Inputs : $x_1 = 1.0$, $x_2 = 0.8$, $x_3 = 0.6$

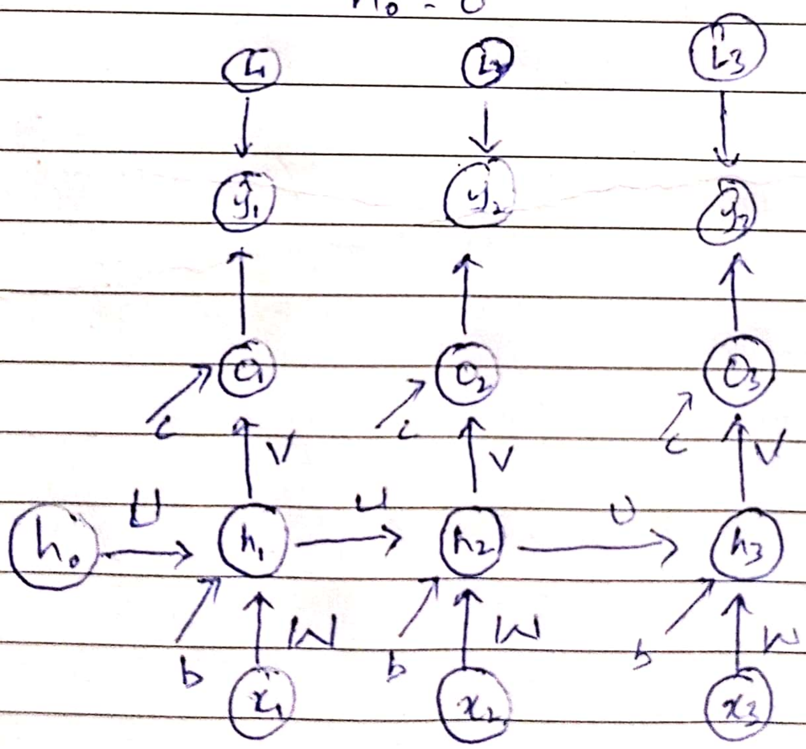
Outputs : $y_1 = 0.5$, $y_2 = 0.6$, $y_3 = 0.7$

Parameters :

- $W = 0.5$ $(i \to h)$
- $U = 0.5$ $(h \to h)$
- $V = 0.5$ $(h \to o)$
- $b = 0.2$ (h)
- $c = 0.1$ (o)

Activation: tanh for hidden, linear for output

$h_0 = 0$



Date______20___

FORWARD PASS

Equations:

$$y_1 = Vh_1 + C$$
$$h_1 = \tanh(Wx_1 + Uh_0 + b)$$

For $t = 1$

$$y_1 = (0.5)(0.604) + 0.1 = \boxed{0.402}$$

$$h_1 = \tanh(0.5 \times 1 + 0 + 0.2) = \tanh(0.7) = 0.604$$

For $t = 2$

$$y_2 = (0.5)(0.716) + 0.1 = \boxed{0.458}$$

$$h_2 = \tanh(0.5 \times 0.8 + 0.5 \times 0.604 + 0.2) = \tanh(0.4 + 0.3 + 0.2) = 0.716$$

For $t = 3$

$$y_3 = (0.5)(0.858) + 0.1 = \boxed{0.529}$$

$$h_3 = \tanh(0.5 \times 0.6 + 0.5 \times 0.716 + 0.2) = \tanh(0.3 + 0.358 + 0.2) = 0.858$$

Loss (MSE):

$$L_1 = \frac{1}{2}(y_1 - \hat{y}_1)^2 = \frac{1}{2}(0.5 - 0.402)^2 = 4.8 \times 10^{-3}$$

$$L_2 = \frac{1}{2}(0.6 - 0.458)^2 = 0.01$$

$$L_3 = \frac{1}{2}(0.7 - 0.529)^2 = 0.01$$

$$L = L_1 + L_2 + L_3 = \boxed{0.024}$$

$\boxed{\text{BACK PROPAGATION}}$

PARAMS : V, U, W, b, c

$\boxed{\text{At time } t = 3}$

$$\hat{y}_3 = Vh_3 + c$$

$$h_3 = \tanh(Wx_3 + Uh_2 + b)$$

$\boxed{\text{First cal for } V}$: Because V has no dependencies on previous hidden states

$$\frac{\partial L_3}{\partial V} = \frac{\partial L_3}{\partial y_3} \frac{\partial y_3}{\partial V} \cancel{\frac{\partial h_3}{\partial h}}$$

