



DISTRIBUTED ALGORITHMS AND  
APPLICATION OF CONGESTION GAME  
FOR COGNITIVE RADIO NETWORKS

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# Abstract

Owing to the proprietary spectrum allocation paradigm and the boom of wireless communications, spectrum scarcity has become an increasingly pressing problem. Cognitive radio is a promising technology to provide high bandwidth for unlicensed users via dynamic spectrum access. Unlicensed users are allowed to utilize licensed spectrum in the following two scenarios, the first, their operation doesn't cause harmful interferences on licensed users, the second, there are no working primary users which are detected with respect to a certain probability. This spectrum usage paradigm brings new spectrum for unlicensed users, nevertheless, it comes at a cost that the unlicensed users need to decide on the spectrum which is available to use, and to take care of the maximum transmission power to avoid harmful interference at primary users carefully. Furthermore, the available spectrum or resources at unlicensed users are dependant upon their locations with respect to primary users whose operation states vary with time, thus the spectrum availability is different at different unlicensed users and varying.

Consequently, as distributed scheme enables unlicensed users to respond quickly to the varying environment with the help of spectrum detection functionality on itself or its neighbourhood, distributed solutions are extensively adopted by the communications systems which are composed with unlicensed users. In this context, a significant amount of research efforts has been dedicated to study the distributed schemes in cognitive radio networks, but designing efficient distributed schemes faces numerous challenges. First, the autonomous decision adopted by the unlicensed users should lead to fast convergence in system scale. Second, unlicensed users tend to be selfish in nature, therefore, the individual decision should not degrade the performance of the system and other users.

In this regard, congestion game becomes a highly suited mathematical tool to model the interaction among the unlicensed users which strive to fulfil their own purposes, as congestion game guarantees convergence into Nash equilibrium and convergence speed is fast when certain conditions are met. But the current application of congestion game is very restricted as it is challenging to formulate a problem into a congestion game. Besides, most of the current research leverage the primitive or standard congestion game model to study the problem with ideal assumptions, which further restricts congestion game's application in problem solving.

In this dissertation, we propose distributed schemes to solve a series of problems which reside from physical layer up to network layer in cognitive radio networks. We expand the application of congestion game and its variants in the problem solving. By designing the suitable utility function which involves local information or certain other information which is easy to obtain, unlicensed users' operation are formulated into a congestion game which permits convergence. We show that the distributed scheme designed with assistance with congestion game model achieve good performance compared with centralized schemes which require big signaling overhead, and the distributed solutions are

stable in the aspects of convergence. For secondary cellular networks which comply to IEEE 802.22 standard, we propose a solution for secondary users to exploit the TV white space with assistance of the centralized database. As one part of the solution, the distributed spectrum allocation problem is formulated into a canonical congestion game, then propose distributed algorithm enlightened by the player behaviours in the game. To form robust clusters against the ungovernable primary users' activity, a distributed clustering scheme is proposed. We formulate the clustering problem into a player specific singleton congestion game, from where a distributed algorithms is derived. As to routing in network layer in cognitive radio ad hoc networks, we propose geographic routing working with spectrum aware virtual coordinate assigned for the unlicensed users. The virtual coordinate integrates the spectrum availability between any pair of neighbouring secondary users, thus geographic routing can easily detour the unlicensed users which are affected by the active primary users.

Acknowledgments.



This thesis is based on the following papers, which are referred in the text respectively.

Di Li; Zhichao Lin; Stoffers, Mirko; Gross, James, "Spectrum aware virtual coordinates assignment and routing in multihop cognitive radio network," IFIP Networking Conference (IFIP Networking), 2015 , vol., no., pp.1,9, 20-22 May 2015 doi: 10.1109/IFIPNetworking.2015.7145337

Di Li; Gross, J., "Distributed TV spectrum allocation for cognitive cellular network under game theoretical framework," Dynamic Spectrum Access Networks (DYSPAN), 2012 IEEE International Symposium on , vol., no., pp.327,338, 16-19 Oct. 2012 doi: 10.1109/DYSPAN.2012.6478156

Di Li; Gross, J., "Robust Clustering of Ad-Hoc Cognitive Radio Networks under Opportunistic Spectrum Access," Communications (ICC), 2011 IEEE International Conference on , vol., no., pp.1,6, 5-9 June 2011 doi: 10.1109/icc.2011.5963426

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# 1

## INTRODUCTION

Due to the propagation characteristics and regulations, only a small portion of electromagnetic spectrum which spans from 8.3 kHz to 3000 GHz is suitable for commercial application. This spectrum is divided into bands (also referred as channels) and allocated to different services. As one kind of limited and precious resource, radio spectrum is strictly regulated by the national administration. The channels are assigned or leased by governments to different operators and entities. In most cases, the operators pay high price for the commercial usage of certain spectrum, and the usage is exclusive for them [39]. This spectrum management policy rules out the unlicensed users to use the spectrum, thus strictly protects the interest of the spectrum licensees. This spectrum access model is referred *exclusive use model* [158], and is the mainstream of spectrum access worldwide, and many existing wireless applications work on the licensed spectrum. For instance, the second-generation (2G) wireless cellular network GSM (global system for mobile communications) in Europe works with GSM-900 band (from 890 MHz to 960 MHz) and GSM-1800 (1710 MHz to 1880 MHz), and the third-generation (3G) wireless cellular network works from 1.8 GHz to 2.4 GHz [121]. The excludability of licensed spectrum can also be open to other user with conditions. The spectrum licences either sell and trade the licensed spectrum, or dynamically use the spectrum within a certain region according to different traffic patterns [150].

Open spectrum access [110] is proposed in 1995 for full openness of entry, which allows access to spectrum through access fees which are determined by demand and supply. It is claimed that open spectrum access brings in benefits, as for spectrum licensees, the fixed costs on securing the licensed spectrum can be converted into marginal costs, and for the market, the incentives for collusive pricing can be eliminated. But as the *openness* here is fully controlled by the licensees, this spectrum usage is still exclusive .

In contrary to exclusive use model, certain spectrum are assigned for open sharing for peer users, the representative example is the unlicensed industrial, scientific, and medical (ISM band) radio band, which supports versatile wireless applications and a prosperous industry, i.e., Wi-Fi. Many spectrum sharing strategies are proposed to cope with the technical challenges [84].

The proliferation of wireless network constantly arises urgent demand on bandwidth and throughput since the first generation of telecommunication in 1980s, which accordingly encourages the efficient use and reuse of the electromagnetic spectrum. The next generation of telecommunication technology 5G, which is reported to come true in 2020, requires a leap forward of spectrum efficiency [76]. The resorted applications under the label of 5G [34], i.e., machine to machine communication, internet of things, etc, requires high speed and lower investment cost, but this is changeling as the achieved capacity approaches Shannon capacity, and most spectrum are licensed. Meanwhile, the actual spectrum usage measurement conducted by FCC tells that at many locations or time, the licensed spectrum is idle [6], and there exists a large number of spectrum bands which have considerable dormant time intervals [18].

To seek more opportunities from spectrum, it is natural to consider opening the licensed spectrum to unlicensed users to use, when the spectrum is not been used by licensed users, or the the licensed users' operation is not hampered. We adopt the terms proposed in [158], and call this spectrum usage policy as *hierarchical access model*, where licensed users are called primary users (PUs), and unlicensed users are named as secondary users (SUs). Hierarchical access model gives a promising solution to the reclamation of new electromagnetic spectrum and the improvement of spectrum efficiency.

## 1.1 Cognitive Radio Network

A cognitive radio network is composed with secondary users and work with the hierarchical access model. Throughout this thesis, we use *cognitive radio networks* to compound the networks composed with secondary users, there are two reasons, firstly, this name explicitly tells a distinctive property of the devices in the networks, which is their cognition ability to their environment, secondly, cognitive radio has become a synonym of the technology employed in the hierarchical spectrum access paradigm in academia and industry in the recent years.

The definition of cognitive radio evolves with the development of radio technology and regulations. We choose two representative definitions to draw a full picture of cognitive radio. Cognitive radio (CR) is firstly proposed by Mitola III who defines the concept of CR in his dissertation [103] as follows:

The point in which personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to detect user communications needs as a function of use context, and to provide radio resources and wireless services most appropriate to those needs.

FCC (Federal Communications Commission in U.S.) describes CR [3] as,

a radio that can change its transmitter parameters based on interaction with the environment in which it operates. . . . This interaction may involve active negotiations with other spectrum users and/or passive sensing and decision making (smart radio) within the radio. The majority of CRs will probably be

SDRs <sup>1</sup>, but a CR does not necessarily use software, nor does it need to be field programmable.

The first definition depicts the functionalities of cognitive radio devices, and the second one mentions the relationships between cognitive radio users and the network which it resides. In this dissertation, we see cognitive radio as a device which is able to sense, detect, learn and monitor the surrounding radio frequency environment, or to access a certain database to retrieve primary users' information, so as to reconfigure its radio operating parameters (e.g., center frequency, bandwidth and transmit power) on the fly to avoid interfering primary users. Cognitive radio may only practice one portion of the aforementioned functionalities according to actual situation. Based on this definition, the cognitive radio network is deemed to be composed with cognitive radio users, whose acronym is CRN.

### 1.1.1 Functionalities of Cognitive Radio

To adapt to the dynamic spectrum environment and make use of the available licensed spectrum, cognitive radio users need to manage the available spectrum by conducting a series of functionalities sequentially, i.e., detecting the available spectrum with spectrum sensing, selecting the proper spectrum for communication, communicate with other secondary users using certain spectrum band, and moving to another channel when the current working channel is not usable due to primary user's appearance. These functions are incorporated into so called cognitive cycle as described in [17].

#### Spectrum Sensing

To know which chunk of spectrum is available is the foundation of spectrum management. In underlay spectrum usage scenario, CR users get to know the available spectrum by means of spectrum sensing. In overlay spectrum usage scenario, secondary users can in principle access all the licensed spectrum. In this procedure, quality of the available channels is determined with several parameters, i.e., band width, operating frequency, path loss, wireless link errors, link layer delay, and the upper bound of interference on the primary user working on that channel, which decides the maximal permissible transmission power of secondary users. Besides, the statistical behaviours of primary users is also an important factor.

Spectrum administration bodies FCC of U.S. [8] and Electronic Communications Committee (ECC) in Europe [9] encourage to adopt both spectrum sensing and location based method.

#### Spectrum Decision

After knowing the available spectrum, CR users select the most appropriate band according to their requirements on quality of service (QoS). This procedure involves considering

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<sup>1</sup>software defined radio is a radio communication system which is able to receive any modulation across a large frequency spectrum, and transmit on desired spectrum band.

the statistical behaviours of the primary users so as to accessing the channel quality fairly. The available licensed spectrum which spans a wide frequency band exhibits different characteristics [90]. Based on the requirements of interested communication, CR users need to identify the characteristics of the spectrum, which include channel quality (channel capacity, error rate, path loss, etc.) [90], channel switching delay [24], and channel holding time, i.e., the expected time duration that the primary users don't occupy the channel before any one occupies again.

Some work in research community models the spectrum availability with stochastic or statistic model, which is helpful when deciding which channel to use. [99] proposes discrete Markov chain and adjusts duty circle models to describe the availability of licensed spectrum for GSM on 900/1800 MHz. [142] models the channel holding time with geometric and log-normal distributions. Statistics of previous sensing results is used to predict spectrum state in the future [83]. Such models provide more complete information on the availability of the licensed channels.

## **Spectrum Sharing**

Then secondary users are to make use of these channels selected in the spectrum decision procedure, and share the spectrum with both the primary users and the other secondary users. As there may be multiple secondary users trying to use the same channel, spectrum sharing is important to coordinate the behaviour of secondary users to avoid deteriorating the performance of secondary users. Spectrum sharing involves choosing proper channels to mitigate co-channel and adjacent interference [138], or adjusting transmission power to achieve promise between transmitter's and other secondary users' performance [133], or adopting a certain media access paradigm to use the spectrum fairly and efficiently. Spectrum sharing also involves consideration on primary users, i.e., when many secondary users work on the same channel, the accumulated interference caused by secondary users could exceed the interference threshold on primary users.

## **Spectrum Mobility**

When one primary user starts to work on a certain channel from idle state, a secondary communicating pair working on the this channel should change their operating channel to another one, where no primary user is detected working on the new channel. This hand off process between channels is called spectrum mobility, which is critical to maintain the communication and network topology for CRN.

### **1.1.2 Spectrum Sharing in CRN**

Spectrum sharing can be divided into two categories, the spectrum sharing between the primary users and secondary users, and the one among the secondary users. The first category is fundamental to the hierarchical access model, as it poses threat of interfering primary users. The second category decides the services that people can obtain from CRN. Thus, spectrum sharing largely defines the challenges and considerations one cognitive radio network faces.

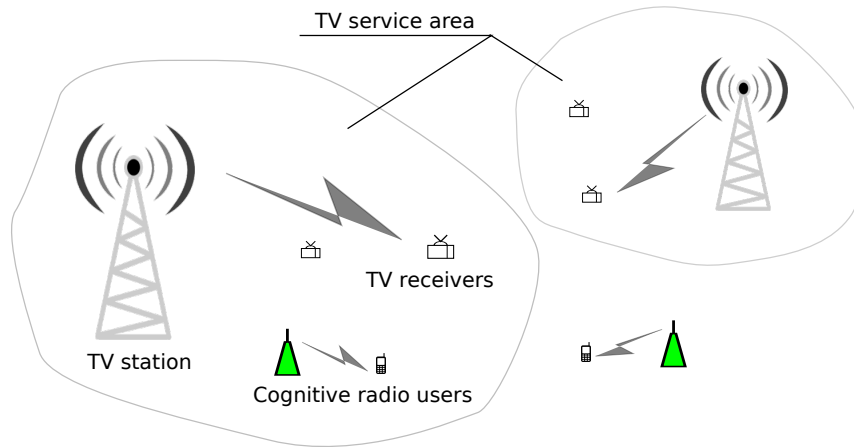


## Spectrum Sharing between primary users and secondary users

As to the spectrum sharing between primary users and secondary users, there are two models, spectrum underlay and overlay. The two models origin from the distinctive understandings of people as to the hierarchical access model, and the different claims on how to effectively protect the primary users' interests.

- **Spectrum underlay:** Working with *spectrum underlay*, secondary users are allowed to transmitting even when some primary users are detected or known to be operating on the same chunk of spectrum, but the interference generated by secondary users on the primary receivers should be under a threshold. Spectrum underlay restricts the secondary users' transmission power, but secondary users are still able to achieve high data rate in short range. When implementing spectrum underlay, secondary users are allowed to operate in the close vicinity of primary users when the interference caused on primary users is taken care [31]. Figure 1.1 shows a spectrum underlay sharing scenario, where some secondary users operate within the TV service area when the caused interference by the secondary users on the TV receiver is lower than threshold.

Spectrum underlay is mainly conducted with a centralized controller, which has global knowledge of primary users' location, the attenuation between all secondary transmitters and all primary receivers. Then the centralized controller calculates the maximal permitted transmission power with certain optimization solutions in a situation where secondary users pose the maximal thread to primary users' operation, where all the secondary users work one the same licensed spectrum. Working with spectrum underlay paradigm, spectrum sensing functionality on secondary users becomes auxiliary [8, 9].



**Figure 1.1** The concept of underlay spectrum usage in a TV service scenario. The hull shows the range of a TV service area, inside and out of which secondary users work on the licensed channels.

- **Spectrum overlay:** The other approach is *spectrum overlay*, where secondary transmitters are only allowed to transmit when the primary users are detected as being idle, and the spectrum should be vacated if a primary user is detected transmitting. Working with this paradigm, secondary users should monitor the spectrum of interest pre-actively to detect primary users' appearance. This kind of spectrum

sharing strictly protects the primary users, but entails burden on secondary users especially on the aspect of spectrum sensing. The detection metrics include received primary users' signal power, spectral correlation or beacons [152]. Spectrum sensing requires sophisticated technologies when primary users' signal is weak, and can be improved by learning technologies or cooperation among multiple secondary users [19]. When spectrum sensing accuracy can be guaranteed above certain threshold, transmission power restrictions can be removed from secondary users.

## **Spectrum sharing among the secondary users**

Among the secondary users, there is also competition on transmission power, spectrum bands or transmission time slots. When we put the consideration on protection of primary users aside, spectrum sharing among the second users is similar with the counterpart problems in the conventional wireless networks, i.e., ad hoc networks, cellular networks, etc.. In CRN, spectrum sharing among the secondary users can be conducted with either cooperative or non-cooperative manner. As to cooperative spectrum sharing, secondary users take into consideration of the caused interference on other secondary users when deciding their spectrum usage. This pattern usually requires cluster structure to facilitate the negotiation of users in a neighbourhood [43]. Whereas with non-cooperative spectrum sharing, secondary users only consider the performance of their own.

### **1.1.3 Operation Models of Spectrum Sharing in CRN**

From the perspective of network operation, spectrum sharing can be classified as centralized and distributed spectrum sharing.

#### **Centralized Scheme**

Centralized spectrum sharing is conducted on a centralized entity to decide the secondary users' strategies to operate in the CRN. There is a considerable number of centralized approaches proposed for spectrum sharing in cognitive radio network, and the global optimality is reported as to different objectives in certain cases. The centralized solution is usually realized by formulating the communication problems into optimization problems. Let us take the resource allocation problem in CRN as example. The resource refers transmission power, spectrum, transmission time, etc.. The objective can be the following, improving the overall SINR of secondary users, mitigation of co-channel interference caused secondary users, minimization of transmission power, etc.. The constraints can be one or several of the following, the interference caused by secondary users on primary users is below a threshold, the transmission power is restricted by the maximum transmission power, or the number of spectrum bands allowed for secondary users to use. There is a good number of research in this regard [14] [64, Chapter 6].

When the formulated problems are linear programming, or convex programming problems, the optimal is easy to obtain. When convexity doesn't hold, there all some skills to convert the problem into convex ones, e.g., a log change of variables turns an apparently non-convex problem into a convex one. When integer variables are involved in the optimization, e.g., in problem of channel assignment, or scheduling, the optimization problem

is very difficult to solve. In this occasion, swarm intelligence algorithms can be applied to obtain sub optimal result. simulated annealing [xxx], generic algorithm [xxx] and ant algorithm etc.

Centralized scheme is suitable in certain scenarios, e.g., when the primary users are TV stations and receivers which work on certain channels for hours of time, spectrum can be seen as constant. When the secondary users access the spectrum in underlay paradigm, they need to take care not to cause more interference than threshold on primary users. In this case the channel usage, i.e., working channel and transmission power, can be decided on the centralized controller. Nevertheless, centralized solution is not suitable in many scenarios of CRN. First, central authority or controller is not available in many CRNs, e.g., CRAHN. Second, when the centralized decision maker exists, centralized solution involves a large number of control messages during network operation. The centralized entity needs to collect spectrum availability sensed on all the secondary users, then computes the spectrum usage strategy before finally distributing the result back to every secondary user. When primary users intensely access the spectrum and result in frequent change of available spectrum, the control overhead becomes a burden for the CRN. Third, Centralized solution need to react to the spectrum variance in the CRN, even when the spectrum changes only happens at a few secondary users. This compiled agility worsens the overhead problem.

## Distributed Scheme

It is the primary users which distinct CRN from other networks. Primary users' usually unexpected operation pattern results in dynamic spectrum availability for secondary users. As distributed schemes exploit mainly local observation, distributed schemes help the secondary users to adapt to the varying environment quickly [156].

Distributed solutions generally don't involve large amount of control overhead. When working with distributed spectrum sharing paradigm, secondary users in CRN decide on their spectrum usage strategies autonomously based on the local information, i.e., either on themselves or from their neighbourhoods. Consequently, distributed scheme usually requires secondary users to exchange information i.e., user ID, channel availability, etc. only with their neighbours, as a result, the overhead involved in the network is reduced. It is reported that control overhead takes account more than 50% of messages in most networks [64]. When the overhead is decreased, spectrum utilization and the number of served users and the network performance can be correspondingly improved.

In certain scenarios, distributed scheme may also make use of centralized entity when it is available and necessary, i.e., secondary users can access the centralized database to retrieve the available channels in IEEE 802.22 cellular networks.

Although distributed scheme results in appealing lightweight computation and overhead, it is actually challenging to design in many scenarios. When the distributed decision is not properly designed, it is possible to trigger endless ripple effect across the network, i.e., one CR's decision on its strategy prompts neighbouring CR users to change strategies, and it changes its strategy again later due to neighbours' new strategies. The distributed schemes designed for CRN can be roughly classified into the following categories.

- Distributed algorithms can be obtained from optimization formulation through decomposition techniques [114]. With this method, secondary users can achieve

global optimal only with local information. The distributed algorithms from decomposition usually involve communication overhead between different subproblems, and the convergence process is hard to decide.

- Online learning.

Online learning lies in the domain of artificial intelligence. The application of online learning in spectrum sharing in CRN includes Q-learning [51], no-regret learning [66, 73], reinforcement learning [78]. Operating with this method, each secondary user seeks for suitable resources to maximize its utility function. Secondary users' operation is in distributed and asynchronized manner, and the CRN ceases to update in a correlated equilibrium. online learning has appealing property on distributed execution, but the convergence speed can not be guaranteed.

- Game theory.

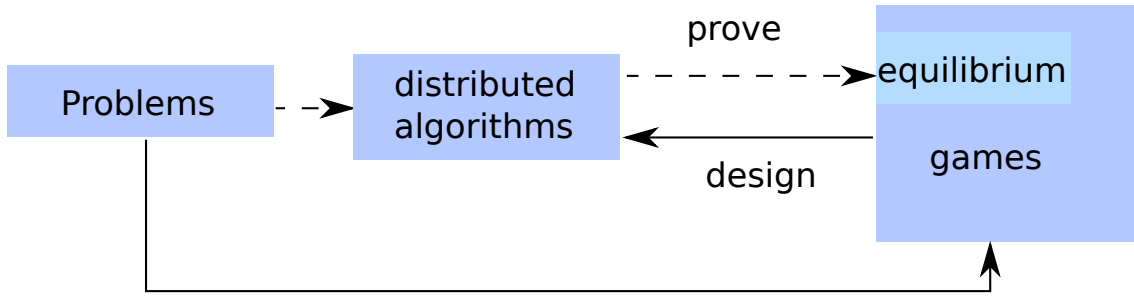
The interests of CR users are conflict, as they compete for limited radio resource. Besides, the CRN operator's interests not aligned with the CR users, e.g., , the operator aims to maximize summed throughput, while CR users only care their own performance. In both cases, we need to formulated these non-cooperative problems into games, and then analyze and solve them as games. Game theory is a branch of applied mathematics, and it is a powerful mathematical tool for studying, modelling and analysing the interactions among individuals. The approaches derived from a game are complementary to the optimization methods, thus game theory serves as an important role for the communication system when the optimization method doesn't work out. We will introduce the application of game theory in CRN in the following section.

## 1.2 Game Theory and CRN

A game consists of three elements: a set of players, a selfish utility for each player, and a set of feasible strategy space for each player. In a game, the players are rational and intelligent decision makers, which are related with one explicit formalized incentive expression (the utility or cost). The players usually have potential conflicting objectives with others. We say the players are rational as they make decisions in compliance with the subjective which is either to maximize the their own expected utilities, or to minimize the expected cost. Player are said to be intelligent as they understand the other players are also rational.

Game theory provides standard procedures to study its equilibriums [100]. The most frequently used concept of equilibrium in game theory is Nash equilibrium (NE). NE defines a stable state of the system, where with the given policies, no player may gain by unilaterally deviating its current strategy. In comparison with optimization which seeks to improve the welfare of the complete network, game theory sheds light on the individual entities in a system. Thus a game can be seen as a collection of optimization problems, each of which maximizes/minimizes the utility/cost of one individual entity in the system. Actually, a distributed scheme can be regarded as a non-cooperative game formulation.

Figure 1.2 illustrates two ways of applying game theory in the problem solving.



**Figure 1.2** The two ways that game theory is involved in problem solving

Along the dashed line and arrow, proposed distributed schemes can be examined under the game theoretic framework. In this way, a distributed algorithm is analysed by looking into the interaction among the users which operate the algorithm, and check whether the system reaches a desirable equilibrium, i.e., the individuals not only improve their own performance but also improve the global welfare of the complete system. In most cases, we can also inverse the this procedure to obtain distributed algorithms in CRN from certain known games [22]. As the solid line with arrow points shows, given a known game which holds preferred properties such like fast convergence to equilibrium, or the game only has a single equilibrium which is also the global optimal, we can formulate our problem in CRN into that kind of game, then modify the best response of the players as the distributed algorithm for the communication entities in our problem.

### 1.2.1 Application of Game Theory in CRN

In the past few years, game theory has been extensively applied to problems in communication and networking [107, 141]. Game theoretic models are used to understand many challenging problems in communication systems, such as resource allocation, topology control, routing, security and so on. NE in CRN means the CR user has no incentive to change its operating parameters e.g., transmission power, spectrum band, time slot, modulation, etc.. We introduce the reasons to apply game theory in communication systems and CRN in details.

#### Communication equipments are rational.

Although current communication equipments involve only a little artificial intelligence, they are supposed to be manufactured and operate based on standards to fulfil certain functions, but selfish behaviour may appear on certain individual equipments to achieve advantages over their peers [100]. In communication system, a device is programmed to maximize the expected utility, which is perfectly rational. For instance, Wi-Fi equipments are manufactured complying the IEEE 802.11 standards. But it is possible that certain manufacturers or the personal who uses the Wi-Fi system (we use station in the following) manipulate certain parameters to obtain performance advantage over other stations in the network. When all the stations in one network are supposed to run distributed coordination function (DCF), i.e., the station must wait a random period of time, which is called contention window (CW), before accessing the media when it senses the media to be busy, a certain selfish station may not choose to wait and is keen on sensing media.

The selfish behaviour causes more collisions for other stations, and possibly results in poor performance in the network.

In this example, the network entities are seen to be self-interested rational, then suitable game theoretic models can guide the behaviours of them, so as to achieve desirable collective behaviours and performance in terms of the system level objective. In Wi-Fi system, game theory facilitates the network operator to issue rules to make the selfish behaviour unprofitable, which curbs the selfish behaviours and lead to NE in the network.

### **Game theory is effective to solve networking problems.**

Algorithms can be retrieved from the analysis of a problem under the game theoretical framework. In the same example of media access in IEEE 802.11 DCF mentioned in the previous item, if stations are allowed to modify the length of contention window, each station will adjust its contention window to obtain the best performance. [42] shows as long as each station greedily updates its CW to maximize certain utility, after certain time the system will stabilize in NE. In the case, the best response process naturally defines an algorithm for Wi-Fi systems. Besides, outcome from game theory is robust [64].

### **Game theory is suitable to apply in cognitive radio network because of the properties of CRN.**

Firstly, as only local information is needed to feed each user, game theory is naturally suitable for designing distributed solutions for cognitive radio networks. Distributed solutions doesn't rely on centralized controller, and each user in the network adopts certain action as response to the other users or environment, this falls in the scope of game theory. Distributed schemes usually lead to Nash equilibrium, which is usually sub optimal. As comparison, although optimization is the common tool to pursue global optimality, when the information is not accurate or adequate enough, or the optimization itself is difficult to solve, the optimized results can be far from optimal.

Secondly, combinatorial nature of communication problems makes game theory as one of a few choices [64]. Many problems in wireless communication involve integer variables, i.e., channel availability, channel assignment, selection of modulation levels or channel coding. It is always challenging to solve combinatorial optimization problems, whereas game theory is natural to describe it in a discrete form.

In this dissertation, we look into congestion game and its applications in CRN. We leave the introduction of game theory and game models in the Chapter 2.

## **1.3 Representative Cognitive Radio Networks and Available Licensed Spectrum**

We introduce two types of cognitive radio networks, which contains the problems we will tackle in this dissertation.

#### 1.3.1 Cognitive Radio Ad Hoc Network

Cognitive radio ad hoc network (CRAHN) is composed with autonomous mobile cognitive radio users which work with overlay spectrum sharing paradigm. CRAHN is one mobile ad hoc network, whereas each mobile user has cognitive ability for spectrum access. Mobile users in CRAHN need to determine which channels are available to use with spectrum sensing, either cooperatively with neighbour users or individually. Mobile users have different knowledge of spectrum availability, which poses new challenges in addition to the ones for the management of mobile ad hoc network. CRAHN can be usually represented by an undirected graph  $G$ . Cognitive radio users constitute the vertices, the edge between two vertices is decided not only by the distance, propagation and attenuation properties between the two vertices, but also the spectrum availability on both vertices, i.e., when they can decode the received signal from each other correctly, and there is common channel available between them on which communication is conducted, then an bidirectional edge is available on graph  $G$ . As to ad hoc network, the graph is constant when users are static. As to cognitive radio ad hoc network, due to primary users' activity, an edge between two vertices is decided by the fact that whether the two vertices can simultaneously access the same licensed channel. Hence, the corresponding graph is dynamic under primary users' operation, which imposes extra difficulties on network organization, routing and many other network functionalities.

As a versatile and flexible network infrastructure, CRAHN is comparatively easy to implement and operate, thus CRAHN is proposed to apply in many scenarios [17]. There are several open issues for CRAHN. For instance, a robust architecture against the network dynamics caused by nodes' mobility, primary users' activity is still due. It is challenging for CRAHN to meet the QoS criteria with the spatial and temporal availability of spectrum [101].

#### 1.3.2 Cognitive Radio Cellular Network

According to Cisco's forecast [47] on mobile data traffic, which published in 2015, the number of mobile-connected devices exceeded the world's population in 2014 and by 2019 there will be nearly 1.5 mobile devices per capita. The monthly global mobile data traffic will surpass 24.3 exabytes by 2019, which is 10 times more than that in 2014. Although there is a strong trend that more traffic load will be offloaded from cellular networks (on to Wi-Fi), cellular networks still face the big challenge and need to explore new resource to meet the demand on increased traffic. The state of the art mobile systems, such as 3GPP Long term Evolution (LTE) and High Speed Packet Access (HSPA) are designed to be very efficient on spectrum utilization, thus cellular networks look into cognitive radio for solution.

#### 1.3.3 CRN Working with TV White Space

TV white space (TVWS) refers to the unused broadcast television spectrum, which is the most feasible spectrum available for large scale utilization for cognitive radio cellular networks. One part of the unused TV spectrum is resulted from the switchover of analog television to digital television, which frees up a large range of TV spectrum. The other

parts are unassigned spectrum and the spectrum which is not being used at a given time in a given geographical area. TV white space is located in the VHF (54-216 MHz) and UHF (470-698 MHz) bands, thus the frequency bands within this spectrum are highly desirable for wireless communications, because they have good properties on propagation, i.e., they have broadband payload capacity, and are able to penetrate obstacles such as trees, buildings, and rugged terrain in non-line-of-sight scenarios. As comparison, a Wi-Fi router's transmission range, which uses 2.4 GHz radio frequency and complies with IEEE 802.11, is around 100 meters under perfect conditions, and the signals can be blocked by barriers, whereas, TVWS band can cover an area of about 10 kilometres in diameter. In 2010, FCC allowed [8] unlicensed devices to operate in the TV white space to provide broadband communication for consumers and business.

Autonomous spectrum sensing is one approach for secondary users to decide the available TV spectrum. Secondary user scans certain parts of the TV spectrum band to detect the presence of existing TV equipments with advanced spectrum sensing techniques and algorithms. This scheme involves complex sensing techniques, and it doesn't work well in worst case sensing scenario, i.e., the TV receivers are hidden nodes [80]. Moreover, it is required to be conservative when deciding the available TV channels leading to an underestimation of available TV channels and causing inefficiencies [63]. Considering the slow change of TV spectrum usage, along with the fact that the spectrum usage by TV stations is scheduled, the geolocation approach together with central database becomes more appealing in TVWS utilization scenario. FCC adopts this solution as the main way, and regulates the secondary usage of TV white space in a prudent manner, including the spectrum bands permitted to use based on their location, the transmission power, the distance away from TV service area and so on.

FCC regulates portable secondary users to operate from channel 21 (512 MHz) to 51 (698 MHz), with the exception of channel 37. As to the fixed secondary users, the allowed spectrum band is from TV channel 2 (54 MHz) to TV channel 51, with TV channels 3, 4 and 37 being prohibited. Thus, the available TV spectrum is about 600 MHz wide. Compared to conventional unlicensed ISM bands in the 2.4 GHz and 5 GHz band, all together TVWS has more to offer. [65] analyses the capacity possibly provided by exploiting free TV channels. Complying with the rigid regulation of FCC, TV white space brings hundreds of kilobits/sec per square kilometre for secondary users when the communicating secondary pair is about 1 km apart. The research shows that interference from TV stations and other co-channel secondary users heavily restricts the capacity. [32] further investigates the possible TVWS usage *within* TV service area (referred to as *gray space*), where the secondary usage does not violate TV receivers. The authors propose another significant amount of TV spectrum, but the interference from TV broadcasters is stronger due to the secondary receivers being closer to TV broadcasters.

The potential applications supported by TVWS are the major driving force for TVWS communications technology. It is envisioned that TVWS can support high-data-rate backbone for fixed stations in large area connectivity, short-range indoor connectivity, seamless connectivity for mobile stations and emergency related equipment. In the following, we give a brief introduction to the governing regulations and industrial standards issued for TVWS usage.



## Co-Existence in TV White Space

Regulatory bodies in different countries have issued requirements on the occupancy of TVWS bands by secondary users respectively. The permitted channels regulated by FCC have been briefly introduced above. Besides, FCC divides the devices operating in the TVWS (TV band devices, or TVBDs) into three categories: Fixed device, Mode I personal/portable device, and Mode II personal/portable device. Every category obeys distinct rules on transmission power, mandatory database access and so on. Fixed and Mode II devices are connected to the database directly (or indirectly by another fixed or Mode II TVBD), Fixed device accesses DB at least once per day, Mode II device has to access DB every 60 seconds, or when it changes location by more than 100 meters. The transmission power for Fixed TVBD is 4 W, and 100 mW for Mode II TVBD. Mode I TVBD operates only under the control of a fixed or Mode II TVBD, and accesses them for available channels every 60 seconds. There is also regulations on the antenna heights and other aspects. OFCOM (being the regulatory authority in UK) opens less TVWS for TVBD and categorizes White Space Devices (WSDs) into Master WSD and Slave WSD which are roughly equivalent to fixed/Mode II TVBD and Mode I TVBD, respectively. ECC (being the regulatory authority in Europe) also adopts Master and Slave structure while utilizing the geolocation and database solution to assign channels and corresponding operating parameters to TVBD. In both FCC and ECC regulations, the available channels are calculated based on the distance between primary and secondary systems together with some propagation model. Hence, the principle behind the notion of channel availability is the presumed received signal strength at potential TV receivers, which should be obviously below a certain level. FCC adopts a fixed transmit power level for secondary systems and assumes that the distance used (resulting from the propagation model) is sufficient to protect the TV receivers. ECC's restriction is more flexible on the transmission power which can be adjusted based on the distance between the secondary user and the TV receivers. For both regulations, TV receivers may become vulnerable when there are multiple secondary equipments transmitting simultaneously. [79] shows that even the regulations integrate an interference margin for multiple secondary users' transmitting, the TV receivers are still vulnerable.

Standardization on secondary usage of TVWS is very active. IEEE 802.19 is a family of IEEE 802 standards in Wireless Coexistence Working Group, among which, IEEE 802.19.1 is for wireless coexistence in TVWS. IEEE 802.11af is standards for WLAN operation in TVWS, IEEE 802.15.4m is for low rate (LR) WPAN operating in TVWS. IEEE 802.22 is for Wireless Regional Area Network (WRAN) using TVWS. As TV band is only 6 or 7 MHz while WLAN channel bandwidth is 40 MHz for state-of-the-art IEEE 802.11n amendment, IEEE 802.11af proposes channel aggregation mechanism for WLAN working with TVWS. Maximum transmit power and spectrum mask of Mode I should be approved by Mode II devices beforehand.

IEEE 802.22 has been the first IEEE standard for wireless regional area network (WRAN) working on TV white space. According to this standard, the system consists of a base station and customer premises equipment (or terminals for short), where terminals are associated with base stations, and are served by them. Recent standard published in Nov. 2010 mentions both, i.e. sensing ability as well as TVWS geolocation and database look-up as schemes for detecting primary systems. Self-coexistence mechanism is also proposed, which provides a TDMA like mechanism for white base stations (WBS) to share TVWS. [68] proposes a distributed solution for power control and channel assignment in

both down-link and up-link communication in a WRAN, but the investigated secondary network is composed with only one base station and multiple terminals.

## 1.4 Problem Statement

As to the various challenging problems in CRN networks, adopting centralized or decentralized solutions is largely dependant on the network types and nature of the problem to be solved. CRAHNs consist of mobile autonomous secondary users and don't have infrastructure, thus the network topology evolves with time, besides, the ungovernable primary users' activity leads to the temporal spectrum availability. These factors make distributed scheme to be the sole choice to manage the CRAHN to convey services. Traditional commercial cellular networks work with licensed spectrum, which operate with centralized manner to provide services meeting QoS criterion. The openness of TV white space attracts private people and organizations to build cellular networks to launch various services. There is one centralized database which is required by regulations and standards, thus it is possible to conduct certain centralized solutions on it. But owners of the cellular networks have different or conflicting interests, thus it may not be possible to let every secondary user to follow the strategy given by the centralized data base [92].

In this dissertation, we meet the challenges in CRN by proposing distributed solutions. We investigate the differences between decentralized and centralized schemes in terms of several criteria, i.e., Performance, overhead, computation time, etc.. Except for the provisioning of services, we are also interested in the mechanism the distributed algorithms. In particular, we look into the following questions, Does the distributed algorithm converge? If the algorithm converges, in which state the algorithm finally ceases, and how robust and quickly the convergence process is? To answer these questions, we turn to game theory, in particular, the recent progress of congestion game models for help.

The concrete technical challenges addressed in this thesis are shown in Figure 1.3, which reside from physical layer to network layer [2]. These questions are individually orthogonal, but as pressing problems in each layers, they together form a test field to check the feasibility of distributed solutions.

OSI Layer	Functionality	Problems	Characteristics of solution
Network Layer	spectrum sharing	routing	distributed, non-cooperative
Link Layer		clustering	distributed, non-cooperative
Physical Layer		spectrum/power allocation in IEEE 802.22 network	distributed, cooperative

**Figure 1.3** Spectrum management and the problems addressed in this thesis

In the following sections, we will introduce the problems in Figure 1.3 in details.

### 1.4.1 Utilization of TV white spectrum

There are several problems with the current regulations and standards proposing on the utilization of TVWS. First, most of these proposals rely on the centralized database to manage the spectrum usage in a centralized manner. This paradigm is not suitable when the TVBD belong to different business bodies, i.e., operators. Thus, a centralized resource allocation is infeasible. Second, the current usage of TVWS is prudent, i.e., on working channel and transmission power as introduced in Section 1.3.3. These conservative measures on the transmission power restrict the full utilization of TV spectrum. Third, although the transmission power of TVBD is restricted, the interference between TVBDs is not given consideration by any regulation or standard. In fact, the channel allocation problem we encountered in this problem is unique as the transmission power for each user and on each channel is different. In other words, the symmetric interaction doesn't exist, thus the solutions proposed for the channel assignment problems in conventional networks, e.g., ad hoc networks, or mesh networks can not be applied any more. The asymmetry disables the heuristic distributed schemes provided in [84], and makes channel allocation problem not to fit into the congestion game model proposed in [95] which is the first paper to discuss channel allocation from the perspective of game theory.

