INSTRUCTIONS:

(a) You may use your brain, and pen/pencil on this exam – and No Other Resources!

(b) You are required to show all your work and provide the necessary explanations to get full credit.

(c) This exam has 4 problems for a total of 100 possible points.

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Extra Credit:

Total:

Problem 1. [20 points]
Consider the matrix A and the vectors v_1, v_2 and v_3:

$$A = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \qquad v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \qquad v_2 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} \qquad \text{and} \qquad v_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

(a) [10 points] Check that v_1, v_2 and v_3 are eigenvectors of the matrix A. What are the eigenvalues of A?

(b) [10 points] Find $A^{2019} v_2 = \underbrace{AA \cdots A}_{2019 \text{ times}} v_2$.

(blank page for your work if you need it)

Problem 2. [30 points]
Consider the matrix

$$A = \begin{bmatrix} 1/2 & 1 \\ 0 & -1/2 \end{bmatrix}$$

(a) [10 points] Find the eigenvalues and eigenvectors of A.

(b) [10 points] Find a formula for A^n.

(c) [10 points] Find $\lim_{n\to\infty} A^n$.

(blank page for your work if you need it)

Problem 3. [25 points]
Factor the symmetric matrix

$$S = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix}$$

into $S = QDQ^T$, where D is diagonal and Q has orthonormal columns.

(blank page for your work if you need it)

Problem 4. [25 points]
For which values of a and b is the matrix

$$A = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & b \\ 0 & 0 & 2 \end{bmatrix}$$

diagonalizable?

(blank page for your work if you need it)

Problem for 5 extra credits

Suppose A is a diagonalizable 3×3 matrix, and has only 1 and -1 as eigenvalues. What is the matrix A^2?