NLP

MID TERM EXAM

Q No 1

1. Co-occurrence Matrix C =

	apple	orange	banana
apple	0	1	1
orange	1	0	3
banana	1	3	2

Vocabulary = {apple, orange, banana}

2. SVD

$$C = U \Sigma V^T$$

$$= \begin{bmatrix} -0.58 & 0.58 & -0.58 \\ -0.58 & -0.82 & 0 \\ -0.58 & 0.23 & 0.78 \end{bmatrix} \begin{bmatrix} 4.24 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.58 \end{bmatrix} \begin{bmatrix} -0.58 & -0.58 & -0.58 \\ 0.58 & -0.82 & 0 \\ -0.58 & 0.23 & 0.78 \end{bmatrix}$$

$$= \begin{bmatrix} 0.29 & -0.12 & -0.95 \\ 0.56 & 0.82 & 0.07 \\ 0.77 & -0.56 & 0.31 \end{bmatrix} \begin{bmatrix} 4.58 & 0 & 0 \\ 0 & 2.19 & 0 \\ 0 & 0 & 0.4 \end{bmatrix} \begin{bmatrix} 0.29 & 0.56 & 0.77 \\ 0.12 & -0.82 & 0.56 \\ 0.95 & -0.07 & -0.31 \end{bmatrix}$$

For $k=2$, we have

$$C = \underbrace{\begin{bmatrix} 0.29 & -0.12 \\ 0.56 & 0.82 \\ 0.77 & -0.56 \end{bmatrix}}_{U_k} \underbrace{\begin{bmatrix} 4.58 & 0 \\ 0 & 2.19 \end{bmatrix}}_{\Sigma_k} \underbrace{\begin{bmatrix} 0.29 & 0.56 & 0.77 \\ 0.12 & -0.82 & 0.56 \end{bmatrix}}_{V_k^T}$$

The 2-dimensional embedding is given as

$$W = U_k \Sigma_k = \begin{bmatrix} 1.3375 & -0.2615 \\ 2.6023 & 1.7965 \\ 3.5337 & -1.224 \end{bmatrix}$$

Q No 2

Co-Occurrence Matrix

$$C = \begin{bmatrix} 0 & 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 1 & 0 \\ 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 1 & 1 & 0 \end{bmatrix} \quad \begin{matrix} \Sigma \\ 4 \\ 4 \\ 5 \\ 4 \\ 3 \end{matrix}$$

Σ : 4 4 5 4 3 | 20

a. The joint probability table is given as:

$$P(w_i, w_j) = \begin{matrix} \\ \text{apple} \\ \text{banana} \\ \text{grape} \\ \text{orange} \\ \text{mango} \end{matrix} \begin{matrix} \begin{matrix} \text{apple} & \text{banana} & \text{grape} & \text{orange} & \text{mango} \end{matrix} \\ \begin{bmatrix} 0 & 2/20 & 1/20 & 0/ & 1/20 \\ 2/20 & 0 & 1/20 & 1/20 & 0 \\ 1/20 & 1/20 & 0 & 2/20 & 1/20 \\ 0 & 1/20 & 2/20 & 0 & 1/20 \\ 1/20 & 0 & 1/20 & 1/20 & 0 \end{bmatrix} \end{matrix} = \begin{bmatrix} 0 & 0.1 & 0.05 & 0 & 0.05 \\ 0.1 & 0 & 0.05 & 0.05 & 0 \\ 0.05 & 0.05 & 0 & 0.1 & 0.05 \\ 0 & 0.05 & 0.1 & 0 & 0.05 \\ 0.05 & 0 & 0.05 & 0.05 & 0 \end{bmatrix}$$

b. Marginal Probabilities

$$P(\text{apple}) = \frac{4}{20} = 0.2 \quad , P(\text{banana}) = \frac{4}{20} = 0.2$$

$$P(\text{grape}) = \frac{5}{20} = 0.25$$

c. PMI

$$P(\text{apple, banana}) = \log_2\left[\frac{P(a,b)}{P(a)P(b)}\right] = \log_2\left[\frac{0.1}{(0.2)(0.2)}\right]$$

