem among the WSDs.

In this paper, we consider the co-existing problem not only between WSDs and incumbent systems, but also among WSDs. After obtaining the the list of available channels and associated maximum permitted transmission power from a WSDB, WSDs need to decide which channel and power level to use. We propose a distribution solution to offload the functionality for co-existing

among the WSDs from the WSDB to the WSDs. There are two reasons for implement distributed solutions. First, distributed solution complies with the current regulations, i.e., it is the WSDs who decide their operation parameters which don't go beyond the information from the WSDB. Second, choosing channel and power is a complex optimization problem. As WSDs' location and types are different, the spectrum available and corresponding maximum transmission powers are accordingly different. Hence, the co-existing problem among the WSDs is a resource allocation problem with special conditions, i.e., different available channels and different transmission power levels. The channel allocation problem, which is a sub-problem of the resource allocation problem, is NP hard. Thus we let the channel and power allocation functionality be completed by the master WSDs.

The rest of the paper is organized as follows. we elucidate the system model in Section II, afterwards related work and problem formulation is presented in Section III. In Section IV, we discuss how to utilize the white space sufficiently by setting the transmission powers based on a convex problem formulation. We analyze the spectrum allocation problem under game theoretical framework and propose an algorithm in Section V, thereafter performance evaluation is presented in Section VI. Finally, we conclude our work and point out directions of future research in Section VII.

2 SYSTEM MODEL AND PROBLEM STATEMENT

We consider a group of WSDs working in an area, and there is one database available. The WSDs work with the master/slave model, among of them, the WSDs which work as masters can access the database to obtain permitted channel and associated maximum transmission power. The WSDs which work as slaves communicate with their corresponding master to obtain the operation parameters. This model is different from that defined by IEEE 802.22 standard at the WSDs don't need to be capable to sense the

2. FCC only requires to return channels.

spectrum. The master WSDs are deemed to be static, and they are interested in payload data communication with their associated terminals. In the rest of the chapter, we use the term master WSD and secondary base station (WBS) interchangeably.

We consider the DTT receivers as incumbent users. To protect them from harmful interference generated by the WSD devices, a series of interference measurement devices are deployed at the contour of the DTT service area, which represent the worst located TV receivers in the TV service areas. For these interference measurement devices, an interference threshold should not be violated by the noise generated by the secondary users.

The database is aware of the location and equipment types of the WSDs and the interference measurement devices. An example of the considered system is shown in Figure 2

As to the notations, the collection of WSDs is denoted as \mathcal{N} where $|\mathcal{N}| = N$. The TV white spectrum includes multiple channels which are denoted as \mathcal{C} , the channels are assumed to be identical in terms of attenuation and shadowing on the same path. When two WSDs working on the same channel, co-channel interference is caused on each. Neighbouring channel interference is not considered in our model. We represent the usage of channel for WSD i with a binary vector $X_i^{|\mathcal{C}| \times 1} = \{\dots, x_{ik}, \dots\} \in \{0, 1\}^{|\mathcal{C}|}$, where $k \in \mathcal{C}$ and binary variable x_{ik} denotes whether channel k is used by user i . As each node can only use one channel, for X_i , there is $\sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1$. $c(i)$ denote the channel being used by a WSD $i \in \mathcal{N}$. The transmission power of WSD i on channel c is P_i^c .

The attenuation between a WBS i and one its associating terminal m is denoted as h_{im} , and the attenuation between the interfering WBS j and m is denoted as h_{jm} . The path loss is dependent on the distance between the corresponding equipments, e.g. $h_{im} = K \cdot d_{im}^{-\alpha}$, where α is the path loss exponent, d_{im} is the distance between i and m , K is a constant which models the reference loss over a single unit of distance. N_0 denotes the thermal noise power. Shadowing without fading is considered in our model. z_{im} models the zero-mean log-normally distributed shadow fading between i and m , and the standard deviation is σ_{SH} .

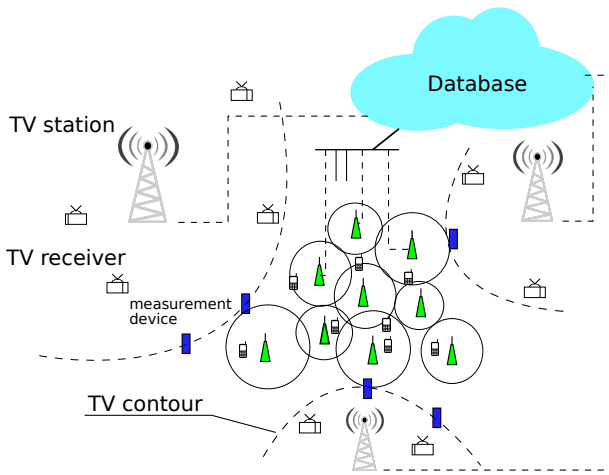


Fig. 2. System model: WBS cells and DTV systems

2.1 Problem Statement

Our goal is to design distributed solution for WSDs to utilize the TVWS. To achieve this, the master WSDs need to choose proper

channel and transmission power level, so that their operation doesn't cause harmful interference to DTTs, and the communication among them is improved.

Given γ_i is the performance metric of the WBSs, in order to ensure the fairness among WBSs, the optimization formulation we are going to solve is shown as Formation 1. Instead of maximizing the sum of the metrics over all WBSs, we minimize the sum of inverted metrics.

$$\begin{aligned} & \text{Minimize} && \sum_{i \in \mathcal{N}} \frac{1}{\gamma_i} \\ & \text{subject to} && \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1 \\ & && P_{i,min}^c \leq P_i^c \leq P_{i,max}^c, c \in \mathcal{C}, i \in \mathcal{N} \end{aligned} \quad (1)$$

where $P_{i,min}^c$ and $P_{i,max}^c$ are the minimal and maximal transmission power of the transmitter of WBS i , where are restricted by the hardware configuration or capabilities. We assume $P_{i,min}^c$ and $P_{i,max}^c$ are identical for all WBSs and over all channels.

As to the performance metric for the QoS provisioning between the WSDs, we use the signal to noise and interference ratio (SINR) on the terminals. SINR is the ratio between the received power of signal of interest and the summed interference experienced by the terminal. The sum of all disturbing radio frequency effects (including interference) on terminal m (we assume the working channel is c) is,

$$f_m^c = \sum_j (P_j^c \cdot h_{jm} \cdot z_{jm}) + N_0, \quad j \in \mathcal{N} \setminus i, c(j) = c \quad (2)$$

where P_j^c denotes the transmission power of interfering WBS j . Then the SINR on end terminal m is,

$$\gamma_m = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{f_m^c} \quad (3)$$

QuasiSINR of WBS

Each WBS's utility is a function of the SINR on all its end terminals, i.e., the utility can be the average SINR at all its terminals. When adopting a function of SINR on all terminals as utility, as the terminals are mobile and they are influenced by many factors, i.e., the type of service provided to the terminals, the utility may diverge from the real performance of the terminals. On the other hand, it is not appropriate to choose one [10] or more fixed terminals, and use their SINRs to represent the SINR for all the other terminals in that cell, because their location could diverge greatly with the locations of the other terminals. Thus, we propose a metric *QuasiSINR* to represent WBS's performance on providing services to its end terminals, which is independent on the actual locations of end terminals.

QuasiSINR of a WBS is the ratio between the weakest signal of interest on a reference point and the summation of the strongest interference caused on the relevant reference point. We need an auxiliary circle to construct the reference point for each WBS, which is shown in Figure 3. As to WBS i , the auxiliary circle is the dashed circle centred at WBS i , whose radius is δ . Assume WBS i and all the other WBSs work on the same channel c , then co-channel interference are caused on its end users by all the other WBSs. The intersection of the auxiliary circle and the connecting line between WBS i and one interfering WBS j , which is shown as red dot, is a reference point which corresponds to the interfering WBS j . There are multiple reference points on the auxiliary circle, which corresponds to the co-channel interfering

WBSs respectively. The power of signal from i on the auxiliary circle, the green dot, is the reference point for the power of signal of interest. We can see both of the reference point of interference and the reference point for the power of signal of interest are largely decided by the radius of the auxiliary circle δ .