$$\frac{\partial}{\partial V}(Vh_3 + c)$$

$$\frac{\partial y_3}{\partial V} = h_3 = \boxed{0.858}$$

$$\frac{\partial L_3}{\partial y_3} = \frac{\partial}{\partial \hat{y}_3} \frac{1}{2}(y - \hat{y})^2$$

$$= (y - \hat{y})(-1)$$

$$= \hat{y} - y$$

$$= 0.529 - 0.7$$

$$\frac{\partial L_3}{\partial y_3} = \boxed{-0.171}$$

$$\frac{\partial L_3}{\partial V} = 0.858 \times -0.171$$

$$= -0.146$$

$$\boxed{\frac{\partial L_3}{\partial V} = -0.146}$$

Date______20___

Cal For c

$$\frac{\partial L_3}{\partial c} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial c} = (-0.171)\left(\frac{\partial y_3}{\partial c}\right)$$

Now,

$$\frac{\partial y_3}{\partial c} = \frac{\partial}{\partial c}(Vh_3 + c) = 1$$

$$\boxed{\frac{\partial L_3}{\partial c} = -0.171}$$

Cal For U

① — $\frac{\partial L_3}{\partial U} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial h_3}\frac{\partial h_3}{\partial U}$ $\qquad \frac{\partial L_3}{\partial y_3} = -0.171$

② - $\frac{\partial h_3}{\partial U} = \frac{\partial h_3}{\partial U} + \frac{\partial h_3}{\partial h_2}\frac{\partial h_2}{\partial U}$ $\qquad \frac{\partial y_3}{\partial h_3} = V = 0.5$

$$= \cancel{\frac{\partial h_3}{\partial U} + \frac{\partial h_3}{\partial h_2}}\Big[$$

$$\frac{\partial h_2}{\partial U} = \frac{\partial h_2}{\partial U} + \frac{\partial h_2}{\partial h_1}\frac{\partial h_1}{\partial U} \longrightarrow \frac{\partial}{\partial U}\left(\tanh(Wx_1 + Uh_0 + b)\right)$$

$$= \text{sech}^2(Wx_1 + Uh_0 + b)\,h_0$$

$$\frac{\partial h_2}{\partial U} = \text{sech}^2(0.4 + 0.3 + 0.1)(0.604) \qquad \frac{\partial h_1}{\partial U} = 0$$

$$= (0.55)(0.604)$$

$$\frac{\partial h_2}{\partial U} = \boxed{0.337}$$

ⓐ

$$\frac{\partial h_2}{\partial U} = \text{sech}^2(Wx_2 + Uh_1 + c)\,h_1$$

$$= \text{sech}^2(0.5\times 0.8 + 0.5\times 0.6 + 0.1)(0.604)$$

$$\frac{\partial h_3}{\partial h_2} = \frac{\partial}{\partial h_2}\left(\tanh\left(W\cancel{x}_3 + Uh_2 + b\right)\right)$$

$$= \operatorname{sech}^2\left(wx_3 + Uh_2 + b\right) \times U$$

$$= \operatorname{sech}^2\left(0.5 \times 0.6 + 0.5 \times 0.716 + 0.2\right) \times 0.5$$

$$= \operatorname{sech}^2\left(0.3 + 0.358 + 0.2\right) \times 0.5$$

$$= \operatorname{sech}^2(0.758) \times 0.5$$

$$= 0.59 \times 0.5$$

$$\boxed{\frac{\partial h_3}{\partial h_2} = 0.29} \quad — \quad (b)$$

$$\frac{\partial h_3}{\partial u} = \frac{\partial}{\partial u}\left(\tanh\left(wx_3 + uh_2 + b\right)\right)$$

$$= \operatorname{sech}^2\left(wx_3 + uh_2 + c\right) \times h_2$$

$$= 0.59 \times 0.716$$

$$\boxed{\frac{\partial h_3}{\partial u} = 0.422} \quad — \quad (c)$$

Now, $\dfrac{\partial h_3}{\partial U} = \dfrac{\partial h_3}{\partial u} + \dfrac{\partial h_3}{\partial h_2}\dfrac{\partial h_2}{\partial u}$

