$$\text{Mean} = E(X) = \sum_{i=0}^{\infty} x_i P_i = \sum_{x=0}^{\infty} x \frac{e^{-\lambda}\lambda^{x}}{x!} =$$

$x = 0 \Rightarrow$ no need for sum

$$= \sum_{x=1}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-1)!} \quad \text{cancel } x$$

$$= e^{-\lambda} \cdot \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \quad \leftarrow \text{took out } \lambda$$

$$= e^{-\lambda} \cdot \lambda \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} \leftarrow \underbrace{\sum \frac{a^x}{x!} = e^{a}}_{\text{use this}}$$

$$= \cancel{e^{-\lambda}} \cdot \lambda \cdot \cancel{e^{\lambda}} = \lambda$$

$$\text{Mean} = \lambda$$

$$Var = \sum x^2 - (\Sigma(x))^2 = \underbrace{\sum x^2 - \Sigma(x)}_{\text{combine}} + \Sigma(x) - (\Sigma x)^2 =$$

$$= \Sigma(x^2 - x) + \underbrace{\Sigma(x)}_{\text{Mean}} - \underbrace{(\Sigma x)^2}_{\text{Mean}^2}$$

$$= \sum_{x=0}^{\infty} x(x-1) \frac{e^{-\lambda}\lambda^{x}}{x!} + \lambda - \lambda^2$$

$x=0$, $x=1$ is zero
cancel $x(x-1)$ from $x!$

$$= \sum_{x=2}^{\infty} \frac{e^{-\lambda}\lambda^{x}}{(x-2)!} + \lambda - \lambda^2 \quad \longrightarrow \text{take } \lambda^2 \text{ out}$$

$$= \lambda^2 e^{-\lambda} \sum_{x=2}^{\infty} \frac{\lambda^{x-2}}{(x-2)!} + \lambda - \lambda^2 \quad \longrightarrow \text{use same approach}$$

$$= \lambda^2 \cancel{e^{-\lambda}} \cdot \cancel{e^{\lambda}} + \lambda - \lambda^2 = \cancel{\lambda^2} - \cancel{\lambda^2} + \lambda = \lambda$$

$P(X=x) \rightarrow$ variable being x from λ parameter

$P(Y=k-x) \rightarrow$ variable being k-x from μ parameter

$k = k-x+x$ $\quad$ k is sum of two independent variable

$P(Z=k) \rightarrow$ probability of k after summation

$$P(Z=k) = \sum_{x=0}^{k} \underbrace{P(X=x, Y=k-x)}_{\text{all possible combinations}}$$

$$= \sum_{x=0}^{k} P(X=x)\, P(Y=k-x) \rightarrow \text{independent}$$

$$= \sum_{x=0}^{k} \frac{\lambda^x e^{-\lambda}}{x!} \frac{\mu^{k-x} e^{-\mu}}{(k-x)!}$$

$$= e^{-(\lambda+\mu)} \sum_{x=0}^{k} \frac{1}{x!(k-x)!} \lambda^x \mu^{k-x} \rightarrow \text{multiply } \frac{k!}{k!}$$

$$= e^{-(\lambda+\mu)} \frac{1}{k!} \underbrace{\sum_{x=0}^{k} \frac{k!}{x!(k-x)!} \cdot \lambda^x \mu^{k-x}}_{\text{Newton's series}}$$

$$= e^{-(\lambda+\mu)} \frac{1}{k!} \cdot (\lambda+\mu)^k = \underbrace{\frac{(\lambda+\mu)^k e^{-(\lambda+\mu)}}{k!}}_{\text{this is Poisson of } \lambda+\mu \text{ parameter}}$$