

$$D'[u, v] = \min(D[u, v], D[u, k] + D[k, v])$$

Here vertex “k” is the intermediate vertex. D[u, k] will provide the cost from source vertex “u” to intermediate vertex “k” and D[k, v] will provide the cost from intermediate vertex “k” to destination vertex “v”. The intermediate vertex could be any vertex of the graph so we will check all the vertices as intermediate vertex.

**Adjacency Matrix:**

	A	B	C	D	E
A	0	3	8	$\infty$	-4
B	$\infty$	0	$\infty$	1	7
C	$\infty$	4	0	$\infty$	$\infty$
D	2	$\infty$	-5	0	$\infty$
E	$\infty$	$\infty$	$\infty$	6	0

**Via vertex “A”:** Use adjacency matrix. The diagonal values and the row & column representing vertex “A” will remain unchanged.

	A	B	C	D	E
A	0	3	8	$\infty$	-4
B	$\infty$	0			
C	$\infty$		0		
D	2			0	
E	$\infty$				0

**Working for other cells:**

$$D'[B, C] = \min(D[B, C], D[B, A] + D[A, C]) \Rightarrow \min(\infty, \infty + 8) = \infty$$

$$D'[B, D] = \min(D[B, D], D[B, A] + D[A, D]) \Rightarrow \min(1, \infty + \infty) = 1$$

$$D'[B, E] = \min(D[B, E], D[B, A] + D[A, E]) \Rightarrow \min(7, \infty - 4) = 7$$

$$D'[C, B] = \min(D[C, B], D[C, A] + D[A, B]) \Rightarrow \min(4, \infty + 3) = 4$$

$$D'[C, D] = \min(D[C, D], D[C, A] + D[A, D]) \Rightarrow \min(\infty, \infty + \infty) = \infty$$

$$D'[C, E] = \min(D[C, E], D[C, A] + D[A, E]) \Rightarrow \min(\infty, \infty - 4) = \infty$$

$$D'[D, B] = \min(D[D, B], D[D, A] + D[A, B]) \Rightarrow \min(\infty, 2 + 3) = 5$$

$$D'[D, C] = \min(D[D, C], D[D, A] + D[A, C]) \Rightarrow \min(-5, 2 + 8) = -5$$

$$D'[D, E] = \min(D[D, E], D[D, A] + D[A, E]) \Rightarrow \min(\infty, 2 + (-4)) = -2$$

$$D'[E, B] = \min(D[E, B], D[E, A] + D[A, B]) \Rightarrow \min(\infty, \infty + 3) = \infty$$

$$D'[E, C] = \min(D[E, C], D[E, A] + D[A, C]) \Rightarrow \min(\infty, \infty + 8) = \infty$$

$$D'[E, D] = \min(D[E, D], D[E, A] + D[A, D]) \Rightarrow \min(6, \infty + \infty) = 6$$

**Resultant matrix calculated via vertex “A”: The changes are highlighted.**

	A	B	C	D	E
A	0	3	8	$\infty$	-4
B	$\infty$	0	$\infty$	1	7
C	$\infty$	4	0	$\infty$	$\infty$
D	2	5	-5	0	-2
E	$\infty$	$\infty$	$\infty$	6	0

**Via vertex “B”:** Now use the most recent matrix i.e., the matrix calculated via vertex “A”. The diagonal cells and the row & column representing vertex “B” will remain unchanged.

	A	B	C	D	E
A	0	3			
B	$\infty$	0	$\infty$	1	7
C		4	0		
D		5		0	
E		$\infty$			0

**Working for other cells:**

$$D'[A, C] = \min (D[A, C], D[A, B] + D[B, C]) \Rightarrow \min (8, 3 + \infty) = 8$$

$$D'[A, D] = \min (D[A, D], D[A, B] + D[B, D]) \Rightarrow \min (\infty, 3 + 1) = 4$$

$$D'[A, E] = \min (D[A, E], D[A, B] + D[B, E]) \Rightarrow \min (-4, 3 + 7) = -4$$

$$D'[C, A] = \min (D[C, A], D[C, B] + D[B, A]) \Rightarrow \min (\infty, 4 + \infty) = \infty$$

$$D'[C, D] = \min (D[C, D], D[C, B] + D[B, D]) \Rightarrow \min (\infty, 4 + 1) = 5$$