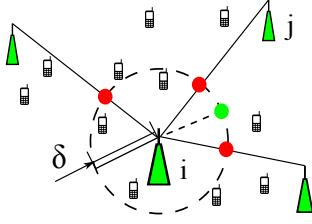


Fig. 3. QuasiSINR is the ratio between the power of signal of interest on the green point with the sum of co-channel interference on the red points

The co-channel interference on the reference point of WBS i from WBS j is,

$$f_{ji}^c = P_j^c \cdot h_{ji} \cdot z_{ji} = P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{ji} \quad (4)$$

where d_{ji} is the distance between WBS i and j , while, h_{ji} and z_{ji} are the attenuation and shadowing from WBS j to the relevant interference reference point. The sum of interference on WBS i ' interference reference points is denoted as f_i^c .

$$f_i^c = \sum_{j \in \mathcal{N}, c(j)=c} f_{ji}^c \quad (5)$$

The power of the signal of interest on auxiliary circle is,

$$\tilde{P}_i^c = P_i^c \cdot h_i \cdot z_i = P_i^c \cdot \delta^{-\alpha} \cdot z_i \quad (6)$$

where h_i and z_i are the attenuation and shadowing from i to any point on the auxiliary circle. Then the quasiSINR of WBS i is denoted as γ_i ,

$$\begin{aligned} \gamma_i &= \frac{\tilde{P}_i^c}{f_i^c + N_0} = \frac{P_i^c \cdot h_i \cdot z_i}{\sum_{j \in \mathcal{N}, c(j)=c} (P_j^c \cdot h_{ji} \cdot z_{ji}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z_i}{\sum_{j \in \mathcal{N}, c(j)=c} (P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{ji}) + N_0} \end{aligned} \quad (7)$$

The radius of the auxiliary circle δ can be adjusted to let WBS foster better service to the terminals in different area. For instance, to take care of the SINR on the border area of the cell, the radius δ can be set as the distance between WBS i and the furthest associated terminal. When the terminals concentrate towards to the WBS, δ can be set smaller to better fit to the terminals' distribution. Because of the auxiliary circle, the interaction between co-channel WBSs are independent on the location of individual end terminals. As a result, the concrete terminals are excluded from the channel and power allocation problem, which simplifies the problem to be solved. Based on Formula 7, a WBS i calculates its quasaiSINR when the locations and transmission powers of other co-channel WBSs, the radius of auxiliary circle and the characteristics of radio environment such as parameters of attenuation and shadowing are known. Such information can be obtained by accessing the database.

When a WBS works on different channels, the co-channel interference received by its end users from other WBSs is different. In order to provide better service to its end users, WBS is motivated to choose the channel which either permits higher

TABLE 2

| Symb. | Description |
|-----------------|--|
| γ_i | QuasiSINR of WBS i |
| f_{ji}^c | Interference caused by WBS j on the interference reference point of WBS i , when they work on channel c |
| f_i^c | The sum of interference caused on the interference reference points of WBS i |
| \tilde{P}_i^c | The power of the signal of interest at the auxiliary circle of WBS i which works on channel c |
| P_i | The transmission power of WBS i |
| h_{ij} | The attenuation from the co-channel interfering WBS j to the corresponding interference reference point of WBS i . |
| h_i | The attenuation from the WBS i to its auxiliary circle. |
| z_{ij} | The shadowing from the co-channel interfering WBS j to its interference reference point of WBS i . |
| z_i | The shadowing from WBS i to its auxiliary circle. |

transmission power or experiences less interference, or the channel compromising the two factors according to Formula 3. Achieving optimal white spectrum allocation in a distributed style is the goal of this work, furthermore, this distributed solution should converge fast and lead to an efficient and stable solution.

2.2 Problem Solution

We obtain the maximal transmission power over each channel for every WBS before dealing with channel and power allocation, afterwards, the transmission power should not exceed this power mask. The maximal transmission power over all channels for each WBS is obtained at the centralized database. As the database has the global info of both the secondary network and primary network, the attenuation and shadowing between any two users are known i.e., attenuation is based on propagation model and shadowing is obtained from measurement, the database is able to guarantee the service of primary users not be interfered when all the WBSs work on the same channel. Meanwhile, as the secondary users belong to different groups of interest, the channel and power allocation should be done in distributed manner.

In summery, we solve the channel and power allocation in downlink communication in IEEE 802.22 network by solving three sequential subproblems:

- Firstly, given a set of secondary WBSs and their geo-locations, the maximum permitted transmission power on each channel for each WBS is determined, so that the interference margin of primary users can not be exceeded no matter how WBSs utilize the spectrum and power resources. In other words, the dynamics in the secondary network is transparent to the primary system.
- Secondly, once the maximum permitted transmission power is determined, WBSs choose their operating channels. Note that this channel assignment problem is different from the works available in literature, where the transmission power is identical over different channels for different WBSs. In this subproblem, the maximum permitted transmission power P_i^c could be different for different channel $c \in \mathcal{C}$ and different WBS $i \in \mathcal{N}$.
- Thirdly, working with the maximal permitted transmission power may not be the optimal in terms of power consumption and the SINR on terminals, thus distributed power adjustment is conducted and the working channel is unchanged.

The solution to the channel allocation subproblem is named as whiteCat (white space Channel allocation).

In the following, we introduce the related works of spectrum and power allocation problem in CRN, especially the works about the usage of TV white space. we also introduce the works related with subproblems mentioned above.

3 RELATED WORKS

3.1 Resource Allocation in CRN

We will emphasis more on the distributed solutions, but in order to give readers a full picture of the solutions as to resource allocation in CRN, we will also introduce the related works on centralized solutions.

Centralized solutions usually solve the formulated constrained optimization problems at a centralized unit. In [11], the objective is to increase the number of supported terminals whose SINRs are above a threshold, and the constrains are to refrain the interference at the primary users within a certain margin. Work [12] minimizes the transmission power and meanwhile makes sure the SINR of terminal is above a threshold, but this work fails to consider the protection of primary users. A heuristic algorithm is proposed in [13], which considers the channel availability and transmission demand of each WBS. The aforementioned two schemes don't consider varying the transmission power.

There is a large variety of distributed solutions. In order to avoid or to alleviate co-channel interference between cells, and to allow arbitrary number of cells to work in IEEE 802.22 network, [14] proposes distributed inter-network spectrum sharing scheme, where contention decisions are made in a distributed way and the winner cells can use the shared channels. But this work doesn't consider the role of transmission power in the co-channel interference. An distributed power allocation (single channel) scheme based on learning for secondary networks is given in [15], where penalty function involving the interference threshold on primary systems is used. [9] discusses power control and channel assignment in both down-link and up-link communication in cellular network. Although the solution is distributed, primary users are required to cooperate with secondary base station in a learning process to decide the transmission power, in addition, there is only one secondary base station considered whereas we need to cope with the multiple cells in our problem. Joint channel-power selection for multiple transmission links (pairs) is investigated in [8]. The authors decompose the Lagrangian dual of the problem, then propose a distributed scheme based on the dual parameters. The scheme converges into pure Nash equilibrium, but in order to facilitate this scheme, monitors are required to watch interference from secondary users, moreover, monitors have to be equipped computational ability and interact with secondary users in the whole process of convergence.

As introduced in Chapter ??, game theory is a powerful tool in designing distributed algorithms. A distributed joint power and channel allocation is proposed in [16], each base station chooses optimal power level and channel to optimize its utility, which results in induced received interference and caused interference on primary users. The execution of this scheme is formulated into an exact potential game. For each base station, after several rounds of best responses in terms of channel and power level, Nash equilibrium is achieved. There are some flaws hindering the application of this scheme. Firstly, the paper doesn't provide means for base stations to obtain the needed information which

is needed to calculate the utility function. Secondly, it is not clear how to calculate the punishment in the utility function, which indicates whether and how much the interference threshold on primary users is violated. Thirdly, the convergence speed of the scheme is not given, in fact, as the problem is formulated into a potential game, converge speed or the number of updates before convergence is a theoretic problem which is still unsolved. Last but not least, as the utility function and the potential in the game are designed as the sum of received and introduced interference, the desired signal power and the punishment, the minimization of this *sum* does indicate meaningful performance metrics, i.e., SINR on terminals, or the total transmission power consumption. In [10], Chen et al. investigate the channel allocation problem in the scenario of TV white space. The channel allocation problem is formulated into a potential game, individual WBS's utility is to maximize the capacity of one single static terminal. Potential game is also adopted in work [17] to design algorithms, which mitigates the adjacent interference. [18] adopts cooperation game to research the coexistence of femtocells. Each femtocell negotiates with neighbouring fremcells, and they form temporary coalition, but the goal of this solution is to allocate resource block in terms of time and transmission power. [12] proposes both centralized and decentralized solutions. Two distributed schemes are proposed, joint channel and power allocation is formulated into a weighted potential game, as an alternative workaround, the problem is solved in two sequential phases.

Distributed algorithm based on Learning is proposed in [19] for LTE to allocate the the resource block in down link, which leads to correlated equilibrium, but slow converge hinders its application.

3.2 Utilization of TV White Space

Here we introduce the solutions proposed on the utilization of TV white space, which includes regulations, proposed standards and recent research advances. In accordance with the regulations of FCC, there are some prototype applications proposed in both cellular network [20], [21] and WiFi-like network [22]. The secondary users access a centralized data base to know the allowed channels and transmission power. Standardization bodies are also working on TVWS utilization, including IEEE 802.22 [23] for Wireless Regional Area Networks (WRAN), IEEE 802.11af [24] for WLAN, IEEE 802.15.4m [25] for 802.15.4 wireless networks in TVWS and 802.19.1 [26] for coexistence methods among local and Metropolitan Area Networks (MAN).

Scientific research on utilization of TVWS goes on in parallel with the regulatory agencies. Feng et al. [27] investigate the business model of TV spectrum utilization in database involved network structure, emphasis on the price policy of the channels approved by FCC. Spectrum sharing in TVWS is formulated as a series of optimization problems. The guarantee that TV receivers should not be affected by the aggregate interferences form TVBDs is one constraint. The objective can be maximizing TVBD's downlink transmission power [2], uplink transmission power [28], or best geographic distribution of TVBDs [29]. A series of works [30], [31], [32], [33], [34] emphasise on interference mitigation among TVBDs via spectrum allocation. Vehicular networks operating with TVWS assisted by TV database and cooperative sensing is discussed in [35]. Work [36] steps further from the database paradigm and makes efforts to utilize the *grey space*, where TVDB is allowed to operate even within the TV service area.

