National University of Computer and Emerging Sciences, Lahore Campus

Course:	Design & Analysis of Algorithms	Course Code:	CS-2009
Program:	BS (Computer/Data Science)	Semester:	Fall 2023
Duration:	60 Minutes	Total Marks:	18
Paper Date:	10-Nov-23	Section:	ALL
Exam:	Midterm 2	Page(s):	6
Name		Roll Number	

Instruction/Notes: Solve it on question paper

Question	1	2	3	4
Marks	/5	/5	/5	/3

Q1) Consider the problem of rod cutting discussed in class. Dry run dynamic programming algorithm on following input for rod length 7. Show all computations and write values in the given array C[n] for memoization. [5 Marks]

Length (i)	1	2	3	4	5	6	7
Price P[i]	2	5	7	12	13	16	17

C[0] = 0

$$C[n] = \max_{1 \le i \le n}(P[i] + C[n-i])$$

Solution:

n	1	2	3	4	5	6	7
C[n]							

Q2) Consider the following recursive algorithm. [1+4 = 5 Marks]

```
/* m and n are lengths of char arrays X and Y respectively */
int Function( char *X, char *Y, int m, int n )
{
   if (m == 0 || n == 0)
      return 0;
   if (X[m-1] == Y[n-1])
      return 1 + Function (X, Y, m-1, n-1);
   else
      return max (Function (X, Y, m, n-1) +1,  Function (X, Y, m-1, n) +1);
}
```

(a) What is time complexity of above algorithm? Show all working

(b) Convert the recursive code given above into bottom up iterative dynamic programming algorithm. Write its time complexity.

Q3) You are given n integers in the range [1, m] and a target T. Find a subset of the n integers whose sum is as close to T as possible without exceeding it. For example,
T= 10,
Set of n integers = {3, 5, 4}
All possible subsets = {3}, {5}, {4}, {3, 5}, {3, 4}, {5, 4}, {3, 5, 4}
Optimal Solution = subset = {5, 4} // The sum is 9, any other subset sum will either exceed 10 or will be less than 9.

Consider following greedy approach for solving this problem. Sort number in descending order. Start adding numbers in subset starting from largest number. Terminate at the number for which the sum exceeds T. Prove this greedy strategy is not optimal by giving counter example. [5 Marks]

Solution:

T =

Set =

Subset given by the Greedy algorithm =

Subset given by the optimal solution =

Q4) This question is about maximum subarray sum problem. We are given an integer array A[0..n) and we have to find the subarray A[i..j) such that the sum of the integers $A[i] + A[i + 1] + \cdot \cdot \cdot + A[j - 1]$ is maximal. If M[j] holds the maximum sum for any segment A[i..j) ending in $j - 1$ how can we efficiently compute the maximum sum for any segment A[i…...j + 1) ending in j? In other words, write the expression (recursive equation) for defining the value of M[j+1].
[Hint: you do not need a loop or pseudocode.] [3 Marks]