$$D'[C, E] = \min (D[C, E], D[C, B] + D[B, E]) \Rightarrow \min (\infty, 4 + 7) = 11$$

$$D'[D, A] = \min (D[D, A], D[D, B] + D[B, A]) \Rightarrow \min (2, 5 + \infty) = 2$$

$$D'[D, C] = \min (D[D, C], D[D, B] + D[B, C]) \Rightarrow \min (-5, 5 + \infty) = -5$$

$$D'[D, E] = \min (D[D, E], D[D, B] + D[B, E]) \Rightarrow \min (-2, 5 + 7) = -2$$

$$D'[E, A] = \min (D[E, A], D[E, B] + D[B, A]) \Rightarrow \min (\infty, \infty + \infty) = \infty$$

$$D'[E, C] = \min (D[E, C], D[E, B] + D[B, C]) \Rightarrow \min (\infty, \infty + \infty) = \infty$$

$$D'[E, D] = \min (D[E, D], D[E, B] + D[B, D]) \Rightarrow \min (6, \infty + 1) = 6$$

**Resultant matrix calculated via vertex “B”:** The changes are highlighted.

	A	B	C	D	E
A	0	3	8	4	-4
B	$\infty$	0	$\infty$	1	7
C	$\infty$	4	0	5	11
D	2	5	-5	0	-2
E	$\infty$	$\infty$	$\infty$	6	0

**Via vertex “C”:** Use the matrix calculated via vertex “B”. The diagonal and the row & column representing vertex “C” will remain unchanged.

	A	B	C	D	E
A	0		8		
B		0	$\infty$		
C	$\infty$	4	0		
D			-5	0	-2
E			$\infty$		0

Only a single change will take place via vertex “C”. All other cells will remain unchanged.

$$D'[D, B] = \min(D[D, B], D[D, C] + D[C, B]) \Rightarrow \min(5, -5 + 4) = -1$$

**Resultant matrix calculated via vertex “C”:**

	A	B	C	D	E
A	0	3	8	4	-4
B	$\infty$	0	$\infty$	1	7
C	$\infty$	4	0	5	11
D	2	-1	-5	0	-2
E	$\infty$	$\infty$	$\infty$	6	0

**Via vertex “D”:** The diagonal cells and the row & column representing vertex “D” will remain unchanged.

	A	B	C	D	E
A	0			4	
B		0		1	
C			0	5	
D	2	-1	-5	0	-2
E				6	0

Changes will take place in the following cells.

$$D'[A, C] = \min (D[A, C], D[A, D] + D[D, C]) \Rightarrow$$

$$D'[B, A] = \min (D[B, A], D[B, D] + D[D, A]) \Rightarrow$$

$$D'[B, C] = \min (D[B, C], D[B, D] + D[D, C]) \Rightarrow$$

$$D'[B, E] = \min (D[B, E], D[B, D] + D[D, E]) \Rightarrow$$

$$D'[C, A] = \min (D[C, A], D[C, D] + D[D, A]) \Rightarrow$$

$$D'[C, E] = \min (D[C, E], D[C, D] + D[D, E]) \Rightarrow$$

$$D'[E, A] = \min (D[E, A], D[E, D] + D[D, A]) \Rightarrow$$

$$D'[E, B] = \min (D[E, B], D[E, D] + D[D, B]) \Rightarrow$$

$$D'[E, C] = \min (D[E, C], D[E, D] + D[D, C]) \Rightarrow$$

**Matrix calculated via vertex “D”: (Complete the table)**

	A	B	C	D	E
A	0	3		4	-4
B		0		1	
C		4	0	5	
D	2	-1	-5	0	-2
E				6	0

**Via vertex “E”:** The diagonal cells and the row & column representing vertex “E” will remain unchanged. Changes will take place in the following cells:

$$D'[A, B] = \min (D[A, B], D[A, E] + D[E, B]) \Rightarrow$$

$$D'[A, C] = \min (D[A, C], D[A, E] + D[E, C]) \Rightarrow$$

$$D'[A, D] = \min (D[A, D], D[A, E] + D[E, D]) \Rightarrow$$

**Matrix calculated via vertex “E”: (Complete the table)**

	A	B	C	D	E
A	0				
B		0			
C			0		
D				0	
E					0