Related Works on Maximal Transmission Power Planning

To protect the TV receivers from harmful interference, the aggregate interference caused by WBSs at the contours of TV receivers should not exceed the interference margin. Work [37] proposes detailed calculations which a geolocation database performs in order to derive location-specific maximum permitted EIRP levels for white-space devices (WSDs) which operate in digital terrestrial TV bands. [2] considers the maximum permitted transmission power for the network which complies with IEEE 802.22 standard. The standard requires a centralized database to store the available channels for each secondary base station, thus centralized scheme can be conducted there after trivial modification. The sufficient condition for the TV receivers not be interfered in the context of TV white space is formulated into a centralized linear programming program (LP) in [2]. The objective function is to maximize the summation of all secondary base stations' transmission power, and the constraints are formed to satisfy the sufficient condition for every interference measuring device for the TV receivers. However, this approach doesn't take the channel assignment problem into account.

Related Works on Channel Allocation with Fixed Transmission Power Level

In our proposed solution, after obtaining the maximum transmission power on each channel, WBSs need to decide one channel to use, and the transmission power is the maximum transmission power, so as to mitigate interference among WBSs and provide the best SINR for their associated end users. Note that in our problem, the transmission power is different for two interfering users when they work on the same channel.

Channel allocation problem dealing with mitigating co-channel interference via channel allocation, which has been attracting plenty of research efforts in the past decade, from multiple channel mesh network [38], Ad hoc network [39] up to cognitive radio network [31], [40].

Channel assignment problem tries to mitigate co-channel interference among users, which can be converted into colouring problem thus is NP hard [38]. Authors of [39] propose heuristic algorithms utilizing best response to improve its welfare, but the transmission power is assumed identical and path loss is deemed as symmetric, which renders this method problematic for our problem where transmission is non-identical and the path loss is asymmetric. [41] formulates channel assignment problem in ad-hoc cognitive radio network into potential game which leads to pure NE, a learning scheme achieving slightly better performance is provided for comparison, but they assume the transmission power is identical and there is no noise in the secondary network, and the proposed random access mechanism demands a huge amount of information to be exchanged, which is a burden for network in ad-hoc structure. [42], [43] investigate the channel allocation problem under game framework in same collision domain, the authors propose algorithms to converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively.

Authors of [39] propose heuristic algorithms utilizing best response based on the welfare on itself to assign channels among users. Simulated annealing is applied to mitigate co-channel interferences in [31]. For the same purpose, no-regret learning [40], [44] is exploit to optimize the choice on channel.

All the available channel allocation schemes are designed under the same assumption, that the transmission power levels are

identical, and the attenuation between any pair is reciprocal. As to our knowledge, there is no work dealing with channel allocation problem where transmission power is different.

4 MAXIMUM PERMITTED TRANSMISSION POWER

The WBSs work in underlay manner and coexist with primary TV stations and receivers, the aggregate generated interference from WBSs on each channel should not exceed the threshold of the TV receivers. We adopt the interference model and the optimization methodology from the work of [4] to plan the maximum transmission power on each channel for WBSs. Having a global view of the propagation parameters, geolocations of WBSs and interference threshold at interference measuring devices which locate on the contour of TV service area, linear programming is implied in the database to calculate the maximum permitted power over each channel.

For WBS $i \in \mathcal{N}$, the maximum transmission power allowed on channel $c \in \mathcal{C}$ is denoted as P_i^c . As to each channel c , the generated interference on each interference measuring device should be within a predefined interference margin I_{pt}^c . The interference margin in a slow fading environment is decided according to [45].

Then the maximum permitted transmission power on channel c for each WBS can be obtained by solving the following optimization problem,

$$\begin{aligned} & \text{Maximize} && \sum_{i \in \mathcal{N}} P_i^c \\ & \text{subject to} && \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z) < I_{pt}^c, \\ & && P_{min}^c \leq P_i^c \leq P_{max}^c \end{aligned} \quad (8)$$

P_{min}^c is the prudent transmission power. P_{max}^c is the maximum transmission power which is restricted by the hardware. z is shadow fading as introduced in 3. Here we only consider the interference caused by WBSs, and omit the interferences from end terminals. Since WBSs' transmission power is higher and their altitude is higher[2], the downlink transmission contributes the major part of interference[46]. The first constraint indicates that the interference margin will not be exceeded even when all the WBSs work on the same channel.

Formula 8 will be solved for each channel $c \in \mathcal{C}$. After solving the $|\mathcal{C}|$ problems, the maximum permitted transmission power vector $\mathbf{P}^c = \{P_1, \dots, P_{|\mathcal{N}|}\}$, $\forall c \in \mathcal{C}$ is obtained.

When working with the same transmission power, the WBSs locating closer to the TV interference measuring devices contribute more to the aggregate interference comparing with the WBSs which locate far from the TV interference measuring devices. Thus when implying linear programming to decide the maximal transmission power, the transmission power used by WBSs which are closer to the TV interference measuring devices is much higher than other WBSs. As a result the maximum permitted transmission power on each channel obtained with LP is seriously unbalanced.

To address this fairness issue, we maximize the sum of the logarithmic value of every WBS's transmission power, and formulate the problem into a convex optimization problem.

$$\begin{aligned} & \text{Maximize} && \sum_{i \in \mathcal{N}} \log P_i^c \\ & \text{subject to} && \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z) < I_{pt}^c, \end{aligned} \quad (9)$$

This optimization will be solved for each channel $c \in C$.

Figure 4 depicts the distribution of maximum permitted transmission power levels obtained in 100 simulations. In each simulation the locations of TV interference measuring devices are randomly decided around the WBSs. In Figure 4, It shows that when applying optimization 8, WBSs' transmission power levels are either the minimum transmission power or the maximum power allowed by the equipment hardware. When applying convex programming, the planned maximum permitted transmission power levels are distributed evenly in between the minimum and maximum power. The gain of SINR on end terminals by applying convex optimization to decide the maximal transmission power is illustrated in the simulation section.

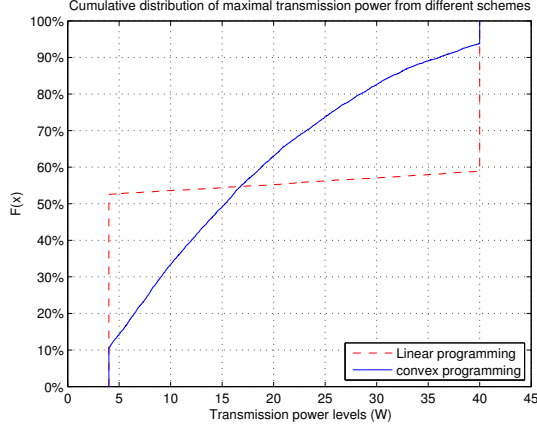


Fig. 4. Distribution of maximum permitted transmission power levels obtained from convex and linear programming formulations

Optimization problem 9 provides the maximum permitted transmission power for each WBS and over each channel. When all the WBSs working on the same channel, the generated interference doesn't exceed the threshold on the interference measurement devices at the contour of TV service area. If there are multiple channels available and WBSs are free to choose their preferred channels, the aggregate interference on one channel will be smaller than that when all WBSs work on that channel. Thus, there is exists a interference margin created by using multiple channels, which provides a room for network dynamics such as new WBS starting to work or increased interference on TV contour due to the variance of broadcast path condition.

5 CHANNEL ALLOCATION WITH FIXED TRANSMISSION POWER

First, we give the centralized solution to obtain the global optimum for this subproblem, then the decentralized scheme under the game theoretic framework is introduced.

5.1 Centralized optimization programming

We formulate the channel allocation problem into a binary quadratic programming problem which can be solved in a centralized way. Let $X_i = \{x_{i1} \cdots x_{ik} \cdots x_{i|C|}\}$ denote the vector of channel usage, there is $|X_i| = |C|$ and binary element x_{ik} represent whether WBS i occupies channel k . For two WBSs i and j , there is,

$$X_i^T X_j = \sum_{k=1}^{|C|} x_{ik} \cdot x_{jk} = \begin{cases} 1 & \text{if } c_i = c_j \\ 0 & \text{if } c_i \neq c_j \end{cases} \quad (10)$$

The power levels across all channels are denoted by a constant vector $P^{|C| \times 1}$, which is possibly nonidentical to WBSs due to different locations. The power used by user i is $P_i^T X_i = \sum_{k=1}^{|C|} P_i^k \cdot x_{ik}$.