### 1.4.2 Robust Clustering in Ad Hoc Cognitive Radio Network

Clustering is an important paving stone for the practical utilization of the unused portions of the licensed spectrum. Clustering secondary users based on geographical proximity and other relevant properties together produces following benefits. First, it is more efficient to solve common control channel (CCC) problem with cluster structure. Dedicated CCC which is allocated to all nodes for the purpose of control information exchange is regarded to be under utilization. Whereas, cluster based approaches group CR nodes into clusters based on their similarity of available unlicensed channels, so that the common channels within each cluster are used to carry the control messages [89]. Second, cluster structure facilitates cooperative sensing and increases the sensing reliability [135]. Third, cluster structure supports coordinated channel switching and simplifies routing in ad-hoc cognitive radio networks [130].

The benefits promised by cluster structure is challenged by the ungovernable activity of primary users. If the members of one cluster have to vacate all the CCCs due to primary users' appearance, the cluster doesn't exist any more. But if the cluster members can migrate to another CCC when one CCC is occupied by primary user, the cluster survives. Thus, a large number of CCCs within cluster means high robustness against primary users. Unfortunately, related works [26, 74, 147, 157] fail to pay attention to this aspect. There is a pressing need for one distributed clustering scheme, which should fulfil the following requirements. First, the execution of the scheme should be finished quickly, so that the scheme can react quickly enough to the primary users' activity. Second, the scheme should result in more CCCs in the formed clusters.

### 1.4.3 Geographic Routing in CRN with Spectrum Aware Virtual Coordinate

Recent measurement in [113] shows the spectrum occupancy doesn't have significant spatial correlations between different locations. It follows that licensed spectrum is used by primary users heavily in some areas, whereas in the other areas licensed spectrum is available over longer timespan for secondary users to use. It is obvious to see that a routing path is better to go through the areas where primary users occupation is lower, as this alleviates or avoids the burden to cope with the fast changing or totally occupied spectrum when forwarding packets potentially with latency requirements. Modified version of geographic routing is able to avoid the primary user influenced area, but it is myopic. Considering the available spectrum is geographically heterogeneous, applying geographic routing like routing schemes in CRAHN is appealing, but the supporting coordinate system is not complete. How to make light weight geographic routing work well in CRN is the problem to be addressed.

### Research Questions

Based on the description on the challenges, we formulate three research questions which will be answered in this thesis.

Question 1 - How to design distributed solution with assistance of game model, so that to make full use of the TV spectrum with the regulated network structure?

Question 2 - How to design distributed solution with assistance of game model, to form robust clusters against primary users' unpredictable activity, so that maintain the benefits brought by cluster structure?

Question 3 - How to make use of the statistic information of the spectrum availability, so as to realize light weight geographic routing in CRN?

## 1.5 Contributions

We briefly introduce the contribution of this dissertation by addressing the aforementioned questions.

### Contribution 1: Utilization of TV white spectrum and application of game model in designing algorithm

In term of providing solution to fully exploit TV white space complying to IEEE 802.22 standard, we propose a complete set of methods to regulate the upper bound of transmission power, working channel and power. Our solution makes full use of the current network structure, strictly protect primary users with a centralized scheme which is conducted in the centralized database. Proposed scheme also considers the necessity of distributable execution which decides the working channel and transmission power.

As to the channel allocation problem, we innovatively formulate this problem in to a canonical congestion game, and design efficient distributed channel selection strategy with the assistance of the centralized database. The successful practice of congestion game in this problem is enlightening for the application of congestion game in other problems where asymmetric interaction exists.

## **Contribution 2: Versatile robust clustering in CRAHN**

Our proposed distributed clustering scheme achieves robust cluster structure against primary users. Furthermore, the clustering algorithm is versatile enough to fulfil other requirements, i.e., fast convergence, less signalling and cluster size control.

Congestion game model is adopted during the algorithm design, which not only contributes the algorithm design directly, but also provides guarantee on convergence and convergence speed.

## **Contribution 3: Spectrum aware virtual coordinates and geographic routing in CRN**

We design spectrum virtual coordinate for CRN, which is not only independent on geographic position, but decided by connection quality among nodes, for instance, channel availability, link quality or hop numbers. With this new coordinate, Euclidean distance between two secondary users reflects the availability of unlicensed channel between them. Then the geographic routing can run with this coordinate system as the spectrum virtual coordinate contributes a large part to the decision on next hop.

## **1.6 Outline**

The structure of this thesis is as follows. In chapter 2, we introduce the theoretical tools used in design algorithms, i.e., game theory and optimization. Chapter 3 introduces the work on utilization of TV white space. The robust clustering scheme is introduced in Chapter 4. In Chapter 5, virtual coordinate based on geographic routing is designed and geographic routing runs on the top of it. Finally, Chapter 7 concludes the thesis by summarizing our contributions and discussing the future work.



# 2

## BACKGROUND

Game theory and optimization techniques have extensive applications in different scenarios. In this dissertation, game theory is the mathematical tool which helps us analyse problems and design algorithms, but we also adopt optimization to achieve the global optima. In this chapter, we introduce game theory and optimization, as to the latter, we in particular introduce the decomposition method which is used to design distributed algorithms.

### 2.1 Introduction of Game

In this section, we give a brief introduction of game theory and congestion game which is applied to solve problems in the dissertation. The notations used in this thesis comply with [109].

A game of normal form can be represented as a tuple  $\Gamma = (\mathcal{N}, (\Sigma_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$ , where

- $\mathcal{N}$  is a finite set of players.
- $\Sigma_i$  is player  $i$ ' set of strategies. Player  $i$  selects one strategy  $\sigma_i \in \Sigma_i$  at one time to play the game.
- $\Sigma = \Sigma_1 \times \cdots \times \Sigma_n$  is the set of states, which denotes all the possible ways that players may pick strategies.
- $\sigma = (\sigma_1, \cdots, \sigma_n)$  is one strategy, which represents an instance of all players' strategy choices.  $\sigma$  is also called one strategy profile, and there is  $\sigma \in \Sigma$ .
- One vector of strategies of player  $i$ ' opponents is expressed as  $\sigma_{-i}$ , and the corresponding strategy profile is denoted as  $\sigma = \{\sigma_i, \sigma_{-i}\}$ .  $u_i(\sigma) = u_i(\sigma_i, \sigma_{-i})$  is the player  $i$ ' outcome in strategy profile  $\sigma$ .

Elements of a game	Components of one CRN
Player $\mathcal{N}$	secondary users
Strategies for player $i$ , $\mathcal{S}_i$	working channels, transmission power, modulation, etc.
Utility of player $i$ , $u_i$	performance in respect of SINR, throughput, etc.

**Table 2.1** Components of problems in CRN and corresponding elements in game

- $u_i : \Sigma \rightarrow \mathbb{R}$  is the utility function of player  $i$ .  $\Sigma_{-i} = \prod_{j \in \mathcal{N} \setminus \{i\}} \Sigma_j$  is the set of states of all the other players except for player  $i$ . As to each player  $i$ , its utility is decided by its choice on strategy  $\sigma_i \in \Sigma_i$ , and is also dependant on the choices of other players  $\sigma_{-i} \in \Sigma_{-i}$ . Utility can also be denoted as  $u_i(\sigma)$  or  $u_i(\sigma_i, \sigma_{-i})$  to stress that the utility is made based on all players' strategies.

Utility function is very important to specify a game, as it gives players preferences on the outcomes with respect to all strategy vectors  $\Sigma$ . The value of the utility can be regarded as payoffs or costs depending on concrete scenarios. Actually, one player maximizes its utility is equivalent to that the player minimizes its cost, note that cost is the reversed utility.

When we want to use game theory to analyse a problem in CRN, it is critical to formulate appropriate components of the problem into corresponding elements of a game. The commonly used formulation is summarized in Table 2.1.

### 2.1.1 Basic Solution Concepts

In this section, we will introduce some basic solution concepts, some of them are used in this thesis.

#### Dominant Strategy Equilibrium

In some games, each player has a unique best strategy, which is independent of the strategies chosen by the other players, then we say a game of this kind has a dominant strategy solution. The mathematical expression is, a strategy vector  $s \in \mathcal{S}$  is a dominant strategy, if for each player  $i$  and each alternate strategy  $s' \in \mathcal{S}$ , there is,

$$u_i(s_i, s'_{-i}) > u_i(s'_i, s'_{-i})$$

$s$  is also called strong dominant strategy, and when the  $>$  can be written as  $\geq$ ,  $s$  is called weak dominate strategy. Note that a dominant strategy solution may not give an optimal payoff to any of the players. This is the case in *prisoner's dilemma*, which is one of the most well known and well studied games. To confess is the dominant solution for both prisoners, which brings them longer time behind bars than that when both of them keep silent<sup>1</sup>. Only a few games have dominant strategy equilibrium, and mechanism design [75] is developed to design games which have dominate strategy equilibrium, and these dominate strategies lead to desirable outcome.

<sup>1</sup>Prisoner's dilemma can be found in almost all the game theory textbooks, thus omitted in this thesis.



## Nash Equilibrium

A desirable solution of games is the one that players choose strategy in accordance with their incentives, minimizing their own cost or maximizing their own payoff/utility. Nash equilibrium successfully captures this property, and is the most discussed and pursued solution concept in game theory.

A strategy vector  $s \in S$  is a *Nash equilibrium* if for any player  $i$  and each alternate strategy  $\sigma'_i$ , there is

$$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i})$$

This means for any player in NE state, no unilateral deviation from its current strategy is more profitable. This also implies that, NE is self enforcing that once players agree on this solution, it is the best interest for every player to stick to its current strategy.

A dominant strategy equilibrium is a Nash equilibrium, but a NE is not necessarily a dominant strategy. There may be multiple NEs in one game, and NE may not be optimal for players. In prisoner dilemma, the dominating strategy is NE but is obviously not the optimal.

Being not unique and possibly sub-optimal, NE is still applied in extremely diverse applications due to the reasons discussed in the beginning of this section. Thus, when pursuing NE as solution, people should answer the question that, what is the gap between NE and global optimal? This can be partially answered by *price of anarchy (PoA)*, the ratio between the worst-case Nash equilibrium to the optimality is used to denote the quality of the solution.

Nash equilibrium is an appealing concept, but it doesn't tell how to reach such a state. Hence, it is important to find an efficient algorithm to reach the equilibrium. The notion of *NP-completeness* is not an appropriate concept of complexity for *finding a Nash equilibrium* problem as NE always exists, nevertheless, theoretical scientists tell *finding a Nash equilibrium* problem is a combinatorial problem and is often very difficult (Chapter 2, [109]). Having said that, in some games, with the special strategy space structure, or players' special behaviours, efficient algorithms to reach NE exist.

## Pure NE and Mixed NE

In the aforementioned games, players deterministically choose one strategy from their strategy sets, then play their chosen strategies and don't involve randomized strategies, then the achieved NE is called pure NE. When players play a game with certain randomization on strategies, and aim to maximize their expected payoff, we call the resulted NE as mixed NE. As to mixed NE, the action of a player is to choose certain strategies according to a probability distribution. Games with finite number of players and strategy set are guaranteed to have NE, whereas, games with an infinite number of players, or games with a finite number of players but they can access to an infinite strategy set may not have NE [109].

In this thesis, we only consider pure NE, as players deterministically play a certain strategy, and the corresponding network components stick to certain operation instead to switch among several different operations based on a vector of probabilities. Other solutions include correlated equilibrium, which also involves probability distribution over strategy vectors.

## Individual Optimization and Game

As to a player in a game, given the strategies of other players  $\sigma_{-i}$ , its utility  $u_i$  is a function of its own strategy  $\sigma_i$ , then the maximization of its payoff or minimization of its cost becomes one optimization problem. Thus, one game comprises  $n$  such optimization problems in total.

## 2.2 Congestion Game

Congestion game is an attractive game model which describes the problem where participants compete for limited resources in a non-cooperative manner, it has good property that Nash equilibrium can be achieved after finite steps of best response dynamic. Congestion game has been used to model certain problems in internet-centric applications or cloud computing, where self-interested clients compete for the centralized resources and meanwhile interact with each other. For example, server selection is involved in distributed computing platforms [41], or users downloading files from cloud, etc. In the following we will introduce an *server matching* [85] problem to illustrate congestion game's application in communication systems.

In accordance with [140], congestion game is a game where players simultaneously allocate sets of resources to minimize their costs, and the cost of a resource is a function of congestion, which is the number of players choosing the resource. Congestion games can be formulated from many problems in realistic world, e.g., minimisation of commuting time on the road for commuters, minimization of energy consumption in mobile cloud computing system [60].

Now we give the formal definition of a congestion game. A congestion game [122] [140] can be expressed by a tuple  $\lambda = (\mathcal{N}, \mathcal{R}, (\Sigma_i)_{i \in \mathcal{N}}, (g_r)_{r \in \mathcal{R}})$ , where

- $\mathcal{N} = \{1, \dots, N\}$  denotes the set of players (each each is labelled with a unique index number)
- $\mathcal{R} = \{1, \dots, m\}$  the set of resources
- $\Sigma_i$  is the set of resources for player to use.
- $\Sigma_{i \in \mathcal{N}} \subseteq 2^{\mathcal{R}}$  is the strategy space of player  $i$ . Under strategy profile  $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)$ , player  $i$  chooses strategy  $\sigma_i \in \Sigma_i$ , and the total number of users using resource  $r$  is  $n_r(\sigma) = |\{i \mid r \in \sigma_i\}|$ .
- The cost  $g_r : \mathbb{N} \rightarrow \mathbb{Z}$  is a function of the number of users for resource  $r$ ,  $g_r^i = \sum_{r \in \sigma_i} g_r(n_r(\sigma))$ .

In our paper,  $g_r^i$  is referred as *congestion* of a game.

We first give a definition of improvement.

**DEFINITION 1:** An *improvement path* is a path  $(\sigma^0, \sigma^1, \dots)$  in which  $u_i(\sigma^k) < u_i(\sigma^{k+1})$ , where  $\sigma^k$  and  $\sigma^{k+1}$  differ in the player  $i$ 's coordination, i.e., only  $i$ 's strategy changes in the strategy  $\sigma = \sigma_1, \sigma_2, \dots, \sigma_{|\mathcal{N}|}$ .

The best response path contains only states for best response improvement steps.

**THEOREM 2.2.1:** *For every congestion game, every sequence of improvement steps is finite. [122]*

This theorem immediately implies the following corollary,

**COROLLARY 2.2.1:** *Every congestion game has at least one pure Nash equilibrium. [122]*

To prove this, let us first have a look at one similar game, potential game. Potential game [105] has already been applied in wireless network and CRN to help solve different problems [58, 88, 108, 111]. Potential game has two desiring properties, the existence of pure NE, and every improvement path is finite, which makes it a suitable model to design distributed algorithms.

## Potential Game

A potential game is a tuple  $\lambda = (\mathcal{N}, (\Sigma_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$ , which satisfies the following, if there exists a function  $\phi : \Sigma \rightarrow \mathbb{R}$ , such that for every  $i \in \mathcal{N}$ , for every  $\sigma_{-i} \in \Sigma_{-i}$ , and every  $\sigma_i, \sigma'_i \in \Sigma_i$ :

$$u_i(\sigma_i, \sigma_{-i}) - u_i(\sigma'_i, \sigma_{-i}) \geq \phi(\sigma_i, \sigma_{-i}) - \phi(\sigma'_i, \sigma_{-i}) \quad (2.1)$$

If equality holds, this game is called an exact potential game, when it doesn't, the game is called an ordinal potential game. It is easy to see that the design of potential function  $\phi$  is the key point to form a potential game. The incentive of any player to change its strategy can be expressed using the potential function, in other words, potential game tracks the changes in the payoff/cost with some player deviates.

Potential game has two important theorems,

- Every finite ordinal potential game has at least one pure strategy Nash equilibrium.
- In every finite ordinal potential game, every improvement path is finite.

### 2.2.1 Convergence Time Towards Nash Equilibrium

As to the Theorem 2.2.1 which implies that for every congestion game, every best response ends in finite steps, we introduce the sketch of the proof of this proposition, as the proof also reveals the number of steps needed to reach a Nash Equilibrium. We first introduce Rosenthal's potential function  $\phi(s) : \Sigma \rightarrow \mathbb{Z}$ , where  $\sigma$  is the strategy profile of all the players, let  $\sigma = \sigma_1, \sigma_2, \dots, \sigma_N$ :

$$\begin{aligned} \phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\ &= \sum_{i \in \mathcal{N}} \sum_{r \in s_i} g_r(n_r^i(\sigma)) \end{aligned} \quad (2.2)$$

$n_r^i(\sigma)$  is the number of players using resource  $r$ , whose indices are smaller than or equal to  $i$ , i.e., from  $\{1, \dots, i\}$ . In the part after the second equality sign,  $\sum_{r \in \sigma_i} g_r(n_r^i(\sigma))$  is a virtual value or cost that player  $i$  have when assuming the resource  $r$  is not used by players whose indices are greater than  $i$ .

The intuitive interpretation of the virtual cost is, according to [140], the cost of each player for choosing the strategy when it is inserted into the game. Let us assume player  $N$  is the last player to be inserted into the game, then  $\sum_{r \in \sigma_N} g_r(n_r^N(\sigma))$  equals to the real cost that player  $n$  takes. When player  $N$  can decrease its cost by switching to another strategy by an unilateral move, then  $\sum_{r \in \sigma_N} g_r(n_r^N(\sigma))$  and its potential decrease by the same amount.

As the potential can be calculated by inserting the players with any sequence, each player will have the same property with play  $N$ , as we just discussed.

In summery, the change of the potential caused by one player's unilateral move from  $\sigma_i$  to  $\sigma'_i$  is equivalent to the change of gain (or loss) of that player.

$$\Delta\phi(\sigma_i \rightarrow \sigma'_i) = g^i(\sigma'_i, \sigma_{-i}) - g^i(\sigma_i, \sigma_{-i}) \quad (2.3)$$

$\sigma_{-i}$  is the strategy profile for all players except for  $i$ . Thus the potential decreases with the update of players monotonically during the convergence process. Most importantly, as the potential of a congestion game is bounded, and every best response move made by a player decreases it at least by 1, the length of any sequence of improvement steps is also finite. Then the Theorem 2.2.1 is proved.

The existence of Nash equilibria gives a natural solution concept for congestion games. Comparing with potential game, congestion game is much easier to connect with the problems in networks or CRN. As long as we can decide the payoff (or cost) happened upon a certain resource, i.e., spectrum bands, time slots, transmission power is monotonic increasing (or decreasing), we can formulate the problem into one congestion game, and we can make use of the property of congestion game to implement best response to achieve Nash equilibrium.

## 2.2.2 Singleton Congestion Game

We introduce a special type of congestion game [77], which implies polynomial number of best response steps towards convergence.

**DEFINITION 2:** A congestion game is called singleton if, for every  $i \in \mathcal{N}$  and every  $\mathcal{R} \in \Sigma_i$ , it holds that  $|\mathcal{R}| = 1$ .

In words, each player allocates one single resource from its strategy set. Although this constraint on the players strategies is very restrictive, there are still  $m^n$  different strategy profiles.

**THEOREM 2.2.2:** In singleton congestion games, all improvement sequences have length  $\mathcal{O}(n^2m)$  [77]

The sketch of the proof is as follows. The upper bound on the number of convergence steps is shown in Formula 2.2. Now we need to approximate this expression. Replace

original delays by smaller integer values without changing the preferences of the player, then calculate an upper bound on the maximum potential with respect to the new delays. For example, if the delay on a certain resource is 15, 50, 100 with respect to 1, 2 and 3 players using that resource, then we manipulate the delays as 1, 2, 3. When the number of resource is  $m$  and number of players is  $n$ , the largest new delay is  $nm$ . Then the Rosenthal's potential in Formula 2.2 has the following relationships with an upper bound.

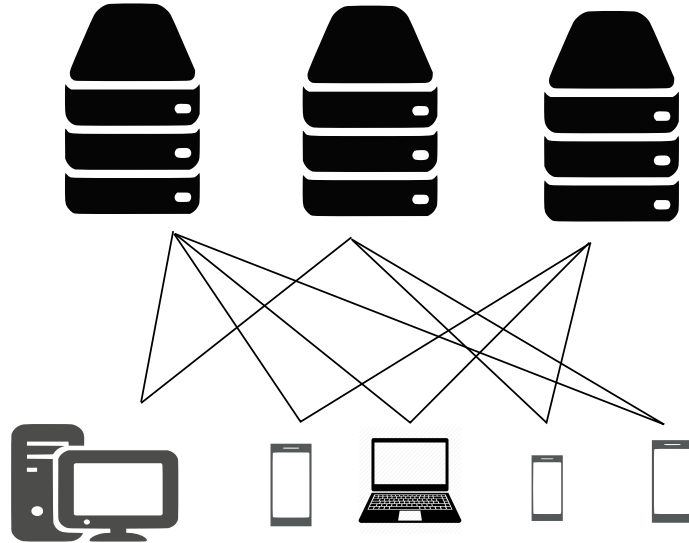
$$\begin{aligned}
 \phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\
 &\leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} (nm) \\
 &\leq n^2 m
 \end{aligned} \tag{2.4}$$

Therefore, the length of improvement sequences is upper-bounded by  $n^2 m$ . The proof in details can be found in [70, 77]

### Example: Server Matching

In order to show how is problem in networks formulated into a game, and from which an effective algorithm is proposed, we introduce a small example with the application of singleton congestion game.

Let us consider a number of self-interested clients and servers as shown in Figure 2.1. Each client is allowed to access one server. The latency of one server is a monotonic increasing function of the *number of clients* attached to it. When clients try to choose one server which has the shortest latency, a congestion game is formed.



**Figure 2.1** An example server matching, one server is possible strategy of the client at the other end of the connecting line

Following the description in Section 2.2, we write the game for this problem,

- The clients constitute the players in the game  $\mathcal{N}$ , and the collection of servers  $\mathcal{M}$  is the strategy space for every player.
- The cost of players in the game is equivalent to the latency for the clients. If we denote the delay happens on the server as  $d(n)$ , where  $n$  is the number of clients using the server, then the delay (cost) for a client (player)  $i$  is  $d(n_r(\sigma))$ , where  $n_r(\sigma)$  and  $r \in \sigma_i$  is the number of clients using the server  $r$ .
- As each user is allowed to use on server, this is a singleton congestion game according to Definition 2.

Then, according to Theorem 2.2.1 and 2.2.2, after maximal  $nm$  times best responses, the system reaches Nash equilibrium.

### 2.2.3 Variants of Congestion Game

In the congestion game we have introduced in Section 2.2, the cost caused on one resource is function of the number of players using that resource. In some variants of congestion game, the cost is decided by some other factors. In *player specific congestion game*, different players may have different delay functions. As to *weighted congestion game*, different players may have different impacts on the delays of the resources they allocate, in other words, there is a weight for every player, and the congestion on a resource is the total weight of all players using that resource.

As to neither player specific congestion game nor weighted congestion game, the finite converge does not hold anymore. But there exists NE for both of Player specific congestion game and weighted congestion game when they are singleton congestion game [56, 102]. When these variant of congestion game are not singleton ones, [13] points that NE exists when the strategy space of players are matroid.

## 2.3 Optimization

As is mentioned in Section 1.1.3, a variety of problems in CRN are formulated and solved by constrained optimization [28, 149, 153]. Table 2.2 summarizes the commonly proposed objectives, involved resources and constraints in the optimization problems in the domain of wireless communication. Part of the contents in the table refers [64]. As to CRN, the interference caused by secondary users on primary users is usually formulated into constraints, which distinguishes the problems in CRN from that in traditional wireless networks.

Convexity is a pursued property in optimization problem solving [35]. As to a convex optimization i.e., minimizing a convex objective function over a convex constraint set, a local optimum is also a global optimum and the duality gap is zero, and there are provably polynomial-time algorithms to solve the convex optimization, i.e., interior-point methods. It is common to conduct optimization in centralized manner in small scale network, but in large scale network, centralized solution is infeasible for several reasons. First, it is hard to obtain the information resides across the network, especially when the information is varying, i.e., the spectrum availability at secondary users in CRN. Second, it is hard for the centralized scheme to scale with the network [114].

### 2.3.1 Decomposition Methods

The decomposition methods are originally proposed to solve very large optimization problems, but as a side effect, it also yields decentralized solution algorithms, which is important in many scenarios of wireless network [36]. In certain cases, decentralized solutions can be interpreted as a set of simple protocols which allow a collection of subsystems to coordinate so that to achieve global optimality.

*Decomposition method* decomposes the original problem into distributively solvable subproblems. Specifically, the decomposition techniques includes *primal decomposition* and *dual decomposition*. The primal decomposition decouples variables, i.e., decomposes the original problem by allocating the available resources to each subproblem, and derived problem formulation is *master problem*. The dual decomposition solves the subproblems and decouples the constraints usually by forming Lagrangian [30, 114]. There is communication between the master problem and the subproblems. The communication is in the form of message passing which introduces overhead in the network. In some cases, however, the communication is implicit in the system, e.g., delay, packet error probability, as these quantities have physical meanings and can be implicitly measured without the need of explicit signaling. The different ways to decouple the resources lead to different algorithms which have different characteristics of the amount of communication overhead and convergence properties.

We take the joint channel and power allocation problem in CRN [136] as an example, to show how is one distributed algorithms obtained by decomposing the optimization problem. To eliminate the impact of the coupling optimization variable, i.e., channel and power levels, primal decomposition decomposes the original problem into two subproblems, channel allocation subproblem and power control subproblem. After solving the first subproblem of channel allocation, the power allocation subproblem is solved using dual-decomposition method, where the Lagrangian variables are calculated in decentralized manner with iterative sub-gradient algorithm. Solving the two subproblems sequentially in one iteration, and the iterations cease when the change of the objective is below a given stopping criterion. When the original problem is assumed convex, the derived distributed algorithms could have a provable convergence to the global optimum. The speed of convergence is difficult to quantify and will depend on how is the original problem is decomposed, i.e., the number of levels in the decomposition, the amount of signaling, and the particular combination of numerical algorithms employed.

	Parameters	Optimization goals	Constraints
Application layer	source-coding rate, buffer priority, packet arrival rate	minimal delay	base layer transmission, strict delay requirement
Network layer	routing path	end to end delay/throughput	maximal hops, security concerns
MAC layer	transmission frequency, transmission priorities	maximal overall throughput, minimal buffer overflow probability	contentions, time/frequency slot
Physical layer	transmission power, modulation, channel coding rate, spectrum bandwidth	minimal over power consumption, maximal throughput, minimal BER, maximize product of bandwidth of power [136]	maximal transmission power, interference on licensed users, available channel coding rate

**Table 2.2:** Optimization problem of cognitive radio networks



# 3

## DISTRIBUTED CHANNEL AND POWER ALLOCATION IEEE 802.22 NETWORKS

### 3.1 Introduction

Secondary users working with TV white space is promising to cope with the scarcity of spectrum resources [8]. Firstly, more unused TV white frequencies become vacant than ever with the ongoing transition from analog to digital broadcasts. Secondly, the frequencies of TV bands enable broadband access over larger geographic ranges compared to higher frequency bands. Nevertheless, services on TV receivers need to be protected with so called interference margin [86] which should not be exceeded by the accumulated interference caused by all secondary users working on the the channel.

FCC and ECC have announced rules on the transmission power of secondary users working in TV white space in US and Europe respectively [8, 9]. FCC requires a minimum distance between secondary user and TV service area, besides, the transmission power for fixed secondary users is set as 4 W, which is a conservative setting. FCC believes with these prudent measures, the interference margin can not be exceeded by interference from secondary users. But it may not be the case when there are multiple secondary equipments transmitting at the same time, which is pointed in [79]. ECC requires the secondary users to adapt their maximum transmission power according to the distance away from the TV receivers.

FCC issued a memorandum [7, 8] in 2010, which removes the mandatory rigid sensing requirements, and prompts the usage of geolocations<sup>1</sup> of secondary users. FCC regulates a centralized database, which registers all the secondary users within one certain area.

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<sup>2</sup>interference margin is the maximal interference caused by secondary users, which doesn't violate TV service.

<sup>1</sup>Geolocation means both geographic location and terrain.

Secondary users can access the database, and can only use the channels assigned by the database. Work [106] follows this rule to obviate spectrum sensing and only relies on the database of TV incumbents to determine the white space availability for secondary users. The authors of [106] demonstrate the feasibility of predicting the available TV spectrum accurately using sophisticated propagation models (Longley-Rice) and geolocations of secondary users. A central database contains the geolocations of all TV stations, then the database calculates the received signal strength index (RSSI) levels of TV UHF signals on all secondary users and accordingly determines the available TV spectrum for them. If RSSI of a channel is below a certain threshold, TV service is regarded not to exist, and the channel is seen available there. The calculated results on channel availability is very close to the measurement results. The work of [106] gives big impetus to the usage of database mode. As in TV white space, the accurate RSSI can be obtained with geolocations and suitable propagation model, given geolocation and appropriate propagation model, secondary users' maximum transmission power can be determined by the central entity according to the interference margin (maximum RSSI level from secondary users) on the TV receivers.

In this chapter, we investigate the usage of TV spectrum in a wireless regional area network which complies with IEEE 802.22 network. The secondary users are assumed to be cellular base stations and associated terminals, all of which work on TV white spectrum. The base station is referred as WBS. Some cellular networks, i.e., GSM or LTE network, work on licensed spectrum and emphasis on providing satisfactory services to their end terminals by choosing proper transmission channel and power. As to cellular network working on TV white spectrum, they have to keep one eye on the primary users to make sure that TV service is not violated, which makes the problem of channel and power selection difficult. With the existence of central database, it is natural to utilize it as a central controller to assign channel and power usage for secondary users, but the secondary users may belong to different commercial groups and they may not content with the assigned resource. Hence, the spectrum sharing of the secondary users in IEEE 802.22 network should be decided in distributed manner and each secondary user takes care of its own interest, i.e., to maximize its preferred utility.

Given all the other WBSs' selection on channel and transmission power, a WBS is interested in choosing the channel which brings it the best performance, i.e., the data rate of its end users. A WBS prefers to choose the channel which experiences the minimum interference, and the transmission power allowed on that channel is higher, so as to obtain better SINR on its terminals and meanwhile maximize their coverage [68, 148]. Nevertheless, high transmission power causes significant co-channel interference to other secondary users operating on the same channel. Hence, a secondary cell has to balance its transmission power and the caused interference on other cells, meanwhile to choose working channel to decrease the experienced interference on its terminals. The goal of this chapter is to protect the primary users from harmful interferences, meanwhile to find a strategy for WBSs to choose channel and power level in order to acquire good SINR on end terminals.

The rest of the chapter is organized as follows. we elucidate the system model in Section II, afterwards related work and problem formulation is presented in Section III. In Section IV, we discuss how to utilize the white space sufficiently by setting the transmission powers based on a convex problem formulation. We analyze the spectrum allocation problem under game theoretical framework and propose an algorithm in Section V, thereafter performance evaluation is presented in Section VI. Finally, we conclude our work and point out directions of future research in Section VII.

## 3.2 System Model and Problem Statement

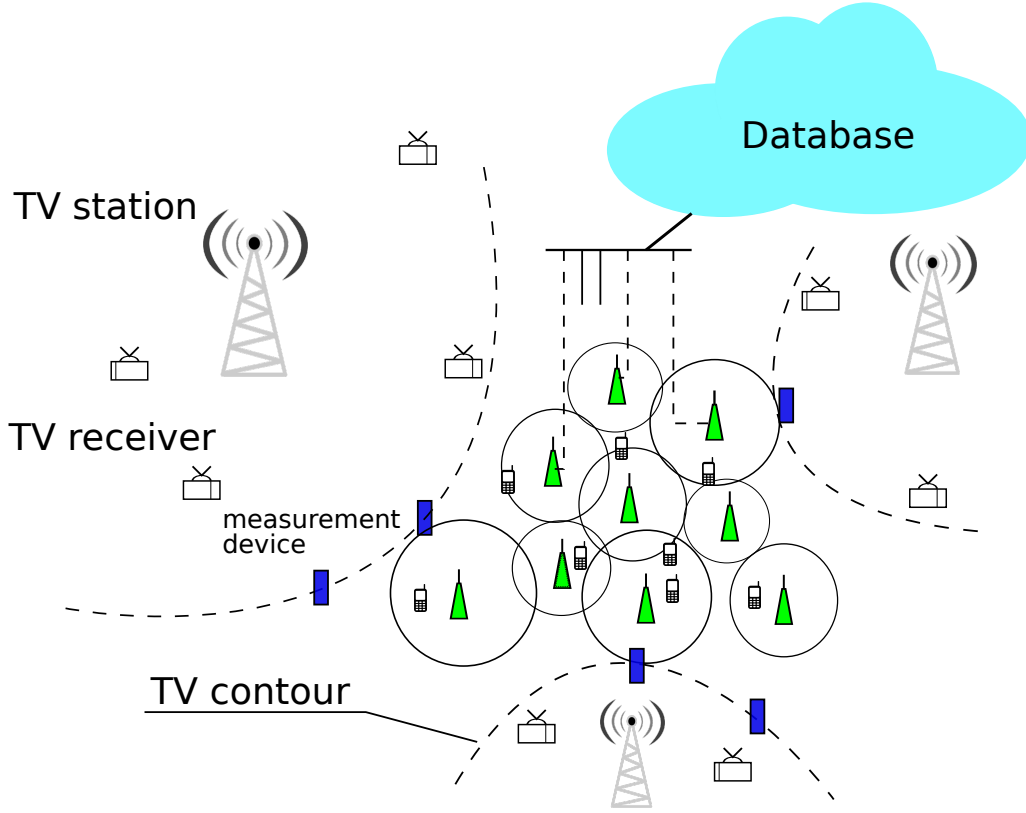
Following the IEEE 802.22 standard, the primary systems considered in this chapter are digital TV (DTV) stations which use the TV spectrum legally. TV stations provide service to passive TV receivers. The secondary users are IEEE 802.22 Wireless Regional Area Network base stations utilizing the TV spectrum with senseless mode [106]. DTV's service should not be interfered by secondary systems. WBSs locate in one area which is surrounded by areas where TV service is delivered. WBSs serve a set of end users/terminals. These secondary systems are distributed over a certain area  $A$  and is surrounded by multiple DTV service areas, as Fig. 3.1 shows. The set of DTV stations is denoted as  $\mathcal{K}$  and the collection of WBSs is denoted as  $\mathcal{N}$  where  $|\mathcal{N}| = N$ . The set of TV white spectrum contains multiple channels which are denoted as  $\mathcal{C}$ , they are assumed to be identical in terms of attenuation and shadowing on the same path. Let  $c(i)$  denote the channel used by a WBS  $i \in \mathcal{N}$ .

When two WBSs working on the same channel, co-channel interference is caused on each, while, neighbouring channel interference is not considered in our model. To simplify the analysis, we assume that each DTV station as well as each WBS utilizes exactly one channel.<sup>2</sup> We represent the usage of channel for WBS  $i$  with a binary vector  $X_i^{|C| \times 1} = \{\dots, x_{ik}, \dots\} \in \{0, 1\}^{|C|}$ , where  $k \in \mathcal{C}$  and binary variable  $x_{ik}$  denotes whether channel  $k$  is used by user  $i$ . As each node can only uses one channel, for  $X_i$ , there is  $\sum_{k=1}^{|C|} x_{ik} = 1$ . The transmission power of WBS  $i$  on channel  $c$  is  $P_i^c$ .

In the rest of the chapter, we use WBS and secondary base station interchangeably. There are interference measurement equipments deployed on the contours of TV service areas (as bold rectangles in Fig. 3.1), which represent the worst located TV receivers in the TV service areas. For these interference measurement devices, an interference threshold should not be violated by the noise generated by the secondary users. The deployment of the interference measurement devices is decided by the TV operators, which are usually along the contour of the area where TV receivers reside. Thus, the locations of interference measurement devices vary according to the concrete location, geographic terrain and possible deployment of secondary networks. WBSs are deemed to be static. We assume the secondary base stations are not under the same operators, thus there is no scheduling mechanism available among WBSs.

WBSs are interested in payload data communication with their associated terminals. As to performance metric for the QoS provisioning, we choose the signal to noise and interference ratio (SINR) on the terminals. SINR is the ratio between the received power of signal of interest and the summed interference experienced by the terminal. Given a WBS  $i$ , another WBS which works on the same channel is denoted as  $\bar{i}$ . As to a certain terminal  $m$  associated to WBS  $i$ , the path losses between  $m$  and the serving WBS  $i$  and interfering WBS  $\bar{i}$  are denoted as  $h_{im}$  and  $h_{\bar{i}m}$  respectively. The path loss is dependent on the distance between the corresponding equipments, e.g.  $h_{im} = K \cdot d_{im}^{-\alpha}$ , where  $\alpha$  is the path loss exponent,  $d_{im}$  is the distance between  $i$  and  $m$ ,  $K$  is a constant which models the reference loss over a single unit of distance.  $N_0$  denotes the thermal noise power. Shadowing without fading is considered in our model.  $z_{\bar{i}m}$  models the zero-mean log-normally distributed shadow fading between  $\bar{i}$  and  $m$ , and the standard deviation is  $\sigma_{SH}$ .

<sup>2</sup>The assumption that one WBS only utilizes one channel is for convenience of analysis. In reality multiple channel usage (channel bonding) is requisite as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfil system requirement.



**Figure 3.1** System model: WBS cells and DTV systems

The sum of all disturbing radio frequency effects (including interference) on terminal  $m$  (we assume the working channel is  $c$ ) is as following,

$$f_m^c = \sum_{\bar{i}} (P_{\bar{i}}^c \cdot h_{\bar{i}j} \cdot z_{\bar{i}m}) + N_0, \quad \bar{i} \in \mathcal{N} \setminus i, c(\bar{i}) = c \quad (3.1)$$

where  $P_{\bar{i}}^c$  denotes the transmission power of interfering WBS  $\bar{i}$ . The SINR on end terminal  $m$  is,

$$\gamma_m = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{f_m^c} \quad (3.2)$$

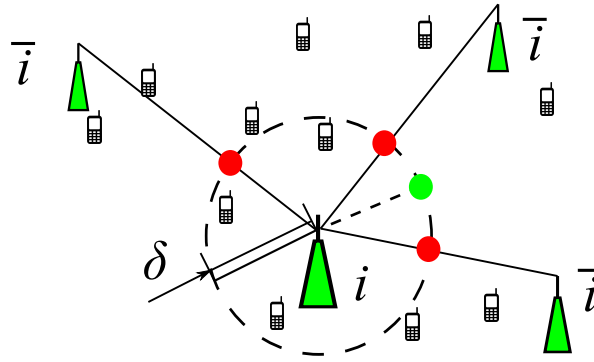
### 3.2.1 Problem Statement

Our goal is to design distributed solution for WBSs to choose channel and transmission power, so as to improve the SINR on their end terminals. Each WBS's utility is a function of certain form of the SINR on all its end terminals, i.e., the utility can be the average SINR at all its terminals. When adopting a function of SINR on all terminals as utility, as the terminals are mobile, and they are influenced by many factors, i.e., the type of service provided to the terminals, the utility may diverge from the real performance of the terminals. On the other hand, it is not appropriate to choose one [44] or more fixed terminals, and use their SINRs to represent the SINR for all the other terminals in that cell, because their location could diverge greatly with the locations of the other terminals. As a result, we propose a metric *QuasiSINR* to represent WBS's performance, which is independent on the actual locations terminals.

