Q1 $4\underline{i}+4\underline{j}+\underline{k}, -4\underline{i}+3\underline{j}-4\underline{k}, 4\underline{i}-\underline{j}-2\underline{k}$

a) We have $A(4,-4,1)$, $B(-4,3,-4)$ $C(4,-1,-2)$

$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$

$= \begin{bmatrix} -4 \\ 3 \\ -4 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -8 \\ 7 \\ -5 \end{bmatrix}$

$= -8\underline{i} + 7\underline{j} - 5\underline{k}$

$\overrightarrow{AC} = OC - OA$

$= \begin{bmatrix} 4 \\ -1 \\ -2 \end{bmatrix} - \begin{bmatrix} 4 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ -3 \end{bmatrix} = 3\underline{j} - 3\underline{k}$

$n = \overrightarrow{AB} \times \overrightarrow{AC}$

$= \begin{vmatrix} i & j & k \\ -8 & 7 & -5 \\ 0 & 3 & -3 \end{vmatrix} = (-21+15)\underline{i} - (24)\underline{j} + (-24)\underline{k}$

$= -6\underline{i} - 24\underline{j} - 24k$

$= \underline{i} + 4\underline{j} + 4\underline{k}$

$d = a \cdot n = (4\underline{i} - 4\underline{j} + \underline{k}) \cdot (\underline{i} + 4\underline{j} + 4\underline{k})$

$= 4 - 16 + 4 = -8$

Equation of plane

$r \cdot n = d$

$r \cdot (\underline{i} + 4\underline{j} + 4\underline{k}) = -8$

$(x\underline{i} + y\underline{j} + z\underline{k}) \cdot (\underline{i} + 4\underline{j} + 4\underline{k}) = -8$

$x + 4y + 4z = -8$

$x + 4y + 4z + 8 = 0$

b)

Per distance $= \frac{d}{|n|} = \frac{8}{\sqrt{1 + 4^2 + 4^2}}$

$= 1.39$

c)

lin OD : $r = a + \lambda b$

$r = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -3 \\ -3 \end{pmatrix}$

Some value of λ

$\begin{pmatrix} 2\lambda \\ 3\lambda \\ -3\lambda \end{pmatrix} - \begin{pmatrix} 1 \\ 4 \\ 4 \end{pmatrix} = -8$

$2\lambda + 12\lambda - 12\lambda = -8$

$2\lambda = -8$

$\lambda = -4$

$r = \begin{pmatrix} 2(-4) \\ 3(-4) \\ -3(-4) \end{pmatrix} = (-8, -12, 12)$

Q3

(a) $AB = 4i - 1j + 1k$

$CD = 0i - j + (3-\lambda)k$

l_1 : line $AB = OA + \lambda AB$

l_2 : line $CD = OC + \mu CD$

$$l_1 : \vec{r_1} = \begin{bmatrix} 97 \\ 4 \\ 1 \end{bmatrix} + \lambda \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix}$$

$$l_2 : \vec{r_2} = \begin{bmatrix} 2 \\ 6 \\ 3 \end{bmatrix} + \mu \begin{bmatrix} 0 \\ -1 \\ 3-\lambda \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_1 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 0 & -1 & 3-\lambda \end{vmatrix}$$

$$b_1 \times b_2 = i(-3+\lambda) - j(12 - 4\lambda) + k(-4)$$

$$|b_1 \times b_2| = \sqrt{\lambda^2 + 9 - 2\lambda + 144 + 16\lambda^2 - 24\lambda + 16}$$

$$= \sqrt{17\lambda^2 - 26\lambda + 169}$$

$$a_2 - a_1 = \begin{bmatrix} 7 \\ 4 \\ -1 \end{bmatrix} - \begin{bmatrix} 2 \\ 6 \\ -3 \end{bmatrix}$$

$$= \begin{bmatrix} 5 \\ -2 \\ 2 \end{bmatrix}$$

$$3 = \frac{(-3+\lambda)5 + 2(12-4\lambda) - 2(4)}{\sqrt{17\lambda^2 - 26\lambda + 169}}$$

$$9(17\lambda^2 - 26\lambda + 169) = (-15 + 5\lambda + 24 - 8\lambda - 8)^2$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda + 24 - 23)^2$$

$$9(17\lambda^2 - 26\lambda + 169) = (7\lambda - 1)^2$$

$$\lambda = 49\lambda^2 + 1 - 24$$

then $\cancel{\lambda}\lambda^2 - 5\lambda + 4 = 0$

b)