Problem 1 can be modeled via general purpose nonlinear optimization:

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^n \frac{\sum_{j \in N, j \neq i} P_j^T X_j (X_i^T X_j) h_{ji} z_{ji} + N_0}{P_i^T X_i h_{ii} z_{ii}} \\ & \text{subject to} && \sum_{k=1}^{|C|} x_{ik} = 1, x_{ik} \in X_i \in \{0, 1\}^{|C|} \end{aligned} \quad (11)$$

Problem 11 is a non-linear problem with binary variables, but it can be reformulated in to a quadratic programming problem as,

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^n \left(\sum_{j \in N, j \neq i} \sum_k \frac{P_{jk} \cdot h_{ji} \cdot z_{ji}}{P_{ik} \cdot h_{ii} \cdot z_{ii}} \cdot x_{jk} \cdot x_{ik} + \sum_k \frac{N_0}{P_{ik} \cdot h_{ii} \cdot z_{ii}} \cdot x_{ik} \right) \\ & \text{subject to} && \sum_{k=1}^{|C|} x_{ik} = 1, x_{ik} \in X_i \in \{0, 1\}^{|C|} \end{aligned} \quad (12)$$

The reformulation is available in Appendix ?? . We use LINDO [47] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimum for the problem. The result of this centralized channel assignment will be evaluated in the simulation section with other schemes.

5.2 Distributed White Space Channel Allocation (With-eCat): Algorithm and Protocol

In this section a distributed scheme for WBSs to allocate channels is proposed, which is named as white space channel allocation technology (With-eCat). With-eCat adopts the best response process, where each WBS (referred as i) chooses the channel which brings the bigger utility u_i as the response of other WBSs' choices on channels. With-eCat is depicted by algorithm 1.

$$u_i = \frac{\sum_{\substack{j \in N, j \neq i, \\ c(j) = c(i)}} f_{ji}}{2 \cdot \bar{P}_i} + \frac{1}{2} \sum_{\substack{j \in N, j \neq i, \\ c(j) = c(i)}} \frac{f_{ij}}{\bar{P}_j} + \sum_{\substack{S: j \in S, \\ c(j) = c(i)}} \frac{N_0}{C \cdot \bar{P}_i} \quad (13)$$

where $f_{ij} = P_i \cdot h_{ij} \cdot z$ and $f_{ji} = P_j \cdot h_{ji} \cdot z$. Note that f_{ij} is the sum of interference on WBS i 's interference reference points. Overlooking the constant coefficient 2, the first item of u_i is a part of the inverted QuasiSINR of station i . To minimize the first item, WBS i needs to choose a channel either permits higher transmission power or experiences less interference, whereas the higher power increases the second item which is a part of inverted QuasiSINR of other co-channel WBSs. Hence, the cost function presents a reasonable comprise between the welfare of one WBS and others.

When WBS only emphasizes on its own utility (e.g. the first part of Formula 13), the best response process doesn't converge. We have following theorem:

THEOREM 5.1: With non-identical transmission power, if every WBS updates its channel based on Algorithm 1 with utility based on its own interests, the process doesn't always converge.

The proof is in Appendix ??.

Algorithm 1: Spectrum selection by WBS i

Input: the distance, path lose and shadowing parameter between WBS i to WBS $j \in \mathcal{N} \setminus i$; radius of auxiliary circle, noise N_0 , total number of WBSs N ; for $j \in \mathcal{N} \setminus i$, the maximal transmission power $P_j^c, c \in C$ and the working channel $c(j)$.

```

1 for  $c \in C \setminus c(i)$  do
2   calculate  $u_i(c)$  based on Formula 13 if  $u_i(c) < u_i(c(i))$ 
   then
3      $c(i) \leftarrow c$ 
4   else
5     keep  $c(i)$  unchanged, reports  $c(i)$  to database
6   end
7 end
8 Notify database of its channel usage, which further notifies
  the other WBSs

```

Some parameters needed to calculate the utility are identical for all WBSs, such as quasi distance e , the total number of WBSs N , number of channels C , attenuation factor α , standard deviation σ_{WBS} in flat shadowing and noise N_0 , albeit the following information is further needed to calculate u_i :

- $\sum_{\substack{j \in \mathcal{N}, j \neq i \\ c(\sigma_j) = c(\sigma_i)}} f_{ji}^c, c \in C$: the received interference on i ' virtual measurement point from other WBSs j working on the same channel for $\forall c \in C$.
- f_{ij}^c : the interference caused by i on j 's virtual measurement point when i works on channel $\forall c \in C$.
- P_j^c : transmission power of j for using $\forall c \in C$.

Unfortunately, it is difficult to get these interferences of interested measured, for station i , it is low efficient to scan all channels and obtain the interferences f_{ji} on virtual measurement point for each channel, furthermore, it is impossible to split the interference f_{ij} from the total interference received on WBS j ' virtual measurement point.

We refer [41] to decide the sequence for WBSs to update their channel. [41] proposes a method like random access mechanism of CSMA/DA, where the access for broadcast medium is changed to getting access to the centralized center to retrieve the current channel usage and update its new channel. All WBSs are able to access the database in one round (with random or predetermined sequence). As WBSs are connected with database, the control messages needed to decide the sequence will not become a burden. Update of channels can happen in the boot phase, or when the quality of services (the SINR on its end users) of WBSs falls below a threshold, or a fixed time duration comes to end, or a new WBS joins in the network.

Similar with [7], we let every WBS store the location information and maximal power map of all other WBSs, i.e., $P_i^c, i \in \mathcal{N}, c \in C$, and each WBS retrieves information about channel usage of other WBSs from centralized base station. After executing Algorithm 1, it reports to centralized database of its channel if it updates the working channel. As the location of WBSs and TV stations and the transmission channel and power of TV stations are usually static (entries of TV station change averagely once in 2 days[7]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent.

5.3 Analysis in Game Theoretical Framework

In this section, We give the proof on whiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [48]. This work reversely engineers the distributed channel allocation schemes proposed in [49], [39], i.e., unifies the algorithms with congestion game. But the problem analysed in [48] assume the transmission power is identical, which is a major difference from the channel allocation problem discussed here.

We have introduced congestion game in Chapter ??, thus we only recap the essence of congestion game here. In congestion game, each player acts selfishly and aims at choosing strategy $\sigma_i \in \Sigma_i$ to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

The Congestion Game Formulated from the Algorithm WhiteCat

We utilize the conception of virtual resource which is firstly introduced in [48]. Virtual resource is a triplet $\{i, j, c\}$, where i, j are two WBSs and $c \in C$ is one channel. This piece of resource is regarded used by i when both i and j use channel c , otherwise, $\{i, j, c\}$ is not used by any WBS.

In the following, we list the element of the congestion game which emulates Algorithm 1. In this section, player and base station are used interchangeably.

- Player i ' strategy space is $\Sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \dots, N\}$, and i has C admissible strategies, one strategy related with channel $c \in C$ is described by the set of virtual resources it uses: $\sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$, note that virtual resource $(i, j, c) \neq (j, i, c)$.
- Under the strategy profile $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)$, player i obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j \in \mathcal{N}, j \neq i \\ c \in C, c(\sigma_i) = c(\sigma_j)}} (g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma))) \quad (14)$$

The transmission power over all channels of player i is $\{p_i^1, p_i^2, \dots, p_i^{|C|}\}$. We define the cost function for virtual resources (i, j, c) as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \frac{f_{ji}}{2\bar{P}_i} + \frac{f_{ij}}{2\bar{P}_j} + \frac{C \cdot N_0}{N \cdot \bar{P}_i} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \quad (15)$$

As resource (i, j, c) only lies in the strategy space of player i and j , thus can only be accessed by this two players. More specifically, according to Formula 15, the cost of resource (i, j, c) is only decided by the number of players using it, which is either 0 or 2. At the first glance, this is a player specific congestion game, as $g_{(i,j,c)}$ is decided by the relevant players' transmission power and inference. But actually the resource (i, j, c) excludes the players except for i and j from using it, thus the cost happened on this resource is only dependant on how many of players from

the set $\{i, j\}$ to use it. Hence, the cost is a function of the number of players using the resource, and this is a canonical congestion game.

Bridging the Game and Algorithm WhiteCat

When we substitute Formula 15 to Formula 14, the total cost for user i under strategy profile σ .

$$\begin{aligned}
 g^i(\sigma) &= \sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} (g(i, j, c)(2) + g(j, i, c)(2)) \\
 &= \sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} \left(\frac{f_{ji}}{\tilde{p}_i} + \frac{f_{ij}}{\tilde{p}_j} + \frac{C \cdot N_0}{N} \left(\frac{1}{\tilde{p}_i} + \frac{1}{\tilde{p}_j} \right) \right) \\
 &= \frac{\sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{p}_i} + \sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{p}_j} + \frac{CN_0}{N} \sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} \left(\frac{1}{\tilde{p}_i} + \frac{1}{\tilde{p}_j} \right) \quad (16) \\
 &= \frac{\sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{p}_i} + \sum_{\substack{j \in N \setminus i \\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{p}_j} + \frac{2CN_0}{N} \sum_{\substack{i \in S \subseteq N \\ S: \forall i \in S \\ c(\sigma_i)=c}} \frac{1}{\tilde{p}_i}
 \end{aligned}$$

where S denotes the set of WBSs whose working channel is the same with WBS i .

