$P(X=x) \rightarrow$ variable being x from λ parameter

$P(Y=k-x) \rightarrow$ variable being k-x from μ parameter

$k = k-x+x$ k is sum of two independent variable

$P(Z=k) \rightarrow$ probability of k after summation

$$P(Z=k) = \sum_{x=0}^{k} \underbrace{P(X=x, Y=k-x)}_{\text{all possible combinations}}$$

$$= \sum_{x=0}^{k} P(X=x)\, P(Y=k-x) \rightarrow \text{independent}$$

$$= \sum_{x=0}^{k} \frac{\lambda^x e^{-\lambda}}{x!} \frac{\mu^{k-x} e^{-\mu}}{(k-x)!}$$

$$= e^{-(\lambda+\mu)} \sum_{x=0}^{k} \frac{1}{x!(k-x)!} \lambda^x \mu^{k-x} \rightarrow \text{multiply } \frac{k!}{k!}$$

$$= e^{-(\lambda+\mu)} \frac{1}{k!} \underbrace{\sum_{x=0}^{k} \frac{k!}{x!(k-x)!} \cdot \lambda^x \mu^{k-x}}_{\text{Newton's series}}$$

$$= e^{-(\lambda+\mu)} \frac{1}{k!} \cdot (\lambda+\mu)^k = \underbrace{\frac{(\lambda+\mu)^k e^{-(\lambda+\mu)}}{k!}}_{\text{this is Poisson of } \lambda+\mu \text{ parameter}}$$