### QuasiSINR of WBS

Instead of improving the SINR on each end users of WBSs, we propose a new metric QuasiSINR to represent the services provided by WBSs to its end users, and try to improve this metric. QuasiSINR of a WBS is the ratio between the weakest signal of interest on a reference point and the summation of the strongest interference caused on the relevant reference point.

We need an auxiliary circle to construct the reference point for each WBS, which is shown in Figure 3.2. The dashed circle centred at WBS  $i$  is the auxiliary circle for WBS  $i$ , whose radius is  $\delta$ . Assume all the WBSs shown in the figure are working on channel  $c$ , thus co-channel interference happen between them. The point of intersection of the connecting line between WBS  $i$  and interfering WBS  $\bar{i}$ , i.e., the red dot on the auxiliary circle, is the reference point for co-channel interference for other WBSs working on the channel  $k$ . The power of signal from  $i$  on the auxiliary circle, the green dot, is the reference point for the power of signal of interest.



**Figure 3.2** QuasiSINR is the ratio between the power of signal of interest on the green point with the sum of co-channel interference on the red points

The co-channel interference on the reference point of WBS  $i$  from channel WBS  $\bar{i}$  is,

$$\tilde{f}_{i\bar{i}}^c = P_{\bar{i}}^c \cdot h_{i\bar{i}} \cdot z_{i\bar{i}} = P_{\bar{i}}^c \cdot (d_{i\bar{i}} - \delta)^{-\alpha} \cdot z_{i\bar{i}} \quad (3.3)$$

where  $d_{i\bar{i}}$  is the distance between WBS  $i$  and  $\bar{i}$ , while,  $h_{i\bar{i}}$  and  $z_{i\bar{i}}$  are the attenuation and shadowing from WBS  $\bar{i}$  to the relevant interference reference point. Based on Formula 3.3, as to WBS  $i$ , when the information is given, which include the locations of other co-channel WBSs, the radius of auxiliary circle and the standard deviation of shadow fading, then the co-channel inference caused on its interference reference points by other co-channel WBSs can be obtained, besides, WBS  $i$  is also aware of the interference it causes on the interference reference points associated with other co-channel WBSs. The power of the signal of interest on auxiliary circle is expressed as,

$$\tilde{P}_i^c = P_i^c \cdot h_i \cdot z_i = P_i^c \cdot \delta^{-\alpha} \cdot z_i \quad (3.4)$$

where  $h_i$  and  $z_i$  are the attenuation and shadowing from  $i$  to any point on the auxiliary circle.

The sum of interference on WBS  $i$ ' interference reference points is denoted as  $\tilde{f}_i$ .

$$\tilde{f}_i = \sum_{j \in \mathcal{N}, c(j)=c} \tilde{f}_{ji} \quad (3.5)$$

Then the quasiSINR of WBS  $i$  is,

$$\begin{aligned}\tilde{\gamma}_i &= \frac{\tilde{P}_i^c}{\tilde{f}_i^k + N_0} \\ &= \frac{P_i^c \cdot h_i \cdot z_i}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot h_{ji} \cdot z_{ji}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{ji}) + N_0}\end{aligned}\quad (3.6)$$

The radius of the auxiliary circle  $\delta$  can be adjusted to let WBS foster better service to the terminals in certain area. For instance, to take care of the SINR on the border area of the cell, the radius  $\delta$  can be set as the distance between WBS  $i$  and the furthest associated terminal. When the terminals concentrate towards to the WBS,  $\delta$  can be set smaller to better fit to the terminals' distribution.

Because of the auxiliary circle, the interaction between co-channel WBSs are independent on the location of individual end terminals. As a result, the concrete terminals are excluded from the channel and power allocation problem, which simplifies the problem to be discussed.

### Calculation Instead of Measurement

According to our system model, WBSs are able to access the central database which stores all WBSs' geolocations i.e., working channel, transmission power, the characteristics of radio frequency environment such as parameters of attenuation and shadowing. To obtain quasiSINR, one WBS doesn't need to measure the signal strength on the reference points which is needed in Formula 3.6, instead, it can make use of the propagation model [79] along with the operating parameters of all the other WBSs, which are stored in the central database to calculate the quasiSINR of any specific WBS.

### Problem Statement in the Form of Optimization

Our goal can be illustrated in the form of an constrained optimization problem. To ensure fairness among WBSs, instead of maximizing the sum of quasiSINR of all WBSs, we minimize the sum of inverted quasiSINR.

$$\begin{aligned}\text{Minimize} \quad & \sum_{i \in \mathcal{N}} \frac{1}{\tilde{\gamma}_i} \\ \text{subject to} \quad & \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1 \\ & P_{i,min}^c \leq P_i^c \leq P_{i,max}^c, c \in \mathcal{C}, i \in \mathcal{N}\end{aligned}\quad (3.7)$$

For a WBS working on different channel, its end terminals receive different levels of interference from other WBSs working on the same channel. In order to provide better service to its end users, WBS is liable to choose either the channel permitting higher transmission power or the channel which experiences less interference, or the channel compromising

the two factors according to Formula 3.2. Achieving optimal white spectrum allocation in a distributed style is the goal of this work, furthermore, this distributed solution should converge fast and lead to an efficient and stable solution.

### 3.3 Problem Decomposition and Related Works

In related works, the protection on primary users is taken care in the same time when channel and power selection are conducted. But according to the current regulations and standards, there exist no communication means between the secondary users and the primary users. Besides, when assuming such communication media is available and preventing primary users from being interfered during secondary users' power and channel allocation, the communication overhead between primary users and second users is considerable.

#### 3.3.1 Problem Decomposition

We obtain the maximal transmission power over each channel for every WBS before dealing with channel and power allocation, afterwards, the transmission power should not exceed this power mask. The maximal transmission power over all channels for each WBS is obtained at the centralized database. As the database has the global info of both the secondary network and primary network, the attenuation and shadowing between any two users are known i.e., attenuation is based on propagation model and shadowing is obtained from measurement, the database is able to guarantee the service of primary users not be interfered when all the WBSs work on the same channel. Meanwhile, as the secondary users belong to different groups of interest, the channel and power allocation should be done in distributed manner.

In summery, we solve the channel and power allocation in downlink communication in IEEE 802.22 network by solving three sequential subproblems:

- Firstly, given a set of secondary WBSs and their geo-locations, the maximum permitted transmission power on each channel for each WBS is determined, so that the interference margin of primary users can not be exceeded no matter how WBSs utilize the spectrum and power resources. In other words, the dynamics in the secondary network is transparent to the primary system.
- Secondly, once the maximum permitted transmission power is determined, WBSs choose their operating channels. Note that this channel assignment problem is different from the works available in literature, where the transmission power is identical over different channels for different WBSs. In this subproblem, the maximum permitted transmission power  $p_i^c$  could be different for different channel  $c \in \mathcal{C}$  and different WBS  $i \in \mathcal{N}$ .
- Thirdly, working with the maximal permitted transmission power may not be the optimal in terms of power consumption and the SINR on terminals, thus distributed power adjustment is conducted and the working channel is unchanged.

The solution to the channel allocation subproblem is named as WhiteCat (white space Channel allocation).

In the following, we introduce the related works of spectrum and power allocation problem in CRN, especially the works about the usage of TV white space. we also introduce the works related with subproblems mentioned above.

### 3.3.2 Resource Allocation in CRN

We will emphasis more on the distributed solutions, but in order to give readers a full picture of the solutions as to resource allocation in CRN, we will also introduce the related works on centralized solutions.

Centralized solutions usually solve the formulated constrained optimization problems at a centralized unit. In [69], the objective is to increase the number of supported terminals whose SINRs are above a threshold, and the constrains are to refrain the interference at the primary users within a certain margin. Work [27] minimizes the transmission power and meanwhile makes sure the SINR of terminal is above a threshold, but this work fails to consider the protection of primary users. A heuristic algorithm is proposed in [117], which considers the channel availability and transmission demand of each WBS. The aforementioned two schemes don't consider varying the transmission power.

There is a large variety of distributed solutions. In order to avoid or to alleviate co-channel interference between cells, and to allow arbitrary number of cells to work in IEEE 802.22 network, [71] proposes distributed inter-network spectrum sharing scheme, where contention decisions are made in a distributed way and the winner cells can use the shared channels. But this work doesn't consider the role of transmission power in the co-channel interference. An distributed power allocation (single channel) scheme based on learning for secondary networks is given in [57], where penalty function involving the interference threshold on primary systems is used. [68] discusses power control and channel assignment in both down-link and up-link communication in cellular network. Although the solution is distributed, primary users are required to cooperate with secondary base station in a learning process to decide the transmission power, in addition, there is only one secondary base station considered whereas we need to cope with the multiple cells in our problem. Joint channel-power selection for multiple transmission links (pairs) is investigated in [148]. The authors decompose the Lagrangian dual of the problem, then propose a distributed scheme based on the dual parameters. The scheme converges into pure Nash equilibrium, but in order to facilitate this scheme, monitors are required to watch interference from secondary users, moreover, monitors have to be equipped computational ability and interact with secondary users in the whole process of convergence.

As introduced in Chapter 1, game theory is a powerful tool in designing distributed algorithms. A distributed joint power and channel allocation is proposed in [111], each base station chooses optimal power level and channel to optimize its utility, which results in induced received interference and caused interference on primary users. The execution of this scheme is formulated into an exact potential game. For each base station, after several rounds of best responses in terms of channel and power level, Nash equilibrium is achieved. There are some flaws hindering the application of this scheme. Firstly, the paper doesn't provide means for base stations to obtain the needed information which is



needed to calculate the utility function. Secondly, it is not clear how to calculate the punishment in the utility function, which indicates whether and how much the interference threshold on primary users is violated. Thirdly, the convergence speed of the scheme is not given, in fact, as the problem is formulated into a potential game, converge speed or the number of updates before convergence is a theoretic problem which is still unsolved. Last but not least, as the utility function and the potential in the game are designed as the sum of received and introduced interference, the desired signal power and the punishment, the minimization of this *sum* does indicate meaningful performance metrics, i.e., SINR on terminals, or the total transmission power consumption. In [44], Chen et al. investigate the channel allocation problem in the scenario of TV white space. The channel allocation problem is formulated into a potential game, individual WBS's utility is to maximize the capacity of one single static terminal. Potential game is also adopted in work [49] to design algorithms, which mitigates the adjacent interference. [87] adopts cooperation game to research the coexistence of femtocells. Each femtocell negotiates with neighbouring femtocells, and they form temporary coalition, but the goal of this solution is to allocate resource block in terms of time and transmission power. [27] proposes both centralized and decentralized solutions. Two distributed schemes are proposed, joint channel and power allocation is formulated into a weighted potential game, as an alternative workaround, the problem is solved in two sequential phases.

Distributed algorithm based on Learning is proposed in [72] for LTE to allocate the resource block in down link, which leads to correlated equilibrium, but slow converge hinders its application.

### 3.3.3 Utilization of TV White Space

Here we introduce the solutions proposed on the utilization of TV white space, which includes regulations, proposed standards and recent research advances. In accordance with the regulations of FCC, there are some prototype applications proposed in both cellular network [53, 119] and WiFi-like network [116]. The secondary users access a centralized data base to know the allowed channels and transmission power. Standardization bodies are also working on TVWS utilization, including IEEE 802.22 [1] for Wireless Regional Area Networks (WRAN), IEEE 802.11af [55] for WLAN, IEEE 802.15.4m [4] for 802.15.4 wireless networks in TVWS and 802.19.1 [5] for coexistence methods among local and Metropolitan Area Networks (MAN).

Scientific research on utilization of TVWS goes on in parallel with the regulatory agencies. Feng et al. [54] investigate the business model of TV spectrum utilization in database involved network structure, emphasis on the price policy of the channels approved by FCC. Spectrum sharing in TVWS is formulated as a series of optimization problems. The guarantee that TV receivers should not be affected by the aggregate interferences from TVBDs is one constraint. The objective can be maximizing TVBD's downlink transmission power [86], uplink transmission power [137], or best geographic distribution of TVBDs [154]. A series of works [33, 45, 58, 128, 151] emphasise on interference mitigation among TVBDs via spectrum allocation. Vehicular networks operating with TVWS assisted by TV database and cooperative sensing is discussed in [48]. Work [50] steps further from the database paradigm and makes efforts to utilize the *grey space*, where TVDB is allowed to operate even within the TV service area.

### Related Works on Maximal Transmission Power Planning

To protect the TV receivers from harmful interference, the aggregate interference caused by WBSs at the contours of TV receivers should not exceed the interference margin. Work [80] proposes detailed calculations which a geolocation database performs in order to derive location-specific maximum permitted EIRP levels for white-space devices (WSDs) which operate in digital terrestrial TV bands. [86] considers the maximum permitted transmission power for the network which complies with IEEE 802.22 standard. The standard requires a centralized database to store the available channels for each secondary base station, thus centralized scheme can be conducted there after trivial modification. The sufficient condition for the TV receivers not be interfered in the context of TV white space is formulated into a centralized linear programming program (LP) in [86]. The objective function is to maximize the summation of all secondary base stations' transmission power, and the constraints are formed to satisfy the sufficient condition for every interference measuring device for the TV receivers. However, this approach doesn't take the channel assignment problem into account.

### Related Works on Channel Allocation with Fixed Transmission Power Level

In our proposed solution, after obtaining the maximum transmission power on each channel, WBSs need to decide one channel to use, and the transmission power is the maximum transmission power, so as to mitigate interference among WBSs and provide the best SINR for their associated end users. Note that in our problem, the transmission power is different for two interfering users when they work on the same channel.

Channel allocation problem dealing with mitigating co-channel interference via channel allocation, which has been attracting plenty of research efforts in the past decade, from multiple channel mesh network [120], Ad hoc network [84] up to cognitive radio network [73, 151].

Channel assignment problem tries to mitigate co-channel interference among users, which can be converted into colouring problem thus is NP hard [120]. Authors of [84] propose heuristic algorithms utilizing best response to improve its welfare, but the transmission power is assumed identical and path loss is deemed as symmetric, which renders this method problematic for our problem where transmission is non-identical and the path loss is asymmetric. [108] formulates channel assignment problem in ad-hoc cognitive radio network into potential game which leads to pure NE, a learning scheme achieving slightly better performance is provided for comparison, but they assume the transmission power is identical and there is no noise in the secondary network, and the proposed random access mechanism demands a huge amount of information to be exchanged, which is a burden for network in ad-hoc structure. [52, 145] investigate the channel allocation problem under game framework in same collision domain, the authors propose algorithms to converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively.

Authors of [84] propose heuristic algorithms utilizing best response based on the welfare on itself to assign channels among users. Simulated annealing is applied to mitigate co-channel interferences in [151]. For the same purpose, no-regret learning [66, 73] is exploited to optimize the choice on channel.

All the available channel allocation schemes are designed under the same assumption, that the transmission power levels are identical, and the attenuation between any pair is

reciprocal. As to our knowledge, there is no work dealing with channel allocation problem where transmission power is different.

### 3.4 The Maximum Permitted Transmission Power

The WBSs work in underlay manner and coexist with primary TV stations and receivers, the aggregate generated interference from WBSs on each channel should not exceed the threshold of the TV receivers. We adopt the interference model and the optimization methodology from the work of [86] to plan the maximum transmission power on each channel for WBSs. Having a global view of the propagation parameters, geolocations of WBSs and interference threshold at interference measuring devices which locate on the contour of TV service area, linear programming is implied in the database to calculate the maximum permitted power over each channel.

For WBS  $i \in \mathcal{N}$ , the maximum transmission power allowed on channel  $c \in \mathcal{C}$  is denoted as  $P_i^c$ . As to each channel  $c$ , the generated interference on each interference measuring device should be within a predefined interference margin  $I_{pt}^c$ . The interference margin in a slow fading environment is decided according to [123].

Then the maximum permitted transmission power on channel  $c$  for each WBS can be obtained by solving the following optimization problem,

$$\begin{aligned} & \text{Maximize} && \sum_{i \in \mathcal{N}} P_i^c \\ & \text{subject to} && \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z) < I_{pt}^c, \\ & && P_{min}^c \leq P_i^c \leq P_{max}^c \end{aligned} \tag{3.8}$$

$P_{min}^c$  is the prudent transmission power.  $P_{max}^c$  is the maximum transmission power which is restricted by the hardware.  $z$  is shadow fading as introduced in 3.2. Here we only consider the interference caused by WBSs, and omit the interferences from end terminals. Since WBSs' transmission power is higher and their altitude is higher [86], the downlink transmission contributes the major part of interference [15]. The first constraint indicates that the interference margin will not be exceeded even when all the WBSs work on the same channel.

Formula 3.8 will be solved for each channel  $c \in \mathcal{C}$ . After solving the  $|\mathcal{C}|$  problems, the maximum permitted transmission power vector  $\mathbf{P}^c = \{P_1, \dots, P_{|\mathcal{N}|}\}, \forall c \in \mathcal{C}$  is obtained.

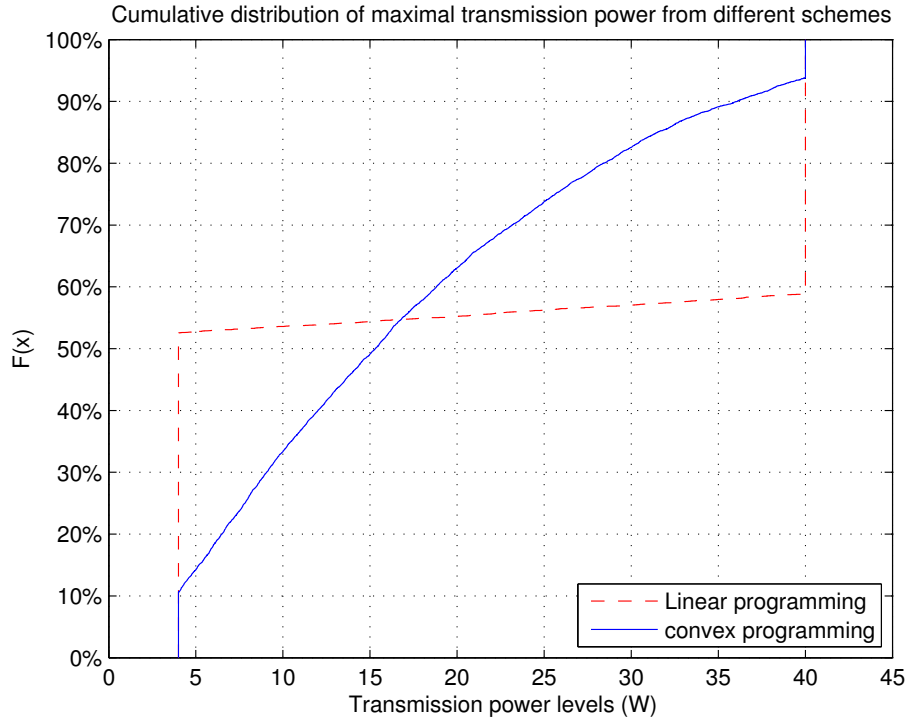
When working with the same transmission power, the WBSs locating closer to the TV interference measuring devices contribute more to the aggregate interference comparing with the WBSs which locate far from the TV interference measuring devices. Thus when implying linear programming to decide the maximal transmission power, the transmission power used by WBSs which are closer to the TV interference measuring devices is much higher than other WBSs. As a result the maximum permitted transmission power on each channel obtained with LP is seriously unbalanced.

To address this fairness issue, we maximize the sum of the logarithmic value of every WBS's transmission power, and formulate the problem into a convex optimization problem.

$$\begin{aligned} & \text{Maximize} && \sum_{i \in \mathcal{N}} \log P_i^c \\ & \text{subject to} && \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z) < I_{pt}^c, \end{aligned} \quad (3.9)$$

This optimization will be solved for each channel  $c \in \mathcal{C}$ .

Figure 3.3 depicts the distribution of maximum permitted transmission power levels obtained in 100 simulations. In each simulation the locations of TV interference measuring devices are randomly decided around the WBSs. In Figure 3.3, It shows that when applying optimization 3.8, WBSs' transmission power levels are either the minimum transmission power or the maximum power allowed by the equipment hardware. When applying convex programming, the planned maximum permitted transmission power levels are distributed evenly in between the minimum and maximum power. The gain of SINR on end terminals by applying convex optimization to decide the maximal transmission power is illustrated in the simulation section.



**Figure 3.3** Distribution of maximum permitted transmission power levels obtained from convex and linear programming formulations

Optimization problem 3.9 provides the maximum permitted transmission power for each WBS and over each channel. When all the WBSs working on the same channel, the generated interference doesn't exceed the threshold on the interference measurement devices at the contour of TV service area. If there are multiple channels available and WBSs are free to choose their preferred channels, the aggregate interference on one channel will be smaller than that when all WBSs work on that channel. Thus, there is exists a interference

margin created by using multiple channels, which provides a room for network dynamics such as new WBS starting to work or increased interference on TV contour due to the variance of broadcast path condition.

## 3.5 Channel Allocation with Fixed Transmission Power

First, we give the centralized solution to obtain the global optimum for this subproblem, then the decentralized scheme under the game theoretic framework is introduced.

### 3.5.1 Centralized optimization programming

We formulate the channel allocation problem into a binary quadratic programming problem which can be solved in a centralized way. Let  $X_i = \{x_{i1} \cdots x_{ik} \cdots x_{i|\mathcal{C}|}\}$  denote the vector of channel usage, there is  $|X_i| = |\mathcal{C}|$  and binary element  $x_{ik}$  represent whether WBS  $i$  occupies channel  $k$ . For two WBSs  $i$  and  $j$ , there is,

$$X_i^T X_j = \sum_{k=1}^{|\mathcal{C}|} x_{ik} \cdot x_{jk} = \begin{cases} 1 & \text{if } c_i = c_j \\ 0 & \text{if } c_i \neq c_j \end{cases} \quad (3.10)$$

The power levels across all channels are denoted by a constant vector  $P^{|\mathcal{C}| \times 1}$ , which is possibly nonidentical to WBSs due to different locations. The power used by user  $i$  is

$$P_i^T X_i = \sum_{k=1}^{|\mathcal{C}|} P_i^k \cdot x_{ik}.$$

Problem 3.7 can be modeled via general purpose nonlinear optimization:

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^n \frac{\sum_{j \in \mathcal{N}, j \neq i} P^T X_j (X_j^T X_i) h_{ji} z + N_0}{P^T X_i} \\ & \text{subject to} && \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1, x_{ik} \in X_i \in \{0, 1\}^{|\mathcal{C}|} \end{aligned} \quad (3.11)$$

Problem 3.11 is a non-linear problem with binary variables, but it can be reformulated in to a quadratic programming problem as,

$$\begin{aligned} & \text{minimize} && \sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k \frac{P_j^k}{P_i^k} \cdot h_{ji} \cdot z \cdot x_{jk} \cdot x_{ik} + \sum_k \frac{N_0}{P_{ik}} \cdot x_{ik} \right) \\ & \text{subject to} && \sum_{k=1}^{|\mathcal{C}|} x_{ik} = 1, x_{ik} \in X_i \in \{0, 1\}^{|\mathcal{C}|} \end{aligned} \quad (3.12)$$

The reformulation is available in Appendix B. We use LINDO [94] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimum for the problem. The result of this centralized channel assignment will be evaluated in the simulation section with other schemes.

### 3.5.2 Distributed White Space Channel Allocation (WitheCat): Algorithm and Protocol

In this section a distributed scheme for WBSs to allocate channels is proposed, which is named as white space channel allocation technology (WitheCat). WitheCat adopts the best response process, where each WBS (referred as  $i$ ) chooses the channel which brings the bigger utility  $u_i$  as the response of other WBSs' choices on channels. WitheCat is depicted by algorithm 1.

$$u_i = \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \tilde{f}_{ji}}{2 \cdot \tilde{P}_i} + \frac{1}{2} \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \frac{\tilde{f}_{ij}}{\tilde{P}_j} + \sum_{\substack{\mathcal{S}: i, j \in \mathcal{S}, \\ c(\sigma_j) = c(\sigma_i)}} \frac{N_0}{C \cdot \tilde{P}_i} \quad (3.13)$$

where  $\tilde{f}_{ij} = P_i \cdot h_{ij} \cdot z$  and  $\tilde{f}_{ji} = P_j \cdot h_{ij} \cdot z$ . Note that  $\tilde{f}_{ij}$  is the sum of interference on WBS  $i$ 's interference reference points. Overlooking the constant coefficient 2, the first item of  $u_i$  is a part of the inverted QuasiSINR of station  $i$ . To minimize the first item, WBS  $i$  needs to choose a channel either permits higher transmission power or experiences less interference, whereas the higher power increases the second item which is a part of inverted QuasiSINR of other co-channel WBSs. Hence, the cost function presents a reasonable comprise between the welfare of one WBS and others.

When WBS only emphasizes on its own utility (e.g. the first part of Formula 3.13), the best response process doesn't converge. We have following theorem:

**THEOREM 3.5.1:** With non-identical transmission power, if every WBS updates its channel based on Algorithm 1 with utility based on its own interests, the process doesn't always converge.

The proof is in Appendix A.

---

**Algorithm 1:** Spectrum selection by WBS  $i$

---

**Input:** the distance, path lose and shadowing parameter between WBS  $i$  to

WBS  $j \in \mathcal{N} \setminus i$ ;

radius of auxiliary circle, noise  $N_0$ , total number of WBSs  $N$ ;

for  $j \in \mathcal{N} \setminus i$ , the maximal transmission power  $P_j^c$ ,  $c \in \mathcal{C}$  and the working channel  $c(j)$ .

- 1 **for**  $c \in \mathcal{C} \setminus c(i)$  **do**
  - 2     calculate  $u_i(c)$  based on Formula 3.13 **if**  $u_i(c) < u_i(c(i))$  **then**
  - 3          $c(i) \leftarrow c$
  - 4     **else**
  - 5         keep  $c(i)$  unchanged
  - 6     **end**
  - 7 **end**
  - 8 Notify database of its channel usage, which further notifies the other WBSs
- 

Some parameters needed to calculate the utility are identical for all WBSs, such as quasi distance  $e$ , the total number of WBSs  $N$ , number of channels  $C$ , attenuation factor  $\alpha$ , standard deviation  $\sigma_{WBS}$  in flat shadowing and noise  $N_0$ , albeit the following information is further needed to calculate  $u_i$ :

- $\sum_{j \in \mathcal{N}, j \neq i, c(\sigma_j)=c(\sigma_i)} \tilde{f}_{ji}^c, c \in \mathcal{C}$ : the received interference on  $i$ ' virtual measurement point from other WBSs  $j$  working on the same channel for  $\forall c \in \mathcal{C}$ .
- $\tilde{f}_{ij}^c$ : the interference caused by  $i$  on  $j$ 's virtual measurement point when  $i$  works on channel  $\forall c \in \mathcal{C}$ .
- $P_j^c$ : transmission power of  $j$  for using  $\forall c \in \mathcal{C}$ .

Unfortunately, it is difficult to get these interferences of interested measured, for station  $i$ , it is low efficient to scan all channels and obtain the interferences  $\tilde{f}_{ji}$  on virtual measurement point for each channel, furthermore, it is impossible to split the interference  $\tilde{f}_{ij}$  from the total interference received on WBS  $j$ ' virtual measurement point.

We refer [108] to decide the sequence for WBSs to update their channel. [108] proposes a method like random access mechanism of CSMA/DA, where the access for broadcast medium is changed to getting access to the centralized center to retrieve the current channel usage and update its new channel. All WBSs are able to access the database in one round (with random or predetermined sequence). As WBSs are connected with database, the control messages needed to decide the sequence will not become a burden. Update of channels can happen in the boot phase, or when the quality of services (the SINR on its end users) of WBSs falls below a threshold, or a fixed time duration comes to end, or a new WBS joins in the network.

Similar with [106], we let every WBS store the location information and maximal power map of all other WBSs, i.e.,  $P_i^c, i \in \mathcal{N}, c \in \mathcal{C}$ , and each WBS retrieves information about channel usage of other WBSs from centralized base station. After executing Algorithm 1, it reports to centralized database of its channel if it updates the working channel. As the location of WBSs and TV stations and the transmission channel and power of TV stations are usually static (entries of TV station change averagely once in 2 days [106]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent.

### 3.5.3 Analysis in game theoretical framework

In this section, We give the proof on whiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [95]. This work reversely engineers the distributed channel allocation schemes proposed in [25,84], i.e., unifies the algorithms with congestion game. But the problem analysed in [95] assume the transmission power is identical, which is a major difference from the channel allocation problem discussed here.

We have introduced congestion game in Chapter 2, thus we only recap the essence of congestion game here. In congestion game, each player acts selfishly and aims at choosing strategy  $\sigma_i \in \Sigma_i$  to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

### The Congestion Game Formulated from the Algorithm WhiteCat

We utilize the conception of virtual resource which is firstly introduced in [95]. Virtual resource is a triplet  $\{i, j, c\}$ , where  $i, j$  are two WBSs and  $c \in \mathcal{C}$  is one channel. This piece of resource is regarded used by  $i$  when both  $i$  and  $j$  use channel  $c$ , otherwise,  $\{i, j, c\}$  is not used by any WBS.

In the following, we list the element of the congestion game which emulates Algorithm 1. In this section, player and base station are used interchangeably.

- Player  $i$ ' strategy space is  $\Sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \dots, N\}$ , and  $i$  has  $C$  admissible strategies, one strategy related with channel  $c \in \mathcal{C}$  is described by the set of virtual resources it uses:  $\sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$ , note that virtual resource  $(i, j, c) \neq (j, i, c)$ .
- Under the strategy profile  $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)$ , player  $i$  obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c=c(\sigma_i)=c(\sigma_j)}} (g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma))) \quad (3.14)$$

The transmission power over all channels of player  $i$  is  $\{p_i^1, p_i^2, \dots, p_i^{|\mathcal{C}|}\}$ . We define the cost function for virtual resources  $(i, j, c)$  as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \frac{\tilde{f}_{ji}}{2\tilde{P}_i} + \frac{\tilde{f}_{ij}}{2\tilde{P}_j} + \frac{C \cdot N_0}{N \cdot \tilde{P}_i} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \quad (3.15)$$

As resource  $(i, j, c)$  only lies in the strategy space of player  $i$  and  $j$ , thus can only be accessed by this two players. More specifically, according to Formula 3.15, the cost of resource  $(i, j, c)$  is only decided by the number of players using it, which is either 0 or 2. At the first glance, this is a player specific congestion game, as  $g_{(i,j,c)}$  is decided by the relevant players' transmission power and inference. But actually the resource  $(i, j, c)$  excludes the players except for  $i$  and  $j$  from using it, thus the cost happened on this resource is only dependant on how many of players from the set  $\{i, j\}$  to use it. Hence, the cost is a function of the number of players using the resource, and this is a canonical congestion game.



### Bridging the Game and Algorithm WhiteCat

When we substitute Formula 3.15 to Formula 3.14, the total cost for user  $i$  under strategy profile  $\sigma$ .

$$\begin{aligned}
g^i(\sigma) &= \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} (g_{(i,j,c)}(2) + g_{(j,i,c)}(2)) \\
&= \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} \left( \frac{\tilde{f}_{ji}}{\tilde{P}_i} + \frac{\tilde{f}_{ij}}{\tilde{P}_j} + \frac{C \cdot N_0}{N} \left( \frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j} \right) \right) \\
&= \frac{\sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} \tilde{f}_{ji}}{\tilde{P}_i} + \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} \frac{\tilde{f}_{ij}}{\tilde{P}_j} + \frac{CN_0}{N} \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} \left( \frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j} \right) \quad (3.16) \\
&= \frac{\sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} \tilde{f}_{ji}}{\tilde{P}_i} + \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j) = c(\sigma_i)}} \frac{\tilde{f}_{ij}}{\tilde{P}_j} + \frac{2CN_0}{N} \sum_{\substack{i \in \mathcal{S} \subset \mathcal{N}, \\ \mathcal{S}: \forall i \in \mathcal{S} \\ c(\sigma_i) = c}} \frac{1}{\tilde{P}_i}
\end{aligned}$$

where  $\mathcal{S}$  denotes the set of WBSs whose working channel is the same with WBS  $i$ .

Now we are going to have a look at the *potential* of the network. According to the expression of Rosenthal's potential in Formula 3.19, the potential is accumulated by adding the players' cost sequentially, in particular, the value which is added is the cost that player experiences when it starts to use the relevant resource, and the value is not changed when other players come to use that resource. Back to our problem, for two WBSs  $i, j \in \mathcal{S}$ , we assume WBS  $i$ 's index is smaller than  $j$ 's index, then the potential increased by  $i$  using the resource  $\{i, j, c\}$  is 0 according to Formula 3.14, and the increase brought in by  $j$  using the resource  $\{i, j, c\}$  is  $g_{(i,j,c)}(2) + g_{(j,i,c)}(2)$ . In other words, for each interfering pair of WBSs, only the WBS with bigger index contributes to the potential. Then the total potential is,

$$\begin{aligned}
G(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i \in \mathcal{N}} \sum_{r \in \sigma_i} g_r(n_r^i(\sigma)) \\
&= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \tilde{f}_{ji}}{\tilde{P}_i} + \frac{CN_0}{N} \sum_{\substack{\mathcal{S} \subset \mathcal{N}, \\ \forall i \in \mathcal{S}, c(\sigma_i) = c}} |\mathcal{S}| \sum_{i \in \mathcal{S}} \frac{1}{\tilde{P}_i} \quad (3.17)
\end{aligned}$$

note that the summation of one WBS's congestion is related to its index.

When players minimize their utilities (cost or potential) illustrated by Formula 3.16, the total congestion in the secondary network given by Formula 3.17 decreases monotonically before reaching one Nash equilibrium. Players' greedy update in the game to minimize its cost Function 3.16, which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm 1 and utility Function 3.13 respectively.

### Gap between the Potential of Game and the Objective

It is natural to raise the question, is the sum of the final utilities of all WBSs exactly the same with the value of potential when the game converges to a Nash equilibrium, which is represented by 3.17? The answer is, they are identical when  $N_0$  is zero, and there will be a little difference when  $N_0$  is not zero. Recall the target objective we want to minimize in Problem 3.7 is,

$$\begin{aligned} \sum_{i \in \mathcal{N}} \frac{\tilde{f}_i}{\tilde{P}_i} &= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \tilde{f}_{ji} + N_0}{\tilde{P}_i} \\ &= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \tilde{f}_{ji}}{\tilde{P}_i} + \sum_{i \in \mathcal{N}} \left( \frac{N_0}{\tilde{P}_i} \right) \end{aligned} \quad (3.18)$$

We notice that only the last items of the objective 3.18 and the potential of the congestion game 3.17 are different. When  $N_0 = 0$ , the potential is exactly the same with the object we want to minimize. When  $N_0 \neq 0$ , if channels are evenly distributed and there is  $C/N^* \mid |\mathcal{S}| = 1$ , then Formula 3.18 and 3.17 are also the same. In both cases, the sum of utilities 3.18 decreases monotonically with every update of WBSs before the system reaches Nash Equilibrium. When  $N_0 \neq 0$  and Formula 3.18 and 3.17 are thus different, the monotonicity on the decrease of sum of utilities 3.18 is not perceived, whereas the system will still cease to NE.

Based on above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed. If we regard one WBS as multiple ones which locate at the same place, and each WBS works on one distinct channel, then the proof on convergence of whiteCat can be applied directly to this case.

Note that the convergence of the game is independent on the the concrete form of the cost function. We adopt the function 3.16 to let the potential of the game be the same with the total utility of all WBSs, so that by executing Algorithm 1, the system objective experiences a monotonic decreasing process before the system reaching NE. The algorithm has potential to solve many other problems, where one user's decision affects others. In this case, the utility of one user can be formulated to incorporate the information of its own utility and others', then the congestion game theory can be used to analogize.

### Communication Overhead of WhiteCat

The problem of channel allocation with different and fixed transmission power is NP hard. WhiteCat is a distributed scheme but certain information of the other WBSs is needed. The centralized base station is piggybacked to provided the needed information. As to one WBS, the number of such inquiries is the number of steps before convergence.

In our formulated congestion game, a player  $i$  is allowed to access up to  $(N - 1)$  resources in the same time, i.e.,  $\{i, j_1, c(i)\}, \{i, j_2, c(i)\} \cdots \{i, j_{N-1}, c(i)\}$ , thus the upper bound of converge steps can not be obtained from the conclusion 3.19 for singleton congestion game. But our problem is special because for each resource, the possible number of players allowed to use each resource is either 2 or 0. Thus we can refer the method used

in Section 2.2.2 to analyse the update times for our problem. Firstly, we sort the cost values in increasing order. Although a WBS

$$\begin{aligned}
 \phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\
 &\leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} n \\
 &\leq n^2 m
 \end{aligned} \tag{3.19}$$

The upper bound of total update steps is  $2n^2$ , thus averagely, the upper bound of update steps for each WBS is  $2n$ .

### 3.6 Variable transmission power after channel allocation

After deciding on the working channel, WBSs operate with the maximum permitted transmission power. As the utility defined in Formula 3.6 is a division of linear function of transmission power and received interference, it is natural to assume that there could exist a vector of transmission power  $\{p_1, p_2, \dots, p_N\}$  where  $p_i < P_{max}^c, \forall i \in \mathcal{N}$ , and the performance doesn't diverge much from the already achieved performance. When using the same utility as Formula 3.6, there is no WBS having the motivation to diverge from the power level (the maximum permitted power) if the other WBSs keep their transmission power unchanged.

We adopt the utility proposed in [132] as our utility function.

$$u = \frac{E \cdot R}{p} (1 - e^{-0.5 \cdot \gamma})^L$$

This new utility is function of both its own transmission power and quasiSINR, thus one WBS doesn't need relevant information from other WBSs. This function has several attracting properties. It is a monotonically increasing function of  $\gamma$  for a fixed transmission power  $p$ . It approaches to 0 when  $\gamma$  increases to infinity, and it is a monotonically decreasing of the transmission power  $p$  for a fixed  $\gamma$ . This function goes to 0 when  $p$  goes to either 0 or infinity. To adopt this algorithm, every WBS keeps on minimizing its utility and finally all the WBSs achieve Nash equilibrium.

### 3.7 Joint Channel and Power Allocation

In the section 3.5, the problem is decomposed into three subproblems which are solved sequentially. The first is solved with linear/convex programming in the centralized database, and the others are solved with distributed schemes. The decomposition of the original problem along with the distributed scheme may yield a result away from the global optimum, so in this section, we propose a centralized scheme which looking for the global optimum results in order to examine the performance of our cascaded channel and power allocation scheme.