Now we are going to have a look at the *potential* of the network. According to the expression of Rosenthal's potential in Formula 19, the potential is accumulated by adding the players' cost sequentially, in particular, the value which is added is the cost that player experiences when it starts to use the relevant resource, and the value is not changed when other players come to use that resource. Back to our problem, for two WBSs $i, j \in S$, we assume WBS i 's index is smaller than j 's index, then the potential increased by i using the resource $\{i, j, c\}$ is 0 according to Formula 14, and the increase brought in by j using the resource $\{i, j, c\}$ is $g(i, j, c)(2) + g(j, i, c)(2)$. In other words, for each interfering pair of WBSs, only the WBS with bigger index contributes to the potential. Then the total potential is,

$$\begin{aligned}
 G(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i \in N} \sum_{r \in \sigma_i} g_r(n_r^i(\sigma)) \\
 &= \sum_{i \in N} \frac{\sum_{\substack{j \in N, j \neq i \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{p}_i} + \frac{CN_0}{N} \sum_{\substack{S \subseteq N \\ S: \forall i \in S \\ c(\sigma_i)=c}} |S| \sum_{i \in S} \frac{1}{\tilde{p}_i} \quad (17)
 \end{aligned}$$

note that the summation of one WBS's congestion is related to its index.

When players minimize their utilities (cost or potential) illustrated by Formula 16, the total congestion in the secondary network given by Formula 17 decreases monotonically before reaching one Nash equilibrium. Players' greedy update in the game to minimize its cost Function 16, which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm 1 and utility Function 13 respectively.

Gap between the Potential of Game and the Objective

It is natural to raise the question, is the sum of the final utilities of all WBSs exactly the same with the value of potential when the game converges to a Nash equilibrium, which is represented by 17? The answer is, they are identical when N_0 is zero, and there

will be a little difference when N_0 is not zero. Recall the target objective we want to minimize in Problem 1 is,

$$\begin{aligned}
 \sum_{i \in N} \frac{f_i}{\tilde{p}_i} &= \sum_{i \in N} \frac{\sum_{\substack{j \in N, j \neq i \\ c(\sigma_j)=c(\sigma_i)}} f_{ji} + N_0}{\tilde{p}_i} \\
 &= \sum_{i \in N} \frac{\sum_{\substack{j \in N, j \neq i \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{p}_i} + \sum_{i \in N} \left(\frac{N_0}{\tilde{p}_i} \right) \quad (18)
 \end{aligned}$$

We notice that only the last items of the objective 18 and the potential of the congestion game 17 are different. When $N_0 = 0$, the potential is exactly the same with the object we want to minimize. When $N_0 \neq 0$, if channels are evenly distributed and there is $C/N * |S| = 1$, then Formula 18 and 17 are also the same. In both cases, the sum of utilities 18 decreases monotonically with every update of WBSs before the system reaches Nash Equilibrium. When $N_0 \neq 0$ and Formula 18 and 17 are thus different, the monotonicity on the decrease of sum of utilities 18 is not perceived, whereas the system will still cease to NE.

Based on above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed. If we regard one WBS as multiple ones which locate at the same place, and each WBS works on one distinct channel, then the proof on convergence of whiteCat can be applied directly to this case.

Note that the convergence of the game is independent on the the concrete form of the cost function. We adopt the function 16 to let the potential of the game be the same with the total utility of all WBSs, so that by executing Algorithm 1, the system objective experiences a monotonic decreasing process before the system reaching NE. The algorithm has potential to solve many other problems, where one user's decision affects others. In this case, the utility of one user can be formulated to incorporate the information of its own utility and others', then the congestion game theory can be used to analogize.

Communication Overhead of WhiteCat

The problem of channel allocation with different and fixed transmission power is NP hard. WhiteCat is a distributed scheme but certain information of the other WBSs is needed. The centralized base station is piggybacked to provided the needed information. As to one WBS, the number of such inquiries is the number of steps before convergence.

In our formulated congestion game, a player i is allowed to access up to $(N - 1)$ resources in the same time, i.e., $\{i, j_1, c(i)\}, \{i, j_2, c(i)\} \cdots \{i, j_{N-1}, c(i)\}$, thus the upper bound of converge steps can not be obtained from the conclusion 19 for singleton congestion game. But our problem is special because for each resource, the possible number of players allowed to use each resource is either 2 or 0. Thus we can refer the method used in Section ?? to analyse the update times for our problem. Firstly, we sort the cost values in increasing order. Although a WBS

$$\begin{aligned}
 \phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\
 &\leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} n \\
 &\leq n^2 m \quad (19)
 \end{aligned}$$

The upper bound of total update steps is $2n^2$, thus averagely, the upper bound of update steps for each WBS is $2n$.

6 VARIABLE TRANSMISSION POWER AFTER CHANNEL ALLOCATION

After deciding on the working channel, WBSs operate with the maximum permitted transmission power. As the utility defined in Formula 7 is a division of linear function of transmission power and received interference, it is natural to assume that there could exist a vector of transmission power $\{p_1, p_2, \dots, p_N\}$ where $p_i < P_{max}^c, \forall i \in N$, and the performance doesn't diverge much from the already achieved performance. When using the same utility as Formula 7, there is no WBS having the motivation to diverge from the power level (the maximum permitted power) if the other WBSs keep their transmission power unchanged.

We adopt the utility proposed in [50] as our utility function.

$$u = \frac{E \cdot R}{p} (1 - e^{-0.5 \cdot \gamma})^L$$

This new utility is function of both its own transmission power and quasiSINR, thus one WBS doesn't need relevant information from other WBSs. This function has several attracting properties. It is a monotonically increasing function of γ for a fixed transmission power p . It approaches to 0 when γ increases to infinity, and it is a monotonically decreasing of the transmission power p for a fixed γ . This function goes to 0 when p goes to either 0 or infinity. To adopt this algorithm, every WBS keeps on minimizing its utility and finally all the WBSs achieve Nash equilibrium.

7 JOINT CHANNEL AND POWER ALLOCATION

In the section 5, the problem is decomposed into three subproblems which are solved sequentially. The first is solved with linear/convex programming in the centralized database, and the others are solved with distributed schemes. The decomposition of the original problem, and the distributed scheme may yield result which is far away from the global optimum, so in this section, we propose a centralized scheme which looks for the global optimum in order to examine the performance of our cascaded and distributed channel and power allocation scheme.

7.1 Centralized Optimization

When we consider to optimize the transmission power and channel jointly, the optimization problem 12 is not quadratic any more and becomes mixed integer non-linear problem, for which no efficient solution exists. We reformulate problem 12 into a mixed binary quadratic optimization problem with some auxiliary variables created, i.e., binary number α , real number β and q , where

$$x_{jk} \cdot x_{ik} = \alpha_{ij}^k \quad (20)$$

$$\beta_{ij}^k = p_j^k \cdot \alpha_{ij}^k \quad (21)$$

$$\frac{1}{p_i^k} = q_i^k \quad (22)$$

Then the optimization problem can be stated as:

minimize

$$\sum_{i=1}^n \left(\sum_{j \in N, j \neq i} \sum_k q_i^k \cdot \beta_{ij}^k \cdot h_{ji} \cdot z + \sum_k N_0 \cdot q_i^k \cdot x_{ik} \right)$$

subject to:

$$\begin{aligned} x_{jk} + x_{ik} - \alpha_{ij}^k &\leq 1 \\ -x_{jk} - x_{ik} + 2 \cdot \alpha_{ij}^k &\leq 0 \\ \beta_{ij}^k - p_j^{k,max} \cdot \alpha_{ij}^k &\leq 0 \\ -\beta_{ij}^k + p_j^{k,min} \cdot \alpha_{ij}^k &\leq 0 \\ \beta_{ij}^k - p_j^k - p_j^{k,min} \cdot \alpha_{ij}^k &\leq -p_j^{k,min} \\ -\beta_{ij}^k + p_j^k + p_j^{k,max} \cdot \alpha_{ij}^k &\leq p_j^{k,max} \\ \sum_{k=1}^{|C|} x_{ik} &= 1, x_{ik} \in X_i \in \{0, 1\}^{|C|} \\ q_i \cdot p_i &= 1 \end{aligned} \quad (23)$$

The objective function is quadratic. We notice that all the constraints except for the last one are linear. The first two constraints realizes Formula 20, the following four constraints realizes Formula 21. Due to the quadratic equality constraint $q_i \cdot p_i = 1$, the optimization problem is non-convex which makes this optimization problem very challenging to solve [51], [52]. Linearisation is possible when the matrix Q in the quadratic is positive definite. As we don't regulate the locations of the WBSs, and the attenuation is random among them, thus the positive definite can not be guaranteed. We use solver LINDO [47] to look for the global optimum.

8 PERFORMANCE EVALUATION

Performance evaluation consists two parts, in the first part white-Cat are compared with other distributed schemes proposed for the problem of channel allocation with different fixed transmission power, where the transmission power levels associated with WBSs and channels are different. In the second part, we compare the distributed joint channel and power allocation solution with other solutions for this problem. To illustrate the structure, we list the contents in Figure 5.

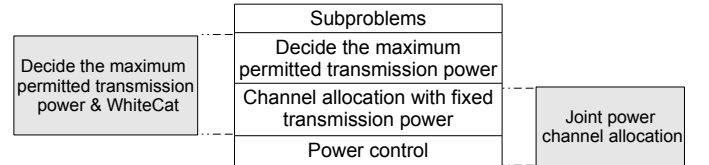


Fig. 5. The evaluation contents in this section, the left part is discussed in Section 8.1 and the right part is in Section 8.2.

