Lecture #23

Question #01 – Find polar co-ordinates of the given rectangular co-ordinates point?

$(3\sqrt{2}, -3\sqrt{2})$

Solution:

$x = 3\sqrt{2}$
$y = -3\sqrt{2}$

converting into polar co-ordinates,
$P(r, \theta)$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{\left(3\sqrt{2}\right)^2 + \left(-3\sqrt{2}\right)^2}$

$r = \sqrt{18 + 18}$

$r = \sqrt{36}$

$r = \boxed{\pm 6}$

$\tan \theta = \dfrac{y}{x}$

$\tan \theta = \dfrac{-3\sqrt{2}}{3\sqrt{2}}$

$\tan \theta = -1$

$\theta = \tan^{-1} -1$

$\theta = 180 - 45$

$\theta = \boxed{135°}$

$\theta = 135° \cdot \dfrac{\pi}{180}$

$\theta = 135° \cdot \dfrac{\pi}{180}$

$\theta = 3 \cdot \dfrac{\pi}{4}$

$\theta = \boxed{\dfrac{3\pi}{4}}$

$\theta = \dfrac{3\pi}{4} + \pi$

$\theta = \boxed{\dfrac{7\pi}{4}}$

$P\left(-6, \dfrac{3\pi}{4}\right) \; and \; \left(6, \dfrac{7\pi}{4}\right) \; Answer$
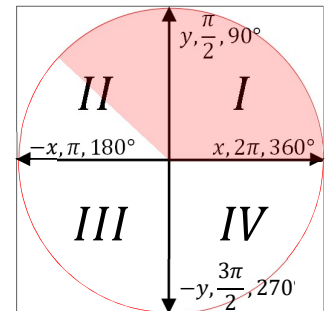
Question #02 – Find polar co-ordinates of the given rectangular co-ordinates point?

$(2, -2)$

Solution:

$x = 2$
$y = -2$

converting into polar co-ordinates,
$P(r, \theta)$

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(2)^2 + (-2)^2}$

$r = \sqrt{4 + 4}$

$r = \sqrt{8}$

$r = \sqrt{4 \cdot 2}$

$r = \boxed{\pm 2\sqrt{2}}$

$\tan \theta = \dfrac{y}{x}$

$\tan \theta = \dfrac{-2}{2}$

$\theta = \tan^{-1} -1$

$\theta = \boxed{135°}$

$\theta = 135° \cdot \dfrac{\pi}{180}$

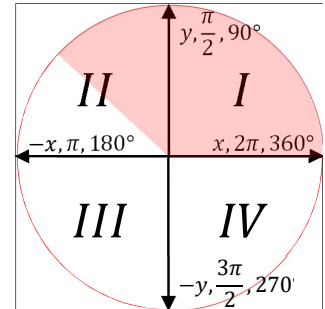$\theta = 3 \cdot \dfrac{\pi}{4}$

$\theta = \boxed{\dfrac{3\pi}{4}}$

$\theta = \dfrac{3\pi}{4} \cdot \pi$

$\theta = \boxed{\dfrac{7\pi}{4}}$

$P\left(-2\sqrt{2}, \dfrac{3\pi}{4}\right)$ and $\left(2\sqrt{2}, \dfrac{7\pi}{4}\right)$ *Answer*

Question #03 – Express the equation $x^2 = 8y$ into polar co-ordinates?

Solution:

using the formula,
$x = r \cos \theta$ and $y = r \sin \theta$

$(r \cos \theta)^2 = 8r \sin \theta$
$r^2 \cos^2 \theta = 8r \sin \theta$
$r \cos^2 \theta = 8 \sin \theta$
$r = \dfrac{8 \sin \theta}{\cos^2 \theta} = \boxed{8 \sec \theta \tan \theta}$ *Answer*

Question #04 – Covert the point $(2, \pi/3)$ from polar to cartesian co-ordinates?

Solution:

$P(r, \theta) = (2, \pi/3)$

using the formula,
$x = r \cos \theta \ and \ y = r \sin \theta$

$x = 2 \cos \dfrac{\pi}{3} = 2 \cdot \dfrac{1}{2} = \boxed{1}$

$y = 2 \sin \dfrac{\pi}{3} = 2 \cdot \dfrac{\sqrt{3}}{2} = \boxed{\sqrt{3}}$

$Therefore, \left(1, \sqrt{3}\right) \ Answer$