Neural Coding
Homework Problems 4.

1. Plot the Fisher Information $J(s)$ for the Bernoulli neuron and for a Poisson, assuming $f(s) = \frac{1}{1+s^2}$, as a function of s. *(2 points)*

2. For the Bernoulli neuron with $f(s) = \frac{1}{1+s^2}$ from previous task.
(a) Compute the minimum discrimination error for the two stimuli $s_1 = 1$ and $s_2 = 2$ assuming equal prior probability for the two stimuli. *(2 points)*
(b) Generalize the computation in (a) to arbitrary s_2. Plot the MDE as a function of $s_2 > 1$. *(1 point)*
(c) Plot the two MDE bounds from the lecture as a function of $s_2 > 1$. *(2 points)*

3. Let the stimulus have one of two values s_1 and s_2 with equal probabilities. Show that the Jensen-Shannon-Information:

$$I_{JS}(s_1, s_2) = \frac{1}{2} D_{KL}\left[p(r|s_1) \,\|p(r)\right] + \frac{1}{2} D_{KL}\left[p(r|s_2) \,\|p(r)\right]$$

Is equal to the mutual information between the response and the class of stimulus $I \in \{1, 2\}$ $(p(I = 1) = 0.5)$

$$MI(r, I) = \sum_{i \in I} \int p(r|i) \log \frac{p(r|i)}{\sum_i p(r|i)p(i)} dr$$

(2 points)

4.* In this task we will try to understand why $d' = \Delta\mu^T \Sigma^{-1} \Delta\mu$ measures the minimal linear discrimination error via

$$LDE = 1 - \Psi\left(d'/2\right),$$

with Ψ a CDF of normal distribution $\Delta\mu = f(s) - f(s+\Delta s)$ for any Gaussian population model with $r \sim \mathcal{N}(f(s), \Sigma(s))$.

(a) Write down the optimal linear classifier for two normal distributions with arbitrary means μ_1, $\boldsymbol{\mu_2}$ covariance matrices Σ_1, Σ_2 and equal class probabilities. *(2 points)*
(b) Show that d' measures the minimal linear discrimination error. *(3 points)*
(c) What are the conditions on the population covariance matrix Σ for the minimal linear discrimination error to be equal to the overall minimal discrimination error? For which population models (Poisson, additive Gaussian, multiplicative Gaussian) are they satisfied? *(2 points)*