The evaluation setting is as follows. A square area which is 60km x 60km is divided into 16 square blocks evenly, for each block there is one WBS locating in the middle of it. Same mount of end terminals distributed in each minor block. The terminals don't necessarily choose the WBS which is nearest to them to obtain service, in stead they choose the WBS to join, which provides the strongest RSSI at them. There is a 20km wide rim area around the square area, where the interference measurement devices for TV receiver are randomly located. The number of such interference

measurement devices is the same with the number of channels in the system, every DTV and WSD device works on one channel.³ The locations of WBSs and TV contours are illustrated in Fig. 6. WBSs' locations are fixed, but the locations of interference measurement devices for TV receivers, the end terminals, and the sequence for WBS to update are randomly decided in each run. Simulations are conducted for 50 times.

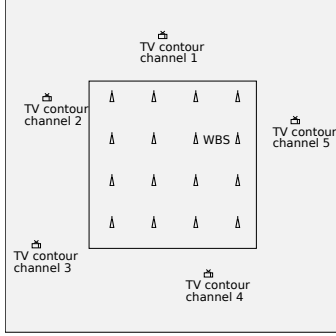


Fig. 6. Layout of WBSs and TV contours

The other parameters are listed in Table 14.

| | |
|---|--------------|
| Number of channels | 5 |
| Number of WBSs | 16 |
| Noise | 10^{-12} W |
| Length of side the square to locate WBSs | 60km |
| Distance between quasai terminal and WBS | 7km |
| Interference threshold on TV contour | 10^{-7} W |
| Path loss factor | 2 |
| Standard deviation in flat shadowing | 8 |
| Minimal WBS transmission power ⁴ | 4W |
| Maximal WBS transmission power | 40W |
| Number of end terminals in network | 800 |

TABLE 3
Simulation parameters

8.1 Maximal Permitted Power Decision and the Distributed Channel Allocation Schemes

In this section, we will firstly evaluate the convex optimization and linear optimization proposed in Section 4 to see which is the better choice to decide the maximum permitted transmission power, the adopted metrics are average power consumption, and SINR on end users where channel allocation is executed. Then with the decided better method, we compare given the power map, how do the channel allocation schemes perform. We compare our proposed channel allocation scheme *whiteCat* with three other distributed schemes, the random allocation scheme, *whiteCase* and No-regret learning, besides, centralized optimization is used to obtain global optima.

3. In reality multiple channel usage (channel bonding) is requisite as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfil system requirement. We need to point out, our scheme can also be applied to the multiple channel usage scenario, in tha case, we regard the WSD working with multiple channels as multiple different WSDs, which work on only one channel, but locate as the same place.

4. minimal and maximal power here denote the power level restricted by the specification of hardware.

- *WhiteCase*: Whitespace channel allocation selfish, where each WBS selfishly updates its channel to achieve the best (as to the considered problem, smallest) possible utility based on Formula ??.
- *Noregret learning*: Each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSs update such mapping dynamically and this approach converges to correlated equilibrium. Please refer the original paper [44] for details.
- *Quadratic optimization*: centralized quadratic optimization introduced in Section 5.1.

The Choice of Radius of Auxiliary Circles, quasiSINR of WBS, and SINR on End Users

The usage of quasiSINR exempts WBSs from taking care the SINR on the end terminals. A WBS's quasiSINR is related with WBS's location and the radius of auxiliary circle. Figure 7 illustrates the effect of using different radii of the auxiliary circle on the data rate can be achieved by end terminals.

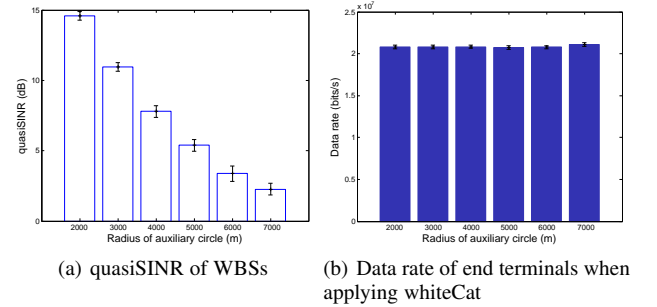


Fig. 7. The effects of different radii of auxiliary circle on end terminals' data rate. Maximum permitted power is obtained by solving convex optimization. WhiteCat is used to assign the channels.

Subfigure 7(a) shows WBSs' quasiSINR decreases when the radii of auxiliary circles increase. Subfigure 7(b) illustrates the choice on radius of auxiliary circle don't influence the performance of whiteCat. In the following simulation, we fixed the radius at 6000 m.

Performance of Channel Allocation Schemes

In Section 4, two different optimization formulations are introduced to obtain the maximum permitted transmission power for WBSs, i.e., convex optimization and linear optimization respectively. In Figure 4, we have seen that the convex optimization generates power levels which distribute evenly between the minimum and maximum transmission power levels configured by the hardware, while, the majority of the power levels generated by linear optimization are either the minimum or maximum transmission power. In this section we run the channel allocation schemes with the maximum permitted power levels obtained from convex and linear optimization respectively. The simulation in this subsection carries twofold meanings. The first is to see which maximum permitted power decision method outperforms the other, the second is to evaluate the performance of the channel allocation schemes. The adopted metrics are the SINR on end terminals and transmission power consumption.

Comparison of the Methods for Maximum Permitted Transmission Power

Figure 8 depicts the power consumption of the channel allocation schemes which work with the two groups of maximum permitted transmission power decided by linear and convex problems respectively. When given maximum permitted transmission power, whiteCat and the centralized optimization scheme consume the least energy. The schemes utilize less transmission power with the maximum permitted transmission power decided by convex optimization. Figure 9 shows the quasiSINR of WBSs. The centralized optimization scheme achieves the highest quasiSINR, because the optimization formulation 11 obtains the global optima.

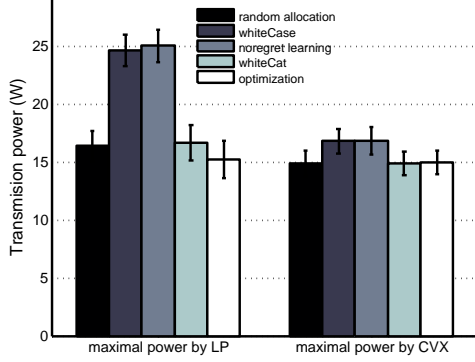


Fig. 8. Power consumed of WBSs by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

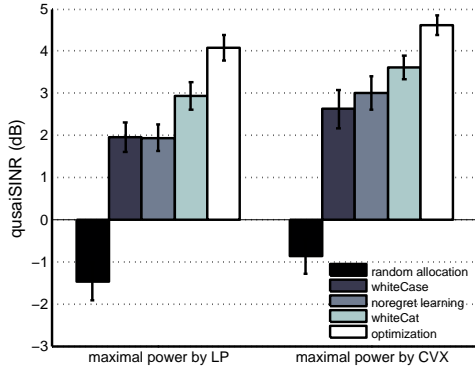


Fig. 9. QuasiSINR of WBSs achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

The average SINR on the end terminals is depicted in Figure 10. When the given maximum permitted transmission power, whiteCat and the centralized optimization achieve similar and the best performance among the schemes. It is also noticed that, the maximum permitted transmission power decided by linear optimization helps the channel allocation schemes achieve better SINR.

The empirical cumulative distribution function curve of SINR on end terminals is drawn in Figure 11. The SINR achieved by WhiteCat and the centralized optimization is stably higher than that obtained from other schemes. For example, the 20% and 80% percentile of the SINR achieved by WhiteCat and the centralized optimization are 0.5 to 1 dB higher than the other channel allocation schemes.

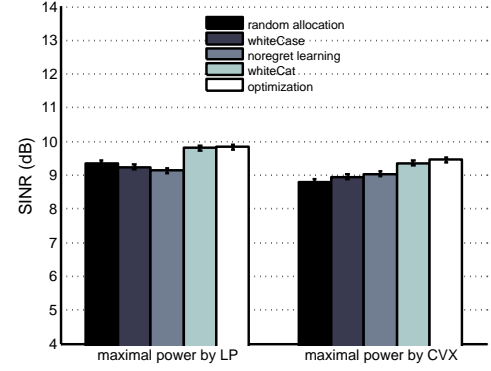


Fig. 10. SINR on end terminals achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

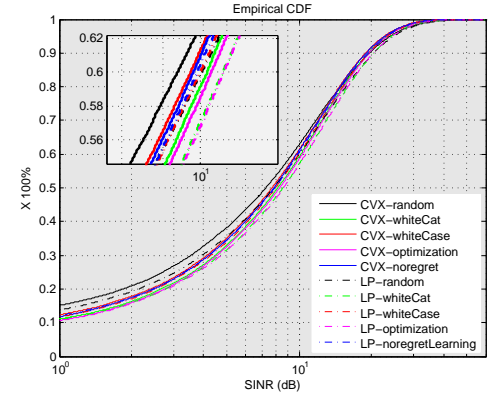


Fig. 11. CDF of SINR on end users obtained by different CA schemes under different methods to decide the maximal transmission power map

Convergence Speed

In the congestion game where scheme whiteCat is derived, each player (WBS) has at most $(n - 1) * |C|$ resources available for usage, thus there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up to 100. Table 4 shows the average number of steps needed before convergence in 100 runs of simulations. As to whiteCat, we account each WBS accessing the base station (refer to 5.2) as *one step*. We compare the convergence speed of WhiteCat with no-regret learning, the scheme derived from potential game [16] and whiteCase. Note that the potential game scheme is to solve joint power and channel allocation problem, as it is developed with game theory, it is reasonable to see its convergence speed. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds).

