INSTRUCTIONS:

(a) You may use your brain, and pen/pencil on this exam – and No Other Resources!

(b) You are required to show all your work and provide the necessary explanations to get full credit.

(c) This exam has 4 problems for a total of 110 possible points.

(d) You have exactly 50 minutes to answer all **4** questions.

Problem 1:

Problem 2:

Problem 3:

Problem 4:

Total:

Problem 1. [45 points] Consider the matrix A and the vector b:

$$A = \begin{bmatrix} 1 & -1 & 2 & 0 \\ 2 & -2 & 4 & 0 \\ 3 & -3 & 7 & 0 \end{bmatrix} \qquad \text{and} \qquad b = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

(a) [10 points] Find a basis for the column space $\mathrm{C}(A)$.

(b) [10 points] Find a basis for the nullspace $\mathrm{N}(A)$.

(c) [10 points] Find a basis for the row space $\mathrm{C}(A^T)$.

(d) [5 points] Find the dimension of the left nullspace $\mathrm{N}(A^T)$.

(e) [10 points] Find the complete solution to $Ax = b$.

(blank page for your work if you need it)

Problem 2. [20 points]

(a) [10 points] Give an example of a 6×3 matrix A with full column rank, and a vector b such that $Ax = b$ is not solvable. For the same matrix A, find another vector c such that $Ax = c$ is solvable.

(b) [10 points] Give an example of a matrix A with full row rank such that the number of solutions to $Ax = b$ is 1, regardless of b.

Problem 3. [25 points] Consider the following 4×6 matrix

$$A = \begin{bmatrix} 1 & 3 & -2 & 0 & 2 & 0 \\ 2 & 6 & -5 & -2 & 4 & -3 \\ 0 & 0 & 5 & 10 & 0 & 15 \\ 2 & 6 & 0 & 8 & 4 & 18 \end{bmatrix},$$

with $\operatorname{rank}(A) = 3$.

(a) [10 points] Show that the following three vectors are in the nullspace of A:

$$v_1 = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \qquad v_2 = \begin{bmatrix} -4 \\ 0 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix} \qquad \text{and} \qquad v_3 = \begin{bmatrix} -2 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$$

(b) [10 points] Show that the vectors v_1, v_2 and v_3 are linearly independent.

(c) [5 points] The dimension of the nullspace of A is ____________, so the vectors v_1, v_2 and v_3 are a __________________ for the nullspace.

Problem 4. [20 points] Are the following statements true or false. If true, please prove it. If false, please give an example where the statement fails.

(a) [10 points] The set of all $n \times n$ matrices that are **NOT** symmetric ($A^T \neq A$) is a subspace.

(b) [10 points] If B is an $n \times n$ invertible matrix, then every vector b in $\mathbb{R}^n$ is a linear combination of the columns of B.