

ASSIGNMENT #1

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Question #1

Find the value of $f_{yxxz}(x, y, z)$, if $f(x, y, z) = x^2 y z e^y + xy + y^2 z + z^2 x y^2$

Solution

Given equation,

$$f_{yxxz}(x, y, z) = ?$$

$$f(x, y, z) = x^2 y z e^y + xy + y^2 z + z^2 x y^2$$

Taking partial derivative of function with respect to y ,

$$f_y = \frac{\partial f}{\partial y} = \frac{\partial(x^2 y z e^y + xy + y^2 z + z^2 x y^2)}{\partial y}$$

Considering x and z as constants,

$$f_y = x^2 z [y \cdot e^y(1) + e^y(1)] + x(1) + (2y)(1)z + z^2 x (2y)(1)$$

$$f_y = x^2 z [y e^y + e^y] + x + 2yz + 2x y z^2$$

Taking partial derivative of f_y with respect to x ,

$$f_{yx} = \frac{\partial f_y}{\partial x} = \frac{\partial(x^2 z [y e^y + e^y] + x + 2yz + 2x y z^2)}{\partial x}$$

Considering y and z as constants,

$$f_{yx} = z [y e^y + e^y] (2x) + 1 + 0 + 2y z^2$$

$$f_{yx} = 2xz (y e^y + e^y) + 1 + 2y z^2$$

Taking partial derivative of f_{yx} with respect to x ,

$$f_{yxx} = \frac{\partial f_{yx}}{\partial x} = \frac{\partial(2xz (y e^y + e^y) + 1 + 2y z^2)}{\partial x}$$

$$f_{yxx} = 2z (y e^y + e^y)$$

Taking partial derivative of f_{yxx} with respect to z ,

$$f_{yxxz} = \frac{\partial f_{yxx}}{\partial z} = \frac{\partial(2z (y e^y + e^y))}{\partial z}$$

$$f_{yxxz} = 2(y e^y + e^y)$$

Answer

$$f_{yxxz} = 2(y e^y + e^y)$$

Question #2

Use dot product to find the angle between the following vectors:

$$\vec{a} = -3j + 4k, \vec{b} = 3i + \sqrt{5}j + \sqrt{2}k$$

Solution

Given equation,

$$\vec{a} = -3j + 4k, \vec{b} = 3i + \sqrt{5}j + \sqrt{2}k$$

By using dot product formula,

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\vec{a} \cdot \vec{b} = (0i + 3j + 4k) \cdot (3i + \sqrt{5}j + \sqrt{2}k)$$

$$\vec{a} \cdot \vec{b} = -3\sqrt{5} + 4\sqrt{2}$$

Taking magnitude of a ,

$$|a| = \sqrt{(0)^2 + (-3)^2 + (4)^2}$$

$$|a| = \sqrt{0 + 9 + 16}$$

$$|a| = \sqrt{25}$$

$$|a| = 5$$

Similarly, taking magnitude of b ,

$$|b| = \sqrt{(3)^2 + (\sqrt{5})^2 + (\sqrt{2})^2}$$

$$|b| = \sqrt{9 + 5 + 2}$$

$$|b| = \sqrt{16}$$

$$|b| = 4$$

Finally,

$$\vec{a} \cdot \vec{b} = |\vec{a}| |\vec{b}| \cos \theta$$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = \cos \theta$$

$$\frac{-3\sqrt{5} + 4\sqrt{2}}{(5)(4)} = \cos \theta$$

$$\frac{-3\sqrt{5} + 4\sqrt{2}}{20} = \cos \theta$$

$$\cos^{-1} \left(\frac{-3\sqrt{5} + 4\sqrt{2}}{20} \right) = \theta$$

$$\cos^{-1}(-0.052) = \theta$$

Answer

$$\theta = 1.62338$$

$$\theta = 93^\circ$$