Table 4 tells that whiteCat is two times faster than the scheme derived from potential game, and 20 times faster than no-regret learning scheme. The relatively smaller confidence interval shows that whiteCat's convergence is not affected by different network configurations. Fast converge is attributed to the working style of WBSs which access the database to get the information of other WBSs, thus the distributable decision involves a part of the global information of the network. Thus, we can see that the speed up of convergence is due to the overhead caused by accessing the database.

Figure 12 depicts one instance of the convergence processes

of three schemes. The Y axis is the summed utility of all WBSs. We can see whiteCat decreases the summed utility constantly, and the channel allocation process ceases after 38 times of updates. Whereas, noregret learning scheme takes 120 steps before convergence, and whiteCase fails to converge.

| Scheme | Average steps | 95% CI | Average time (s) |
|--------------------|---------------|--------|------------------|
| whiteCat | 58 | 5.6 | 2 |
| noregret | 1916 | 1541 | 144 |
| PotentialGame [16] | 120 | 10 | 4 |
| optimization-LINDO | - | - | 40 |
| whiteCase | 4587 | 2742 | 50 |

TABLE 4

Convergence speed of the distributed channel allocation schemes. As to the distributed scheme, the time involved to communicate with database is not considered and included.

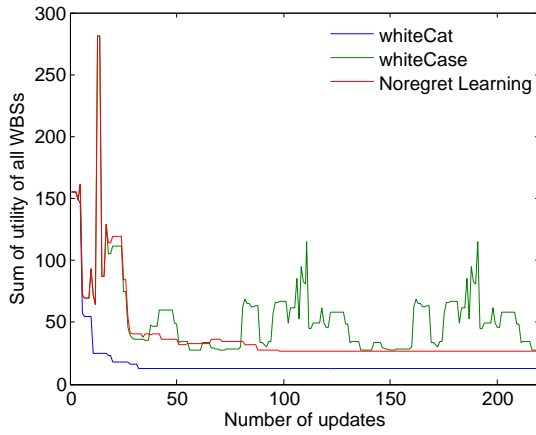


Fig. 12. Convergence process of three different schemes in one simulation.

Stability of SINR in Convergence Process

WBS provides service to end users in the process of channel allocation. A certain SINR corresponds to certain transmission configurations like modulation type and data rate. The oscillation of SINR resulted from WBS changing the working channel during the convergence process may cause reconfiguration, reduced throughput or delay variance, which is not preferred. We propose a metric *Cost of Oscillation* (COS) to represent the stability of SINR in the converging process. Assuming each update of channel takes 1 time unit, the variance of SINR of end user i at time $t + 1$ is

$$\Delta\gamma_i(t+1) = \left| \frac{\gamma_i(t+1) - \gamma_i(t)}{\gamma_i(t)} \right|$$

The COS value for one network applied with a certain channel allocation scheme is,

$$COS = \sum_{t=1}^T \sum_{i \in N} \Delta\gamma_i(t) \quad (24)$$

$\gamma_i(0)$ is the SINR for i before starting channel allocation. The variance of SINR in channel allocation process is shown in table 5 from which we can see WhiteCat achieves only 6% of oscillation on SINR compared with No-regret approach.

| Scheme | COS | 95% confidence interval |
|-----------|--------|-------------------------|
| WhiteCat | 8850 | 2984 |
| No-regret | 145460 | 1541 |
| WhiteCase | 246790 | 168050 |

TABLE 5

Variance of SINR during the convergence process

8.2 Performance of Joint Power and Channel Allocation

As introduced in section 6, after channel allocation is conducted, transmission power is adjusted in a distributive manner. In other words, power and channel allocation is executed with two cascaded distributed schemes. As comparisons, we implement two joint power channel allocation schemes. One is centralized optimization introduced in section 7.1, which is used as upper bound in the comparison. The other comparison is distributed joint power and channel allocation scheme [16] which is introduced in Section ??, we name it as *potentialGame*. We need to point it out that, scheme *potentialGame* doesn't aim to improve the SINR on end terminals, but on the sum of produced and received interferences. The performance of joint channel and power allocation schemes are presented in Fig. 13, 14 and 15 in terms of total utility, power consumption and achieved SINR on end users respectively.

Figure 13 illustrates the comparison of the cascaded solutions, i.e., channel allocation and the following power control, in terms of the total utility in the network. We can see that our proposed scheme *whiteCat+dpa*, the cascaded channel and power allocation method falls behind the cascaded channel allocation optimization and power allocation, and the joint channel and power allocation optimization, but outperforms all the other distributed solutions. Note that the *potentialGame* method along with power control results the worst performance, the reason is the objective adopted by *potentialGame* is to minimize the sum of received interference in the network, thus the performance on summed utility demonstrates randomness. Figure 15 draws the CDF of SINR on end users when applying different channel allocation and power allocation schemes. It is clear that our proposed approach achieves the best among the distributed schemes, and only worse than the schemes which involves centralized optimization.

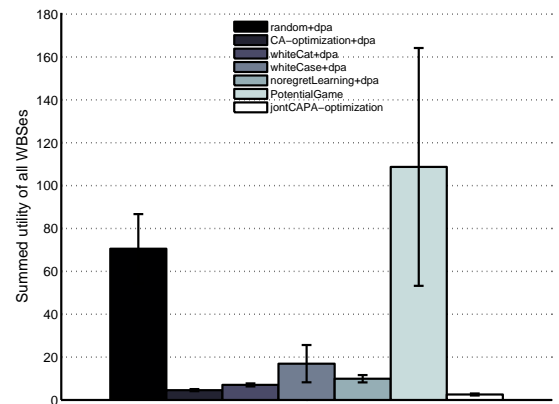


Fig. 13. Summed utility of all WBSs, which is the objective in problem 1. dpa in legend represents distributed power allocation

