k Heads in n tosses

Binomial Prob Distribution

p : prob of getting heads

$(1-p)$: " " " tails

$$P(n,k) = \binom{n}{k} p^k (1-p)^{n-k}$$

$O(n)$

H _ _ H _ H

H H H

$\binom{5}{3}$ ways of getting 3 heads out of 5 tosses

k Heads in n tosses
Sub-problem definition & Recurrence

→ n coins

→ Each has head prob. p_i : p_1, p_2, ..., p_n

$(1-p_1)$, $(1-p_2)$ . .. , $(1-p_n)$

$\binom{n}{k} \le \binom{n}{n/2} \approx n^{n/2}$

$P[n, k]$

each of the $\binom{4}{3}$ combinations of getting 3 heads in 4 tosses has a diff. prob.

H H H T

$p_1 p_2 p_3 (1-p_4)$

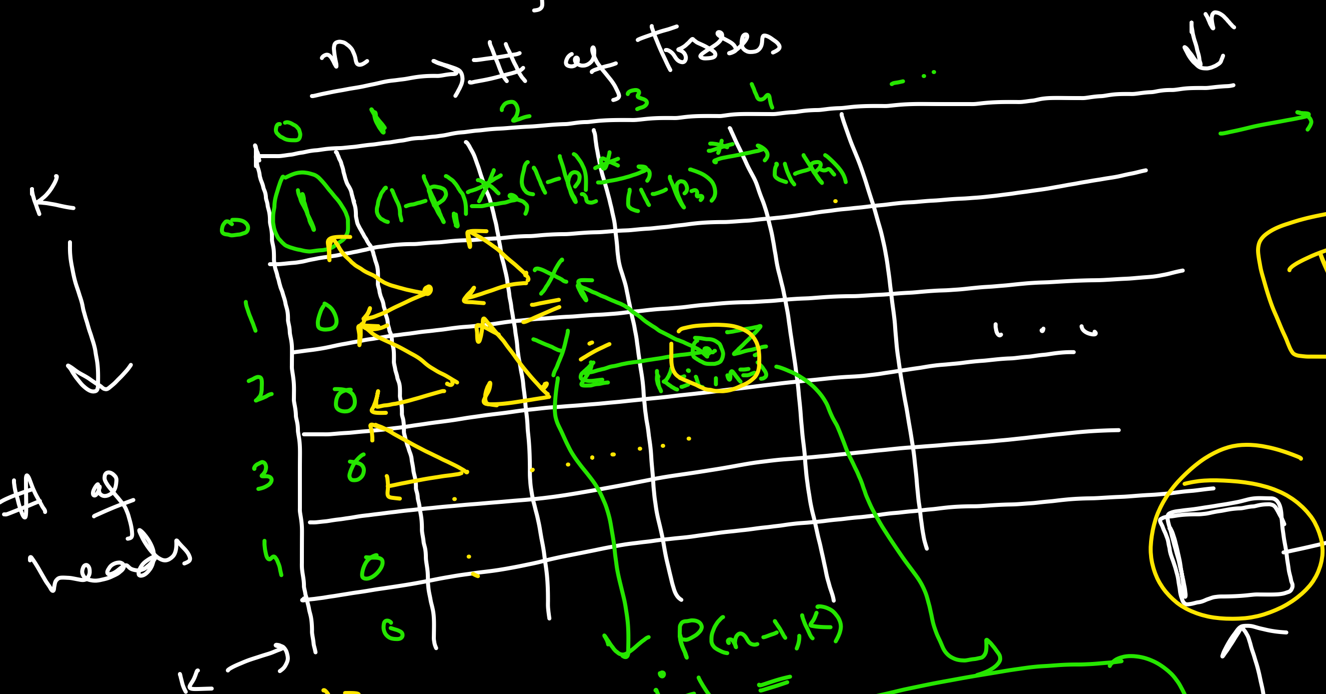
H T H H

$p_1 (1-p_2) p_3 p_4$

k Heads in n tosses
Auxiliary Storage

$n \to$ # of tosses , $k \to$ # of heads

$P(n, k)$



$\to O(n)$ $P[0,0] = 1$

$\boxed{T(n, K) = O(nK)}$

read the ans.

$P(n,K)$ final

$\binom{n}{K} \cdot n$

$Z = X \cdot p_3 + Y \cdot (1-p_3)$

→ nK boxes to fill

→ each single box needs O(1) time

$O(nK)$

Recurrence.

↳ $P[n, K] :=$ prob of K heads in n tosses.

$P[n, K-1]$

$P[n-1, K]$

$P[n-1, K-1]$

$\vdots$

How many sub-problems

$O(n*K)$

$$P[10, 5] = P[9, 5] * (1 - p_5) + P[9, 4] * p_5$$

4 heads / 9 tosses — 1 head / 10^{th} toss

5 heads / 9 tosses — 0 heads / 10^{th} Toss

$P[10, 5]$

$P[10, 4]$

$P[9, 4]$

$P[9, 3]$

$\vdots$

$F[n]$ | $mss[i]$

$lis[i]$ | $h[i]$

$$\boxed{P(n,K) = \underline{P(n-1, K-1) * P_n} + \underline{P(n-1, K) * (1-P_n)}}$$

$$P(1,1) = P_1$$

$$n \quad K$$

$$\boxed{P(0,0) = 1}$$

$$\boxed{\begin{gathered} P(n,K) = 0 \\ K > n \end{gathered}}$$