Lecture #30

Question #01 – Determine whether the differential is exact or not?

$dz = 4xy^3dx + 6x^2y^2dy$

Solution:

$P = 4xy^3$
$Q = 6x^2y^2$

taking partial derivative of both,

$\frac{\partial P}{\partial y} = 4x(3y^2)$
$\frac{\partial P}{\partial y} = \boxed{12xy^2}$

$\frac{\partial Q}{\partial x} = 6(2x)y^2$
$\frac{\partial Q}{\partial x} = \boxed{12xy^2}$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}\ exact\ differential$

Question #02 – Determine whether the differential is exact or not?

$dz = (5x + 9xy)dx + 5x^3dy$

Solution:

$P = 5x + 9xy$
$Q = 5x^3$

taking partial derivative of both,

$\frac{\partial P}{\partial y} = 5x + 9xy$
$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}5x + \frac{\partial}{\partial y}9xy$
$\frac{\partial P}{\partial y} = 0 + 9x\frac{\partial}{\partial y}y$
$\frac{\partial P}{\partial y} = 0 + 9x(1)$
$\frac{\partial P}{\partial y} = \boxed{9x}$

$\frac{\partial Q}{\partial x} = 5x^3$
$\frac{\partial Q}{\partial x} = 5(3x^2)$
$\frac{\partial Q}{\partial x} = \boxed{15x^2}$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}\ not\ exact\ differential$

Question #03 – Determine whether the differential is exact or not? If so, find z?

$dz = 3x(xy - 2)dx + (x^3 + 2y)dy$

Solution:

$P = 3x(xy - 2)$
$Q = x^3 + 2y$

taking partial differential of both,

$\frac{\partial P}{\partial y} = 3x^2y - 6x$
$\frac{\partial P}{\partial y} = \frac{\partial}{\partial y}3x^2y - \frac{\partial}{\partial y}6x$
$\frac{\partial P}{\partial y} = 3x^2\frac{\partial}{\partial y}y - 0$
$\frac{\partial P}{\partial y} = \boxed{3x^2}$

$\frac{\partial Q}{\partial x} = x^3 + 2y$
$\frac{\partial Q}{\partial x} = \frac{\partial}{\partial x}x^3 + \frac{\partial}{\partial x}2y$
$\frac{\partial Q}{\partial x} = 3x^2 + 0$
$\frac{\partial Q}{\partial x} = \boxed{3x^2}$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ $exact\ differential$

taking indefinite integrals of both,

$z = \int Pdx$
$z = \int 3x(xy - 2)dx$
$z = \int (3x^2y - 6x)dx$
$z = \int 3x^2y\,dx - \int 6x\,dx$
$z = 3y\int \frac{x^{2+1}}{3} - 6\int \frac{x^{1+1}}{2}dx$
$z = \frac{3yx^3}{3} - \frac{6x^2}{2}$
$z = \boxed{x^3y - 3x^2 + C}$

$z = \int Qdy$
$z = \int (x^3 + 2y)\,dy$
$z = \int x^3dy + \int 2y\,dy$
$z = x^3\int \frac{y^{0+1}}{1}dy + 2\int \frac{y^{1+1}}{2}dy$
$z = x^3y + 2\frac{y^2}{2} + C$
$z = \boxed{x^3y + y^2 + C}$

comparing right-hand side of both equation,

$C = -3x^2$
$C = y^2$
$z = \boxed{x^3y - 3x^2 + y^2}$ $Answer$