REFERENCES

- [1] "In the matter of unlicensed operation in the TV broadcast bands: second memorandum opinion and order, federal communications commission, fcc 10-174," http://hraunfoss.fcc.gov/edocs_public/attachmatch/DA-11-131A1.pdf, Jan. 2011, visited July 2012.
- [2] K. Koufos, K. Ruttik, and R. Jantti, "Controlling the interference from multiple secondary systems at the TV cell border," in *Proc. of IEEE PIMRC 2011*, Sep. 2011.
- [3] "Technical and operational requirements for the possible operation of cognitive radio systems in the 'white spaces' of the frequency band 470-790 MHz, Cardiff," <http://www.erdocdb.dk/docs/doc98/official/Pdf/ECCRep159.pdf>, Jan. 2011, visited July 2012. [Online]. Available: <http://www.erdocdb.dk/docs/doc98/official/Pdf/ECCRep159.pdf>
- [4] R. J  ntti, J. Kertt  la, K. Koufos, and K. Ruttik, "Aggregate interference with fcc and ecc white space usage rules: case study in finland," in *Proc. of IEEE DySPAN 2011*, May 2011.
- [5] "Google spectrum database," visited Juni 2016. [Online]. Available: <https://www.google.com/get/spectrumdatabase/>
- [6] "FCC frees up vacant TV airwaves for   Super Wi-Fi   Technologies and Other Technologies," <http://hraunfoss.fcc.gov>, Sep. 2010, visited July 2012. [Online]. Available: http://hraunfoss.fcc.gov/edocs_public/attachmatch/DOC-301650A1.pdf
- [7] R. Murty, R. Chandra, T. Moscibroda, and V. Bahl, "Senseless: A database-driven white spaces network," in *Proc. of IEEE DySpan 2011*, May 2011.
- [8] Y. Wu and D. Tsang, "Distributed power allocation algorithm for spectrum sharing cognitive radio networks with qos guarantee," in *Proc. of IEEE INFOCOM 2009*, Apr. 2009.
- [9] A. Hoang, Y. Liang, and M. Islam, "Power control and channel allocation in cognitive radio networks with primary users' cooperation," *IEEE Transactions on Mobile Computing*, vol. 9, no. 3, pp. 348–360, Mar. 2010. [Online]. Available: <http://dx.doi.org/10.1109/TMC.2009.136>
- [10] X. Chen and J. Huang, "Game theoretic analysis of distributed spectrum sharing with database," in *In Proceedings of 2012 IEEE 32nd International Conference on Distributed Computing Systems (ICDCS)*, June 2012, pp. 255–264.
- [11] A. T. Hoang and Y.-C. Liang, "Downlink channel assignment and power control for cognitive radio networks," *Wireless Communications, IEEE Transactions on*, vol. 7, no. 8, pp. 3106–3117, August 2008.
- [12] S. Bahramian and B. Khalaj, "Joint dynamic frequency selection and power control for cognitive radio networks," in *Telecommunications, 2008. ICT 2008. International Conference on*, June 2008, pp. 1–6.
- [13] C. Passiatore and P. Camarda, "A centralized inter-network resource sharing (CIRS) scheme in IEEE 802.22 cognitive networks," in *Proc. of 2011 The 10th IFIP Annual Mediterranean Ad Hoc Networking Workshop (Med-Hoc-Net)*, June 2011, pp. 17–24.
- [14] W. Hu, M. Gerla, G. Vrantis, and G. Pottie, "Efficient, flexible, and scalable inter-network spectrum sharing and communications in cognitive IEEE 802.22 networks," in *In Proceedings of First International Workshop on Cognitive Radio and Advanced Spectrum Management 2008*, Feb 2008, pp. 1–5.
- [15] A. Galindo-Serrano and L. Giupponi, "Aggregated interference control for cognitive radio networks based on multi-agent learning," in *Proc. of CrownCom 2009*, Jun. 2009.
- [16] N. Omidvar and B. Khalaj, "A game-theoretic approach for joint channel and power allocation in downlink of cellular cognitive radio networks," in *Proc. of IEEE PIMRC 2012*, Sept 2012, pp. 1198–1202.
- [17] J. Elias and M. Krunz, "Distributed spectrum management in TV white space cognitive radio networks," in *Proc. of IFIP Networking Conference, 2015*, May 2015, pp. 1–8.
- [18] R. Langar, S. Secci, R. Boutaba, and G. Pujolle, "An operations research game approach for resource and power allocation in cooperative femto-cell networks," *IEEE Transactions on Mobile Computing*, vol. 14, no. 4, pp. 675–687, April 2015.
- [19] J. Huang and V. Krishnamurthy, "Cognitive base stations in LTE/3GPP femtocells: A correlated equilibrium game-theoretic approach," *IEEE Transactions on Communications*, vol. 59, no. 12, pp. 3485–3493, Dec. 2011.
- [20] M. I. Rahman, A. Behravan, H. Koorapaty, J. Sachs, and K. Balachandran, "License-exempt LTE systems for secondary spectrum usage: scenarios and first assessment," in *Dyspan 2011, IEEE*, May 2011.
- [21] X. Feng, J. Zhang, and Q. Zhang, "Database-assisted multi-AP network on TV white spaces: Architecture, spectrum allocation and ap discovery," in *Proc. of IEEE DySPAN 2011*, may 2011, pp. 265–276.
- [22] T. M. R. M. Paramvir Bahl, Ranveer Chandra and M. Welsh, "White space networking with Wi-Fi like connectivity," in *Proc. of ACM SIGCOMM 2009*, Aug. 2009.
- [23] "IEEE 802.22 wireless regional area networks," <http://www.ieee802.org/22/Technology/>.
- [24] A. Flores, R. Guerra, E. Knightly, P. Ecclesine, and S. Pandey, "IEEE 802.11af: a standard for TV white space spectrum sharing," *Communications Magazine, IEEE*, vol. 51, no. 10, pp. 92–100, October 2013.
- [25] "TV white space amendment to 802.15.4," <http://www.ieee802.org/15/pub/TG4m.html>.
- [26] "Wireless coexistence in the TV white space," <http://ieee802.org/19/pub/TG1.html>.
- [27] X. Feng, Q. Zhang, and J. Zhang, "A hybrid pricing framework for tv white space database," *IEEE Transactions on Wireless Communications*, vol. 13, no. 5, pp. 2626–2635, May 2014.
- [28] X. Tao, Z. Zhao, and H. Zhang, "Location information based interference control for cognitive radio network in TV white spaces," in *Proc. of IEEE WCNC 2013*, pp. 3614–3619.
- [29] L. Zhang, L. Yuan, Z. Feng, and Z. Feng, "Mutual-interference-aware available spectrum resource distribution in TV white space," in *Proc. of IEEE PIMRC 2013*, Sept 2013, pp. 2743–2747.
- [30] X. Chen and J. Huang, "Game theoretic analysis of distributed spectrum sharing with database," in *Proc. of 2012 IEEE 32nd International Conference on Distributed Computing Systems (ICDCS)*, June 2012, pp. 255–264.
- [31] B. Ye, M. Nekovee, A. Pervez, and M. Ghavami, "TV white space channel allocation with simulated annealing as meta algorithm," in *Proc. of 2012 7th International ICST Conference on Cognitive Radio Oriented Wireless Networks and Communications*, June 2012, pp. 175–179.
- [32] K. Bian and J.-M. Park, "A coexistence-aware spectrum sharing protocol for 802.22 WRANs," in *Proceedings of 18th International Conference on Computer Communications and Networks*, Aug 2009, pp. 1–6.
- [33] S. Sengupta, R. Chandramouli, S. Brahma, and M. Chatterjee, "A game theoretic framework for distributed self-coexistence among IEEE 802.22 networks," in *Proc. of IEEE GLOBECOM 2008*, 30 2008-dec. 4 2008, pp. 1–6.
- [34] V. Gardellin, S. Das, and L. Lenzi, "A fully distributed game theoretic approach to guarantee self-coexistence among wrans," in *Proc. of IEEE INFOCOM, Computer Communications Workshops*, march 2010, pp. 1–6.
- [35] M. Di Felice, A. Ghandhour, H. Artail, and L. Bononi, "Integrating spectrum database and cooperative sensing for cognitive vehicular networks," in *Proc. of IEEE VTC 2013 Fall*, Sept 2013, pp. 1–7.
- [36] B. Ellingsaeter, H. Bezabih, J. Noll, and T. Maseng, "Using TV receiver information to increase cognitive white space spectrum," in *Processings of 2012 IEEE International Symposium on Dynamic Spectrum Access Networks (DySPAN)*, Oct 2012, pp. 131–141.
- [37] H. Karimi, "Geolocation databases for white space devices in the uhf tv bands: Specification of maximum permitted emission levels," in *New Frontiers in Dynamic Spectrum Access Networks (DySPAN), 2011 IEEE Symposium on*, May 2011, pp. 443–454.
- [38] A. Raniwala and T. Chiueh, "Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network," in *Proc. of IEEE INFOCOM 2005*, Mar. 2005.
- [39] B. Ko, V. Misra, J. Padhye, and D. Rubenstein, "Distributed channel assignment in multi-radio 802.11 mesh networks," in *Proc. of IEEE WCNC 2007*, Mar. 2007.
- [40] J. Huang, Q. Zhu, V. Krishnamurthy, and T. Basar, "Distributed correlated q-learning for dynamic transmission control of sensor networks," in *Proc. of IEEE ICASSP 2010*, Mar. 2010.
- [41] N. Nie and C. Comaniciu, "Adaptive channel allocation spectrum etiquette for cognitive radio networks," in *Proc. of IEEE DySpan 2005*, Nov. 2005.
- [42] M. Felegyhazi, M. Cagalj, S. Bidokhti, and J.-P. Hubaux, "Non-cooperative multi-radio channel allocation in wireless networks," in *Proc. of IEEE INFOCOM 2007*, May 2007.
- [43] F. Wu, S. Zhong, and C. Qiao, "Globally optimal channel assignment for non-cooperative wireless networks," in *Proc. of IEEE INFOCOM 2008*, Apr. 2008.
- [44] S. Hart and A. Colell, "A simple adaptive procedure leading to correlated equilibrium," *Econometrica*, vol. 68, no. 5, pp. 1127–1150, 2000. [Online]. Available: <http://dx.doi.org/10.2307/2999445>
- [45] K. Ruttik, K. Koufos, and R. Jantti, "Model for computing aggregate interference from secondary cellular network in presence of correlated shadow fading," in *In Proceedings of IEEE PIMRC 2011*, Sept 2011, pp. 433–437.
- [46] N. Ahmed, V. Shrivastava, A. Mishra, S. Banerjee, S. Keshav, and K. Papagiannaki, "Interference mitigation in enterprise w lans through speculative scheduling," in *Proc. of ACM MobiCom 2007*, Sep. 2007. [Online]. Available: <http://doi.acm.org/10.1145/1287853.1287900>

- [47] Y. Lin and L. Schrage, "The global solver in the lindo api," *Optimization Methods Software*, vol. 24, no. 4-5, pp. 657–668, Aug. 2009. [Online]. Available: <http://dx.doi.org/10.1080/10556780902753221>
- [48] M. Liu and Y. Wu, "Spectrum sharing as congestion games," in *Proc. of Allerton Conference 2008*, Sep. 2008.
- [49] B. Babadi and V. Tarokh, "Distributed dynamic frequency allocation in wireless networks under time-varying user activities," in *In Proceedings of IEEE Sarnoff Symposium 2008*, April 2008, pp. 1–5.
- [50] V. Shah, N. Mandayam, and D. Goodman, "Power control for wireless data based on utility and pricing," in *Proc. of IEEE PIMRC 1998*, vol. 3, Sep 1998, pp. 1427–1432 vol.3.
- [51] H. Hmam, "Quadratic optimisation with one quadratic equality constraint," DTIC Document, Tech. Rep., 2010.
- [52] M. Salahi, "Convex optimization approach to a single quadratically constrained quadratic minimization problem," *Central European Journal of Operations Research*, vol. 18, no. 2, pp. 181–187, 2010.