Plane ABD when $\lambda = 1$

so $D = 2i + 7j + k$

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ -5 & 3 & 2 \end{vmatrix}$$

$$= i(-2-3) - j(8+5) + k(12-5)$$

$$= -5i - 13j + 7k$$

$$eq = -5(2) - 13(7)s + 7(1)t$$

$$= -10 - 91s + 7t$$

for plane π_2, when $n = 4$

$$AB \times AD = \begin{vmatrix} i & j & k \\ 4 & -1 & 1 \\ 5 & 3 & 5 \end{vmatrix}$$

$$= -8i - 15j + 17k$$

$$= -8(x-11) - 15(y-3) + 17(z-0)$$

$$= 8x + 15y - 17z - 133$$

(c)

$$\theta = \cos^{-1}\left[\frac{|n_1 \cdot n_2|}{|n_1||n_2|}\right]$$

$$n_1 \cdot n_2 = 4 + 195 + 19$$

$$\theta = \cos^{-1} \frac{318}{\sqrt{25+169+9} \cdot \sqrt{64+225+289}}$$

$$\theta = \cos^{-1} \frac{318}{374.56}$$

$$\theta = 31.89^{\circ}$$

Q3

$l_1 = t\hat{i} + \hat{j}$ $\quad -2\underline{i} - \underline{j}$

$l_2 = \hat{j} + k$ $\quad -2\underline{j} + k$

The shortest distance b/w l_1 and l_2 is $\sqrt{21}$

$r_1 = OA + \lambda AB$

$r_2 = OA + \lambda AB$

$r_1 = t\hat{i} + \hat{j} + \lambda 2\hat{i} - \hat{j}$

$$L_1 = \vec{r_1} = \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix} + \lambda \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix}$$

$$L_2 = \vec{r_2} = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} + u \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$$

$$D = \frac{(b_1 \times b_2) \cdot (a_2 - a_1)}{|b_1 \times b_2|}$$

$$b_1 \times b_2 = \begin{vmatrix} i & j & k \\ -2 & -1 & 0 \\ 0 & -2 & 1 \end{vmatrix}$$

$$= \hat{i}(-1) - \hat{j}(-2) + \hat{k}(4)$$

$$= -\hat{i} + 2\hat{j} + 4\hat{k}$$

$$|b_1 \times b_2| = \sqrt{(-1)^2 + (2)^2 + (4)^2}$$

$$= \sqrt{1 + 4 + 16}$$

$= \sqrt{21}$

$$(a_2 - a_1) = \begin{bmatrix} 0 \\ 1 \\ t \end{bmatrix} - \begin{bmatrix} t \\ 1 \\ 0 \end{bmatrix}$$

$$= -t\hat{i} + t\hat{k}$$

$$D = \frac{(-\hat{i} + 2\hat{j} + 4\hat{k}) \cdot (-t\hat{i} + t\hat{k})}{\sqrt{21}}$$

$$\sqrt{21} = \frac{t + 4t}{\sqrt{21}}$$

$$21 = t + 4t$$

$$5t = 21$$

$$t = \frac{21}{5}$$

(b) $\vec{r_1} = \frac{21}{5}\hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j})$

$\vec{r_2} = \hat{j} - \frac{21}{5}k + \mu(-2\hat{j} + \hat{k})$

$\pi_1 = r = a + \lambda \overline{AB} + \mu \overline{AC}$

$\vec{r} = -\frac{21}{5}\hat{i} + \hat{j} + \lambda(-2\hat{i} - \hat{j}) + \mu(-2\hat{j} + \hat{k})$

(c) $\lambda_2 = 5x - 6y + 5z = 0$

$\lambda_2 = \frac{x_2 - 0}{0} \; 7 \; \lambda_2 = \frac{y - 1}{-2}$

$\lambda_2 = \frac{2 - 4 \cdot 2}{1}$

$$\cos\theta = \frac{a_1b_1 + a_2b_2 + a_3b_3}{|a||b|}$$

$$a \cdot b = \begin{bmatrix} 0 \\ 1 \\ \frac{21}{5} \end{bmatrix} \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$$

