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# **Distributed Algorithms and Application of Congestion Game for Cognitive Radio Networks**

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# Abstract

Owing to the proprietary spectrum allocation paradigm and the boom of wireless communications, spectrum scarcity has become an increasingly pressing problem. Cognitive radio is a promising technology to provide high bandwidth for unlicensed users via dynamic spectrum access. Unlicensed users are allowed to utilize licensed spectrum in the following two scenarios, the first, their operation doesn't cause harmful interferences on licensed users, the second, there are no working primary users which are detected with respect to a certain probability. This spectrum usage paradigm brings new spectrum for unlicensed users, nevertheless, it comes at a cost that the unlicensed users need to decide on the spectrum which is available to use, and to take care of the maximum transmission power to avoid harmful interference at primary users carefully. Furthermore, the available spectrum or resources at unlicensed users are dependent upon their locations with respect to primary users whose operation states vary with time, thus the spectrum availability is different at different unlicensed users and varying.

Consequently, as distributed scheme enables unlicensed users to respond quickly to the varying environment with the help of spectrum detection functionality on itself or its neighbourhood, distributed solutions are extensively adopted by the communications systems which are composed with unlicensed users. In this context, a significant amount of research efforts has been dedicated to study the distributed schemes in cognitive radio networks, but designing efficient distributed schemes faces numerous challenges. First, the autonomous decision adopted by the unlicensed users should lead to fast convergence in system scale. Second, unlicensed users tend to be selfish in nature, therefore, the individual decision should not degrade the performance of the system and other users.

In this regard, congestion game becomes a highly suited mathematical tool to model the interaction among the unlicensed users which strive to fulfil their own purposes, as congestion game guarantees convergence into Nash equilibrium and convergence speed is fast when certain conditions are met. But the current application of congestion game is very restricted as it is challenging to formulate a problem into a congestion game. Besides, most of the current research leverage the primitive or standard congestion game model to study the problem with ideal assumptions, which further restricts congestion game's application in problem solving.

In this dissertation, we propose distributed schemes to solve a series of problems which reside from physical layer up to network layer in cognitive radio networks. We expand the application of congestion game and its variants in the problem solving. By designing the suitable utility function which involves local information or certain other information which is easy to obtain, unlicensed users' operation are formulated into a congestion game which permits convergence. We show that the distributed schemes designed with the assistance of congestion game model achieve comparable performance compared with the

centralized schemes which require big signaling overhead, and the distributed solutions are stable in the aspects of convergence. For secondary cellular networks which comply with IEEE 802.22 standard, we propose a solution for secondary users with assistance of the centralized database to exploit the TV spectrum which is underutilized. As one part of the solution, the distributed spectrum allocation problem is formulated into a canonical congestion game, then propose distributed algorithm enlightened by the player behaviours in the game. To form robust clusters against the ungovernable primary users' activity, a distributed clustering scheme is proposed. We formulate the clustering problem into a player specific singleton congestion game, from where a distributed algorithms is derived. As to routing in network layer in cognitive radio ad hoc networks, we propose geographic routing working with spectrum aware virtual coordinate assigned for the unlicensed users. The virtual coordinate integrates the spectrum availability between any pair of neighbouring secondary users, thus geographic routing can easily detour the unlicensed users which are affected by the active primary users.

## ACKNOWLEDGMENTS



This thesis is based on the following papers, which are referred in the text respectively.

Di. Li, Erwin. Fang, and James. Gross, "Robust clustering for ad hoc cognitive radio network," Transactions on Emerging Telecommunications Technologies, vol. 29, no. 5, p. e3285, Feb. 2018.

Di Li; Zhichao Lin; Mirko Stoffers; James Gross, "Spectrum aware virtual coordinates assignment and routing in multihop cognitive radio network," IFIP Networking Conference (IFIP Networking), 2015 , vol., no., pp.1,9, 20-22 May 2015 doi: 10.1109/IFIPNetworking.2015.7145337

Di Li; James Gross, "Distributed TV spectrum allocation for cognitive cellular network under game theoretical framework," Dynamic Spectrum Access Networks (DYSPAN), 2012 IEEE International Symposium on , vol., no., pp.327,338, 16-19 Oct. 2012 doi: 10.1109/DYSPAN.2012.6478156

Di Li; James Gross, "Robust Clustering of Ad-Hoc Cognitive Radio Networks under Opportunistic Spectrum Access," Communications (ICC), 2011 IEEE International Conference on , vol., no., pp.1,6, 5-9 June 2011 doi: 10.1109/icc.2011.5963426

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# 1

## Introduction

Wireless networks utilize radio waves instead of wires to carry information, which makes seamless coverage and mobility possible to realize. After decades of fast growth, there have been many wireless standards designed for different purposes. For example, wireless personal-area network (WPAN) connects devices which locate within a short range of 10 meters. The WPAN specifications i.e., Zigbee, Bluetooth are specified by IEEE 802.15 standards. Wireless local-area network (WLAN) enables mobile user to connect to a local area network (LAN) through wireless connection within a range of one hundred meters, where the technologies are specified by the IEEE 802.11 group of standards. WiFi is a successful realization of WLAN technology, which is based on IEEE 802.11 standard. Wireless metropolitan-area network (WMAN) provides connection with broadband speeds and larger coverage (from one hundred meters to several kilometres), where the technologies adopted are specified by IEEE 802.16. Third generation (3G) mobile communication systems have seen widespread deployment around the globe, which provide people with higher data rates and better quality of service (QoS). These technologies and standards, along with many others [122], fit different needs of various applications.

Due to the propagation characteristics and regulations, only the electromagnetic spectrum which spans from 8.3 kHz to 3000 GHz is suitable for commercial application. These spectrum is divided into bands (also referred as channels) and allocated to different networks and services. For example, WiFi and Bluetooth devices communicate in the 2.4 GHz radio frequency (RF) which is also known as industrial, scientific, and medical radio band (ISM band). Wimax networks work from 2 GHz to 66 GHz in the RF spectrum. As one kind of limited and precious resource, radio spectrum is strictly regulated by the national administration. The channels are assigned or leased by governments to different operators and entities. Usually the operators need to pay high price for the exclusive commercial usage of certain spectrum [44]. This spectrum management policy rules out the unlicensed users to use the spectrum, thus strictly protects the interest of the spectrum licensees. This spectrum access model is referred as *exclusive use model* [180]. Many existing wireless applications work on the licensed spectrum. For instance, the second-generation (2G) wireless cellular network GSM (global system for mobile communications) in Europe works with GSM-900 band (from 890 MHz to 960 MHz) and GSM-1800

(1710 MHz to 1880 MHz), and the third-generation (3G) wireless cellular network works from 1.8 GHz to 2.4 GHz [143].

Under certain conditions, the licensed spectrum which is granted to an operator or institute can also be accessed by other users. Open spectrum access [128] is proposed in 1995 for full openness of entry, which allows access to spectrum through access fees which are determined by demand and supply. It is claimed that open spectrum access yields benefits, as for spectrum licensees, the fixed costs on securing the licensed spectrum can be converted into marginal costs, and for the market, the incentives for collusive pricing can be eliminated. But as the *openness* here is fully controlled by the licensees, this spectrum usage is still exclusive .

In contrary to the exclusive use model, certain spectrum are assigned for open sharing for peer users, the representative example is the unlicensed ISM radio band, which supports versatile wireless applications and a prosperous industry, i.e., Wi-Fi.

The proliferation of wireless network constantly arises urgent demand on bandwidth and throughput since the first generation of telecommunication in 1980s, which accordingly encourages the efficient use and reuse of the electromagnetic spectrum. The next generation of telecommunication technology 5G, which is reported to come true in 2020, requires a leap forward of spectrum efficiency [89]. The resorted applications under the label of 5G [39], i.e., machine to machine communication, internet of things, etc, requires high speed and lower investment cost, but this is changeling as the achieved capacity approaches Shannon capacity, and most of the spectrum which are suitable to be used are already licensed. Meanwhile, the actual spectrum usage measurement conducted by FCC indicates that at many locations, the licensed spectrum is idle for most of the time [8], and there exists a large number of spectrum bands which have considerable dormant time intervals [21].

To seek more opportunities from spectrum, it is natural to consider opening the licensed spectrum to the unlicensed users when the spectrum is not used by the licensed users or the the licensed users' operation is not hampered. We adopt the terms proposed in [180] and call this spectrum usage policy as *hierarchical access model*, where licensed users are called primary users (PUs), and unlicensed users are named as secondary users (SUs). Hierarchical access model gives a promising solution to the reclamation of new electromagnetic spectrum and the improvement of spectrum efficiency.

## 1.1 Cognitive Radio Network

A cognitive radio network is composed with secondary users and work with the hierarchical access model. Throughout this thesis, we use *cognitive radio networks* to compound the networks composed with secondary users, there are two reasons, firstly, this name explicitly tells a distinctive property of the devices in the networks, which is their cognition ability to their environment, secondly, cognitive radio has become a synonym of the technology employed in the hierarchical spectrum access paradigm in academia and industry in the recent years.

The definition of cognitive radio evolves with the development of radio technology and regulations. We choose two representative definitions to draw a full picture of cognitive

radio. Cognitive radio (CR) is firstly proposed by Mitola III who defines the concept of CR in his dissertation [119] as follows:

The point in which personal digital assistants (PDAs) and the related networks are sufficiently computationally intelligent about radio resources and related computer-to-computer communications to detect user communications needs as a function of use context, and to provide radio resources and wireless services most appropriate to those needs.

FCC (Federal Communications Commission in U.S.) describes CR [5] as,

a radio that can change its transmitter parameters based on interaction with the environment in which it operates. . . . This interaction may involve active negotiations with other spectrum users and/or passive sensing and decision making (smart radio) within the radio. The majority of CRs will probably be SDRs <sup>1</sup>, but a CR does not necessarily use software, nor does it need to be field programmable.

The first definition depicts the functionalities of cognitive radio devices, and the second one mentions the relationships between cognitive radio users and the network which it resides. In this dissertation, we see cognitive radio as a device which is able to sense, detect, learn and monitor the surrounding radio frequency environment, or to access a certain database to retrieve primary users' information, so as to reconfigure its radio operating parameters (e.g., center frequency, bandwidth and transmit power) on the fly to avoid interfering primary users. Cognitive radio may only practice one portion of the aforementioned functionalities according to actual situation. Based on this definition, the cognitive radio network is deemed to be composed with cognitive radio users, whose acronym is CRN.

Throughout this dissertation, we use several different terms to denote cognitive radio devices, i.e., , cognitive radio users, secondary users, or secondary nodes in different contexts. In particular, when we want to stress the devices' cognitive ability on their environment, we use the term cognitive user or CR user, when we discuss spectrum sharing, and want to indicate the passiveness of the devices in terms of spectrum, we call them secondary users. If the CRN is depicted as a graph, and we emphasis on the network topology, then the term node will be used.

### 1.1.1 Functionalities of Cognitive Radio

To adapt to the dynamic spectrum environment and make use of the available licensed spectrum, cognitive radio users should carry out the following functionalities which are incorporated into a so called cognitive cycle as described in [20].

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<sup>1</sup>software defined radio is a radio communication system which is able to receive any modulation across a large frequency spectrum, and transmit on desired spectrum band.

### 1.1.1.1 Spectrum Sensing

To know which portion of the licensed spectrum is available is fundamental. In underlay spectrum usage scenario, CR users get to know the available spectrum by means of spectrum sensing. In overlay spectrum usage scenario, secondary users can in principle access all the licensed spectrum. In this procedure, quality of the available channels is determined with several parameters, i.e., bandwidth, operating frequency, path loss, wireless link errors, link layer delay, and the upper bound of interference on the primary user working on that channel, which decides the maximal permissible transmission power of secondary users. Besides, the statistical behaviours of primary users is also an important factor.

Spectrum administration bodies FCC of U.S. [10] and Electronic Communications Committee (ECC) in Europe [11] encourage to adopt both spectrum sensing and location based method.

### 1.1.1.2 Spectrum Decision

After knowing the available spectrum, CR users select the most appropriate band according to their requirements on quality of service (QoS). This procedure involves considering the statistical behaviours of the primary users so as to accessing the channel quality fairly. The available licensed spectrum which spans a wide frequency band exhibits different characteristics [103]. Based on the requirements of interested communication, CR users need to identify the characteristics of the spectrum, which include channel quality (channel capacity, error rate, path loss, etc.) [103], channel switching delay [27], and channel holding time, i.e., the expected time duration that the primary users don't occupy the channel before any one occupies again.

Some works in research community model the spectrum availability with stochastic or statistical model, which is helpful when deciding which channel to use. [113] proposes discrete Markov chain and adjusts duty cycle models to describe the availability of licensed spectrum for GSM on 900/1800 MHz. [163] models the channel holding time with geometric and log-normal distributions. Statistics of previous sensing results is used to predict spectrum state in the future [96]. Such models provide more complete information on the availability of the licensed channels.

### 1.1.1.3 Spectrum Sharing

Then secondary users are to make use of these channels selected in the spectrum decision procedure, and share the spectrum with both the primary users and the other secondary users. As there may be multiple secondary users trying to use the same channel, spectrum sharing is important to coordinate the behaviour of secondary users to avoid deteriorating the performance of secondary users. Spectrum sharing involves choosing proper channels to mitigate co-channel and adjacent interference [159], or adjusting transmission power to reach compromise between transmitter's and other secondary users' performance [154], or adopting a certain media access paradigm to use the spectrum fairly and efficiently. Spectrum sharing also involves the concerns from primary users, i.e., when many secondary users work on the same channel, the accumulated interference caused by secondary users could exceed the interference threshold on primary users.



#### 1.1.1.4 Spectrum Mobility

When one primary user starts to work on a certain channel from idle state, a secondary communicating pair working on the this channel should change their operating channel to another one, where no primary user is detected working on the new channel. This hand off process between channels is called spectrum mobility, which is critical to maintain the communication and network topology for CRN.

### 1.1.2 Spectrum Sharing in CRN

Spectrum sharing can be divided into two categories, the spectrum sharing between the primary users and secondary users, and the one among the secondary users. The first category is fundamental to the hierarchical access model, as it poses threat of interfering primary users. The second category decides the services that people can obtain from CRN. Thus, spectrum sharing largely defines the challenges and considerations one cognitive radio network faces.

#### 1.1.2.1 Spectrum Sharing between Primary and Secondary Users

As to the spectrum sharing between primary users and secondary users, there are two models, spectrum underlay and overlay. The two models origin from the distinctive understandings of people as to the hierarchical access model, and the different claims on how to effectively protect the primary users' interests.

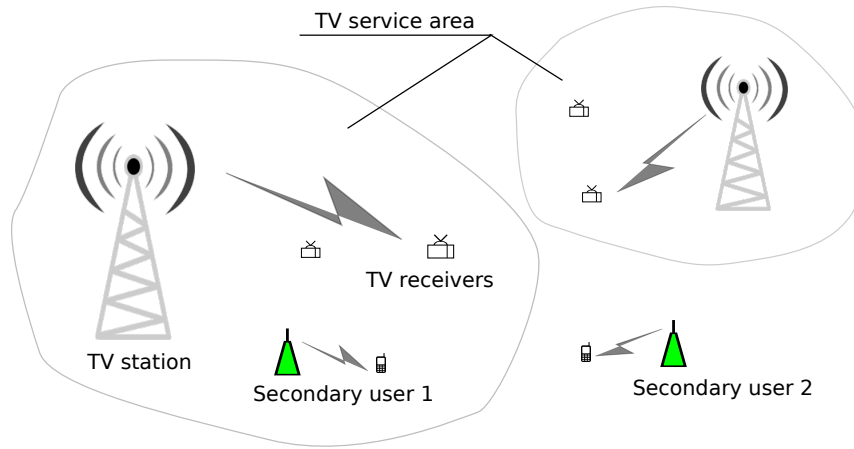
- Spectrum underlay: Working with *spectrum underlay*, secondary users are allowed to transmitting even when primary users are detected (or known via other means) to be operating on the same chunk of spectrum, as long as the interference generated by secondary users on the primary receivers is under a threshold. Spectrum underlay restricts the secondary users' transmission power, but secondary users are still able to achieve high data rate in short range. When implementing spectrum underlay, secondary users are allowed to operate in the close vicinity of primary users when the interference caused on primary users is carefully watched [36]. For instance, a there is a device need to monitor the interference level caused on primary users, and the secondary user need to be aware of this information via certain means [81].

Spectrum underlay is usually conducted in a centralized controller, which has global knowledge of primary users' locations, the attenuation between all secondary transmitters and all primary receivers. Then the centralized controller obtains the maximum permitted transmission power by solving an optimization problem where the interference generated by secondary users doesn't exceed the thread of primary users serves as one constraint. Working with spectrum underlay paradigm, spectrum sensing functionality on secondary users becomes auxiliary [10, 11].

- Spectrum overlay: The other approach is *spectrum overlay*, where secondary transmitters are only allowed to transmit when the primary users are detected as being idle, and the spectrum should be vacated if a primary user is detected transmitting. This paradigm requires secondary users to sense the spectrum of interest

pre-actively to detect primary users' appearance, accordingly entails burden on secondary users especially on the aspect of spectrum sensing. The spectrum sensing metrics include received primary users' signal power, spectral correlation or beacons [174]. Spectrum sensing requires sophisticated technologies when primary users' signal is weak, and can be improved by learning technologies or cooperation among multiple secondary users [22]. When spectrum sensing accuracy can be guaranteed above certain threshold, transmission power restrictions can be removed from secondary users.

In summary, spectrum overlay can be seen as time division among primary and secondary users, as comparison, spectrum underlay can be seen to be space (or power) division. Figure 1.1 shows a spectrum sharing scenario where the primary users are the TV systems. Secondary user 1 works in spectrum overlay by detecting and then using the spectrum, secondary user 2 works in spectrum underlay by refraining its transmission power.



**Figure 1.1** An illustration of underlay and overlay spectrum usage. The hull shows the range of a TV service area.

### 1.1.2.2 Spectrum Sharing among the Secondary Users

Among the secondary users, there is also competition on transmission power, spectrum bands or transmission time slots. When we put the consideration on protection of primary users aside, spectrum sharing among the second users is similar with the counterpart problems in the conventional wireless networks, i.e., ad hoc networks, cellular networks, etc.. In CRN, spectrum sharing among the secondary users can be conducted with either cooperative or non-cooperative manner. As to cooperative spectrum sharing, secondary users take into consideration of the caused interference on other secondary users when deciding their spectrum usage. This pattern usually requires cluster structure to facilitate the negotiation of users in a neighbourhood [49]. Whereas with non-cooperative spectrum sharing, secondary users only consider the performance of their own.

From the perspective of network operation, spectrum sharing can be classified as centralized and distributed spectrum sharing.

## 1.2 Solutions in CRN and Game Theory

We introduce the problems and corresponding solutions in CRN, and the application of game theory into the problem solving.

### 1.2.1 The Solutions in CRN

We introduced the solutions in CRN, which are categorised as centralized and distributed.

#### 1.2.1.1 Centralized Scheme

Centralized spectrum sharing is conducted on a centralized entity to decide the secondary users' strategies to operate in the CRN. There is a considerable number of centralized approaches proposed for spectrum sharing in cognitive radio network, and the global optimality is reported as to different objectives in certain cases. The centralized solution is usually realized by formulating the communication problems into optimization problems. Let us take the resource allocation problem in CRN as example. The resource refers transmission power, spectrum, transmission time, etc.. The objective can be the following, improving the overall SINR of secondary users, mitigation of co-channel interference caused secondary users, minimization of transmission power, etc.. The constraints can be one or several of the following, the interference caused by secondary users on primary users is below a threshold, the transmission power is restricted by the maximum transmission power, or the number of spectrum bands allowed for secondary users to use. There is a good number of research in this regard [17] [75, Chapter 6].

When the formulated problems are linear programming, or convex programming problems, the optimal is easy to obtain. When convexity doesn't hold, there are some skills to convert the problem into convex ones, e.g., a log change of variables turns an apparently non-convex problem into a convex one. When integer variables are involved in the optimization, e.g., in problem of channel assignment, or scheduling, the optimization problem is very difficult to solve. In this occasion, swarm intelligence algorithms can be applied to obtain sub optimal result, i.e., simulated annealing [117, 158], generic algorithm [48] and ant algorithm [78].

Centralized scheme is suitable in certain scenarios, e.g., when the primary users are TV stations and receivers which work on certain channels for hours of time, spectrum can be seen as constant. When the secondary users access the spectrum in underlay paradigm, they need to take care not to cause more interference than threshold on primary users. In this case the channel usage, i.e., working channel and transmission power, can be decided on the centralized controller. Nevertheless, centralized solution is not suitable in many scenarios of CRN. First, central authority or controller is not available in many CRNs, e.g., cognitive radio ad hoc network (CRAHN). Second, when the centralized decision maker exists, centralized solution involves a large number of control messages during network operation. The centralized entity needs to collect spectrum availability sensed on all the secondary users, then computes the spectrum usage strategy before finally distributing the result back to every secondary user. When primary users intensely access the spectrum and result in frequent change of available spectrum, the control overhead becomes a burden for the CRN. Third, Centralized solution need to react to the spectrum

variance in the CRN, even when the spectrum changes only happens at a few secondary users. This compiled agility worsens the overhead problem.

### 1.2.1.2 Distributed Scheme

It is the primary users which distinct CRN from other networks. Primary users' usually unexpected operation pattern results in dynamic spectrum availability for secondary users. As distributed schemes exploit mainly local observation, distributed schemes help the secondary users to adapt to the varying environment quickly [178].

Distributed solutions generally don't involve large amount of control overhead. When working with distributed spectrum sharing paradigm, secondary users in CRN decide on their spectrum usage strategies autonomously based on the local information, i.e., either on themselves or from their neighbourhoods. Consequently, distributed scheme usually requires secondary users to exchange information i.e., user ID, channel availability, etc. only with their neighbours, as a result, the overhead involved in the network is reduced. It is reported that control overhead takes account more than 50% of messages in most networks [75]. When the overhead is decreased, spectrum utilization and the number of served users and the network performance can be correspondingly improved.

In certain scenarios, distributed scheme may also make use of centralized entity when it is available and necessary, i.e., secondary users can access the centralized database to retrieve the available channels in IEEE 802.22 cellular networks.

Although distributed scheme results in appealing lightweight computation and overhead, it is actually challenging to design in many scenarios. When the distributed decision is not properly designed, it is possible to trigger endless ripple effect across the network, i.e., one CR's decision on its strategy prompts neighbouring CR users to change strategies, and it changes its strategy again later due to neighbours' new strategies. The distributed schemes designed for CRN can be roughly classified into the following categories.

- Distributed algorithms can be obtained from optimization formulation through decomposition techniques [134]. With this method, secondary users can achieve global optimal only with local information. The distributed algorithms from decomposition usually involve communication overhead between different subproblems, and the convergence process is hard to decide.
- Online learning. Online learning lies in the domain of artificial intelligence. The application of online learning in spectrum sharing in CRN includes Q-learning [61], no-regret learning [77,86], reinforcement learning [91]. Operating with this method, each secondary user seeks for suitable resources to maximize its utility function. Secondary users' operation is in distributed and asynchronized manner, and the CRN ceases to update in a correlated equilibrium. online learning has appealing property on distributed execution, but the convergence speed can not be guaranteed.
- Game theory. We will give comprehensive introduction in the following section.

## 1.2.2 Game Theory and Communication Systems

Game theory is the study of mathematical models of conflict and cooperation between intelligent rational decision-makers [?]. In the past few years, game theory has been extensively applied to problems in communication and networking systems [125, 162]. Due to the following reasons, game theory is adopted to understand many challenging problems in communication systems, such as resource allocation, topology control, routing, security and so on.

- **Communication Equipments are Rational.** Current communication equipments are supposed to be manufactured and operate based on standards to fulfil certain functions, but selfish behaviour may appear on certain individual equipments to achieve advantages over their peers [114]. For instance, a device in a communication system can be programmed on purpose to maximize the expected utility, which is selfish but is also regarded as *rational* from the perspective of the user using that device.

For example, Wi-Fi equipments are manufactured complying with the IEEE 802.11 standards. But it is possible that certain manufacturers or the personal who uses the Wi-Fi system (we use station in the following) manipulate certain parameters to obtain performance advantage over other stations in the network. When all the stations in one network are supposed to run distributed coordination function (DCF), i.e., each station must wait for a random period of time, which is called contention window (CW), before accessing the media when it senses the media to be busy, one certain selfish station may not choose to wait but is keen on sensing media. The selfish behaviour causes more collisions with other stations, and possibly results in poor performance in the network. In this example, the selfish network entities are seen to be self-interested rational, then suitable game theoretic models can guide the behaviours of them, so as to achieve desirable collective behaviours and performance in terms of the system level objective. In Wi-Fi system, game theory facilitates the network operator to issue rules to make the selfish behaviour unprofitable, which curbs the selfish behaviours and lead to NE in the network.

- **Game Theory is Effective to Solve Networking Problems.** Algorithms can be retrieved from the analysis of a problem under the game theoretical framework. In the same example of media access in IEEE 802.11 DCF mentioned in the previous item, if stations are allowed to modify the length of contention window, each station will adjust its contention window to obtain the best performance. [47] shows as long as each station greedily updates its CW to maximize certain utility, after certain time the system will stabilize in NE. In the case, the best response process naturally defines an algorithm for Wi-Fi systems. Besides, outcome from game theory is robust [75].

## 1.2.3 Game Theory and CRN

Cognitive radio network is a domain where conflicts are prevalent. The interests of CR users are conflicting because they compete for limited radio resource. Besides, the CRN operator's interests are not aligned with the CR users, e.g., the operator aims to maximize summed throughput, while CR users only care their own performance. In both cases,

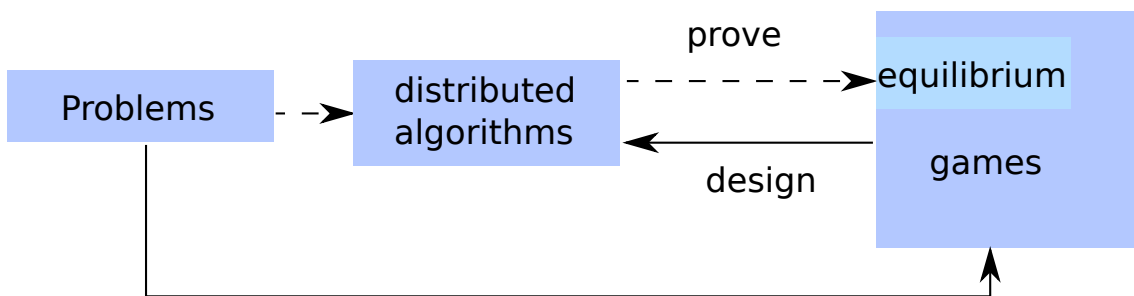
we need to formulate these non-cooperative problems into games, and then analyse and solve them as games.

In CRN, as introduced in Section 1.2.1.2, there is usually only local information is available to secondary users, then game theory is naturally suitable for designing distributed solutions for cognitive radio networks. Distributed solutions doesn't rely on centralized controller, and each user in the network adopts certain action as response to the other users or environment, this falls in the scope of game theory. As comparison, although optimization is the common tool to pursue global optimal, when the information is not accurate or adequate, or the optimization itself is difficult to solve, the optimized results will diverge from the optimal. Besides, the combinatorial nature of communication problems makes game theory as one of a few choices [75]. Many problems in wireless communication involve integer variables, i.e., channel availability, channel assignment, selection of modulation levels or channel coding. It is challenging to solve certain combinatorial optimization problems, whereas game theory is natural to describe it in a discrete form.

The most frequently used concept of equilibrium in game theory is Nash equilibrium (NE). NE defines a stable state of the system, where with the given policies, no player may gain by unilaterally deviating its current strategy. Nash equilibrium is usually sub optimal. NE in CRN means the CR user has no incentive to change its operating parameters e.g., transmission power, spectrum band, time slot, modulation, etc.. In comparison with optimization which seeks to improve the welfare of the complete network, game theory sheds light on the individual entities in a system. Thus a game can be seen as a collection of optimization problems, each of which maximizes/minimizes the utility/cost of one individual entity in the system. Actually, a distributed scheme can be regarded as a non-cooperative game formulation.

### 1.2.3.1 Application of Game in CRN

Game theory provides standard procedures to study its equilibriums [114]. Figure 1.2 illustrates two ways of applying game theory in the problem solving.



**Figure 1.2** The two ways that game theory is involved in problem solving

Along the dashed line and arrow, proposed distributed schemes can be examined under the game theoretic framework. In this way, a distributed algorithm is analysed by looking into the interaction among the users which operate the algorithm, and check whether the system reaches a desirable equilibrium, i.e., the individuals not only improve their own performance but also improve the global welfare of the complete system. In most cases, we can also inverse the this procedure to obtain distributed algorithms in CRN from certain known games [25]. As the solid line with arrow points shows, given a known

game which holds preferred properties such like fast convergence to equilibrium, or the game only has a single equilibrium which is also the global optimal, we can formulate our problem in CRN into that kind of game, then modify the best response of the players as the distributed algorithm for the communication entities in our problem.

In this dissertation, we look into congestion game and its applications in CRN. We leave the introduction of game theory and game models in the Chapter 2.

## 1.3 Problem Statement and Research Scope

As to the various challenging problems in CRN networks, adopting centralized or decentralized solutions is largely dependant on the natures of the networks, and the concrete problems to be addressed. CRAHNs consist of mobile autonomous secondary users and don't have infrastructure, besides, the network topology evolves with time due to users' mobility, and the ungovernable primary users' activity leads to temporal spectrum availability. Because of these factors, secondary users execute concurrently based on the local information, then the distributed scheme becomes the natural choice to manage the CRAHN to convey services. The opening of TV white space attracts people and organizations to build cellular networks to launch various services. A data base is required to register the fixed and mode II TVBDs [106], but it is not supposed to afford the computation on the resources for each TVBD. Besides, the operation of TVBDs is carried out by different users or operators, thus it is not feasible for the centralized entity to coordinate the devices and assign the network sources to them. Hence distributed scheme is also necessary in the utilization of TV white spectrum.

As introduce in Sections 1.2 and 1.2.1.2, game theory is suitable to design distributed schemes. Suitable game models can be used to analyse distributed scheme in terms of the final state of the system after convergence, and the speed to reach the equilibrium. In this thesis, we propose distributed schemes to solve the pressing problems about utilizing the unlicensed spectrum. Except for the provisioning of services, we are also interested in the mechanism the distributed algorithms. In particular, we look into the following questions, Does the distributed algorithm converge? If the algorithm converges, in which state the algorithm finally ceases, and how robust and quick the convergence process is? To answer these questions, we turn to game theory, in particular, the recent progress of congestion game models for help.

### 1.3.1 Research Questions

Based on the description of the challenges in cognitive radio networks, we formulate two research questions which will be answered in this thesis.

Question 1 - Given a problem, when distributed algorithms are necessary because of the specific properties of that problem, how to guarantee the convergence and performance? In particular, how to design utility function to make use of the game model when designing the algorithm?

Question 2 - What are the strengths and weaknesses of the distributed algorithm compared with other schemes, in particular, the centralized scheme?

### 1.3.1.1 Contributions

In this thesis, we answer the aforementioned research questions as contributions.

**Contribution C1** - We solve the problems in CRN by applying the congestion game in a pioneering manner, and design distributed schemes accordingly. Under the framework of the congestion game, we investigate the convergence property and overhead of the proposed distributed algorithms, and demonstrate how to refrain the overhead and fasten the convergence process.

**Contribution C2** - We conduct comprehensive comparison between the proposed scheme and the other distributed schemes and also centralized scheme with extensive performance evaluations, and point out the pros and cons of adopting distributed scheme to solve the problems in CRN.

These contributions are illustrated in the process of solving three concrete technical problems. Figure 1.3 illustrate the relation between contributions and the concrete technical problems to be addressed in this thesis.

	Q1: How to design distributed algorithm with converges fast	Q2: Comparison between distributed and centralized schemes
	C1: Algorithm design with congestion game	C2: Performance evaluation
P1: Utilization of TV white spectrum	S1: Distributed channel allocation	
P2: Robust cluster structure under primary users' activity		
P3: Light weight routing		
	S3: Spectrum aware virtual coordinates	

**Figure 1.3** Overview of the problems and contributions, and the concrete problems addressed in this thesis.

### 1.3.2 Research Scope

This thesis doesn't target designing new games, instead, it exploits the known congestion games [16, 136] to solve the pressing challenges existing in cognitive radio networks. We apply congestion game to solve channel allocation and network structure formation problems by designing suitable utility functions for the network units.

The concrete technical challenges addressed in this thesis are shown in Figure 1.4, which reside in physical layer, media access layer (MAC) layer and network layer [4]. These questions are individually orthogonal, but as pressing problems in each layers, they together form a test field to check the feasibility of distributed solutions.

#### 1.3.2.1 Problem 1: Utilization of TV White Spectrum

There are several problems in the current regulations and standards which are proposed on the utilization of TVWS. First, most of these proposals rely on the centralized database



OSI Layer	Functionality	Problems	Characteristics of solution
Network Layer	overlay	routing	distributed, non-cooperative
Link Layer	spectrum sharing overlay	clustering	distributed, non-cooperative
Physical Layer	spectrum sensing spectrum decision underlay	spectrum/power allocation in IEEE 802.22 network	distributed, cooperative

**Figure 1.4** Spectrum management and the examples discussed in this thesis, congestion game is applied in two of them.

to manage the spectrum usage in a centralized manner. This paradigm is not suitable when the TVBD belong to different business bodies, i.e., operators. Thus, a centralized resource allocation is infeasible. Next, the current usage of TVWS is prudent, i.e., on working channel and transmission power as introduced in Section 2.1.3. These conservative measures on the transmission power restrict the full utilization of TV spectrum. Third, although the transmission power of TVBD is restricted, the interference between TVBDs is not given consideration by any regulation or standard. In fact, the channel allocation problem we encountered in this problem is unique as the transmission power for each user and on each channel is different. In other words, the symmetric interaction doesn't exist, thus the solutions proposed for the channel assignment problems in conventional networks, e.g., ad hoc networks, or mesh networks don't work any more. The asymmetry disables the heuristic distributed schemes provided in [97], and makes channel allocation problem not to fit into the congestion game model proposed in [109] which is the first paper to discuss channel allocation from the perspective of game theory.

### 1.3.2.2 Problem 2: Robust Cluster Structure in Ad Hoc Cognitive Radio Network

Clustering is an important paving stone for the practical utilization of the unused portions of the licensed spectrum. Forming clusters with the secondary users which have proximity of geography and other properties produces several benefits. First, it is more efficient to solve common control channel (CCC) problem with cluster structure. Dedicated and static CCC which is allocated to all users is regarded to be under utilization and contrary to the opportunistic access paradigm. Whereas, cluster based approaches group CR users into clusters based on their similarity of available unlicensed channels, so that different common channels are used in various clusters to convey control messages [102]. Second, cluster structure facilitates cooperative sensing and increases the sensing reliability [156]. Third, cluster structure supports coordinated channel switching and routing in ad-hoc cognitive radio networks [152].

The benefits promised by cluster structure is challenged by the ungovernable (from the perspective of secondary users) activity of primary users. If the members of one cluster have to vacate all the CCCs due to primary users' appearance, the cluster can not be maintained any more. But the cluster survives if all the cluster members can migrate to another CCC after one CCC is occupied by primary user. Thus, a large number of

CCCs within cluster means high robustness against primary users. Unfortunately, related works [29, 87, 168, 179] fail to pay attention to this issue. There is a pressing need for one distributed clustering scheme, which should fulfil the following requirements. First, the execution of the scheme should be finished quickly, so that the scheme can react quickly enough to the primary users' activity. next, the scheme should achieve a proper compromise between the number of CCCs and cluster sizes.

### 1.3.2.3 Problem 3: Light Weight Routing in CRN

Recent measurement in [133] shows the spectrum occupancy doesn't have significant spatial correlations between different locations. It follows that licensed spectrum is used by primary users heavily in some areas, whereas in the other areas licensed spectrum is available over longer timespan for secondary users to use. It is obvious to see that a routing path is better to go through the areas where primary users occupation is lower, as this alleviates or avoids the burden to cope with the fast changing or totally occupied spectrum when forwarding packets potentially with latency requirements. Modified version of geographic routing is able to avoid the primary user influenced area, but it is myopic. considering the available spectrum is geographically heterogeneous, applying geographic routing alike routing schemes in CRAHN is appealing, but the supporting coordinate system is not complete. How to make light weight geographic routing work well in CRN is the problem to be addressed.

## 1.4 Outline

The structure of this thesis is as follows. In chapter 2, we introduce the theoretical tools used in design algorithms, i.e., game theory and optimization. Chapter 3 introduces the work on utilization of TV white space. The robust clustering scheme is introduced in Chapter 4. In Chapter 5, virtual coordinate based on geographic routing is designed and geographic routing runs on the top of it. Finally, Chapter 7 concludes the thesis by summarizing our contributions and discussing the future work.

# 2

## Background

In this chapter, we introduce several representative cognitive radio networks, which have open questions to be answered in the following chapters. As tool to solve the problem, game model and optimization techniques are introduced.

### 2.1 Cognitive Radio Networks

We introduce two types of cognitive radio networks, which contains the problems we will tackle in this dissertation.

#### 2.1.1 Cognitive Radio Ad Hoc Network

CRAHN is composed with autonomous mobile cognitive radio users which work CRAHN is one mobile ad hoc network, whereas each mobile user has cognitive ability for spectrum access [20]. Mobile users in CRAHN work with overlay spectrum sharing paradigm, thus need to determine which channels are available to use with spectrum sensing, either cooperatively with neighbour users or individually. Mobile users have different knowledge of spectrum availability, which poses new challenges in addition to the ones for the management of mobile ad hoc network. CRAHN can usually be represented by an undirected graph  $G$ . Cognitive radio users constitute the vertices, the edge between two vertices is decided not only by the distance, propagation and attenuation properties between the two vertices, but also the spectrum availability on both vertices, i.e., when they can decode the received signal from each other correctly, and there is common channel available between them on which communication is conducted, then an bidirectional edge is available on graph  $G$ . As to ad hoc network, the graph is constant when users are static. As to cognitive radio ad hoc network, due to primary users' activity, an edge between two vertices is decided by the fact that whether the two vertices can simultaneously access the same

licensed channel. Hence, the corresponding graph is dynamic under primary users' operation, which imposes extra difficulties on network organization, routing and many other network functionalities.

As a versatile and flexible network infrastructure, CRAHN is comparatively easy to implement and operate, thus CRAHN is proposed to apply in many scenarios [20]. There are several open issues for CRAHN. For instance, a robust architecture against the network dynamics caused by user' mobility, primary users' activity is still due. It is challenging for CRAHN to meet the QoS criteria with the spatial and temporal availability of spectrum [116].

### 2.1.2 Cognitive Radio Cellular Network

According to Cisco's forecast [54] on mobile data traffic, which published in 2015, the number of mobile-connected devices exceeded the world's population in 2014 and by 2019 there will be nearly 1.5 mobile devices per capita. The monthly global mobile data traffic will surpass 24.3 exabytes by 2019, which is 10 times more than that in 2014. Although there is a strong trend that more traffic load will be offloaded from cellular networks (on to Wi-Fi), cellular networks still face the big challenge and need to explore new resource to meet the demand on increased traffic. The state of the art mobile systems, such as 3GPP Long term Evolution (LTE) and High Speed Packet Access (HSPA) are designed to be very efficient on spectrum utilization, thus cellular networks look into cognitive radio for solution.

### 2.1.3 CRN Working in TV White Space

TV white space (TVWS) refers to the unused broadcast television spectrum, which is the most feasible spectrum available for large scale utilization for cognitive radio cellular networks. One part of the unused TV spectrum is resulted from the switchover of analog television to digital television, which frees up a large range of TV spectrum. The other parts are unassigned spectrum and the spectrum which is not being used at a given time in a given geographical area. TV white space is located in the VHF (54-216 MHz) and UHF (470-698 MHz) bands, thus the frequency bands within this spectrum are highly desirable for wireless communications, because they have good properties on propagation, i.e., they have broadband payload capacity, and are able to penetrate obstacles such as trees, buildings, and rugged terrain in non-line-of-sight scenarios. As comparison, a Wi-Fi router's transmission range, which uses 2.4 GHz radio frequency and complies with IEEE 802.11, is around 100 meters under perfect conditions, and the signals can be blocked by barriers, whereas, TVWS band can cover an area of about 10 kilometres in diameter. In 2010, FCC allowed [10] unlicensed devices to operate in the TV white space to provide broadband communication for consumers and business.

