INSTRUCTIONS:

(a) You may use your brain, and pen/pencil on this exam – and No Other Resources!

(b) You are required to show all your work and provide the necessary explanations to get full credit.

(c) This exam has 5 problems for a total of 100 possible points.

(d) You have exactly 50 minutes to answer all **5** questions.

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Problem 5:

Total:

Problem 1. [15 points]
Suppose A is some 4×4 matrix. You want to transform this into a new 4×4 matrix B by doing operations on the rows. For each part, (i) write down the matrix E such that $B = EA$. Also, (ii) say whether E is invertible (you don't need to compute E^{-1}, just say whether E is invertible or not).

(a) [5 points] Swap the second and fourth rows of A.

$$E = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

The matrix E is ____________________

(b) [5 points] Add the second row to the third row, and subtract twice the second row from the fourth row.

$$E = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

The matrix E is ____________________

(c) [5 points] Keep the first and third rows the same, divide the second row by 2, and divide the fourth row by -5.

$$E = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

The matrix E is ____________________

Problem 2. [20 points]
The matrix A has a remarkably simple inverse. Find A^{-1} by Gauss-Jordan elimination.

$$A = \begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad A^{-1} = \begin{bmatrix} & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$

Problem 3. [20 points]
Suppose that A and B are two symmetric matrices, and C is some other matrix (possibly nonsymmetric), all of the same size $n \times n$. Which of the following matrices are certainly symmetric? Justify your answer.

(a) [10 points] $A^2 - B^2$

(b) [10 points] $C^T - C$

Problem 4. [25 points]
Consider the following system of linear equations depending on a parameter k

$$\begin{aligned} x_1 \qquad\qquad\quad +3x_3 &= \quad 1 \\ -x_1 + x_2 \qquad\quad -x_3 &= \quad 1 \\ x_1 + x_2 + (k^2+1)x_3 &= k+1 \end{aligned}$$

[15 points] Use <u>elimination</u> to transform it into an upper triangular system

Now answer the following two questions (no justification needed).

(a) [5 points] For which value(s) of k does the system have infinitely many solutions?

(b) [5 points] For which value(s) of k does the system have no solutions?

Problem 5. [20 points] Are the following statements true or false. If true, please prove it. If false, please give an example where the statement fails.

(a) [10 points] The matrix A is $n \times n$ and the matrix B is $n \times m$. If A is invertible and $AB = 0$, then $B = 0$ (the $n \times m$ zero matrix).

(b) [10 points] If A is an upper triangular matrix, then A is invertible.