$$= 1.3219$$

$$PPMI = (1.3219)$$

$$P(\text{apple, grape}) = \log_2\left[\frac{P(a,g)}{P(a)P(g)}\right] = \log_2\left[\frac{0.05}{(0.2)(0.25)}\right] = 0$$

$$PPMI = 0$$

$$P(\text{banana, grape}) = \log_2\left[\frac{P(b,g)}{P(b)P(g)}\right] = \log_2\left[\frac{0.05}{(0.2)(0.25)}\right] = 0$$

$$PPMI = 0$$

Q No3

a. $z_1 = w_1 x + b_1$, $f_1 = \tanh(z_1)$, $z_2 = w_2 f_1 + b_2$, $f_2 = \tanh(z_2)$

$$\hat{y} = f_2$$

b. Forward Pass

$z_1 = 0.7$, $f_1 = 0.6044$, $z_2 = 0.6209$, $f_2 = 0.5517 = \hat{y}$, $x = 2$, $y = 1$

$$L = \frac{1}{2}(y - \hat{y})^2 = 0.1005$$

c. Backward Pass

$$\frac{\partial L}{\partial f_2} = -\frac{1}{2}(y - f_2) = -0.2241$$

$$\frac{\partial f_2}{\partial w_2} = \frac{\partial}{\partial w_2} \tanh(w_2 f_1 + b_2) = \text{Sech}^2(w_2 f_1 + b_2) \cdot f_1 = 0.4204$$

$$\frac{\partial f_2}{\partial b_2} = \frac{\partial}{\partial b_2} \tanh(w_2 f_1 + b_2) = \text{sech}^2(w_2 f_1 + b_2) = 0.6956$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial f_1}{\partial w_1} \frac{\partial f_2}{\partial f_1} \frac{\partial L}{\partial f_2}$$

$$\frac{\partial f_2}{\partial f_1} = \frac{\partial}{\partial f_1} \tanh(w_2 f_1 + b_2) = \text{Sech}^2(w_2 f_1 + b_2) \cdot w_2 = 0.1391$$

$$\frac{\partial f_1}{\partial w_1} = \frac{\partial}{\partial w_1} \tanh(w_1 x + b_1) = \text{sech}^2(w_1 x + b_1) \cdot x = 1.2695$$

$$\frac{\partial L}{\partial b_1} = \frac{\partial f_1}{\partial b_1} \frac{\partial f_2}{\partial f_1} \frac{\partial L}{\partial f_2}$$

$$\frac{\partial f_1}{\partial b_1} = \frac{\partial}{\partial b_1} \tanh(w_1 x + b_1) = \text{sech}^2(w_1 x + b_1) = 0.6347$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial f_2}{\partial w_2} \frac{\partial L}{\partial f_2} = -0.0942, \quad \frac{\partial L}{\partial b_2} = \frac{\partial f_2}{\partial b_2} \frac{\partial L}{\partial f_2} = -0.1559$$

$$\partial L/\partial w_1 = \frac{\partial f_2}{\partial w_1} \frac{\partial f_2}{\partial f_1} \frac{\partial L}{\partial f_2} = -0.0396, \quad \frac{\partial L}{\partial b_1} = \frac{\partial f_1}{\partial b_1} \frac{\partial f_2}{\partial f_1} \frac{\partial L}{\partial f_2} = -0.0198$$

Weights Update

$$w_1^1 = w_1^0 - \eta \frac{\partial L}{\partial w_1}\Big|_{w_1^0}$$

$$= 0.1 - 0.01(-0.0396)$$

$$= 0.104$$

$$b_1^1 = b_1^0 - \eta \frac{\partial L}{\partial b_1}\Big|_{b_1^0}$$

$$= 0.5 - 0.1(-0.0198)$$

$$= 0.502$$

$$w_2^1 = w_2^0 - \eta \frac{\partial L}{\partial w_2}\Big|_{w_2^0}$$

$$= 0.2 - 0.1(-0.0942)$$

$$= 0.2094$$

$$b_2^1 = b_2^0 - \eta \frac{\partial L}{\partial b_2}\Big|_{b_2^0}$$

$$= 0.5 - 0.1(-0.1559)$$

$$= 0.5156$$

$y =$