# Large Int. Multiplication

$n=4$

$x = 12|34$

$= 12 \times 10^2 + 34$

$$\boxed{\begin{aligned} x &= 10^{n/2} x_L + x_R \\ y &= 10^{n/2} y_L + y_R \end{aligned}}$$

## High School Method

$n=4$

$$\begin{array}{rl} 1011 & x \\ \times\, 1121 & y \\ \hline 1011 & \\ 2022\text{X} & \\ 1011\text{XX} & \\ 1011\text{XXX} & \leftarrow \text{shift} \\ \hline \end{array}$$

$T(n) = O(n^2)$

loops
(no rec)

$X = \underbrace{X_L}_{n/2}\,|\,\underbrace{X_R}_{n/2}$

$Y = \underbrace{Y_L}_{n/2}\,|\,\underbrace{Y_R}_{n/2}$

## Version I

$(a+b)(c+d)$

$$xy = (10^{n/2} x_L + x_R)(10^{n/2} y_L + y_R)$$

$$= 10^n x_L y_L + 10^{n/2}(x_L y_R + x_R y_L) + x_R y_R$$

$+$ : $O(n)$ op.

$-$ : $O(n)$ op.

$10^n$ : $O(n)$ op.

Re-write with 3 mults.

$$\left.\begin{aligned} T(n) &= 4T\left(\tfrac{n}{2}\right) + \underset{5}{cn}, \quad T(1) = c \\ &= O(n^2) \end{aligned}\right\} + \text{recursive overhead.}$$

## Version II

$$\begin{aligned} T(n) &= \underline{3T}\left(\tfrac{n}{2}\right) + \underset{8}{c'n}, \quad T(1) = c \\ &= \boxed{O(n^{1.58})} \end{aligned}$$

**See**: the iterative version of MergeSort.

$$\sqrt{n} = n^{1/2}$$

$$\simeq \frac{n^2}{\boxed{\sqrt{n}}}$$

$$\begin{aligned} X \cdot Y &= (10^{n/2} X_L + X_R)(10^{n/2} Y_L + Y_R) \\ &= 10^n X_L Y_L + \boxed{10^{n/2}(X_L Y_R + X_R Y_L)} + X_R Y_R \\ &= 10^n \underline{P_1} + 10^{n/2}\left[(X_L + X_R)(Y_L + Y_R) - P_1 - P_2\right] + \underline{P_2} \end{aligned}$$

$p_1 \leftarrow X_L Y_L$ ✓

$p_2 \leftarrow X_R Y_R$ ✓

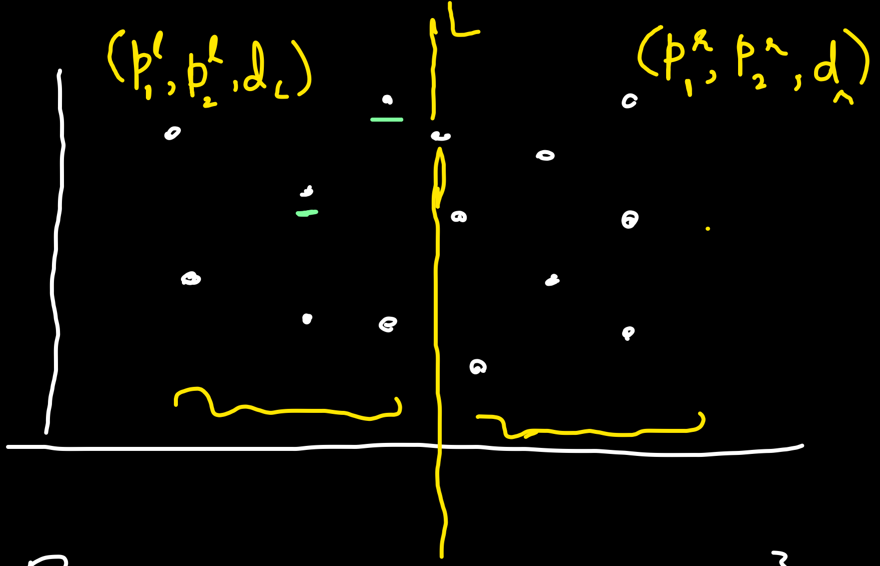
## Karatsuba's Algo

$$\boxed{O(n \lg n)}$$

$$\left.\begin{matrix} 3 + 4i \\ 2 + 5i \end{matrix}\right]$$

$$=$$

# Closest Pair of Points on the XY-plane



$$d(p_1, p_2) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$$

## Naive Algo

test all pairs of points

$\binom{n}{2}$ candidates

$\simeq n^2 = O(n^2)$

→ Pre-sort points on
→ X-coordinates
→ Algo on y-coordinates
→ (in a different array)

$O(n\log n)$

$O(n\log n)$ Algo possible through D&C

**Base Case:** 2 points $T(2) = c$

↪ return their distance.