Autonomous spectrum sensing is one approach for secondary users to decide the available TV spectrum. Secondary user scans certain parts of the TV spectrum band to detect the presence of existing TV equipments with advanced spectrum sensing techniques and algorithms. This scheme involves complex sensing techniques, and it doesn't work well in worst case sensing scenario, i.e., the TV receivers are hidden nodes [93]. Moreover, it is required to be conservative when deciding the available TV channels leading to an

underestimation of available TV channels and causing inefficiencies [74]. Considering the slow change of TV spectrum usage, along with the fact that the spectrum usage by TV stations is scheduled, the geolocation approach together with central database becomes more appealing in TVWS utilization scenario. FCC adopts this solution as the main way, and regulates the secondary usage of TV white space in a prudent manner, including the spectrum bands permitted to use based on their location, the transmission power, the distance away from TV service area and so on.

The potential applications supported by TVWS are the major driving force for TVWS communications technology. It is envisioned that TVWS can support high-data-rate backbone for fixed stations in large area connectivity, short-range indoor connectivity, seamless connectivity for mobile stations and emergency related equipment. In the following, we give a brief introduction to the governing regulations and industrial standards issued for TVWS usage.

### 2.1.3.1 Regulations and Standards on TV White Spectrum

Regulatory bodies in different countries have issued requirements on the occupancy of TVWS bands by secondary users respectively.

FCC regulates portable secondary users to operate from channel 21 (512 MHz) to 51 (698 MHz), with the exception of channel 37. As to the fixed secondary users, the allowed spectrum band is from TV channel 2 (54 MHz) to TV channel 51, with TV channels 3, 4 and 37 being prohibited. Thus, the available TV spectrum is about 600 MHz wide. Compared to conventional unlicensed ISM bands in the 2.4 GHz and 5 GHz band, all together TVWS has more to offer. [76] analyses the capacity possibly provided by exploiting free TV channels. Complying with the rigid regulation of FCC, TV white space brings hundreds of kilobits/sec per square kilometre for secondary users when the communicating secondary pair is about 1 km apart. The research shows that interference from TV stations and other co-channel secondary users heavily restricts the capacity. [37] further investigates the possible TVWS usage *within* TV service area (referred to as *gray space*), where the secondary usage does not violate TV receivers. The authors propose another significant amount of TV spectrum, but the interference from TV broadcasters is stronger due to the secondary receivers being closer to TV broadcasters. Besides, FCC divides the devices operating in the TVWS (TV band devices, or TVBDs) into three categories: *fixed device*, *mode I personal/portable device*, and *mode II personal/portable device*. Every category obeys distinct rules on transmission power, mandatory database access and so on. Fixed and mode II devices are connected to the database directly (or indirectly by another fixed or mode II TVBD), fixed device accesses database at least once per day, mode II device has to access database every 60 seconds, or when it changes location by more than 100 meters. The transmission power for fixed TVBD is 4 W, and 100 mW for mode II TVBD. Mode I TVBD operates only under the control of a fixed or mode II TVBD, and accesses them for available channels every 60 seconds. There is also regulations on the antenna heights and other aspects.

OFCOM (the regulatory authority in UK) opens less TVWS for TVBD and categorizes White Space Devices (WSDs) into *master WSD* and *slave WSD* which are roughly equivalent to fixed/mode II TVBD and mode I TVBD, respectively. ECC (the regulatory authority in Europe) also adopts Master and Slave structure while utilizing the geolocation and database solution to assign channels and corresponding operating parameters to TVBD.

In both FCC and ECC regulations, the available channels are calculated based on the distance between primary and secondary systems together with propagation model. Hence, the principle behind the notion of channel availability is the presumed received signal strength at potential TV receivers, which should be obviously below a certain level. FCC adopts a fixed transmit power level for secondary systems and assumes that the distance adopted (resulting from the propagation model) is sufficient to protect the TV receivers. ECC's restriction is more flexible on the transmission power which can be adjusted based on the distance between the secondary user and the TV receivers. For both regulations, TV receivers may become vulnerable when there are multiple secondary equipments transmitting simultaneously. [92] shows that even the regulations integrate an interference margin for multiple secondary users' transmitting, the TV receivers are still vulnerable.

Standardization on secondary usage of TVWS is very active. IEEE 802.19 is a family of IEEE 802 standards in Wireless Coexistence Working Group, among which, IEEE 802.19.1 [7] is for coexistence methods among local and Metropolitan Area Networks (MAN). IEEE 802.11af [64] is standards for WLAN operation in TVWS, IEEE 802.15.4m [6] is for low rate (LR) WPAN operating in TVWS. IEEE 802.22 [3] is for Wireless Regional Area Networks (WRAN) using TVWS. As TV band is only 6 or 7 MHz while WLAN channel bandwidth is 40 MHz for state-of-the-art IEEE 802.11n amendment, IEEE 802.11af proposes channel aggregation mechanism for WLAN working with TVWS. Maximum transmit power and spectrum mask of mode I should be approved by the mode II devices beforehand.

IEEE 802.22 is the first IEEE standard for wireless regional area network (WRAN) working on TV white space. According to this standard, the system consists of a base station and customer premises equipment (or terminals for short), where terminals are associated with base stations, and are served by them. Recent standard published in Nov. 2010 mentions both, i.e. sensing ability as well as TVWS geolocation and database look-up as schemes for detecting primary systems. Self-coexistence mechanism is also proposed, which provides a TDMA like mechanism for white base stations (WBS) to share TVWS.

## 2.2 Introduction of Game

Game theory is a branch of applied mathematics, and it is a powerful mathematical tool for studying, modelling and analysing the interactions among individuals. The approaches derived from a game are complementary to the optimization methods, thus game theory serves as an important role for the communication system when the optimization method doesn't work out. In this dissertation, game theory is the mathematical tool which helps us analyse problems and design algorithms, but we also adopt optimization to achieve the global optima. In the following sections, we will introduce game theory (the notations used in this thesis comply with [127]) and optimization techniques. As to the latter, we in particular introduce the decomposition method which is used to design distributed algorithms.

A game consists of three elements: a set of players, a selfish utility for each player, and a set of feasible strategy space for each player. In a game, the players are rational and intelligent decision makers, which are related with one explicit formalized incentive expression (the utility or cost). The players usually have potential conflicting objectives with others. We say the players are rational as they make decisions in compliance with

Elements of a game	Components of one CRN
Player $\mathcal{N}$	secondary users
Strategies for player $i$ , $\mathcal{S}_i$	working channels, transmission power, modulation, etc.
Utility of player $i$ , $u_i$	performance in respect of SINR, throughput, etc.

**Table 2.1** Components of problems in CRN and corresponding elements in game

the subjective which is either to maximize the their own expected utilities, or to minimize the expected cost. Player are said to be intelligent as they understand the other players are also rational.

A game of normal form can be represented as a tuple  $\Gamma = (\mathcal{N}, (\Sigma_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$ , where

- $\mathcal{N}$  is a finite set of players.
- $\Sigma_i$  is player  $i$ ' set of strategies. Player  $i$  selects one strategy  $\sigma_i \in \Sigma_i$  at one time to play the game.
- $\Sigma = \Sigma_1 \times \cdots \times \Sigma_n$  is the set of states, which denotes all the possible ways that players may pick strategies.
- $\sigma = (\sigma_1, \dots, \sigma_n)$  is one strategy, which represents an instance of all players' strategy choices.  $\sigma$  is also called one strategy profile, and there is  $\sigma \in \Sigma$ .
- One vector of strategies of player  $i$ ' opponents is expressed as  $\sigma_{-i}$ , and the corresponding strategy profile is denoted as  $\sigma = \{\sigma_i, \sigma_{-i}\}$ .  $u_i(\sigma) = u_i(\sigma_i, \sigma_{-i})$  is the player  $i$ ' outcome in strategy profile  $\sigma$ .
- $u_i : \Sigma \rightarrow \mathbb{R}$  is the utility function of player  $i$ .  $\Sigma_{-i} = \prod_{j \in \mathcal{N} \setminus \{i\}} \Sigma_j$  is the set of states of all the other players except for player  $i$ . As to each player  $i$ , its utility is decided by its choice on strategy  $\sigma_i \in \Sigma_i$ , and is also dependant on the choices of other players  $\sigma_{-i} \in \Sigma_{-i}$ . Utility can also be denoted as  $u_i(\sigma)$  or  $u_i(\sigma_i, \sigma_{-i})$  to stress that the utility is made based on all players' strategies.

Utility function is very important to specify a game, as it gives players preferences on the outcomes with respect to all strategy vectors  $\Sigma$ . The value of the utility can be regarded as payoffs or costs depending on concrete scenarios. Actually, one player maximizes its utility is equivalent to that the player minimizes its cost, note that cost is the reversed utility.

When we want to use game theory to analyse a problem in CRN, it is critical to formulate appropriate components of the problem into corresponding elements of a game. The commonly used formulation is summarized in Table 2.1.

### 2.2.1 Basic Solution Concepts

In this section, we will introduce some basic solution concepts, some of them are used in this thesis.

### 2.2.1.1 Dominant Strategy Equilibrium

In some games, each player has one unique best strategy, which is independent of the strategies chosen by the other players, then we say a game of this kind has a dominant strategy solution. The mathematical expression is as follows, when a strategy vector  $s \in \mathcal{S}$  is a dominant strategy, for each player  $i$  and each alternate strategy  $s' \in \mathcal{S}$ , there is,

$$u_i(s_i, s'_{-i}) > u_i(s'_i, s'_{-i})$$

in this case,  $s$  is also called strong dominant strategy, and when the  $>$  can be written as  $\geq$ ,  $s$  is called weak dominate strategy. Note that a dominant strategy solution may not give an optimal payoff to any of the players. This is the case in *prisoner's dilemma*, which is one of the most well known and studied games (Chapter 1, [127]). To confess is the dominant solution for both prisoners, which brings them longer time behind bars than that when both of them keep silent<sup>1</sup>. Only a few games have dominant strategy equilibrium, and mechanism design [88] is developed to design games which have dominate strategy equilibrium, and these dominate strategies lead to desirable outcome.

### 2.2.1.2 Nash Equilibrium

A desirable solution of games is the one that players choose strategy in accordance with their incentives, minimizing their own cost or maximizing their own payoff/utility. Nash equilibrium successfully captures this property, and is the most discussed and pursued solution concept in game theory.

A strategy vector  $s \in \mathcal{S}$  is a *Nash equilibrium* if for any player  $i$  and each alternate strategy  $\sigma'_i$ , there is

$$u_i(\sigma_i, \sigma_{-i}) \geq u_i(\sigma'_i, \sigma_{-i})$$

This means for any player in NE state, no unilateral deviation from its current strategy is more profitable. This also implies that, NE is self enforcing that once players agree on this solution, it is the best interest for every player to stick to its current strategy.

A dominant strategy equilibrium is a Nash equilibrium, but a NE is not necessarily a dominant strategy. There may be multiple NEs in one game, and NE may not be optimal for players. In prisoner dilemma, the dominating strategy is NE but is obviously not the optimal. Being not unique and possibly sub-optimal, NE is still applied in extremely diverse applications due to the reasons discussed in the beginning of this section. Thus, when pursuing NE as solution, people should answer the question that, what is the gap between NE and global optimal? This can be partially answered by *price of anarchy* (*PoA*), the ratio between the worst-case Nash equilibrium to the optimality is used to denote the quality of the solution.

### 2.2.1.3 Complexity of Obtaining NE

Nash equilibrium is an appealing concept, but it doesn't tell how to obtain such a state. Hence, it is important to find an efficient algorithm to reach the equilibrium. The notion of

<sup>1</sup>Prisoner's dilemma can be found in almost all the game theory textbooks, thus omitted in this thesis.



*NP-completeness* is not an appropriate concept of complexity for the problem of *finding a Nash equilibrium*, because NE always exists, nevertheless, theoretical scientists tell that *finding a Nash equilibrium* problem is a combinatorial problem and can be very difficult to solve (Chapter 2, [127]). Having said that, in some games, with the special strategy space structure, or players' special behaviours, there exist efficient algorithms to reach NE.

In the aforementioned games, players deterministically choose one strategy from their strategy sets, then play their chosen strategies and don't involve randomized strategies, then the achieved NE is called pure NE. When players play a game with certain randomization on strategies, and aim to maximize their expected payoff, we call the resulted NE as mixed NE. As to mixed NE, the action of a player is to choose certain strategies according to a probability distribution. Games with finite number of players and strategy set are guaranteed to have NE, whereas, the games with infinite number of players, or the games with a finite number of players but the players can access an infinite strategy set may not have NE [127].

In this thesis, we always look for pure NE, as players deterministically play a certain strategy, and the corresponding network components stick to certain operation instead to switch among several different operations based on a vector of probabilities. Other solutions include correlated equilibrium, which also involves probability distribution over strategy vectors.

## 2.3 Congestion Game

Congestion game is an attractive game model which describes the problem where participants compete for limited resources in a non-cooperative manner, it has good property that Nash equilibrium can be achieved after finite steps of best response dynamic, i.e., each player choose strategy to maximizes/minimizes its utility/cost with respect to the other players' strategies. Congestion game has been used to model certain problems in internet-centric applications or cloud computing, where self-interested clients compete for the centralized resources and meanwhile interact with each other. For example, server selection is involved in distributed computing platforms [46], or users downloading files from cloud, etc. In the following we will introduce an *server matching* [98] problem to illustrate congestion game's application in communication systems.

In accordance with [161], congestion game is a game where players simultaneously allocate sets of resources to minimize their costs, and the cost of a resource is a function of congestion, which is the number of players choosing the resource. Congestion games can be formulated from many problems in realistic world, e.g., minimisation of commuting time on the road for commuters, minimization of energy consumption in mobile cloud computing system [69].

Now we give the formal definition of a congestion game. A congestion game [145] [161] can be expressed by a tuple  $\lambda = (\mathcal{N}, \mathcal{R}, (\Sigma_i)_{i \in \mathcal{N}}, (g_r)_{r \in \mathcal{R}})$ , where

- $\mathcal{N} = \{1, \dots, N\}$  denotes the set of players (each each is labelled with a unique index number)
- $\mathcal{R} = \{1, \dots, m\}$  the set of resources

- $\Sigma_i$  is the set of resources for player to use.
- $\Sigma_{i \in \mathcal{N}} \subseteq 2^{\mathcal{R}}$  is the strategy space of player  $i$ . Under strategy profile  $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)$ , player  $i$  chooses strategy  $\sigma_i \in \Sigma_i$ , and the total number of users using resource  $r$  is  $n_r(\sigma) = |\{i \mid r \in \sigma_i\}|$ .
- The cost  $g_r : \mathbb{N} \rightarrow \mathbb{Z}$  is a function of the number of users for resource  $r$ ,  $g_r^i = \sum_{r \in \sigma_i} g_r(n_r(\sigma))$ .

In this thesis,  $g_r^i$  is referred as *congestion* of a game.

We first give a definition of improvement.

**DEFINITION 1:** An improvement path is a path  $(\sigma^0, \sigma^1, \dots)$  in which  $u_i(\sigma^k) < u_i(\sigma^{k+1})$ , where  $\sigma^k$  and  $\sigma^{k+1}$  differ in the player  $i$ 's coordination, i.e., only  $i$ 's strategy changes in the strategy  $\sigma = \sigma_1, \sigma_2, \dots, \sigma_{|\mathcal{N}|}$ .

The best response path contains only states for best response improvement steps.

**THEOREM 2.3.1:** For every congestion game, every sequence of improvement steps is finite. [145]

This theorem immediately implies the following corollary,

**COROLLARY 2.3.1:** Every congestion game has at least one pure Nash equilibrium. [145]

To prove this, let us first have a look at one similar game, potential game. Potential game [123] has already been applied in wireless network and CRN to help solve different problems [67, 101, 126, 129]. Potential game has two desiring properties, the existence of pure NE, and every improvement path is finite, which makes it a suitable model to design distributed algorithms.

### 2.3.1 Potential Game

A potential game is a tuple  $\lambda = (\mathcal{N}, (\Sigma_i)_{i \in \mathcal{N}}, (u_i)_{i \in \mathcal{N}})$ , which satisfies the following, if there exists a function  $\phi : \Sigma \rightarrow \mathbb{R}$ , such that for every  $i \in \mathcal{N}$ , for every  $\sigma_{-i} \in \Sigma_{-i}$ , and every  $\sigma_i, \sigma'_i \in \Sigma_i$ :

$$u_i(\sigma_i, \sigma_{-i}) - u_i(\sigma'_i, \sigma_{-i}) \geq \phi_i(\sigma_i, \sigma_{-i}) - \phi_i(\sigma'_i, \sigma_{-i}) \quad (2.1)$$

If equality holds, this game is called an exact potential game, when it doesn't, the game is called an ordinal potential game. It is easy to see that the design of potential function  $\phi$  is the key point to form a potential game. The incentive of any player to change its strategy can be expressed using the potential function, in other words, potential game tracks the changes in the payoff/cost with some player deviates.

Potential game has two important theorems,

- Every finite ordinal potential game has at least one pure strategy Nash equilibrium.
- In every finite ordinal potential game, every improvement path is finite.

### 2.3.2 Convergence Time Towards Nash Equilibrium

As to the Theorem 2.3.1 which implies that for every congestion game, every best response ends in finite steps, we introduce the sketch of the proof of this proposition, as the proof also reveals the number of steps needed to reach a Nash Equilibrium. We first introduce Rosenthal's potential function  $\phi(s) : \Sigma \rightarrow \mathbb{Z}$ , where  $\sigma$  is the strategy profile of all the players, let  $\sigma = \sigma_1, \sigma_2, \dots, \sigma_N$ :

$$\begin{aligned}\phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\ &= \sum_{i \in \mathcal{N}} \sum_{r \in s_i} g_r(n_r^i(\sigma))\end{aligned}\tag{2.2}$$

$n_r^i(\sigma)$  is the number of players using resource  $r$ , whose indices are smaller than or equal to  $i$ , i.e., from  $\{1, \dots, i\}$ . In the part after the second equality sign,  $\sum_{r \in s_i} g_r(n_r^i(\sigma))$  is a virtual value or cost that player  $i$  have when assuming the resource  $r$  is not used by players whose indices are greater than  $i$ .

The intuitive interpretation of the virtual cost is, according to [161], the cost of each player for choosing the strategy when it is inserted into the game. Let us assume player  $N$  is the last player to be inserted into the game, then  $\sum_{r \in s_N} g_r(n_r^N(\sigma))$  equals to the real cost that player  $n$  takes. When player  $N$  can decrease its cost by switching to another strategy by an unilateral move, then  $\sum_{r \in s_N} g_r(n_r^N(\sigma))$  and its potential decrease by the same amount.

As the potential can be calculated by inserting the players with any sequence, each player will have the same property with play  $N$ , as we just discussed.

In summery, the change of the potential caused by one player's unilateral move from  $\sigma_i$  to  $\sigma'_i$  is equivalent to the change of gain (or loss) of that player.

$$\Delta\phi(\sigma_i \rightarrow \sigma'_i) = g^i(\sigma', \sigma_{-i}) - g^i(\sigma, \sigma_{-i})\tag{2.3}$$

$\sigma_{-i}$  is the strategy profile for all players except for  $i$ . Thus the potential decreases with the update of players monotonically during the convergence process. Most importantly, as the potential of a congestion game is bounded, and every best response move made by a player decreases it at least by 1, the length of any sequence of improvement steps is also finite. Then the Theorem 2.3.1 is proved.

The existence of Nash equilibria gives a natural solution concept for congestion games. Comparing with potential game, congestion game is much easier to connect with the problems in networks or CRN. As long as we can decide the payoff (or cost) happened upon a certain resource, i.e., spectrum bands, time slots, transmission power is monotonic increasing (or decreasing), we can formulate the problem into one congestion game, and we can make use of the property of congestion game to implement best response to achieve Nash equilibrium.

### 2.3.3 Singleton Congestion Game

We introduce a special type of congestion game [90], which implies polynomial number of best response steps towards convergence.

**DEFINITION 2:** A congestion game is called *singleton* if, for every  $i \in \mathcal{N}$  and every  $\mathcal{R} \in \Sigma_i$ , it holds that  $|\mathcal{R}| = 1$ .

In words, each player allocates one single resource from its strategy set. Although this constraint on the players strategies is very restrictive, there are still  $m^n$  different strategy profiles.

**THEOREM 2.3.2:** In singleton congestion games, all improvement sequences have length  $\mathcal{O}(n^2m)$  [90]

The sketch of the proof is as follows. The upper bound on the number of convergence steps is shown in Formula 2.2. Now we need to approximate this expression. Replace original delays by smaller integer values without changing the preferences of the player, then calculate an upper bound on the maximum potential with respect to the new delays. For example, if the delay on a certain resource is 15, 50, 100 with respect to 1, 2 and 3 players using that resource, then we manipulate the delays as 1, 2, 3. When the number of resource is  $m$  and number of players is  $n$ , the largest new delay is  $nm$ . Then the Rosenthal's potential in Formula 2.2 has the following relationships with an upper bound.

$$\begin{aligned}
 \phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\
 &\leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} (nm) \\
 &\leq n^2m
 \end{aligned} \tag{2.4}$$

Therefore, the length of improvement sequences is upper-bounded by  $n^2m$ . The proof in details can be found in [83,90]

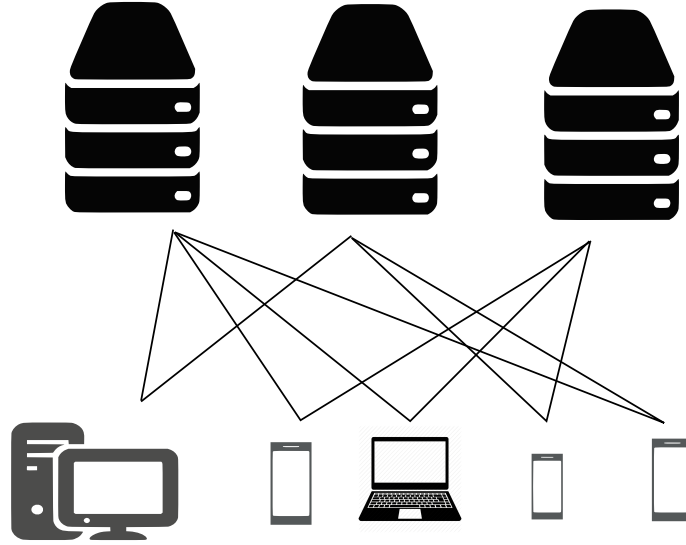
### 2.3.3.1 Example: Server Matching

In order to show how is problem in networks formulated into a game, and from which an effective algorithm is proposed, we introduce a small example with the application of singleton congestion game.

Let us consider a number of self-interested clients and servers as shown in Figure 2.1. Each client is allowed to access one server. The latency of one server is a monotonic increasing function of the *number of clients* attached to it. When clients try to choose one server which has the shortest latency, a congestion game is formed.

Following the description in Section 2.3, we formulate the congestion game for this problem,

- The clients constitute the players in the game  $\mathcal{N}$ , and the collection of servers  $\mathcal{M}_i$  is the strategy space for player  $i \in \mathcal{N}$ .



**Figure 2.1** An example server matching problem. A client is allowed to access only one server. The line illustrates that the corresponding server is within the client's strategy space.

- The cost of a player in the game is equivalent to the latency experienced by the client in the problem. If we denote the delay happens on the server  $\alpha$  as  $d_\alpha(n)$ , where  $n$  is the number of clients accessing the server, then the delay (cost) for a client (player)  $i$  is  $d_\alpha(n_r(\sigma))$ , where  $n_r(\sigma)$ ,  $r \in \sigma_i$  is the number of clients using the server  $r$ .
- As each user is allowed to use on server, this is a singleton congestion game according to Definition 2.

*Solution:* The solution for clients to match the servers is explicitly given by the proof of 2.3.2. When the clients adopt best response strategy, i.e., each player chooses the server which causes it the shortest delay with respect to other clients' choices, after a number of updates on the servers, the system reaches at NE, i.e., the players cease to switch as they have the shortest delay in that state of NE. According to Theorem 2.3.1 and 2.3.2, the algorithm converges at Nash equilibrium after maximal  $nm$  times of best responses.

### 2.3.4 Variants of Congestion Game

In the congestion game we have introduced in Section 2.3, the cost caused on one resource is function of the number of players using that resource. In some variants of congestion game, the cost is decided by some other factors. In *player specific congestion game*, different players may have different delay functions. As to *weighted congestion game*, different players may have different impacts on the delays of the resources they allocate, in other words, there is a weight for every player, and the congestion on a resource is the total weight of all players using that resource.

As to neither player specific congestion game nor weighted congestion game, the finite converge does not hold anymore. But there exists NE for both of Player specific congestion game and weighted congestion game when they are singleton congestion game [65, 118]. When these variant of congestion game are not singleton ones, [16] points that NE exists when the strategy space of players are matroid.

## 2.4 Optimization

A variety of problems in CRN are formulated and solved by constrained optimization [32, 170, 176]. As to a player in a game, given the strategies of other players  $\sigma_{-i}$ , its utility  $u_i$  is a function of its own strategy  $\sigma_i$ , then the maximization of its payoff or minimization of its cost becomes one optimization problem. Thus, one game comprises  $n$  such optimization problems in total.

Table 2.2 summarizes the commonly proposed objectives, involved resources and constraints in the optimization problems in the domain of wireless communication. Part of the contents in the table refers [75]. As to CRN, the interference caused by secondary users on primary users is usually formulated into constraints, which distinguishes the problems in CRN from that in traditional wireless networks.

Convexity is a pursued property in optimization problem solving [40]. As to a convex optimization i.e., minimizing a convex objective function over a convex constraint set, a local optimum is also a global optimum and the duality gap is zero, and there are provably polynomial-time algorithms to solve the convex optimization, i.e., interior-point methods. It is common to conduct optimization in centralized manner in small scale network, but in large scale network, centralized solution is infeasible for several reasons. First, it is hard to obtain the information resides across the network, especially when the information is varying, i.e., the spectrum availability at secondary users in CRN. Second, it is hard for the centralized scheme to scale with the network [134].

### 2.4.1 Decomposition Methods

The decomposition methods are originally proposed to solve very large optimization problems, but as a side effect, it also yields decentralized solution algorithms, which is important in many scenarios of wireless network [41]. In certain cases, decentralized solutions can be interpreted as a set of simple protocols which allow a collection of subsystems to coordinate so that to achieve global optimality.

*Decomposition method* decomposes the original problem into distributively solvable subproblems. Specifically, the decomposition techniques includes *primal decomposition* and *dual decomposition*. The primal decomposition decouples variables, i.e., decomposes the original problem by allocating the available resources to each subproblem, and derived problem formulation is *master problem*. The dual decomposition solves the subproblems and decouples the constraints usually by forming Lagrangian [35, 134]. There is communication between the master problem and the subproblems. The communication is in the form of message passing which introduces overhead in the network. In some cases, however, the communication is implicit in the system, e.g., delay, packet error probability, as these quantities have physical meanings and can be implicitly measured without the need of explicit signaling. The different ways to decouple the resources lead to different algorithms which have different characteristics of the amount of communication overhead and convergence properties.

We take the joint channel and power allocation problem in CRN [157] as an example, to show how is one distributed algorithms obtained by decomposing the optimization problem. To eliminate the impact of the coupling optimization variable, i.e., channel and

power levels, primal decomposition decomposes the original problem into two subproblems, channel allocation subproblem and power control subproblem. After solving the first subproblem of channel allocation, the power allocation subproblem is solved using dual-decomposition method, where the Lagrangian variables are calculated in decentralized manner with iterative sub-gradient algorithm. Solving the two subproblems sequentially in one iteration, and the iterations cease when the change of the objective is below a given stopping criterion. When the original problem is assumed convex, the derived distributed algorithms could have a provable convergence to the global optimum. The speed of convergence is difficult to quantify and will depend on how is the original problem is decomposed, i.e., the number of levels in the decomposition, the amount of signaling, and the particular combination of numerical algorithms employed.

	Parameters	Optimization goals	Constraints
Application layer	source-coding rate, buffer priority, packet arrival rate	minimal delay	base layer transmission, strict delay requirement
Network layer	routing path	end to end delay/throughput	maximal hops, security concerns
MAC layer	transmission frequency, transmission priorities	maximal overall throughput, minimal buffer overflow probability	contentions, time/frequency slot
Physical layer	transmission power, modulation, channel coding rate, spectrum bandwidth	minimal over power consumption, maximal throughput, minimal BER, maximize product of bandwidth of power [157]	maximal transmission power, interference on licensed users, available channel coding rate

**Table 2.2:** Optimization problem of cognitive radio networks



# 3

## Spectrum Allocation in TV White Space complying with ECC Rulings

### Abstract

In this chapter, we will see the application of congestion game in solving the channel allocation problem in the context of TV white space. The channel allocation problem we will address is a general problem, as the transmission power is not identical for every transmitter and on each channel, actually, the transmission power could be unique for each transmitter-channel combination. With the suitable utility function designed for transmitters, the behaviours of the transmitters can be described by a congestion game. The algorithm of channel allocation is derived from the dynamics of the transmitter in the game, which reaches Nash equilibrium quickly.

Furthermore, we provide a complete solution to fully exploit TV white space complying with IEEE 802.22 standard. We propose a centralized methods to regulate the upper bound of transmission power, so that to strictly protect the primary users. The the distributed channel allocation and power control are conducted sequentially.

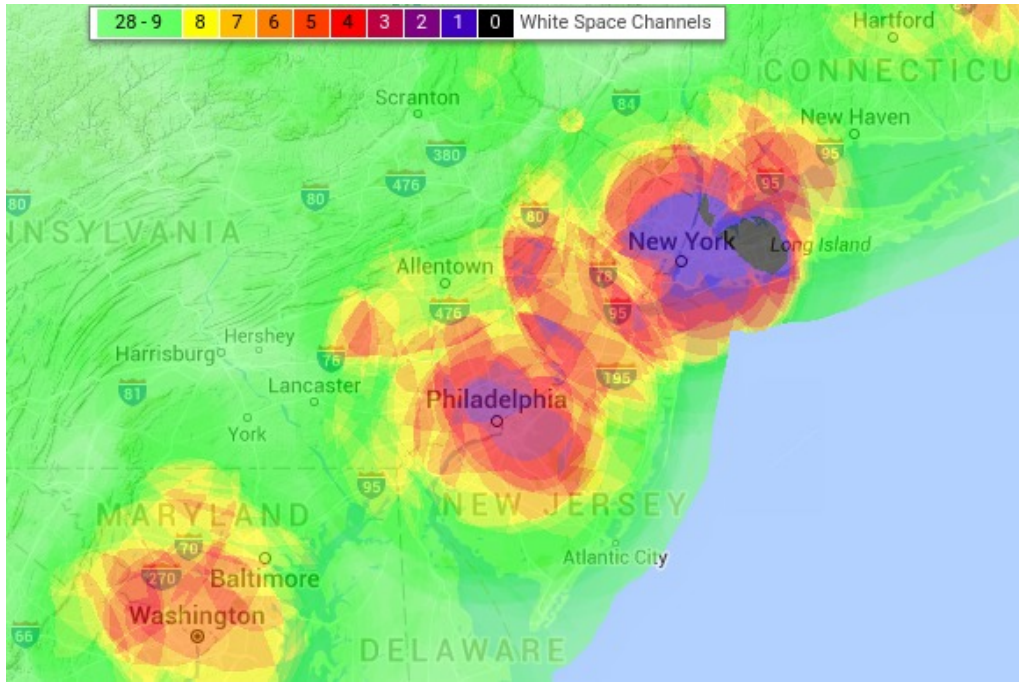
### 3.1 Introduction

Secondary users working with TV white space is promising to cope with the scarcity of spectrum resources [10]. Firstly, more unused TV white frequencies become vacant than ever with the ongoing transition from analog to digital broadcasts. Secondly, the frequencies of TV bands enable broadband access over larger geographic ranges compared to higher frequency bands. Nevertheless, services on TV receivers need to be protected with so called interference margin <sup>1</sup> [99] which should not be exceeded by the accumulated interference caused by all secondary users working on the the channel.

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<sup>1</sup>interference margin is the maximal interference caused by secondary users, which doesn't violate TV service.

FCC and ECC have announced rules on the transmission power of secondary users working in TV white space in US and Europe respectively [10, 11]. FCC requires a minimum distance between secondary user and TV service area, besides, the transmission power for fixed secondary users is set as 4 W, which is a conservative setting. FCC believes with these prudent measures, the interference margin can not be exceeded by interference from secondary users. But it may not be the case when there are multiple secondary equipments transmitting at the same time, which is pointed in [92]. ECC requires the secondary users to adapt their maximum transmission power according to the distance away from the TV receivers.



**Figure 3.1** Variability of available channels in a densely populated area. This figure is obtained from [1]

FCC issued a memorandum [9, 10] in 2010, which removes the mandatory rigid sensing requirements, and prompts the usage of geolocations<sup>2</sup> of secondary users. FCC regulates a centralized database, which registers all the secondary users within one certain area. Secondary users can access the database, and can only use the channels assigned by the database. Work [124] follows this rule to obviate spectrum sensing and only relies on the database of TV incumbents to determine the white space availability for secondary users. The authors of [124] demonstrate the feasibility of predicting the available TV spectrum accurately using sophisticated propagation models (Longley-Rice) and geolocations of secondary users. A central database contains the geolocations of all TV stations, then the database calculates the received signal strength index (RSSI) levels of TV UHF signals on all secondary users and accordingly determines the available TV spectrum for them. If RSSI of a channel is below a certain threshold, TV service is regarded not to exist, and the channel is seen available there. The calculated results on channel availability is very close to the measurement results. The work of [124] gives big impetus to the usage of database mode. As in TV white space, the accurate RSSI can be obtained with geolocations and suitable propagation model, given geolocation and appropriate propagation

<sup>2</sup>Geolocation means both geographic location and terrain.

model, secondary users' maximum transmission power can be determined by the central entity according to the interference margin (maximum RSSI level from secondary users) on the TV receivers.

In this chapter, we investigate the usage of TV spectrum in a wireless regional area network which complies with IEEE 802.22 network. The secondary users are assumed to be cellular base stations and associated terminals, all of which work on TV white spectrum. The base station is referred as WBS. Some cellular networks, i.e., GSM or LTE network, work on licensed spectrum and emphasis on providing satisfactory services to their end terminals by choosing proper transmission channel and power. As to cellular network working on TV white spectrum, they have to keep one eye on the primary users to make sure that TV service is not violated, which makes the problem of channel and power selection difficult. With the existence of central database, it is natural to utilize it as a central controller to assign channel and power usage for secondary users, but the secondary users may belong to different commercial groups and they may not contend with the assigned resource. Hence, the spectrum sharing of the secondary users in IEEE 802.22 network should be decided in distributed manner and each secondary user takes care of its own interest, i.e., to maximize its preferred utility.

Given all the other WBSs' selection on channel and transmission power, a WBS is interested in choosing the channel which brings it the best performance, i.e., the data rate of its end users. A WBS prefers to choose the channel which experiences the minimum interference, and the transmission power allowed on that channel is higher, so as to obtain better SINR on its terminals and meanwhile maximize their coverage [81, 169]. Nevertheless, high transmission power causes significant co-channel interference to other secondary users operating on the same channel. Hence, a secondary cell has to balance its transmission power and the caused interference on other cells, meanwhile to choose working channel to decrease the experienced interference on its terminals. The goal of this chapter is to protect the primary users from harmful interferences, meanwhile to find a strategy for WBSs to choose channel and power level in order to acquire good SINR on end terminals.

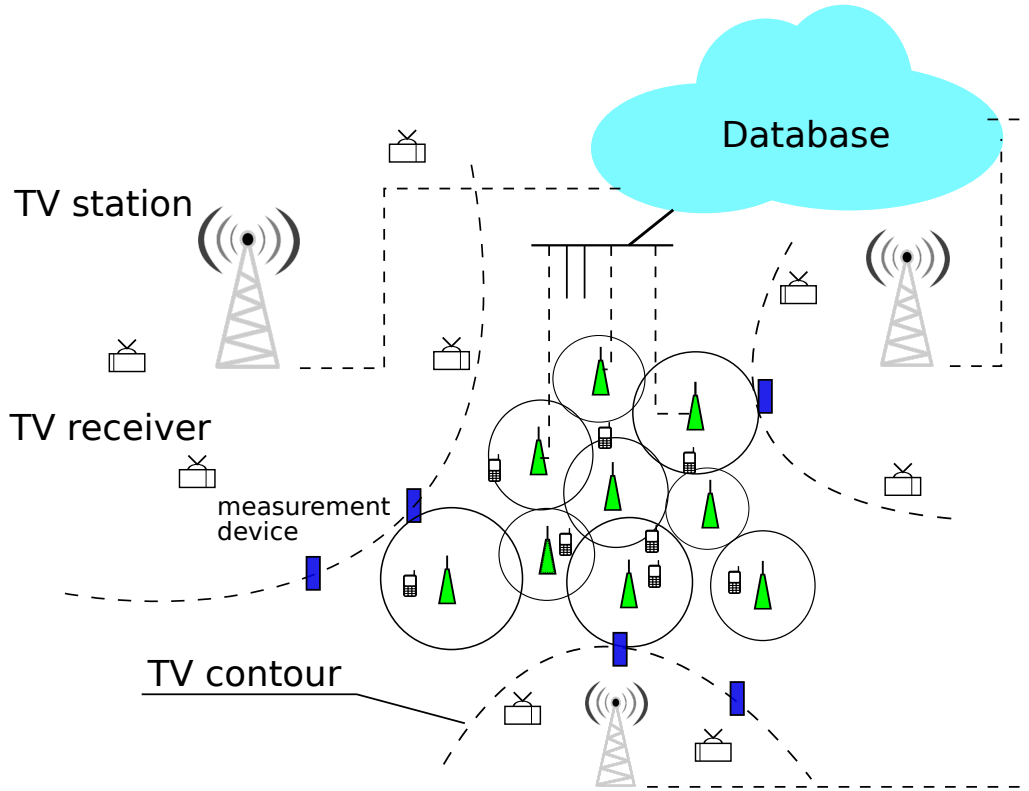
The rest of the chapter is organized as follows. we elucidate the system model in Section II, afterwards related work and problem formulation is presented in Section III. In Section IV, we discuss how to utilize the white space sufficiently by setting the transmission powers based on a convex problem formulation. We analyze the spectrum allocation problem under game theoretical framework and propose an algorithm in Section V, thereafter performance evaluation is presented in Section VI. Finally, we conclude our work and point out directions of future research in Section VII.

## 3.2 System Model and Problem Statement

We consider an IEEE 802.22 compliant cellular network where the fixed white space devices work as base station and provide broadband access to their terminals. The network is illustrated in Figure 3.2. We call the fixed white space devices which work as base stations as White space Base Stations (WBSs), they are located in one area which is surrounded by digital TV stations and receivers. Several critical points are deployed in the vicinity of the digital TV receivers which are the most vulnerable to the interference caused by the white space devices. WBSs work in underlay manner and coexist with the digital TV stations and receivers, the aggregate interference generated by WBSs should

not exceed the threshold on each channel at each critical point. As to the WBSs, the out-of-band emission is regard as trivial, therefore, we only consider co-channel interference among the WBSs. To simplify the analysis, we assume that each digital TV station as well as each WBS utilizes exactly one channel.<sup>3</sup>

As to the notations, the set of WBSs is denoted as  $\mathcal{N}$  where  $|\mathcal{N}| = N$ . The TVWS spectrum bands are denoted as set  $\mathbb{C}$ . We represent the usage of channel for WBS  $i$  with a binary vector  $X_i^{|\mathbb{C}| \times 1} = \{x_i^1, \dots, x_i^k, \dots, x_i^{|\mathbb{C}|}\} \in \{0, 1\}^{|\mathbb{C}|}$ , where  $k \in \mathbb{C}$  and binary variable  $x_i^k$  denotes whether channel  $k$  is used by user  $i$ . All the WBSs work with the channels approved by the WSDB, they operate with a channel from the approved ones after choosing it, thus we omit the time index in the channel usage. As each node can only uses one channel, for  $X_i$ , there is  $\sum_{k=1}^{|\mathbb{C}|} x_i^k = 1$ . The transmission power of WBS  $i$  on channel  $c$  is  $P_i^c$ .  $c(i)$  denotes the channel used by a WBS  $i \in \mathcal{N}$ .



