Neural Coding
Homework Problems 3.

1. Show that $E\left[(x-c)^2\right] = (c - E[x])^2 + Var[x]$. *(2 points)*

2. Let $X \sim Poisson(\lambda)$. (a) Show that $Var[X] = \lambda$. *(2 points)*

(b) Plot in Python the p.m.f. of X for $\lambda \in 1, 3, 5, 50$. What do you observe about the shape of the p.m.f. as λ grows? *(1 point)*

(c) What are the parameters of the best matching Normal distribution for a given Poisson distribution with parameter λ? No formal derivation is required here, use your intuition and verify by plotting (for the same values of λ as above). *(2 points)*

3. Compute the Fisher Information J_x for a Bernouilli neuron with arbitrary tuning curve $f(x)$ i.e.

$$p(r = 1|f(x)) = B(f(x)).$$

(2 points)

4.* Let the rate of the homogeneous Poisson process μ be drawn randomly from the Gamma distribution with parameters $\alpha = m \in \mathbb{N}$ and β. Density of the Gamma distribution is given by

$$\rho(\mu) = g(\mu|\alpha, \beta) = \frac{\mu^{\alpha-1} e^{-\beta\mu}}{\beta^{-\alpha}\Gamma(\alpha)}.$$

Show that the spike count distribution $P(k) = \int_0^\infty \mathcal{P}_{Poisson}(k|\mu)\rho(\mu)d\mu$ obtained from marginalization over the parameter μ is a negative binomial distribution. *(3 points)*