Putting values a, b, c

$$= 0.422 + (0.29)(0.337)$$

$$= 0.422 + 0.09$$

$$\boxed{\frac{\partial h_3}{\partial U} = 0.51}$$

CamScanner

Date ______ 20___

Now,

$$\frac{\partial L_3}{\partial U} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial h_3}\frac{\partial h_3}{\partial U}$$

$$= (-0.171)(0.5)(0.51) = -0.04$$

$$\boxed{\frac{\partial L_3}{\partial U} = -0.04}$$

$\boxed{\text{Cal For W}}$

$$\frac{\partial L_3}{\partial W} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial h_3}\frac{\partial h_3}{\partial W}$$

$$\frac{\partial h_3}{\partial W} = \frac{\partial h_3}{\partial W} + \frac{\partial h_3}{\partial h_2}\frac{\partial h_2}{\partial W}$$

$$\frac{\partial h_2}{\partial W} = \frac{\partial h_2}{\partial W} + \frac{\partial h_2}{\partial h_1}\frac{\partial h_1}{\partial W}$$

$$\frac{\partial h_1}{\partial W} = \frac{\partial}{\partial W}\left(\tanh(wx_1 + uh_0 + b)\right)$$

$$= \operatorname{sech}^2(wx_1 + uh_0 + b)x_1$$

$$= \operatorname{sech}^2(0.5 + 0 + 0.1)(1)$$

$$\boxed{\frac{\partial h_1}{\partial W} = 0.71}$$

$$\frac{\partial h_2}{\partial h_1} = \frac{\partial}{\partial h_1}\left(\tanh(wx_2 + uh_1 + b)\right)$$

$$= \operatorname{sech}^2(wx_2 + uh_1 + c)\cdot u$$

$$\frac{\partial h_2}{\partial h_1} = \operatorname{sech}^2(0.4 + 0.3 + 0.1)\cdot 0.5 \boxed{= 0.27}$$

$$\frac{\partial h^2}{\partial W} = \frac{\partial}{\partial W}\left(\tanh\left(Wx_2 + Uh_1 + b\right)\right)$$

$$= \operatorname{sech}^2\left(Wx_2 + Uh_1 + b\right) x_2$$

$$= \operatorname{sech}^2\left(0.4 + 0.3 + 0.2\right) \times 0.8$$

$$= 0.55 \times 0.8 = \boxed{0.44}$$

$$\frac{\partial h^2}{\partial W} = 0.44 + 0.27 \times 0.71 = \boxed{0.63}$$

$$\frac{\partial h_3}{\partial h_2} = 0.29$$

$$\frac{\partial h_3}{\partial W} = \frac{\partial}{\partial W}\left(\tanh\left(Wx_3 + Uh_2 + b\right)\right)$$

$$= 0.59 \times 0.6 = \boxed{0.354}$$

$$\frac{\partial h_3}{\partial W} = \frac{\partial h_3}{\partial W} + \frac{\partial h_3}{\partial h_2}\frac{\partial h_2}{\partial W}$$

$$\boxed{\frac{\partial h_3}{\partial W} = 0.354 + 0.29 \times 0.63 = 0.536}$$

$$\frac{\partial L_3}{\partial W} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial h_3}\frac{\partial h_3}{\partial W}$$

$$= (-0.171)(0.5)(0.536)$$

$$\boxed{\frac{\partial L_3}{\partial W} = -0.04}$$

Date______20___

$\boxed{\text{Cal for } b}$

$$\frac{\partial L_3}{\partial b} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial h_3}\frac{\partial h_3}{\partial b}$$

$$\frac{\partial h_3}{\partial b} = \frac{\partial h_3}{\partial b} + \frac{\partial h_3}{\partial h_2}\frac{\partial h_2}{\partial b}$$

$$\frac{\partial h_2}{\partial b} = \frac{\partial h_2}{\partial b} + \frac{\partial h_2}{\partial h_1}\frac{\partial h_1}{\partial b}$$

$$\frac{\partial h_1}{\partial b} = \frac{\partial}{\partial b}\left(\tanh(wx_1 + uh_0 + b)\right)$$

$$= \text{sech}^2(wx_1 + uh_0 + b) \cdot 1 = \boxed{0.71}$$

$$\frac{\partial h_2}{\partial h_1} = 0.27$$

$$\frac{\partial h_2}{\partial b} = \text{sech}^2(wx_2 + uh_1 + b)1 = \boxed{0.55}$$

$$\frac{\partial h_2}{\partial b} = 0.55 + 0.27 \times 0.71 = 0.55 + 0.19$$

$$\boxed{\frac{\partial h_2}{\partial b} = 0.74}$$

$$\boxed{\frac{\partial h_3}{\partial h_2} = 0.29}$$

$$\frac{\partial h_3}{\partial b} = \text{sech}^2(wx_3 + uh_2 + b)\ 1 = 0.59$$

$$\boxed{\frac{\partial h_3}{\partial b} = 0.59}$$

$$\frac{\partial h_3}{\partial b} = 0.59 + (0.29)(0.74)$$

$$= 0.59 + 0.21 = \boxed{0.8 = \frac{\partial h_3}{\partial b}}$$

$$\frac{\partial L_3}{\partial b} = \frac{\partial L_3}{\partial y_3}\frac{\partial y_3}{\partial h_3}\frac{\partial h_3}{\partial b}$$