### 3.7.1 Centralized optimization

When we consider to optimize the transmission power and channel jointly, the optimization problem 3.12 is not quadratic any more and becomes mixed integer non-linear problem, for which no efficient solution exists. We reformulate problem 3.12 into a mixed binary quadratic optimization problem with some auxiliary variables created, i.e., binary number  $\alpha$ , real number  $\beta$  and  $q$ , where

$$x_{jk} \cdot x_{ik} = \alpha_{ij}^k \quad (3.20)$$

$$\beta_{ij}^k = p_j^k \cdot \alpha_{ij}^k \quad (3.21)$$

$$\frac{1}{p_i^k} = q_i^k \quad (3.22)$$

Then the optimization problem can be stated as:

minimize

$$\sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k q_i^k \cdot \beta_{ij}^k \cdot h_{ji} \cdot z + \sum_k N_0 \cdot q_i^k \cdot x_{ik} \right)$$

subject to:

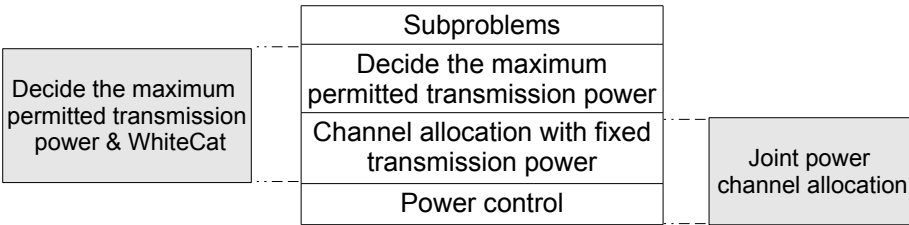
$$\begin{aligned} x_{jk} + x_{ik} - \alpha_{ij}^k &\leq 1 \\ -x_{jk} - x_{ik} + 2 \cdot \alpha_{ij}^k &\leq 0 \\ \beta_{ij}^k - p_j^{k,max} \cdot \alpha_{ij}^k &\leq 0 \\ -\beta_{ij}^k + p_j^{k,min} \cdot \alpha_{ij}^k &\leq 0 \\ \beta_{ij}^k - p_j^k - p_j^{k,min} \cdot \alpha_{ij}^k &\leq -p_j^{k,min} \\ -\beta_{ij}^k + p_j^k + p_j^{k,max} \cdot \alpha_{ij}^k &\leq p_j^{k,max} \\ \sum_{k=1}^{|\mathcal{C}|} x_{ik} &= 1, x_{ik} \in \{0, 1\}^{|\mathcal{C}|} \\ q_i \cdot p_i &= 1 \end{aligned} \quad (3.23)$$

The objective function is quadratic, but the quadratic equality constraint makes this optimization problem very challenging to solve. Linearisation is possible when Q is positive definite 'Quadratic Optimisation with One Quadratic Equality Constraint'. As we don't regulate the locations of the WBSs, and the attenuation is random among them, thus the positive definite can not be guaranteed. We use solver Lindo to look for the global optimum. This form of optimization is non linear, but several algorithms are available to obtain the global optimum [67, 126].

all the constraints except for the last one are linear. The first two constraints realizes Formula 3.20, the following four constraints realizes Formula 3.21. Due to the quadratic equality constraint  $q_i \cdot p_i = 1$ , the optimization problem is non-convex and only a few solvers are able to solve it by using global searching. One workaround to avoid global searching is to linearise the equality constraint. After linearising constraint  $q_i \cdot p_i = 1$ , the problem becomes mixed integer quadratic problem, and can be solved with Gurobi.

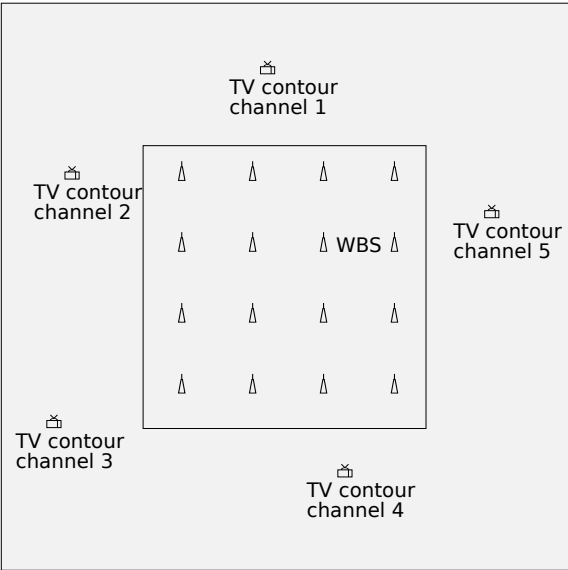
### 3.8 Performance Evaluation

Performance evaluation consists two parts, in the first part whiteCat are compared with other distributed schemes proposed for the problem of channel allocation with different fixed transmission power, where the transmission power levels associated with WBSs and channels are different. In the second part, we compare the distributed joint channel and power allocation solution with other solutions for this problem. To illustrate the structure, we list the contents in Figure 3.4.



**Figure 3.4** The evaluation contents in this section, the left part is discussed in Section 3.8.1 and the right part is in Section 3.8.2.

The evaluation setting is as follows. A square area which is 60km x 60km is divided into 16 square blocks evenly, for each block there is one WBS locating in the middle of it. Same mount of end terminals distributed in each minor block. The terminals don't necessarily choose the WBS which is nearest to them to obtain service, in stead they choose the WBS to join, which provides the strongest RSSI at them. There is a 30km wide rim area around the square area, where TV contours are randomly located. The TV station which protected by TV contour works only with one channel. The locations of WBSs and TV contours are illustrated in Fig. 3.5. The other parameters are listed in



**Figure 3.5** Layout of WBSs and TV contours

Table 3.14.

Number of channels	5
Number of WBSs	16
Noise	$10^{-12}\text{W}$
Length of the square to locate WBSs	60km
Distance between quasis terminal and WBS	7km
Interference threshold on TV contour	$10^{-7}\text{W}$
Path loss factor	2
Standard deviation in flat shadowing	8
Minimal WBS transmission power	4W
Maximal WBS transmission power	40W
Number of end terminals in network	800

**Table 3.1** Simulation parameters

### 3.8.1 Maximal Permitted Power Decision and the Distributed Channel Allocation Schemes

In this section, we will firstly evaluate the convex optimization and linear optimization proposed in Section 3.4 to see which is the better choice to decide the maximum permitted transmission power, the adopted metrics are average power consumption, and SINR on end users where channel allocation is executed. Then with the decided better method, we compare given the power map, how do the channel allocation schemes perform. We compare our proposed channel allocation scheme *whiteCat* with three other distributed schemes, the random allocation scheme, *whiteCase* and No-regret learning, besides, centralized optimization is used to obtain global optima.

- *WhiteCase*: Whitespace channel allocation selfish, where each WBS selfishly updates its channel to achieve the best (as to the considered problem, smallest) possible utility based on Formula A.1.
- *No-regret learning*: Each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSs update such mapping dynamically and this approach converges to correlated equilibrium. Please refer the original paper [66] for details.
- *Quadratic optimization*: centralized quadratic optimization introduced in Section 3.5.

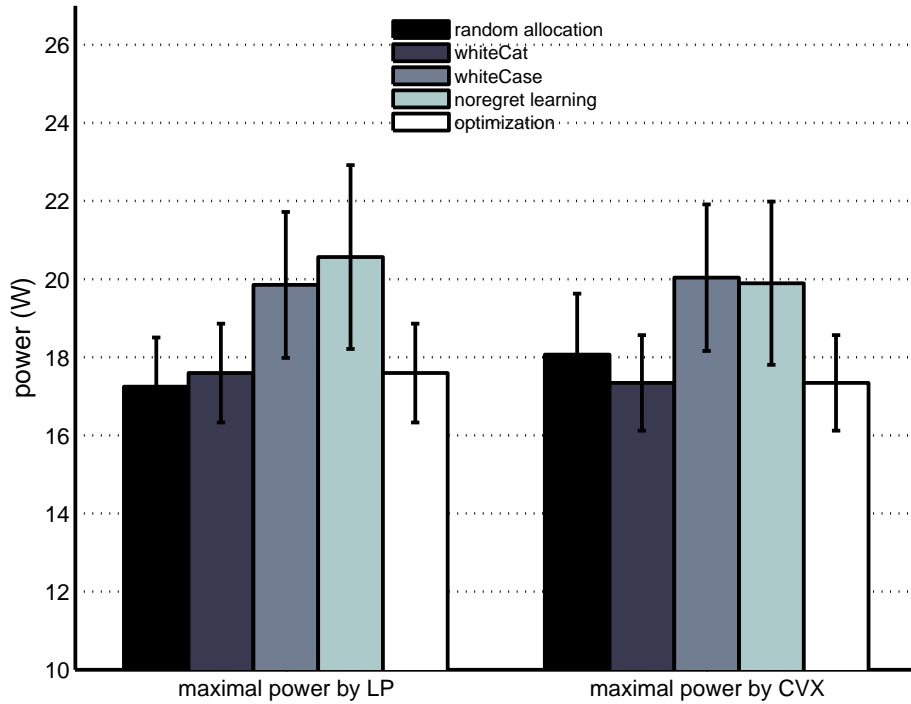
### Comparison of the Methods for Maximum Permitted Transmission Power

In Section 3.4, two different optimization formulations are introduced to calculate the maximum permitted transmission power, i.e., convex optimization and linear optimization respectively. In Fig. 3.3, we have seen that the convex optimization generates power levels which distribute evenly between the minimum and maximum possible transmission power, while, the majority of the power levels generated by linear optimization are either the minimum or maximum possible transmission power. In this section we will run the channel allocation schemes with the power levels obtained from convex and linear optimization respectively, in order to answer two questions, the first is which maximum

permitted power decision outperforms the other, and the second is to see the performance of the channel allocation schemes. The adopted metrics are the SINR on end terminals and transmission power consumption.

In each simulation, WBSs' locations are fixed whereas the location of interference measurement devices for TV receivers, end terminals and the sequence for WBS to update are randomly decided. Simulations are conducted for 100 times. Fig. 3.6 depicts the power consumptions of the channel allocation schemes working with the two groups of maximum transmission power decided by linear and convex problems respectively.

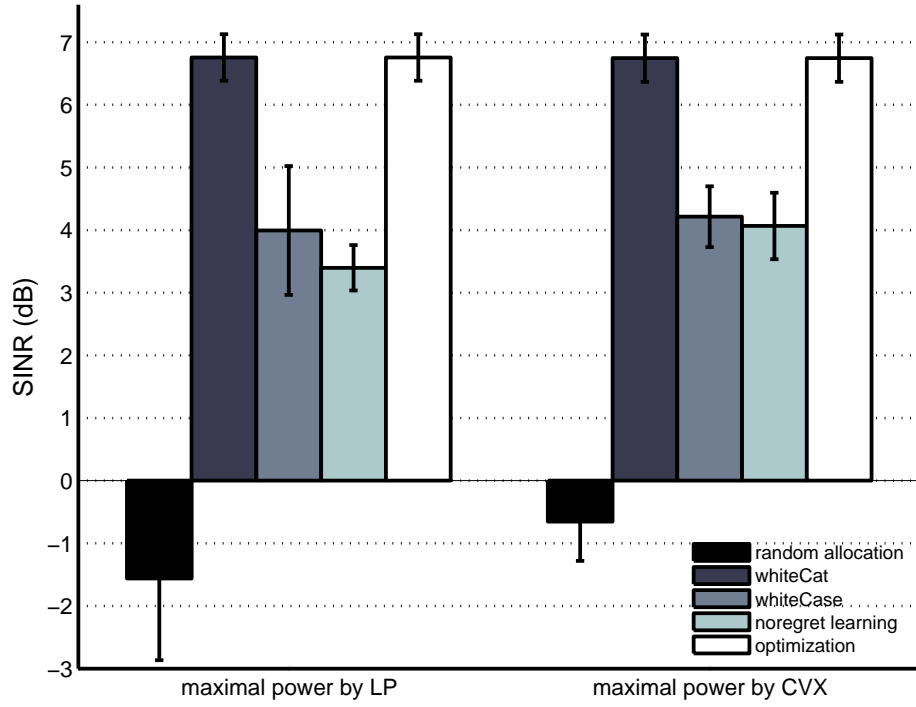
and 3.7 show that when convex formulation is applied to decide the maximal power map, all the four distributed spectrum allocation schemes consume less transmission power consumption by 15% to 30% and achieve better QuasiSINR. The cumulative distribution function curve of SINR on end terminals is drawn in Figure 3.8, where the X axis represents SINR level, and the Y axis shows the cumulative proportion of end terminals whose SINR equals or smaller than that level. The curves show that except for the random method, all three other distributed schemes perform better in low SINR area ( $\text{SINR} < 10\text{dB}$ ) with convex formulation, while worse in the high SINR area. Hence we adopt convex formulation to decide the maximal transmission power in the following simulation.



**Figure 3.6** Power consumed by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

### Comparison of Channel Allocation Schemes

After deciding the maximum permitted transmission power on each channel for each WBS, the data center distributes this maximum power map to all WBSs, and trigger the procedure of distributed channel allocation.



**Figure 3.7** QuasiSINR achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

The performance of the five spectrum allocation approaches working with the maximum transmission power map calculated from convex formulation is elucidated in the right part of both Fig. 3.6 and 3.7. We can see that whiteCat consumes 12% less transmission power than WhiteCase and No-regret learning schemes, whereas better QuasiSINR is obtained. The cumulative distribution function curve of SINR on end terminals with convex programming is presented in Fig. 3.8 with solid curves. It can be seen that for any cumulative proportion under 90%, the corresponding SINR level caused by whitecat on end terminals is slightly (around 0.5-1 dB) but stably higher than that obtained by WhiteCase and No-regret schemes, and 3 dB higher than that in random scheme.

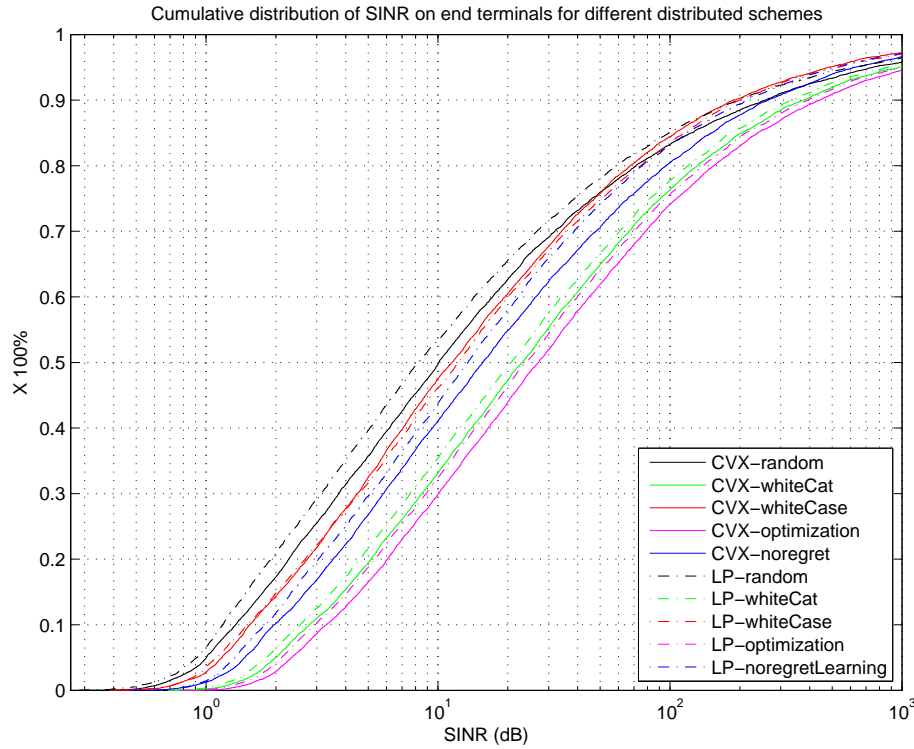
In each run of simulation, average value of the 20 % end terminals with the worst SINR is recorded, and the averaged such value over 100 simulations is illustrated in Figure 3.9 which shows whiteCat achieves better performance for the worst suffered end terminals than WhiteCase and No-regret approaches.

### Convergence Speed

In the congestion game, each player has at most  $(n-1) * |\mathcal{C}|$  resources available for usage, so there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up to 100.

As to whiteCat, we account each WBS accessing the base station (refer to 3.5.2) as *one step*. We compare the convergence speed of WhiteCat with no-regret learning, the scheme derived from potential game [111] and whiteCase. Note that the potential game scheme is to solve joint power and channel allocation problem, as it is developed with game theory, it is reasonable to see its convergence speed. We record the number steps before convergence.





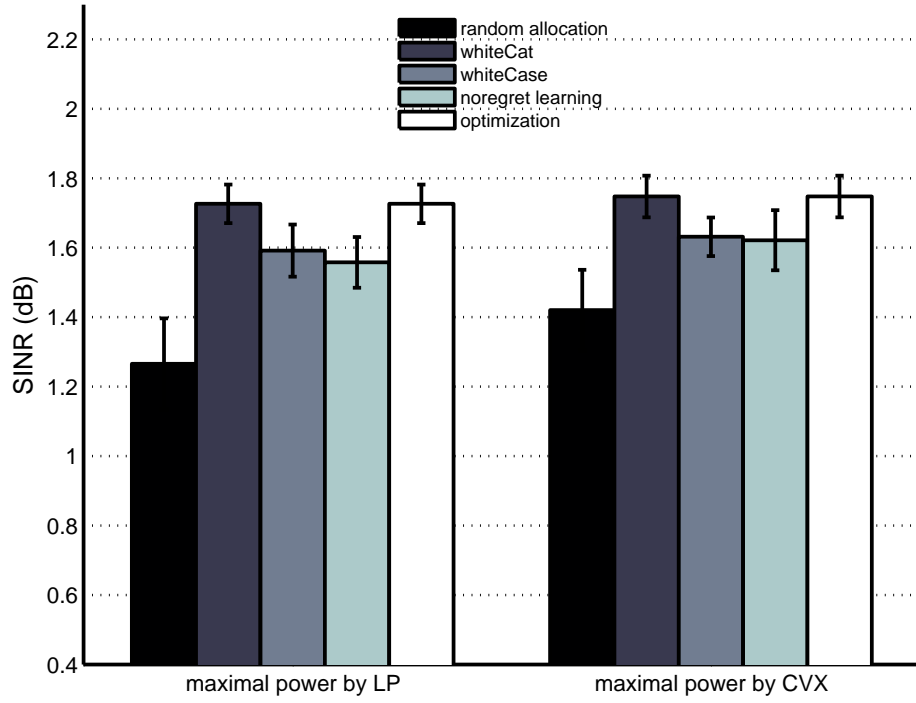
**Figure 3.8** CDF of SINR on end users obtained by different CA schemes under different methods to decide the maximal transmission power map

Table 3.2 shows the average number of steps needed before convergence in 100 runs of simulations. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds). The table tells that whiteCat is two times faster than the scheme derived from potential game, and 20 times faster than no-regret learning scheme. The relatively smaller confidence interval shows that whiteCat's convergence is not affected by different network conditions. Fast converge is attributed to WBSs accessing the database, so that the distributable decision involves a part of the global information. Thus, we can see that the speed up of convergence is due to the overhead caused by accessing the database.

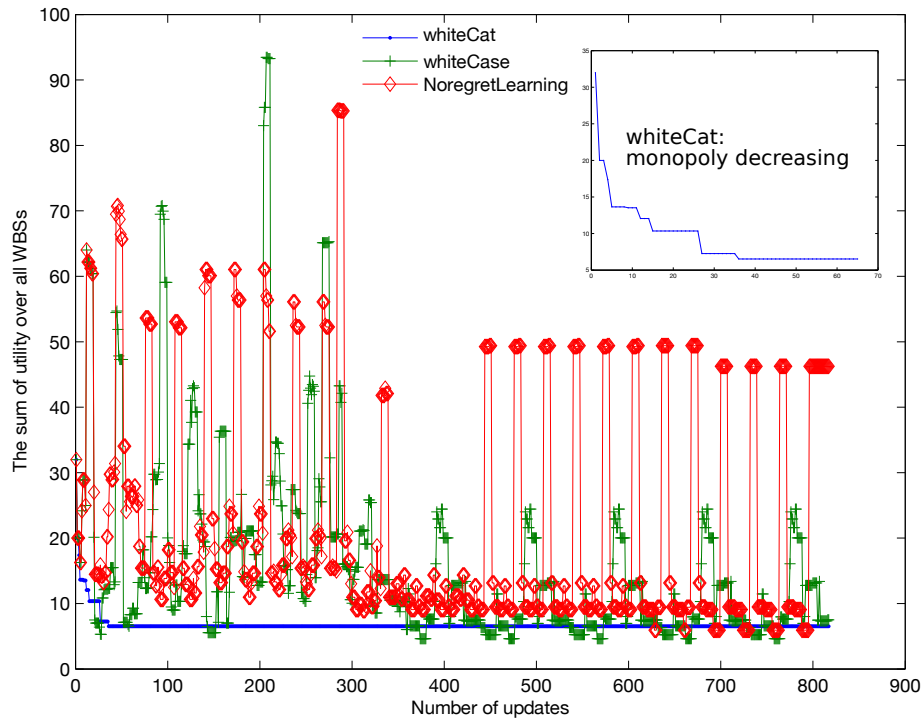
As to average running time for each convergence with Matlab, Whitecat is much faster than other schemes. Figure 3.10 depicts one instance of the convergence processes of three schemes.

Scheme	Average steps	95% CI	Average time <sup>3</sup> (s)
whiteCat	58	5.6	2
no-regret	1916	1541	144
PotentialGame [111]	120	10	4
optimization-LINDO	-	-	40
whiteCase	4587	2742	50

**Table 3.2** Convergence speed of the distributed channel allocation schemes



**Figure 3.9** The average SINR of the 20% worst end terminals



**Figure 3.10** Convergence with three different schemes in one simulation instance

### Stability of SINR in Convergence Process

WBS provides service to end users in the process of channel allocation. A certain SINR corresponds to certain transmission configurations like modulation type and data rate. The oscillation of SINR resulted from WBS changing the working channel during the convergence process may cause reconfiguration, reduced throughput or delay variance, which is not preferred. We propose a metric *Cost of Oscillation* (COS) to represent the

stability of SINR in the converging process. Assuming each update of channel takes 1 time unit, the variance of SINR of end user  $i$  at time  $t + 1$  is

$$\Delta\gamma_i(t+1) = \left| \frac{\gamma_i(t+1) - \gamma_i(t)}{\gamma_i(t)} \right|$$

The COS value for one network applied with a certain channel allocation scheme is,

$$COS = \sum_{t=1}^T \sum_{i \in \mathcal{N}} \Delta\gamma_i(t) \quad (3.24)$$

$\gamma_i(0)$  is the SINR for  $i$  before starting channel allocation. The variance of SINR in channel allocation process is shown in table 3.3 from which we can see WhiteCat achieves only 6% of oscillation on SINR compared with No-regret approach.

Scheme	COS	95% confidence interval
Whitecat	8850	2984
No-regret	145460	1541
Whitecase	246790	168050

**Table 3.3** Variance of SINR during the convergence process

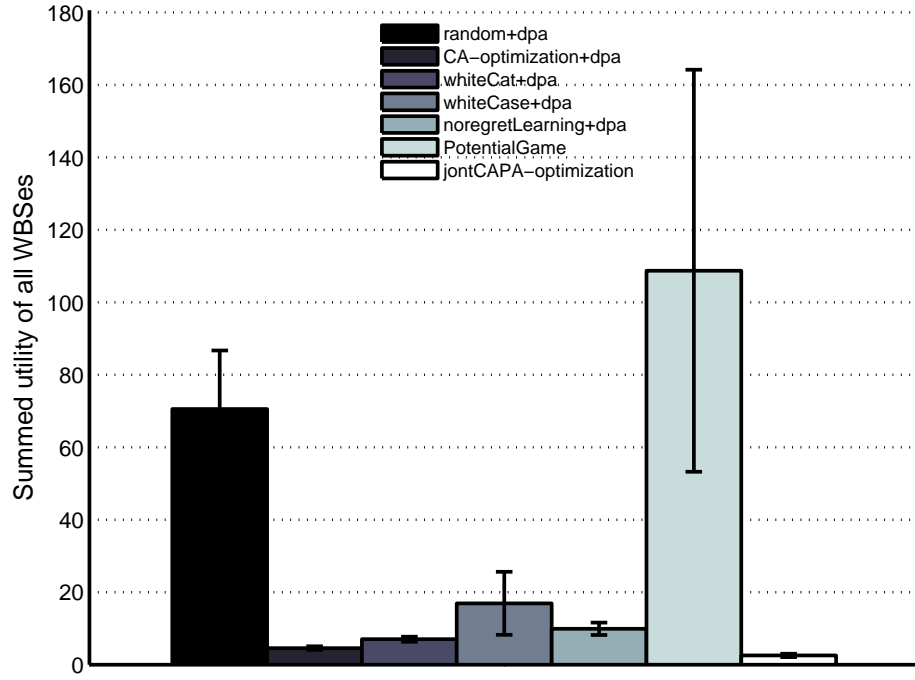
### 3.8.2 Performance of Joint Power and Channel Allocation

As introduced in section 3.6, after channel allocation is conducted, transmission power is adjusted in a distributive manner. In other words, power and channel allocation is executed with two cascaded distributed schemes. As comparisons, we implement two joint power channel allocation schemes. One is centralized optimization introduced in section 3.7.1, which is used as upper bound in the comparison. The other comparison is distributed joint power and channel allocation scheme [111] which is introduced in Section 3.3. We need to point it out that, scheme proposed in [111] doesn't aim to improve the SINR on end terminals, but on the sum of produced and received interferences. The performance of joint channel and power allocation schemes are presented in Fig. 3.11, 3.12 and 3.13 in terms of total utility, power consumption and achieved SINR on end users respectively.

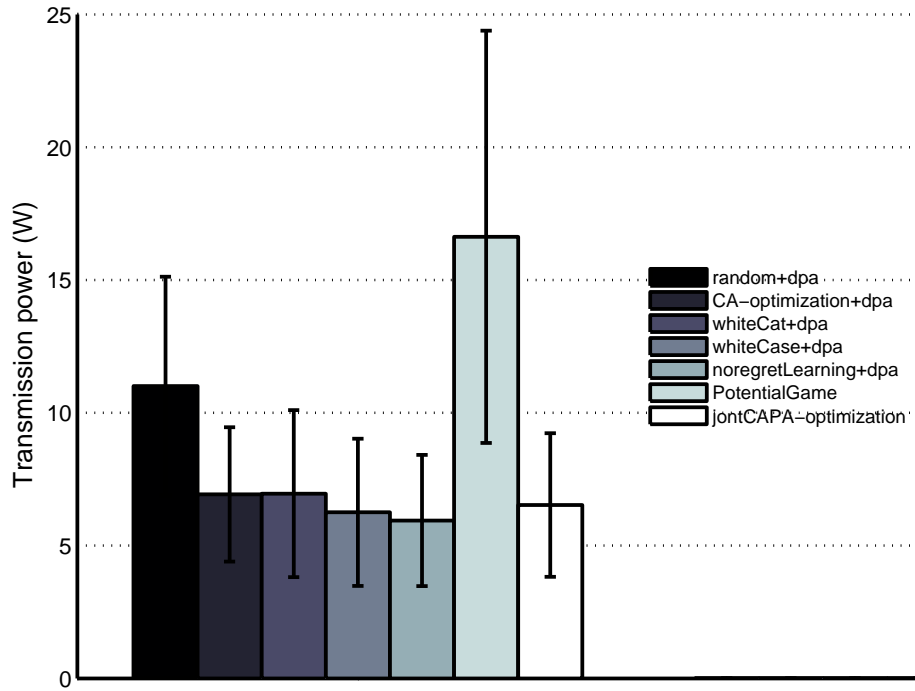
In Figure 3.11, we see that our proposed scheme *whiteCat+dpa*, the cascaded channel and power allocation method falls behind the cascaded channel allocation optimization and power allocation, and the joint channel and power allocation optimization, but outperforms other solutions. Figure 3.13 illustrates clearly that our proposed scheme approaches the two centralised schemes in providing SINR to end users.

## 3.9 Conclusion

We propose a set of solutions on the utilization of TV white space. The method to decide the maximum permitted transmission power which complies with IEEE 802.22 standard

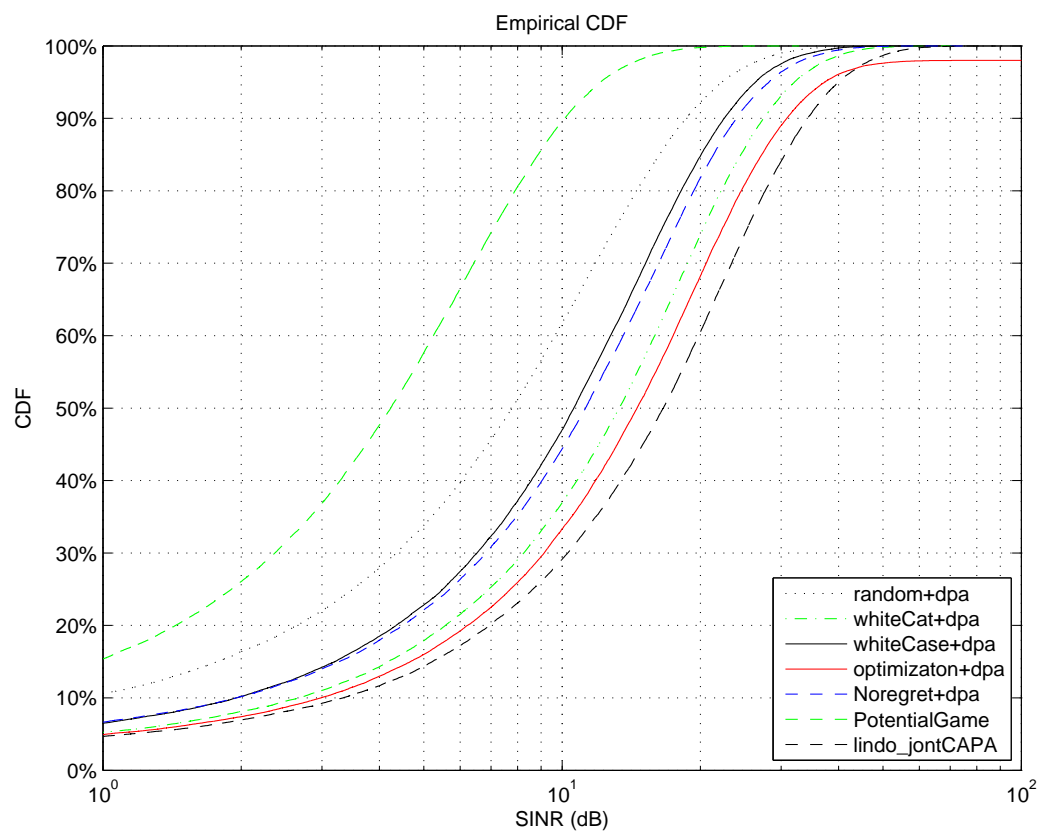


**Figure 3.11** Summed utility of all WBSs, which is the objective in problem 3.7. dpa in legend represents distributed power allocation



**Figure 3.12** Average transmission power of one WBS

is proposed. Then a distributed channel allocation algorithm is designed based on congestion game, where WBS can distributively choose the working channel which is associated with maximum permitted transmission power. A transmission power control procedure is conducted afterwards. This solution makes full use of the infrastructure, the centralized database, and distributes the computation and decision making afterwards on channel and transmission power. This scheme outperforms several other related works, and has comparable performance with the centralized solution.



**Figure 3.13** Cumulative distribution of SINR on end users after channel and power allocation



# 4

## ROBUST CLUSTERING of AD HOC COGNITIVE RADIO NETWORK

### 4.1 Introduction

As introduced in Chapter 1, efficient spectrum sensing is identified as to be critical to the success of cognitive radio networks [124]. Cooperative spectrum sensing is able to effectively cope with noise uncertainty and channel fading, thus remarkably improves the sensing accuracy. [19]. Collaborative sensing relies on the consensus of CR users within certain area, and decreases considerably the false sensing reports caused by fading and shadowing of reporting channel. Clustering, i.e., forming adjacent secondary users as a collectivity to perform certain tasks together, is regarded as an effective method used in cooperative spectrum sensing [135, 157]. Clustering is also efficient to enable all CR users<sup>1</sup> within the same cluster to stop payload transmission on the operating channel and initiate the sensing process, so that the all the CR nodes within the one cluster are able to vacate the channel swiftly when primary users are detected by at least one CR node residing in the cluster [143]. With cluster structure, as CR users can be notified by cluster head (CH) or other cluster members about the possible collision, the possibility for them to interfere neighbouring clusters is reduced [118]. Clustering algorithm has also proposed to support routing in cognitive ad-hoc networks [11].

Except for the advantages brought in by clustering, there is an issue on forming clusters in cognitive radio network. As the activity of primary users is controlled by licensed operators which are generally not known to CR nodes, the communication on licensed channels between CR nodes is not guaranteed. For a pair of communicating CR nodes, whenever a primary user is detected to be operating on the working channel, CR nodes have to retreat that working channel. The affected CR nodes switch to one other idle channel if there are available idle channels, if not, the communication has to be stopped. As a result, when coexisting with primary users, the ability for one pair of CR nodes to maintain communication on licensed channels is totally decided by primary users' activity.

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<sup>1</sup>User and node are used interchangeably in this chapter

Although the formed clusters is vulnerable under the influence of primary users, and there is little possibility to change primary users' operation, there is something can be done by secondary users in the process of forming clusters. One cluster is sustained when there is at least one common channel available for all members in that cluster, so that the cluster can operate on the licensed channel. As the increase of channel occupation by primary users is assumed to be random, a cluster with more channels will survive with higher probability. Thus the number of available common channels in the cluster indicates robustness of it when facing ungovernable influence from primary users. It is not difficult to see that forming clusters with different neighbours leads to different amount of common channels in the clusters. As a result, how to form the clusters plays an important role on the robustness of clusters in CRN.

To solely pursue cluster robustness against the primary users' activity, i.e., to achieve more common channels within clusters, the ultimately best clustering strategy is ironically that each node constitutes one single node clusters. Apparently this contradicts our motivation of proposing cluster in cognitive radio network. This contradiction indicates that, the robustness discussed in terms of number of common channels carries little meaning when the sizes of formed clusters are not given consideration. Besides, cluster size plays import roles in certain aspects. For instance, cluster size is one decisive factor in power preservation [20, 74], and it is also influence the accuracy of cooperative spectrum sensing [147]. Hence, cluster size should be given consideration when discussing cluster robustness against primary users.

In this chapter, a decentralized clustering approach ROSS is proposed to cover the issue of robustness of clusters in CRN. ROSS is able to form clusters with desired sizes, and the generated clusters are more robust than other robust clustering schemes, i.e., more secondary users residing in clusters against increasing influence from primary users. Compared to previous work, ROSS involves much less control messages, and the generated clusters are significantly more robust. ROSS selects cluster head through coordination within its neighborhood, and then cluster membership is decided locally and its fast convergence is proved under game theoretic framework. On the basis of ROSS, we propose a light weighted version ROSS-DFA which requires exchanging less overheads. Throughout this chapter, we refer both ROSS-DGA, ROSS-DFA, and the other versions with size control feature, as *variants of ROSS*.

The rest of chapter is organized as follows. After reviewing related work in section 4.2, we present our system model in Section 4.3. Then we introduce our clustering scheme ROSS and its variants in section 4.4. Centralized scheme is given discussion in section 4.5. Performance evaluation is in section 4.6. Finally, we conclude our work and point out direction future research in section 4.7.

## 4.2 Related Work

Prior to the emergence of open spectrum access, as an important method to manage network, clustering has been proposed in for ad hoc networks [29, 81, 93], wireless mesh networks [11], and wireless sensor networks [11] and . In ad hoc and mesh networks, the major focus of clustering is to preserve connectivity (under static channel conditions) or to improve routing. In the context of sensor networks, the emphasis of clustering has



been on longevity and coverage. Overhead generated by clustering in ad hoc network is analysed in [134, 146].

As to cognitive radio networks, clustering schemes are also proposed, which target different aspects. Work [147] improves spectrum sensing ability by grouping the CR users with potentially best detection performance into the same cluster. Clustering scheme [74] obtains the best cluster size which minimizes power consumption caused by communication within and among clusters. [74] proposes clustering strategy in cognitive radio network, which looks into the relationship between cluster size and power consumption and accordingly controlling the cluster size to decrease power consumption. Cogmesh is proposed in [43] to construct clusters by the neighbour nodes which share local common channels, and by interacting with neighbour clusters, a mesh network in the context of open spectrum sharing is formed. Robustness issue is not considered by this clustering approach. [144] targets on the QoS poisoning and energy efficiency. This approach first decides on the relay nodes which minimize transmission power consumption, then the chosen nodes become cluster heads and clusters are formed in a dynamic coalition process. This work emphasis on power efficiency and doesn't take into account the channel availability and the issue of robustness of the formed clusters. In [26, 157], the channel available to the largest set of one-hop neighbours is selected as common channel which yields a partition of the CRN into clusters. This approach minimizes the set of distinct frequency bands (and hence, the set of clusters) used as common channels within the CRN. However, bigger cluster sizes generally lead to less options within one cluster to switch to if the common channel is reclaimed by a primary node. Hence, this scheme does not provide robustness to formed clusters. [23] deploys cluster structure in order to implement common channel control, medium access with multiple channel and channel allocation. The node with the maximum number of common channels within its  $k$ -hop neighborhood is chosen as cluster head, but how to avoid one node appearing in multiple clusters is not given consideration.

Clustering robustness is considered in [89, 96]. Authors [89, 96] emphasis on improving the numbers of common channels within clusters, in order to strengthening robustness of clusters, but the perused metric is not examined or proved to be able to maintain cluster structure under vigorous intensity of primary users' activities. The authors consider the balance between the number of idle common channels within cluster and cluster size and propose an algorithm that increases the number of common channels within clusters. One drawback of this scheme is, in order to increase the number of common channels within clusters, the scheme excludes certain CR nodes from the formed clusters, so that isolated nodes have to form clusters themselves. Besides, this scheme leads to a high variance on the size of clusters, which is not desired in certain applications as discussed in [23, 74].

## 4.3 System Model

Let us consider a two dimensional area where primary and secondary users coexist together. They share the licensed channels according to the spectrum overlay model as introduced in Section 1.1.2, where secondary transmitters are only allowed to transmit when the primary users are detected as being idle, and the spectrum should be vacated if a primary user is detected transmitting. The set of primary users and secondary users are presented by  $\mathcal{P}$  and  $\mathcal{N}$  separately, there are  $|\mathcal{P}| = P$  and  $|\mathcal{N}| = N$ . Cognitive radio

ad hoc network is constituted by all the secondary users in  $\mathcal{N}$ . The collection of non-overlapping licensed frequency bands is denoted as  $\mathcal{F}$  with  $|\mathcal{F}| = F$ , which is shared by the primary and secondary users. We assume that primary users have a relatively low variation in activity (periods of activity and inactivity in the range of seconds or minutes). Primary users access the allocated channels in  $\mathcal{F}$  according to its need without sending any explicit notification to secondary users. Secondary users conduct spectrum sensing independently, by which the secondary users validate the channels to be available or not.

<sup>2</sup> The available channels sensed on secondary user  $i$  is denoted by  $V_i$  and  $|V_i| \leq F$ .