**Figure 3.2** System model: WBS cells and digital TV (DTV) systems

For a terminal  $m$  which is associated to WBS  $i$ , the attenuation between WBS  $i$  and  $m$  is denoted as  $h_{im}$ . For the attenuation, we only take path loss and shadowing into account in the following. The path loss is dependent on the distance between the corresponding equipment, e.g.  $h_{im} = K \cdot d_{im}^{-\alpha}$ , where  $\alpha$  is the path loss exponent,  $d_{im}$  is the distance between  $i$  and  $m$ , while  $K$  is a constant which models the reference loss over a single unit of distance. Shadowing without fading is considered in our model.  $z_{im}$  models the

<sup>3</sup>The assumption that one WBS only utilizes one channel is for convenience of analysis. In reality multiple channel usage (channel bonding) is requirement as one single TV channel's bandwidth is 6 MHz which is not adequate for a WBS to fulfill terminal demands.

zero-mean log-normally distributed shadow fading between  $i$  and  $m$ , with the standard deviation  $\sigma_{\text{SH}}$ .  $N_0$  denotes the thermal noise power. The SINR at end terminal  $m$  is,

$$\gamma_m = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{f_m^c} = \frac{P_i^c \cdot h_{im} \cdot z_{im}}{\sum_{j \in \mathcal{N} \setminus i, c(j)=c} (P_j^c \cdot h_{jm} \cdot z_{jm}) + N_0} \quad (3.1)$$

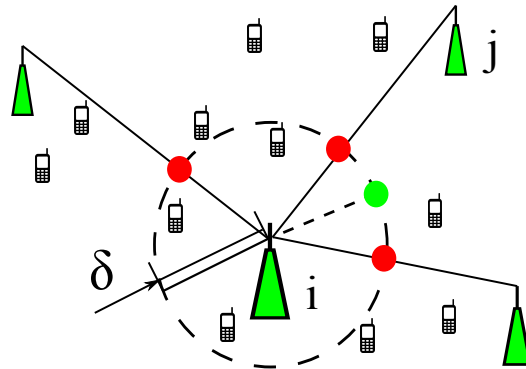
where  $P_j^c$  denotes the transmission power of interfering WBS  $j$ .

In our model, we only assume the WBSs, which work with high transmission power, as the potential interfering unlicensed devices to the DTV service, meanwhile, WBSs are interested in providing broadband access to their associated terminals. Our goal in this paper is to assign TVWS channel to each WBS so as to improve the signal to noise and interference ratio (SINR) of their associated end terminals, meanwhile complying with the ECC regulations. The channels in  $\mathbb{C}$  are assumed to be identical in terms of attenuation and shadowing on the same path. A WBS's utility is a function of the SINR on all its end terminals, i.e., the average SINR at all its terminals.

### 3.2.1 Problem Formulation

When we look for a metric for WBS, which represents the SINR on end users, it is not appropriate to choose only one terminal, as done in [50], or multiple fixed terminals to represent the all the terminals in the cell, because the locations of the chosen terminal could diverge greatly with respect to the other terminals. Thus, we propose a metric *QuasiSINR* to represent WBS's performance in terms of SINR on its end terminals, which is independent on the actual locations of end terminals.

Our goal is to minimize the sum of inverted SINR the WBSs  $\sum_{i \in \mathcal{N}} \frac{1}{\gamma_i}$ . We minimize the sum of inverted quasiSINR instead of maximizing the sum of quasiSINR in order to ensure the fairness among WBSs, . With an auxiliary circle centered at the discussed WBS, which is shown as dashed circle in Figure 3.3, QuasiSINR is the ratio between the power of signal of interest on the circle and the summation of the strongest power from the interfering WBSs on the auxiliary circle.



**Figure 3.3** Assuming the radius of the auxiliary circle is  $\delta$  and all the WBSs work on the a channel, then QuasiSINR is a quotient where the divided is the WBS  $i$ 's power on the auxiliary circle (i.e., the green pot), and the divisor is the summation of the interfering power on the red pots.

The quasiSINR of WBS  $i$  is denoted as  $\gamma_i$ ,

$$\begin{aligned}\gamma_i &= \frac{P_i^c \cdot h_{i \rightarrow i\text{'s auxiliary circle}} \cdot z_{i \rightarrow i\text{'s auxiliary circle}}}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot h_{j \rightarrow i\text{'s auxiliary circle}} \cdot z_{j \rightarrow i\text{'s auxiliary circle}}) + N_0} \\ &= \frac{P_i^c \cdot \delta^{-\alpha} \cdot z_{i \rightarrow \text{auxiliary circle}}}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot (d_{ji} - \delta)^{-\alpha} \cdot z_{j \rightarrow \text{auxiliary circle}}) + N_0}\end{aligned}\quad (3.2)$$

In the following paper, when we talk about the channel and power allocation with respect to WBS, the notation  $h_{ij}$  denotes the attenuation between WBS  $i$  to the auxiliary circle of WBS  $j$ .  $h_i$  denotes the attenuation between WBS  $i$  to its own auxiliary circle. Then  $\gamma_i$  becomes,

$$\gamma_i = \frac{P_i^c \cdot h_i \cdot z_i}{\sum_{\substack{j \neq i, j \in \mathcal{N} \\ c(j)=c(i)}} (P_j^c \cdot h_{ji} \cdot z_{ji}) + N_0}\quad (3.3)$$

The abbreviations and notations used in this paper are found in Table 4.1. The radius of the auxiliary circle  $\delta$  can be adapted to foster better service to the terminals in certain area, i.e., a larger radius  $\delta$  will take care of the SINR on the terminals reside far away and vice visa.

Abbr. Symbol	Description
TVWS	TV white spaces
WSDB	white space database
WBS	white space base stations
$\gamma$	QuasiSINR
$f_{ji}^c$	The co-channel interference caused by WBS $j$ on the auxiliary circle of WBS $i$ , $c$ is the working channel for both
$f_i^c$	The sum of interference caused on the auxiliary circle of WBS $i$
$P_i^c$	The maximal permitted Tx power of WBS $i$ on channel $c$ (ECC solution)
$h_{ij}$	The attenuation between WBS $i$ to the auxiliary circle of WBS $j$ .
$h_i$	The attenuation between WBS $i$ to its won auxiliary circle.
$z_{ij}$	The shadowing from WBS $i$ to the auxiliary circle of WBS $j$ .
$z_i$	in Section 3.5.2, the shadowing from WBS $i$ to its own auxiliary circle.
cp	Critical point

**Table 3.1** Notations

In our model, WBSs access the WSDB and obtain the transmission parameters, i.e., working channel, transmission power of the other WBSs, the attenuation characteristics between itself and all the other WBSs, and vice visa. WBSs calculate their QuasiSINRs with these information respectively.

## 3.3 Related Works

In the section we will introduce the related works of spectrum and power allocation problem in CRN and TV white space.

### 3.3.1 Resource Allocation in CRN

Resource allocation problem is often formulated into different constrained optimization problems, and then get solved in centralized manner. In [82], the objective is to increase the number of supported terminals whose SINRs are above a threshold, and the constraints are to refrain the interference at the primary users within a certain margin. Work [31] minimizes the transmission power and meanwhile makes sure the SINR of terminal is above a threshold, but this work fails to consider the protection of primary users. A heuristic algorithm is proposed in [138], which assigns channels to the WRAN in such a way to avoid harmful interference and based on the continuous changes of the spectrum availability.

There is a large variety of distributed solutions. In order to avoid or to alleviate co-channel interference between cells, and to allow arbitrary number of cells to work in IEEE 802.22 network, [84] proposes distributed inter-network spectrum sharing scheme, where contention decisions are made in a distributed way and the winner cells can use the shared channels. But this work doesn't consider the role of transmission power in the co-channel interference. An distributed power allocation (single channel) scheme based on learning for secondary networks is given in [66], where penalty function involving the interference threshold on primary systems is used. [81] discusses power control and channel assignment in both down-link and up-link communication in cellular network. Although the solution is distributed, primary users are required to cooperate with secondary base station in a learning process to decide the transmission power, in addition, there is only one secondary base station considered whereas we need to cope with the multiple cells in our problem. Joint channel-power selection for multiple transmission links (pairs) is investigated in [169]. The authors decompose the Lagrangian dual of the problem, then propose a distributed scheme based on the dual parameters. The scheme converges into pure Nash equilibrium, but in order to facilitate this scheme, monitors are required to watch interference from secondary users, moreover, monitors have to be equipped computational ability and interact with secondary users in the whole process of convergence. Distributed algorithm based on Learning is proposed in [85] for LTE to allocate the resource block in down link, which leads to correlated equilibrium, but slow converge hinders its application.

As introduced in Chapter 1, game theory is a powerful tool in designing distributed algorithms. A distributed joint power and channel allocation is proposed in [129], each base station chooses optimal power level and channel to optimize its utility, which results in induced received interference and caused interference on primary users. The execution of this scheme is formulated into an exact potential game. For each base station, after several rounds of best responses in terms of channel and power level, Nash equilibrium is achieved. There are some flaws hindering the application of this scheme. Firstly, the paper doesn't provide means for base stations to obtain the needed information which is needed to calculate the utility function. Secondly, it is not clear how to calculate the punishment in the utility function, which indicates whether and how much the interference

threshold on primary users is violated. Thirdly, the convergence speed of the scheme is not given, in fact, as the problem is formulated into a potential game, converge speed or the number of updates before convergence is a theoretic problem which is still unsolved. Last but not least, as the utility function and the potential in the game are designed as the sum of received and introduced interference, the desired signal power and the punishment, the minimization of this *sum* does indicate meaningful performance metrics, i.e., SINR on terminals, or the total transmission power consumption. [100] adopts cooperation game to research the coexistence of femtocells. Each femtocell negotiates with neighbouring femtocells, and they form temporary coalition, but the goal of this solution is to allocate resource block in terms of time and transmission power. [31] proposes both centralized and decentralized solutions. Two distributed schemes are proposed, joint channel and power allocation is formulated into a weighted potential game, as an alternative workaround, the problem is solved in two sequential phases.

### 3.3.2 Utilization of TV White Space

Here we introduce the solutions proposed on the utilization of TV white space, which includes regulations, proposed standards and recent research advances. In accordance with the regulations of FCC, there are some prototype applications proposed in both cellular network [63, 141] and WiFi-like network [137]. The secondary users access a centralized data base to know the allowed channels and transmission power. A series of works [38, 51, 67, 150, 172] emphasis on interference mitigation among white space devices via spectrum allocation. Vehicular networks operating with TVWS assisted by TV database and cooperative sensing is discussed in [57]. Work [60] steps further from the database paradigm and makes efforts to utilize the *grey space*, where TVDB is allowed to operate even within the TV service area.

Authors in [15, 56, 76, 80] have proposed different approaches for assessment of TVWS capacity under FCC and ECC regulations respectively. Thereafter, a lot of works which comply with the FCC regulations are proposed to investigate the spectrum sharing issues in the coexistence of white space devices. Hesar et al. [79] maximize the Shannon capacity of the cellular networks working on TV white space. The solution seeks the trade-off between wide band and co-channel interference, and a centralized heuristic scheme is proposed to improve the Shannon capacity on the location of the secondary base stations. Yang et al. [171] and Gopal et al. [71] work towards throughput maximization of a CS-MA/CA based WiFi like network in TVWS under aggregate interference. The authors of [33] and [173] formulate the secondary networks into conflict graphs. Targeting at WiFi like white space device networks, Ying et al. [173] increases the percentage of nodes served and the number of assigned channels. Bansal et al. of [33] proposed a improved graph coloring scheme to improve the fairness and throughput. [30, 72, 144] devote themselves to the heterogeneous unlicensed networks, i.e., the white space devices operates on heterogeneous network technologies.

Game theory draws a lot of attention to utilize the unlicensed spectrum. With identical transmission power and symmetric path attenuation, Nie et al. [126] formulate channel assignment problem in ad-hoc cognitive radio network into a potential game which leads to pure NE, the authors of [62, 166] propose algorithms which converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively. Omidvar et al. [129] use potential game to propose a distributed joint power and channel allocation in



cognitive radio network. Although the scheme is not tailored for TVWS, protection on the primary users are considered. Potential game is adopted in work [59] to mitigate the adjacent interference meanwhile bonding multiple channels, where the white space devices comply with the FCC rulings. In [50], Chen et al. formulate the channel allocation problem in TV white space into a potential game where individual WBS's utility is to maximize the capacity of one static terminal. In [50], Chen et al. investigate the channel allocation problem in the scenario of TV white space. The channel allocation problem is formulated into a potential game, individual WBS's utility is to maximize the capacity of one single static terminal. Potential game is also adopted in work [58] to design algorithms, which mitigates the adjacent interference.

### 3.3.3 Related Works on Channel Allocation with Fixed Transmission Power Level

In our work, after obtaining the channel-power map, WBSs need to decide one channel and the associated transmission power to use. As the transmission power could be different for two interfering transmitters, we actually investigate a problem which is different from the available channel allocation problems. Here we are going to review the channel allocation schemes where transmission power levels are identical.

Channel allocation is adopted to mitigate the co-channel interference, which has been attracting plenty of research efforts in the past decade. Channel assignment problem is converted into colouring problem thus is NP hard [142]. Authors of [97] propose heuristic algorithms utilizing best response to improve its welfare, but the transmission power is assumed identical and path loss is deemed as symmetric, which renders this method problematic for our problem where transmission is non-identical and the path loss is asymmetric. [126] formulates channel assignment problem in ad-hoc cognitive radio network into potential game which leads to pure NE, a learning scheme achieving slightly better performance is provided for comparison, but they assume the transmission power is identical and there is no noise in the secondary network, and the proposed random access mechanism demands a huge amount of information to be exchanged, which is a burden for network in ad-hoc structure. [62, 166] investigate the channel allocation problem under game framework in same collision domain, the authors propose algorithms to converge to pure Nash equilibrium (NE) and strongly dominate strategy equilibrium respectively. Simulated annealing is applied to mitigate co-channel interferences in [172]. For the same purpose, no-regret learning [77, 86] is exploit to optimize the choice on channel.

To our knowledge, through the work [2012 DySPAN], we are the first to deal with channel allocation with different transmission power.

## 3.4 The Maximum Permitted Transmission Power

As WBSs work in underlay manner and coexist with primary TV stations and receivers, to protect the TV receivers from harmful interference, the aggregate interference caused by WBSs at the contours of TV receivers should not exceed the interference margin. Work [93] proposes detailed calculations which a geolocation database performs in order to derive location-specific maximum permitted EIRP levels for white-space devices

(WSDs) which operate in digital terrestrial TV bands. [99] calculates the maximum permitted transmission power for the network which complies with IEEE 802.22 standard. The standard requires a centralized database to store the available channels for each secondary base station, thus centralized scheme can be conducted there after trivial modification. The sufficient condition that the TV receivers are not interfered is formulated into a constraint in a centralized linear programming program (LP), where the objective function is to maximize the sum of transmission powers of all WBSs. We adopt the interference model and the optimization methodology in [99] to plan the maximum transmission power on each channel for WBSs in our work. With the global view of the propagation parameters, geolocations of WBSs and interference threshold at the critical points, linear programming is conducted in the TVDB to calculate the maximum permitted transmission power on all channels for each WBS. In the following sections, we call the maximum permitted transmission power over all channels as *channel-power map*.

For WBS  $i \in \mathcal{N}$ , the maximum transmission power allowed on channel  $c \in \mathcal{C}$  is denoted as  $P_i^c$ . As to each channel  $c$ , the generated interference on each interference measuring device should be within a predefined interference margin  $I_{pt}^c$ . The interference margin in a slow fading environment is decided according to [146].

Then the maximum permitted transmission power on channel  $c$  for each WBS can be obtained by solving the following optimization problem,

$$\begin{aligned}
 & \text{Maximize} && \sum_{i \in \mathcal{N}} P_i^c \\
 & \text{subject to} && \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z) < I_{pt}^c, \\
 & && P_{min}^c \leq P_i^c \leq P_{max}^c
 \end{aligned} \tag{3.4}$$

$P_{min}^c$  is the prudent transmission power.  $P_{max}^c$  is the maximum transmission power which is restricted by the hardware.  $z$  is shadow fading as introduced in 3.1. Here we only consider the interference caused by WBSs, and omit the interferences from end terminals. Since WBSs' transmission power is higher and their altitude is higher [99], the downlink transmission contributes the major part of interference [18]. The first constraint indicates that the interference margin will not be exceeded even when all the WBSs work on the same channel.

Formula 3.4 will be solved for each channel  $c \in \mathcal{C}$ . After solving the  $|\mathcal{C}|$  problems, the maximum permitted transmission power vector  $\mathbf{P}^c = \{P_1, \dots, P_{|\mathcal{N}|}\}, \forall c \in \mathcal{C}$  is obtained.

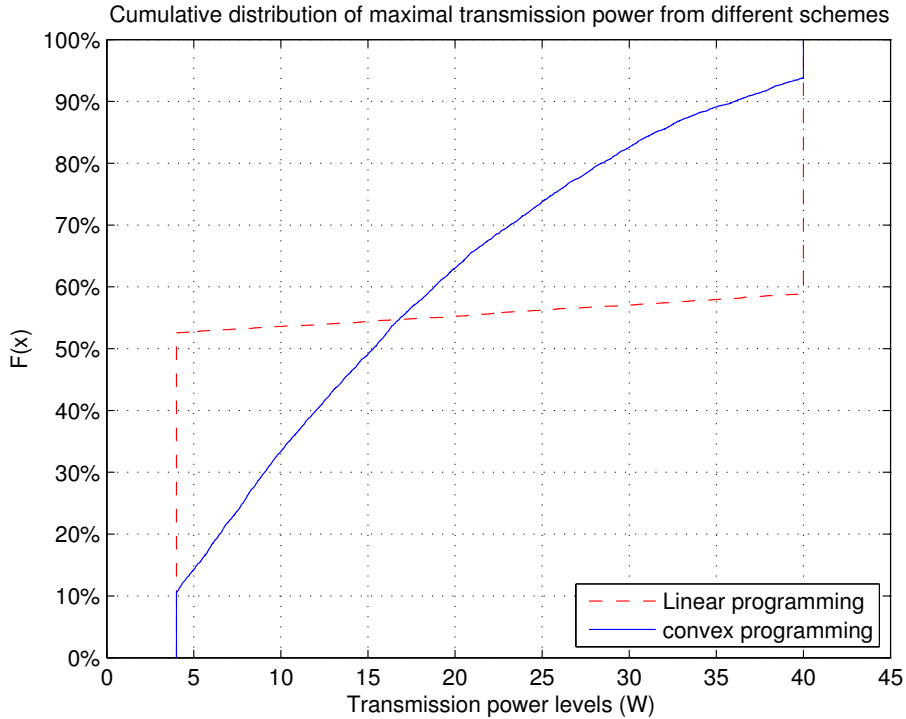
When working with the same transmission power, the WBSs locating closer to the TV interference measuring devices contribute more to the aggregate interference comparing with the WBSs which locate far from the TV interference measuring devices. Thus when implying linear programming to decide the maximal transmission power, the transmission power used by WBSs which are closer to the TV interference measuring devices is much higher than other WBSs. As a result the maximum permitted transmission power on each channel obtained with LP is seriously unbalanced.

To address this fairness issue, we maximize the sum of the logarithmic value of every WBS's transmission power, and formulate the problem into a convex optimization problem.

$$\begin{aligned} & \text{Maximize} && \sum_{i \in \mathcal{N}} \log P_i^c \\ & \text{subject to} && \sum_{i \in \mathcal{N}} (P_i^c \cdot h_{i,pt} \cdot z) < I_{pt}^c, \end{aligned} \quad (3.5)$$

This optimization will be solved for each channel  $c \in \mathcal{C}$ .

Figure 3.4 depicts the distribution of maximum permitted transmission power levels obtained in 100 simulations. In each simulation the locations of TV interference measuring devices are randomly decided around the WBSs. In Figure 3.4, It shows that when applying optimization 3.4, WBSs' transmission power levels are either the minimum transmission power or the maximum power allowed by the equipment hardware. When applying convex programming, the planned maximum permitted transmission power levels are distributed evenly in between the minimum and maximum power. The gain of SINR on end terminals by applying convex optimization to decide the maximal transmission power is illustrated in the simulation section.



**Figure 3.4** Distribution of maximum permitted transmission power levels obtained from convex and linear programming formulations

Optimization problem 3.5 provides the maximum permitted transmission power for each WBS and over each channel. When all the WBSs working on the same channel, the generated interference doesn't exceed the threshold on the interference measurement devices at the contour of TV service area. If there are multiple channels available and WBSs are free to choose their preferred channels, the aggregate interference on one channel will be smaller than that when all WBSs work on that channel. Thus, there is exists a interference

margin created by using multiple channels, which provides a room for network dynamics such as new WBS starting to work or increased interference on TV contour due to the variance of broadcast path condition.

### 3.5 Channel Allocation with Single Channel

First, we give the centralized solution to obtain the global optimum for this subproblem, then the decentralized scheme under the game theoretic framework is introduced.

#### 3.5.1 Centralized Optimization Programming

We formulate the channel allocation problem into a binary quadratic programming problem which can be solved in a centralized way. Let  $X_i = \{x_{i1} \cdots x_{ik} \cdots x_{i|C|}\}$  denote the vector of channel usage, there is  $|X_i| = |C|$  and binary element  $x_{ik}$  represent whether WBS  $i$  occupies channel  $k$ . For two WBSs  $i$  and  $j$ , there is,

$$X_i^T X_j = \sum_{k=1}^{|C|} x_{ik} \cdot x_{jk} = \begin{cases} 1 & \text{if } c_i = c_j \\ 0 & \text{if } c_i \neq c_j \end{cases} \quad (3.6)$$

The power levels across all channels are denoted by a constant vector  $P^{|C| \times 1}$ , which is possibly nonidentical to WBSs due to different locations. The power used by user  $i$  is

$$P_i^T X_i = \sum_{k=1}^{|C|} P_i^k \cdot x_{ik}.$$

Problem 4.4 can be modeled via general purpose nonlinear optimization:

$$\begin{aligned} & \text{minimize} \quad \sum_{i=1}^n \frac{\sum_{j \in \mathcal{N}, j \neq i} P_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{P_i^T X_i h_{ii} z_{ii}} \\ & \text{subject to} \quad \sum_{k=1}^{|C|} x_{ik} = 1, x_{ik} \in X_i \in \{0, 1\}^{|C|} \end{aligned} \quad (3.7)$$

Problem 3.7 is a non-linear problem with binary variables, but it can be reformulated in to a quadratic programming problem as,

$$\begin{aligned} & \text{minimize} \quad \sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k \frac{P_{jk} \cdot h_{ji} \cdot z_{ji}}{P_{ik} \cdot h_i \cdot z_i} \cdot x_{jk} \cdot x_{ik} + \sum_k \frac{N_0}{P_{ik} \cdot h_i \cdot z_i} \cdot x_{ik} \right) \\ & \text{subject to} \quad \sum_{k=1}^{|C|} x_{ik} = 1, x_{ik} \in X_i \in \{0, 1\}^{|C|} \end{aligned} \quad (3.8)$$

The reformulation is available in Appendix B. We use LINDO [108] which is a state of art non-linear problem solver to solve the problem, which employs Branch-And-Reduce method to get the global optimum for the problem. The result of this centralized channel assignment will be evaluated in the simulation section with other schemes.

### 3.5.2 Distributed White Space Channel Allocation (WitheCat): Algorithm and Protocol

In this section a distributed scheme for WBSs to allocate channels is proposed, which is named as white space channel allocation technology (WitheCat). WitheCat adopts the best response process, where each WBS (referred as  $i$ ) chooses the channel which brings the bigger utility  $u_i$  as the response of other WBSs' choices on channels. WitheCat is depicted by algorithm 1.

$$u_i = \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} f_{ji}}{2 \cdot \tilde{P}_i} + \frac{1}{2} \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \sum_{\substack{\mathcal{S}: i, j \in \mathcal{S}, \\ c(\sigma_j) = c(\sigma_i)}} \frac{N_0}{C \cdot \tilde{P}_i} \quad (3.9)$$

where  $f_{ij} = P_i \cdot h_{ij} \cdot z$  and  $f_{ji} = P_j \cdot h_{ij} \cdot z$ . Note that  $f_{ij}$  is the sum of interference on WBS  $i$ 's interference reference points. Overlooking the constant coefficient 2, the first item of  $u_i$  is a part of the inverted QuasiSINR of station  $i$ . To minimize the first item, WBS  $i$  needs to choose a channel either permits higher transmission power or experiences less interference, whereas the higher power increases the second item which is a part of inverted QuasiSINR of other co-channel WBSs. Hence, the cost function presents a reasonable comprise between the welfare of one WBS and others.

When WBS only emphasizes on its own utility (e.g. the first part of Formula 3.9), the best response process doesn't converge. We have following theorem:

**THEOREM 3.5.1:** With non-identical transmission power, if every WBS updates its channel based on Algorithm 1 with utility based on its own interests, the process doesn't always converge.

The proof is in Appendix A.

---

**Algorithm 1:** Spectrum selection by WBS  $i$ 


---

**Input:** the distance, path lose and shadowing parameter between WBS  $i$  to

WBS  $j \in \mathcal{N} \setminus i$ ;

radius of auxiliary circle, noise  $N_0$ , total number of WBSs  $N$ ;

for  $j \in \mathcal{N} \setminus i$ , the maximal transmission power  $P_j^c$ ,  $c \in \mathcal{C}$  and the working channel  $c(j)$ .

- 1 **for**  $c \in \mathcal{C} \setminus c(i)$  **do**
  - 2     calculate  $u_i(c)$  based on Formula 3.9 **if**  $u_i(c) < u_i(c(i))$  **then**
  - 3          $c(i) \leftarrow c$
  - 4     **else**
  - 5         keep  $c(i)$  unchanged
  - 6     **end**
  - 7 **end**
  - 8 Notify database of its channel usage, which further notifies the other WBSs
- 

Some parameters needed to calculate the utility are identical for all WBSs, such as quasi distance  $e$ , the total number of WBSs  $N$ , number of channels  $C$ , attenuation factor  $\alpha$ , standard deviation  $\sigma_{WBS}$  in flat shadowing and noise  $N_0$ , albeit the following information is further needed to calculate  $u_i$ :

- $\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} f_{ji}^c, c \in \mathcal{C}$ : the received interference on  $i$ ' virtual measurement point from other WBSs  $j$  working on the same channel for  $\forall c \in \mathcal{C}$ .
- $f_{ij}^c$ : the interference caused by  $i$  on  $j$ 's virtual measurement point when  $i$  works on channel  $\forall c \in \mathcal{C}$ .
- $P_j^c$ : transmission power of  $j$  for using  $\forall c \in \mathcal{C}$ .

Unfortunately, it is difficult to get these interferences of interested measured, for station  $i$ , it is low efficient to scan all channels and obtain the interferences  $f_{ji}$  on virtual measurement point for each channel, furthermore, it is impossible to split the interference  $f_{ij}$  from the total interference received on WBS  $j$ ' virtual measurement point.

We refer [126] to decide the sequence for WBSs to update their channel. [126] proposes a method like random access mechanism of CSMA/DA, where the access for broadcast medium is changed to getting access to the centralized center to retrieve the current channel usage and update its new channel. All WBSs are able to access the database in one round (with random or predetermined sequence). As WBSs are connected with database, the control messages needed to decide the sequence will not become a burden. Update of channels can happen in the boot phase, or when the quality of services (the SINR on its end users) of WBSs falls below a threshold, or a fixed time duration comes to end, or a new WBS joins in the network.

Similar with [124], we let every WBS store the location information and maximal power map of all other WBSs, i.e.,  $P_i^c, i \in \mathcal{N}, c \in \mathcal{C}$ , and each WBS retrieves information about channel usage of other WBSs from centralized base station. After executing Algorithm 1, it reports to centralized database of its channel if it updates the working channel. As the location of WBSs and TV stations and the transmission channel and power of TV stations are usually static (entries of TV station change averagely once in 2 days [124]), except for the channel usage in the network, the change of the other data stored in WBS is infrequent.

### 3.5.3 Analysis in Game Theoretical Framework

In this section, We give the proof on whiteCat's convergence in the framework of congestion game theory. Formulating a spectrum sharing problem into a congestion game and the concept of *virtual resources* are firstly proposed in [109]. This work reversely engineers the distributed channel allocation schemes proposed in [28,97], i.e., unifies the algorithms with congestion game. But the problem analysed in [109] assume the transmission power is identical, which is a major difference from the channel allocation problem discussed here.

We have introduced congestion game in Chapter 2, thus we only recap the essence of congestion game here. In congestion game, each player acts selfishly and aims at choosing strategy  $\sigma_i \in \Sigma_i$  to minimize their individual cost. The gain (loss) caused by any player's unilateral move is exactly the same as the gain (loss) in the potential, which may be viewed as a global objective function. For problems where the potential of the problem is the same with the summation of the cost of all users, the cost function can be used as a utility function directly. This equivalence doesn't exist in our problem, but by carefully choosing the cost function for players, we can make sure that the change of individuals' cost is in the same direction with that of the global utility.

### The Congestion Game Formulated from the Algorithm WhiteCat

We utilize the conception of virtual resource which is firstly introduced in [109]. Virtual resource is a triplet  $\{i, j, c\}$ , where  $i, j$  are two WBSs and  $c \in \mathcal{C}$  is one channel. This piece of resource is regarded used by  $i$  when both  $i$  and  $j$  use channel  $c$ , otherwise,  $\{i, j, c\}$  is not used by any WBS.

In the following, we list the element of the congestion game which emulates Algorithm 1. In this section, player and base station are used interchangeably.

- Player  $i$ ' strategy space is  $\Sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c, c = 1, 2, \dots, N\}$ , and  $i$  has  $C$  admissible strategies, one strategy related with channel  $c \in \mathcal{C}$  is described by the set of virtual resources it uses:  $\sigma_i = \{(i, j, c), j \in \mathcal{N}, j \neq i, c(\sigma_j) = c\}$ , note that virtual resource  $(i, j, c) \neq (j, i, c)$ .
- Under the strategy profile  $\sigma = (\sigma_1, \sigma_2, \dots, \sigma_N)$ , player  $i$  obtains a total cost of

$$g^i(\sigma) = \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c=c(\sigma_i)=c(\sigma_j)}} (g_{(i,j,c)}(n_{(i,j,c)}(\sigma)) + g_{(j,i,c)}(n_{(j,i,c)}(\sigma))) \quad (3.10)$$

The transmission power over all channels of player  $i$  is  $\{p_i^1, p_i^2, \dots, p_i^{|\mathcal{C}|}\}$ . We define the cost function for virtual recourses  $(i, j, c)$  as follows,

$$g_{(i,j,c)}(k) = \begin{cases} \frac{f_{ji}}{2\tilde{P}_i} + \frac{f_{ij}}{2\tilde{P}_j} + \frac{C \cdot N_0}{N \cdot \tilde{P}_i} & \text{if } k = 2 \\ 0 & \text{otherwise} \end{cases} \quad (3.11)$$

As resource  $(i, j, c)$  only lies in the strategy space of player  $i$  and  $j$ , thus can only be accessed by this two players. More specifically, according to Formula 3.11, the cost of resource  $(i, j, c)$  is only decided by the number of players using it, which is either 0 or 2. At the first glance, this is a player specific congestion game, as  $g_{(i,j,c)}$  is decided by the relevant players' transmission power and inference. But actually the resource  $(i, j, c)$  excludes the players except for  $i$  and  $j$  from using it, thus the cost happened on this resource is only dependant on how many of players from the set  $\{i, j\}$  to use it. Hence, the cost is a function of the number of players using the resource, and this is a canonical congestion game.

### Bridging the Game and Algorithm WhiteCat

When we substitute Formula 3.11 to Formula 3.10, the total cost for user  $i$  under strategy profile  $\sigma$ .

$$\begin{aligned}
g^i(\sigma) &= \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c=c(\sigma_j)=c(\sigma_i)}} (g_{(i,j,c)}(2) + g_{(j,i,c)}(2)) \\
&= \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j)=c(\sigma_i)}} \left( \frac{f_{ji}}{\tilde{P}_i} + \frac{f_{ij}}{\tilde{P}_j} + \frac{C \cdot N_0}{N} \left( \frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j} \right) \right) \\
&= \frac{\sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{CN_0}{N} \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j)=c(\sigma_i)}} \left( \frac{1}{\tilde{P}_i} + \frac{1}{\tilde{P}_j} \right) \quad (3.12) \\
&= \frac{\sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{\substack{j \in \mathcal{N} \setminus i, \\ c(\sigma_j)=c(\sigma_i)}} \frac{f_{ij}}{\tilde{P}_j} + \frac{2CN_0}{N} \sum_{\substack{i \in \mathcal{S} \subset \mathcal{N}, \\ \mathcal{S}: \forall i \in \mathcal{S} \\ c(\sigma_i)=c}} \frac{1}{\tilde{P}_i}
\end{aligned}$$

where  $\mathcal{S}$  denotes the set of WBSs whose working channel is the same with WBS  $i$ .

Now we are going to have a look at the *potential* of the network. According to the expression of Rosenthal's potential in Formula 3.15, the potential is accumulated by adding the players' cost sequentially, in particular, the value which is added is the cost that player experiences when it starts to use the relevant resource, and the value is not changed when other players come to use that resource. Back to our problem, for two WBSs  $i, j \in \mathcal{S}$ , we assume WBS  $i$ 's index is smaller than  $j$ 's index, then the potential increased by  $i$  using the resource  $\{i, j, c\}$  is 0 according to Formula 3.10, and the increase brought in by  $j$  using the resource  $\{i, j, c\}$  is  $g_{(i,j,c)}(2) + g_{(j,i,c)}(2)$ . In other words, for each interfering pair of WBSs, only the WBS with bigger index contributes to the potential. Then the total potential is,

$$\begin{aligned}
G(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) = \sum_{i \in \mathcal{N}} \sum_{r \in \sigma_i} g_r(n_r^i(\sigma)) \\
&= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j)=c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \frac{CN_0}{N} \sum_{\substack{\mathcal{S} \subset \mathcal{N}, \\ \forall i \in \mathcal{S}, c(\sigma_i)=c}} |\mathcal{S}| \sum_{i \in \mathcal{S}} \frac{1}{\tilde{P}_i} \quad (3.13)
\end{aligned}$$

note that the summation of one WBS's congestion is related to its index.

When players minimize their utilities (cost or potential) illustrated by Formula 3.12, the total congestion in the secondary network given by Formula 3.13 decreases monotonically before reaching one Nash equilibrium. Players' greedy update in the game to minimize its cost Function 3.12, which ceases finally in pure Nash Equilibrium. The strategy and cost function of players in the game is transplanted as Algorithm 1 and utility Function 3.9 respectively.



### Gap between the Potential of Game and the Objective

It is natural to raise the question, is the sum of the final utilities of all WBSs exactly the same with the value of potential when the game converges to a Nash equilibrium, which is represented by 3.13? The answer is, they are identical when  $N_0$  is zero, and there will be a little difference when  $N_0$  is not zero. Recall the target objective we want to minimize in Problem 4.4 is,

$$\begin{aligned} \sum_{i \in \mathcal{N}} \frac{f_i}{\tilde{P}_i} &= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} f_{ji} + N_0}{\tilde{P}_i} \\ &= \sum_{i \in \mathcal{N}} \frac{\sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(\sigma_j) = c(\sigma_i)}} f_{ji}}{\tilde{P}_i} + \sum_{i \in \mathcal{N}} \left( \frac{N_0}{\tilde{P}_i} \right) \end{aligned} \quad (3.14)$$

We notice that only the last items of the objective 3.14 and the potential of the congestion game 3.13 are different. When  $N_0 = 0$ , the potential is exactly the same with the object we want to minimize. When  $N_0 \neq 0$ , if channels are evenly distributed and there is  $C/N^* \mid \mathcal{S} \mid = 1$ , then Formula 3.14 and 3.13 are also the same. In both cases, the sum of utilities 3.14 decreases monotonically with every update of WBSs before the system reaches Nash Equilibrium. When  $N_0 \neq 0$  and Formula 3.14 and 3.13 are thus different, the monotonicity on the decrease of sum of utilities 3.14 is not perceived, whereas the system will still cease to NE.

Based on above analysis, we can see the assumption that each WBS only occupies one channel can be easily removed. If we regard one WBS as multiple ones which locate at the same place, and each WBS works on one distinct channel, then the proof on convergence of whiteCat can be applied directly to this case.

Note that the convergence of the game is independent on the the concrete form of the cost function. We adopt the function 3.12 to let the potential of the game be the same with the total utility of all WBSs, so that by executing Algorithm 1, the system objective experiences a monotonic decreasing process before the system reaching NE. The algorithm has potential to solve many other problems, where one user's decision affects others. In this case, the utility of one user can be formulated to incorporate the information of its own utility and others', then the congestion game theory can be used to analogize.

### Communication Overhead of WhiteCat

The problem of channel allocation with different and fixed transmission power is NP hard. WhiteCat is a distributed scheme but certain information of the other WBSs is needed. The centralized base station is piggybacked to provided the needed information. As to one WBS, the number of such inquiries is the number of steps before convergence.

In our formulated congestion game, a player  $i$  is allowed to access up to  $(N - 1)$  resources in the same time, i.e.,  $\{i, j_1, c(i)\}, \{i, j_2, c(i)\} \cdots \{i, j_{N-1}, c(i)\}$ , thus the upper bound of converge steps can not be obtained from the conclusion 3.15 for singleton congestion game. But our problem is special because for each resource, the possible number of players allowed to use each resource is either 2 or 0. Thus we can refer the method used

in Section 2.3.3 to analyse the update times for our problem. Firstly, we sort the cost values in increasing order. Although a WBS

$$\begin{aligned}
 \phi(\sigma) &= \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} g_r(i) \\
 &\leq \sum_{r \in \mathcal{R}} \sum_{i=1}^{n_r(\sigma)} n \\
 &\leq n^2 m
 \end{aligned} \tag{3.15}$$

The upper bound of total update steps is  $2n^2$ , thus averagely, the upper bound of update steps for each WBS is  $2n$ .

### 3.6 Channel Allocation with Multiple Channels

Both of the proposed centralized and distributed channel allocation schemes, which are initially designed for single channel allocation, can be easily adapted to solve the multiple TV channel allocation problem which complies with ECC rulings.

As to the distributed scheme allocating multiple channels for WBSs, we regard a WBS working on a certain channel as a *logic* WBS, in other words, a WBSs operating on multiple channels can be seen as multiple co-location logic WBSs and each of them operates on a single channel respectively. The logic WBSs work complying with the algorithm 1, and the convergence also applies in this process.

As to the centralized solution, the constraint in the optimization formulation 3.8 needs to be changed as,

$$\sum_{k=1}^{|C|} x_{ik} = w, x_{ik} \in X_i \in \{0, 1\}^{|C|}$$

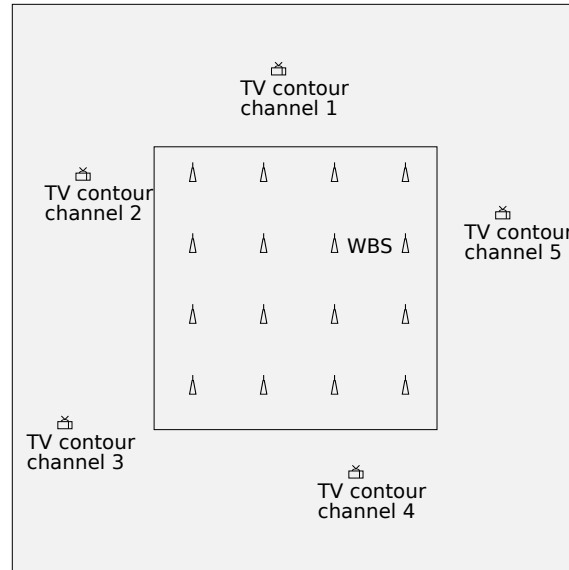
where  $w$  is the number of channels for each WBS to operate.

