INSTRUCTIONS:

(a) You may use your brain, and pen/pencil on this exam – and No Other Resources!

(b) You are required to show all your work and provide the necessary explanations to get full credit.

(c) This exam has 5 problems for a total of 100 possible points.

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

Problem 1. [25 points]
Find the complete solution to $Ax = b$

$$A = \begin{bmatrix} 1 & -1 & 0 & 3 & 3 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & -1 & 0 & 3 & 4 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 2 \\ 0 \\ 2 \\ 0 \end{bmatrix}$$

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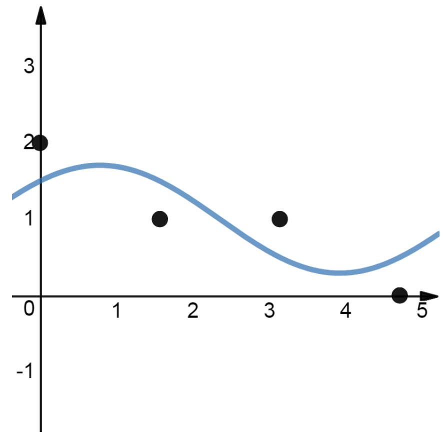
Problem 2. [25 points]
Solve a least squares problem to find the trigonometric polynomial

$$p(x) = a + b\sin(x) + c\cos(x)$$

that best fits the points:

$(0, 2)$
$(\pi/2, 1)$
$(\pi, 1)$
$(3\pi/2, 0)$



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Problem 3. [15 points]
An $n \times n$ **semi-magic square** is an $n \times n$ matrix in which the sum along each row and each column is a constant (called the line-sum of the semi-magic square). For example, 2×2 semi-magic squares with line-sums 0, 2 and -4, respectively, are

$$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \qquad \text{and} \qquad \begin{bmatrix} -3 & -1 \\ -1 & -3 \end{bmatrix}.$$

Consider the subspace S consisting of all 2×2 semi-magic squares. Find a basis for S.

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Problem 4. [15 points]
Consider the following matrix A and vector b

$$A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad b = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}.$$

(a) [10 points] Find the projection matrix P onto the column space of A.

(b) [5 points] Find the projection of the vector b onto the column space of A.

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Problem 5. [20 points]
Find an orthonormal basis for the column space of the matrix

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}.$$

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