One dedicated control channel is assumed to be available for all the secondary nodes to exchange control messages with neighbours in the process of cluster formation. The control channel could be in ISM band or other reserved spectrum which is exclusively used for transmitting control messages. Note that the assumption of dedicated control channel is to simplify the discussion so that we can focus on the kernel of this chapter, the robustness of clusters. Actually, the control messages involved in the clustering process can be conveyed on available licensed channels through rendezvous process by channel hopping [62,155]. Secondary users have the same transmission range on both licensed and control channel, and the communication links are reciprocal. Over the control channel, secondary users exchange their spectrum sensing results  $V_i$  with one hop neighbours.

As to two secondary users, when the distance between them is smaller than secondary users' transmission range, we assume the pair is able to communicate on both control channel and licensed channel, and the both are considered to be neighbours of each other. When licensed channels are available on two neighbouring secondary nodes in the same time, payload communication can be conducted on one or multiple licensed channels. Any user outside the transmission range of the transmitter can not receive data from it, i.e., any secondary user outside primary users' transmission ranges can not detect their existence, whereas a secondary user can always communicate with secondary sender, or detect the operating primary user, when it locates within the secondary sender or the operating primary user's transmission range. As the transmission range of primary users is limited and secondary users are at different locations, secondary users have different views on the occupancy of the spectrum (apart from the fact that there might be false negatives in the sensing process), i.e.,  $V_i \neq V_j$ , for  $i \neq j$ .

As the assumed 0/1 state of connectivity is solely based on the distance between secondary users, CRN can be represented by a graph  $\mathcal{G}(I, \mathcal{E})$ , where  $\mathcal{E} = \{(i, j, v) | i, j \in \mathcal{N} \wedge v \in V_i \wedge v \in V_j\}$  is wireless link between any secondary node  $i$  and its neighbour  $j$  with licensed channel  $v$ . Due to relatively low primary user dynamics, time index is omitted here. For secondary node  $i$ , its neighborhood  $Nb_i$  consists of all the secondary users locating within its transmission range, regardless whether common licensed channels exist or not. In the rest of this chapter, *channel* only refers to the licensed spectrum except when control channel is particularly mentioned.

## Clustering

A cluster  $C$  is composed with one cluster head and cluster members, which satisfies the following conditions:

---

<sup>2</sup>The spectrum availability can be validated with a certain probability of detection. Spectrum sensing/validation is out of the scope of this thesis.

- Cluster head  $H_C$  is able to communicate with any cluster member directly, i.e., for any cluster member  $i \in C$ , there is  $i \in Nb_{H_C}$ .
- There exists at least one common licensed channel for the cluster, i.e.,  $\cap_{i \in C} V_i \neq \emptyset$ .

Cluster head coordinates the activities of cluster members, i.e., notifies all the members to evade a channel if the channel is sensed by one cluster member to be occupied by primary users, or notify the members to use a different channel for payload communication. Cluster is denoted as  $C_i$  where its cluster head is  $i$ . We refer to the common channels within a cluster  $C$  by the term *common control channels* (CCC) and denote this set of channels by set  $K_C$ .  $K_C = \cap_{i \in C_i} V_i$ , and  $k_C = |K_C|$  is the number of common channels for cluster  $C$ . CR users can be mobile, thus clustering is performed periodically, or when the current cluster can not maintain because of no licensed channels are available.

Symbol	Description
$\mathcal{P}, \mathcal{N}$	complete collection of primary and secondary users
$\mathcal{F}$	set of non-overlapping channels in the scenario
$Nb_i$	node $i$ 's neighborhood
$C$	a cluster
$C_i$	a cluster whose cluster head is $i$ but in Section 4.5, $C_i$ is $i$ th cluster among the legitimate clusters.
$V_i$	set of available channels on CR node $i$
$V_C$	set of available common channels of cluster $C$
$V_{C_i}$	set of available common channels of cluster $C_i$
$H_C$	cluster head of a cluster $C$
$\delta$	desired cluster size
$S_i$	set of claiming clusters, each of which includes debatable node $i$ after phase I
$K_{C_i}$	set of common channels within cluster $C_i$
$k_{C_i}$	number of common channels of cluster $C_i$

**Table 4.1** Notations in robust clustering problem

## 4.4 Distributed Coordination Framework: Clustering Algorithm

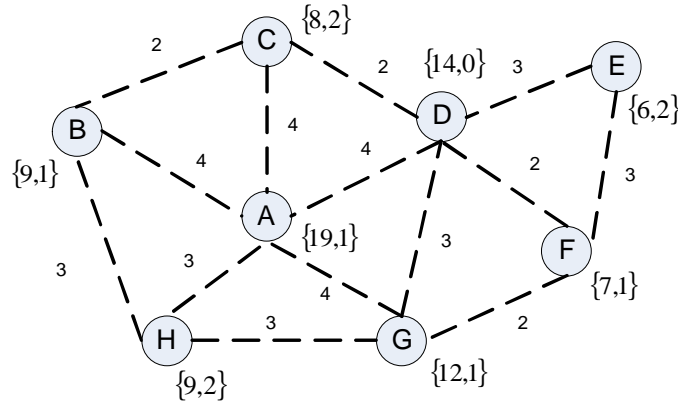
In this section, we present the new clustering scheme named as ROSS (RObust Spectrum Sharing). Briefly, ROSS is based on the spectrum sensing results of all its neighbours along with the spectrum sensing result on itself, then utilizes local similarity of the available channels to form clusters. ROSS consists of two cascaded phases: *cluster formation* and *membership clarification*. We will describe them sequentially.

#### 4.4.1 Phase I - Cluster Formation

After spectrum sensing and communication with neighbours, every CR node is aware of the available channels on itself and all its neighbours. We propose two metrics for each CR node  $i$  to characterize the channel availability between it and its neighborhood.

*Individual connectivity degree  $D_i$ :*  $D_i = \sum_{j \in Nb_i} |V_i \cap V_j|$ , which denotes the sum of the pairwise common channels of node  $i$ . It is an indicator of node  $i$ 's adhesive property to the CRN.

*Social connectivity degree  $G_i$ :*  $G_i = |\bigcap_{j \in Nb_i \cup i} V_j|$ , which is the number of common channels of CR node  $i$  and its neighbours.  $G_i$  represents the ability of  $i$  to form a robust cluster with its neighborhood. Figure 4.1 illustrates an example CRN where the corresponding individual and social connectivity degrees are calculated and shown for each node. Individual connectivity degree  $D_i$  and social connectivity degree  $G_i$  together



**Figure 4.1** Connectivity graph and the connectivity vector  $\{D_i, G_i\}$  on each node. The available channels sensed by each CR node are:  $V_A = \{1, 2, 3, 4, 5, 6, 10\}$ ,  $V_B = \{1, 2, 3, 5, 7\}$ ,  $V_C = \{1, 3, 4, 10\}$ ,  $V_D = \{1, 2, 3, 5\}$ ,  $V_E = \{2, 3, 5, 7\}$ ,  $V_F = \{2, 4, 5, 6, 7\}$ ,  $V_G = \{1, 2, 3, 4, 8\}$ ,  $V_H = \{1, 2, 5, 8\}$ . Dashed lines indicates two end nodes are within transmission range of each other. Each edge is labelled by the number of common channels between the two ends.

form the *connectivity vector*, with which we can proceed to introduce the first phase of algorithm ROSS. The algorithm of phase I can be sketched like this: cluster heads are determined firstly, then accordingly, clusters are formed on the basis of the cluster head's neighbourhood.

#### Determining Cluster Heads and Form the Initial Clusters

In this phase, each CR node decides whether it is cluster head by comparing its connectivity vector with its neighbours. When CR node  $i$  has the bigger individual connectivity degree than any neighbours except those already become cluster heads, then node  $i$  becomes cluster head. In other words, CR node  $i$  becomes cluster head if  $D_i > D_k, \forall k \in Nb_i \setminus CHs$  ( $CHs$  donate the cluster heads existing in  $Nb_i$ ). If there is another CR node  $j$  in its neighborhood has the same individual connectivity degree, i.e.,  $D_j = D_i$  and  $D_j > D_k, \forall k \in Nb_j \setminus \{CHs \cup i\}$ , then the node out of  $\{i, j\}$  with higher social connectivity degree becomes cluster head, and the other one becomes member of it. If  $G_i = G_j$  as

well, node ID is used to break the tie, i.e., the one with smaller node ID takes precedence and becomes cluster head.

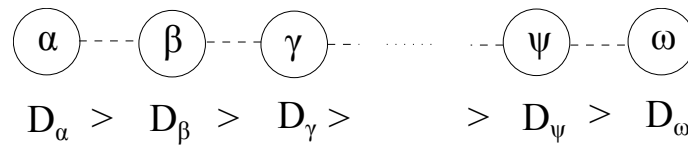
After receiving the notification from a cluster head, a CR node, e.g.,  $i$ , is aware it is one member of a cluster, then CR user  $i$  sets its individual connectivity degree to zero, which is smaller than all the possible individual connectivity degrees in the CR network. Then node  $i$  broadcasts its new individual connectivity degree to all its neighbours. When a CR node  $i$  is associated to multiple clusters,  $D_i$  is still set to zero. When a cluster member is excluded from a cluster, that CR user's individual connectivity degree is restored to the original value which is further broadcast to its neighbours. The purpose of this temporary manipulation on individual connectivity degree is to let the CR nodes out side of the cluster be possible to become cluster heads, so that every CR node either becomes cluster head or a member of at least one cluster. In other words, with this manipulation on its individual connectivity degree, every CR node in the network will be involved in the clustering process, the proof will be given later. The final states of all the CR users in the CRN are described in the following theorem. Figure 4.1 provides an example about how do CR nodes decide cluster heads. Node  $B$  and  $H$  have same individual connectivity degree,  $D_B = D_H$ , but as  $G_H = 2 > G_B = 1$ , node  $H$  becomes cluster head. In Figure 4.1, the cluster  $C_H$  is  $\{H, B, A, G\}$ .

The pseudo code for deciding cluster head and forming initial clusters is in Algorithm 2.

We have the following theorem to show that as long as a secondary user's connectivity the CRN is not zero, the secondary user will always be integrated into a certain cluster.

**THEOREM 4.4.1:** *Given a CRN, every secondary user is included into at least one cluster within  $N$  steps.*

**Proof.** Note the formed cluster can be a singleton cluster, i.e., cluster size is 1. To simplify the proof, we assume secondary users have unique individual connectivity degrees. This is fair as there are other metrics to break the tie according to Algorithm 2, i.e., the social connectivity degrees and node ID. Assume there are some secondary users which are not assigned to any cluster and node  $\alpha$  is one of them. As node  $\alpha$  is not contained in any cluster, there must be at least one node  $\beta \in Nb_\alpha$ , with  $D_\alpha < D_\beta$ . Otherwise, node  $\alpha$  is eligible to form a cluster. Then, as node  $\beta$  form a cluster and include node  $\alpha$ , we can repeat the analysis on node  $\alpha$ , and deduce that node  $\beta$  has at least one neighbouring node  $\gamma$  with  $D_\gamma > D_\beta$ , and this series of nodes with monotonically increasing  $D_i$  might continue to grow and ceases finally because both of the individual connectivity degree and the total number of nodes are limited. The formed node series is shown as Figure 4.2.



**Figure 4.2** The node series in the proof, the deduction starts from node  $\alpha$

Now we find the node  $\omega$  is in the end of this series. As  $\omega$  is the end node and does not have neighbouring nodes with bigger individual connectivity degree  $D$ ,  $\omega$  becomes cluster head and incorporate all its one-hop neighbours, including the node before it in the node series (here we assume that every new formed cluster has common channels). After that,

**Algorithm 2:** Cluster head determination and initial cluster formation

**Input:** Unclustered CR node  $i$  which is aware of  $D_j$  and  $G_j$ ,  $j \in Nb_i$ , and the ID of CRs which have be decided to be cluster heads,  $ID_{CH}$ ,  $CH \in Nb_i$ .

Empty sets  $\tau_1, \tau_2, \tau_3, \tau_4, \tau_5$

**Result:** Whether or not  $i$  is cluster head

```

1  for CR node  $j \in Nb_i \setminus CHs$  do
2      if  $D_i == D_j$  then
3           $\tau_1 \leftarrow j$ 
4      else
5          if  $D_j < D_i$  then
6               $\tau_2 \leftarrow j$ 
7          end
8      end
9  end
10 if  $\tau_2 \neq \emptyset$  then
11      $i$  is not CH; break;
12 else if  $\tau_1$  is  $\emptyset$  then
13      $i$  is CH; break;
14 else
15     for  $\forall k \in \tau_1$  do
16         if  $\nexists m \in Nb_k \setminus CHs$ , such that  $D_m < D_k$  then
17              $\tau_3 \leftarrow k$ 
18         end
19     end
20     if  $\tau_3$  is  $\emptyset$  then
21          $i$  is CH; break;
22     else
23         for  $\forall n \in \tau_3$  do
24             if  $G_n > G_i$  then
25                  $\tau_4 \leftarrow n$ 
26             else
27                 if  $G_n == G_i$  then
28                      $\tau_5 \leftarrow n$ 
29                 end
30             end
31         end
32         if  $\tau_4 \neq \emptyset$  then
33              $i$  is not CH; break;
34         else if  $\tau_4$  is  $\emptyset$  and  $\tau_5 \neq \emptyset$  then
35             if  $ID_i < ID_r, \forall r \in \tau_5$  then
36                  $i$  is CH; break;
37             end
38         else
39              $i$  is CH; break;
40         end
41     end
42 end
43 if  $i$  is cluster head then
44      $D_j, j \in Nb_i \setminus CHs$  is changed as a big positive value  $M$ ;
45 end

```

/\*  $\tau_1 \neq \emptyset, \tau_1$  is  $\emptyset$  \*/

/\*  $\tau_4$  and  $\tau_5$  are  $\emptyset$  \*/

the node recruited into cluster will set its connection degree  $D$  to zero, which enables the node further down in the list to become a cluster head. In this way, all the nodes in the series are included in at least one cluster in an inverse sequence. This result contradicts the initial assumption and proves the claim stated above. Meanwhile, through this proof, we know that within at most  $N$  steps, all nodes will become a part of certain clusters.  $\square$

According to Theorem 4.4.1, we can assign reasonable amount of time for phase I to complete.

### Guarantee Availability of Common Control Channel

After deciding itself being cluster head, CR node broadcasts to notify its neighbours on control channel, meanwhile,  $i$ 's initial cluster is formed immediately, which is  $i$ 's neighborhood except for those nodes which have become cluster heads, i.e.,  $C_i = (Nb_i \setminus CHs) \cup i$ . The reason for say the previous step generates initial clusters is, it is possible that certain formed clusters don't poses any common control channels. This problem can be solved with the following method.

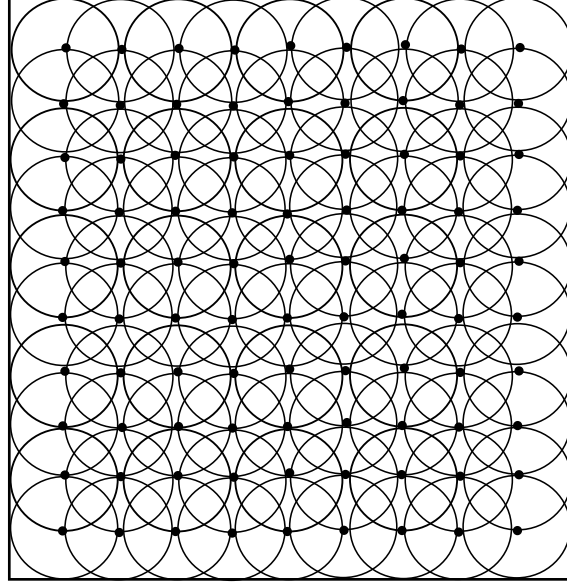
As smaller cluster size increases the number of common channels within the cluster, certain nodes are eliminated until there is at least one common channel. The elimination of nodes is performed according to an ascending list of nodes sorted by their number of common channels with the cluster head, which means, the cluster member which has the least number of common channels with the cluster head is excluded first. If there are nodes having the same number of common channels with cluster head, the node whose elimination brings in more common channels will be excluded. If this criterion meets a tie, the tie will be broken by deleting the node with smaller ID. It is possible that the cluster head excludes all its neighbours and resulting itself to be the cluster. The pseudo code for cluster head to obtain at least one common channel is shown in Algorithm 3. As to the nodes eliminated in this procedure, they become either cluster heads or get included into other clusters later on according to Theorem 4.4.1.

### Cluster Size Control in Dense CRN

In the introduction section, we have stated that cluster size should be given consideration to justify the pursuing of robust clusters, here we illustrate the pressing necessity to control cluster size when CRN becomes dense via theoretical analysis and simulation, and provide solution to it.

Assume CR nodes and primary users are evenly distributed and primary users occupy the licensed channels randomly, i.e., both CR nodes density and channel availability in the CRN is homogeneous. Based on Algorithm 2, cluster heads are the CR nodes which possess the biggest individual connectivity degrees in their neighborhood respectively, and they are surrounded by CR nodes. The clusters formed are the neighborhood of cluster heads, which is decided by the transmission range and network density. When the CRN is extremely dense, consider a cluster with cluster head of CR node  $i$ , based on the rule for cluster head selection Algorithm 2, the nearest cluster head could locate just outside node  $i$ 's neighborhood or its transmission range. An instance of this situation is shown in Figure 4.3. In the figure, black dots represent cluster heads, the circles denotes the

transmission ranges of those cluster heads. Cluster members are not shown in the figure. Let  $l$  be the length of side of simulation plan square, and  $r$  be CR's transmission radius.



**Figure 4.3** Clusters formation in extremely dense CRN. Black dots are cluster heads, cluster members are not drawn.

Based on the aforementioned analysis and geometry illustration as shown in Figure 4.3, we can give an estimate on the maximum number of generated clusters. The estimated number of cluster heads is the product of the number of cluster heads in row and line,  $l/r * l/r = l^2/r^2$ .

Now we verify the estimation with simulation. We distribute CR users and primary users randomly on a square plan, and set  $r = 10, l = 50$ . Network density is increased by adding more CR users. For each network scale, simulation is run for 50 times. Figure 4.4 shows the number of formed clusters. With the increase of CR users in the network, network density increases linearly (see the right hand side Y axis, which indicates the number of neighbours.), and the number of formed clusters also increases and approaches to the upper bound of 25 which complies with the estimation. The confidence rate is 95% in the figure.

Both the analysis and simulation show that with the increase of network density, the cluster size also increases. In case of dense network where cluster size is large, there is substantial burden on cluster heads to manage the cluster members, which is a challenge for resource limited cluster heads. As a result, certain measures are needed to prevent the network size to increase with the increasing network density. This task falls on cluster head. To control cluster size, cluster heads prune their cluster members if sizes are greater than the desired size  $\delta$ . Given desired size as  $\delta$ , cluster head excludes members sequentially, whose absence leads to the maximum increase of common channels within the cluster. This process ends when the size of resultant cluster is at most  $\delta$  and at least one CCC is available. This procedure is similar with guaranteeing CCC available to be available in cluster, thus the algorithm is also given in Algorithm 3. The nodes eliminated in this procedure later become a member or cluster head according to Theorem 4.4.1.

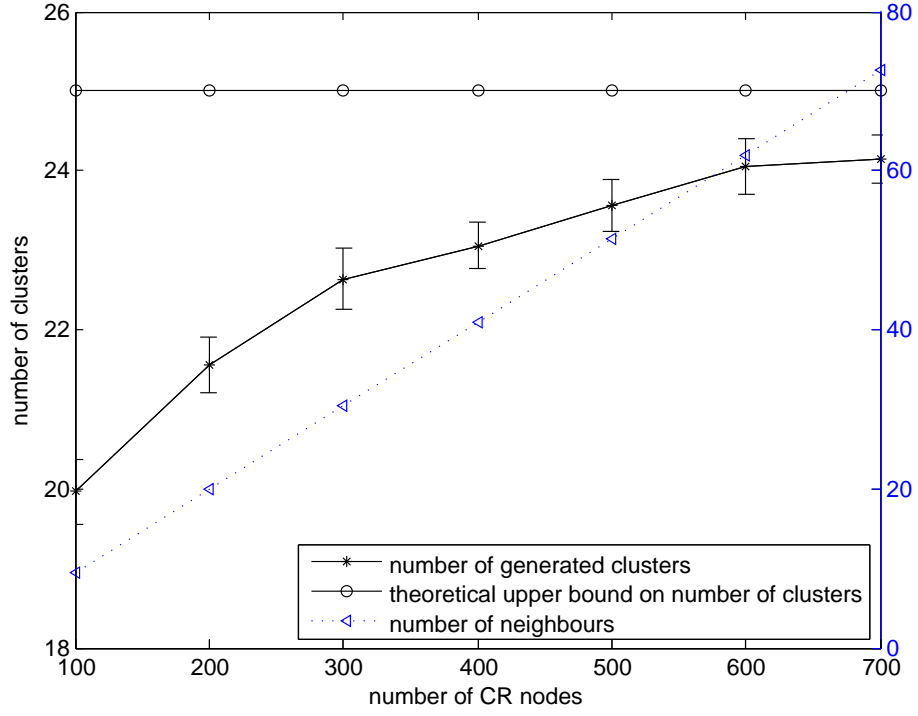


**Algorithm 3:** CCC guarantee and cluster size control by cluster head**Input:** Initial cluster formed, empty sets  $\mathcal{T}_1, \mathcal{T}_2$ **Output:** cluster has at least CCC, and satisfies the requirement on cluster size/\* When to guarantee available CCCs, execute from line 1,  
when to control cluster size, execute from line 2 \*/

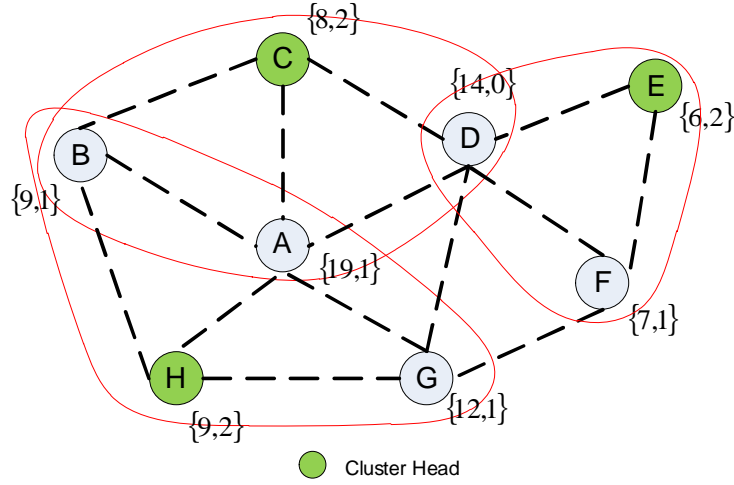
```

1 while  $V_C = \emptyset$  do
2   while  $|C| > \delta + 1$  do
3     calculate  $\lambda = \min_{i \in C, i \neq H_C} (|K_{H_C} \cap K_i|)$ ;
4     for each  $i \in C \setminus H_C$  do
5       if  $|V_{H_C} \cap V_i| == \lambda$  then
6          $\mathcal{T}_1 \leftarrow i$ 
7       end
8     end
9     if  $|\mathcal{T}_1| == 1$  then
10      delete node  $i$  from  $C$ ;
11      break;
12    else
13      calculate  $\mu = \max_{i \in \mathcal{T}_1} (|\cap_{j \in C \setminus i} V_j| - |\cap_{j \in C} V_j|)$ ;
14      for each  $i \in \mathcal{T}_1$  do
15        if  $|\cap_{j \in C \setminus i} V_j| - |\cap_{j \in C} V_j| == \mu$  then
16           $\mathcal{T}_2 \leftarrow i$ 
17        end
18      end
19      if  $|\mathcal{T}_2| == 1$  then
20        delete node  $i$  from  $C$ ;
21        break;
22      else
23        delete  $i \in \mathcal{T}_2$ , which has the highest ID;
24      end
25    end
26 end

```



**Figure 4.4** Number of clusters formed

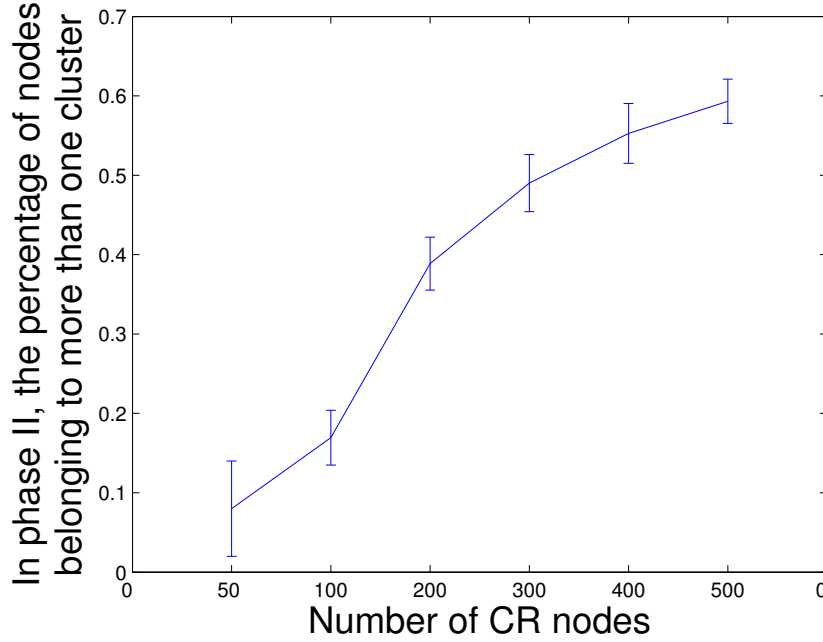


**Figure 4.5** Clusters formation after the first phase of ROSS. Some nodes remain debatable nodes after the first phase.

#### 4.4.2 Phase II - Membership Clarification

After applying ROSS phase I on the example in Figure 4.1, we get the clusters shown in Figure 4.5. We notice there are several nodes, i.e.,  $A, B, D$ , are included into more than one cluster. We refer these nodes as *debatable nodes* as their cluster affiliations are not clear, and the clusters which include debatable node  $i$  are called *claiming clusters* of node  $i$ , and are represented as  $S_i$ . Actually, debatable nodes extensively exist in CRN with larger scale. Figure 4.6 shows the percentage of debatable nodes increases with the scaling of CRN network.

Debatable nodes should be exclusively associated with one cluster and removed from the other claiming clusters, this procedure is called cluster membership clarification. We will introduce the solution for cluster membership clarification in the following.



**Figure 4.6** The number of debatbale nodes after the first phase of ROSS. The unit of Y axis is 100%

### Distributed Greedy Algorithm

After Phase I, debatable nodes, e.g.,  $i$  needs to decide which cluster  $C \in S_i$  to stay, and leaves the others. The principle for debatable node  $i$  to choose one claiming cluster is to result in the greatest increase of common channels in all its claiming clusters. Node  $i$  communicates with all the cluster heads whose clusters are in  $S_i$ , and is aware of the vector of common channels of the claiming clusters, then  $i$  is able to calculate how many more common channels in one certain claiming cluster if  $i$  leaves that cluster. Based on this calculation,  $i$  decides in which claiming cluster to stay and leaves the other claiming clusters. If there exists one cluster  $C \in S_i$ , by leaving from which the clusters in  $S_i$  obtain the minimum increased common channels than leaving any other claiming clusters, then  $i$  chooses to stay in cluster  $C$ . When there comes a tie in terms of the increase of common channels among multiple claiming clusters,  $i$  chooses to stay in the cluster whose cluster head shares more common channels with  $i$ . In case there are multiple claiming clusters demonstrating the same on the aforementioned metrics, node  $i$  chooses to stay in the smallest claiming cluster. IDs of cluster heads will be used to break tie if the previous rule doesn't decide on the unique cluster to stay.

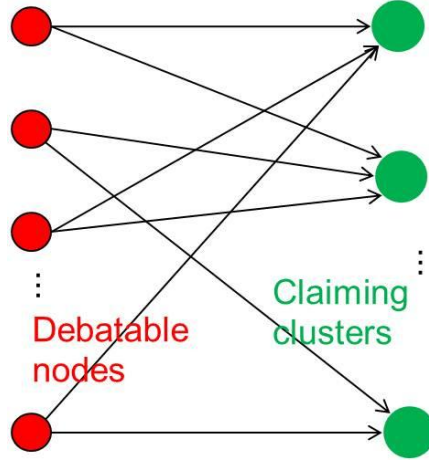
Algorithm for debatable node  $i$  to decide which claiming cluster to stay is described as Algorithm 4. To conduct Algorithm 4, debatable node  $i$  needs to know the necessary information about its claiming clusters, i.e.,  $V_C$ ,  $V_{H_C}$ ,  $|C|$ ,  $C \in S_i$ , which are respectively the set of available channels in  $C$ , the set of available channels on  $C$ ' cluster head  $H_C$ , and  $C$ ' cluster size. Node  $i$  decides which cluster to stay based on Algorithm 4, then

notifies all its claiming clusters, and retrieves the updated information of the necessary information  $V_C, V_{H_C}, |C|$ , where  $C \in S_i$ .

This procedure raises the concern on infinite chain effect that debatable nodes update their choices based on other debatable nodes' choices, and never cease. Assume debatable node  $i$  locates in one cluster  $C \in S_i$ , and  $C$  could have more than one debatable node except for  $i$ . Let  $i$  out of  $C$ 's debatable nodes to make decision on the cluster to stay first. Then the choices of those debatable nodes except for  $i$  change  $C$ 's membership, which possibly further triggers node  $i$  to alter its previous decision. Thus, we must answer this question raised when implementing ROSS-DGA. In the following we show that the process of membership clarification can be formulated into a singleton congestion game, and a equilibrium state is reached after a finite number of best response updates.

### Bridging ROSS-DGA with Congestion Game

In this part, we illustrate that when debatable nodes decide on the exclusive clusters to stay, in particular, the To formulate the problem of membership clarification for the debatable nodes in the context of a game, we look at the problem from a different perspective. In the new perspective, the debatable nodes are regarded as isolated and don't belong to any cluster, which means the clusters they used to belong to become their neighbouring clusters. Then as to each debatable node, the previous problem to decide which clusters to leave becomes a new problem that which cluster to join. In this new problem, debatable node  $i$  (note now  $i \notin S_i$ ) chooses one cluster  $C$  out of  $S_i$  to join if the decrement of common channels in cluster  $C$  is the smallest in  $S_i$ , and the decrement of CCCs in cluster  $C$  is  $\sum_{C \in S_i} \Delta |K_C| = \sum_{C \in S_i} (|K_C| - |K_{C \cup i}|)$ . The relation between debatable nodes and claiming clusters is shown in Figure 4.7.



**Figure 4.7** Debatable nodes and claiming clusters

In the following, the debatable nodes constitute the players, and we show that the decision of debatable nodes to clarify their membership can be mapped to the behaviour of the players in a *player-specific singleton congestion game* when proper cost function is given.

The game to be constructed can be represented by a 4-tuple  $\Gamma = (\mathcal{P}, \mathcal{R}, (\sum_i)_{i \in \mathcal{N}}, \Delta |K_C^i|)$ , where elements in  $\Gamma$  are given as below,

---

**Algorithm 4:** Debatable node  $i$  decides its affiliation, chooses one claiming cluster to stay and leaves all the other claiming clusters

---

**Input:** all claiming clusters  $C \in S_i$

**Output:** one cluster  $C$

```

1 calculate  $\lambda = \text{Min}_{C \in S_i} (|K_{C \setminus i}| - |K_C|)$ ;
2 define set  $\mathcal{C}_1$ ;
3 for each  $C \in S_i$  do
4     if  $|K_{C \setminus i}| - |K_C| == \lambda$  then
5          $\mathcal{C}_1 \leftarrow C$ 
6     end
    /* metric is the increase of CCCs due to  $i'$  departing */
7 end
8 if  $|\mathcal{C}_1| == 1$  then
9     choose cluster  $C$ ;
10    break;
11 else
12    calculate  $\mu = \text{Max}_{C \in \mathcal{C}_1} (V_{H_C} \cap V_i)$ ;
13    define set  $\mathcal{C}_2$ ;
14    for each  $C \in \mathcal{C}_1$  do
15        if  $V_{H_C} \cap V_i == \mu$  then
16             $\mathcal{C}_2 \leftarrow C$ 
17        end
        /* metric is the number of common channels between  $i$ 
        and cluster head of demanding cluster */
18    end
19    if  $|\mathcal{C}_2| == 1$  then
20        choose cluster  $C$ ;
21        break;
22    else
23        calculate  $\nu = \min_{C \in \mathcal{C}_2} |C|$ ;
24        define set  $\mathcal{C}_3$ ;
25        for each  $C \in \mathcal{C}_2$  do
26            if  $|C| == \nu$  then
27                 $\mathcal{C}_3 \leftarrow C$ 
28            end
            /* metric is cluster size */
29        end
30        if  $|\mathcal{C}_3| == 1$  then
31            choose cluster  $C$ ;
32            break;
33        else
34            choose the  $C \in \mathcal{C}_3$ , which has the highest ID;
35        end
36    end
37 end
38 Node  $i$  notifies  $H_C$  which is cluster head of  $C$  of its affiliation decision, cluster  $C$ 
then accepts it.

```

---

- $\mathcal{P}$ , the set of players of the game, which are the debatable nodes after phase I in our clustering problem.
- $\mathcal{R} = \cup S_i, i \in \mathcal{P}$ , denotes the set of resources for players to choose,  $S_i$  is the set of claiming clusters of node  $i$ .  $\mathcal{R}$  is the set of claiming clusters after phase I in our clustering problem.
- As to the strategy space  $\sum_i$  of player  $i \in \mathcal{N}$ , there is  $\sum_i \subseteq 2^{[S_i]}$ . As one debatable node is supposed to choose one claiming cluster in our problem, thus only one resource is allocated for  $i$ , accordingly this congestion game is a singleton game.
- The utility (cost) function  $f(C)$  of resource  $C \in \mathcal{R}$ , (or to say  $f(r)$  of resource  $r \in \mathcal{R}$ ) is  $\Delta|K_C^i|$  which represents the decrement of CCCs in cluster  $C$  caused by debatable node  $i$  joining in it. As to cluster  $C \in S_i$ , the decrement of CCCs caused by enrolment of debatable nodes is  $\sum_{i: C \in S_i, i \rightarrow C} \Delta|K_C^i|$ .  $i \rightarrow C$  means  $i$  joins in cluster  $C$ . Obviously this function is non-decreasing with respect to the number of nodes joining in cluster  $C$ .

The utility function is not purely decided by the number of players (debatable nodes) as that in a canonical congestion game, as in this game the channel availability on debatable nodes is different. Given two same sized groups of debatable nodes, when the nodes are not completely the same (neither are the channel availabilities on these nodes), the cost happened on one claiming cluster could be different if the two groups of debatable nodes join in that cluster respectively. Hence, this game is called player specific. In this game, every player greedily updates its strategy (choosing one claiming cluster to join) if joining in a different claiming cluster minimizes the decrement of CCCs  $\sum_{i: C \in S_i} \Delta|K_C^i|$ , player's strategy in the game is exactly the same with the behaviour of debatable node in membership clarification phase, which is described by Algorithm 4.

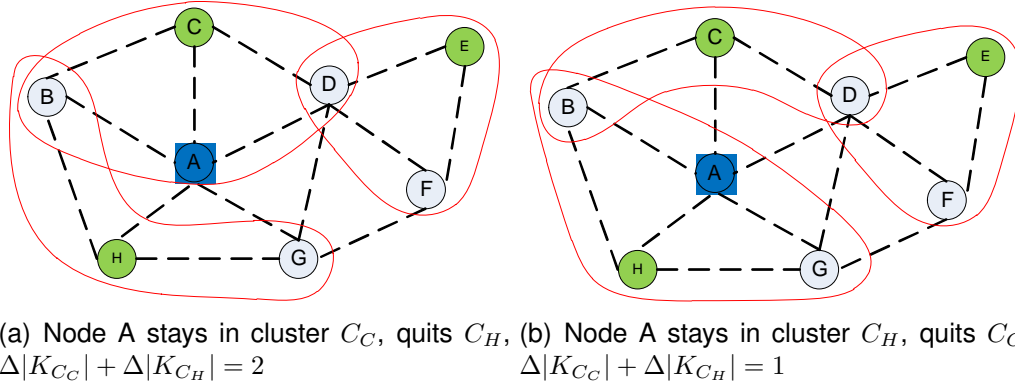
As to singleton congestion game, there exists pure equilibria which can be reached with best response update [13]. As introduced in Section 2.2.2, the number of steps before convergence is  $\mathcal{O}(n^2 * m)$ , where  $n$  is the number of players, and  $m$  is the number of resources. In our problem, the players are the debatable nodes, and the resources are the claiming clusters (or clusters heads). Thus the number of steps is  $\mathcal{O}(N^3)$ . In fact, the actual number of steps is much smaller than  $N^3$ . The amount of debatable nodes is illustrated in Figure 4.5, which is between 10% to 80% of the total number of CR nodes in the network. The number of clusters heads, as discussed in the part of cluster size in Section 4.4.1, is decided by the network density and the CR node's transmission range. Only a small part of the CR nodes become cluster heads, as in the example shown in Figure 4.4, the number of clusters is only 3.4% to 20% of the total number of CR nodes.

### Distributed Fast Algorithm (DFA)

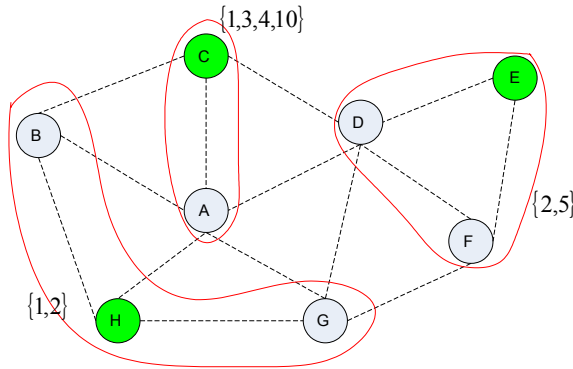
The convergence speed of DGA is large recalling that the number of steps is of  $\mathcal{O}(N^3)$ . Here we propose a faster algorithm DFA which is especially suitable for CRN where channel availability change dynamically and re-clustering is necessary. In DFA, each debatable node executes only one iteration of Algorithm 4 (by setting 'the current value' in Line 14 to zero). Every cluster includes all its debatable nodes, thus the membership

is static and debatable nodes can make decisions simultaneously without considering the change of membership of its claiming clusters.

For example, in Figure 4.5, node  $A$ 's claiming clusters are cluster  $C_C, C_H \in S_A$ , their members are  $\{A, B, C, D\}$  and  $\{A, B, H, G\}$  respectively. The two possible strategies of node  $A$ 's clarification is illustrated in Figure 4.8. In Figure 4.8(a), node  $A$  staying in  $C_C$  and leaving  $C_H$  brings 2 more CCC into  $S_A$ , which is more than that brought by another strategy showed in 4.8(b). After the decisions made similarly by the other debatable nodes  $B$  and  $D$ , the final clusters formed are shown in Figure 4.9.