### 3.7 Performance Evaluation

Performance evaluation consists with three parts. In the first part we investigate the choice of radius of the auxiliary circle. Then the comparison between our proposed schemes which work with single channel and the other distributed schemes are conducted. In this process, the channel-power maps which are obtained from linear and convex optimization are used respectively. In the last part, we compare the proposed schemes working with different amount of channels with a distributed scheme which is designed complying with FCC rulings.

The evaluation setting is as follows and some parameters are listed in Table 3.10. A square area which is 60km x 60km is divided evenly into 16 square blocks. There is one WBS sitting in the middle of each block, where its end terminals are distributed within the same

block. There is a 20km wide rim area around the square area, where the critical points for the DTV receivers are randomly located. The locations of WBSs and TV contours are illustrated in Fig. 3.5. WBSs' locations are fixed, but the end terminals, and the sequence for WBS to update are randomly decided in each run. In each run the critical points for the digital TV service are located at different places, so that the power-channel map for every WBS is different in different runs. Each simulation was repeated 50 times and the mean along with its 95 percent confidence interval is plotted for every measurement.



**Figure 3.5** Layout of WBSs and TV contours

The other parameters are listed in Table 3.10.

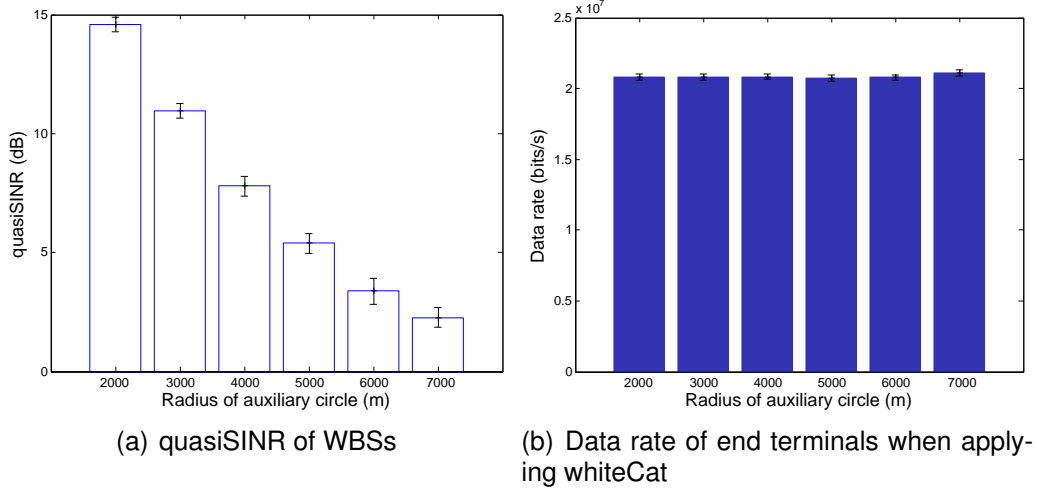
Number of channels	4
Number of WBSs	9, 16
Noise	$10^{-13}$ Watt
Side length of area hosting WBSs	60 km
Auxiliary circle radius	1 km
Inf. threshold on critical point	$10^{-8}$ , $5 \times 10^{-8}$ Watt
Path loss factor	2
Standard deviation in flat shadowing	8
Number of end terminals per cell	10
Min. WBS Tx power	1 Watt
Max. WBS Tx power	4 Watt
Number of simulation runs	50

**Table 3.2** Simulation parameters

### 3.7.1 The Choice of Radius of Auxiliary Circles, quasiSINR of WBS, and SINR on End Users

The usage of quasiSINR exempts WBSs from taking care the SINR on the end terminals. A WBS's quasiSINR is related with WBS's location and the radius of auxiliary circle.

Figure 3.6 illustrates the effect of using different radii of the auxiliary circle on the data rate can be achieved by end terminals.



**Figure 3.6** The effects of different radii of auxiliary circle on end terminals' data rate. Maximum permitted power is obtained by solving convex optimization. WhiteCat is used to assign the channels.

Subfigure 3.6(a) shows WBSs' quasiSINR decreases when the radii of auxiliary circles increase. Subfigure 3.6(b) illustrates the choice on radius of auxiliary circle don't influence the performance of whiteCat. In the following simulation, we fixed the radius at 6000 m.

### 3.7.2 Performance of Channel Allocation Schemes Operating with Single Channel

The comparison schemes are *whiteCase* and No-regret learning, besides, centralized optimization is used to obtain global optima.

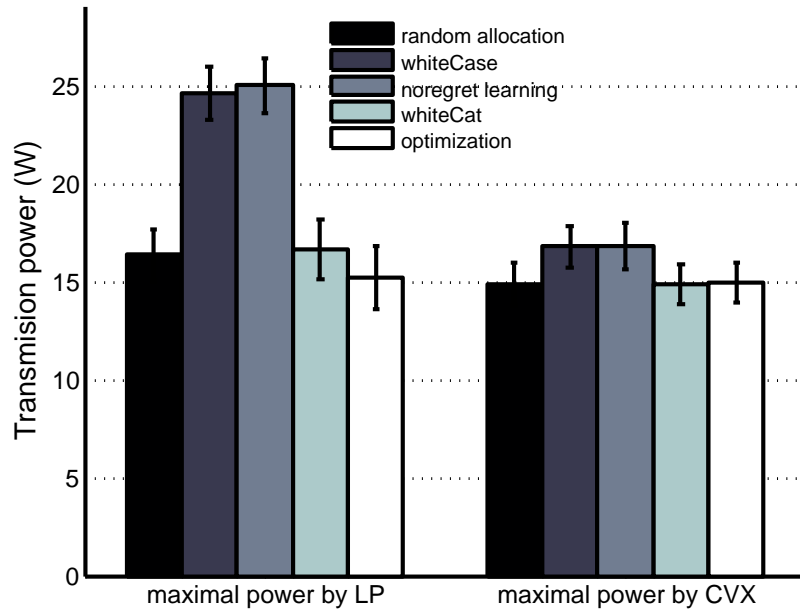
- *WhiteCase*: Whitespace channel allocation selfish, where each WBS selfishly updates its channel to achieve the best (as to the considered problem, smallest) possible utility based on Formula A.1.
- *Noregret learning*: Each WBS maps the probability of choosing each strategy to a certain proportion of the regret which the WBS may have if it doesn't choose that strategy, and the WBS choose the strategy with the biggest probability. WBSs update such mapping dynamically and this approach converges to correlated equilibrium. Please refer the original paper [77] for details.
- *Quadratic optimization*: centralized quadratic optimization introduced in Section 3.5.1.

In Section 3.4, two different optimization formulations are introduced to obtain the channel-power map for WBSs, i.e., convex optimization and linear optimization respectively. In Figure 3.4, we have seen the convex optimization generates power levels which distribute evenly between the minimum and maximum transmission power levels configured by the

hardware, while, the majority of the power levels generated by linear optimization are either the minimum or maximum transmission power. The simulation in this subsection carries twofold meanings. The first is to see which channel-power map outperforms the other, the second is to evaluate the performance of the channel allocation schemes. The adopted metrics are the SINR on end terminals and transmission power consumption.

### 3.7.2.1 Comparison of the Methods for Maximum Permitted Transmission Power

Figure 3.12 depicts the power consumption of the channel allocation schemes which work with the two groups of maximum permitted transmission power decided by linear and convex problems respectively. When given maximum permitted transmission power, whiteCat and the centralized optimization scheme consume the least energy. The schemes utilize less transmission power with the maximum permitted transmission power decided by convex optimization. Figure 3.8 shows the quasiSINR of WBSs. The centralized optimization scheme achieves the highest quasiSINR, because the optimization formulation 3.7 obtains the global optima.

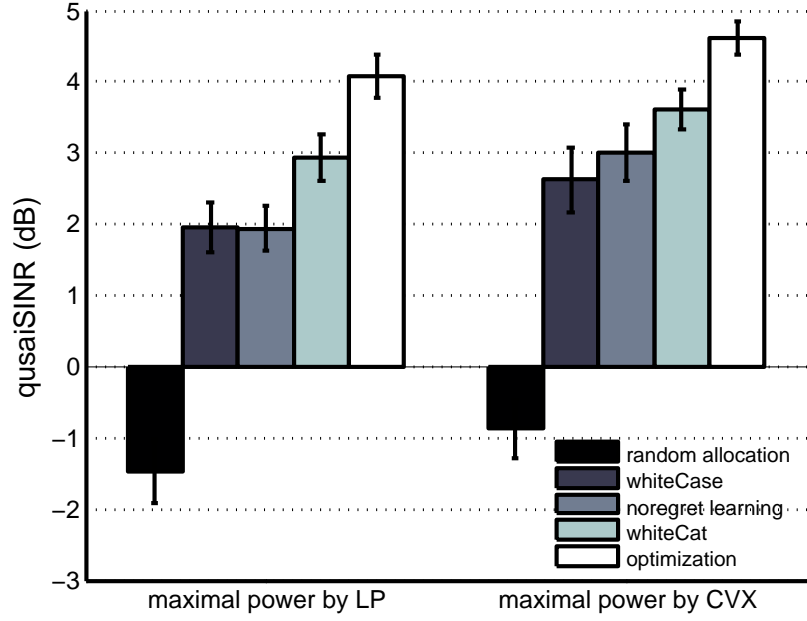


**Figure 3.7** Power consumed of WBSs by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

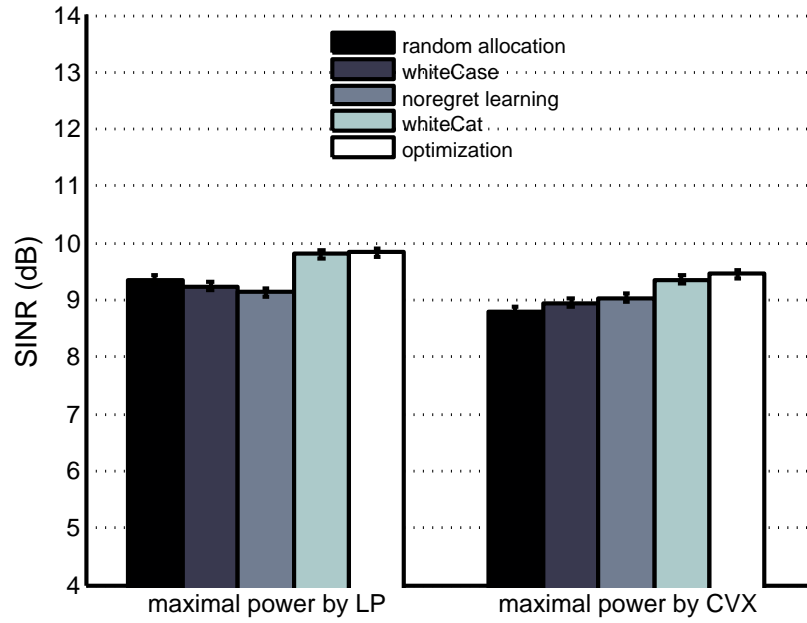
The average SINR on the end terminals is depicted in Figure 3.9. When the given maximum permitted transmission power, whiteCat and the centralized optimization achieve similar and the best performance among the schemes. It is also noticed that, the maximum permitted transmission power decided by linear optimization helps the channel allocation schemes achieve better SINR.

The empirical cumulative distribution function curve of SINR on end terminals is drawn in Figure 3.10.

The SINR achieved by WhiteCat and the centralized optimization is stably higher than that obtained from other schemes. For example, the 20% and 80% percentile of the SINR



**Figure 3.8** QuasiSINR of WBSs achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

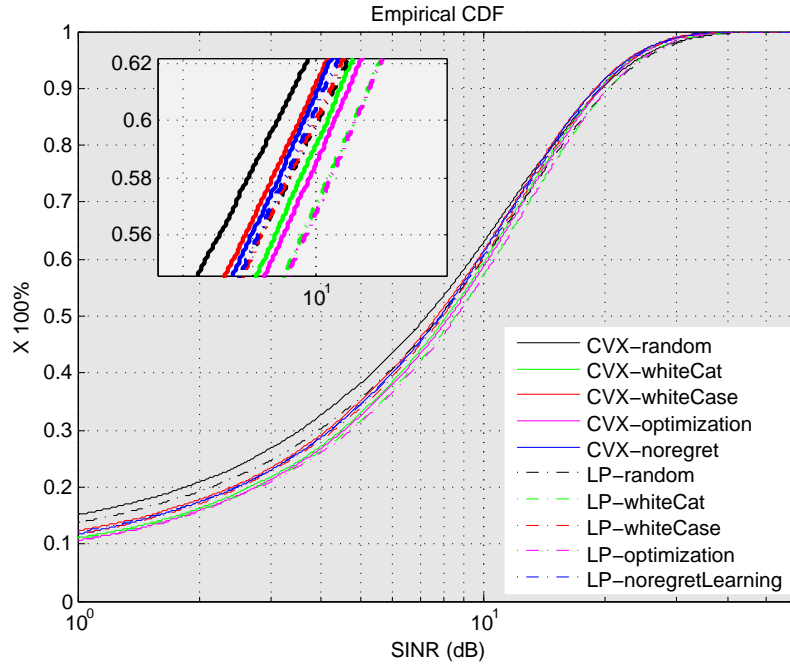


**Figure 3.9** SINR on end terminals achieved by different distributed spectrum allocation schemes under different ways deciding the maximal transmission power map

achieved by WhiteCat and the centralized optimization are 0.5 to 1 dB higher than the other channel allocation schemes.

### 3.7.2.2 Convergence Speed

In the congestion game where scheme whiteCat is derived, each player (WBS) has at most  $(n-1) * |\mathcal{C}|$  resources available for usage, thus there is no polynomial steps converging to NE, while, simulation shows the algorithm can quickly converge to NE when the number of WBS is up to 100. Table 3.3 shows the average number of steps needed before conver-



**Figure 3.10** CDF of SINR on end users obtained by different CA schemes under different methods to decide the maximal transmission power map

gence in 100 runs of simulations. As to whiteCat, we account each WBS accessing the base station (refer to 3.5.2) as *one step*. We compare the convergence speed of WhiteCat with no-regret learning, the scheme derived from potential game [129] and whiteCase. Note that the potential game scheme is to solve joint power and channel allocation problem, as it is developed with game theory, it is reasonable to see its convergence speed. As there is no guarantee for WhiteCase to converge, we stop the channel allocation process after 16000 steps (1000 rounds).

Table 3.3 tells that whiteCat is two times faster than the scheme derived from potential game, and 20 times faster than no-regret learning scheme. The relatively smaller confidence interval shows that whiteCat's convergence is not affected by different network configurations. Fast converge is attributed to the working style of WBSs which access the database to get the information of other WBSs, thus the distributable decision involves a part of the global information of the network. Thus, we can see that the speed up of convergence is due to the overhead caused by accessing the database.

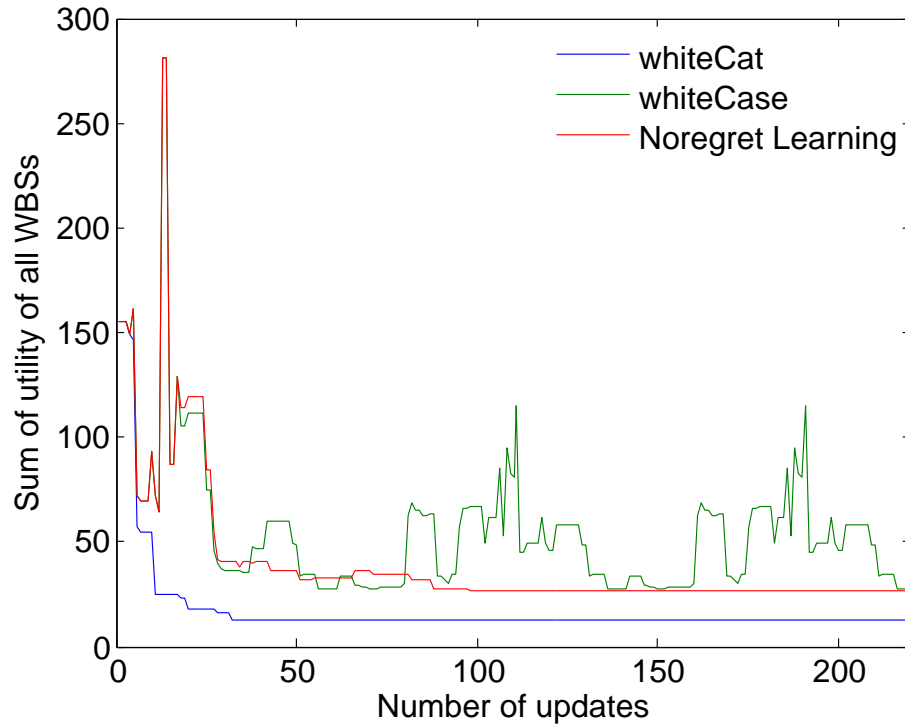
Figure 3.11 depicts one instance of the convergence processes of three schemes. The Y axis is the summed utility of all WBSs. We can see whiteCat decreases the summed utility constantly, and the channel allocation process ceases after 38 times of updates. Whereas, noregret learning scheme takes 120 steps before convergence, and whiteCase fails to converge.

### 3.7.3 Performance of Channel Allocation Schemes Operating with Single Channel

As comparison, we implement the centralized scheme proposed in [79] (code can be found at [2]), which is designed complying with FCC regulations. Some adaptations are made to comply with the ECC rulings, 1) When a channel is chosen, the WBS transmits with the

Scheme	Average steps	95% CI	Average time (s)
whiteCat	58	5.6	2
noregret	1916	1541	144
PotentialGame [129]	120	10	4
optimization-LINDO	-	-	40
whiteCase	4587	2742	50

**Table 3.3** Convergence speed of the distributed channel allocation schemes. As to the distributed scheme, the time involved to communicate with database is not considered and included.

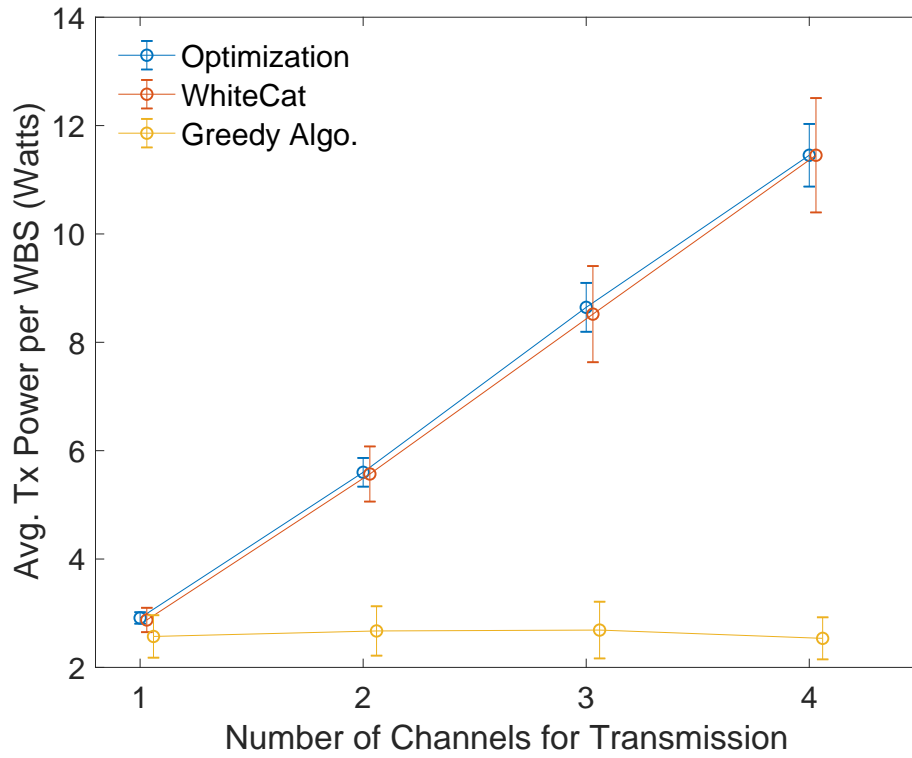


**Figure 3.11** Convergence process of three different schemes in one simulation.

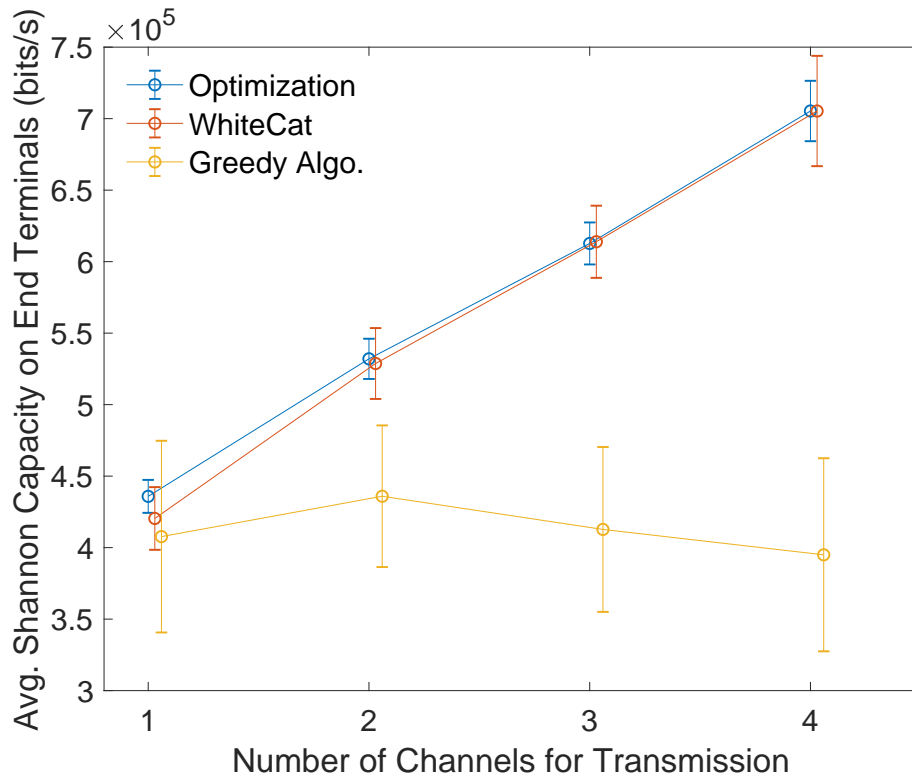
maximal permitted transmission power permitted on that channel instead of an identical power for all the WBSs; 2) Auxiliary circle is introduced into the scheme, and the SINR is not the quotient of the transmission power and the interference on the WBSs' locations, but the power of signal and interference on the auxiliary circles.

The first group of simulation is conducted with 9 WBSs which locate as a 3 X 3 array, there are 4 TVWS channels. Figure 3.12 and 3.13 depict the average transmission power of all the WBSs and average capacity over all the end user when working with different amount of channels. Figure 3.14 and 3.15 illustrate the average transmission power and capacity for each WBS over all simulation runs. The two proposed schemes have similar performances which increase linearly with the number of TVWS channels in use. The greedy scheme consumes as less power as our proposed schemes when single channel is used, because many WBSs are in idle state by adopting the greedy scheme. The average Shannon capacity is also comparable to our proposed schemes. When more channels are allowed, our proposed schemes clearly outperform the greedy scheme in terms of achieved Shannon capacity, with the cost of high transmission power.



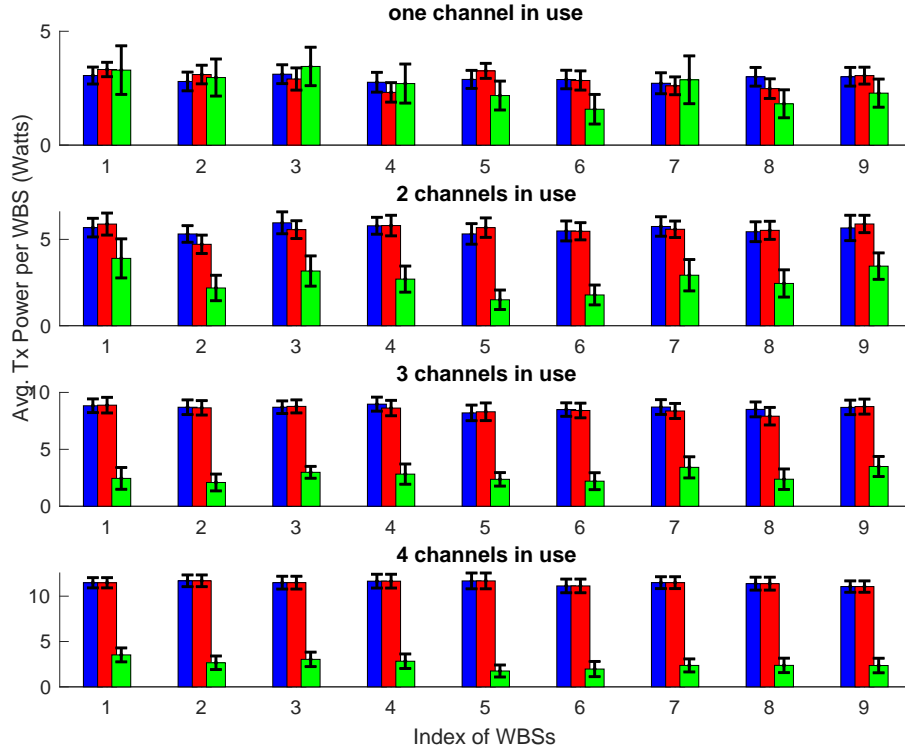


**Figure 3.12** Average transmission power of all WBSs, 9 WBSs, 4 channels.

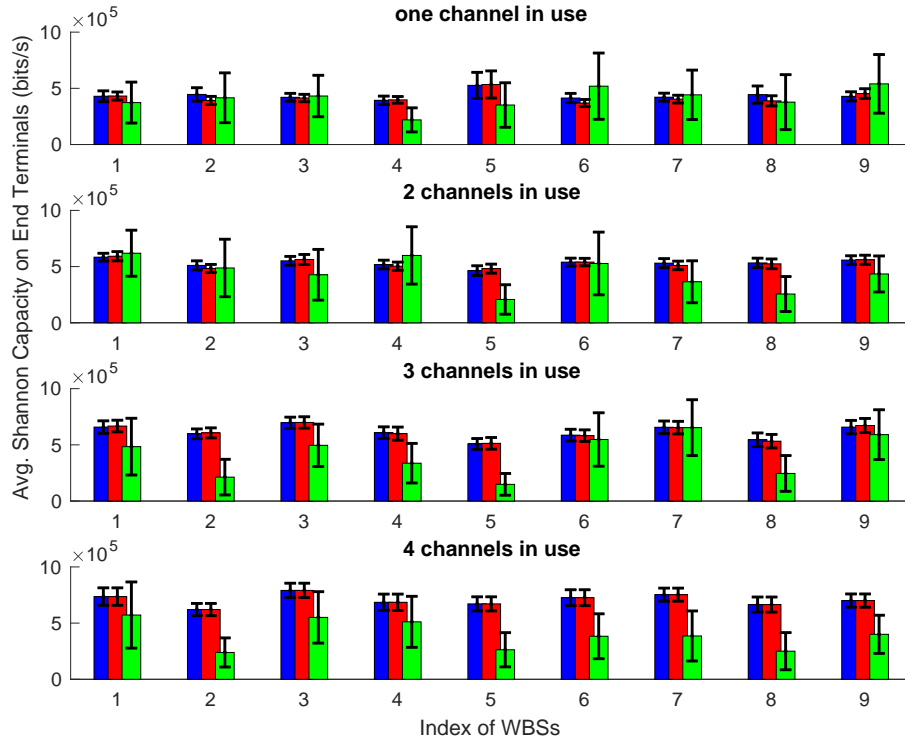


**Figure 3.13** Average capacity over all end terminals, 9 WBSs, 4 channels.

The second group of simulation is done with 16 WBSs, which is a denser scenario than group 1. The average transmission power of WBSs and average capacity on end users, as shown in 3.16 and 3.17, are similar when implying the proposed centralized and distributed schemes. Figure 3.18 and Figure 3.19 depict the average transmission power and



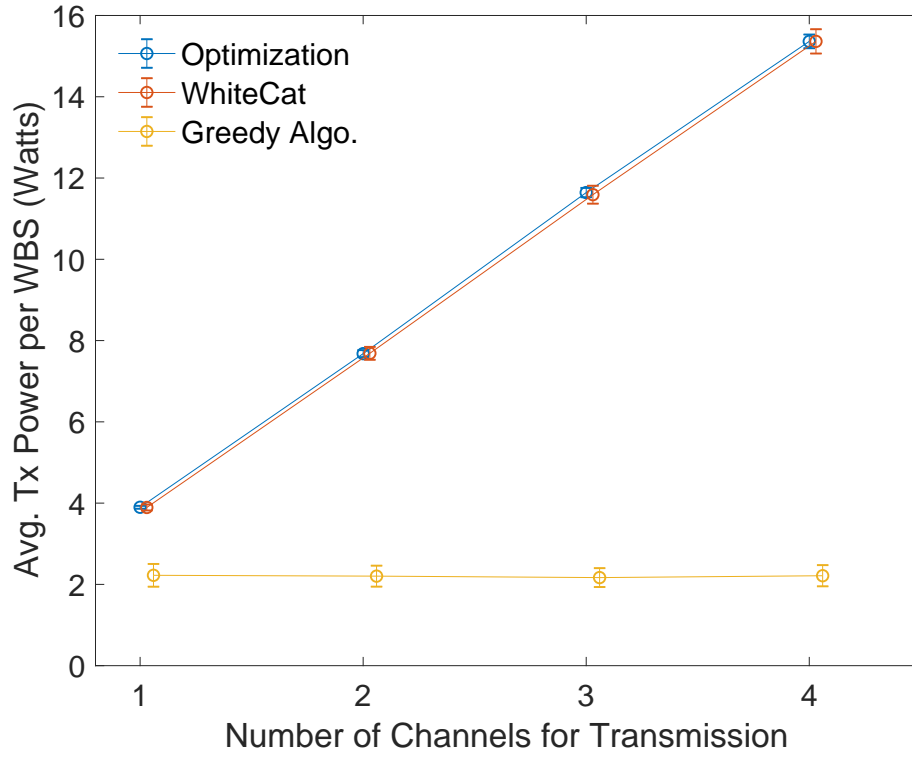
**Figure 3.14** Average transmission power of each WBS, 9 WBSs, 4 available channels. Increasing number of channels (1 to 4 channels) are used from top to bottom.



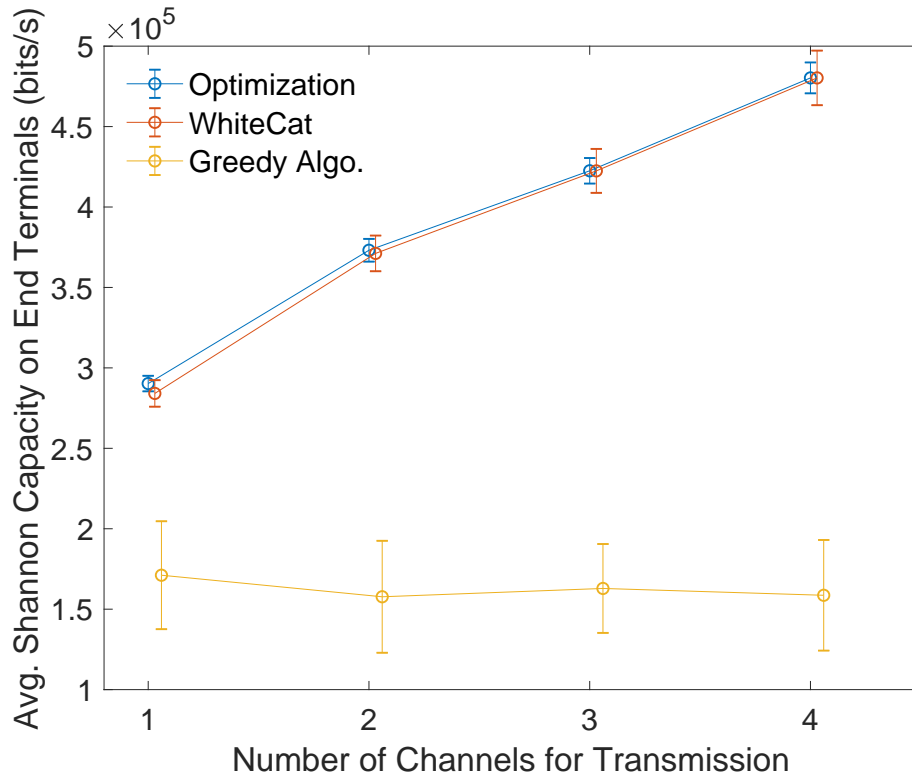
**Figure 3.15** Average capacity of end terminals in each WBS's cell, 9 WBSs, 4 available channels. Increasing number of channels (1 to 4 channels) are used from top to bottom.

Shannon capacity in each WBS cell. Comparing with group 1, less transmission power is consumed by all the three investigated schemes, and meanwhile the achieved capacity is less. This is because the network is denser in group 2 and the co-channel interference has

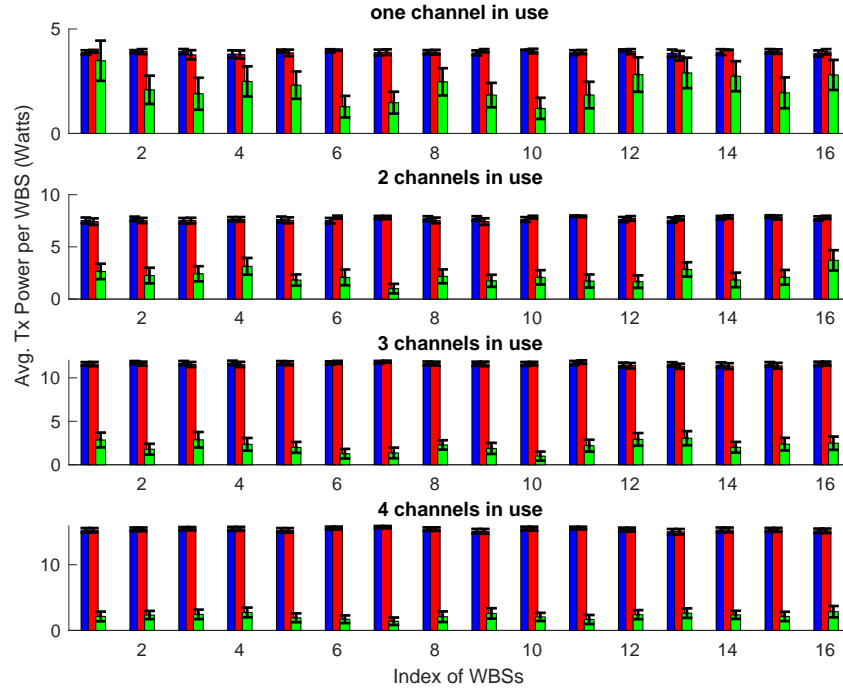
bigger impact on the WBSs. The greedy scheme is liable to generate more idle WBSs, as a result, both of the transmission power and Shannon capacity are less than our proposed schemes.



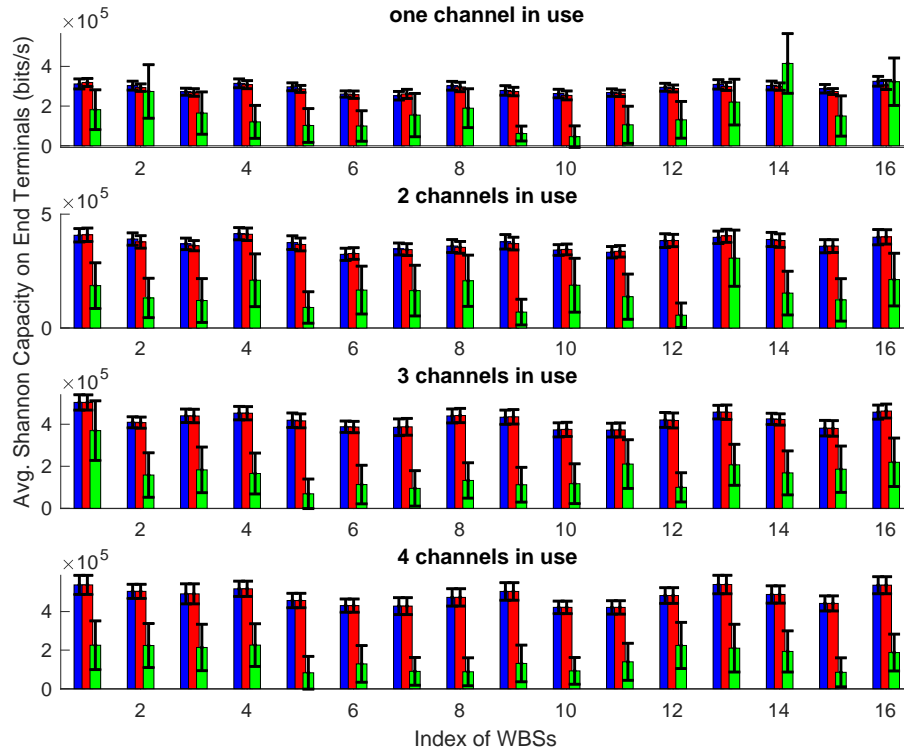
**Figure 3.16** Average transmission power of WBSs, 16 WBSs, 4 channels.



**Figure 3.17** Average capacity on end terminals, 16 WBSs, 4 channels.



**Figure 3.18** Average transmission power of WBSs, 16 WBSs, 4 available channels. Increasing number of channels (1 to 4 channels) are used from top to bottom.



**Figure 3.19** Average capacity on end terminals, 16 WBSs, 4 available channels. Increasing number of channels (1 to 4 channels) are used from top to bottom.

## 3.8 Conclusions

Congestion game is applied to analyse the channel allocation problem, where transmission power is not necessarily identical. The proposed algorithm which is derived from the best response of the congestion game converges quickly, and achieves better performance than other distributed schemes. Without consider the communication latency between WBSs and the database, this distributed scheme executes much faster than the centralized scheme.

In particular, we investigate the channel allocation problem in the context of utilization of TV white space. Except for channel allocation, we also propose solutions for transmission power control for the cellular network which complies with IEEE 802.22 standard.



# 4

## Congestion Game in Robust Clustering of CRN

### Abstract

In the process of forming clusters, every secondary user needs to decide with whom to form a cluster, or which cluster to join. Congestion game model is adopted to analyse this process, which not only contributes the algorithm design directly, but also provides guarantee on reaching Nash Equilibrium and convergence speed.

Our proposed distributed clustering scheme outperforms comparison scheme in terms of robustness against primary users, convergence speed and volume of control messages. Furthermore, the clustering algorithm is versatile enough to fulfil other requirements such like fast convergence and cluster size control.

### 4.1 Introduction

Cognitive radio (CR) is a promising technology to solve the spectrum scarcity problem [120]. Licensed users access the spectrum allocated to them whenever there is information to be transmitted. In contrast, unlicensed users can only access the licensed spectrum after validating the channel is unoccupied by licensed users, where spectrum sensing [175] plays an important role in this process. In this hierarchical spectrum access model [180], the licensed users are also called primary users (PU), and the CR users are known as secondary users and constitute the cognitive radio networks (CRN). As to the operation of CRN, efficient spectrum sensing is identified to be critical to the success of cognitive radio networks [147]. Cooperative spectrum sensing is able to effectively cope with noise uncertainty and channel fading, thus remarkably improving the sensing accuracy [22]. Collaborative sensing relies on the consensus of CR users within certain area, and decreases considerably the negative probability due to the random channel effects like fading and shadowing. In this regard, clustering is regarded as an effective method in cooperative spectrum sensing [156, 179], as a cluster forms adjacent secondary users as a

collectivity to perform spectrum sensing together. Clustering is also efficient to enable all CR devices within the same cluster to stop payload transmission on the operating channel and initiate the sensing process, so that all the CR users<sup>1</sup> within the one cluster are able to vacate the channel swiftly when primary users are detected by at least one CR node residing in the cluster [164]. With cluster structure, as CR users can be notified by cluster head (CH) or other cluster members about the possible collision, the possibility for them to interfere neighbouring clusters is reduced [139]. Clustering algorithm has also proposed to support routing in cognitive ad-hoc networks [13].