$$= (-0.171)(0.5)(0.8)$$

$$\boxed{\frac{\partial L_3}{\partial b} = -0.06}$$

Gradiant of

So, / Losses at $t = 3$:

$$\frac{\partial L_3}{\partial V} = -0.146$$

$$\frac{\partial L_3}{\partial c} = -0.171$$

$$\frac{\partial L_3}{\partial U} = -0.04$$

$$\frac{\partial L_3}{\partial W} = -0.04$$

$$\frac{\partial L_3}{\partial b} = -0.06$$

FOR TIME STEP : $T=2$

$$y_2 = Vh_2 + c$$
$$h_2 = \tanh(Wx_2 + Uh_1 + b)$$

First, cal for V & C

For V

$$\frac{\partial L_2}{\partial V} = \frac{\partial L_2}{\partial y_2}\frac{\partial y_2}{\partial V}$$

$$\frac{\partial y_2}{\partial V} = \frac{\partial}{\partial V}(Vh_2 + c) = h_2 = 0.51$$

$$\frac{\partial L_2}{\partial \hat{y}_2} = \frac{\partial}{\partial \hat{y}_2}\left[\frac{1}{2}(y-\hat{y})^2\right] = \hat{y}_2 - y_2 = 0.458 - 0.6 = -0.14$$

$$\boxed{\frac{\partial L_2}{\partial V} = (0.51)(-0.14) = -0.07}$$

For C

$$\frac{\partial L_2}{\partial c} = \frac{\partial L_2}{\partial y_2}\frac{\partial y_2}{\partial c}$$

$$\frac{\partial y_2}{\partial c} = \frac{\partial}{\partial c}(Vh_2 + c) = 1$$

$$\boxed{\frac{\partial L_2}{\partial c} = (-0.14)(1) = -0.14}$$

For U

$$\frac{\partial L_2}{\partial u} = \frac{\partial L_2}{\partial y_2}\frac{\partial y_2}{\partial h_2}\frac{\partial h_2}{\partial u}$$

$$\frac{\partial h_2}{\partial u} = \frac{\partial h_2}{\partial u} + \frac{\partial h_2}{\partial h_1}\frac{\partial h_1}{\partial u} = 0.337 \quad \text{(From prev calculations)}$$

$$\frac{\partial}{\partial h_2}(Vh_2 + c) = V = \boxed{0.5 = \frac{\partial y_2}{\partial h_2}}$$

$$\frac{\partial L_2}{\partial y_2} = -0.14 \quad \text{(From previous calculations)}$$

So,

$$\frac{\partial L_2}{\partial u} = (-0.14)(0.5)(0.33) = \boxed{-0.23}$$

For W

$$\frac{\partial L_2}{\partial W} = \frac{\partial L_2}{\partial y_2}\frac{\partial y_2}{\partial h_2}\frac{\partial h_2}{\partial W}$$

$$\frac{\partial h_2}{\partial W} = \frac{\partial h_2}{\partial W} + \frac{\partial h_2}{\partial h_1}\frac{\partial h_1}{\partial W} = 0.63 \quad \text{(From prev cal)}$$

$$\frac{\partial L_2}{\partial W} = (-0.14)(0.5)(0.63)$$

$$\boxed{\frac{\partial L_2}{\partial W} = -0.04}$$

$\boxed{\text{For } b}$

$$\frac{\partial L_2}{\partial b} = \frac{\partial L_2}{\partial y_2} \frac{\partial y_2}{\partial h_2} \frac{\partial h_2}{\partial b}$$

$$\frac{\partial h_2}{\partial b} = \frac{\partial h_2}{\partial b} + \frac{\partial h_2}{\partial h_1} \frac{\partial h_1}{\partial b} = 0.74 \quad (\text{Prev cal})$$

$$\frac{\partial L_2}{\partial b} = (-0.14)(0.5)(0.74) = \boxed{-0.05}$$

All gradiants of Losses :

$$\frac{\partial L_2}{\partial V} = -0.07$$

$$\frac{\partial L_2}{\partial c} = -0.14$$

$$\frac{\partial L_2}{\partial U} = -0.23$$

$$\frac{\partial L_2}{\partial W} = -0.04$$

$$\frac{\partial L_2}{\partial b} = -0.05$$

Date_______ 20___

$$\boxed{\text{For TIME STEP : } T = 1}$$

$$y_1 = Vh_1 + c$$
$$h_1 = \tanh(Wx_1 + Uh_0 + b)$$

$$\boxed{\text{For } c}$$

$$\frac{\partial L_1}{\partial c} = \frac{\partial L_1}{\partial y_1}\frac{\partial y_1}{\partial c}$$