**Figure 4.8** Membership clarification: possible cluster formations decided by node  $A$ 's different choices



**Figure 4.9** Final formation of clusters, common channels for each cluster is shown.

## 4.5 Centralized Clustering Scheme

The centralized clustering scheme aims to form clusters with desired sizes, meanwhile the total number of common channels of all clusters is maximized. In the following, we refer this problem as *centralized clustering* for short, and the problem definition is as follows,

**DEFINITION 3:** *Centralized clustering in CRN.*

*Given a cognitive radio network  $\mathcal{N}$  where nodes are indexed from 1 to  $N$  sequentially. Based on certain correlation, certain secondary users constitute one cluster  $C$ .  $1 \leq |C| \leq k$  where  $|C|$  is size of cluster  $C$  and  $k$  is positive integer. We name the collection of such clusters as  $\mathcal{S} = \{C_1, C_2, \dots, C_{|\mathcal{S}|}\}$  (the subscript  $i$  is the unique index of cluster in  $\mathcal{S}$ , not the ID of cluster head of relevant cluster),  $\mathcal{S}$  has following properties:  $\bigcup_{1 \leq i \leq l} C_i = N$  and  $V_{C_i} \neq \emptyset$  for any  $i$  which satisfies  $1 \leq i \leq l$ .*

*Following condition distinguish the centralized clustering problem discussed in this thesis. The number of common channels is denoted as  $f$  which is  $|V_C|$  if  $|V_C| > 1$ , and  $f = 0$  if  $|V_C| = 1$ . The question of this problem is to find a subcollection  $\mathcal{S}' \subseteq \mathcal{S}$ , so that  $\bigcup_{C_j \in \mathcal{S}'} C_j = N$ , and  $C'_j \cap C_j = \emptyset$  for  $C'_j, C_j \in \mathcal{S}'$ , so that  $\sum_{C \in \mathcal{S}'} f$  is maximized. The decision version of centralized clustering in CRN is to ask whether exist  $\mathcal{S}' \subseteq \mathcal{S}$ , so that  $\sum_{C \in \mathcal{S}'} f \geq \lambda$  where  $\lambda$  is a real number.*

### Complexity of Clustering Problem

In the following part of this section, we will discuss the complexity of centralized clustering problem and provide a centralized solution for it. We put the definition of weighted k-set packing problem here as it will be used in the analysis on the complexity of our problem.

**DEFINITION 4:** *Weighted k-set packing.*

*Given a set  $\mathcal{G}$  which contains finite number of positive integers, and a collection of set  $\mathcal{Q} = \{s_1, s_2, \dots, s_m\}$ , where for each element  $s_r, 1 \leq r \leq m$ , there is  $s_r \subseteq \mathcal{G}, 1 \leq |s_r| \leq k$ , and  $s_r$  has an associated weight which is positive real number. The question is whether exists a collection  $\mathcal{S} \subseteq \mathcal{Q}$ , where  $\mathcal{S}$  contains only disjoint sets and the total weight of sets in  $\mathcal{S}$  is greater than  $\lambda$ . Weighted k-set packing is NP-hard when  $k \geq 3$ . [59]*

**THEOREM 4.5.1:** *CRN clustering problem is NP-hard, when the maximum size of clusters  $k \geq 3$ .*

**Proof.** To see centralized clustering problem is NP-hard, we reduce the NP-hard problem *weighted k-set packing* to it.

To complete the reduction, we need to conduct following two steps:

- step 1: Show there exists a polynomial algorithm  $\sigma$ , by which any instance (e.g.,  $\mathcal{S}$ ) of a weighted k-set packing can be transformed to instance  $\sigma(\mathcal{S})$  for centralized clustering.
- step 2: Show that  $\mathcal{S}$  is a yes instance of weighted k-set packing if and only if  $\sigma(\mathcal{S})$  is an yes instance for CRN clustering problem.



We continue using the notation introduced in problem definition. Let set  $\mathcal{G}$  contains  $N$  positive integer numbers which are indexed from 1 to  $N$  sequentially. Assume one instance  $\mathcal{S}$  of weighted k-set packing is a collection of disjoint sets  $\mathcal{Q} = \{s_1, s_2, \dots, s_m\}$ , each set is composed by certain amount of elements in  $\mathcal{G}$ .  $\omega$  indicates the weight for each set  $s$ ,  $\omega : \mathcal{S} \rightarrow \mathbb{Z}^+$ .

The polynomial algorithm  $\sigma$  consists of three transformations.

- In the first transformation, for each set  $s_i$  of instance  $\mathcal{S}$ , the elements are duplicated, for instance, given  $s_i = \{1, 4, 6\}$ , the dummy set  $s'_i$  is  $\{1, 1, 4, 4, 6, 6\}$ . By doing this, we obtain the dummy sets and constitute the dummy instance  $\mathcal{S}'$  based on  $\mathcal{S}$ . The purpose of this transformation is to eliminate the single element set in  $\mathcal{S}$ . The weight of set is unchanged after this transformation, i.e.,  $\omega(s_i) = \omega(s'_i)$ . After this transformation, there is no set with only one element. This transformation requires  $\sum_{s_i \in \mathcal{S}} |s_i|$  steps.
- In the immediate following second transformation, we transform the dummy instance  $\mathcal{S}'$  to an instance for CRN clustering problem. Given an instance  $\mathcal{S}'$ , we retrieve all the elements which appear in it, and map each of those elements into one CR node, i.e., each integer corresponds to one CR node, particularly, that integer becomes the CR node's ID. As to duplicated elements, we also map them into a CR node, thus there exist CR nodes with the same ID. As a result, these CR nodes constitute a collection of CR nodes, but note that they have not constituted one CRN yet as there are not connections drawn among them. Connections in CRN under this context is decided by physical conditions, which says the corresponding CR nodes have common channels and close enough to communicate with each other. This transformation requires  $2 \cdot \sum_{s_i \in \mathcal{S}} |s_i|$  steps.
- Mere isolated nodes don't constitute network, thus we add connections in CRN based on the sets in  $\mathcal{S}'$  sequentially. For each set  $s' \in \mathcal{S}'$ , we add connection between two CR nodes if their IDs are in  $s'$ . There is also connection between the CR node and its dummy node. The number of common channels of the CR nodes equals to the weight of set  $s'$ . No connection exists between two CR nodes if their IDs don't appear in one set in  $\mathcal{Q}$ . Afterwards, the CR node whose ID doesn't appear in any set in  $\mathcal{S}'$  becomes single node clusters, according to the definition of clustering problem in CRN, the number of common channels is 0. This procedure requires  $\sum_{s'_i \in \mathcal{S}'} |s'_i|$  steps to map sets in  $\mathcal{S}'$  into CRN and connections, and at most  $N$  steps to complement the single node clusters in CRN.

The number of common channels of cluster  $f$  is non-decreasing function of cluster size, while, the weight of set in weighted k-set packing problem doesn't have this property. In weighted k-set packing, the weight of a set with smaller size could be larger than a set with more elements. But this difference doesn't hinder the transformation and we use an example to explain. Assume two sets in  $\mathcal{S}$  are  $s_1 = \{1, 2\}$  and  $s_2 = \{1, 2, 3, 4\}$ , their weights are 3 and 5 respectively. Their dummy sets are  $s'_1 = \{1, 1, 2, 2\}$  and  $s'_2 = \{1, 1, 2, 2, 3, 3, 4, 4\}$  and their new weights are 3 and 5 as before. The connections mapped to CRN are contradictory to reality, as the number of common channels of CR node group  $\{1, 1, 2, 2\}$  can only be smaller than that of  $\{1, 1, 2, 2, 3, 3, 4, 4\}$ . We let this contradiction in the process of mapping happen because it will be eliminated later: no matter one instance  $\mathcal{S}$  for weighted k-set packing results in *yes* or *no*, at most only one set of  $s'_1$  and  $s'_2$  is chosen, then

we can safely delete the connections based on the deleted set from the CRN, and the contradiction is eliminated.

We have crossed the hurdle of finding one polynomial algorithm  $\sigma$  to transform instance of weighted k-set packing to an instance for clustering in CRN. Now we look into the step 2 in reduction.

When the instance  $\mathcal{S}$  for weighted k-set packing contains one solution, i.e., there is a group of sets in  $\mathcal{S}$ , whose sum weights is greater than  $\lambda$ , then in the CRN which is mapped from  $\mathcal{S}'$ , the sum number of common channels of the clusters which correspond to the selected sets in  $\mathcal{S}$  and  $\mathcal{S}'$ , is greater than  $\lambda$ .

When there is no solution out of set  $\mathcal{G}$  for weighted k-set packing, let's assume the maximum sum of weights of all instances is  $\sum_{s_i \in \mathcal{S}} \omega(s_i) = \delta < \lambda$ . The dummy set of each  $s_i \in \mathcal{S}$  is mapped to cluster of CR nodes. Definition of CRN clustering regulates that the number of common channels is 0 when the cluster has only one node. As to  $|s_i| = 1$ , the mapped cluster has two nodes, with one of them is the dummy CR node. Then number of common channels is on longer 0 but equals to the weight of corresponding set  $s_i$ . Then the sum number of common channels of the clusters in CRN is  $\delta < \lambda$ , thus, there is no clustering solution for the mapped CRN.

After proving weighted k-set packing can be reduced to centralized clustering in CRN, we can say the latter problem is NP-hard.  $\square$

### 4.5.1 Centralized Optimization

As there is no efficient algorithm to solve clustering problem in CRN, we adopt binary linear programming to solve the problem. Note that binary linear programming is in NP-complete.

Given a CRN  $N$  and desired cluster size  $\delta$ , we get a collection of clusters  $G$ , where clusters satisfy the conditions of clusters in Section 4.3, and the sizes of clusters are  $1, 2, \dots, \delta$ . Note that the legitimate clusters include the singleton ones, which guarantees the partition of any network is feasible. With  $n = |N|$ ,  $g = |G|$ , we construct a  $g \times n$  matrix  $Q_{g \times n}$ . Each element  $q_{ij} = |k_{C_i}|$  if  $j \in C_i$ , and  $q_{ij} = 0$  if  $j \notin C_i$ . In other words, Each non-zero element  $q_{ij}$  denotes the number of common channel of the cluster  $i$  where node  $j$  resides.

$$Q = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & \dots & j & \dots & n-1 & n \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ \vdots \\ g \end{matrix} & \begin{pmatrix} k_1 & k_1 & 0 & \dots & \dots & \dots & 0 & 0 \\ k_2 & 0 & k_2 & \dots & \dots & \dots & 0 & 0 \\ \vdots & \vdots & & \vdots & & \vdots & & \\ 0 & k_i & 0 & \dots & \dots & \dots & k_i & 0 \\ \vdots & \vdots & & \vdots & \dots & \dots & & \vdots \\ \vdots & 0 & 0 & \dots & \dots & \dots & k_{i'} & 0 \\ k_g & & & \vdots & \dots & \dots & & \vdots \end{pmatrix} \end{matrix}$$

**Figure 4.10** Matrix  $Q$ ,

We also have one  $G \times N$  binary matrix  $X$ , the element of the matrix is binary variable  $x_{ij}$ ,  $i = 1, \dots, G, j = 1, \dots, N$ .  $x_{ij} = 1$  denotes cluster  $i$  is one partition chosen by the clustering scheme,  $x_{ij} = 0$  means this partition is not adopted.

$$\begin{aligned} \min_{x_{ij}} \quad & \sum_{j=1}^g \sum_{i=1}^n (-x_{ij}q_{ij} + w_i * \text{cost}(\delta)) \\ \text{subject to} \quad & \sum_{i=1}^g x_{ij} = 1, j = 1, \dots, n \\ & \sum_{j=1}^n x_{ij} = \delta * (1 - w_i), j = 1, \dots, g \\ & x_{ij} \text{ and } w_j \text{ are binary variables.} \\ & i \in \{1, 2, \dots, g\}, j \in \{1, 2, \dots, n\} \end{aligned}$$

As the resultant clusters are with certain desired sizes, we try to maximize the sum of products of cluster size and number of common channels in the objective function. The second item of objective function denotes the *punishment* for choosing the cluster whose size is not  $\delta$ . We design  $\text{cost}(\delta)$  as follows,

$$\text{cost}(\delta) = \begin{cases} 0 & \text{if } |C_i| = \delta \\ \alpha_1 & \text{if } |C_i| = \delta - 1 \\ \alpha_2 & \text{if } |C_i| = \delta - 2 \\ \dots & \end{cases}$$

where  $\alpha_i > 0$  and increases with  $i$  getting larger. Choice of  $\alpha_i$  affects the resultant clusters.

Constraint 3.2 restricts that node  $j$  resides in exactly one cluster. In constraint 3.3,  $w_j$  is an auxiliary binary variable,  $w_j = 0$  denotes cluster  $j$  is chosen in the solution. When  $w_i = 1$ ,  $i$ th cluster is not chosen according to constrain 3.3, then the objective function suffers certain *loss*.

This is a linear binary optimization problem, which is solved by function *bintprog* provided in MATLAB.

Take CRN in Figure 4.1 for example. As  $|N| = 8$ , we let the cluster size  $\delta$  to be either 2 or 3 so that the partition of network is possible. A collection of clusters  $G$  is built, where the clusters satisfy the conditions for cluster in Section 4.3 and the sizes of clusters are 1, 2 and 3.  $G = \{\{A\}, \{B\}, \dots, \{B, C\}, \{B, A\}, \{B, H\}, \dots, \{B, A, C\}, \{B, H, C\}, \{A, D, C\}, \dots\}$ , and  $G = 38$ .

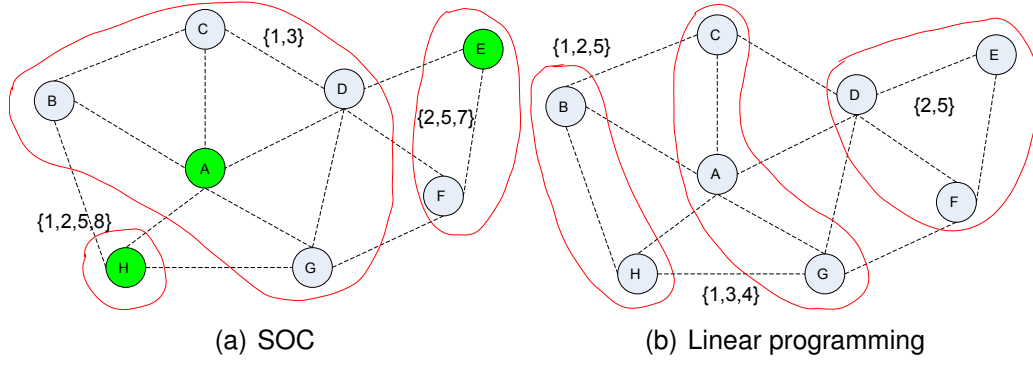
The clustering result of binary linear programming is  $\{\{D, E, F\}, \{A, C, G\}, \{H, G\}\}$ , the number of common channels is  $\{2, 3, 3\}$ . The solution from ROSS is  $\{\{B, H, G\}, \{C, A\}, \{D, E, F\}\}$ , the number of common channels is  $\{2, 4, 2\}$ . By applying SOC, the clustering result is  $\{A, B, C, D, G\}, \{E, F\}, \{H\}$ .

The final clusterings of the example CRN by SOC and linear programming are as follows,

As to the average number of common channel, the results of ROSS, LP and SOC are 2.66, 2.66, and 3 respectively. Note there is one singleton cluster  $C_H$  generated. When the singleton cluster  $\{E\}$  is excluded, the average number of common channels of SOC drops to 2.5.

## 4.6 Performance Evaluation

In this section, we evaluate the performances of all the variants of ROSS, i.e., ROSS-DGA and ROSS-DFA, and that with cluster size control functionalities. The latter is referred



**Figure 4.11** Final clustering of the example CRN

as ROSS-x-DGA/ROSS-x-DFA, where  $x$  is the desired cluster size. Note that ROSS generates clusters on based of cluster heads' neighbourhood, thus the desired cluster size is smaller than the average neighbourhood size. We choose SOC as comparison scheme. To the best of our knowledge, SOC [89] is the only work emphasizing on the robustness of clustering structure from all previous work on clustering in CRN. The authors of [89] compared SOC with other schemes based on the average number of common channels within each cluster, on which SOC outperforms other schemes by 50%-100%. This is because the schemes except for SOC are designed either for ad hoc network without consideration of channel availability [29], or for CRN but just considering basic connection among CR nodes [157]. Hence, we only compare the two versions of our scheme ROSS-DGA and ROSS-DFA with SOC to show the merits of ROSS, and also compare with the centralized scheme to see the gap with the global optima. We will investigate the following metrics:

- *Average number of common channels per un-singleton cluster.* This metric shows the robustness of the current un-singleton clusters. Note that this metric along is biased and misleading, because the CR nodes with more channels could be formulated as singleton clusters. This happens in SOC solution, whose objective is to improve the average number of common channel over *all* clusters, i.e., including the singleton clusters, thus many CR nodes with more channels are turned into clusters.
- *Number of unclustered CRs with moderate and vigorous intensity of primary users' activities.* This is a straight forward metric on the robustness of clusters. With this metric, we investigate the performance of different schemes under different availability of spectrum in the CRN.
- *Cluster sizes.* Specific clusters size is pursued in many applications due to energy preservation and the system design [74]. We will present the distribution of CRs residing in the formed clusters, and the number of generated clusters through multiple simulations.
- *Amount of control messages involved.*

The simulation is conducted with C++. Certain number of CRs and PUs are deployed within a square whose edge is 100 m. We adopt the round disk model to simulate transmission. Transmission ranges of CR and PU are 10 and 30 respectively. As to CRs, the CR node residing within another CR node's transmission range is seen as neighbour of

that CR node. If CR node locating within one PU's transmission range, the CR node is not allowed to use the channel which is being used by that PR. The number of licensed channels in simulation is 10, each PU is operating on each channel with probability of 50%.

There are two parts of simulation, in the first part, we investigate the gap between the distributed schemes with the centralized scheme. As there is no polynomial time solution available to solve the centralized problem, we adopt a small network to compare the performances of the ROSS, SOC and the centralized solution. In the second part, we increase the network scale and change network density to thoroughly compare the two distributed schemes.

### 4.6.1 Centralized Schemes vs. Decentralized Schemes

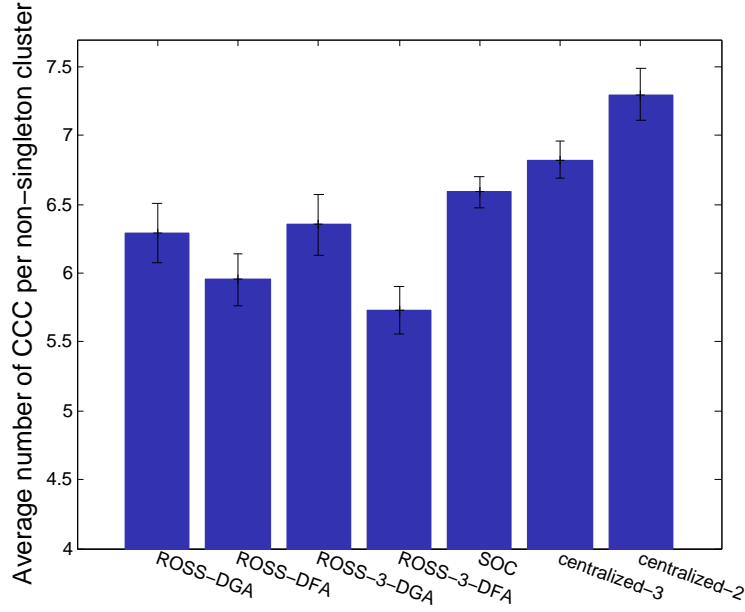
Coinciding with the system model in Section 4.3, 10 primary users and 20 CR users are dropped randomly (with uniform distribution) within some area of size  $A^2$ , where we set the transmission ranges of primary and CR users to  $A/3$ . There are  $P = 10$  available channels. With this setting, the average number of neighbours of one CR user is around 5. Each primary user randomly occupies one channel, and CR users are assumed to be able to sense the existence of primary users and identify available channels. When clustering scheme is executed, around 7 channels are available on each CR node. All primary and CR users are assumed to be static during the process of clustering. Performance results are averaged over 50 randomly generated topologies with equal parameters. The desired cluster size is 3. The confidence interval shown in figure corresponds to 95% confidence level.

#### Number of Common Control Channels in Non-singleton Clusters

Figure 4.12 shows the average number of common channel of non-singleton clusters, as the singleton clusters (in other words unclustered nodes) don't execute any functionalities of clusters, which are has be discussed in Section 4.1. From Figure 4.12, we can see centralized schemes outperform distributed schemes. SOC achieves the most number of CCC than all the variants of ROSS. The reason is, SOC is liable to group the neighbouring CRs which share the most abundant spectrum together, no matter how many of them are, thus the number of CCC of the formed clusters is higher. But this method leaves considerable number of CRs which have less spectrum not in any clusters. As to the variants of ROSS, the procedure of debatable nodes greedily looking for better affiliation improves the number of CCC, thus ROSS-DGA with and without size control outperform ROSS-DFA and its size control version respectively. We also notice that, the size control feature doesn't affect the number of CCC for both ROSS-DGA and ROSS-DFA. This is because the desired cluster size happens to be the average size of clusters generated by ROSS-DGA and ROSS-DFA, then the size control functionality doesn't play effect to increase the number of CCCs.

#### Survival Rate of Clusters with Increasing Primary Users

With the number of PRs in CRN increases, or their operation becomes more intensive, some CCCs will no longer be available. If there is no common control channels available



**Figure 4.12** Number of common channels for non-singleton clusters, the numbers in the names of schemes annotate the desired cluster size.

any more because of the new added PRs, the cluster is regarded as destroyed and the former cluster member CRs become unclustered CRs or in other words singleton clusters.

We investigate the robustness of the formed clusters when they co-exist with primary users whose intensity of activities are varying. After the clusters are formed under the influence of the initial 10 PRs, extra 100 PRs are sequentially added into the network. The transmission range and channel occupancy of the new PU is the same with the previous ones, i.e., transmission range is  $A/3$ , and one channel out of 10 is randomly chosen to operate. Figure 4.13 shows the number of unclustered CRs with the increase of PRs, which indicates the vulnerability of clusters under varying surrounding of licensed spectrum.

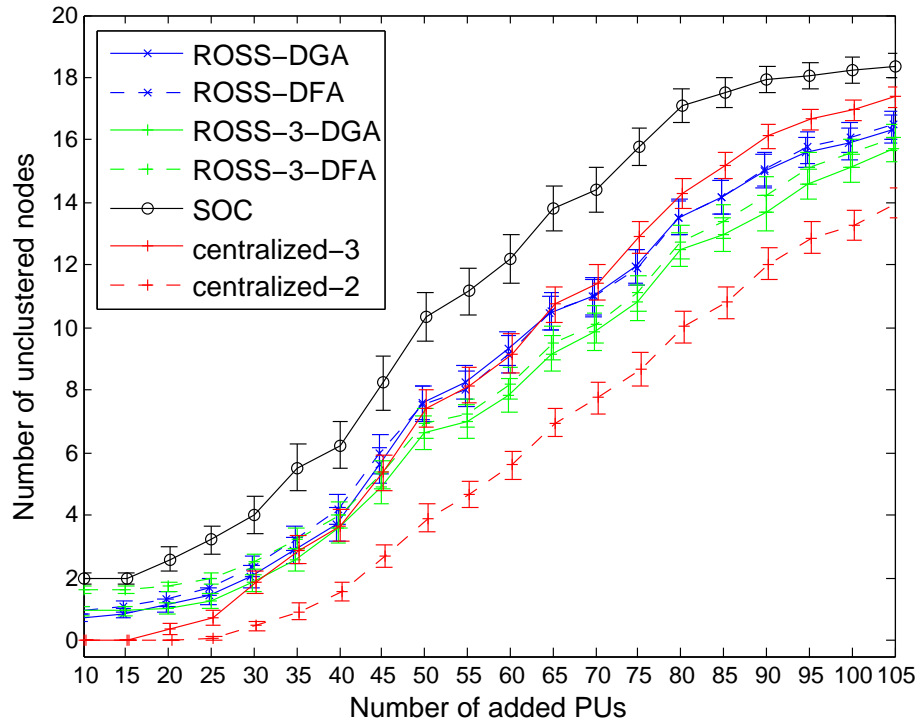
We obtain three conclusions corresponding to three comparisons shown in this figure,

- Centralized scheme with cluster size of 2 produces the most robust clusters, and SOC results in the most vulnerable clusters. Centralized scheme with cluster size of 3 achieves less unclustered CRs than variants of ROSS when the number of PRs is 10~30, when number of PUs is 30~60, same amount of unclustered CRs are generated with variants of ROSS. When there are 75 and more new PRs, centralized scheme with cluster size of 3 results in more unclustered CR nodes than variants of ROSS. Size control feature makes both ROSS-DGA and ROSS-DFA outperform themselves without size control when number of new PRs is greater than 50.

The reason that centralized scheme with cluster size of 3 does not completely excel variants of ROSS is due to the favourable achievement of it: the uniformly sized clusters. As distributed schemes, variants of ROSS generate considerable amount of smaller clusters which are more likely to survive when PRs' activities become intense. The comparison on cluster sizes will be given in details in 4.6.1.

- ROSS with size control is better than the other two distributed schemes. The size control decreases the clusters size and makes the clusters more robust when under PRs' activity.

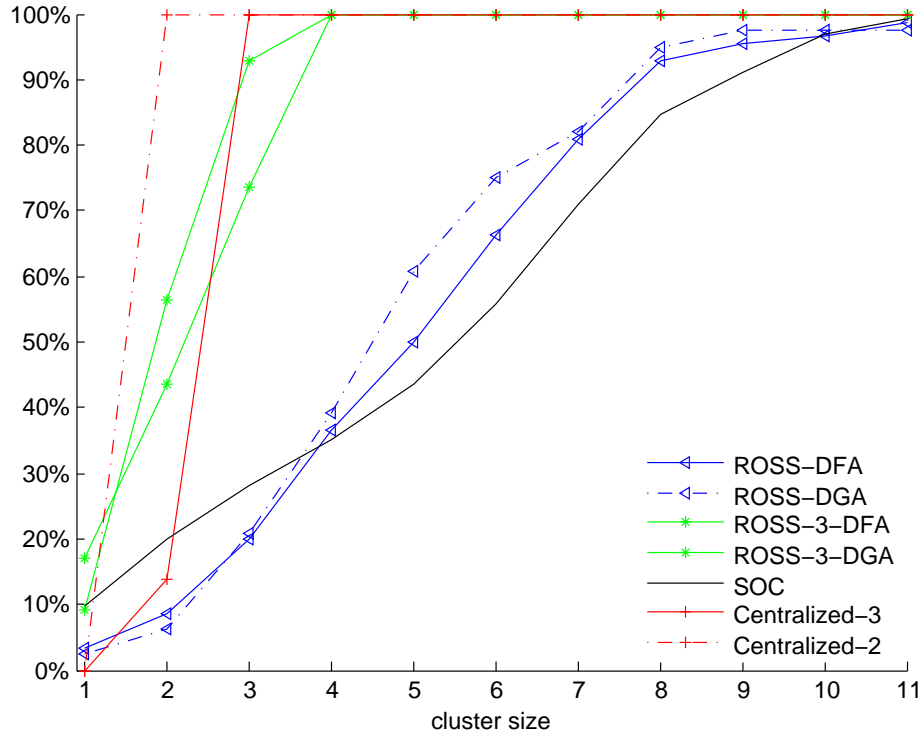
- Greedy algorithm improves survival rate. ROSS-DGA improves the survival rate of ROSS-DFA, so does ROSS-DGA with size control against ROSS-DFA with size control. This comply with the observation on number of CCC in section 4.6.1. As the debatable CRs greedily update their affiliation with demanding clusters, and the metric for updating is the maximum increase of CCCs of the demanding clusters, the average number of common channels is improved (shown in Figure 4.12), then the robustness of clusters is enhanced. Meanwhile, sizes of more clusters become smaller also contributes more robustness.



**Figure 4.13** Number of CRs which are not included in any clusters

### Cluster Size Control

Figure 4.14 shows the number of CRs residing in certain sized clusters. The centralized schemes are able to form clusters which strictly satisfy the requirement on cluster sizes. When the desired size is 2, each generated cluster has two members. When the desired size is 3, in average only 3 CRs are formed into 2 node clusters. When ROSS-3-DFA is applied, most number of CRs are in 3 node clusters, nevertheless, slightly less nodes are found in 2 node and 4 node clusters, there are also considerable number of singleton clusters. ROSS-3-DGA decreases the clusters sizes and results in more 2 node clusters, the second most CRs are found in 3 node clusters. ROSS-DGA and ROSS-DFA generate rather even distribution of nodes with different sizes, whereas SOC results in more CRs unclustered or clusters of large sizes. Figure 4.14 shows distributed clustering schemes are not able to control cluster sizes perfectly, but ROSS-DGA and ROSS-DFA eliminate the clusters whose size diverges largely with the desired one, i.e., single node clusters and clusters with size of 13 and 14. Particularly, size control enable both ROSS-DGA and ROSS-DFA to achieve clusters whose sizes demonstrate certain homogeneity, i.e., cluster sizes vary from 1 to 4. But there are considerable number of single node clusters, which is due to the cluster pruning discussed in section 4.5.



**Figure 4.14** Cumulative distribution of CRs residing in clusters with different sizes, as to ROSS with size control feature, the desired cluster size is 3. The average number of neighbours is 4.8.

### Control Signalling Overhead

As to any variants of ROSS, there are two phases, in the first phase, clusters are formed, in the second phase, cluster membership is decided so that each node only resides in one cluster. Control message exchanges between CR nodes are involved in both phases.

In this section we compare the amount of control messages involved for clustering in different schemes, e.g., centralized scheme, ROC, ROSS-DGA, ROSS-DFA and those with size control feature. In order to highlight the amount of control signalling only for clustering, we omit the control messages used for neighbourhood discovery, which are regarded the same for all schemes, and only compare the number of control messages brought in by the features of the schemes. The control message here refers both broadcast and unicast.

As to variants of ROSS, in the first phase, after each node broadcasts their new knowledge on spectrum robustness, cluster is automatically formed by cluster head which is decided from consensus by comparing the spectrum robustness with neighbours, then the cluster head broadcasts message containing its ID and the available channels in its cluster. As to ROSS with size control feature, there are same amount of cluster heads with ROSS without enabling size control feature, and the cluster head broadcasts the available channels of the pruned cluster. Afterwards in the second phase, membership clarification of debatable nodes is conducted. Debatable node informs the cluster which it going to stay and the cluster head broadcasts message about its new cluster. As to SOC, each node needs to maintain one cluster, the final clusters are formed after three rounds of comparisons and cluster mergers, while as to ROSS, only debatable nodes need to communicate with cluster heads to clarify their membership.



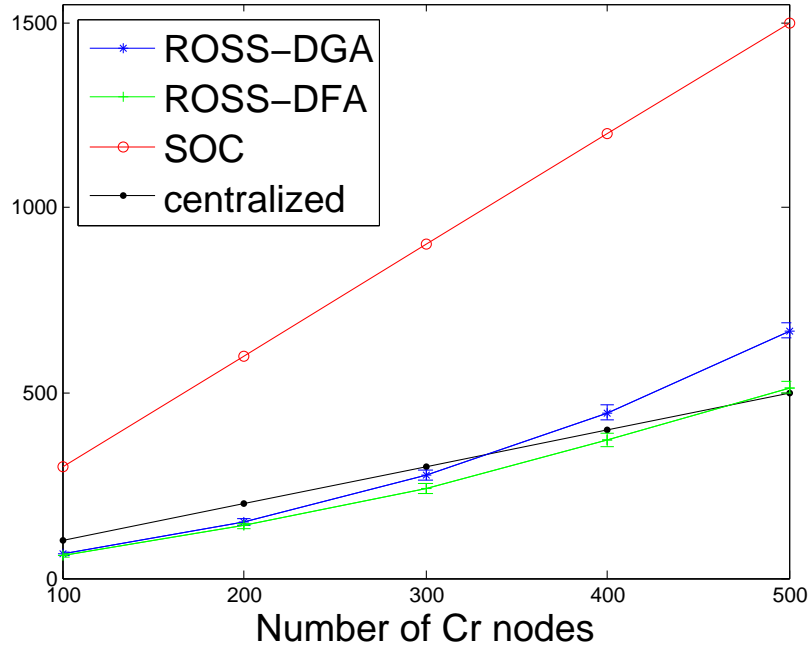
Worst case protocol complexities. We assume that the protocols execute synchronously. We compare the Time Complexity (TC), defined as the number of steps required to perform a protocol operation, and the Communication Complexity (CC), defined as the number of broadcast in performing the operation.

The complexity parameters are the number of nodes  $n$  in network, number of clusters  $h$ .

The quantitative analysis of amount of control overhead and the size of messages are illustrated in Table 4.2,

Scheme	Number of broadcast	Content of message
ROSS-DGA, ROSS-x-DGA	$h+2*m^2c$ (upper bound)	$ID_{H_C}$ and $V_C$ for $h + m^2c$ times, notification to join in one cluster for $m^2c$ times
ROSS-DFA, ROSS-x-DFA	$h + 2m$ (upper bound)	$ID_{H_C}$ and $V_C$ for $h + m$ times, notification to join in one cluster for $m$ times
SOC	$3 * n$	$\{V_i\}, i \in M \subseteq Nb_i$
Centralized	$n$	$\{C\}$

**Table 4.2** Singalling overhead. Notations:  $n$ -number of CR nodes in CRN,  $h$ -number of cluster heads,  $m$ -number of debatable nodes,  $c$ -number of demanding clusters,  $\delta$ -desired cluster size

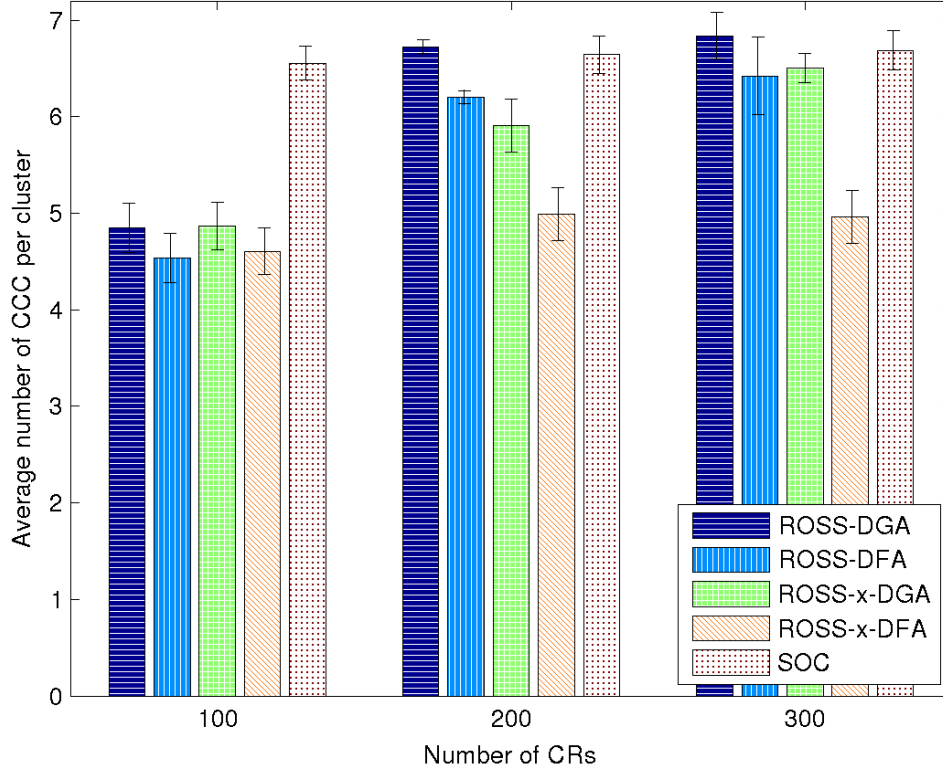


**Figure 4.15** Number of control messages

## 4.6.2 Comparison between Distributed Schemes

In this section we investigate the performances of distributed clustering schemes in CRN with different network scales and densities. The transmission range of CR is  $A/10$ , PR's transmission range is  $A/5$ . The number of PU is 30. The number of CR is 100, 200 and 300, and the average number of neighbours of each CR is 9.5, 20, and 31.

### Number of CCC per Non-singleton Clusters



**Figure 4.16** Number of common channels of non-singleton clusters. As to ROSS with size control feature, we adopt  $x = 6$  when  $N = 100$ ,  $x = 12$  when  $N = 200$ ,  $x = 21$  when  $N = 300$ , which is around  $2/3$  of the number of average neighbours

Figure 4.16 illustrates the average number of CCCs of the non-singleton clusters. It shows when  $N = 100$ , variants of ROSS have 30% less CCCs than SOC, but this gap is decreased significantly when  $N$  is 200 and 300, i.e., when  $N = 300$ , number of CCCs achieved by ROSS variants (except for ROSS-x-DFA) is almost the same with that resulted from SOC.

This means SOC performs better on average number of CCCs per non-singleton clusters when network is sparse<sup>3</sup>. When the network becomes denser, even this metric favours SOC as discussed in the beginning of Section 4.6, ROSS-DGA achieves even more CCCs than SOC, and ROSS-DFA and ROSS-x-DGA increase the number of CCCs visibly.

### Survival Rate of Clusters with Increasing Primary Users

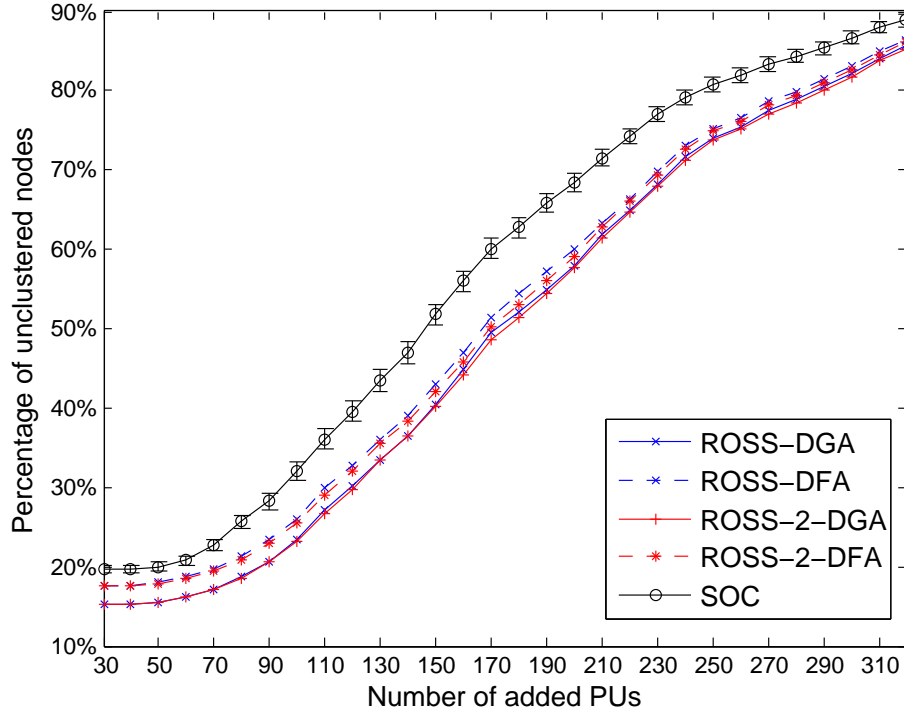
In this part of simulation, we investigate the robustness of clusters by increasing the PRs working on certain channels.