The communication within a cluster is conducted in the spectrum which is available for every member in that cluster. Usually there are multiple unlicensed channels available for all the members in a cluster, which are referred as *common control channels* (CCC). When one or several members can not use one certain CCC because primary users are detected to appear on that channel, this channel will be excluded from the set of CCCs, in particular, if this channel is the working channel, then all the cluster members switch to another channel in the set of CCCs. In the context of CRN, as the activity of primary users is controlled by licensed operators which are generally not known to CR users, the availability of CCCs for the formed clusters is totally decided by primary users' activity. In other words, the availability of CCCs for clusters is passive and can not guaranteed. In CRN, one cluster survives the influence of primary users when at least one CCC is available in that cluster. As the primary users' operation is assumed to be unknown to the CR users, a cluster formed with more CCCs will survive with higher probability. Thus the number of CCCs in one cluster indicates the robustness of it when facing the unpredictable influence from primary users.

To solely pursue cluster robustness against the primary users' activity, i.e., to achieve more common channels within clusters, the ultimately best clustering strategy is ironically that every node constitutes one cluster (called as *singleton cluster* in this paper). Apparently this contradicts our motivation of proposing cluster in cognitive radio network to enable cooperative decision making. This contradiction indicates that, the robustness discussed in terms of number of common channels carries little meaning when the sizes of formed clusters are not given consideration. Besides, cluster size plays import roles in certain aspects. For instance, cluster size is one decisive factor in power preservation [23, 87], and it also influences the accuracy of cooperative spectrum sensing [168].

In this paper, a decentralized clustering approach ROSS (RObust Spectrum Sharing) is proposed to cover the issues of robustness and size control of clusters in CRN. ROSS is able to form clusters with desired sizes, and when compared with previous works, ROSS involves smaller signalling overhead, and the generated clusters are significantly more robust against the primary users which appear after the clusters are formed. We also propose the light weighted versions of ROSS, which involve less overheads and thus are more suitable for fast deployment. Throughout this paper, we refer the clustering schemes on the basis of ROSS as *variants of ROSS*, i.e., the fast versions, or that with size control feature.

The rest of paper is organized as follows. After reviewing related work in section 4.2, we present the system model in Section 4.3. In Section 4.4, problem formulation is presented and the problem is thoroughly analysed. The centralized clustering scheme and the our

<sup>1</sup>The term *user* and *node* are used interchangeably in this paper, in particular, user is used when its networking or cognitive ability are discussed or stressed, and node is used when the network topology is discussed.



proposed clustering scheme ROSS are introduced in Section 4.5.3 and 4.5 respectively. Extensive performance evaluation is in section 4.6. Finally, we conclude our work and point out direction future research in section 4.7.

## 4.2 Related Work

Prior to the emergence of cognitive radio, clustering has been proposed as an important method to manage network in ad hoc networks [34, 94, 107], wireless mesh networks and sensor networks [13]. In ad hoc and mesh networks, the major focus of clustering is to preserve connectivity (under static channel conditions) or to improve routing. In sensor networks, the emphasis of clustering is on longevity and coverage. Overhead generated by clustering in ad hoc network is analysed in [155, 167].

As to cognitive radio networks, clustering schemes are also proposed, which target different aspects. Work [168] improves spectrum sensing ability by grouping the CR users with potentially best detection performance into the same cluster. Clustering scheme [87] obtains the best cluster size which minimizes power consumption caused by communication within and among clusters. [87] proposes clustering strategy in cognitive radio network, which looks into the relationship between cluster size and power consumption and accordingly controlling the cluster size to decrease power consumption. Cogmesh is proposed in [49] to construct clusters with the neighbouring nodes which share local common channels. This method pays attention to the communication among the formed clusters, but cluster robustness is not considered. [165] targets on the QoS poisoning and energy efficiency. This approach first decides on the relay nodes which minimize transmission power consumption, then the chosen nodes become cluster heads and clusters are formed in a dynamic coalition process. This work emphasis on power efficiency and doesn't take into account the channel availability and the issue of robustness of the formed clusters. In [29, 179], the channel available to the largest set of one-hop neighbours is selected as common channel which yields a partition of the CRN into clusters. This approach minimizes the set of distinct frequency bands (and hence, the set of clusters) used as common channels within the CRN. However, bigger cluster sizes generally lead to less options within one cluster to switch to if the common channel is reclaimed by a primary node. Hence, this scheme does not provide robustness to formed clusters. [26] deploys cluster structure in order to implement common channel control, medium access with multiple channel and channel allocation. The node with the maximum number of common channels within its k-hop neighborhood is chosen as cluster head, but how to avoid one node appearing in multiple clusters is not given consideration. A event-driven clustering scheme is proposed for cognitive radio sensor network in [131]. The cluster heads are selected according to node degree, spectrum availability and the distance to the sink, and the clusters are no longer available after the event ends.

Clustering robustness is considered in [102, 110, 115]. The first two propose a distributed scheme where the metric is the product of cluster size and the number of common control channels. This scheme involves both cluster size and the number of CCCs, but it is inherently flawed. With the metric, cluster could be formed only due to one factor of the two, e.g. a spectrum rich node will exclude its neighbour to form a cluster by itself. Besides, this scheme leads to a high variance on the size of clusters, which is not desired in certain applications as discussed in [26, 87]. [115] presents a heuristic method to form clusters,

although the authors claim robustness is one goal to achieve, they actually uphold the minimum number of formed clusters.

### 4.3 System Model

We consider a set of cognitive radio users  $\mathcal{N}$  and a set of primary users distributed on a two-dimensional Euclidean plane. These users share a number of non-overlapping licensed channels according to the spectrum overlay model. The set of these licensed channels is denoted as  $\mathcal{K}$ . As secondary users, the CR users are allowed to transmit on a channel  $k \in \mathcal{K}$  only if no primary user is detected being accessing channel  $k$ . Further, we consider a *cognitive radio ad-hoc network* which consists of all secondary users and does not contain any primary user.

Secondary users conduct spectrum sensing independently and sequentially on all licensed channels. We assume that every node can detect the presence of primary user on each channel with certain accuracy.<sup>2</sup> We denote  $K_i \subseteq \mathcal{K}$  as the set of available channels for  $i$ . We adopt the unit disk model [55] for the transmission of both primary and CR users. Both primary users and CR users have fixed transmission ranges respectively, and all the channels are regarded to be identical in terms of signal propagation. If a CR node locates within the transmission range of primary user  $p$ , that CR node is not allowed to use the channel  $k(p)$ .

We assume that in addition to the licensed channels, there is one dedicated control channel. This control channel could be in ISM band or other reserved spectrum which is exclusively used for transmitting control messages. Actually, the control messages involved in the clustering process can also be transmitted on the available licensed channels through a rendezvous process by channel hopping [73, 177], i.e., two neighbouring nodes establish communication on the same channel. Over the control channel, a secondary user  $i$  can exchange its spectrum sensing result  $K_i$  to any  $i' \in \text{Nb}(i)$ . It is available for any secondary node  $i$  to exchange control messages with any other node in its neighborhood  $\text{Nb}(i)$  during the cluster formation phase.  $\text{Nb}(i)$  is simply defined as the set of nodes located within the transmission range of  $i$ .

If a secondary user  $i$  is not in the transmission range of a primary user  $p$ ,  $i$  can certainly not detect the presence of  $p$ . As the transmission range of primary users is limited and secondary users are located at different locations, different secondary users may have different views of the spectrum availability, i.e., for any  $i, i' \in \mathcal{N}$ ,  $K_i = K_{i'}$  does not necessarily hold. As the assumed 0/1 state of connectivity is solely based on the Euclidean distance between secondary users,

A cognitive radio network can be represented as an undirected graph  $G = (\mathcal{N}, E)$ , where  $E \subseteq \mathcal{N} \times \mathcal{N}$  such that  $\{i, i'\} \in E$  if, and only if, there exists a channel  $k \in \mathcal{K}$  with  $k \in K_i \cap K_{i'}$ . Note that we consider the channel availability only for *one* snapshot of time. For the rest of this paper the word channel is referred to licensed channel, if the control channel is not explicitly mentioned.

<sup>2</sup>The spectrum availability can be validated with a certain probability of detection. Spectrum sensing/validation is out of the scope of this paper.

## 4.4 Robust Clustering Problem in CRN

In this section, we describe the definition of cluster in the context of CRNs, and the problem of robust clustering.

### 4.4.1 Cluster in CRN

A cluster  $C \subseteq \mathcal{N}$  is a set of secondary nodes consisting of a cluster head  $h_C$  and a number of cluster members. The cluster head is able to communicate with any cluster member directly. In other terms, for any cluster member  $i \in C$ ,  $i \in \text{Nb}(h_C)$  holds.

Cluster is denoted as  $C(i)$  when its cluster head is  $i$ . Cluster size of  $C(i)$  is written as  $|C(i)|$ .  $K(C) = \cap_{i \in C} K_i$ ,  $K(C)$  denotes the set of common control channels in cluster  $C$ . Clustering is performed periodically, as secondary users are mobile and the channel availability on secondary users are changing as primary users change their operation state.

Symbol	Description
$\mathcal{N}$	collection of CR users in a CRN
$N$	number of CR users in a CRN, $N =  \mathcal{N} $
$\mathcal{K}$	set of licensed channels
$k(i)$	the working channel of user $i$
$\text{Nb}(i)$	the neighborhood of CR node $i$
$C(i)$	a cluster whose cluster head is $i$
$K_i$	the set of available channels at CR node $i$
$K(C(i))$	the set of available CCCs of cluster $C(i)$
$h_C$	the cluster head of a cluster $C$
$\delta$	desired cluster size
$S_i$	a set of claiming clusters, each of which includes debatable node $i$ after phase I
$d_i$	individual connectivity degree of CR node $i$
$g_i$	social connectivity degree of CR node $i$
$C_i$	the $i$ th legitimate cluster (only appear in Sec. 4.5.3)

**Table 4.1** Notations in robust clustering problem

### 4.4.2 Robust Clustering Problem

In the introduction section, we have stated that cluster size should be given consideration to justify the concept of robustness of clusters, i.e., without specifying requirement on cluster sizes, small clusters will be generated to obtain more CCCs. Except for cooperative sensing, clusters need to conduct some other functionalities. When cluster size is large, there will be substantial burden on cluster heads to manage the cluster members, which is a challenge for resource limited cluster heads, thus the cluster size should fall in a desired range [52, 140].

The centralized clustering scheme aims to form clusters with desired sizes, meanwhile the total number of CCCs of all clusters is maximized. In the following, we refer this problem as *centralized clustering*, and give the formal problem definition.

**DEFINITION 3:** *Robust clustering problem in CRN.*

Given a cognitive radio network  $\mathcal{N}$  where nodes are indexed from 1 to  $N$  sequentially. Based on certain correlation, certain secondary users constitute one cluster  $C$ .  $1 \leq |C| \leq k$  where  $|C|$  is the size of cluster  $C$  and  $k$  is a positive integer. We name the collection of such clusters as  $\mathcal{S} = \{C_1, C_2, \dots, C_{|\mathcal{S}|}\}$ , where  $\mathcal{S}$  satisfies the following properties:  $\bigcup_{1 \leq i \leq |\mathcal{S}|} C_i = N$  and  $K(C(i)) \neq \emptyset$  for any  $i$  which satisfies  $1 \leq i \leq |\mathcal{S}|$ .

We give a new definition of the number of CCCs, where the number of common control channels is  $|K(C)|$  if  $|C| > 1$ , and is zero when  $|C| = 1$ . We use  $f(C)$  to denote the number of CCCs of a cluster  $C$  in the new definition.

The centralized clustering problem is to find a subcollection  $\mathcal{S}' \subseteq \mathcal{S}$ , so that  $\bigcup_{C_j \in \mathcal{S}'} C_j = N$ , and  $C_{j'} \cap C_j = \emptyset$  for  $C_{j'}, C_j \in \mathcal{S}'$  and  $j' \neq j$ , so that  $\sum_{C \in \mathcal{S}'} f(C)$  is maximized. The decision version of centralized clustering in CRN is to ask whether there exists a non-empty  $\mathcal{S}' \subseteq \mathcal{S}$ , so that  $\sum_{C \in \mathcal{S}'} f \geq \lambda$  where  $\lambda$  is a real number.

#### 4.4.3 Complexity of the Robust Clustering Problem

In this section we investigate the complexity of robust clustering problem. Theorem 4.4.1 tells centralized clustering problem in CRN is one NP-hard problem.

**THEOREM 4.4.1:** *CRN clustering problem is NP-hard, when the maximum size of clusters  $k \geq 3$ .*

The proof is in the following.

**Proof.** We put the definition of weighted  $k$ -set packing problem here as it to be used in the analysis on the complexity of the centralised clustering problem.

**DEFINITION 4:** *Weighted  $k$ -set packing.*

Given a finite set  $\mathcal{G} = \{g_1, \dots, g_N\} \subset \mathbb{N}$  of non-negative integers and a collection of sets  $\mathcal{Q} = \{S_1, S_2, \dots, S_m\}$  such that  $S_i \subseteq \mathcal{G}$  for every  $1 \leq i \leq m$ . Each set  $S \in \mathcal{Q}$  is associated with a weight  $\omega(S) \in \mathbb{R}$ . Further, we are given a threshold value  $\lambda \in \mathbb{N}$ .

The question is whether there exists a collection  $\mathcal{S} \subseteq \mathcal{Q}$  such that  $\mathcal{S}$  contains only pairwise disjoint sets, i.e., for all  $S, S' \in \mathcal{S}$  with  $S \neq S'$  it holds that  $S \cap S' = \emptyset$ , and the total weight of the sets in  $\mathcal{S}$  is greater than  $\lambda$ , i.e.,  $\sum_{S \in \mathcal{S}} \omega(S) > \lambda$ .

Weighted  $k$ -set packing is NP-hard when  $k \geq 3$ . [68]

To prove that the centralized clustering problem is NP-hard, we reduce the NP-hard problem *weighted  $k$ -set packing* to it to prove the former is at least as hard as the latter. We show the existence of a polynomial-time algorithm  $\sigma$  that transforms any instance  $\mathcal{I}$  of a weighted  $k$ -set packing into an instance  $\sigma(\mathcal{I})$  of centralized clustering problem.

W.l.o.g. let set  $\mathcal{G} = \{1, \dots, N\}$ . The polynomial algorithm  $\sigma$  consists of three steps.

- In the first step, we transform the instance  $\mathcal{I}$  to  $\mathcal{I}'$  by adding dummy elements into each set in  $\mathcal{I}$ . Formally, for each set  $s_i \in \mathcal{I}$ , we create  $s'_i = s_i \times \{1, 2\}$ . Then we change the dummy elements by adding  $N$  on them. The purpose of this transformation is to eliminate the set in  $\mathcal{I}$ , which has single element. The weight of these sets remain unchanged, i.e.,  $\omega(s'_i) = \omega(s_i)$ .

- Mapping the elements in  $\mathcal{I}'$  to CR nodes on a two-dimensional Euclidean plane, and mapping the sets in  $\mathcal{I}'$  to clusters. There is a bijection between the union of sets in  $\mathcal{I}'$  to the CRN which consists of the CR nodes, in particular, each element one-to-one corresponds to the CR node whose node ID equals to the element. We further regulate that, each set in  $\mathcal{I}'$  has a corresponding cluster, i.e., there is a bijection between a set in  $\mathcal{I}'$  and a set of node IDs in a cluster, besides, the number of CCCs is equal to the weight of the corresponding set.
- In this step, we transform the instance  $\mathcal{I}'$  to a clustering proposal for CRN. We adjust the locations of the CR nodes which are mapped from the dummy elements in the instance  $\mathcal{I}'$ , and let them beside the CR nodes whose ID is exact  $N$  smaller. Because of the dummy nodes, the clustering solution which corresponds to  $\mathcal{I}'$  doesn't have singleton cluster. This transformation requires  $2 \cdot \sum_{s_i \in \mathcal{I}'} |s_i|$  steps. Now we look back to the elements in  $\mathcal{Q}$ , which don't appear in  $\mathcal{I}'$ . We one-to-one map these elements to CR nodes, and arbitrarily put these CR nodes in the plane, and these CR nodes become single node clusters. According to the definition of clustering problem in CRN, the number of CCCs in these single node clusters is 0. These singleton clusters and the clusters mapped from  $\mathcal{I}'$  constitute a clustering proposal, and finding these singleton clusters requires at most  $N$  steps. An example is shown in Table 4.2.

$\mathcal{N}$	$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$\mathcal{Q}$	$\{\{1\}, \{1, 5\}, \{1, 2, 4\}, \{2, 3\}, \{4\}\}$
Instance for Weighted k-set packing	$\{\{1\}, \{2, 3\}, \{4\}\}$
Instance with dummy elements	$\{\{1, 11\}, \{2, 12, 3, 13\}, \{4, 14\}\}$
Instance for clustering solution (dashed circles are dummy nodes)	

Table 4.2

We have crossed the hurdle of finding one polynomial algorithm  $\sigma$  which transforms an instance of weighted k-set packing to an instance of the clustering problem in CRN. When  $\mathcal{I}$  is not an instance for weighted k-set packing problem due to the existence of joint sets, the corresponding cluster strategy is not a solution for the centralized clustering problem, as there are overlapped clusters.

When there are only disjoint sets in an instance  $\mathcal{I}$  for weighted k-set packing, the sum weight is identical to the sum number of CCCs in the CRN mapped from  $\mathcal{I}'$ , even  $\mathcal{I}$  contains sets which only have one element. Thus, when a instance  $\mathcal{I}$  for k-set packing problem is true, i.e., the sum of weights is greater than  $\lambda$ , then in the CRN which is mapped from  $\mathcal{I}'$ , the summed number of CCCs of the clusters is greater than  $\lambda$ . In the other way around, when an instance is false for weighted k-set packing problem, the summed number of CCCs of the clusters in the mapped CRN is smaller than  $\lambda$ , and there is no clustering solution satisfying the centralized clustering problem.

Thus, weighted k-set packing can be reduced to clustering problem in CRN, then the latter problem is of NP-hard.  $\square$

In the following sections, both distributed and centralized solutions will be introduced.

## 4.5 Distributed Clustering Algorithm: ROSS

The cluster formation of one CRN can be obtained by solving the proposed optimization problem, but centralized scheme is inherently not suitable for the CRN which involves dynamics, e.g., the mobility of CR users and the variance of the available spectrum due to primary users' operation. In this section we introduce the distributed clustering scheme ROSS. ROSS consists of two cascaded phases: *cluster formation* and *membership clarification*. With ROSS, CR nodes form clusters based on the proximity of the available spectrum in their neighbourhood. Afterwards, CR nodes belong to one certain cluster.

### 4.5.1 Phase I - Cluster Formation

After conducting spectrum sensing and communication with neighbours, every CR node is aware of the available channels available for themselves as well as for all its neighbors. For each CR user  $i$ , two metrics are proposed to characterize the spectrum proximity between  $u$  and its neighborhood:

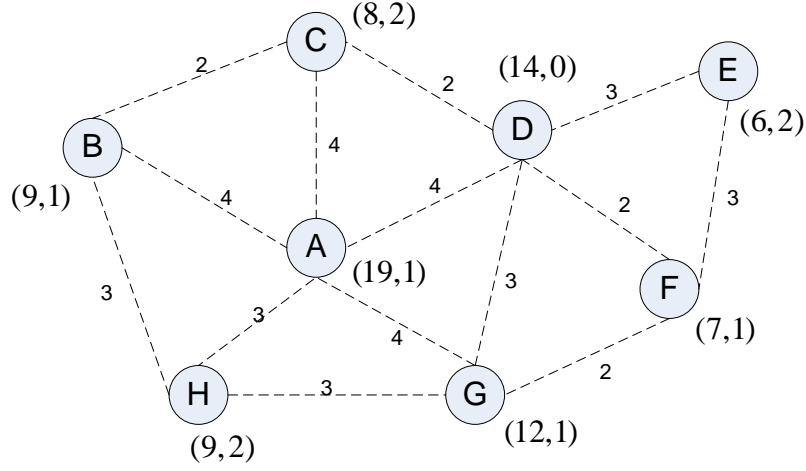
- *Individual connectivity degree  $d_i$* :  $d_i = \sum_{j \in \text{Nb}(i)} |K_i \cap K_j|$ . It denotes the sum of the numbers of common controls channels between node  $i$  and every neighbour. It is an indicator of node  $i$ 's adhesive property to the CRN.
- *Social connectivity degree  $g_i$* :  $g_i = |\bigcap_{j \in \text{Nb}(i) \cup i} K_j|$ . It is the number of common channels available to  $i$  and all its neighbors.  $g_i$  represents the ability of  $i$  to form a robust cluster with its neighbours.

Individual connectivity degree  $d_i$  and social connectivity degree  $g_i$  together form the *connectivity vector*. Figure 4.1 illustrates an example CRN where each node's connectivity vector is shown.

After introducing the connectivity vector, we proceed to introduce the first phase of algorithm ROSS. To put it briefly, in this phase cluster heads are determined in the beginning. Then clusters are formed on the basis of the cluster heads' neighborhoods.

#### 4.5.1.1 Determining Cluster Heads and Form the Initial Clusters

In this phase, each CR node decides whether it is a cluster head by comparing its connectivity vector with its neighbors. When CR node  $i$  has lower individual connectivity degree than any neighbors except for those which have already become cluster heads (the appearance of cluster heads will be explained in Section 4.5.2), then node  $i$  becomes clusters head. If there is another CR node  $j$  in its neighborhood which has the same individual connectivity degree as  $i$ , i.e.,  $d_j = d_i$  and  $d_j < d_k, \forall k \in \text{Nb}(j) \setminus \{\text{CHs} \cup i\}$ , then the



**Figure 4.1** Connectivity graph of the example CRN and the connectivity vector  $(d_i, g_i)$  for each node. The sets of the indices of the available channels sensed by each node are:  $K_A = \{1, 2, 3, 4, 5, 6, 10\}$ ,  $K_B = \{1, 2, 3, 5, 7\}$ ,  $K_C = \{1, 3, 4, 10\}$ ,  $K_D = \{1, 2, 3, 5\}$ ,  $K_E = \{2, 3, 5, 7\}$ ,  $K_F = \{2, 4, 5, 6, 7\}$ ,  $K_G = \{1, 2, 3, 4, 8\}$ ,  $K_H = \{1, 2, 5, 8\}$ . Dashed edge indicates the end nodes are within each other's transmission range.

node out of  $\{i, j\}$  with higher social connectivity degree becomes cluster head. The other nodes become a member of that cluster. If  $g_i = g_j$  as well, the node ID is used to break the tie, i.e., the one with smaller node ID becomes the cluster's head. The node which becomes cluster head broadcasts a message of its eligibility of being cluster head to notify its neighbours, and claims its neighbourhood as its cluster. The pseudo code for the cluster head decision and the initial cluster formation is shown in Algorithm 2.

After receiving the notification from a cluster head, a CR node  $i$  is aware that it becomes a member of a cluster. Consequently,  $i$  sets its individual connectivity degree to a positive number  $M > |\mathcal{K}| \cdot N$ . Then  $i$  broadcasts its new individual connectivity degree to all its neighbours. We manipulate the individual connectivity degree of the CR nodes which are included in certain clusters. Hence, nodes located outside of the a formed cluster can possibly become cluster heads or can also be included into other clusters. When a CR node  $i$  is associated to multiple clusters, i.e.,  $i$  has received multiple notifications of cluster head eligibility from different CR nodes,  $d_i$  is still set to  $M$ . We have the following theorem to show that as long as a secondary user's individual connectivity degree is greater than zero, that secondary user will eventually be integrated into a certain cluster, or it eventually becomes a cluster head.

**THEOREM 4.5.1:** *Given a CRN, it takes at most  $N$  steps that every secondary user either becomes cluster head, or gets included into at least one cluster.*

Here, by *step* we mean one secondary user executing Algorithm 4.5.1 for one time.

**Proof.** We consider a CRN which can be represented as a connected graph. To simplify the discussion, we assume the secondary users have unique individual connectivity degrees. Each user has an identical ID and a social connectivity degree. This assumption is fair as the social connectivity degrees and node ID are used to break ties in Algorithm 2, when the individual connectivity degrees are unique, it is not necessary to use the former two metrics.

---

**Algorithm 2:** ROSS phase I: cluster head determination and initial cluster formation for Unclustered CR node  $i$

---

**Input:**  $d_j, g_j, j \in Nb_i \setminus CHs$ . Empty sets  $\tau_1, \tau_2$

**Result:** Returning 1 means  $i$  is cluster head, then  $d_j$  is set to 0,  $j \in Nb_i \setminus CHs$ .  
returning 0 means  $i$  is not CH.

```

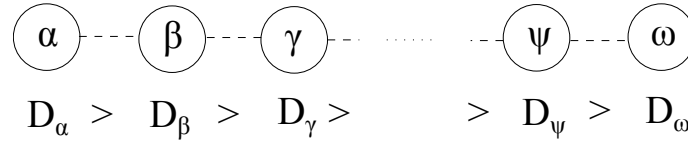
1 if  $\nexists j \in Nb_i \setminus CHs$ , such that  $d_i \geq d_j$  then
2   | return 1;
3 end
4 if  $\exists j \in Nb_i \setminus CHs$ , such that  $d_i > d_j$  then
5   | return 0;
6 else
7   | if  $\nexists j \in Nb_i \setminus CHs$ , such that  $d_j == d_i$  then
8     |  $\tau_1 \leftarrow j$ 
9   | end
10 end
11 if  $\nexists j \in \tau_1$ , such that  $g_i \leq g_j$  then
12   | return 1;
13 end
14 if  $\exists j \in \tau_1$ , such that  $g_i < g_j$  then
15   | return 0;
16 else
17   | if  $\nexists j \in \tau_1$ , such that  $g_j == g_i$  then
18     |  $\tau_2 \leftarrow j$ 
19   | end
20 end
21 if  $ID_i$  is smaller than any  $ID_j, j \in \tau_2 \setminus i$  then
22   | return 1;
23 end
24 return 0;

```

---



For the sake of contradiction, let us assume there exist some secondary user  $\alpha$  which is not included into any cluster. Then there is at least one node  $\beta \in \text{Nb}_\alpha$  such that  $d_\alpha > d_\beta$ . According to Algorithm 2,  $\delta$  is not included in any clusters, because otherwise  $d_\beta = M$ , a large positive integer. Now, we distinguish between two cases: If  $\beta$  becomes cluster head, node  $\alpha$  is included, the assumption is not true. If  $\beta$  is not a cluster head, then  $\beta$  is not in any cluster, we can repeat the previous analysis made on node  $\alpha$ , and deduce that node  $\beta$  has at least one neighbouring node  $\gamma$  with  $d_\gamma < d_\beta$ . Till now, when there is no cluster head identified, the unclustered nodes, i.e.,  $\alpha, \beta$  form a linked list, where their connectivity degrees monotonically decrease. But this list will not continue to grow, because the minimum individual connectivity degree is zero, and the length of this list is upper bounded by the total number of nodes in the CRN. An example of the formed node series is shown as Figure 4.2.



**Figure 4.2** The node series discussed in the proof of Theorem 4.5.1, the deduction begins from node  $\alpha$

In this example, node  $\omega$  is at the tail of the list. As  $\omega$  does not have neighboring nodes with lower individual connectivity degree,  $\omega$  becomes a cluster head. Then  $\omega$  incorporates all its one-hop neighbours (here we assume that every newly formed cluster has common channels), including the nodes which precede  $\omega$  in the list. The nodes which join a cluster set their individual connection degrees to  $M$ , which enables the node immediately precede in the list to become a cluster head. In this way, cluster heads are generated from the tail of list to the head of the list, and all the nodes in the list are in at least one cluster, which contradicts the assumption that  $\alpha$  is not included in any cluster.

If we see a secondary user *becoming a cluster head*, or *becoming a cluster member* as one step, as the length of the list of secondary users is not larger than  $N$ , there are  $N$  steps for this scenario to form the initial clusters.

□

The procedure of the proof also illustrates the time needed to conduct Algorithm 4.5.1. Consider an extreme scenario, where all the secondary nodes sequentially execute Algorithm 2, i.e., they constitute one list as discussed in the example in the proof. If one step can be finished within certain time  $T$ , then the total time needed for the network to conduct Algorithm 4.5.1 is  $N * T$ . In other scenarios, as Algorithm 2 can be executed concurrently by secondary users which locate in different places, the needed time can be considerably reduced. Let us apply Algorithm 2 to the example shown in Figure 4.1. Node  $B$  and  $H$  have the same individual connectivity degree, i.e.,  $d_B = d_H$ . As  $g_H = 2 > g_B = 1$ , node  $H$  becomes the cluster head and cluster  $C_H$  is  $\{H, B, A, G\}$ .

#### 4.5.1.2 Guarantee the Availability of Common Control Channel

After executing phase I of ROSS, it is possible that certain formed clusters don't own CCCs. As decreasing cluster size increases the number of CCCs within the cluster, for

those clusters having no CCCs, certain nodes need to be eliminated to gain at least one CCC. The sequence of elimination is performed according to an ascending list of nodes which are sorted by the number of common channels between the nodes and the cluster head. In other words, the cluster member which has the least number of common channels with the cluster head is excluded first. If there are multiple nodes having the same number of common channels with the cluster head, the node whose elimination brings in more common channels will be excluded. If this criterion meets a tie, the tie will be broken by deleting the node with smaller node ID. It is possible that the cluster head excludes all its neighbours and resulting in a *singleton cluster* which is composed by itself. The pseudo code for cluster head to obtain at least one common channel is shown in Algorithm 3. As to the nodes eliminated in this procedure, they restore their original individual connectivity degrees, and become either cluster heads or get included into other clusters afterwards according to Theorem 4.5.1.

---

**Algorithm 3:** ROSS phase I: cluster head guarantees the availability of CCC (start from line 1) / cluster size control (start from line 2)

---

**Input:** Cluster  $C$ , empty sets  $\tau_1, \tau_2$

**Output:** Cluster  $C$  has at least one CCC, or satisfies the requirement on cluster size

---

```

1 while  $K_C = \emptyset$  do
2   while  $|C| > \delta$  do
3     if  $\exists$  only one  $i \in C \setminus H_C, i = \arg \min(|K_{H_C} \cap K_i|)$  then
4        $C = C \setminus i$ 
5     else
6        $\exists$  multiple  $i$  which satisfies  $i = \arg \min(|K_{H_C} \cap K_i|)$ 
7        $\tau_1 \leftarrow i$ 
8     end
9     if  $\exists$  only one  $i \in \tau_1, i = \arg \max(|\cap_{j \in C \setminus i} K_j| - |\cap_{j \in C} K_j|)$  then
10       $C = C \setminus i$ 
11    else
12       $C = C \setminus i$ , where  $i = \arg \min_{i \in \tau_1} ID_i$ 
13    end
14  end
15 end

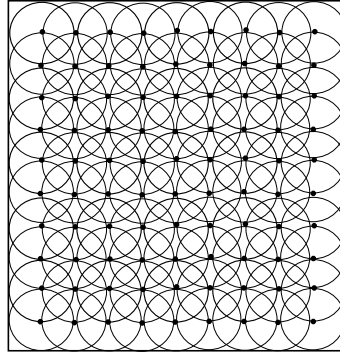
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#### 4.5.1.3 Cluster Size Control in Dense CRN

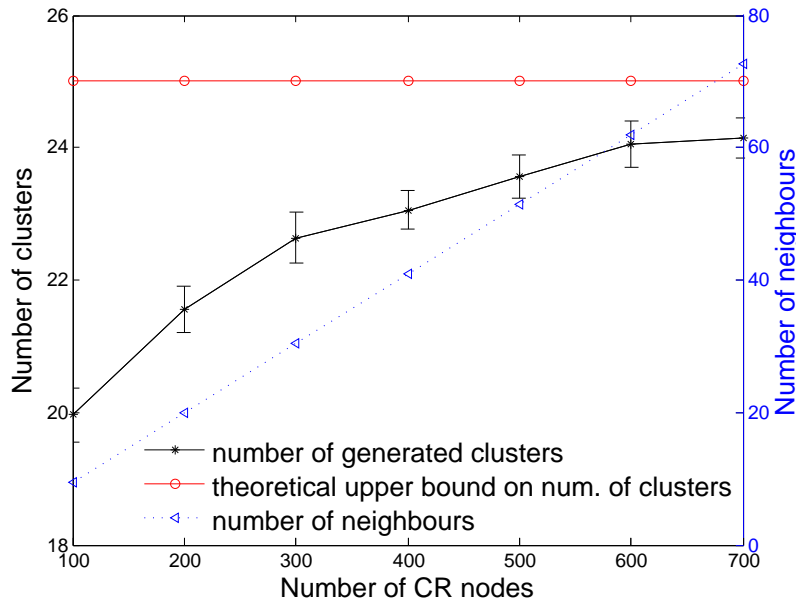
In the following we illustrate the pressing necessity to control the cluster size when CRN becomes dense via both theoretical analysis and simulation. Assuming the CR users and PUs are evenly distributed and PUs occupy the licensed channels randomly, then both CR nodes density and channel availability in the CRN can be seen to be spatially homogeneous. The formed clusters are the neighbourhoods of cluster heads, and the neighbourhood is decided by the transmission range and network density. We consider a cluster  $C(i)$  where  $i$  is CH in a dense CRN. When we don't consider the CHs which could appear within  $i$ 's neighbourhood in the procedure of guaranteeing CCCs, according to Algorithm 2, the nearest cluster heads could locate just outside node  $i$ 's transmission range. An instance of this situation is shown in Figure 4.3. In the figure, black dots represent

cluster heads, the circles denotes the transmission ranges of cluster heads. Cluster members are not shown in the figure. Let  $l$  be the length of side of simulation plan square, and



**Figure 4.3** Clusters formation in extremely dense CRN. Black dots are cluster heads, cluster members are not drawn.

$r$  be CR's transmission radius. Based on the aforementioned analysis and geometry illustration as shown in Figure 4.3, we give an estimate on the maximum number of generated clusters, which is the product of the number of cluster heads in one row and that number in one line,  $l/r * l/r = l^2/r^2$ . Given  $r=10$  and  $l=50$ , the maximum number of clusters is 25. The number of clusters in the simulation is shown in Figure 4.4. Simulation is run for 50 times and the confidence interval is 95%. With the increase of CR users, network density (the average number of neighbours) increases linearly, and the number of clusters approaches to 25 which complies with the estimation.

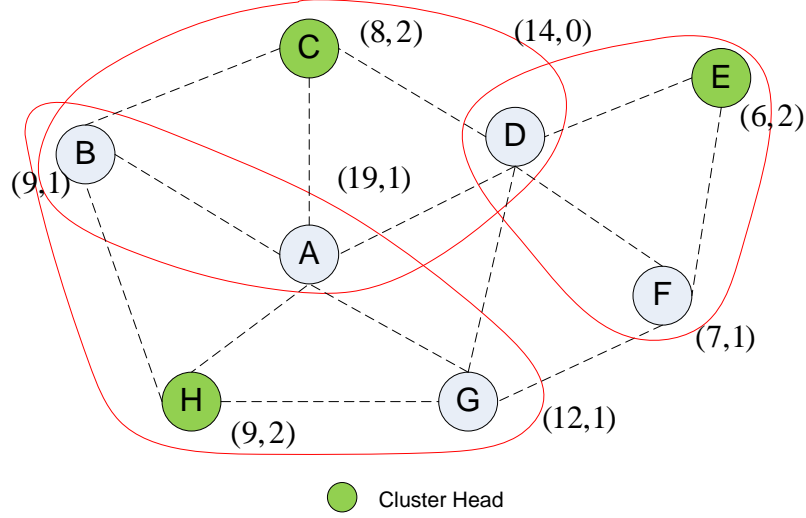


**Figure 4.4** The correlation between the number of formed clusters and network density.

Both analysis and simulation show that when applying ROSS, after the number of clusters saturates with the increase of network density, the cluster size increases linearly with the network density, thus certain measures are needed to curb this problem.

This task falls to the cluster heads. To control the cluster size, cluster heads prune their cluster members to achieve the desired cluster size. The desired size  $\delta$  is decided based on the capability of the CR users and the tasks to be conveyed. Given the desired size

$\delta$ , a cluster head excludes members sequentially according to the following principle, the absence of one cluster member leads to the maximum increase of common channels within the cluster. This process ends when the size of resultant cluster is  $\delta$  and at least one CCC is available. This procedure is similar with that to guarantee CCCs in cluster, thus the algorithm can reuse Algorithm 3.



**Figure 4.5** Clusters formation after the phase I of ROSS. Some nodes are debatable nodes, i.e., belonging to more than one cluster.

## 4.5.2 Phase II - Membership Clarification

After applying phase I of ROSS to the example in Figure 4.1, the resulted clusters are shown in Figure 4.5. We notice nodes  $A, B, D$  are included in more than one cluster. We refer these nodes as *debatable nodes* as their cluster affiliations are not clear, and the clusters which include the debatable node  $i$  are called *claiming clusters* of node  $i$ , and are denoted as  $S_i$ . Actually, debatable nodes extensively exist in CRN with larger scale. Debatable nodes should be exclusively associated with only one cluster and be removed from the other claiming clusters, this procedure is called *cluster membership clarification*. We will introduce the solution for cluster membership clarification in the following.

### 4.5.2.1 Distributed Greedy Algorithm (DGA)

After Phase I, debatable nodes, e.g.,  $i$  needs to decide one cluster  $C \in S_i$  to stay, and thereafter leaves the rest others in  $S_i$ . The principle for debatable node  $i$  to choose one claiming cluster is that its decision can result in the greatest increase of common channels in all its claiming clusters. Since node  $i$  is a neighbour of all the cluster heads in  $S_i$ , node  $i$  is aware of the channel availability on these claiming cluster heads, and the common control channels in these claiming clusters. With these information, node  $i$  is able to calculate how many more CCCs will be produced in one claiming cluster if  $i$  leaves that cluster. If there exists one cluster  $C \in S_i$ , when  $i$  leaves this cluster brings the least increased CCCs than leaving any other claiming clusters, then  $i$  chooses to stay in cluster  $C$ . When there comes a tie, among the relevant claiming clusters,  $i$  chooses to stay in the cluster whose cluster head shares the most CCCs with  $i$ . In case there are multiple

claiming clusters demonstrating the same on the aforementioned criteria, node  $i$  chooses to stay in the claiming cluster which has the smallest size. Node IDs of cluster heads will be used to break tie if the previous rules could not decide on the unique claiming cluster to stay. The pseudo code of this algorithm is described as Algorithm 4. After deciding its membership, debatable node  $i$  notifies all its claiming clusters, and retrieves the updated information of the claiming clusters, e.g.,  $K(C)$ ,  $K_{h_C}$ ,  $|C|$ , where  $C \in S_i$ .

---

**Algorithm 4:** Debatable node  $i$  decides its affiliation in phase II of ROSS
 

---

**Input:** all claiming clusters  $C \in S_i$

**Output:** one cluster  $C \in S_i$ , node  $i$  notifies all its claiming clusters in  $S_i$  about its affiliation decision.

