Neural Coding
Homework Problems 5.

1. Consider two population of N neurons $f_j(x) + \eta_j$ with $j = 1, \ldots, N$ and i.i.d. Gaussian noise $n_j \sim \mathcal{N}(0, \sigma^2)$ with different tuning curves in both populations.
Population (A):

$$f_j(s) = f_{\max} s.$$

Population (B):

$$f_j(s) = \begin{cases} N f_{\max} \left(s - \frac{j-1}{N}\right) & , \frac{j-1}{N} < s < \frac{j}{N} \\ 0 & , \text{ else} \end{cases}$$

Consider the stimulus interval to be $s \in (0, 1)$
Hint: We are considering the whole population code, so we have the population response defined as $(\mathbf{r}|s)$. However in the case A. the responses of individual neurons are independent, so $P(\mathbf{r}|s) = \prod_{k=1}^{N} P(r_k|s)$. Another useful thing to remember: logarithm (needed for Fisher information) transforms multiplication into sum
(a) Set $N = 10$ and $f_{\max} = 20$. Plot the tuning function of neuron 5 for codes A and B. *(1 point)*
(b) Plot the Fisher Information J_s for both codes for the settings in (a). *(1 point)*
(c*) Compute the Fisher Information J_s for both codes (in the general case not restricted to specific parameter settings). Which code is better and why? *(3 points)*

2. Let $X = (X_1, \ldots, X_N)$ be a vector of random variables, e.g. the spike counts of neurons in a given time window. **(a)** Show that

$$\mathrm{Var}\left[\sum_i X_i\right] = \sum_i \mathrm{Var}\left[X_i\right] + \sum_i \sum_{j \neq i} \mathrm{Cov}\left[X_i, X_j\right]$$

(3 points)
(b) Assume that X_i have identical variances σ^2 and all pairs X_i, X_j have identical correlations ρ. Show that

$$\mathrm{Var}\left[\frac{1}{N} \sum_i X_i\right] = \frac{\sigma^2}{N}(1 + \rho(N-1))$$

Which proves equation from the lecture. *(2 points)*

(c*) Show that for even N

$$\mathrm{Var}\left[\frac{1}{N}\sum_{i=1}^{N}(-1)^i X_i\right] = \frac{\sigma^2}{N}(1-\rho)$$

(3 points)