Figure 4.17 shows the increasing trend of singleton clusters, or to say, unclustered nodes, with the increase of PRs. SOC generates around 10 more singleton clusters than the

<sup>3</sup>this is also observed in the evaluation in Section 4.6.1 where  $N = 20$

variants of ROSS, which accounts for 10% of the total CR nodes. The confidence intervals of the variants of ROSS are not shown in the figure as they overlap, and we only show the average values. It can be seen that greedy algorithms result in slightly less singleton clusters than their counterparts.

Figure 4.18 shows a denser CRN where  $N = 300$ . SOC noticeably causes more singleton clusters than ROSS variants, except that ROSS-3-DFA results in more singleton clusters when PRs are few. The reason is that ROSS-3-DFA conducts cluster membership clarification for only once, which causes large number of singleton clusters, while, in ROSS-3-DGA increase the size of smaller clusters through debatable nodes' repeated updates thus drastically decreases the number of singleton clusters.



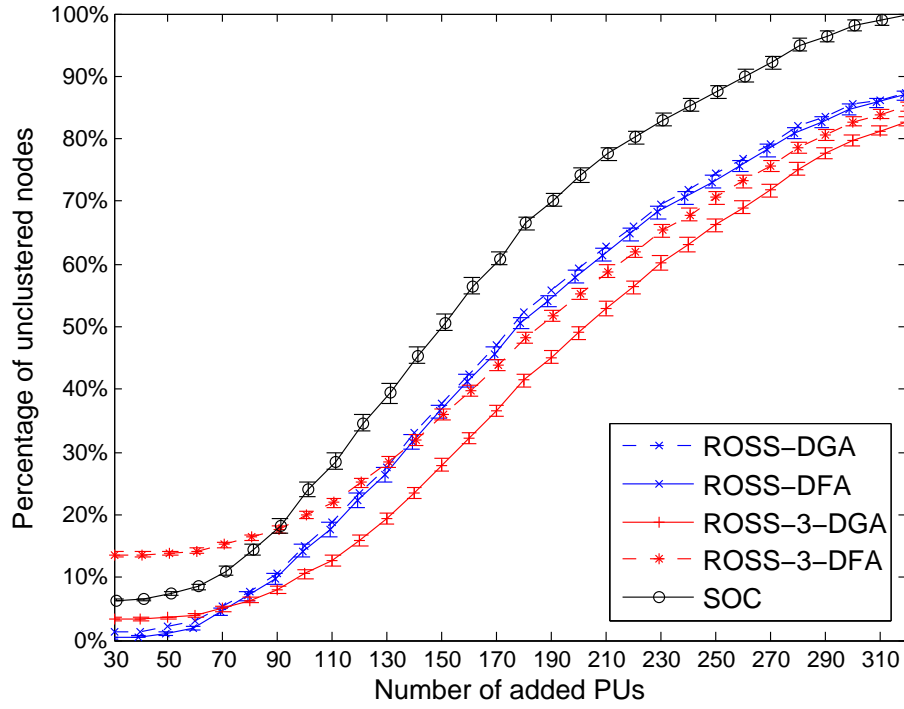
**Figure 4.17** Number of CRs which are not included in any clusters,  $N = 100$

From the Figure 4.17 and 4.18, we can conclude that the greedy versions of ROSS are more robust than their counterpart variants of ROSS. When the network is denser, the improvement on cluster sizes and robustness by the greedy search in the membership clarification phase is more obvious.

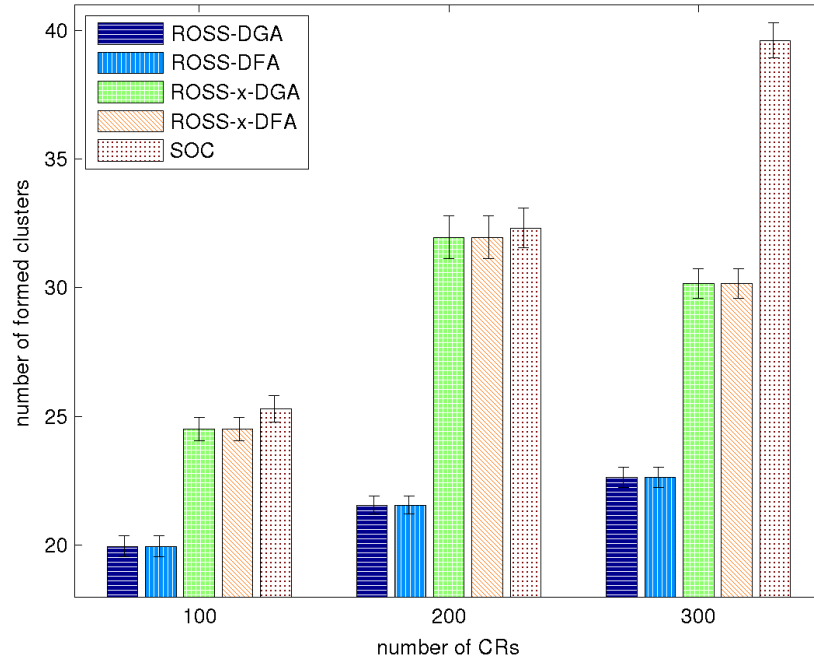
### Cluster Size Control

The number of formed clusters is shown in Fig. 4.19. When the network scales up, the number of formed clusters by ROSS increases by smaller margin. This result coincides with the analysis in Section 4.5, that with ROSS, the number of formed clusters saturates when the network scales. When the network becomes denser, more clusters are generated by SOC compared with ROSS variants. To better understand the distribution of the sizes of formed clusters, we depict the cluster sizes with cumulative distribution. In this group of evaluation, the number of PRs is 30.

Figures 4.20 4.21 4.22 illustrate the distribution of CR nodes which reside in clusters with certain sizes, where the CRN has different densities, i.e., the number of CRs is 100, 200



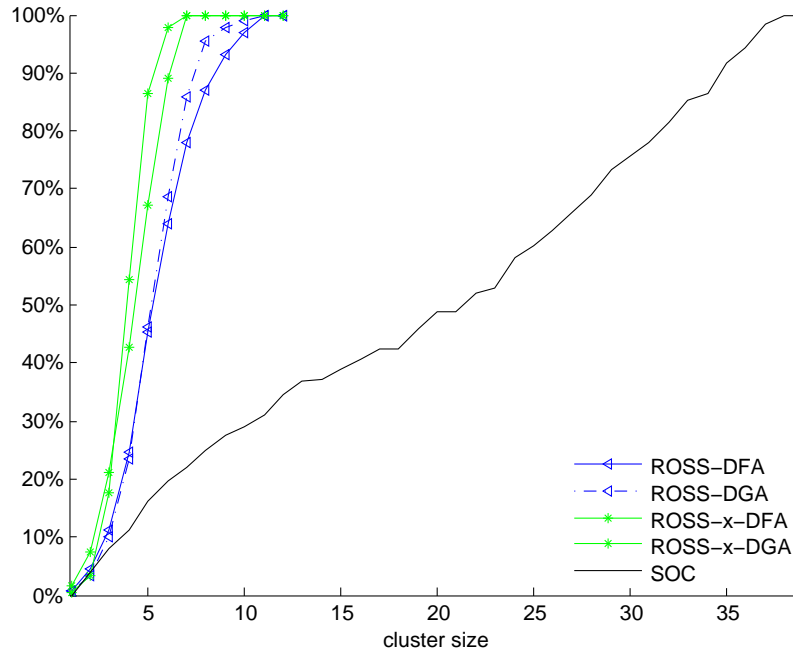
**Figure 4.18** Number of CRs which are not included in any clusters,  $N = 300$



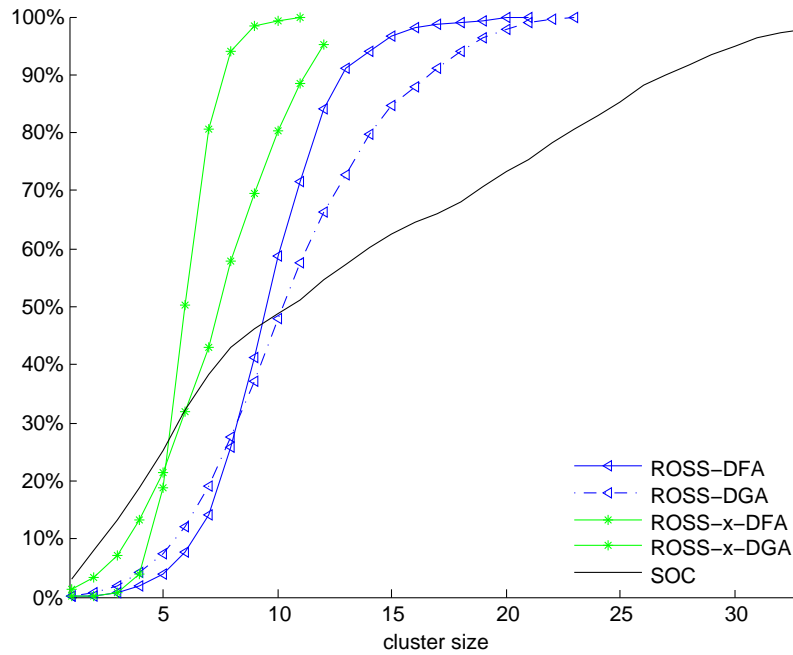
**Figure 4.19** Number of formed clusters

and 300 respectively. The desired cluster sizes are 5, 8 and 13, which are around 45% of the average number of neighbours in respective CRNs. When variants of ROSS are applied, the schemes with size control features generate clusters whose sizes are evenly distributed between 2 and the respective desired size. This means the size control feature effectively restricts the size of clusters, which is able to prevent the clusters from un-governable growing with the increase of network density. The sizes of clusters generated

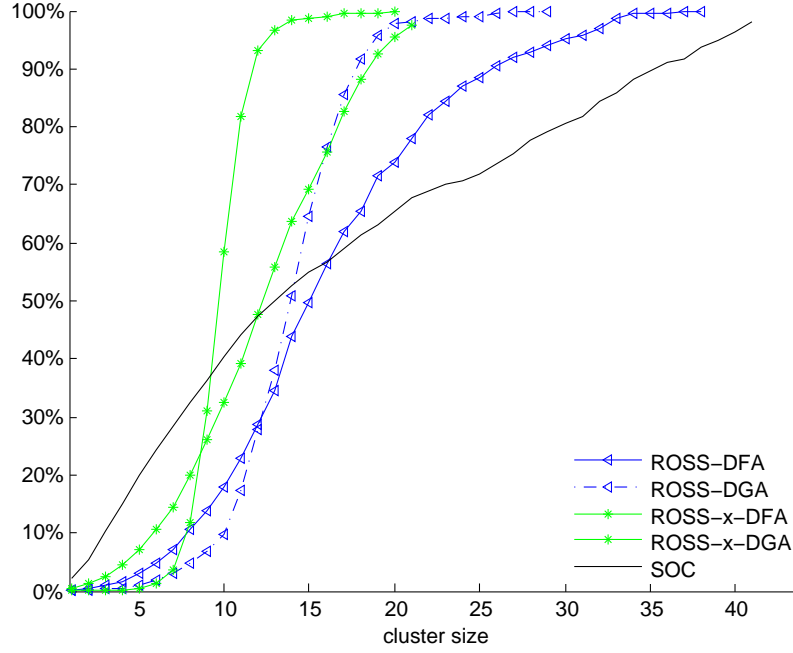
by ROSS-DGA and ROSS-DFA span a wider range than that with feature control feature. But the range where most of the CRs (80% which exclude the CR nodes residing in small and big clusters) resides centres a value, which is roughly the half of the average number of neighbours. Overlooking the three Figures, we can see ROSS-DGA and ROSS-DFA show similar behaviour on cluster sizes. The clusters generated from SOC demonstrate randomness on cluster sizes, which diverge strongly.



**Figure 4.20** Cumulative distribution of CRs residing in clusters with different sizes. Desired cluster size  $x$  is 5. 100 CRs, 30 PRs in network, the average number of neighbours of CR node is 9.5.



**Figure 4.21** Cumulative distribution of CRs residing in clusters with different sizes. Desired cluster size  $x$  is 7. 200 CRs, 30 PRs in network, the average number of neighbours of CR node is 20



**Figure 4.22** Cumulative distribution of CRs residing in clusters with different sizes. Desired cluster size  $x$  is . 300 CRs, 30 PRs, the average number of neighbours is 30

## 4.7 Conclusions

We investigate extensively the robust clustering problem in CRN, which is important to form clusters which maintains unbroken to the greatest extent possible under primary users' activity. We prove the NP hardness of the problem and one distributed and light weighted clustering scheme ROSS-DGA is proposed. The clusters resulted from ROSS-DGA and its faster version ROSS-DFA are less vulnerable compared with other distributed clustering schemes, and demonstrates similar survival rate with centralized scheme under primary users' influence. An light weighted cluster size control mechanism is contained in both ROSS-DGA and ROSS-DFA, which is advantageous for cooperative sensing and network operation with clusters. Furthermore, considerable less control messages are generated when compared with other clustering schemes.

The shortage of ROSS scheme is it doesn't generate big clusters whose sizes exceed the cluster head's neighbourhood. This problem is attributed to fact that ROSS forms clusters on the basis of cluster head's neighbourhood, and doesn't involve interaction with the nodes outside its neighbourhood. In the other way around, forming big cluster which extends out side of cluster head's neighbourhood has very limited applications, because multiple hop communication and coordination are required manage this kind of big clusters.

# 5

## **SPECTRUM AWARE VIRTUAL COORDINATE ASSIGNMENT AND ROUTING IN MULTIHOP COGNITIVE RADIO NETWORK**

### **5.1 Introduction**

Primary users' activity demonstrates different patterns [82], consequently the availability of licensed spectrum exhibits different dynamics accordingly. In certain scenarios the licensed spectrum occupancy stays available for fairly long time, e.g., TV white space [106]. In that case the licensed spectrum occupancy can be seen as static during a long period of time. In other scenarios primary users' states change frequently, but measurements [113, 142] show that the percentage of time that licensed spectrum is occupied at a specific location or during a certain period of time doesn't change, i.e. in city downtown during the work time, the duty cycle of spectrum occupancy by cellular network is stable.

To fully exploit the potential of the secondary spectrum, it is crucial to investigate routing in dynamic spectrum environment. The dynamic availability of spectrum causes frequent break down of links between secondary users, and leads to prevalent topology changes, which makes spectrum aware routing difficult but essential [40].

Recent measurement in [113] shows the spectrum occupancy doesn't have significant spatial correlations between different locations. It follows that licensed spectrum is used by primary users heavily in some areas, whereas in the other areas licensed spectrum is available over longer timespan for secondary users to use. It is obvious to see that a routing path is better to go through the areas where primary users occupation is lower, as this alleviates or avoids the burden to cope with the changing or totally occupied spectrum when forwarding packets potentially with latency requirements. Geographic routing is a

natural choice to realize this geography sensitive routing path. Geographic routing is light weight regarding the determination of next hop, and achieves high scalability in various wireless networks [12]. Merely knowing the geographic locations of its neighbours and the destination, a node is able to locally choose the next hop which has the smallest distance to the destination. However, in CRN dynamic link state renders geographic routing unsuccessful since packets are forwarded to the destination along the shortest path rather than avoiding areas heavily influenced by primary users.

To enable geographic routing in CRN, in this chapter we propose SAViC, spectrum aware virtual coordinates for secondary users in multi-channel multi-hop CRN. The virtual coordinate is independent of real geographic position, and has been proposed to represent the properties of the media like, link quality [21] or hop numbers [38]. Following this line of thought, our proposed virtual coordinate represents the spectrum occupancy of primary users. On top of this, we propose the geographic routing scheme which decides the next hop with Euclidean distance metric, and detours the areas affected by primary users, or cuts through the area with lower spectrum occupancy. With SAViC, geographic routing imposes little computation on deciding the next hop, and requires less communication cost transmitting packet to next hop. As to our knowledge, this is the first work integrating the spectrum usage by primary users into network coordinates in order to support geographic routing in CRN, which carries meanings especially for those resource restricted devices which want to work with licensed frequency band. The remainder of the chapter is organized as follows, after reviewing related work in Section II, system model is introduced in Section III. Assignment of SAViC is explained in Section IV, followed by opportunistic access during transmission in Section V. Section VI gives performance evaluation, concluding remarks are given in the last section.

## 5.2 Related Work

When secondary users are static and primary users' operation activity is known, i.e., primary users occupy a certain channel for long time, or they occupy a channel with fixed probability, then centralized routing schemes for CRN can be designed [112]. But as centralized scheme requires sensing result from each secondary user in the network, thus suffers from any change of channel state of secondary users [10], besides, one centralized controller is needed to calculate the routing path on the basis of collected information from the network [112, 115].

Considerable amount of distributed schemes are proposed to cope with routing in CRN where spectrum state is usually considered to be rapid changing. [37] proposes CAODV (Cognitive Ad-hoc On-demand Distance Vector) and let each CR node explore all channels and store route for each available channel. CAODV requires frequent message exchange between secondary users to maintain the up to date connections on each channel due to PU's activities, which is a burden for secondary user when primary users' activity is intense. [127] improving the DSR scheme (Dynamic Source Routing) by letting RREQ messages record spectrum availability, link quality and congestion possibility along routing paths, but it also suffers from frequently changing channel state.

To cope with the rapid change of channel state, some routing schemes abandon routing table and let the transmitter decide the next hop for each single packet based on spectrum state between transmitter and neighbours. When there is packet to send, secondary user



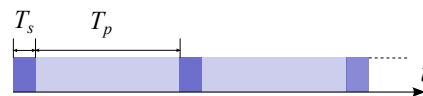
evaluates channel availability based on the statistics of sensing history [98], or the prediction on channel availability in the forthcoming time slot [97], then secondary user chooses the favoured channel and next hop node to send out the packet. Distance to the destination is also a consideration for choosing next hop. Such per-packet forwarding paradigm reacts swiftly on the fast changing channel state, but it requires more powerful computation power on secondary users. Firstly, that scheme produces high computation complexity on determining the channel and next hop node, secondly, specifically designed MAC mechanism and large amount of control messages are needed to coordinate the communication between the sender and the potential next hop nodes, these aspects make it uneconomic for many networks, e.g. wireless sensor networks operating with licensed spectrum [131]. Furthermore, as this kind of routing paradigm emphasizes on finding the maximal transmission opportunity of secondary spectrum, the selection on preferred channel decreases the scope of next hop neighbours, thus it may yield route which does not reach the destination [82, 125].

The solution provided by Chowdhury et al. [46] improves geographic routing in multiple channel CRN by introducing circumventing mechanism, i.e., the source node launches geographic routing on each channel, and every routing path bypasses the next hop, which is chosen based on geographic routing metric (e.g., Euclidean distance), if it is affected by primary user, the routing packet chooses a neighbour of that node free from primary user's affection so as to avoid the primary user affected area. Such routing is conducted on all channels, afterwards a path merge process is undertaken and one path with alternating channel is finally formed with consideration of end to end delay. *SEARCH* adopts routing table and doesn't involve frequent overhead exchanges. As the decision of the next hop is largely decided by the channel availability on the time point of decision, the node chosen as next hop may not be able to work after a short while due to primary user's reappearance. Thus source node needs to periodically launch route request to update the routing path which may have been invalid. As a result, this scheme works well when the primary user's activity is infrequent, but when it goes tense, the frequent invalidity of nodes due to lack of available spectrum seriously deteriorates routing performance.

### 5.3 System Model

We consider a CRN composed with secondary users which are randomly and statically deployed in a plane. There are orthogonal licensed channels denoted by set  $C$ , and secondary user is allowed to use any of them if no primary user is detected on that channel by the secondary user. One common control channel (CCC) in license-exempt band is available for all secondary users to exchange control messages. Only one licensed channel is used for payload transmission. Primary users are static, and they occupancy spectrum in a constant manner, e.g., the percentage of time that they access a certain channel is static in any period of time.

Proactive spectrum sensing is conducted locally and periodically as Figure 5.1 shows.



**Figure 5.1** Sensing duration  $T_s$  and sensing period  $T_p$

The sensing duration  $T_s$  includes both detection time in physical layer and the decision synchronisation time. Sensing period  $T_p$  is the time between two successive sensing durations. If the channel is sensed as busy<sup>1</sup> in sensing duration, which means at least one primary user in the vicinity is in ON state, we say the state of primary user is labelled as ON in the following sensing period  $T_p$ . If not a single primary user is sensed, primary users in vicinity in the following sensing period is said to be in state OFF. Secondary user senses each licensed channel for time  $T_{access}$  with round robin scheduling, and records statistics of ON/OFF states of that channel at its place.

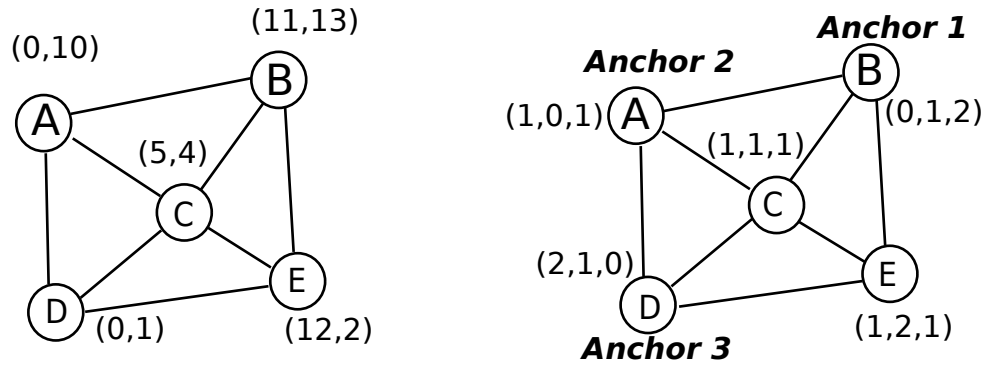
## 5.4 Spectrum Aware Virtual Coordinates

In this section, we firstly introduce how spectrum aware virtual coordinate is assigned, then we introduce two normalized spectrum utilities on secondary user adopted in the virtual coordinate assignment process. One of them is called *normalized spectrum availability* which is on the basis of duty cycle of primary users' absence, the other is called *normalized longest blocking time* which as the name tells, is based on the lengths of time durations that primary users are detected.

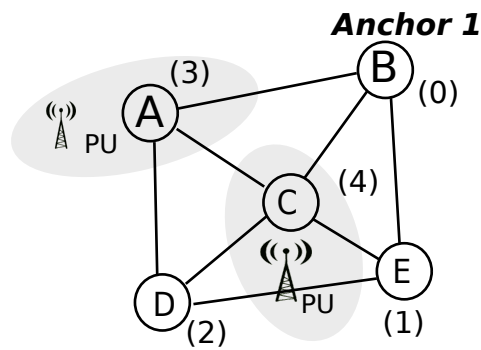
Virtual coordinate has been proposed in sensor and ad hoc networks [21, 38]. In the left part of Figure 5.2, nodes are labelled with physical positions. The right hand side part shows the same network assigned triplet virtual coordinate for each node according to virtual coordinate assignment protocol (VCap) [38], where each element of the coordinate denotes the minimal number of hops away from corresponding anchor. This kind of virtual ordinate belongs to tree based virtual coordinates, and is obtained based on anchors which locate at the edge of network. Anchor messages are broadcast from anchors, each of them contains a counter recording the number of hops travelled. The minimum counter of the arriving anchor messages constitutes the corresponding element of the virtual coordinate on the arrival node. Except for the hope numbers away from certain anchor node, virtual coordinate can also be composed with link quality [21] in wireless sensor networks. The hop based virtual coordinate is independent on actual physical position, but supports greedy geographic routing successfully [21, 38]. For example, when the source-destination is B and D, and Euclidean distance calculated with virtual coordinate is adopted in the routing decision, then the greedy geographic routing achieves the same routing path in both networks:  $B \rightarrow C \rightarrow D$ . This path is one with the shortest traversal distance.

In this chapter, we propose licensed spectrum aware virtual coordinate in CRN, which enables geographic routing to find the path with better available spectrum. Figure 5.3 shows one CRN where secondary users are assigned virtual coordinate according to anchor 1. The transmission opportunity of the nodes locating within primary users' transmission range, e.g., node A and C, is decreased due to sporadic spectrum, as a result, the cost for packet transmission, e.g., transmission delay and energy consumption, is increased. We integrate this obstacle caused by spectrum scarcity to transmission into virtual coordinate.

<sup>1</sup>Concrete sensing techniques are not discussed in this chapter.



**Figure 5.2** Left: nodes with physical locations, Right: nodes with doublet virtual coordinate, each element in the virtual coordinate is the number of hops away from corresponding anchor. Connecting lines denote the communication is possible.



**Figure 5.3** A network under primary users' influence assigned with SAViC, only anchor 1 is adopted. Flooding of anchor messages is not shown.

### 5.4.1 Assign Spectrum Aware Virtual Coordinate

As to SAViC, anchors broadcast anchor messages which flood over the network and result in virtual coordinate for each secondary user. Several anchors are needed to assign unique virtual coordinate for each secondary user. How to select anchors is out of the scope of this chapter. In the following, we introduce how is virtual coordinate decided on each node with respect to anchors.

Each secondary user maintains its virtual coordinate which is one  $r$ -tuple where each element contained corresponds to one anchor, and the tuple length is  $r$ . The elements of virtual coordinate on each node is set as big positive value. An anchor message is generated on the anchor, which contains a *counter* whose value is set as 0. The anchor message is broadcast on control channel. The influence of primary users on a secondary user is quantified as spectrum utility  $\lambda$  on that secondary user. The bigger value indicates heavier spectrum occupation by primary users.  $\lambda$  will be discussed in Section 5.4.2 and 5.4.3.

When a node  $i$  receives an anchor message from the  $t$ th anchor,  $i$  compares the  $t$ th element in its current virtual coordinate with the sum of the  $\lambda$  and *counter*1 which is contained in the arriving anchor message. If the sum is greater than the  $t$ th element in its current virtual coordinate, which indicates that the path traversed by this anchor message exposes to more active primary users, node  $i$  drops the anchor message. If the sum is smaller than the  $t$ th element in its current virtual coordinate, the node set the  $t$ th element as the sum and updates *counter*1 contained in the anchor message before forwarding it. This process is presented as Algorithm 5. The process ceases within a period of time.

**LEMMA 5.4.1:** *As to one anchor, the number of times for each node to forward anchor message is bounded by  $g$ , where  $g$  is the number of one hop neighbours of anchor.*

**LEMMA 5.4.2:** *The counter value of one anchor message is increased when it is forwarded.*

**Proof.** This is proved by the lines 18-20 in Algorithm 5. □

**LEMMA 5.4.3:** *One anchor message affiliated with one anchor accesses the one secondary user at most twice.*

**Proof.** Assume one node is accessed by the same anchor message for two times. When the anchor message arrives at the secondary user, let's say  $i$ , for the first time, there is  $vc > counter + \lambda$ , and both of the  $vc$  and *counter* in the anchor message are updated to be  $counter + \lambda$ . After being forwarded, the anchor message travels at least one other secondary user before arriving  $i$ , and the counter is greater than the counter value when it is forwarded from  $i$  according to Lemma 5.4.2, which means, current counter value is greater than  $\lambda_i$ , and the anchor message is dropped. Thus, this anchor message accesses secondary user  $i$  for at most two times. □

**THEOREM 5.4.1:** *The number of anchor message corresponding to one anchor is bound by  $2 * g * n$ , where  $n$  is the number of secondary users.*

**Proof.** This is proved by Lemma 5.4.1 and 5.4.3. □

According to Theorem 5.4.1, every secondary users obtains virtual coordinate which respects to all anchors after a finite time duration.

---

**Algorithm 5:** Secondary user  $i$  obtains one element  $vc_i$  in its VC with respect to an anchor

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**Input:**  $vc_i = M$ ,  $M$  is one big positive number

---

```

2 if  $i$  is anchor then
4    $vc_i = 0$ ;
6   set  $counter1 = \lambda_i$  in anchor message;
8   broadcast anchor message;
9 end
11 if receive anchor message then
13   if  $counter + \lambda_i \geq vc_i$  then
15     drop anchor message;
16   else
18      $vc_i = counter + \lambda_i$ ;
20     set  $counter = vc_i$  in anchor message;
22     broadcast anchor message;
23   end
24 end

```

---

### 5.4.2 Normalized Spectrum Availability on Secondary User

Based on the statistics of primary user's ON/OFF states in time duration  $T_{assessment}$  which contains multiple  $T_s$ , secondary user  $i$  characterizes the likelihood that one licensed channel, say  $k$ , is available at its own position with *duty cycle*, which is

$$\gamma_i^k = \frac{\Delta_{\text{OFF}}}{\Delta_{\text{OFF}} + \Delta_{\text{ON}}}, \quad (5.1)$$

where  $\Delta_{\text{OFF}}$  is the number of sensing periods when channel  $k$  is sensed as OFF in  $T_{assessment}$ . To implement SAViC whose resultant Euclidean distance between two nodes reflects both influence from primary users and distance in terms of hops, we need to design a normalized quantified spectrum availability  $\lambda_i$ .

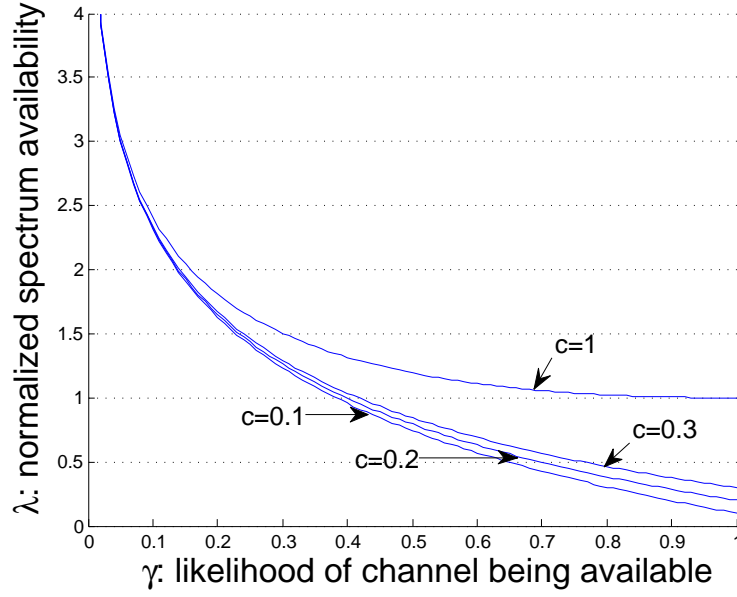
#### Single licensed channel

When there is only one licensed channel in CRN (the superscript of channel  $\lambda$  is omitted), the normalized spectrum availability on node  $i$  is proposed as,

$$\lambda_i = -\ln \gamma_i + c \cdot \gamma_i \quad (5.2)$$

With Formula 5.2, when one anchor message which originates from anchor  $X$  is forwarded from node  $a$  to  $b$  without being dropped, the distance based on virtual coordinate reflects both the spectrum availability and geographic distance in terms of hops between the two nodes. Based on Algorithm 5 and Formula 5.2, the distance in dimension X is,

$$\begin{aligned}
 |x_b - x_a| &= \sum_{i \in P_{(a,b)}} (-\ln \gamma_i + c \cdot \gamma_i) \\
 &= -\ln \left( \prod_{i \in P_{(a,b)}} \gamma_i \right) + c \cdot \sum_{i \in P_{(a,b)}} \gamma_i
 \end{aligned} \quad (5.3)$$



**Figure 5.4** Normalized spectrum availability with respect to the likelihood of spectrum being available one a node

here  $x_a$  and  $x_b$  are virtual coordinates of node  $a$  and  $b$  in dimension  $X$  respectively.  $P_{(a,b]} = (\dots, b)$  denotes the list of nodes after  $a$  and till  $b$ , which forward the same anchor message.

The reason to choose the form of Formula 5.2 is as follows. As Formula 5.3 shows, the first item is logarithm of the product of consecutive spectrum availability likelihood of the nodes in  $P_{(a,b]}$ . The product is the likelihood that one message travels from node  $a$  to  $b$  without hampered by primary users, which is an important property we want to integrated into our virtual coordinate system. The fist item will be infinity when the spectrum is occupied by primary users during all the time, where  $\gamma = 0$ . In this case, infinity can be replaced with a large positive value which results in a large  $\lambda$ . The second item denotes number of hops, which can be seen clearly when  $\gamma_i = 0$  for node  $i \in P_{(a,b]}$ .

As  $\lambda_i$  needs to be monotonically decreasing with respect to  $\gamma_i$ , so that the less spectrum availability results in bigger cost for communication, thus there should be

$$\frac{\partial \lambda_i}{\partial \gamma_i} = c - \frac{1}{\gamma_i} < 0 \quad (5.4)$$

hence the tuning parameter  $c$  should be smaller than 1. In the simulation part, we choose  $c = 0.2$  so that  $\lambda$  visibly reflects the changes of  $\gamma$  when  $\gamma$  is not too small, as Figure 5.4 shows.

### Multiple licensed channels

When multiple licensed channels are allowed to use without interfering primary users, one node can switch to an another channel which is at present available to send or forward packet, then the normalized channel availability is,

$$\gamma_i = 1 - \prod_{k=1}^{|C|} (1 - \gamma_i^k) \quad (5.5)$$

Based on Formula 5.2, the normalized spectrum availability on node  $i$  when multiple secondary channels are available is,

$$\begin{aligned}\lambda_i &= -\ln \gamma_i + c \cdot \gamma_i \\ &= -\ln\left(1 - \prod_{k=1}^{|C|} (1 - \gamma_i^k)\right) + c \cdot \left(1 - \prod_{k=1}^{|C|} (1 - \gamma_i^k)\right)\end{aligned}\quad (5.6)$$

### 5.4.3 Normalized Longest Blocking Time on Secondary Users

Channel utility introduced in previous subsection characterizes the likelihood that secondary user is allowed to forward packets, but it fails to reflect the availability of spectrum in a finer granularity of time. For instance, in the period of time  $T_{access}$  to access the spectrum availability, one channel which frequently changes between state ON and OFF due to primary users' violent operation may have the same likelihood of available spectrum with the channel where primary user sojourns on state ON for long time. This difference has direct consequence on delay when the likelihood of spectrum availability on PU affected secondary nodes is homogeneous.

Let  $T_{ON}^k$  be the length of time period that channel  $k$  is not available, there is  $T_{ON}^k = n \cdot (T_s + T_p)$ , where  $n$  is the number of consecutive sensing duration that channel  $k$  is sensed as busy.  $T_{ON}^k$  is recorded within  $T_{access}$ , and we use  $\tau^k = \overline{T_{ON}^k}$  to denote the average value of the time duration that channel  $k$  is occupied by primary user, which is the maximum time period that secondary user is blocked from sending/forwarding.

#### Single licensed channel

In single licensed channel scenario, the normalized maximum blocking time on node  $i$  is (superscript  $k$  is omitted),

$$\lambda_i = f(\tau_i) = \gamma_i \cdot \tau_i + b \cdot e^{-\gamma_i \cdot \tau_i} \quad (5.7)$$

The first item is the product of blocking time and the duty cycle of available spectrum, note that we assume the duty cycle is identical for any PU affected secondary user and thus can be regarded as constant. As to the secondary users which locate out of any primary user's transmission range, there is  $\tau_i = 0$ , then  $\lambda_i = b$ ,  $\lambda_i$  denotes hop count in this case. This is the reason that the second item is needed.

Same with the analysis in section 5.4.2, when one anchor message travels through path  $P_{(a,b)}$ , the distance on the corresponding coordinate dimension is the sum of the normalized longest blocking time, which is the function of the sum of maximum blocking time on the cascaded nodes on the trajectory of anchor message,

$$\begin{aligned}|x_b - x_a| &= \sum_{i \in P_{(a,b)}} (\gamma_i \cdot \tau_i + b \cdot e^{-\gamma_i \cdot \tau_i}) \\ &= \gamma_i \cdot \sum_{i \in P_{(a,b)}} \tau_i + b \cdot e^{-\gamma_i \cdot \sum_{i \in P_{(a,b)}} \tau_i}\end{aligned}\quad (5.8)$$

Normalized longest blocking time  $\lambda$  is monotonically increasing with  $\tau_i$ , which requires

$$\begin{aligned} \frac{\partial \lambda_i}{\partial \tau_i} &= \gamma_i - \gamma_i \cdot b \cdot e^{-\tau_i \cdot \gamma_i} > 0 \\ b &< e^{\gamma_i \cdot \tau_i} \end{aligned} \quad (5.9)$$

then we set the tuning parameter  $b$  as 1.

### Multiple licensed channels

In multiple licensed channel scenario,  $\tau_i$  equals to the smallest maximum blocking time over all secondary channels on node  $i$ ,

$$\tau_i = \min \tau_i^x, x \in C \quad (5.10)$$

The normalized maximum blocking time on node  $i$  is as Formula 5.7 shows.

In remainder of this chapter, the virtual coordinate based on normalized spectrum utility is referred as *spectrum availability based VC*, and The virtual coordinate based on normalized maximum blocking time is denoted as *blocking time based VC* out of convenience.

When  $\lambda$  on secondary nodes is identical, the resultant SAViC appears to be similar with hop based virtual coordinate. In reality, as the measurement shows in [113], heterogeneity of spectrum usage by primary users is very normal, besides, the two kinds of virtual coordinates make it easier to find out such heterogeneity. [113] also shows within certain frequency band, primary users' activity is stable for hours, e.g., cellular network. When primary user's operation pattern changes, e.g., occupy spectrum with increased duty cycle, then SAViC needs to be reimplemented.

## 5.5 Geographic Routing and Opportunistic Spectrum Access

Although spectrum aware virtual coordinate is the main concern of this chapter, we also introduce the geographic routing to be used as it affects the routing result directly. With geographic routing, packet sender/forwarder chooses the neighbour which has smaller Euclidean distance to the destination. The distance between node  $i$  and destination  $d$  is  $\sqrt{(x_d - x_i)^2 + (y_d - y_i)^2 + (z_d - z_i)^2}$ , when virtual coordinate can be denoted as  $\{x, y, z\}$ . A trivial improvement on greedy geographic routing is implemented in network layer to mitigate the dead end problem. When routing protocol reaches dead end node  $u$  which is closest to destination,  $u$  adds its ID to the packet as taboo before forwarding the packet to  $v$  which is closest to the destination in its neighborhood. The packet will not be sent to the nodes whose IDs appear to be taboos.

Buffer is implemented on each node, where packets stay temporally when no unoccupied licensed spectrum is available. Secondary user resends buffered packet every period of time, and drops it if there is still no available channel after trying for 10 times.

In multiple channel CRN, after one node deciding on the next hop via geographic routing, which channel to use needs to be answered. This problem involves considerations from



many aspects, such as minimizing channel switch cost [125], mitigating co-channel interferences [91] etc.. We adopt a lightweight heuristic method in this chapter. When there is packet to send and the next hop is decided, packet sender chooses the channel in descending sequence with channel's metric, i.e., likelihood of channel availability, or blocking time. The sender chooses the channel with the best metric, then conducts spectrum sensing in the immediately following sensing duration to determine the channel's usability. If the channel is sensed as free to use, sender transmits *request\_channel\_x* to the next hop on the control channel, when it receives the answering message *channel\_x\_available* from that node, it starts communication on channel *x* in the following sensing period. If the channel is sensed to be busy before or among the transmission, or it receives *channel\_x\_unavailable* message from next hop node, the sender moves to the channel with the second best metric, and conduct the same procedure as described above.