```

1 while  $i$  has not chosen the cluster, or  $i$  has joined cluster  $\tilde{C}$ , but
    $\exists C' \in S_i, C' \neq \tilde{C}$ , which has  $|K(C' \setminus i)| - |K(C')| < |K(C \setminus i)| - |K(C)|$  do
2   if  $\exists$  only one  $C \in S_i, C = \arg \min(|K(C \setminus i)| - |K(C)|)$  then
3     return  $C$ ;
4   else
5      $\exists$  multiple  $C \in S_i$  which satisfies  $C = \arg \min(|K(C \setminus i)| - |K(C)|)$ ;
6      $\tau_1 \leftarrow C$ ;
7   end
8   if  $\exists$  only one  $C \in \tau_1, C = \arg \max(K_{h_C} \cap K_i)$  then
9     return  $C$ ;
10  else
11     $\exists$  multiple  $C \in S_i$  which satisfies  $C = \arg \max(K_{h_C} \cap K_i)$ ;
12     $\tau_2 \leftarrow C$ ;
13  end
14  if  $\exists$  only one  $C \in \tau_2, C = \arg \min |C|$  then
15    return  $C$ ;
16  else
17    return  $\arg \min_{C \in \tau_2} h_C$ ;
18  end
19 end

```

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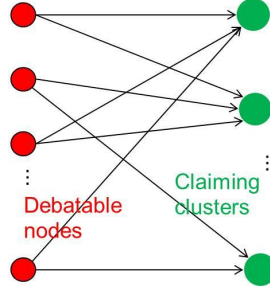
In the following we show that the process of membership clarification can be formulated into a game, and a equilibrium is reached after a finite number of best response updates made by the debatable nodes.

#### 4.5.2.2 Bridging ROSS-DGA with Congestion Game

Game theory is a powerful mathematical tool for studying, modelling and analysing the interactions among individuals. A game consists of three elements: a set of players, a selfish utility for each player, and a set of feasible strategy space for each player. In a game, the players are rational and intelligent decision makers, which are related with one explicit formalized incentive expression (the utility or cost). Game theory provides standard procedures to study its equilibriums [114]. In the past few years, game theory has been extensively applied to problems in communication and networking [125, 162]. Congestion game is an attractive game model which describes the problem where participants compete for limited resources in a non-cooperative manner, it has good property that Nash

equilibrium can be achieved after finite steps of best response dynamic, i.e., each player choose strategy to maximizes/minimizes its utility/cost with respect to the other players' strategies. Congestion game has been used to model certain problems in internet-centric applications or cloud computing, where self-interested clients compete for the centralized resources and meanwhile interact with each other. For example, server selection is involved in distributed computing platforms [46], or users downloading files from cloud, etc.

To formulate the debatable nodes' membership clarification into the desired congestion game, we observe this process from a different (or opposite) perspective. From the new perspective, the debatable nodes are regarded to be isolated and don't belong to any cluster, in other words, their claiming clusters become clusters which are beside them. Now for the debatable nodes, the previous problem of deciding which clusters to leave becomes a new problem that which cluster to join. In the new problem, debatable node  $i$  chooses one cluster  $C$  out of  $S_i$  to join if the decrease of CCCs in cluster  $C$  is the smallest in  $S_i$ , and the decrease of CCCs in cluster  $C$  is  $\sum_{C \in S_i} \Delta|K(C)| = \sum_{C \in S_i} (|K(C)| - |K(C \cup i)|)$ . The interaction between the debatable nodes and the claiming clusters is shown in Figure 4.6.



**Figure 4.6** Debatable nodes and claiming clusters

In the following, we show that the decision of debatable nodes to clarify their membership can be mapped to the behaviour of the players in a *player-specific singleton congestion game* when proper cost function is given. The game to be constructed is represented with a 4-tuple  $\Gamma = (\mathcal{P}, \mathcal{R}, \sum_{i \in \mathcal{P}}, f)$ , and the elements in  $\Gamma$  are explained below,

- $\mathcal{P}$ , the set of players in the game, which are the debatable nodes in our problem.
- $\mathcal{R} = \cup S_i, i \in \mathcal{P}$ , denotes the set of resources for players to choose, in our problem,  $S_i$  is the set of claiming clusters of node  $i$ , and  $\mathcal{R}$  is the set of all claiming clusters.
- Strategy space  $\sum_i, i \in \mathcal{P}$  is the set of claiming clusters  $S_i$ . As debatable node  $i$  is supposed to choose only one claiming cluster, then only one piece of resource will be allocated to  $i$ .
- The utility (cost) function  $f(C)$  as to a resource  $C$ .  $f(C) = \Delta|K^i(C)|, C \in S_i$ , which represents the decrease of CCCs in cluster  $C$  when debatable node  $i$  joins  $C$ . As to cluster  $C \in S_i$ , the decrease of CCCs caused by the enrolment of debatable nodes is  $\sum_{i: C \in S_i, i \rightarrow C} \Delta|K^i(C)|$ .  $i \rightarrow C$  means  $i$  joins cluster  $C$ . Obviously this function is non-decreasing with respect to the number of nodes joining cluster  $C$ .

The utility function  $f$  is not purely decided by the number of players accessing the resource (debatable nodes join claiming clusters), which happens in a canonical

congestion game. The reason is in this game the channel availability on debatable nodes is different. Given two same groups of debatable nodes and their sizes are the same, when the nodes are not completely the same (neither are the channel availabilities on these nodes), the cost happened on one claiming cluster could be different if the two groups of debatable nodes join that cluster respectively. Hence, this congestion game is player specific [16]. In this game, every player greedily updates its strategy (choosing one claiming cluster to join) if joining a different claiming cluster minimizes the decrease of CCCs  $\sum_{i:C \in S_i} \Delta |K^i(C)|$ , and a player's strategy in the game is exactly the same with the behaviour of a debatable node in the membership clarification phas.

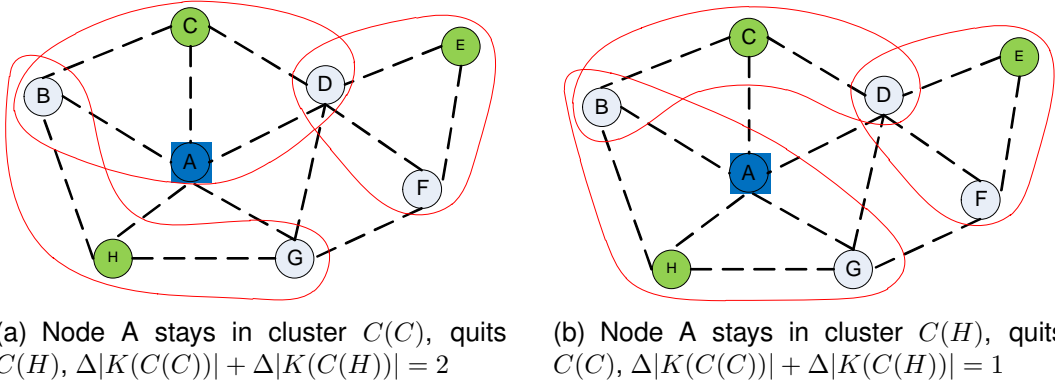
As to singleton congestion game, there exists a pure equilibria which can be reached with the best response update, and the upper bound for the number of steps before convergence is  $n^2 * m$  [16], where  $n$  is the number of players, and  $m$  is the number of resources. In our problem, the players are the debatable nodes, and the resources are the claiming clusters. Thus the upper bound of the number of steps can be expressed as  $\mathcal{O}(N^3)$ .

In fact, the number of steps which are actually involved in this process is much smaller than  $N^3$ , as both  $n$  and  $m$  are considerably smaller than  $N$ . The percentage of debatable nodes in  $\mathcal{N}$  is illustrated in Figure 4.14, which is between 10% to 60% of the total number of CR nodes in the network. The number of clusters heads, as discussed in Section 4.5.1, is dependent on the network density and the CR node's transmission range. As shown in Figure 4.4, the cluster heads take up only 3.4% to 20% of the total number of CR nodes.

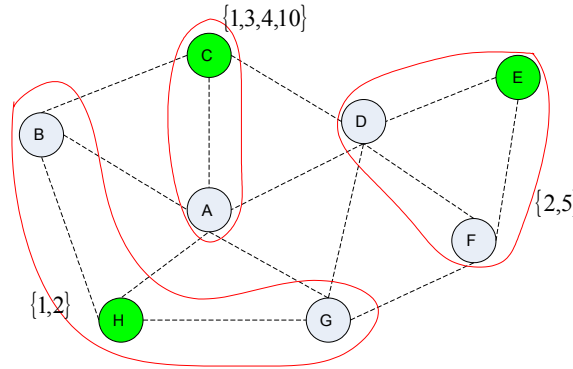
#### 4.5.2.3 Distributed Fast Algorithm (DFA)

We propose a faster version of ROSS, ROSS-DFA, which differs from ROSS-DGA in the second phase. With ROSS-DFA, debatable nodes decide their respective cluster heads once. The debatable nodes consider their claiming clusters to include all their debatable nodes, thus the membership of claiming clusters is static and all the debatable nodes can make decision simultaneously without considering the change of membership of their claiming clusters. As ROSS-DFA is quicker than ROSS-DGA, the former is especially suitable for the CRN where the channel availability changes dynamically and re-clustering is necessary. To run ROSS-DFA, debatable node executes only one loop in Algorithm 4.

Now we apply both ROSS-DGA and ROSS-DFA to the toy network in Figure 4.5 which has been applied the phase I of ROSS. In the network, node  $A$ 's claiming clusters are cluster  $C(C), C(H) \in S_A$ , their members are  $\{A, B, C, D\}$  and  $\{A, B, H, G\}$  respectively. The two possible strategies of node  $A$  is illustrated in Figure 4.7. In Figure 4.7(a), node  $A$  staying in  $C(C)$  and leaving  $C(H)$  brings 2 more CCCs to  $S_A$ , which is more than that brought by another strategy showed in 4.7(b). After the decisions made similarly by the other debatable nodes  $B$  and  $D$ , the final clusters are formed as shown in Figure 4.8.



**Figure 4.7** Membership clarification: possible cluster formations caused by node A's different choices



**Figure 4.8** Final formation of clusters.  $K(C(C))$ ,  $K(C(E))$ ,  $K(C(H))$  are shown beside corresponding clusters.

### 4.5.3 Centralized Solution for Robust Clustering

As the robust clustering problem in CRN is NP hard, there is no polynomial algorithm to solve the problem, i.e., maximizing the sum of CCCs in all non-singleton clusters. Thus, we formulate the problem into a binary linear programming problem, where the objective function and the constraints are heuristic, in particular, the objective is different from the object in the Definition 3.

Given a CRN  $\mathcal{N}$  and desired cluster size  $\delta$ , we obtain a collection of clusters  $\mathcal{G}$  which contains all the *legitimate* clusters, and the sizes of these clusters are  $1, 2, \dots, \delta$ . Legitimate clusters are the clusters which satisfy the conditions in Section 4.4.1. Note that the legitimate clusters include the *singleton* ones, i.e., the cluster which has only one CR node, so that we can guarantee the partition of any network is always feasible. With  $N = |\mathcal{N}|$ ,  $G = |\mathcal{G}|$ , we construct a constant  $G \times N$  matrix  $Q_{G \times N}$ . The element of matrix  $Q$  is  $q_{ij}$ , where the subscript  $i$  is the index of legitimate cluster, and  $j$  is the node ID of one CR node. There are  $i \in \{1, 2, \dots, G-1, G\}$ , and  $j \in \{1, 2, \dots, N-1, N\}$ . Element  $q_{ij} = |K(C_i)|$  if node  $j \in C_i$ , and  $q_{ij} = 0$  if  $j \notin C_i$ . In other words, each non-zero element  $q_{ij}$  denotes the number of CCCs of the cluster  $i$  where node  $j$  resides.



$$\begin{matrix}
& 1 & 2 & 3 & \dots & j & \dots & N-1 & N \\
\begin{matrix} 1 \\ 2 \\ \vdots \\ i \\ \vdots \\ \vdots \\ G \end{matrix} & \begin{pmatrix} |K(C_1)| & |K(C_1)| & 0 & \dots & \dots & \dots & 0 & 0 \\ |K(C_2)| & 0 & |K(C_2)| & \dots & \dots & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & |K(C_i)| & 0 & \dots & \dots & \dots & |K(C_i)| & 0 \\ \vdots & \vdots & \vdots & \vdots & \dots & \dots & \vdots & \vdots \\ \vdots & \vdots & 0 & 0 & \dots & \dots & |K(C'_i)| & 0 \\ |K(C_G)| & \dots & \vdots & \dots & \dots & \vdots & \vdots & \vdots \end{pmatrix}
\end{matrix}$$

**Figure 4.9** An example of Matrix  $Q$ , its rows correspond to all legitimate clusters, and columns correspond to the CR nodes in the CRN.

We build a  $G \times N$  binary variable matrix  $X$ , which illustrates the clustering solution. The element of matrix  $X$  is binary variable  $x_{ij}$ ,  $i = 1, \dots, G, j = 1, \dots, N$ . Now, we can formulate the optimization problem as follows,

$$\begin{aligned}
\min_{x_{ij}} \quad & \sum_{j=1}^N \sum_{i=1}^G (-x_{ij} q_{ij} + (1 - w_i) * p) \\
\text{subject to} \quad & \sum_{i=1}^G x_{ij} = 1, \text{ for } \forall j = 1, \dots, N \\
& \sum_{j=1}^N x_{ij} = |C_i| * (1 - w_i), \text{ for } \forall i = 1, \dots, G \\
& x_{ij} \text{ and } w_i \text{ are binary variables.} \\
& i \in \{1, 2, \dots, G\}, \quad j \in \{1, 2, \dots, N\}
\end{aligned}$$

The objective is a sum of two components, the first component is the sum of products of cluster size and the corresponding number of CCCs, which is the sole metric adopted by the scheme SOC [102]. The second component is the *punishment* for choosing the clusters whose sizes are not  $\delta$ . In fact, the second part is particularly designed to eliminates the drawbacks of SOC, i.e., SOC produces a large number of singleton clusters and a few large clusters. In practical computation, we minimize the opposite of the first part, and the punishment is a positive value. The first constraint restricts each node  $j$  to reside in exactly one cluster. In the second constraint,  $w_i$  is an auxiliary binary variable which denotes whether cluster  $C_i$  is chosen by the solution, in particular,

$$w_i = \begin{cases} 0 & \text{if } i\text{th legitimate cluster } C_i \text{ is chosen} \\ 1 & \text{if } i\text{th legitimate cluster } C_i \text{ is not chosen} \end{cases}$$

The second constraint regulates that when the  $i$ th legitimate cluster  $C_i$  is chosen, the number of elements which equal to 1 in the  $i$ th row of the matrix  $X$  is  $|C_i|$ . Now we explain how does the mechanism of the punishment in the objective work. The parameter  $p$  is defined as follows,

$$p = \begin{cases} 0 & \text{if } |C_i| = \delta \\ \alpha_1 & \text{if } |C_i| = \delta - 1 \\ \alpha_2 & \text{if } |C_i| = \delta - 2 \\ \dots & \end{cases}$$

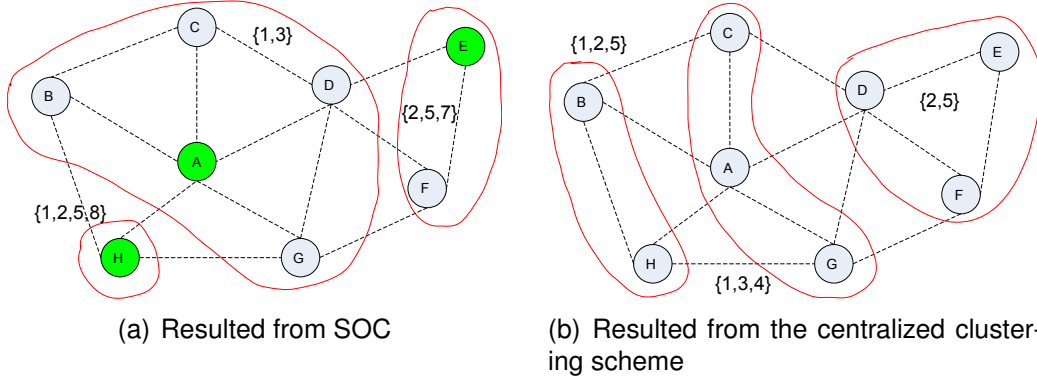
where  $\alpha_i > 0$  and increases when  $|C_i|$  diverges from  $\delta$ . Because of  $w_i$ , any chosen cluster ( $w = 0$ ) brings certain *punishment*. When the chosen cluster's size is desired size  $\delta$ , the punishment is zero. In contrary, when the chosen cluster's size diverges from  $\delta$ , the

objective function suffers *loss*. In particular, when  $w_i = 0$  and  $|C_i| = 1$ , the punishment is the most severe. This design doesn't follow the definition of  $f(C)$  in Definition 3 strictly, where  $f(C_i) = 0$  when  $|C_i| = 1$ , but our design echoes the definition by exerting the most severe punishment on the singleton clusters in the clustering solution. Choice of  $\alpha_i$  affects the resultant clusters.

#### 4.5.4 Apply the Centralized and Comparison Distributed Schemes in the Example CRN

After introducing the resulted clusters from ROSS in Fig. 4.8, we apply the other robust clustering schemes in the same example CRN in Figure 4.1. As to the centralized robust clustering scheme, we let the desired cluster size  $\delta$  be 3. A collection of clusters  $\mathcal{G}$  is obtained, which contains all the clusters satisfying the conditions of cluster in Section 4.4.1 and the sizes of clusters are 1, 2 or 3.  $\mathcal{G} = \{\{A\}, \{B\}, \dots, \{B, C\}, \{B, A\}, \{B, H\}, \dots, \{B, A, C\}, \{B, H, C\}, \{A, D, C\}, \dots\}$ , and  $G = |\mathcal{G}| = 38$ . When  $\alpha_1$  and  $\alpha_1$  are set as 0.2 and 0.8, the formed clusters are shown in Fig. 4.10(b). The resulted clustering solutions from SOC is shown in Fig. 4.10(a).

As to the average number of CCCs, the results of ROSS (including both ROSS-DGA and ROSS-DFA), centralized and SOC are 2.66, 2.66, and 3 respectively. Note there is one singleton cluster  $C(H)$  generated by SOC, which is not preferred. When we take no account of the singleton clusters, then the average number of common channels of SOC drops to 2.5.



**Figure 4.10** Final clusters formed in the example network when being applied with SOC and the centralized clustering scheme.

## 4.6 Performance Evaluation

The schemes involved in the simulation are listed as follows,

- ROSS without size control, i.e., ROSS-DGA and ROSS-DFA.
- ROSS- $\delta$ -DGA and ROSS- $\delta$ -DFA, which are the variants of ROSS with the size control feature.  $\delta$  is the desired cluster size.

- SOC [102], one distributed clustering scheme pursuing cluster robustness. To the best of our knowledge, SOC is the only work emphasizing on the robustness of clustering structure among the related works.
- Centralized robust clustering scheme. The formulated optimization is an integer linear optimization problem, which is solved by the function *bintprog* provided in MATLAB.

The authors of [102] compared SOC with other schemes in terms of the average number of CCCs of the formed cluster, on which SOC outperforms other schemes by 50%-100%. SOC's comparison schemes are designed either for ad hoc network without consideration of channel availability [34], or for CRN but just considering connection among CR nodes [179]. Thus SOC is chosen to be the only distributed scheme as comparison, besides, we also compare ROSS with the centralized scheme. We investigate the schemes with the following metrics.

- **Average number of CCCs per non-singleton cluster.** Non-singleton cluster refers the cluster whose cluster size is larger than 1. Comparing with the metric adopted by SOC [102], which is the average number of CCCs per cluster without excluding the singleton clusters, this metric provides an more accurate description of the robustness of the non-singleton cluster. The larger number of CCCs per non-singleton clusters means these clusters have higher probability to survive when the primary users' operation becomes more intense. Although this metric doesn't disclose the information about the CR nodes which are not included in any non-singleton clusters, we still examine this metric as the number of CCCs is involved in the utility which is adopted by all robust clustering schemes.
- **Cluster sizes.** The distribution of CRs residing in the formed clusters with different sizes is presented.
- **Robustness of the clusters against newly added PUs.** We increase the number of PUs to challenge the clusters, and count the number of *unclustered* CR nodes which are the CR nodes which are not included in any non-singleton clusters. This metric indicates the robustness of clusters from a more practical point of view, i.e., as to the clusters formed for a given CRN and spectrum availability, how many CR nodes can still make use of the clusters when the spectrum availability decreases.
- **Amount of control messages involved.** We investigate the number of control messages involved in the clustering process.

Simulation consists of two parts, in the first part, we investigate the performance of centralized scheme, and the gap between the distributed schemes and the centralized scheme. This part is conducted in a small network, as there is no polynomial time solution available to solve the centralized problem. In the second part, we investigate the performance of the proposed distributed schemes in the CRN with different scales and densities.

We give a brief introduction to the settings which are common for both simulation parts. CRs and PUs are deployed on a two-dimensional Euclidean plane. The number of licensed channels is 10, each PU is operating on each channel with probability of 50%. CR

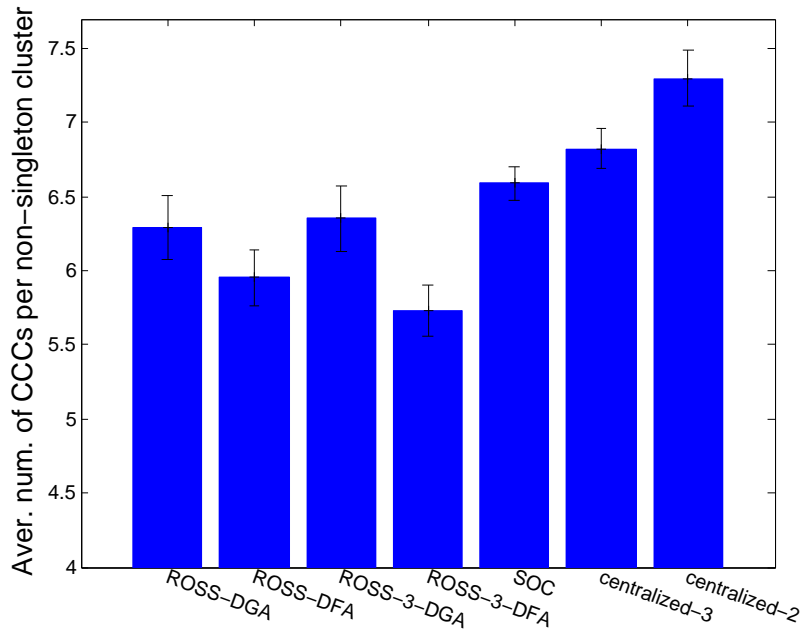
users are assumed to be able to sense the existence of primary users and identify available channels. All primary and CR users are assumed to be static during the process of clustering. The simulation is written in C++, and the performance results are averaged over 50 randomly generated topologies, and the confidence interval corresponds to 95% confidence level.

#### 4.6.1 Centralized Schemes vs. Decentralized Schemes

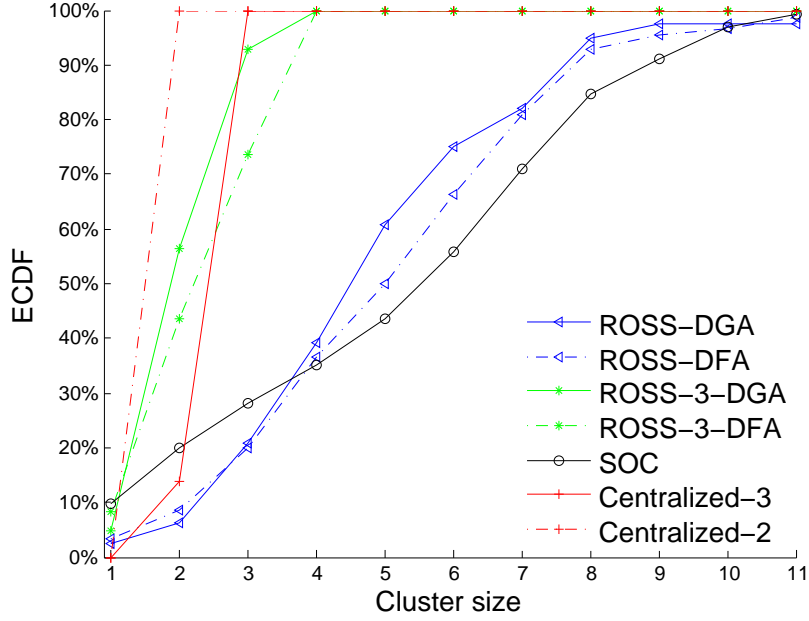
There are 10 primary users and 20 CR users are dropped randomly (with uniform distribution) within a square area of size  $A^2$ , where we set the transmission ranges of primary and CR users to  $A/3$ . When clustering scheme is executed, around 7 channels are available on each CR node. The desired cluster size  $\delta$  is 3, the parameters used in the *punishment* for choosing the clusters with undesired sizes are set as follows,  $\alpha_1 = 0.4$ ,  $\alpha_2 = 0.6$ .

##### 4.6.1.1 Number of CCCs in Non-singleton Clusters

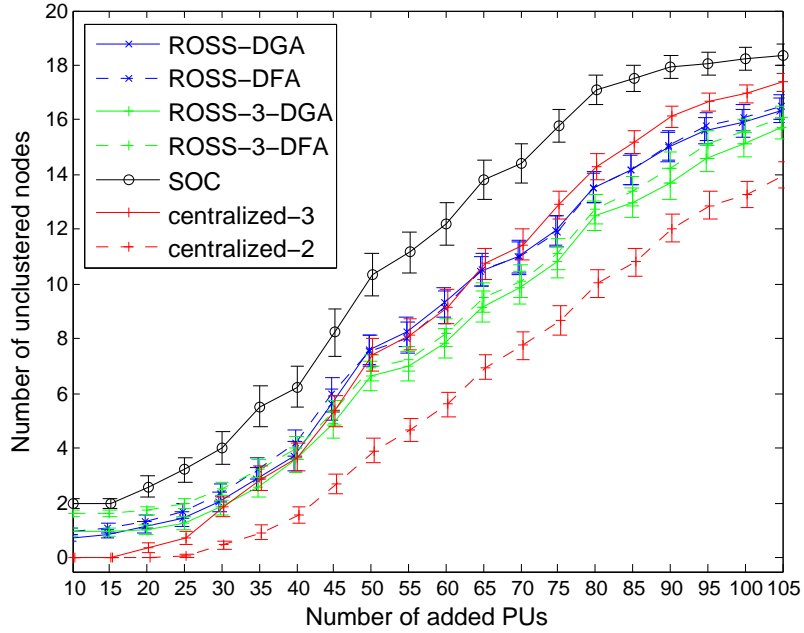
From Figure 4.11, we can see the centralized schemes outperform the distributed schemes. As to the distributed schemes, SOC achieves the best than all the variants of ROSS. The reason is, SOC is liable to group the neighbouring CRs which share the most abundant spectrum together, no matter how many of them are, thus the number of CCC of the formed clusters is higher. We have discussed the flaw of this metric as it doesn't convey the number of unclustered CR nodes, in fact, SOC generates the most unclustered CRs, which can be seen when we discuss the performance on the number of unclustered CR nodes. As to the variants of ROSS, we notice that the greedy mechanism increases CCCs in non-singleton clusters significantly.



**Figure 4.11** Number of common channels of non-singleton clusters



**Figure 4.12** Cumulative distribution of CRs residing in clusters with different sizes



**Figure 4.13** Number of unclustered CRs with decreasing spectrum availability

#### 4.6.1.2 Cluster Size Control

Figure 4.12 depicts the empirical cumulative distribution of the CRs residing in certain sized clusters, from which we have two conclusions. The first, SOC generates more unclustered CR nodes than other schemes. The centralized schemes produce no unclustered CR nodes, ROSS-DGA/DFA generate 3% unclustered nodes, as comparison, 10% of nodes are unclustered when applying SOC. ROSS-DGA and ROSS-DFA with size control feature generate 5%-8% unclustered CR nodes, which is due to the cluster pruning procedure (discussed in section 4.5.1.3). Second, the centralized schemes and cluster size control mechanism of RPSS generate clusters which satisfy the requirement on cluster size strictly. As to ROSS-DFA and ROSS-DFA with size control feature, CR nodes re-

side averagely in clusters whose sizes are 2, 3 and 4. The sizes of clusters resulted from ROSS-DGA and ROSS-DFA are disperse, but appear to be better than SOC, i.e., the 50% percentiles for ROSS-DGA, ROSS-DFA and SOC are 4.5, 5, and 5.5, and the 90% percentiles for the three schemes are 8, 8, and 9.

#### 4.6.1.3 Robustness of the clusters against newly added PUs

We add PUs in the CRN to decrease the available spectrum, and observe the number of unclustered CR nodes. Clusters are formed with the presence of 10 PUs in the beginning, then extra 20 batches of PUs are added sequentially, where each batch includes 5 PUs.

Figure 4.13 shows with the increase of PUs, certain clusters disappear and the number of unclustered CR nodes increases. Among the robust clustering schemes, the centralized scheme with desired size of two generates the most robust clusters, meanwhile, SOC results in the most vulnerable clusters. The centralized scheme with desired size of three doesn't outperform the variants of ROSS, because the centralized scheme peruses clusters with size of 3 at the expenses of sanctifying CCCs. In contrary, the variants of ROSS generate some smaller clusters which are more likely to survive when PUs' activities become intense.

#### 4.6.1.4 Control Signalling Overhead

In this section we compare the overhead of signalling involved in different clustering schemes. We omit the control messaged involved in neighbourhood discovery, which is the premise for any clustering scheme. According to [105], the message complexity is defined as the number of messages used by all nodes. To have the same criterion to compare, we count *the number of transmissions of control messages*, without distinguishing they are sent with broadcast or unicast. This metric is synonymous with *the number of updates* discussed in Section 4.5.

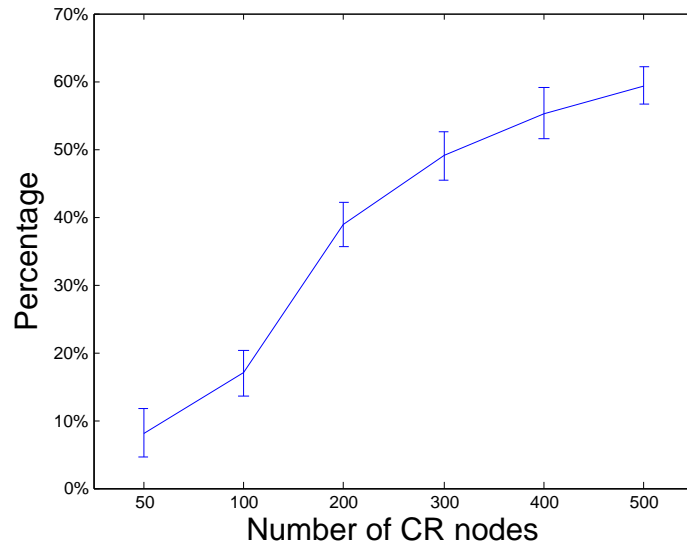
As to ROSS, the control messages are generated in both phases. In the first phase, when a CR node decides itself to be the cluster head, it broadcasts a message containing its ID, cluster members and the set of CCCs in its cluster. In the second phase, a debatable node broadcasts its affiliation to inform its claiming clusters, then the CHs of the claiming clusters broadcast message about the new cluster members if they are changed due to the debatable node's decision. The total number of the decisions involved in cluster formation has been analysed in Theorem 4.5.1 and Section 4.5.2.2 respectively.

Comparison scheme SOC involves three rounds of execution. In the first two rounds, every CR node maintains its own cluster and seek to integrate neighbouring clusters, or joins one of them. The final clusters are obtained in the third round. In each round, every CR node is involved in comparisons and cluster mergers.

The centralized scheme is conducted at the centralized control device, but it involves two phases of control message transmission. The first phase is information aggregation, in which every CR node's channel availability and neighborhood is transmitted to the centralized controller. The second phase is broadcasting, where the clustering solution is disseminated to every CR node. We adopt the algorithm proposed in [130] to broadcast and gather information as the algorithm is simple and self-stabilizing. This scheme needs

building a backbone structure to support the communication, and we apply ROSS to generate cluster heads which serve as the backbone, besides, the debatable nodes as used as the gateway nodes between the backbone nodes. As the backbone is built once and support transmission for multiple times, the messages involved in the clustering process are not counted. As to the process of information gathering, we assume that every cluster member sends the spectrum availability and its ID to its cluster head, which further forwards the message to the controller, thus the number of transmission is  $N$ . As to the process of dissemination, in an extreme situation where all the gateway and the backbone nodes broadcast, the number of transmission is  $h + m$ , where  $h$  is the number of cluster heads and  $m$  is number of debatable nodes.

The number of control messages which are involved in both ROSS and the centralized scheme is related with the number of debatable nodes. Figure 4.14 shows the percentage of debatable nodes when the CRN network becomes denser, from which we can obtain the value of  $m$ . The message complexity, quantitative analysis of the number of control