1 point $T(1) = c$ }

↪ return $\infty$

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**Combine**

$d_l \le d_r \longrightarrow d_l$ is better } so is $d_l$ or $d_r$ necessarily

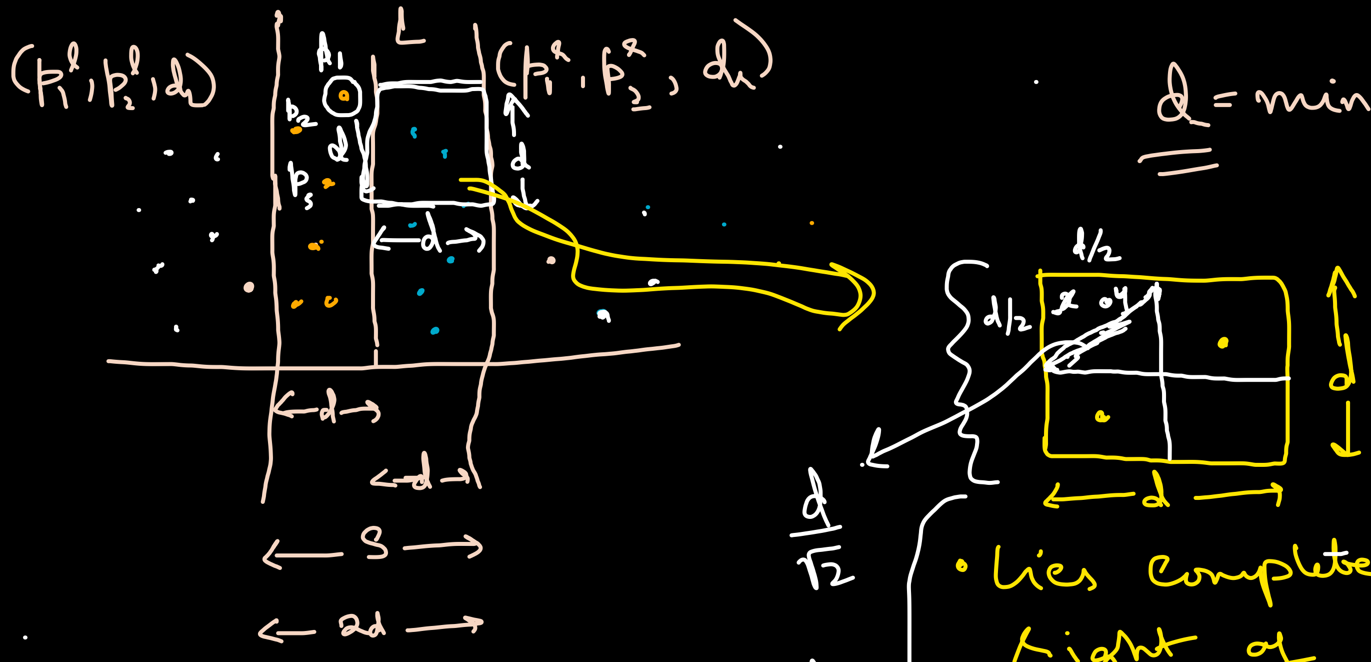
$d_r < d_l \longrightarrow d_r$ is better } the overall optimal?

**Goal:**

$$T(n) = 2T\left(\frac{n}{2}\right) + \underline{cn} \longrightarrow \text{combine time}$$

$$= O(n \log n)$$

$d = \min(d_r, d_l)$



$$\sqrt{\frac{d^2}{4}+\frac{d^2}{4}} = \frac{d}{\sqrt{2}}$$

- Lies completely to the right of line L
- No two points inside this box can be closer than d dist.

$\times\ d(x,y) \le \frac{d}{\sqrt{2}}$
not possible

Cannot have more than 4 points.

$\frac{d}{\sqrt{2}} < d$