## 5.6 Performance Evaluation

In this section, we present the performance of geographic routing together with SAViC. Both virtual coordinates based on metrics of spectrum availability and blocking time respectively are evaluated. Prior to that, the set-up of simulation is introduced.

### 5.6.1 Simulation Setup

In this section, we introduce the deployment of the primary users to generate various spectrum availability in the network, then introduce the important parameters in simulation. Different from [38] where simulation is conducted without considering any activities in MAC and physical layer, simulation in this chapter deploys a wireless environment which is close to reality, e.g., interferences and channel shadowing are involved.

#### Primary Users

Measurements [61, 104] show that the sufficiently accurate statistic model of spectrum occupancy can be given by a Markovian process. In simulation, primary user alternates state between ON and OFF as a two-state discrete time Markov chain (2DTMC). The probability that it changes from one state to the other, or stays in the same state is called transition probability. Transition probability further decides the stationary probability of 2DTMC, which represents the percentage of time that primary user is in state ON or OFF in a long run. The relationship between stationary probability  $\Pi = \{\pi_{\text{ON}}, \pi_{\text{OFF}}\}$  and duty cycle  $\gamma$  is,

$$\lim_{T_{\text{assessment}} \rightarrow \infty} \gamma = \pi_{\text{OFF}} \quad (5.11)$$

Transition probability also decides the continuous sojourning time of primary user on a certain state, which affects the longest blocking time sensed by secondary users. Hence, by adjusting transition probability, we can let primary user operate with desired intensity, i.e. stationary probability for being in state OFF, or continuous sojourning time of being on state ON. We denote stationary probability of state OFF as  $P_{\text{OFF}}$ , and the maximal blocking time as  $T$ . In the following, we only use  $P_{\text{OFF}}$  and  $T$  to define primary user's

dynamics, and omit mentioning the transition probability. The time unite of the DTMC for primary user to follow is 0.1s.

As spectrum availability based and longest blocking time based virtual coordinates are designed for CRN which is influence by certain primary user activity, we design two primary user distributions. As a result, we design two categories of primary user settings to evaluate the routing performance assisted by the two categories of virtual coordinate respectively.

- As to spectrum availability based virtual coordinate, two primary users are located in the CRN which can not affect all the nodes in CRN, as shown in Figure 5.6(a).
- For blocking time based virtual coordinate, network is evenly covered by primary users which have the same duty cycle, but some primary users have different blocking time with the others, as Figure 5.9 shows.

When multiple channel scenario is to be investigated, existing primary users simply start to work with current and additional channels, and there is no new primary users appear.

### Parameter Setting

Simulation is conducted with INET framework provided by OMNeT++ simulator [139], which comprises both generation of SAViC and following geographic routing. Secondary users are randomly distributed in a square area and 6 nodes which locate at the edge are deployed as anchors.

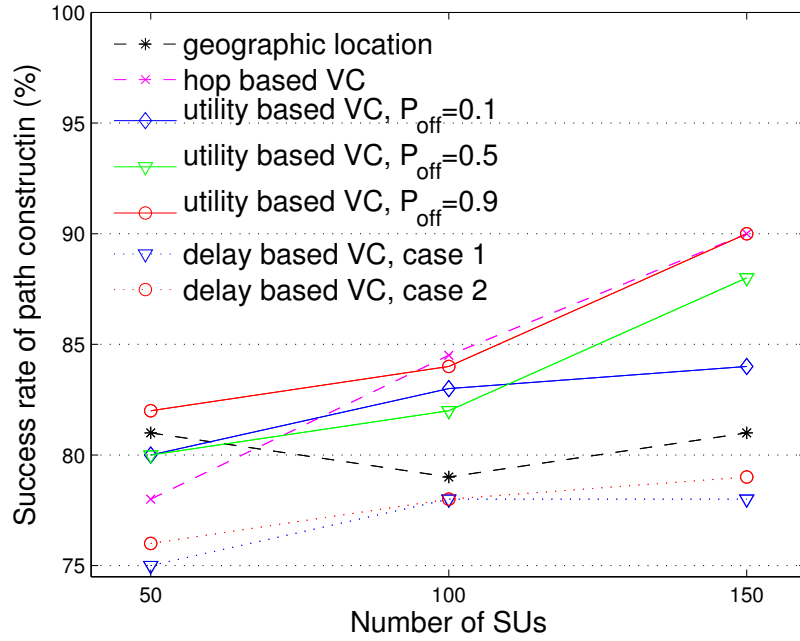
## 5.6.2 Success Rate of Geographic Routing on Finding Path

We evaluate SAViC's reachability, i.e. given the virtual coordinate of destination, geographic routing forwards packet from source to the node with the desired virtual coordinate. The comparisons are real geographic location, and hop based virtual coordinate according to VCap [38]. We deploy 6 anchors and then there is no duplicated virtual coordinate among the resultant virtual coordinate system.

In order to evaluate the effectiveness of coordinates to support geographic routing, we design different configurations of primary users. As to duty cycle based VC, two primary users are randomly deployed in the network. As to delay based VC, we configure 9 primary users to evenly cover the network, among of them, 4 primary users have different maximal blocking time with the rest. Under one configuration of primary users, 1000 random CRN is generated and in each CRN one far departed pair of source and destination is chosen to test.

Figure 5.5 shows, spectrum availability based virtual coordinate supports geographic routing to achieve similar reachability with hop based virtual coordinate<sup>2</sup>, which is better than that with real geographic location. Blocking time based virtual coordinate performs a little bit worse than other coordinates. In summary, after integrating the primary user's influence, SAViC supports geographic routing to achieve comparable success rate of path construction with conventional virtual coordinate and real geographic location.

<sup>2</sup>The numerical result of hop based virtual coordinate coincides with the simulation result presented in [38] under the same network density



**Figure 5.5** Reachability of different virtual coordinates, the average number of neighbors is 6, 12, and 14

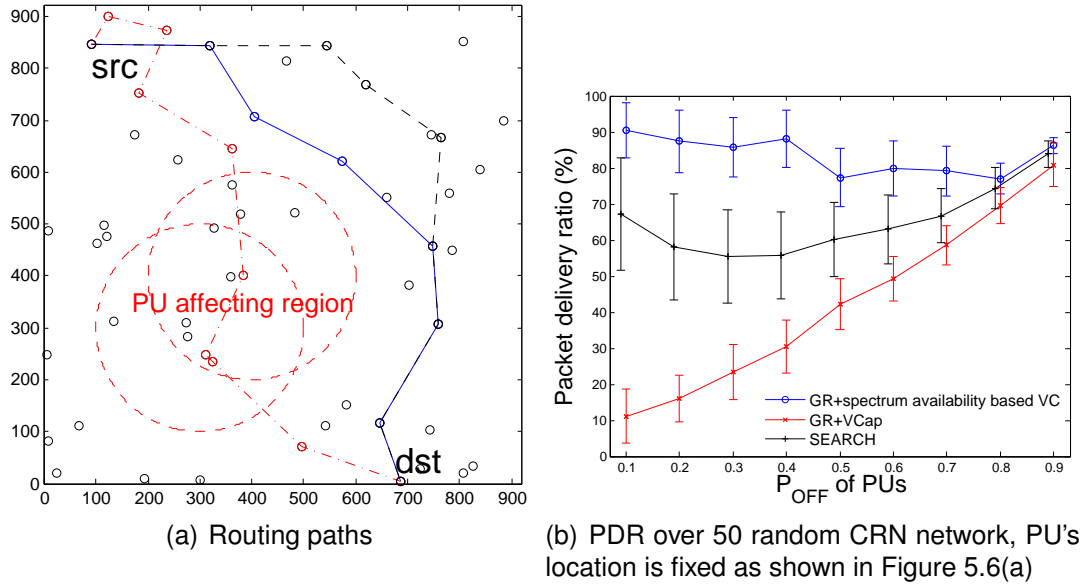
### 5.6.3 Routing Performance

We sequentially present the routing performance of SAViC based on spectrum availability and blocking time respectively. In more details, spectrum availability SAViC is compared with hop based virtual coordinate VCap and *SEARCH*. The reason to choose *SEARCH* [46] is it is on the basis of geographic routing and utilizes routing table in the interval of updates, thus it requires less computation ability and overhead exchanges. The time interval for *SEARCH* to update routing tables of the nodes on routing path is 5s. Both single and multiple licensed channel scenarios are investigated for the three solutions.

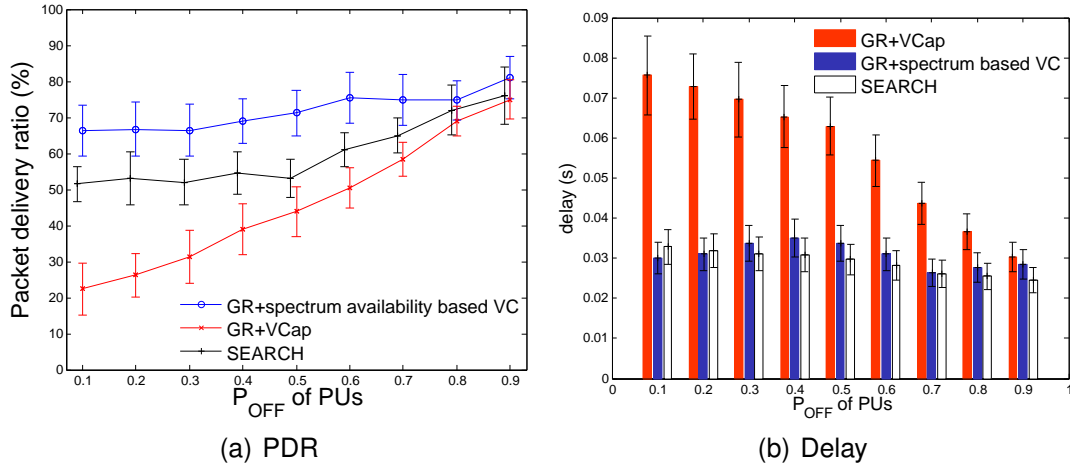
#### Duty Cycle Based Virtual Coordinate

We start by looking into the performance of SAViC in single channel scenario.

We start with a case study, two primary users locate at the centers of dashed cycles as shown in Figure 5.6(a). VC based on spectrum availability and VCap (hop is its metric) are assigned to secondary users separately. The red dashed path in Figure 5.6(a) is formed by geographic routing with VCap, which cuts across the primary users' affecting area and thus suffers great packet loss. The black dash and blue solid paths are formed with spectrum availability based virtual coordinate, the two paths are formed when primary user's working intensities  $P_{OFF}$  is 0.1 and 0.9 respectively. These two paths vividly illustrate that utility based virtual coordinate successfully integrates the spectrum scarcity in CRN network, and decomposes a large part of routing decision. The paths of *SEARCH* is not drawn here as the routing path is possible to change after path update. We keep the primary users in the middle of the network, for each activity intensity, 50 CRNs where secondary users are randomly located are generated. Figure 5.6(b) shows the PDR of spectrum availability based virtual coordinate is high except for a minor decline when



**Figure 5.6** Routing paths and corresponding PDR



**Figure 5.7** Packet delivery ratio with single secondary channel, over 50 randomly located CR nodes and PUs

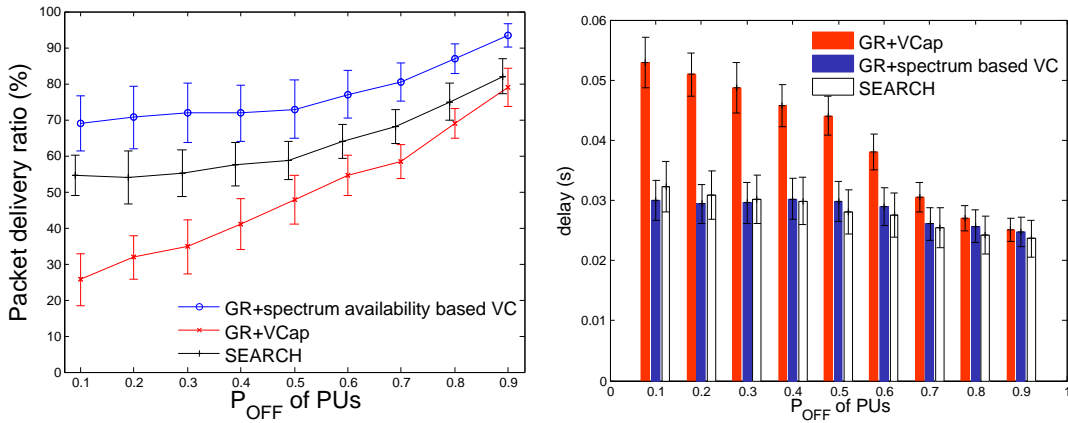
$P_{OFF}$  is between 0.5 and 0.8, which is contradictory to the monotonically increasing trend of hop based virtual coordinate. This can be explained by the path snapshot in Figure 5.6(a). When channel is sensed to be scarce (primary users access channel intensively), path generated is far away from the affected area and circumvents completely. When primary users become less intensive, routing path moves closer to that area. In other words, the weaker dynamics of primary users attracts path and result in packet drop. When  $P_{OFF}$  approaches to 1, spectrum availability based virtual coordinate becomes actual hop based virtual coordinate as the link metric in formula 5.2 becomes zero.

The paradox that more licensed spectrum leads to worse PDR can also be observed on *SEARCH*, whose PDR curve declines first and increases later on. When channel is heavily utilized by primary users, the routing request is more likely to encounter operating primary user, then a node out of the primary user affecting area is chosen as next hop, so that the path experiences less packet loss (with the price of more hops). When primary users become less intense, routing request is more likely to traverse the affected areas, as a

result, the routing path experiences packet loss due to the primary users in that area before next route update.

Figure 5.7(a) shows the PDR when both primary and secondary users' locations are random. SAViC's performance deteriorates because the source and destination may be influenced by primary users, so that a path completely detour the primary users' area is impossible. where geographic routing has no means to detour the affected areas. In figure 5.7(b), SAViC and SEARCH achieve lower delay although forwarding more packets, which means SAViC is effective to facilitate geographic routing to avoid PU affecting areas.

Now we introduce the routing performance in multiple channel scenario, where two licensed channels available, but only one is allowed for payload transmission.



(a) PDR over 50 CRNs where SUs and PUs are randomly located (b) PDR over 50 CRNs where SUs and PUs are randomly located

**Figure 5.8** Packet delivery ratio with multiple secondary channel scenario

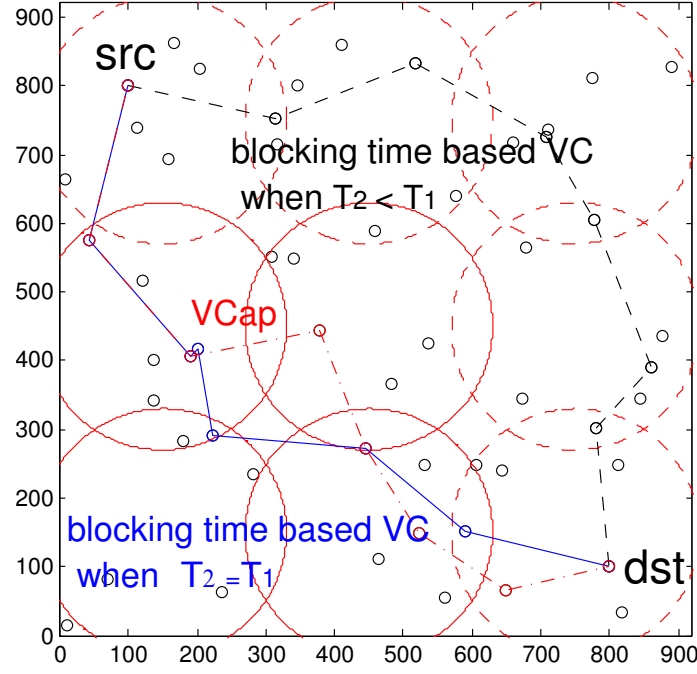
In this part of simulation, we follow the setting of single channel scenario, except that secondary users have at most two licensed channels.

Thanks to the second channel, the packet delivery ratio is increased as shown in Figure 5.8(a), and delay is decreased as depicted in Figure 5.6(a). SAViC still outperform the other schemes especially in the aspect of PDR.

### Longest Blocking Time Based Virtual Coordinate

As discussed in section 5.4.3, spectrum availability based virtual coordinate doesn't reflect the sparsity or abundance of spectrum well when the likelihood of spectrum availability is homogeneous in CRN. A CRN working with single licensed channel in Figure 5.9 is used to show the fail of spectrum availability based virtual coordinate. Two items are used in the following to make the analysis tidy.

$T_1$	Maximal blocking time of primary users whose transmission ranges are solid cycles in Figure 5.9
$T_2$	Maximal blocking time of the other primary users



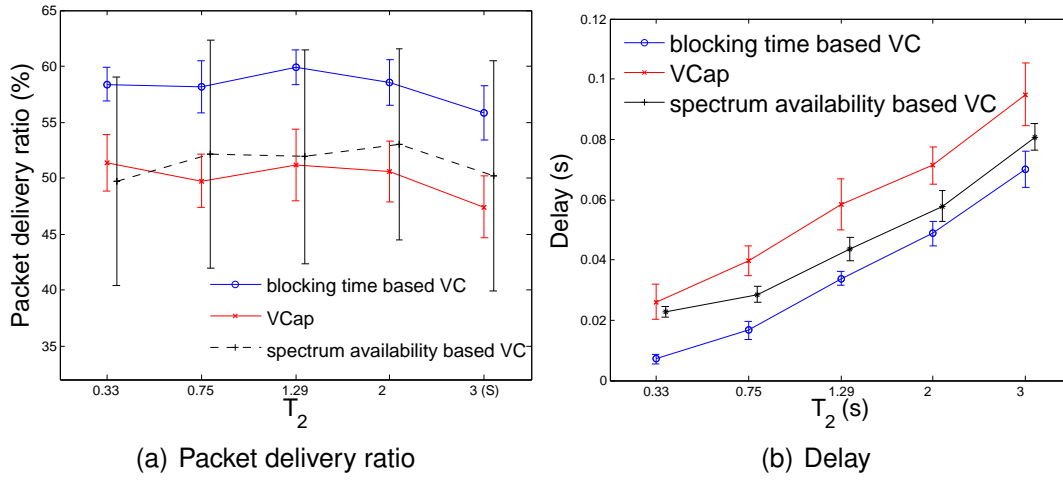
**Figure 5.9** Routing paths in one network,  $T_1 = 3s$ . cycles denote the transmission range of primary users.

In the network, 9 primary users evenly distributed,  $P_{\text{OFF}} = 0.5$  for each of them. For the primary users denoted by the solid cycles, maximal blocking time  $T_1 = 3s$ , and  $T_2 = 1s$  and  $3s$  for the other primary users. When  $T_2 < T_1$ , the resultant routing path is in black and dashed, which goes through area where primary users have shorter maximal blocking time. When  $T_2 = T_1$ , the resultant routing path largely converges with the path with VCap.

The ineffectiveness of spectrum availability based virtual coordinate in case of identical  $P_{\text{OFF}}$  is observed in Figure 5.9. In this case a different characteristic, i.e., the longest blocking time, which shows the geographically diverse characteristics of spectrum can be used. In our simulation,  $P_{\text{OFF}} = 0.9$  for all primary users, but they are diverse on sojourn time, i.e.  $T_1$  of primary user is  $3s$ , and  $T_2$  is shorter. We randomize the location of secondary users in 50 networks, and present the performance of blocking time based virtual coordinate to show its superiority on decreased end to end delay and PDR. In this part of simulation, we don't show the result of *SEARCH*, as it performs as bad as geographic routing with hop based virtual coordinate. The reason is the widespread primary users seriously hamper the routing requests to arrive at destination, consequentially most paths for forwarding the packets can not be constructed successfully.

Figure 5.10(b) shows as  $T_2$  increases from  $0.33s$  to  $3s$ , the delay of successfully delivered packets also increases for both blocking time based virtual coordinate and VCap, but a constant gap exists in between. Whereas, the delay of spectrum availability based virtual coordinate is random as respect to sojourn time, the reason is the routing metric in this scenario doesn't involve blocking time imposed by primary users.

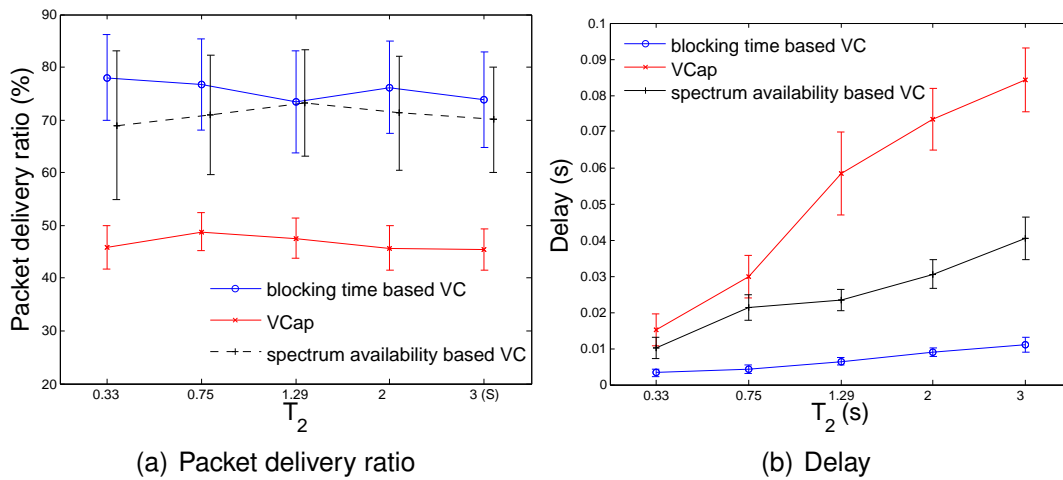
The packet delivery ratio shown in Figure 5.10(a) is constant with both blocking time based virtual coordinate and VCap, because all the primary users have the same  $P_{\text{OFF}}$  which is  $0.9$ . Particularly, blocking time based virtual coordinate achieves higher packet delivery ratio than the other two virtual ordinates, the reason is when the former is ap-



**Figure 5.10** Geographic routing in single secondary channel scenario,  $P_{\text{OFF}} = 0.9$  for all primary users,  $T_1 = 3s$ ,  $T_2$  varies.

plied, less packets are dropped from buffer as the time of being blocked is shorter for the secondary users on the path.

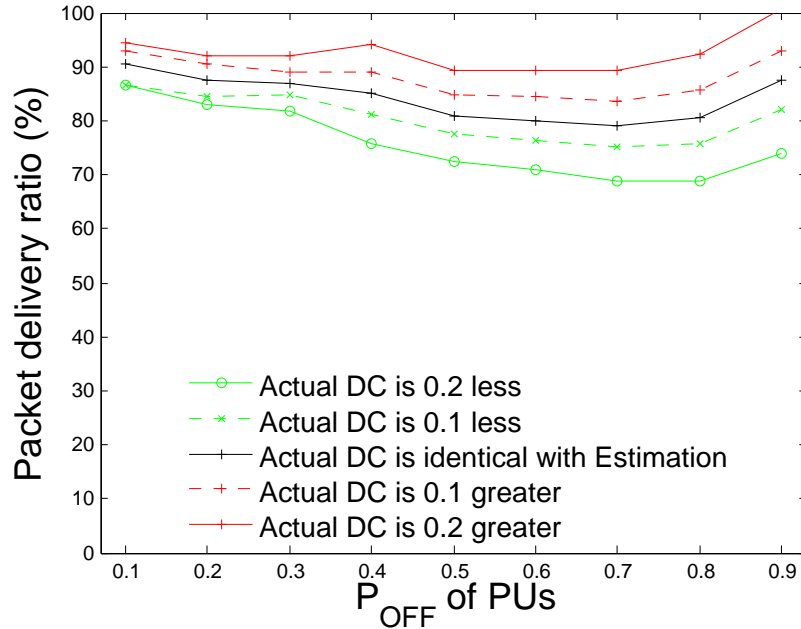
Now we have a look at the CRN with two licensed channels. As to performance of delay, because of the second available channel, blocking time based virtual coordinate achieves very delay, in contrast, spectrum availability based virtual coordinate still demonstrates obvious randomness, as is shown in Figure 5.11(b). Compare Figure 5.11(a) and 5.10(a), we can see the packet delivery ratio in two channel network is obviously higher than that in single channel network, as the second channel provides extra transmission opportunities. Blocking time based virtual coordinate achieves up to 10% better performance than that with spectrum availability based VC, the reason is packets in buffer have greater likelihood to be sent out before getting dropped.



**Figure 5.11** Geographic routing in two secondary channel scenario,  $P_{\text{OFF}} = 0.9$  for all primary users,  $T_1 = 3s$ ,  $T_2$  varies.

### 5.6.4 Sensitivity of SAViC to Estimation Errors

In this Section, we evaluate the performance of SAViC in the presence of erroneous estimates about the primary user activity. In order to induce a particular amount of errors, we artificially add errors to duty cycle of primary user activity by directly modifying the implementation of 2 state markov chain. By doing this a real life scenario is built, where estimation errors are expected. The simulation is conducted with the same configuration with Figure 5.6(a). Figure 5.12 shows even the errors are significant, i.e., DC is modified by 25% to 100%, the corresponding performance varies only at most 13%.



**Figure 5.12** In the setting of Figure 5.6(a), performance of DC based virtual coordinate is not affected greatly by the erroneous estimation

## 5.7 Conclusion

The proposed virtual coordinate SAViC reshapes the topology of cognitive radio network based on sensing results of spectrum availability. As SAViC adjusts the distance between nodes based on the communication obstruction caused by primary users, the virtual coordinate comprises a part of the routing decision, so that geographic routing is able to detour the areas seriously affected by primary user. Geographic routing with SAViC greatly simplifies the computation and communication burden on each secondary user involved in routing in CRN. Together with SAViC, geographic routing achieves better performances than other geographic routing designed for CRN through extensive simulation. This paradigm of routing is especially suitable for CRN network where the resource limited CR nodes can only support geographic routing. This work emphasises on avoiding primary users' influence with geographic routing, and doesn't consider the interference issue among the secondary users, which should be addressed in the future work.



# 6

## Conclusion and Future work

The goal of this thesis is to design paradigms and algorithms that enable individual cognitive radio to act autonomously, and finally converge to an equilibrium state, meanwhile, the performance of cognitive radio users are improved. In this dissertation, we solve a series of problems residing from layer 1 to layer 3 in the OSI model [2] of CRN with distributed solutions, and employ game theory to seek the guarantee on convergence and Nash equilibrium.

### 6.1 Contribution

We propose a complete strategy of power and spectrum allocation in IEEE 802.22 networks. We make full use of the centralized database which is required by the standard and relevant regulations. The maximal transmission power on channel for each WBS is decided in the database by solving an optimization problem. Then a two stage channel power allocation scheme is proposed in a distributed manner, where the database is involved to assist interaction between secondary users. This work lies in layer 1, after deciding the maximal transmission power on each secondary cellular base station, we formulate the distributed spectrum allocation problem in TV white space scenario (a special CRN where primary user is TV station which operates according to a slow and pre-decided schedule) into a canonical congestion game, then propose distributed algorithm corresponding to the behaviour of player in the game.

As to improving the accuracy of spectrum sensing with clusters, we designed a distributed clustering scheme ROSS. ROSS is lightweight, thus is suitable CRAHN where the network connection varies due to the mobility of spectrum and secondary users. ROSS increases the number of common available channels within one cluster, thus improves the robustness of formed clusters against primary users, so that cooperative sensing can make the best use of clusters. This problem can be regarded to lie in layer 2.

In layer 3, we propose a lighted weighted routing scheme for CRN. Spectrum aware virtual coordinate is proposed, thus light weighted geographic routing can be used to decide the next hop.

## 6.2 Future Work

Building on the top of the our contributions, game theory can help to design distributed solution for further problems in CRN. In this section, we describe the future work, and sketch the solutions briefly.

### Structureless Diffusion Based Routing in Game Theoretical Framework in Event-driven Cognitive Radio Sensor Network

One future work is to propose a structure less data fusion paradigm for cognitive radio sensor network. Sensor nodes are deployed to sense physical phenomena of interests (PoIs), and they transmit the PoIs to the sink node. Cognitive radio capability is transplanted onto sensor network [16], so as to alleviate collision and decrease delay. But congestion is still difficult to be totally eliminated by merely providing more spectrum. Congestion occurs in wireless sensor network when incoming traffic load exceeds the capacity. Variant solutions [129] have been proposed to mitigate congestion, e.g., implementing flow control to avoid transmitting to the downstream node where congestion is detected, changing the MAC behaviour to give the congested sensor larger possibility to transmit, adding rely nodes to reroute data around the congested nodes.

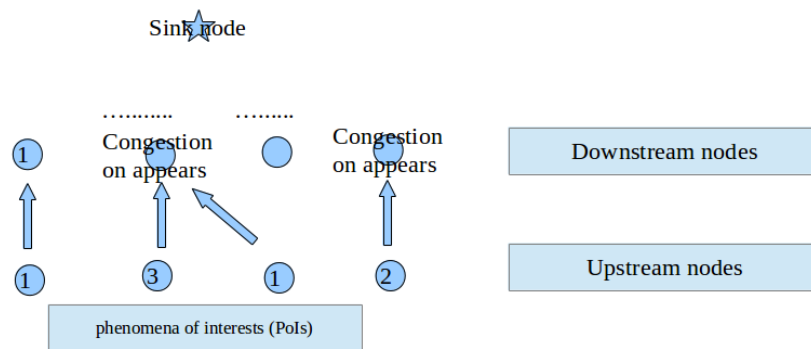
When there are multiple upstream nodes<sup>1</sup> transmitting to sink node, and there are several downstream nodes are available to be relies, then which portion of the data on the upstream node should be sent to which downstream node becomes a question. As shown in Figure 6.1, several nodes are downstream nodes for node 2 and 3. We have proposed routing solution for end to end communication in CRN, where geographic routing is conducted with spectrum aware virtual coordinate. As to multiple to one communication, simply using our proposed scheme is problematic, as we need to consider the possible collision caused by multiple transmission, priority of services and possible congestion appearing on the relying nodes.

#### Problem Statement

The basic idea is from the *server matching* introduced in Section 2.

Delay of the packets is used to represent the congestion. This delay could be the average of delay on all packets, or a function of delay of individual packet. Although it is not clear how to granulate the data based on their priorities, it is clear that the total congestion (delay) is a increasing function of the amount of incoming data. When there are only one group of downstream nodes and one group of upstream nodes involved, and the upstream nodes have finite number of transmission strategy (a certain portion is sent to a certain

<sup>1</sup>Downstream nodes of one node  $i$  represent the nodes relying data from node  $i$  towards to the sink node, upstream nodes of node  $i$  means the nodes sending data to node  $i$ .



**Figure 6.1** Multiple to one communication

downstream node), then the convergence will be brought by best response strategy. But the converge is not guaranteed when consider within a bigger picture, as the downstream nodes can become upstream nodes when they send data to their downstream nodes.

The questions needed to be answered are as follows,

- What is the strategy for each sensor node to assign its data to different downstream nodes.
- As this problem is a cascaded of congestion games, is there equilibrium?

because the structured techniques incur relatively higher overheads to maintain the network structure.

## Other Future Research Works

- As to channel and power allocation in IEEE 802.22 network, cooperation can be brought to improve performance.
- As a light weight clustering scheme which can cope with mobility of both spectrum and users, ROSS can be used to support routing and resource allocation.



# 7

## Appendix



# A

## Proof of Theorem 3.5.1

For selfish best response approach, the utility function is set as follows,

$$u_i = \frac{\sum_{c(i)=c(i)} \tilde{f}_{ji} + N_0}{P_i \cdot h_{ii}} \quad (\text{A.1})$$

**Proof.** In order to simplify the proof, we assume  $N_0 = 0$ . Consider one WBS  $i$  executing algorithm 1 with utility A.1, and updates its channel from  $c_i$  to  $c'_i$ , we denote  $u'_k, k \in \mathcal{N}$  as the utility of WBS  $k$  when  $i$  chooses channel  $c'_i$ , accordingly, the summation of utilities of all WBSs after  $i$  changing to  $c'_i$  is  $U' = \sum_{\forall k \in \mathcal{N}, c(i)=c'_i} u'_k$ .

$$\begin{aligned} U' &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u'_j \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} (u_j + (u'_j - u_j)) \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u_j + \sum_{j \in \mathcal{N}, j \neq i} (u'_j - u_j) \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u_j + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} (u'_j - u_j) + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} (u'_j - u_j) \\ &\quad + \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(j) \neq c'_i, c(j) \neq c_i}} (u'_j - u_j) \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u_j + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{\tilde{f}_{ij}}{\tilde{P}_j} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{\tilde{f}_{ij}}{\tilde{P}_j} \right) \end{aligned} \quad (\text{A.2})$$

where,

$$\begin{aligned} u'_i &= u_i + \Delta u_i(c_i \rightarrow c'_i) \\ &= u_i + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{\tilde{f}_{ji}}{\tilde{P}_i} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{\tilde{f}_{ji}}{\tilde{P}_i} \right) \end{aligned} \quad (\text{A.3})$$

bring A.3 into A.2, we get,

$$\begin{aligned} U' &= U + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{\tilde{f}_{ji}}{\tilde{P}_i} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{\tilde{f}_{ji}}{\tilde{P}_i} \right) \\ &\quad + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{\tilde{f}_{ij}}{\tilde{P}_j} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{\tilde{f}_{ij}}{\tilde{P}_j} \right) \end{aligned} \quad (\text{A.4})$$

According to algorithm 1, the summation of second and third items, which is the variance of  $i$ ' utility, is negative. If we can confirm the summation of fourth of the last four items is negative, the whole utility of the network decreases with  $i$ ' each update. For simplification, we assume that the channel is symmetric, which means,  $h_{ij} = h_{ji}$ , and  $z$  is identical among all WBSs. Then, the problem we want to confirm is equivalent to the following: Given the in-equation with  $n, m$  are natural numbers

$$\sum_{i=1}^m \alpha_i < \sum_{i=1}^n \beta_i, \quad (\text{A.5})$$

Prove the following in-equation is correct or not,

$$\sum_{i=1}^m \left( \alpha_i + \frac{1}{\alpha_i} \right) < \sum_{i=1}^n \left( \beta_i + \frac{1}{\beta_i} \right), \quad (\text{A.6})$$

We propose a small contradiction to prove A.6 is not true. When  $m = 2, n = 1$ , and  $\alpha_1 = 1, \alpha_2 = 0.5, \beta = 2.1$ , we can see that although  $\sum_{i=1}^m \alpha_i = 1.5 < \sum_{i=1}^n \beta_i = 2.1$ , there is  $\sum_{i=1}^m \left( \alpha_i + \frac{1}{\alpha_i} \right) = 4.5 > \sum_{i=1}^n \left( \beta_i + \frac{1}{\beta_i} \right) = 2.58$ . hence, with WBS's update, it is possible that  $U' > U$ , thus there is no monotonically convergence by utilizing A.1.

□

Notice that the last four items in A.4 is exactly the change of summation of utilities of all WBSs after  $i$ ' update if WhiteCat is executed, hence the monotonic convergence of WhiteCat is proved here analytically if noise is considered to be zero. If noise is considered, we can follow the conclusion in the end of 3.5.3 that WhiteCat converges without monotonicity.



# B

## Deviation of Problem 3.11

We reformulate the objective problem 3.11 which is a binary non-linear programming to binary quadratic programming as follows,

$$\begin{aligned}
 & \sum_{i=1}^n \frac{\sum_{j \in \mathcal{N}, j \neq i} P^T X_j (X_j^T X_i) h_{ji} z + N_0}{P^T X_i} \\
 &= \sum_{i=1}^n \left( \frac{\sum_{j \in \mathcal{N}, j \neq i} \sum_k (P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z) + \sum_k N_0 \cdot x_{ik}}{P^T X_i} \right) \\
 &= \sum_{i=1}^n \left( \frac{\sum_{j \in \mathcal{N}, j \neq i} \sum_k (P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z)}{P^T X_i} + \frac{\sum_k N_0 \cdot x_{ik}}{P^T X_i} \right) \\
 &= \sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k \frac{(P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z)}{P^T X_i} + \sum_k \frac{N_0 \cdot x_{ik}}{P^T X_i} \right)
 \end{aligned} \tag{B.1}$$

we now simplify the first item in the parenthesis. If we assume secondary base station  $i$  is working on channel  $m$ , then there is  $x_{im} = 1$ , then

$$\begin{aligned}
 \frac{P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z}{P^T X_i} &= \frac{P_{jk} \cdot x_{jk} \cdot x_{im} \cdot h_{ji} \cdot z}{P_{im} \cdot x_{im}} \\
 &= \frac{P_{jk} \cdot x_{jk} \cdot h_{ji} \cdot z}{P_{im}}
 \end{aligned} \tag{B.2}$$

other wise, formula B.2 equals to 0.

Similarly, for the second item in the bracket,

$$\frac{N_0 \cdot x_{ik}}{P^T X_i} = \frac{N_0}{P_{ik}} \cdot x_{ik} \tag{B.3}$$

then, formula B.1 becomes,

$$\sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k \frac{P_{jk}}{P_{ik}} \cdot h_{ji} \cdot z \cdot x_{jk} \cdot x_{ik} + \sum_k \frac{N_0}{P_{ik}} \cdot x_{ik} \right) \quad (\text{B.4})$$

which is a binary quadratic programming problem.

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# Glossary

- 2TDMC** two-state discrete time Markov chain. 105
- 5G** 5th generation mobile networks. 2
- CCC** Common control channel. 15, 67, 72
- CH** cluster head. 63
- COS** Cost of oscillation. 54
- CR** Cognitive radio. 2
- CRAHN** Cognitive radio ad hoc network. 11
- CRN** Cognitive radio network. 3
- CW** Contention window. 9
- DCF** Distributed coordination function. 9
- DSR** Dynamic source routing. 96
- DTV** digital TV. 31
- ECC** Electronic communications committee in Europe. 3, 29
- FCC** Federal communications commission in U.S.. 29
- GSM** Global system for mobile communications. 1
- HSPA** High speed packet access. 11
- ISM band** Industrial, scientific, and medical. 1
- LP** linear programming program. 38
- LTE** 3GPP Long term Evolution. 11
- MAN** Metropolitan area networks. 37
- NE** Nash equilibrium. 8
- PDA** Personal digital assistant. 2
- PoA** price of anarchy. 21
- PU** Primary user. 2
- QoS** Quality of service. 31
- ROSS** Robust spectrum sharing. 67
- ROSS-DFA** Robust spectrum sharing-distributed fast algorithm. 64
- ROSS-DGA** Robust spectrum Sharing-distributed greedy algorithm. 64
- RSSI** Received signal strength index. 30, 49
- SINR** signal-to-noise-and-interference ratio. 31, 32
- SU** Secondary user. 2
- TC** Time complexity. 89
- TVBD** TV band devices. 12
- TVWS** TV white space. 11
- UHF** Ultra high frequency. 30
- VCap** virtual coordinate assignment protocol. 98
- WBS** White base station. 13
- WhiteCat** White space channel allocation. 36
- WRAN** Wireless regional area network. 37