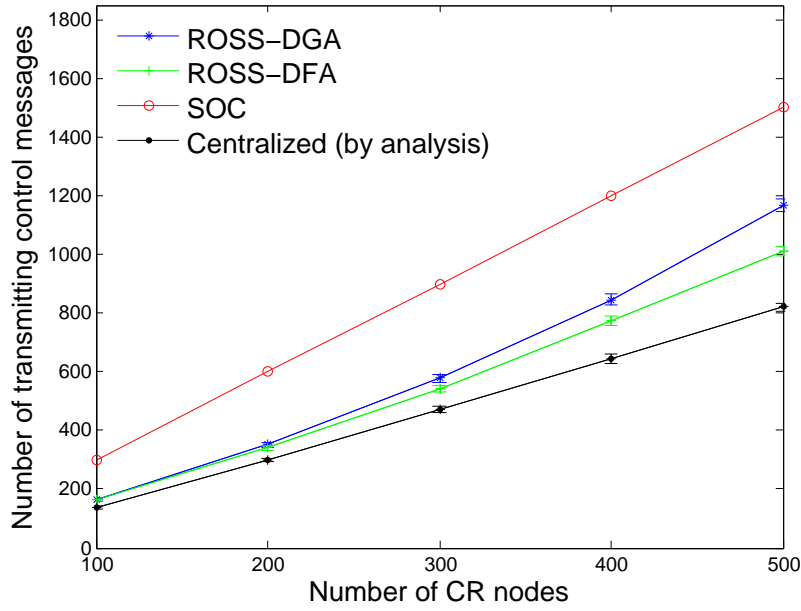
**Figure 4.14** The percentage of debatable nodes after phase I of ROSS.

messages involved in clustering, and the size of control messages are shown in Table 4.3. Figure 4.15 shows the number of transmissions of SOC, the upper bound of the number of transmissions for ROSS, and the analytical number of transmissions of the centralised scheme.

## 4.6.2 Comparison Between Distributed Schemes

In this section we investigate the performances of distributed clustering schemes in CRN with different network scales and densities. The transmission range of CR is  $A/5$ , PR's transmission range is  $2A/5$ . The initial number of PU is 30. The desired sizes adopted are listed in the Table 4.4, which is about 60% of the average number of neighbours.

Scheme	Message Complexity	Quantitative number of messages	Content of message (size of message)
ROSS-DGA, ROSS- $\delta$ -DGA	$\mathcal{O}(N^3)$ (worst case)	$h + 2 * m^2 d$ (upper bound)	Phase I: notification from cluster head (1 byte), new individual connectivity degree (1 byte); Phase II: update of debatable nodes' affiliation (1 byte), claiming cluster $i$ ' new membership ( $ C(i) $ bytes), $K(C(i))$ ( $ P $ bytes), $i \in \mathcal{N}$
ROSS-DFA, ROSS- $\delta$ -DFA	$\mathcal{O}(N)$ (worst case)	$h + 2m$ (upper bound)	
SOC	$\mathcal{O}(N)$	$3 * N$	
Centralized	$\mathcal{O}(N)$	$h + m + N$ (upper bound) [130]	$\{C\}$ ( $ C_i  * N$ bytes)

**Table 4.3** Signalling overhead**Figure 4.15** Number of control messages. Note the curves of ROSS-DGA and ROSS-DFA are the upper bounds

#### 4.6.2.1 Number of CCCs per Non-singleton Clusters

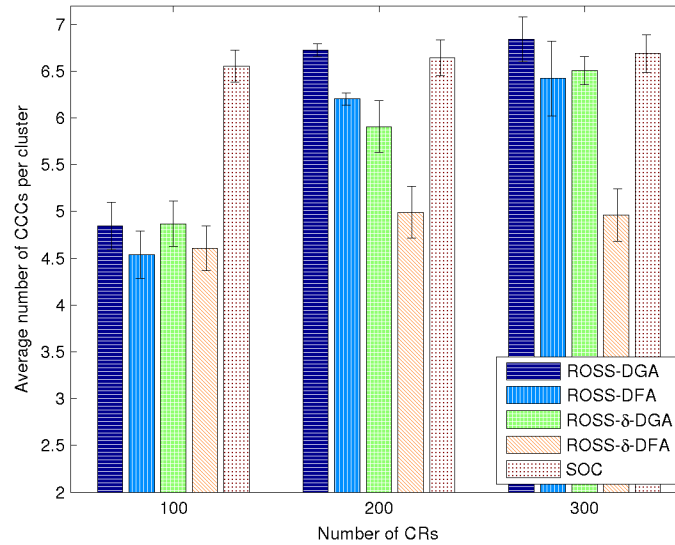
Figure 4.16 shows the average number of CCCs of the non-singleton clusters. We notice that SOC achieves the most CCCs per non-singleton cluster, although the lead over the variants of ROSS shrinks significantly when  $N$  increases.

#### 4.6.2.2 Robustness of the clusters against newly added PUs

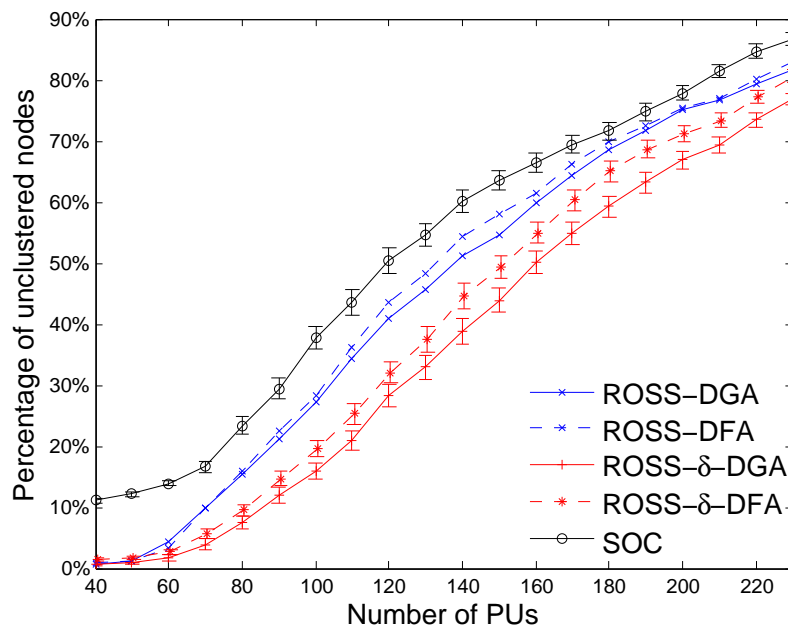
We add extra 20 batches of PUs sequentially in the CRN, where each batch includes 10 PUs. Figure 4.17 and 4.18 show that when  $N = 100$  and 200, more unclustered CR nodes appear in the CRN where SOC is applied. When the network becomes denser, ROSS-DGA/DFA generate slightly more unclustered CR nodes than SOC when new PUs are not many, but SOC's performance deteriorates quickly when the number of PUs becomes larger. We only show the average values of the variants of ROSS as their confidence

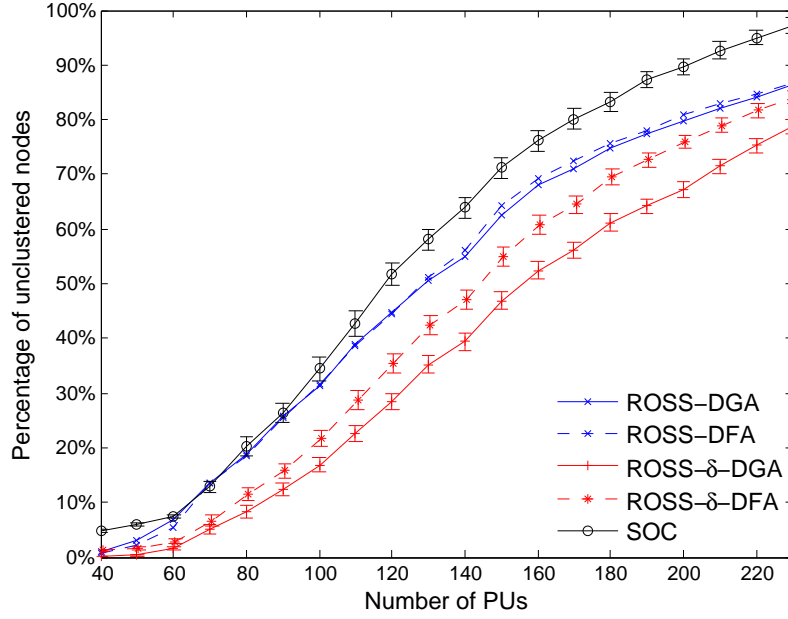


Number of CRs	100	200	300
Average num. of neighbours	9.5	20	31
Desired size $\delta$	6	12	20

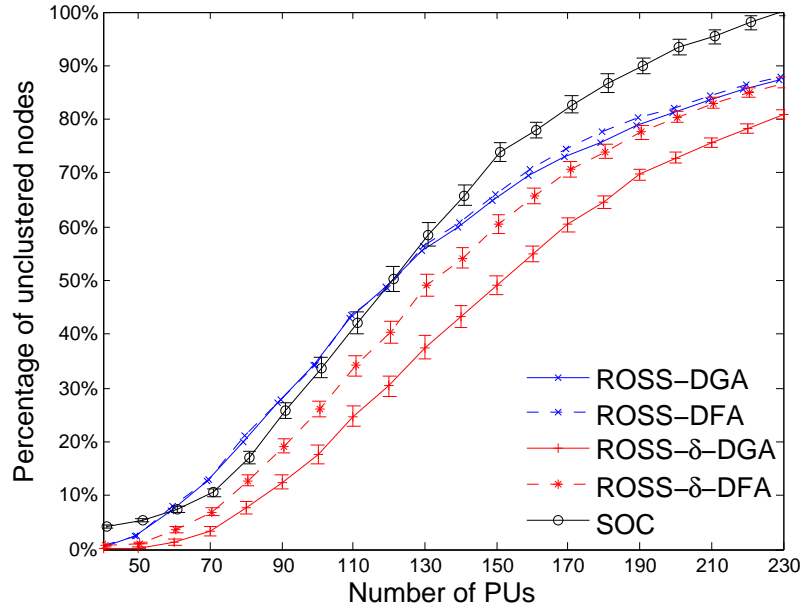
**Table 4.4****Figure 4.16** Number of common channels of non-singleton clusters.

intervals overlap. When applying ROSS with size control mechanism, significantly less unclustered CR nodes are generated. Besides, the greedy mechanism moderately strengthens the robustness of the clusters.

**Figure 4.17** Percentage of CR nodes which are not included in any non-singleton clusters, 100 CRs



**Figure 4.18** Percentage of CR nodes which are not included in any non-singleton clusters, 200 CRs

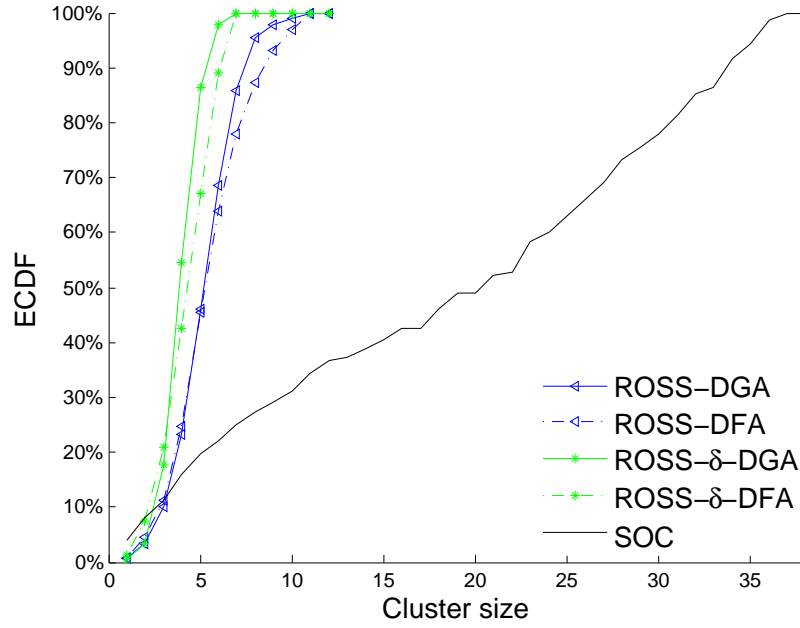


**Figure 4.19** Percentage of CR nodes which are not included in any non-singleton clusters, 300 CRs

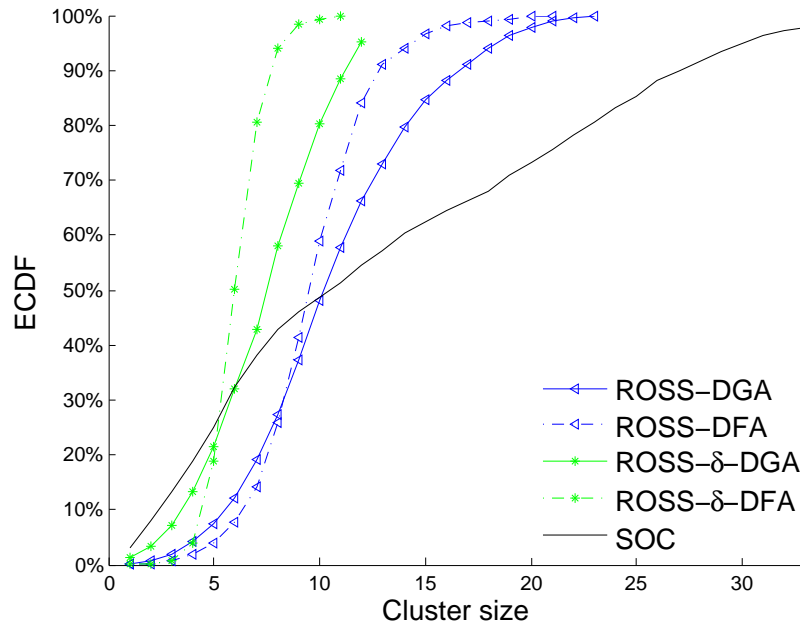
#### 4.6.2.3 Cluster Size Control

Figure 4.23 shows when the network density scales up, the number of formed clusters by ROSS increases by smaller margin, and that generated by SOC increases linearly. This result coincides with the analysis in Section 4.5.1.3. To better understand the distribution of the sizes of formed clusters, we depict the empirical cumulative distribution of CR nodes which reside in clusters with certain sizes in CRNs in Figures 4.20 4.21 4.22.

The sizes of clusters generated by ROSS-DGA and ROSS-DFA span a wider range than ROSS with size control feature. Most of the generated clusters are smaller than the average number of neighbours, which is roughly equal with the 95% percentile of the ROSS-



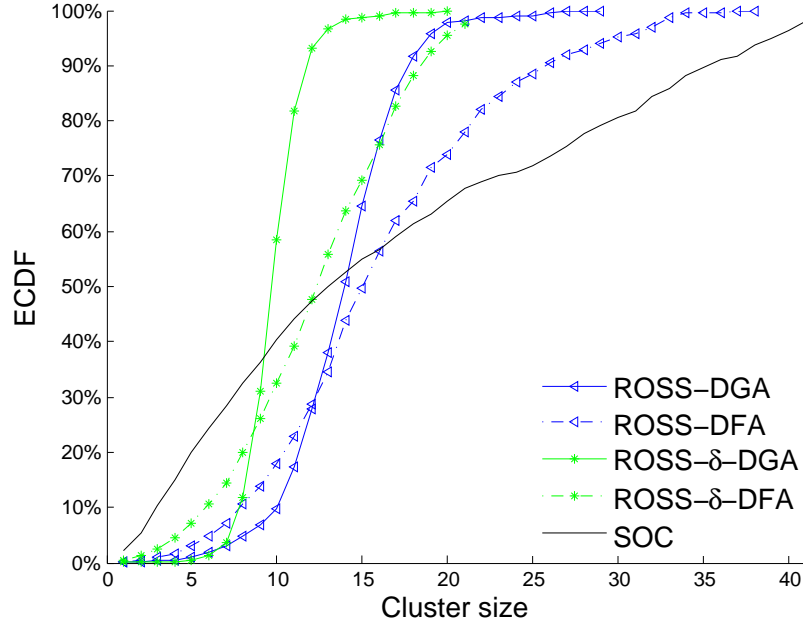
**Figure 4.20** Cumulative distribution of CRs residing in clusters with different sizes, 100 CRs, 30 PUs in network



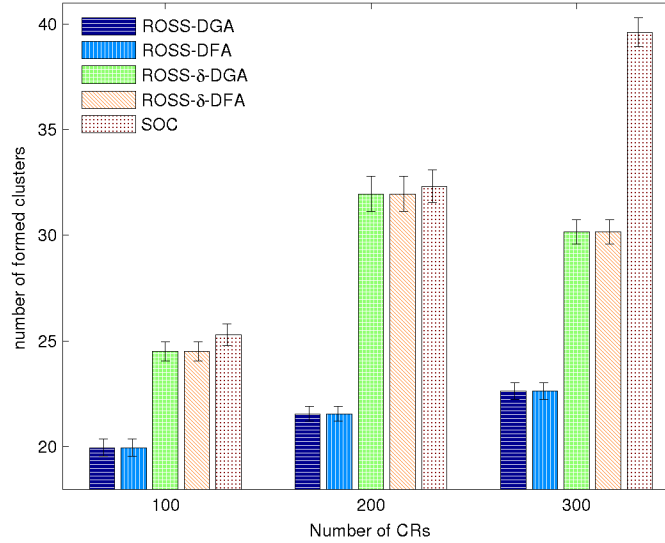
**Figure 4.21** Cumulative distribution of CRs residing in clusters with different sizes, 200 CRs, 30 PUs in network

DGA curve. The 50% percentile of the ROSS-DGA curve is roughly the desired size  $\delta$ . When the variants of ROSS with size control feature are applied, the sizes of the most generated clusters are smaller than  $\delta$ . As to the curves of SOC, the 95% percentiles are 36, 30, and 40 in respective networks.

Overiewing the resulted clusters sizes, we conclude that the sizes of the clusters generated by ROSS are limited by the network density, the sizes of the clusters formed by ROSS with size control feature are restricted by the desired size. In contrary, the clusters generated from SOC demonstrate strong divergence on cluster sizes.



**Figure 4.22** Cumulative distribution of CRs residing in clusters with different sizes, 300 CRs, 30 PUs in network



**Figure 4.23** The number of formed clusters.

### 4.6.3 Insights from the Simulation

The centralized clustering scheme can form the clusters which satisfy the requirement on cluster size strictly, and the clusters are robust against the newly added PUs, besides, it generates the smallest control overhead in the process of cluster formation.

As distributed schemes, ROSS outperform the comparison scheme SOC considerably on three metrics. ROSS generates much less singleton clusters than SOC, and the resulted clusters are robust than SOC in front of newly added PUs. The signalling overhead involved in ROSS is about half of that needed for SOC, and the signalling messages are much shorter than the latter. The sizes of the clusters which are generated by ROSS

are restricted by the network density, which demonstrate smaller discrepancy than that of SOC. Besides, ROSS achieves similar performance to the centralized scheme, i.e., the cluster sizes obtained by ROSS with size control feature are limited by the desired size, and the cluster robustness is similar when applying the variants of ROSS and the centralized scheme respectively. As to the variants of ROSS, the greedy mechanism in ROSS-DGA helps to improve the performance on cluster size and cluster robustness at the cost of mildly increased signalling overhead.

We also notice that the metric that the number of CCCs per non-singleton cluster doesn't indicate the robustness of clusters as shown in Figure 4.13 and ??, although it is adopted as the metric in the formation of clusters.

## 4.7 Conclusions

In this chapter we design a distributed clustering scheme with the singleton congestion model, which forms robust clusters against primary users' effect. Through simulation and theoretical analysis, we find that distributed scheme achieves similar performance with centralized optimization in terms of cluster survival ratio and number of control messages. This chapter investigates the robust clustering problem in CRN extensively, and proves the NP hardness of this problem. A Light weighted clustering scheme ROSS is proposed, on the basis of which, we propose the fast version scheme and the scheme which generate clusters with desired sizes. These schemes outperform other distributed clustering scheme in terms of both cluster survival ratio and the number of control messages.

The shortage of ROSS scheme is it doesn't generate big clusters whose sizes exceed the cluster head's neighbourhood. This problem is attributed to fact that ROSS forms clusters on the basis of cluster head's neighbourhood, and doesn't involve interaction with the nodes outside its neighbourhood. In the other way around, forming big cluster which extends out side of cluster head's neighbourhood has very limited applications, because multiple hop communication and coordination are required manage this kind of big clusters.



# 5

## Decentralized Virtual Coordinate Assignment and Routing

### Abstract

In order to facilitate light weight geographic routing, we design spectrum virtual coordinate for CRN, which is not only independent on geographic position, but decided by connection quality among nodes, for instance, channel availability, link quality or hop numbers. With this new coordinate, Euclidean distance between two secondary users reflects the availability of unlicensed channel between them. Then the geographic routing can run with this coordinate system as the spectrum virtual coordinate contributes a large part to the decision on next hop.

### 5.1 Introduction

Primary users' activity demonstrates different patterns [95], consequently the availability of licensed spectrum exhibits different dynamics accordingly. In certain scenarios the licensed spectrum occupancy stays available for fairly long time, e.g., TV white space [124]. In that case the licensed spectrum occupancy can be seen as static during a long period of time. In other scenarios primary users' states change frequently, but measurements [133, 163] show that the percentage of time that licensed spectrum is occupied at a specific location or during a certain period of time doesn't change, i.e. in city downtown during the work time, the duty cycle of spectrum occupancy by cellular network is stable.

To fully exploit the potential of the secondary spectrum, it is crucial to investigate routing in dynamic spectrum environment. The dynamic availability of spectrum causes frequent break down of links between secondary users, and leads to prevalent topology changes, which makes spectrum aware routing difficult but essential [45].

Recent measurement in [133] shows the spectrum occupancy doesn't have significant spatial correlations between different locations. It follows that licensed spectrum is used by

primary users heavily in some areas, whereas in the other areas licensed spectrum is available over longer timespan for secondary users to use. It is obvious to see that a routing path is better to go through the areas where primary users occupation is lower, as this alleviates or avoids the burden to cope with the changing or totally occupied spectrum when forwarding packets potentially with latency requirements. Geographic routing is a natural choice to realize this geography sensitive routing path. Geographic routing is light weight regarding the determination of next hop, and achieves high scalability in various wireless networks [14]. Merely knowing the geographic locations of its neighbours and the destination, a node is able to locally choose the next hop which has the smallest distance to the destination. However, in CRN dynamic link state renders geographic routing unsuccessful since packets are forwarded to the destination along the shortest path rather than avoiding areas heavily influenced by primary users.

To enable geographic routing in CRN, in this chapter we propose SAViC, spectrum aware virtual coordinates for secondary users in multi-channel multi-hop CRN. The virtual coordinate is independent of real geographic position, and has been proposed to represent the properties of the media like, link quality [24] or hop numbers [43]. Following this line of thought, our proposed virtual coordinate represents the spectrum occupancy of primary users. On top of this, we propose the geographic routing scheme which decides the next hop with Euclidean distance metric, and detours the areas affected by primary users, or cuts through the area with lower spectrum occupancy. With SAViC, geographic routing imposes little computation on deciding the next hop, and requires less communication cost transmitting packet to next hop. As to our knowledge, this is the first work integrating the spectrum usage by primary users into network coordinates in order to support geographic routing in CRN, which carries meanings especially for those resource restricted devices which want to work with licensed frequency band. The remainder of the chapter is organized as follows, after reviewing related work in Section II, system model is introduced in Section III. Assignment of SAViC is explained in Section IV, followed by opportunistic access during transmission in Section V. Section VI gives performance evaluation, concluding remarks are given in the last section.

## 5.2 Related Work

When secondary users are static and primary users' operation activity is known, i.e., primary users occupy a certain channel for long time, or they occupy a channel with fixed probability, then centralized routing schemes for CRN can be designed [132]. But as centralized scheme requires sensing result from each secondary user in the network, thus suffers from any change of channel state of secondary users [12], besides, one centralized controller is needed to calculate the routing path on the basis of collected information from the network [132, 135].

Considerable amount of distributed schemes are proposed to cope with routing in CRN where spectrum state is usually considered to be rapid changing. [42] proposes CAODV (Cognitive Ad-hoc On-demand Distance Vector) and let each CR node explore all channels and store route for each available channel. CAODV requires frequent message exchange between secondary users to maintain the up to date connections on each channel due to PU's activities, which is a burden for secondary user when primary users' activity is intense. [149] improving the DSR scheme (Dynamic Source Routing) by letting RREQ



messages record spectrum availability, link quality and congestion possibility along routing paths, but it also suffers from frequently changing channel state.

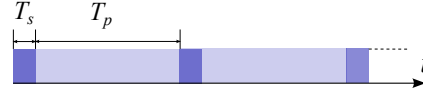
To cope with the rapid change of channel state, some routing schemes abandon routing table and let the transmitter decide the next hop for each single packet based on spectrum state between transmitter and neighbours. When there is packet to send, secondary user evaluates channel availability based on the statistics of sensing history [112], or the prediction on channel availability in the forthcoming time slot [111], then secondary user chooses the favoured channel and next hop node to send out the packet. Distance to the destination is also a consideration for choosing next hop. Such per-packet forwarding paradigm reacts swiftly on the fast changing channel state, but it requires more powerful computation power on secondary users. Firstly, that scheme produces high computation complexity on determining the channel and next hop node, secondly, specifically designed MAC mechanism and large amount of control messages are needed to coordinate the communication between the sender and the potential next hop nodes, these aspects make it uneconomic for many networks, e.g, wireless sensor networks operating with licensed spectrum [153]. Furthermore, as this kind of routing paradigm emphasizes on finding the maximal transmission opportunity of secondary spectrum, the selection on preferred channel decreases the scope of next hop neighbours, thus it may yield route which does not reach the destination [95, 148].

The solution provided by Chowdhury et al. [53] improves geographic routing in multiple channel CRN by introducing circumventing mechanism, i.e., the source node launches geographic routing on each channel, and every routing path bypasses the next hop, which is chosen based on geographic routing metric (e.g., Euclidean distance), if it is affected by primary user, the routing packet chooses a neighbour of that node free from primary user's affection so as to avoid the primary user affected area. Such routing is conducted on all channels, afterwards a path merge process is undertaken and one path with alternating channel is finally formed with consideration of end to end delay. *SEARCH* adopts routing table and doesn't involve frequent overhead exchanges. As the decision of the next hop is largely decided by the channel availability on the time point of decision, the node chosen as next hop may not be able to work after a short while due to primary user's reappearance. Thus source node needs to periodically launch route request to update the routing path which may have been invalid. As a result, this scheme works well when the primary user's activity is infrequent, but when it goes tense, the frequent invalidity of nodes due to lack of available spectrum seriously deteriorates routing performance.

## 5.3 System Model

We consider a CRN composed with secondary users which are randomly and statically deployed in a plane. There are orthogonal licensed channels denoted by set  $C$ , and secondary user is allowed to use any of them if no primary user is detected on that channel by the secondary user. One common control channel (CCC) in license-exempt band is available for all secondary users to exchange control messages. Only one licensed channel is used for payload transmission. Primary users are static, and they occupancy spectrum in a constant manner, e.g., the percentage of time that they access a certain channel is static in any period of time.

Proactive spectrum sensing is conducted locally and periodically as Figure 5.1 shows.



**Figure 5.1** Sensing duration  $T_s$  and sensing period  $T_p$

The sensing duration  $T_s$  includes both detection time in physical layer and the decision synchronisation time. Sensing period  $T_p$  is the time between two successive sensing durations. If the channel is sensed as busy<sup>1</sup> in sensing duration, which means at least one primary user in the vicinity is in ON state, we say the state of primary user is labelled as ON in the following sensing period  $T_p$ . If not a single primary user is sensed, primary users in vicinity in the following sensing period is said to be in state OFF. Secondary user senses each licensed channel for time  $T_{access}$  with round robin scheduling, and records statistics of ON/OFF states of that channel at its place.

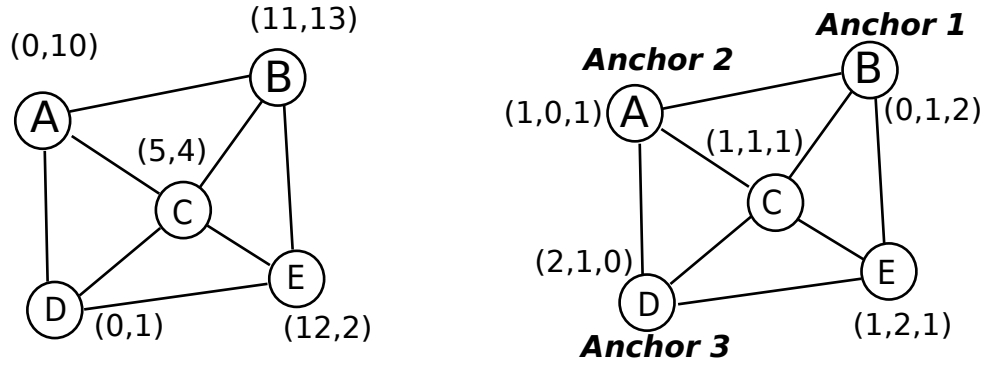
## 5.4 Spectrum Aware Virtual Coordinates

In this section, we firstly introduce how spectrum aware virtual coordinate is assigned, then we introduce two normalized spectrum utilities on secondary user adopted in the virtual coordinate assignment process. One of them is called *normalized spectrum availability* which is on the basis of duty cycle of primary users' absence, the other is called *normalized longest blocking time* which as the name tells, is based on the lengths of time durations that primary users are detected.

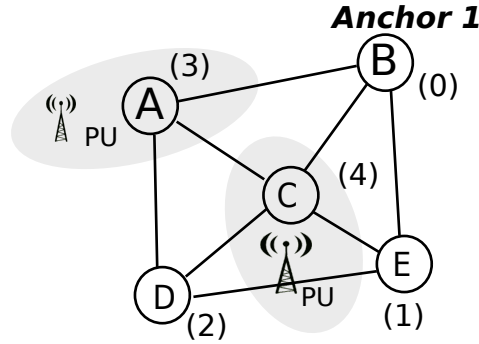
Virtual coordinate has been proposed in sensor and ad hoc networks [24, 43]. In the left part of Figure 5.2, nodes are labelled with physical positions. The right hand side part shows the same network assigned triplet virtual coordinate for each node according to virtual coordinate assignment protocol (VCap) [43], where each element of the coordinate denotes the minimal number of hops away from corresponding anchor. This kind of virtual ordinate belongs to tree based virtual coordinates, and is obtained based on anchors which locate at the edge of network. Anchor messages are broadcast from anchors, each of them contains a counter recording the number of hops travelled. The minimum counter of the arriving anchor messages constitutes the corresponding element of the virtual coordinate on the arrival node. Except for the hope numbers away from certain anchor node, virtual coordinate can also be composed with link quality [24] in wireless sensor networks. The hop based virtual coordinate is independent on actual physical position, but supports greedy geographic routing successfully [24, 43]. For example, when the source-destination is B and D, and Euclidean distance calculated with virtual coordinate is adopted in the routing decision, then the greedy geographic routing achieves the same routing path in both networks:  $B \rightarrow C \rightarrow D$ . This path is one with the shortest traversal distance.

In this chapter, we propose licensed spectrum aware virtual coordinate in CRN, which enables geographic routing to find the path with better available spectrum. Figure 5.3 shows one CRN where secondary users are assigned virtual coordinate according to anchor 1. The transmission opportunity of the nodes locating within primary users' transmission range, e.g., node A and C, is decreased due to sporadic spectrum, as a result, the cost for

<sup>1</sup>Concrete sensing techniques are not discussed in this chapter.



**Figure 5.2** Left: nodes with physical locations, Right: nodes with doublet virtual coordinate, each element in the virtual coordinate is the number of hops away from corresponding anchor. Connecting lines denote the communication is possible.



**Figure 5.3** A network under primary users' influence assigned with SAViC, only anchor 1 is adopted. Flooding of anchor messages is not shown.

packet transmission, e.g., transmission delay and energy consumption, is increased. We integrate this obstacle caused by spectrum scarcity to transmission into virtual coordinate.

### 5.4.1 Assign Spectrum Aware Virtual Coordinate

As to SAViC, anchors broadcast anchor messages which flood over the network and result in virtual coordinate for each secondary user. Several anchors are needed to assign unique virtual coordinate for each secondary user. Anchors are located at the edges of network and there is distance between each pair of them. The number of anchors is  $r$ . How to decide their locations is out of the scope of this chapter. In the following, we introduce how is virtual coordinate decided on each node with respect to anchors.

A secondary user's virtual coordinate is one  $r$ -tuple where each element corresponds to one anchor. Before the assignment, the elements of virtual coordinate on each node are set as a large positive value. An anchor message is generated on each anchor and broadcast on control channel. The anchor message contains a *counter* whose value is set as 0. The influence of primary users on a secondary user is quantified as spectrum utility  $\lambda$  on that secondary user. The larger value indicates heavier spectrum occupation by primary users. Quantization of  $\lambda$  will be discussed in Section 5.4.2 and 5.4.3.

When a node  $i$  receives an anchor message from the  $t$ th anchor,  $i$  obtains the value of *counter*<sub>1</sub> which is contained in the arriving anchor message. Then  $i$  compares its  $t$ th element in its current virtual coordinate with the sum of the spectrum utility  $\lambda$  and *counter*.

When its current  $t$ th element in its current virtual coordinate is less than the sum, node  $i$  drops the anchor message. The rational behind this operation is, the sum of  $\lambda$  on the current node and *counter* from the arriving anchor message indicates the spectrum availability along the path which begins from the anchor node to the current node, thus when the sum is larger in value, the path traversed by this anchor message exposes to more active primary users and thus dropped. When the sum is less than the  $t$ th element in its current virtual coordinate, the node replaces its  $t$ th element with the sum, and updates *counter* contained in the anchor message before forwarding it. This process is presented as Algorithm 5.

The number of anchor messages involved in this process is linear to the network size, which is supported by the following proofs.

**LEMMA 5.4.1:** *For each secondary user, the number of times to forward anchor messages which are associated with one specific anchor is bounded by  $g$ , where  $g$  is the number of one hop neighbours of that anchor.*

**Proof.** Assuming a extreme situation, where □

The counter value of one anchor message is increased after it is forwarded. This is can be seen from the lines 10-12 in Algorithm 5.

**LEMMA 5.4.2:** *One anchor message affiliated with one anchor visits any secondary user at most twice.*

**Proof.** Assume one node is accessed by the same anchor message for two times. When the anchor message arrives at the secondary user, let's say  $i$ , for the first time, there is  $vc > counter + \lambda$ , and both of the *vc* and *counter* in the anchor message are updated to be  $counter + \lambda$ . After being forwarded, the anchor message travels at least one other secondary user before arriving  $i$ , and the counter is greater than the counter value when it is forwarded from  $i$  according to Lemma 5.4.1, which means, current counter value is greater than  $\lambda_i$ , and the anchor message is dropped. Thus, this anchor message accesses secondary user  $i$  for at most two times. □

**THEOREM 5.4.1:** *The number of anchor message corresponding to one anchor is bound by  $2 * g * n$ , where  $n$  is the number of secondary users.*

**Proof.** This is proved by Lemma 5.4.1 and 5.4.2. □

According to Theorem 5.4.1, every secondary users obtains virtual coordinate which respects to all anchors after a finite time duration.

## 5.4.2 Normalized Spectrum Availability on Secondary Users

Based on the statistics of primary user's ON/OFF states in time duration  $T_{assessment}$  which contains multiple  $T_s$ , secondary user  $i$  characterizes the likelihood that one licensed channel, say  $k$ , is available at its own position with *duty cycle*, which is

$$\gamma_i^k = \frac{\Delta_{OFF}}{\Delta_{OFF} + \Delta_{ON}}, \quad (5.1)$$

---

**Algorithm 5:** Assignment of  $vc_i$  for secondary user  $i$  with respect to one anchor

**Input:**  $vc_i = M$ ,  $M$  is one big positive number

---