$$\frac{\partial L_1}{\partial y_1} = \frac{\partial}{\partial y_1}\left[\frac{1}{2}(y-\hat{y})^2\right] = (\hat{y}_1 - y_1) = 0.402 - 0.5$$
$$= \boxed{-0.102}$$

$$\frac{\partial y_1}{\partial c} = \frac{\partial}{\partial c}(Vh_1 + c) = 1$$

$$\boxed{\frac{\partial L_1}{\partial c} = -0.102}$$

$$\boxed{\text{For } V}$$

$$\frac{\partial L_1}{\partial V} = \frac{\partial L_1}{\partial y_1}\frac{\partial y_1}{\partial V}$$

$$\frac{\partial y_1}{\partial V} = \frac{\partial}{\partial V}(Vh_1 + c) = h_1 = 0.604$$

$$\boxed{\frac{\partial L_1}{\partial V} = (-0.102)(0.604) = -0.06}$$

Date ______ 20__

$\boxed{\text{for } U}$

$$\frac{\partial L_1}{\partial U} = \frac{\partial L_1}{\partial y_1} \frac{\partial y_1}{\partial h_1} \frac{\partial h_1}{\partial U}$$

$$\frac{\partial h_1}{\partial U} = \frac{\partial}{\partial U}\left(\tanh\left(Wx_1 + Uh_0 + b\right)\right)$$

$$= \text{sech}^2\left(Wx_1 + Uh_0 + b\right) h_0$$

$$= 0 \rightarrow \text{since } h_0 = 0$$

$$\frac{\partial L_1}{\partial U} = 0$$

$\boxed{\text{for } W}$

$$\frac{\partial L_1}{\partial W} = \frac{\partial L_1}{\partial y_1} \frac{\partial y_1}{\partial h_1} \frac{\partial h_1}{\partial W}$$

$$\frac{\partial}{\partial W}\left(\tanh\left(Wx_1 + Uh_0 + b\right)\right)$$

$$= \text{sech}^2\left(Wx_1 + Uh_0 + b\right) x_1$$

$$\boxed{= 0.71}$$

$$\frac{\partial}{\partial h_1}\left(Vh_1 + c\right) = V = \boxed{0.5}$$

$$\frac{\partial L_1}{\partial W} = (-0.102)(0.5)(0.71) \boxed{= -0.036}$$

Date______20___

$\boxed{\text{For } b}$

$$\frac{\partial L_1}{\partial b} = \frac{\partial L_1}{\partial y_1} \frac{\partial y_1}{\partial h_1} \frac{\partial h_1}{\partial b} = -0.102 \times 0.5 \times 0.71$$

$$\frac{\partial L_1}{\partial b} = -0.036$$

So now, taking Sum of all Gradients.

$$\frac{\partial L}{\partial c} = -0.102 - 0.14 - 0.171 = -0.413$$

$$\frac{\partial L}{\partial u} = -0.04 - 0.23 + 0 = -0.27$$

$$\frac{\partial L}{\partial w} = -0.036 - 0.04 - 0.04 = -0.116$$

$$\frac{\partial L}{\partial v} = -0.146 - 0.07 - 0.06 = -0.276$$

$$\frac{\partial L}{\partial b} = -0.036 - 0.05 - 0.06 = -0.146$$

Date ______ 20__

Updating Weights of Parameters.

Considering $\eta = 0.1$

$$C_2 = C_1 - \eta \frac{\partial L}{\partial c} = 0.1 - (0.1)(-0.413)$$
$$= 0.1 + 0.041 = \boxed{0.141}$$

$$b_2 = b_1 - \eta \frac{\partial L}{\partial b} = 0.2 - (0.1)(-0.146)$$
$$= 0.2 + 0.014 = \boxed{0.214}$$

$$V_2 = V_1 - \eta \frac{\partial L}{\partial V} = 0.5 - (0.1)(-0.276)$$
$$= 0.5 + 0.027 = \boxed{0.527}$$

$$W_2 = W_1 - \eta \frac{\partial L}{\partial W} = 0.5 - (0.1)(-0.116)$$
$$= 0.5 + 0.011 = \boxed{0.511}$$

$$U_2 = U_1 - \eta \frac{\partial L}{\partial U} = 0.5 - (0.1)(-0.27)$$
$$= 0.5 + 0.027 = \boxed{0.527}$$