Lecture #29

Question #01 – Write expression of the arc-length of the curve by the vector function?

$\vec{r}(t) = (5t - \ln 3t)\hat{\imath} + (t^2 + 7)\hat{\jmath}\ where\ 0 \le t \le 4$

Solution:

$\frac{dx}{dt} = \frac{d}{dt}5t - \frac{d}{dt}\ln 3t$

$\frac{dx}{dt} = 5 - \frac{1}{3t}\cdot\frac{d}{dt}3t$

$\frac{dx}{dt} = 5 - \frac{1}{3t}\cdot 3$

$\frac{dx}{dt} = \boxed{5 - \frac{3}{3t}}$

$\frac{dy}{dt} = \frac{d}{dt}t^2 + \frac{d}{dt}7$

$\frac{dy}{dt} = 2t + 0$

$\frac{dy}{dt} = \boxed{2t}$

using arc-length formula,

$$Arc\ length = \int_0^4 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\ dt}$$

$$Arc\ length = \int_0^4 \sqrt{\left(5 - \frac{3}{3t}\right)^2 + (2t)^2 dt}$$

Question #02 – Find the arc-length of the curve?

$\vec{r}(t) = 2\sin t\,\hat{\imath} + 2\cos t\,\hat{\jmath}\ where\ 0 \le t \le 2\pi$

Solution:

$\frac{dx}{dt} = 2\frac{d}{dt}\sin t$

$\frac{dx}{dt} = \boxed{2\cos t}$

$\frac{dy}{dt} = 2\frac{d}{dt}\cos t$

$\frac{dy}{dt} = \boxed{-2\sin t}$

using arc-length formula,

$$Arc\ length = \int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\ dt}$$

$$Arc\ length = \int_0^{2\pi} \sqrt{(2\cos t)^2 + (-2\sin t)^2 dt}$$

$$Arc\ length = \int_0^{2\pi} \sqrt{4\cos^2 t + 4\sin^2 t\ dt}$$

$$Arc\ length = \int_0^{2\pi} \sqrt{4(\cos^2 t + \sin^2 t)dt}$$

$$Arc\ length = \int_0^{2\pi} 2\sqrt{(\cos^2 t + \sin^2 t)dt}$$

$$Arc\ length = \int_0^{2\pi} 2\sqrt{(1)dt}$$

$$Arc\ length = \int_0^{2\pi} 2dt$$

$$Arc\ length = 2\int_0^{2\pi} \frac{t^{0+1}}{1}dt$$

$$Arc\ length = \left[\frac{2t}{1}\right]_0^{2\pi}$$

$$Arc\ length = \boxed{4\pi}\ Answer$$

Question #03 – Find the arc-length of the given curve?

$\vec{r}(t) = (2 + 5t)\hat{\imath} + (3 - 6t)\hat{\jmath}\ where\ 2 \leq t \leq 3$

Solution:

$\frac{dx}{dt} = \frac{d}{dt}2 + \frac{d}{dt}5t$
$\frac{dx}{dt} = 0 + 5(1)$
$\frac{dx}{dt} = \boxed{5}$

$\frac{dx}{dt} = \frac{d}{dt}3 - \frac{d}{dt}6t$
$\frac{dx}{dt} = 0 - 6(1)$
$\frac{dx}{dt} = \boxed{6}$

using arc-length formula,

$Arc\ length = \int_2^3 \sqrt{(5)^2 + (6)^2 dt}$

$Arc\ length = \int_2^3 \sqrt{25 + 36\ dt}$

$Arc\ length = \int_2^3 \sqrt{61\ dt}$

$Arc\ length = \sqrt{61}\int_2^3 dt$

$Arc\ length = \sqrt{61}\int_2^3 \frac{t^{0+1}}{1}$

$Arc\ length = \sqrt{61} \cdot \left[\frac{t}{1}\right]_2^3$
$Arc\ length = \sqrt{61} \cdot [3 - 2]$
$Arc\ length = \sqrt{61} \cdot [1]$
$Arc\ length = \boxed{\sqrt{61}}\ Answer$

Question #04 – Find the intersecting points of the curve?

$y = x\ and\ y = \sqrt{x}$

Solution:

$y = x \rightarrow eq(i)$
$y = \sqrt{x} \rightarrow eq(ii)$

comparing equation (i) and equation (ii),
$x = \sqrt{x}$

taking square both the sides,
$x^2 = x$
$x^2 - x = 0$
$x(x - 1) = 0$
$x = 0\ and\ x - 1 = 0$
$Answer \rightarrow \boxed{x = 0}\ \boxed{x = 1}$

Question #05 – Given the equation of two curves, find the intersecting point of curves?

$y = -6 - x^2 \; and \; y = 4 - 7x$

Solution:

$y = -6 - x^2 \rightarrow eq(i)$
$y = 4 - 7x \rightarrow eq(ii)$

comparing equation (i) and equation (ii),
$-6 - x^2 = 4 - 7x$
$x^2 - 7x + 6 + 4 = 0$
$x^2 - 7x + 10 = 0$

using middle term break method,
$x^2 - 2x - 5x + 10 = 0$
$x(x-2) - 5(x-2) = 0$
$(x-2)(x-5) = 0$
$x - 2 = 0 \; and \; x - 5 = 0$
$Answer \; \rightarrow \; \boxed{x = 2} \boxed{x = 5}$

using quadratic equation formula,
$x^2 - 7x + 10 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)}$$

$$x = \frac{7 \pm \sqrt{49 - 40}}{2}$$

$$x = \frac{7 \pm \sqrt{9}}{2}$$

$$x = \frac{7 \pm 3}{2}$$

either,

$$x = \frac{7 - 3}{2}$$

$$x = \frac{4}{2}$$

$Answer \; \boxed{x = 2}$

or,

$$x = \frac{7 + 3}{2}$$

$$x = \frac{10}{2}$$

$Answer \; \boxed{x = 5}$

Question #06 – Given the equation of two curves, find the intersecting points of curve?

$y = 4 - x \; and \; y = 4 - x^2$

Solution:

$y = 4 - x \; \rightarrow eq(i)$
$y = 4 - x^2 \; \rightarrow eq(ii)$

comparing equation (i) and equation (ii),
$4 - x = 4 - x^2$
$x^2 - x - 4 + 4 = 0$
$x^2 - x = 0$
$x(x - 1) = 0$
$x = 0 \; and \; x - 1 = 0$
$Answer \; \rightarrow \boxed{x = 0} \boxed{x = 1}$