```

1 if  $i$  is anchor then
2    $vc_i = 0$ ;
3   set  $counter1 = \lambda_i$  in anchor message;
4   broadcast anchor message;
5 end
6 if  $i$  receives anchor message then
7   if  $counter + \lambda_i \geq vc_i$  then
8     drop anchor message;
9   else
10     $vc_i = counter + \lambda_i$ ;
11    set  $counter = vc_i$  in anchor message;
12    broadcast anchor message;
13  end
14 end

```

---

where  $\Delta_{\text{OFF}}$  is the number of sensing periods when channel  $k$  is sensed as OFF in  $T_{\text{assessment}}$ . To implement SAViC whose resultant Euclidean distance between two nodes reflects both influence from primary users and distance in terms of hops, we need to design a normalized quantified spectrum availability  $\lambda_i$ .

#### 5.4.2.1 Single Licensed Channel

When there is only one licensed channel in CRN (the superscript of channel  $\lambda$  is omitted), the normalized spectrum availability on node  $i$  is proposed as,

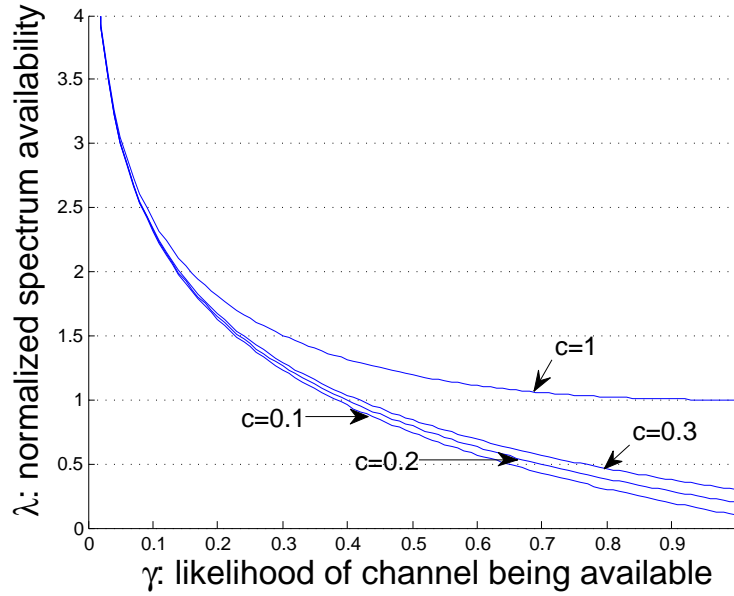
$$\lambda_i = -\ln \gamma_i + c \cdot \gamma_i \quad (5.2)$$

With Formula 5.2, when one anchor message which originates from anchor  $X$  is forwarded from node  $a$  to  $b$  without being dropped, the distance based on virtual coordinate reflects both the spectrum availability and geographic distance in terms of hops between the two nodes. Based on Algorithm 5 and Formula 5.2, the distance in dimension  $X$  is,

$$\begin{aligned}
 |x_b - x_a| &= \sum_{i \in P_{(a,b)}} (-\ln \gamma_i + c \cdot \gamma_i) \\
 &= -\ln \left( \prod_{i \in P_{(a,b)}} \gamma_i \right) + c \cdot \sum_{i \in P_{(a,b)}} \gamma_i
 \end{aligned} \quad (5.3)$$

here  $x_a$  and  $x_b$  are virtual coordinates of node  $a$  and  $b$  in dimension  $X$  respectively.  $P_{(a,b)} = (\dots, b)$  denotes the list of nodes after  $a$  and till  $b$ , which forward the same anchor message.

The reason to choose the form of Formula 5.2 is as follows. As Formula 5.3 shows, the first item is logarithm of the product of consecutive spectrum availability likelihood of the nodes in  $P_{(a,b)}$ . The product is the likelihood that one message travels from node  $a$  to  $b$  without hampered by primary users, which is an important property we want to integrated



**Figure 5.4** Normalized spectrum availability with respect to the likelihood of spectrum being available one a node

into our virtual coordinate system. The first item will be infinity when the spectrum is occupied by primary users during all the time, where  $\gamma = 0$ . In this case, infinity can be replaced with a large positive value which results in a large  $\lambda$ . The second item denotes number of hops, which can be seen clearly when  $\gamma_i = 0$  for node  $i \in P_{(a,b)}$ .

As  $\lambda_i$  needs to be monotonically decreasing with respect to  $\gamma_i$ , so that the less spectrum availability results in bigger cost for communication, thus there should be

$$\frac{\partial \lambda_i}{\partial \gamma_i} = c - \frac{1}{\gamma_i} < 0 \quad (5.4)$$

hence the tuning parameter  $c$  should be smaller than 1. In the simulation part, we choose  $c = 0.2$  so that  $\lambda$  visibly reflects the changes of  $\gamma$  when  $\gamma$  is not too small, as Figure 5.4 shows.

#### 5.4.2.2 Multiple Licensed Channels

When multiple licensed channels are allowed to use without interfering primary users, one node can switch to another channel which is at present available to send or forward packet, then the normalized channel availability is,

$$\gamma_i = 1 - \prod_{k=1}^{|C|} (1 - \gamma_i^k) \quad (5.5)$$

Based on Formula 5.2, the normalized spectrum availability on node  $i$  when multiple secondary channels are available is,

$$\begin{aligned} \lambda_i &= -\ln \gamma_i + c \cdot \gamma_i \\ &= -\ln(1 - \prod_{k=1}^{|C|} (1 - \gamma_i^k)) + c \cdot (1 - \prod_{k=1}^{|C|} (1 - \gamma_i^k)) \end{aligned} \quad (5.6)$$

### 5.4.3 Normalized Longest Blocking Time on Secondary Users

Channel utility introduced in previous subsection characterizes the likelihood that secondary user is allowed to forward packets, but it fails to reflect the availability of spectrum in a finer granularity of time. For instance, in the period of time  $T_{access}$  to access the spectrum availability, one channel which frequently changes between state ON and OFF due to primary users' violent operation may have the same likelihood of available spectrum with the channel where primary user sojourns on state ON for long time. This difference has direct consequence on delay when the likelihood of spectrum availability on PU affected secondary nodes is homogeneous.

Let  $T_{ON}^k$  be the length of time period that channel  $k$  is not available, there is  $T_{ON}^k = n \cdot (T_s + T_p)$ , where  $n$  is the number of consecutive sensing duration that channel  $k$  is sensed as busy.  $T_{ON}^k$  is recorded within  $T_{access}$ , and we use  $\tau^k = \overline{T_{ON}^k}$  to denote the average value of the time duration that channel  $k$  is occupied by primary user, which is the maximum time period that secondary user is blocked from sending/forwarding.

#### 5.4.3.1 Single Licensed Channel

In single licensed channel scenario, the normalized maximum blocking time on node  $i$  is (superscript  $k$  is omitted),

$$\lambda_i = f(\tau_i) = \gamma_i \cdot \tau_i + b \cdot e^{-\gamma_i \cdot \tau_i} \quad (5.7)$$

The first item is the product of blocking time and the duty cycle of available spectrum, note that we assume the duty cycle is identical for any PU affected secondary user and thus can be regarded as constant. As to the secondary users which locate out of any primary user's transmission range, there is  $\tau_i = 0$ , then  $\lambda_i = b$ ,  $\lambda_i$  denotes hop count in this case. This is the reason that the second item is needed.

Same with the analysis in section 5.4.2, when one anchor message travels through path  $P_{(a,b)}$ , the distance on the corresponding coordinate dimension is the sum of the normalized longest blocking time, which is the function of the sum of maximum blocking time on the cascaded nodes on the trajectory of anchor message,

$$\begin{aligned} |x_b - x_a| &= \sum_{i \in P_{(a,b)}} (\gamma_i \cdot \tau_i + b \cdot e^{-\gamma_i \cdot \tau_i}) \\ &= \gamma_i \cdot \sum_{i \in P_{(a,b)}} \tau_i + b \cdot e^{-\gamma_i \cdot \sum_{i \in P_{(a,b)}} \tau_i} \end{aligned} \quad (5.8)$$

Normalized longest blocking time  $\lambda$  is monotonically increasing with  $\tau_i$ , which requires

$$\begin{aligned} \frac{\partial \lambda_i}{\partial \tau_i} &= \gamma_i - \gamma_i \cdot b \cdot e^{-\tau_i \cdot \gamma_i} > 0 \\ b &< e^{\gamma_i \cdot \tau_i} \end{aligned} \quad (5.9)$$

then we set the tuning parameter  $b$  as 1.

### 5.4.3.2 Multiple Licensed Channels

In multiple licensed channel scenario,  $\tau_i$  equals to the smallest maximum blocking time over all secondary channels on node  $i$ ,

$$\tau_i = \min \tau_i^x, x \in C \quad (5.10)$$

The normalized maximum blocking time on node  $i$  is as Formula 5.7 shows.

In remainder of this chapter, the virtual coordinate based on normalized spectrum utility is referred as *spectrum availability based VC*, and The virtual coordinate based on normalized maximum blocking time is denoted as *blocking time based VC* out of convenience.

When  $\lambda$  on secondary nodes is identical, the resultant SAViC appears to be similar with hop based virtual coordinate. In reality, as the measurement shows in [133], heterogeneity of spectrum usage by primary users is very normal, besides, the two kinds of virtual coordinates make it easier to find out such heterogeneity. [133] also shows within certain frequency band, primary users' activity is stable for hours, e.g., cellular network. When primary user's operation pattern changes, e.g., occupy spectrum with increased duty cycle, then SAViC needs to be reimplemented.

## 5.5 Geographic Routing and Opportunistic Spectrum Access

Although spectrum aware virtual coordinate is the main concern of this chapter, we also introduce the geographic routing to be used as it affects the routing result directly. With geographic routing, packet sender/forwarder chooses the neighbour which has smaller Euclidean distance to the destination. The distance between node  $i$  and destination  $d$  is  $\sqrt{(x_d - x_i)^2 + (y_d - y_i)^2 + (z_d - z_i)^2}$ , when virtual coordinate can be denoted as  $\{x, y, z\}$ . A trivial improvement on greedy geographic routing is implemented in network layer to mitigate the dead end problem. When routing protocol reaches dead end node  $u$  which is closest to destination,  $u$  adds its ID to the packet as taboo before forwarding the packet to  $v$  which is closest to the destination in its neighborhood. The packet will not be sent to the nodes whose IDs appear to be taboos.

Buffer is implemented on each node, where packets stay temporally when no unoccupied licensed spectrum is available. Secondary user resends buffered packet every period of time, and drops it if there is still no available channel after trying for 10 times.

In multiple channel CRN, after one node deciding on the next hop via geographic routing, which channel to use needs to be answered. This problem involves considerations from many aspects, such as minimizing channel switch cost [148], mitigating co-channel interferences [104] etc.. We adopt a lightweight heuristic method in this chapter. When there is packet to send and the next hop is decided, packet sender chooses the channel in descending sequence with channel's metric, i.e., likelihood of channel availability, or blocking time. The sender chooses the channel with the best metric, then conducts spectrum sensing in the immediately following sensing duration to determine the channel's usability. If the channel is sensed as free to use, sender transmits *request\_channel\_x* to the next hop on the control channel, when it receives the answering message *channel\_x\_available*



from that node, it starts communication on channel  $x$  in the following sensing period. If the channel is sensed to be busy before or among the transmission, or it receives *channel\_x\_unavailable* message from next hop node, the sender moves to the channel with the second best metric, and conduct the same procedure as described above.

## 5.6 Performance Evaluation

In this section, we present the performance of geographic routing together with SAViC. Both virtual coordinates based on metrics of spectrum availability and blocking time respectively are evaluated. Prior to that, the set-up of simulation is introduced.

### 5.6.1 Simulation Setup

In this section, we introduce the deployment of the primary users to generate various spectrum availability in the network, then introduce the important parameters in simulation. Different from [43] where simulation is conducted without considering any activities in MAC and physical layer, simulation in this chapter deploys a wireless environment which is close to reality, e.g., interferences and channel shadowing are involved.

#### 5.6.1.1 Primary Users

Measurements [70, 121] show that the sufficiently accurate statistic model of spectrum occupancy can be given by a Markovian process. In simulation, primary user alternates state between ON and OFF as a two-state discrete time Markov chain (2DTMC). The probability that it changes from one state to the other, or stays in the same state is called transition probability. Transition probability further decides the stationary probability of 2DTMC, which represents the percentage of time that primary user is in state ON or OFF in a long run. The relationship between stationary probability  $\Pi = \{\pi_{\text{ON}}, \pi_{\text{OFF}}\}$  and duty cycle  $\gamma$  is,

$$\lim_{T_{\text{assessment}} \rightarrow \infty} \gamma = \pi_{\text{OFF}} \quad (5.11)$$

Transition probability also decides the continuous sojourning time of primary user on a certain state, which affects the longest blocking time sensed by secondary users. Hence, by adjusting transition probability, we can let primary user operate with desired intensity, i.e. stationary probability for being in state OFF, or continuous sojourning time of being on state ON. We denote stationary probability of state OFF as  $P_{\text{OFF}}$ , and the maximal blocking time as  $T$ . In the following, we only use  $P_{\text{OFF}}$  and  $T$  to define primary user's dynamics, and omit mentioning the transition probability. The time unite of the DTMC for primary user to follow is 0.1s.

As spectrum availability based and longest blocking time based virtual coordinates are designed for CRN which is influence by certain primary user activity, we design two primary user distributions. As a result, we design two categories of primary user settings to evaluate the routing performance assisted by the two categories of virtual coordinate respectively.

- As to spectrum availability based virtual coordinate, two primary users are located in the CRN which can not affect all the nodes in CRN, as shown in Figure 5.6(a).
- For blocking time based virtual coordinate, network is evenly covered by primary users which have the same duty cycle, but some primary users have different blocking time with the others, as Figure 5.9 shows.

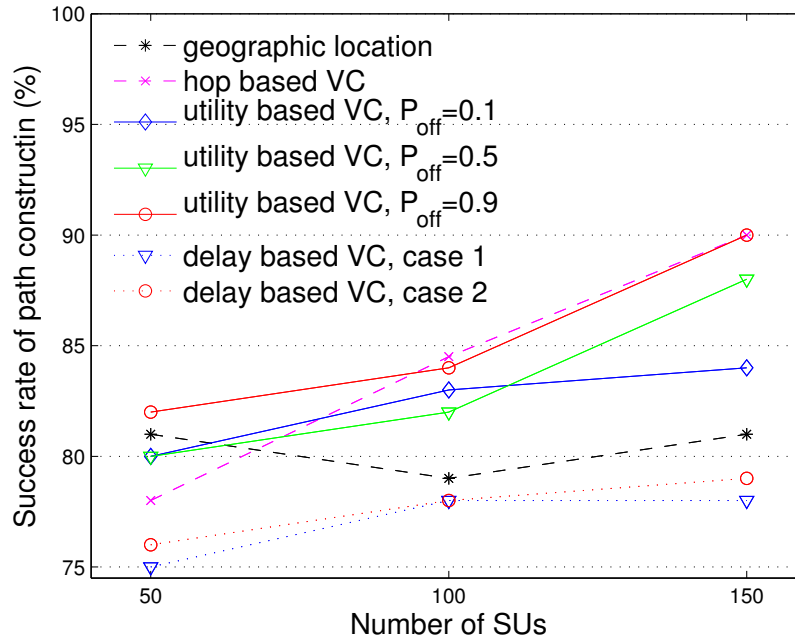
When multiple channel scenario is to be investigated, existing primary users simply start to work with current and additional channels, and there is no new primary users appear.

### 5.6.1.2 Parameter Setting

Simulation is conducted with INET framework provided by OMNeT++ simulator [160], which comprises both generation of SAViC and following geographic routing. Secondary users are randomly distributed in a square area and 6 nodes which locate at the edge are deployed as anchors.

### 5.6.2 Success Rate of Geographic Routing on Finding Path

We evaluate SAViC's reachability, i.e. given the virtual coordinate of destination, geographic routing forwards packet from source to the node with the desired virtual coordinate. The comparisons are real geographic location, and hop based virtual coordinate according to VCap [43]. We deploy 6 anchors and then there is no duplicated virtual coordinate among the resultant virtual coordinate system.



**Figure 5.5** Reachability of different virtual coordinates, the average number of neighbors is 6, 12, and 14

In order to evaluate the effectiveness of coordinates to support geographic routing, we design different configurations of primary users. As to duty cycle based VC, two primary

users are randomly deployed in the network. As to delay based VC, we configure 9 primary users to evenly cover the network, among of them, 4 primary users have different maximal blocking time with the rest. Under one configuration of primary users, 1000 random CRN is generated and in each CRN one far departed pair of source and destination is chosen to test.

Figure 5.5 shows, spectrum availability based virtual coordinate supports geographic routing to achieve similar reachability with hop based virtual coordinate<sup>2</sup>, which is better than that with real geographic location. Blocking time based virtual coordinate performs a little bit worse than other coordinates. In summary, after integrating the primary user's influence, SAViC supports geographic routing to achieve comparable success rate of path construction with conventional virtual coordinate and real geographic location.

### 5.6.3 Routing Performance

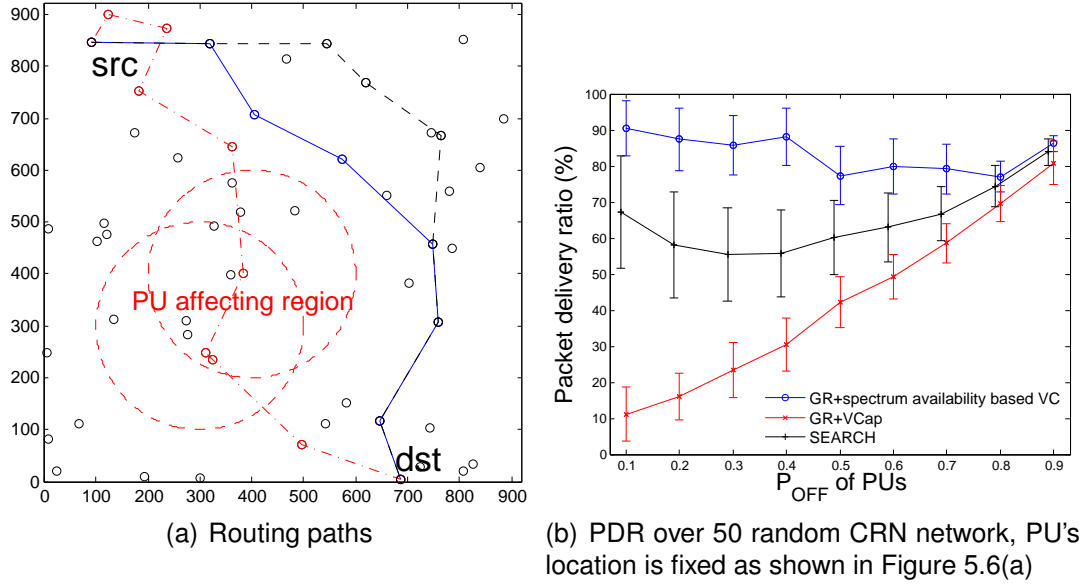
We sequentially present the routing performance of SAViC based on spectrum availability and blocking time respectively. In more details, spectrum availability SAViC is compared with hop based virtual coordinate VCap and *SEARCH*. The reason to choose *SEARCH* [53] is it is on the basis of geographic routing and utilizes routing table in the interval of updates, thus it requires less computation ability and overhead exchanges. The time interval for *SEARCH* to update routing tables of the nodes on routing path is 5s. Both single and multiple licensed channel scenarios are investigated for the three solutions.

#### 5.6.3.1 Duty Cycle Based Virtual Coordinate

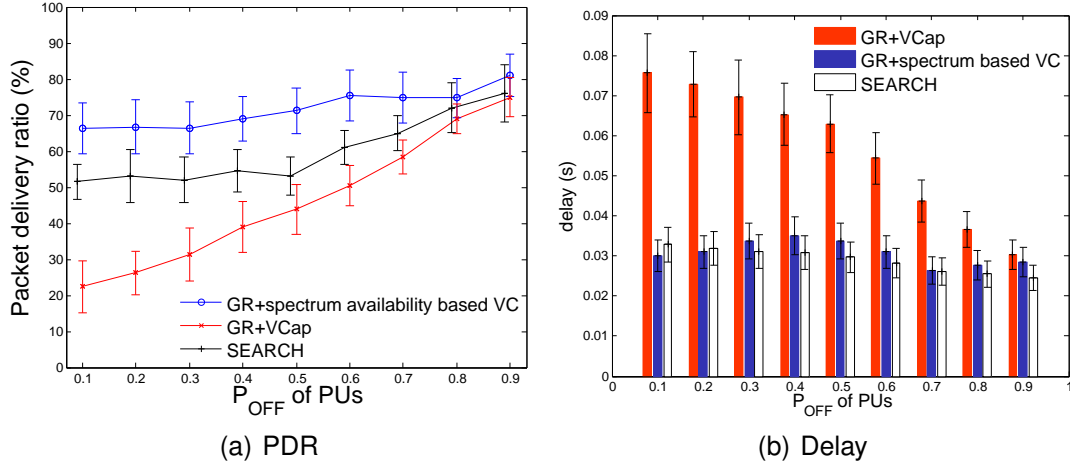
We start by looking into the performance of SAViC in single channel scenario.

We start with a case study, two primary users locate at the centers of dashed cycles as shown in Figure 5.6(a). VC based on spectrum availability and VCap (hop is its metric) are assigned to secondary users separately. The red dashed path in Figure 5.6(a) is formed by geographic routing with VCap, which cuts across the primary users' affecting area and thus suffers great packet loss. The black dash and blue solid paths are formed with spectrum availability based virtual coordinate, the two paths are formed when primary user's working intensities  $P_{\text{OFF}}$  is 0.1 and 0.9 respectively. These two paths vividly illustrate that utility based virtual coordinate successfully integrates the spectrum scarcity in CRN network, and decomposes a large part of routing decision. The paths of *SEARCH* is not drawn here as the routing path is possible to change after path update. We keep the primary users in the middle of the network, for each activity intensity, 50 CRNs where secondary users are randomly located are generated. Figure 5.6(b) shows the PDR of spectrum availability based virtual coordinate is high except for a minor decline when  $P_{\text{OFF}}$  is between 0.5 and 0.8, which is contradictory to the monotonically increasing trend of hop based virtual coordinate. This can be explained by the path snapshot in Figure 5.6(a). When channel is sensed to be scarce (primary users access channel intensively),

<sup>2</sup>The numerical result of hop based virtual coordinate coincides with the simulation result presented in [43] under the same network density



**Figure 5.6** Routing paths and corresponding PDR



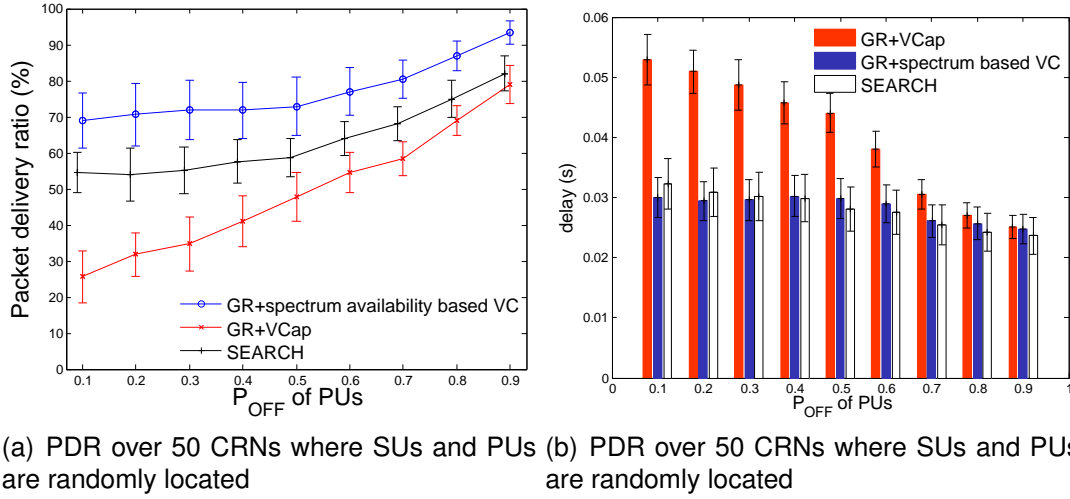
**Figure 5.7** Packet delivery ratio with single secondary channel, over 50 randomly located CR nodes and PUs

path generated is far away from the affected area and circumvents completely. When primary users become less intensive, routing path moves closer to that area. In other words, the weaker dynamics of primary users attracts path and result in packet drop. When  $P_{OFF}$  approaches to 1, spectrum availability based virtual coordinate becomes actual hop based virtual coordinate as the link metric in formula 5.2 becomes zero.

The paradox that more licensed spectrum leads to worse PDR can also be observed on *SEARCH*, whose PDR curve declines first and increases later on. When channel is heavily utilized by primary users, the routing request is more likely to encounter operating primary user, then a node out of the primary user affecting area is chosen as next hop, so that the path experiences less packet loss (with the price of more hops). When primary users become less intense, routing request is more likely to traverse the affected areas, as a result, the routing path experiences packet loss due to the primary users in that area before next route update.

Figure 5.7(a) shows the PDR when both primary and secondary users' locations are random. SAViC's performance deteriorates because the source and destination may be influenced by primary users, so that a path completely detour the primary users' area is impossible. where geographic routing has no means to detour the affected areas. In figure 5.7(b), SAViC and SEARCH achieve lower delay although forwarding more packets, which means SAViC is effective to facilitate geographic routing to avoid PU affecting areas.

Now we introduce the routing performance in multiple channel scenario, where two licensed channels available, but only one is allowed for payload transmission.



**Figure 5.8** Packet delivery ratio with multiple secondary channel scenario

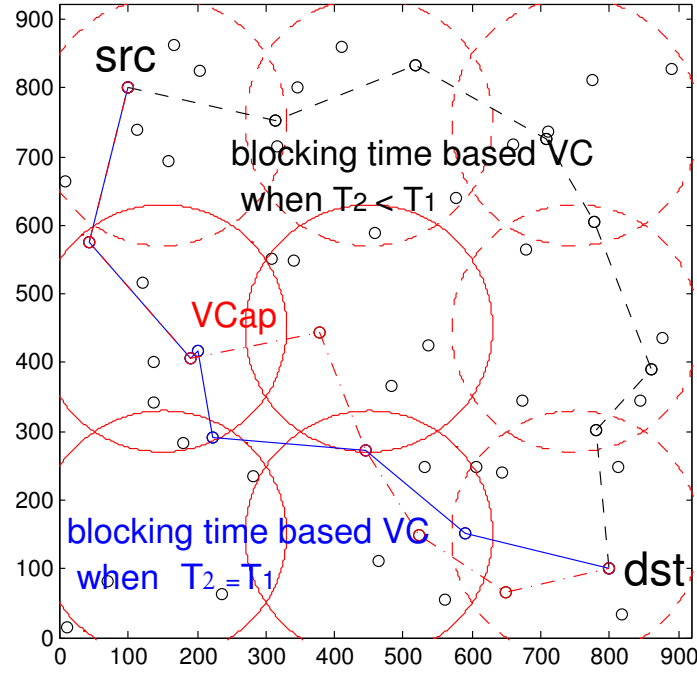
In this part of simulation, we follow the setting of single channel scenario, except that secondary users have at most two licensed channels.

Thanks to the second channel, the packet delivery ratio is increased as shown in Figure 5.8(a), and delay is decreased as depicted in Figure 5.6(a). SAViC still outperform the other schemes especially in the aspect of PDR.

### 5.6.3.2 Longest Blocking Time Based Virtual Coordinate

As discussed in section 5.4.3, spectrum availability based virtual coordinate doesn't reflect the sparsity or abundance of spectrum well when the likelihood of spectrum availability is homogeneous in CRN. A CRN working with single licensed channel in Figure 5.9 is used to show the fail of spectrum availability based virtual coordinate. Two items are used in the following to make the analysis tidy.

$T_1$	Maximal blocking time of primary users whose transmission ranges are solid cycles in Figure 5.9
$T_2$	Maximal blocking time of the other primary users



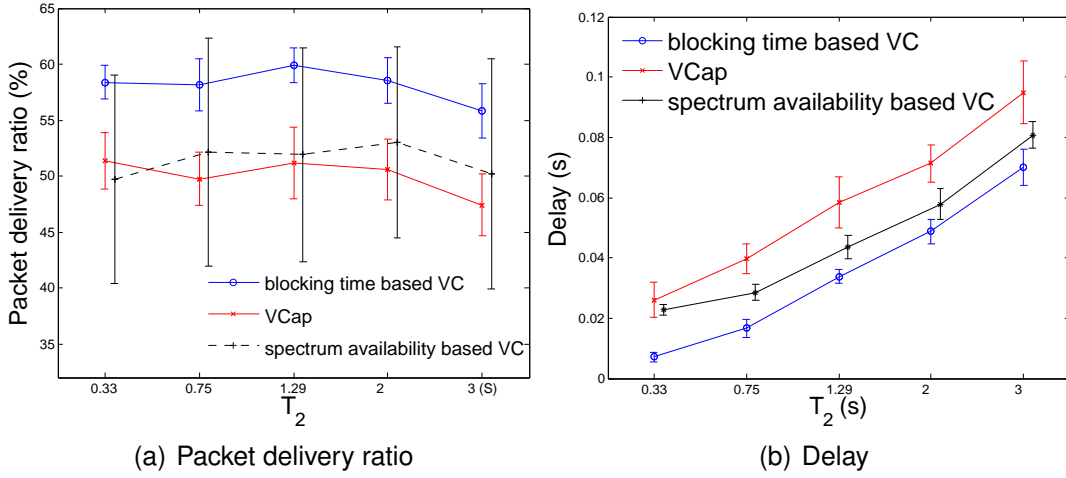
**Figure 5.9** Routing paths in one network,  $T_1 = 3s$ . cycles denote the transmission range of primary users.

In the network, 9 primary users evenly distributed,  $P_{\text{OFF}} = 0.5$  for each of them. For the primary users denoted by the solid cycles, maximal blocking time  $T_1 = 3s$ , and  $T_2 = 1s$  and  $3s$  for the other primary users. When  $T_2 < T_1$ , the resultant routing path is in black and dashed, which goes through area where primary users have shorter maximal blocking time. When  $T_2 = T_1$ , the resultant routing path largely converges with the path with VCap.

The ineffectiveness of spectrum availability based virtual coordinate in case of identical  $P_{\text{OFF}}$  is observed in Figure 5.9. In this case a different characteristic, i.e., the longest blocking time, which shows the geographically diverse characteristics of spectrum can be used. In our simulation,  $P_{\text{OFF}} = 0.9$  for all primary users, but they are diverse on sojourn time, i.e.  $T_1$  of primary user is  $3s$ , and  $T_2$  is shorter. We randomize the location of secondary users in 50 networks, and present the performance of blocking time based virtual coordinate to show its superiority on decreased end to end delay and PDR. In this part of simulation, we don't show the result of *SEARCH*, as it performs as bad as geographic routing with hop based virtual coordinate. The reason is the widespread primary users seriously hamper the routing requests to arrive at destination, consequentially most paths for forwarding the packets can not be constructed successfully.

Figure 5.10(b) shows as  $T_2$  increases from  $0.33s$  to  $3s$ , the delay of successfully delivered packets also increases for both blocking time based virtual coordinate and VCap, but a constant gap exists in between. Whereas, the delay of spectrum availability based virtual coordinate is random as respect to sojourn time, the reason is the routing metric in this scenario doesn't involve blocking time imposed by primary users.

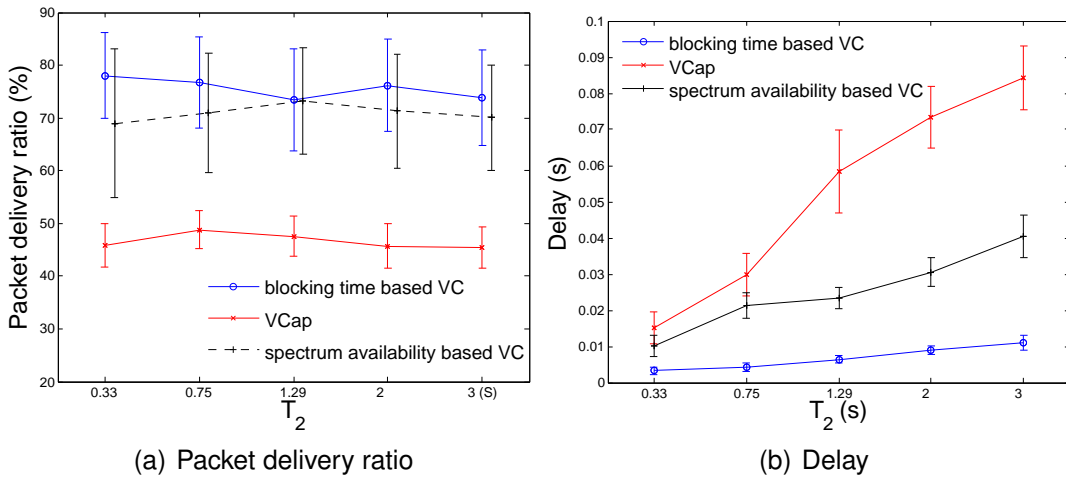
The packet delivery ratio shown in Figure 5.10(a) is constant with both blocking time based virtual coordinate and VCap, because all the primary users have the same  $P_{\text{OFF}}$  which is  $0.9$ . Particularly, blocking time based virtual coordinate achieves higher packet delivery ratio than the other two virtual ordinates, the reason is when the former is ap-



**Figure 5.10** Geographic routing in single secondary channel scenario,  $P_{\text{OFF}} = 0.9$  for all primary users,  $T_1 = 3s$ ,  $T_2$  varies.

plied, less packets are dropped from buffer as the time of being blocked is shorter for the secondary users on the path.

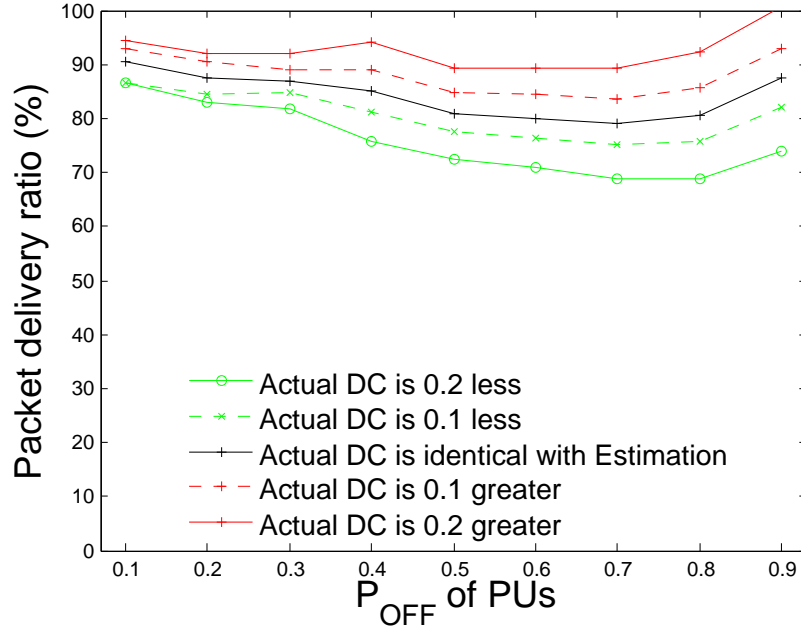
Now we have a look at the CRN with two licensed channels. As to performance of delay, because of the second available channel, blocking time based virtual coordinate achieves very delay, in contrast, spectrum availability based virtual coordinate still demonstrates obvious randomness, as is shown in Figure 5.11(b). Compare Figure 5.11(a) and 5.10(a), we can see the packet delivery ratio in two channel network is obviously higher than that in single channel network, as the second channel provides extra transmission opportunities. Blocking time based virtual coordinate achieves up to 10% better performance than that with spectrum availability based VC, the reason is packets in buffer have greater likelihood to be sent out before getting dropped.



**Figure 5.11** Geographic routing in two secondary channel scenario,  $P_{\text{OFF}} = 0.9$  for all primary users,  $T_1 = 3s$ ,  $T_2$  varies.

### 5.6.4 Sensitivity of SAViC to Estimation Errors

In this Section, we evaluate the performance of SAViC in the presence of erroneous estimates about the primary user activity. In order to induce a particular amount of errors, we artificially add errors to duty cycle of primary user activity by directly modifying the implementation of 2 state markov chain. By doing this a real life scenario is built, where estimation errors are expected. The simulation is conducted with the same configuration with Figure 5.6(a). Figure 5.12 shows even the errors are significant, i.e., DC is modified by 25% to 100%, the corresponding performance varies only at most 13%.



**Figure 5.12** In the setting of Figure 5.6(a), performance of DC based virtual coordinate is not affected greatly by the erroneous estimation

## 5.7 Conclusions

The proposed virtual coordinate SAViC reshapes the topology of cognitive radio network based on sensing results of spectrum availability. As SAViC adjusts the distance between nodes based on the communication obstruction caused by primary users, the virtual coordinate comprises a part of the routing decision, so that geographic routing is able to detour the areas seriously affected by primary user. We obtain the upper bound of the number of control messages (anchor messages) in the process of coordinate assignment, which is linear with the network size, network density and the number of anchors.

Geographic routing with SAViC greatly simplifies the computation and communication burden on each secondary user involved in routing in CRN. Together with SAViC, geographic routing achieves better performances than other geographic routing designed for CRN through extensive simulation. This paradigm of routing is especially suitable for CRN network where the resource limited CR nodes can only support geographic routing. This work emphasises on avoiding primary users' influence with geographic routing, and doesn't consider the interference issue among the secondary users, which should be addressed in the future work.



# 6

## Conclusion and Future Work

The goal of this thesis is to design paradigms and algorithms that enable individual cognitive radio to act autonomously, and finally converge to an equilibrium state, meanwhile, the performance of cognitive radio users are improved. In this dissertation, we solve a series of problems residing from layer 1 to layer 3 in the OSI model [4] of CRN with distributed solutions, and employ game theory to seek the guarantee on convergence and Nash equilibrium.

### 6.1 Contribution

We propose a complete strategy of power and spectrum allocation in IEEE 802.22 networks. We make full use of the centralized database which is required by the standard and relevant regulations. The maximal transmission power on channel for each WBS is decided in the database by solving an optimization problem. Then a two stage channel power allocation scheme is proposed in a distributed manner, where the database is involved to assist interaction between secondary users. This work lies in layer 1, after deciding the maximal transmission power on each secondary cellular base station, we formulate the distributed spectrum allocation problem in TV white space scenario (a special CRN where primary user is TV station which operates according to a slow and pre-decided schedule) into a canonical congestion game, then propose distributed algorithm corresponding to the behaviour of player in the game.

As to improving the accuracy of spectrum sensing with clusters, we designed a distributed clustering scheme ROSS. ROSS is lightweight, thus is suitable CRAHN where the network connection varies due to the mobility of spectrum and secondary users. ROSS increases the number of common available channels within one cluster, thus improves the robustness of formed clusters against primary users, so that cooperative sensing can make the best use of clusters. This problem can be regarded to lie in layer 2.

In layer 3, we propose a lighted weighted routing scheme for CRN. Spectrum aware virtual coordinate is proposed, thus light weighted geographic routing can be used to decide the next hop.

## 6.2 Future Work

Building on the top of the our contributions, game theory can help to design distributed solution for further problems in CRN. In this section, we describe the future work, and sketch the solutions briefly.

### Structureless Diffusion Based Routing in Game Theoretical Framework in Event-driven Cognitive Radio Sensor Network

One future work is to propose a structure less data fusion paradigm for cognitive radio sensor network. Sensor nodes are deployed to sense physical phenomena of interests (PoIs), and they transmit the PoIs to the sink node. Cognitive radio capability is transplanted onto sensor network [19], so as to alleviate collision and decrease delay. But congestion is still difficult to be totally eliminated by merely providing more spectrum. Congestion occurs in wireless sensor network when incoming traffic load exceeds the capacity. Variant solutions [151] have been proposed to mitigate congestion, e.g., implementing flow control to avoid transmitting to the downstream node where congestion is detected, changing the MAC behaviour to give the congested sensor larger possibility to transmit, adding rely nodes to reroute data around the congested nodes.

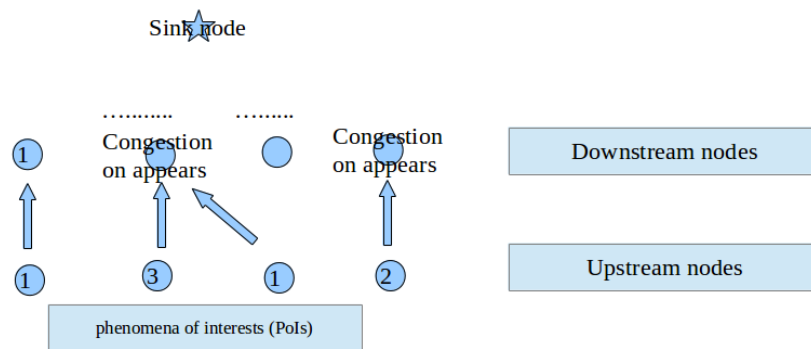
When there are multiple upstream nodes<sup>1</sup> transmitting to sink node, and there are several downstream nodes are available to be relies, then which portion of the data on the upstream node should be sent to which downstream node becomes a question. As shown in Figure 6.1, several nodes are downstream nodes for node 2 and 3. We have proposed routing solution for end to end communication in CRN, where geographic routing is conducted with spectrum aware virtual coordinate. As to multiple to one communication, simply using our proposed scheme is problematic, as we need to consider the possible collision caused by multiple transmission, priority of services and possible congestion appearing on the relying nodes.

#### Problem Statement

The basic idea is from the *server matching* introduced in Section 2.

Delay of the packets is used to represent the congestion. This delay could be the average of delay on all packets, or a function of delay of individual packet. Although it is not clear how to granulate the data based on their priorities, it is clear that the total congestion (delay) is a increasing function of the amount of incoming data. When there are only one group of downstream nodes and one group of upstream nodes involved, and the upstream nodes have finite number of transmission strategy (a certain portion is sent to a certain

<sup>1</sup>Downstream nodes of one node  $i$  represent the nodes relying data from node  $i$  towards to the sink node, upstream nodes of node  $i$  means the nodes sending data to node  $i$ .



**Figure 6.1** Multiple to one communication

downstream node), then the convergence will be brought by best response strategy. But the converge is not guaranteed when consider within a bigger picture, as the downstream nodes can become upstream nodes when they send data to their downstream nodes.

The questions needed to be answered are as follows,

- What is the strategy for each sensor node to assign its data to different downstream nodes.
- As this problem is a cascaded of congestion games, is there equilibrium?

because the structured techniques incur relatively higher overheads to maintain the network structure.

## Other Future Research Works

- As to channel and power allocation in IEEE 802.22 network, cooperation can be brought to improve performance.
- As a light weight clustering scheme which can cope with mobility of both spectrum and users, ROSS can be used to support routing and resource allocation.



# 7

## Appendix



# A

## Proof of Theorem 3.5.1

For selfish best response approach, the utility function is set as follows,

$$u_i = \frac{\sum_{c(i)=c(i)} f_{ji} + N_0}{P_i \cdot h_i \cdot z_i} \quad (\text{A.1})$$

**Proof.** In order to simplify the proof, we assume  $N_0 = 0$ . Consider one WBS  $i$  executing algorithm 1 with utility A.1, and updates its channel from  $c_i$  to  $c'_i$ , we denote  $u'_k, k \in \mathcal{N}$  as the utility of WBS  $k$  when  $i$  chooses channel  $c'_i$ , accordingly, the summation of utilities of all WBSs after  $i$  changing to  $c'_i$  is  $U' = \sum_{\forall k \in \mathcal{N}, c(i)=c'_i} u'_k$ .

$$\begin{aligned} U' &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u'_j \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} (u_j + (u'_j - u_j)) \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u_j + \sum_{j \in \mathcal{N}, j \neq i} (u'_j - u_j) \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u_j + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} (u'_j - u_j) + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} (u'_j - u_j) \\ &\quad + \sum_{\substack{j \in \mathcal{N}, j \neq i, \\ c(j) \neq c'_i, c(j) \neq c_i}} (u'_j - u_j) \\ &= u'_i + \sum_{j \in \mathcal{N}, j \neq i} u_j + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{f_{ij}}{\tilde{P}_j} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{f_{ij}}{\tilde{P}_j} \right) \end{aligned} \quad (\text{A.2})$$

where,

$$\begin{aligned} u'_i &= u_i + \Delta u_i(c_i \rightarrow c'_i) \\ &= u_i + \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{f_{ji}}{\tilde{P}_i} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{f_{ji}}{\tilde{P}_i} \right) \end{aligned} \quad (\text{A.3})$$

bring A.3 into A.2, we get,

$$\begin{aligned}
 U' = U &+ \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{f_{ji}}{\tilde{P}_i} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{f_{ji}}{\tilde{P}_i} \right) \\
 &+ \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c'_i}} \left( \frac{f_{ij}}{\tilde{P}_j} \right) - \sum_{\substack{j \in \mathcal{N}, \\ j \neq i, \\ c(j)=c_i}} \left( \frac{f_{ij}}{\tilde{P}_j} \right)
 \end{aligned} \tag{A.4}$$

According to algorithm 1, the summation of second and third items, which is the variance of  $i$ ' utility, is negative. If we can confirm the summation of fourth of the last four items is negative, the whole utility of the network decreases with  $i$ ' each update. For simplification, we assume that the channel is symmetric, which means,  $h_{ij} = h_{ji}$ , and  $z$  is identical among all WBSs. Then, the problem we want to confirm is equivalent to the following: Given the in-equation with  $n, m$  are natural numbers

$$\sum_{i=1}^m \alpha_i < \sum_{i=1}^n \beta_i, \tag{A.5}$$

Prove the following in-equation is correct or not,

$$\sum_{i=1}^m \left( \alpha_i + \frac{1}{\alpha_i} \right) < \sum_{i=1}^n \left( \beta_i + \frac{1}{\beta_i} \right), \tag{A.6}$$

We propose a small contradiction to prove A.6 is not true. When  $m = 2, n = 1$ , and  $\alpha_1 = 1, \alpha_2 = 0.5, \beta = 2.1$ , we can see that although  $\sum_{i=1}^m \alpha_i = 1.5 < \sum_{i=1}^n \beta_i = 2.1$ , there is  $\sum_{i=1}^m \left( \alpha_i + \frac{1}{\alpha_i} \right) = 4.5 > \sum_{i=1}^n \left( \beta_i + \frac{1}{\beta_i} \right) = 2.58$ . hence, with WBS's update, it is possible that  $U' > U$ , thus there is no monotonically convergence by utilizing A.1.

□

Notice that the last four items in A.4 is exactly the change of summation of utilities of all WBSs after  $i$ ' update if WhiteCat is executed, hence the monotonic convergence of WhiteCat is proved here analytically if noise is considered to be zero. If noise is considered, we can follow the conclusion in the end of 3.5.3 that WhiteCat converges without monotonicity.



# B

## Deviation of Problem 3.7

We reformulate the objective problem 3.7 which is a binary non-linear programming to binary quadratic programming as follows,

$$\begin{aligned}
 & \sum_{i=1}^n \frac{\sum_{j \in \mathcal{N}, j \neq i} \mathbf{P}_j^T X_j (X_j^T X_i) h_{ji} z_{ji} + N_0}{\mathbf{P}_i^T X_i h_i z_i} \\
 &= \sum_{i=1}^n \left( \frac{\sum_{j \in \mathcal{N}, j \neq i} \sum_k (P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot x_{jk} \cdot h_{ji} \cdot z_{ji}) + \sum_k N_0 \cdot x_{ik}}{\mathbf{P}_i^T X_i h_i z_i} \right) \\
 &= \sum_{i=1}^n \left( \frac{\sum_{j \in \mathcal{N}, j \neq i} \sum_k (P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z_{ji})}{\mathbf{P}_i^T X_i h_i z_i} + \frac{\sum_k N_0 \cdot x_{ik}}{\mathbf{P}_i^T X_i h_i z_i} \right) \\
 &= \sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k \frac{(P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z_{ji})}{\mathbf{P}_i^T X_i h_i z_i} + \sum_k \frac{N_0 \cdot x_{ik}}{\mathbf{P}_i^T X_i h_i z_i} \right)
 \end{aligned} \tag{B.1}$$

If we assume WBS  $i$  is working on channel  $m$ , then there is  $x_{im} = 1$ , then the first item in the parenthesis becomes,

$$\begin{aligned}
 & \frac{P_{jk} \cdot x_{jk} \cdot x_{ik} \cdot h_{ji} \cdot z_{ji}}{\mathbf{P}_i^T X_i h_i z_i} = \frac{P_{jk} \cdot x_{jk} \cdot x_{im} \cdot h_{ji} \cdot z_{ji}}{P_{im} \cdot x_{im} \cdot h_i \cdot z_i} \\
 &= \frac{P_{jk} \cdot x_{jk} \cdot x_{im} \cdot h_{ji} \cdot z_{ji}}{P_{im} \cdot h_i \cdot z_i}
 \end{aligned} \tag{B.2}$$

Similarly, for the second item in the parenthesis,

$$\frac{N_0 \cdot x_{ik}}{\mathbf{P}_i^T X_i h_i z_i} = \frac{N_0}{P_{ik} h_i z_i} \cdot x_{ik} \tag{B.3}$$

then, formula B.1 becomes,

$$\sum_{i=1}^n \left( \sum_{j \in \mathcal{N}, j \neq i} \sum_k \frac{P_{jk} \cdot h_{ji} \cdot z_{ji}}{P_{ik} \cdot h_i \cdot z_i} \cdot x_{jk} \cdot x_{ik} + \sum_k \frac{N_0}{P_{ik} \cdot h_i \cdot z_i} \cdot x_{ik} \right) \quad (\text{B.4})$$

which is a binary quadratic programming problem.

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# Glossary

- 2TDMC** two-state discrete time Markov chain. 101
- 3G** Third generation. 1
- 5G** 5th generation mobile networks. 2
- CCC** Common control channel. 15, 63, 68
- CH** cluster head. 59
- COS** Cost of oscillation. 54
- CR** Cognitive radio. 3
- CRAHN** Cognitive radio ad hoc network. 7
- CRN** Cognitive radio network. 3
- CW** Contention window. 10
- DCF** Distributed coordination function. 10
- DSR** Dynamic source routing. 92
- DTV** digital TV. 31
- ECC** Electronic communications committee in Europe. 4, 29
- FCC** Federal communications commission in U.S.. 29
- GSM** Global system for mobile communications. 1
- HSPA** High speed packet access. 12
- ISM band** Industrial, scientific, and medical. 1
- LP** linear programming program. 38
- LTE** 3GPP Long term Evolution. 12
- MAC** Media Control Layer. 14
- MAN** Metropolitan area networks. 37
- NE** Nash equilibrium. 9
- PDA** Personal digital assistant. 3
- PoA** price of anarchy. 21
- PU** Primary user. 2
- QoS** Quality of service. 1, 31
- RF** Radio Frequency. 1
- ROSS** Robust spectrum sharing. 63
- ROSS-DFA** Robust spectrum sharing-distributed fast algorithm. 60
- ROSS-DGA** Robust spectrum Sharing-distributed greedy algorithm. 60
- RSSI** Received signal strength index. 30, 49
- SINR** signal-to-noise-and-interference ratio. 31, 32
- SU** Secondary user. 2
- TC** Time complexity. 85
- TVBD** TV band devices. 13
- TVWS** TV white space. 12
- UHF** Ultra high frequency. 30
- VCap** virtual coordinate assignment protocol. 94
- WBS** White base station. 14
- WhiteCat** White space channel allocation. 36
- WLAN** Wireless local-area network. 1
- WMAN** Wireless metropolitan-area network. 1
- WPAN** Wireless personal-area network. 1
- WRAN** Wireless regional area network. 37