INSTRUCTIONS:

(a) You may use your brain, and pen/pencil on this exam – and No Other Resources!

(b) You are required to show all your work and provide the necessary explanations to get full credit.

(c) This exam has 5 problems for a total of 100 possible points.

(d) You have exactly 50 minutes to answer all **5** questions.

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

Problem 1. [25 points]
The 3×3 matrix A reduces to the identity matrix $\left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{smallmatrix}\right]$ by the following three row operations (in order):

Subtract twice the first row from the second row.

Add the first row to the third row.

Subtract the second row from the third row.

(a) [10 points] Find the LU factorization of A

(b) [5 points] Solve the linear system $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

(c) [10 points] Find A^{-1}

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Problem 2. [15 points]
Find all values of a for which the following matrix is invertible:

$$A = \begin{bmatrix} a & 2 & 3 \\ a & a & 4 \\ a & a & a \end{bmatrix}$$

Problem 3. [20 points]
Are the following statements true or false. If true, please prove it. If false, please give an example where the statement fails.

(a) [10 points] If $A^2 = A$, then $A = 0$ or $A = I_n$.

(b) [10 points] The matrix $2I_n - 5AA^T$ is symmetric.

Problem 4. [20 points]
Consider the following four vectors

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ -k \end{bmatrix} \qquad v_2 = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \qquad v_3 = \begin{bmatrix} -2k \\ 6 \\ 2 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$$

(a) [10 points] For which values of k is the vector b a linear combination of v_1, v_2 and v_3?

(b) [10 points] Write the vector b as a linear combination of v_1, v_2 and v_3 when $k = 0$.

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Problem 5. [20 points]

A <u>subspace</u> of a vector space V is a subset of V that satisfies two requirements: If v and w are in the subspace and c is any scalar, then (i) $v + w$ is in the subspace (ii) cv is in the subspace.

(a) [10 points] Show that the set of 3×3 matrices satisfying $A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is not a subspace of $\mathbb{R}^{3\times 3}$.

(b) [10 points] Show that the set of 3×3 permutation matrices is not a subspace of $\mathbb{R}^{3\times 3}$.

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