$$= -6 + 29.4$$

$$= 23.4$$

$$|a| = \sqrt{(1)^2 + \left(\frac{21}{5}\right)^2}$$

$$= \sqrt{1 + 17.64}$$

$$|a| = 4.3$$

$$|b| = \sqrt{(5)^2 + (6)^2 + (7)^2}$$

$$|b| = 10.49$$

$$\theta = \cos^{-1}\left(\frac{23.4}{4.3 \times 10.49}\right)$$

$$\theta = \cos^{-1}\frac{23.4}{45.11}$$

$$\theta = 59.34^\circ$$

d) $\bar{n}_1 = \begin{bmatrix} -\frac{21}{5} \\ 1 \\ 0 \end{bmatrix}, \bar{n}_2 = \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix}$

$$\bar{n}_1 \cdot \bar{n}_2 = \begin{bmatrix} -\frac{21}{5} \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 5 \\ -6 \\ 7 \end{bmatrix} = -21 - 6 = -27$$

$$|a| = \sqrt{\left(\frac{21}{5}\right)^2 + (1)^2} = \sqrt{17.6441} = 4.3$$

$$|b| = \sqrt{(5)^2 + (-6)^2 + (7)^2} = 10.49$$

$$\theta = \cos^{-1} \frac{-27}{4.3 \times 10.49}$$

$$\theta = 126.78$$

acute angle:-

$$\phi = 180 - 126.78$$

$$\phi = 53.23^{\circ}$$

Q5

(a)

$$= \left(\frac{-2-6}{2}\right), \left(\frac{-1-3}{2}\right)$$

$$= \frac{-8}{2}, \frac{-4}{2}$$

$$= (-4, -2)$$

eq of circle $= (x+h)^2 + (y-k)^2 = r^2$

$$(x+4)^2 + (y+2)^2 = r^2$$

$$(x, y) = (-2, -1)$$

$$(-2+4)^2 + (-1+2)^2 = r^2$$

$$(2)^2 + (1)^2 = r^2$$

$$4+1 = r^2$$

$$r^2 = 5$$

Put the values

$$(x+4)^2 + (y+2)^2 = 5$$

b) eq of circle

$$(x-h)^2 + (y-k)^2 = r^2$$

let

$$r = 0, y = b$$

at point $(4,0)$

$$(4)^2 + (-b)^2 = r^2$$

$$16 + b^2 = r^2 \quad \text{—} \ 1$$

at point $(0,2)$

$$(0)^2 + (2-b)^2 = r^2$$

$$(2-b)^2 = r^2 \quad \text{—} \ 2$$

Compare eq 1 and 2

$$16 + b^2 = (2-b)^2$$

$$16 + b^2 - b^2 - 4 + 4b = 0$$

$$12 + 4b = 0$$

$$4b = -12$$

$$\boxed{b = -3}$$

Put the values in eq 1

So, $r^2 = (4)^2 + (-3)^2$

$$r^2 = 16 + 9$$

$$r = \pm 5$$

$$r = 5$$

(c) $y^2 = 100x$

$y^2 = 100x$

Compare with

$y^2 = 4ax$

$4a = 100$

$a = 25$

eq of directrix $= x = -a$

$x = -25$

(d) $x^2 = 24y$

Compare with

$x^2 = 4ay$

$4a = 24$

$a = 8$

So focus is $F(a,0) = F(0,0)$

and eq of directrix is

$x = -a$

$x = -8$

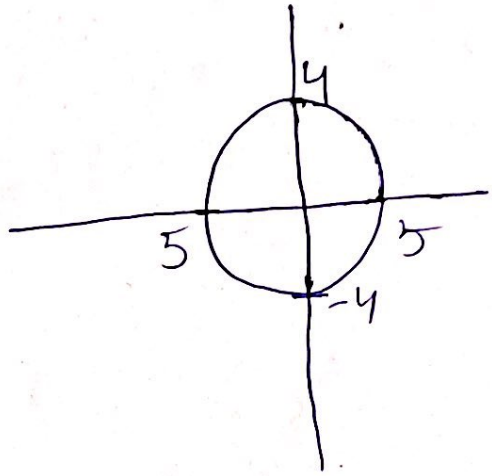
e)

Compare with

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x^2}{5^2} + \frac{y^2}{4^2}$

$a = 5$

$b = 4$



$c = \sqrt{a^2 - b^2} = \sqrt{25 - 16}$

$= \pm 3$

$F_1 = (3, 0)$

$F_2 = (-3, 0)$

length of major axis $= 2a$

$= 2(5) = 10$

f)

major axis $= 10$

minor axis $= 8$

$2a = 10$

$a = 5$

$2b = 8$

$b = 4$

eq of ~~of~~ ellipse along x-axis

$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1$

$\left(\frac{x}{5}\right)^2 + \left(\frac{y}{5}\right)^2 = 1$