Foundation for Advancement of Science & Technology

Solving Recurrance

Recurrance Tree

$$T(n) = T(n/2) + O(n^3)$$

$$\begin{array}{cc}
n^3 & n^3 \\
| & \\
\left(\frac{n}{2}\right)^3 & \left(\frac{n}{2^1}\right)^3 \\
| & \\
\left(\frac{n}{4}\right)^3 & \left(\frac{n}{2^2}\right)^3 \\
| & \\
\left(\frac{n}{8}\right)^3 & \left(\frac{n}{2^3}\right)^3 \\
\vdots & \vdots \\
(1) & \left(\frac{n}{2^i}\right)^3
\end{array}$$

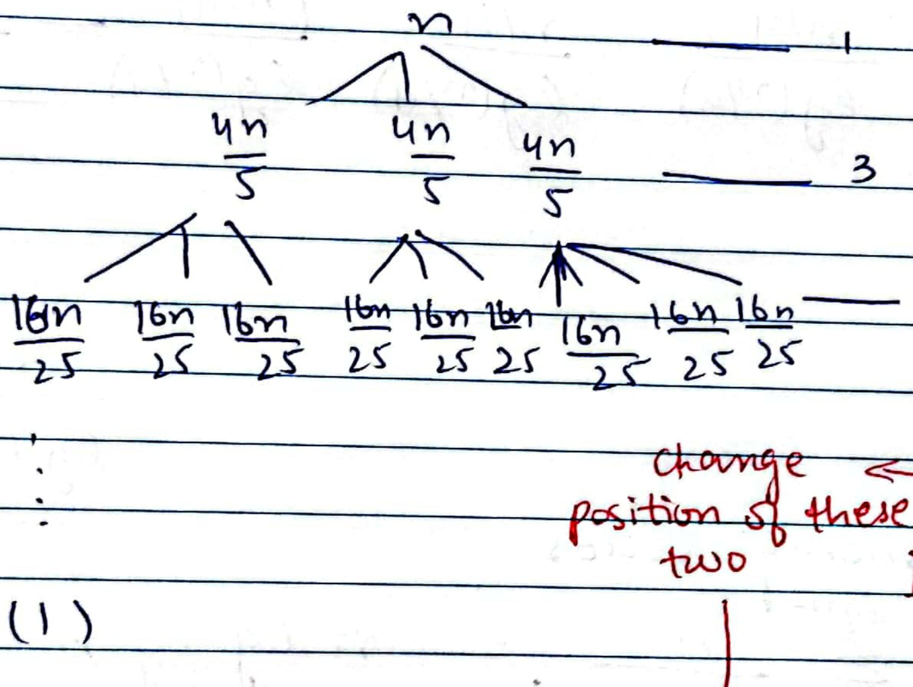
$$n^3 + n^3\left(\frac{1}{2^1}\right)^3 + n^3\left(\frac{1}{2^2}\right)^3 + n^3\left(\frac{1}{2^3}\right)^3 + \dots + n^3\left(\frac{1}{2^i}\right)^3$$

$$n^3 + n^3\left(\frac{1}{2^3}\right)^1 + n\left(\frac{1}{2^3}\right)^2 + n^3\left(\frac{1}{2^3}\right)^3 + \dots + n^3\left(\frac{1}{2^3}\right)^i$$

common ratio $= \frac{1}{8} < 1$

geometric series $\Rightarrow \dfrac{n^3}{1 - 1/8} = O(n^3)$

$T(n) = 3T\left(\frac{4n}{5}\right) + \Theta(1)$



(cuz each node costs 1)

geometric series

$1 + 3^1 + 3^2 + \cdots + 3^{\log_{\boxed{5/4}} n}$

→ jis factor se divide ho raha ho wo

change position of these two

$\dfrac{\boxed{3}^{\log_{5/4} \boxed{n}} - 1}{3 - 1}$

$\dfrac{n^{\log_{5/4} 3} - 1}{3 - 1}$

$\boxed{n^{4.9}}$

replace 'n' by this value in each step of tree

$$T(n) = 2T\left(\frac{n}{2}\right) + \frac{n}{\lg n}$$

$$\frac{n}{\lg n} \quad \text{———} \quad \frac{n}{\lg n}$$

$$\frac{n/2}{\lg(n/2)} \qquad \frac{n/2}{\lg(n/2)} \quad \text{———} \quad \frac{n/2}{\lg(n)-1} \; *$$

$$\frac{n/4}{\lg(n/4)} \qquad \frac{n/4}{\lg(n/4)} \qquad \frac{n/4}{\lg(n/4)} \qquad \frac{(n/4)}{\lg(n/4)} \quad \text{———} \quad \frac{n/2^2}{\lg(n)-2}$$

$\vdots$

(1)

$$\vdots$$

$$\frac{n/2^i}{\lg(n)-i} \; *$$

By Harmonic series

$$\sum_{i=0}^{\lg n - 1} \frac{n}{\lg(n)-i} = n \lg \lg n$$

$$* \quad \frac{n/2}{\lg(n/2)} = \frac{n/2}{\lg n - \lg 2} = \frac{n/2}{\lg n - 1}$$

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$T(n) = 2T(n-1) + \theta(1)$

$n-1$ — 1

$n-2$ $\quad$ $n-2$ — 2 $\quad$ 2^1

$n-3$ $\quad$ $n-3$ $\quad$ $n-3$ $\quad$ $n-3$ — 4 $\quad$ 2^2

$\vdots$

(1) — 2^i

$\boxed{2^n}$

$$T(N) = \frac{N}{4} + \frac{3N}{4} + N^2$$

N^2 — N^2

$\left(\frac{N}{4}\right)^2$ $\left(\frac{3N}{4}\right)^2$ — $\frac{N^2}{4^2} + \frac{9N^2}{4^2} = \frac{10}{16} N^2$

$\left(\frac{N}{16}\right)^2$ $\left(\frac{3N}{16}\right)^2$ $\left(\frac{3n}{16}\right)^2$ $\left(\frac{9N}{16}\right)^2$ — $\frac{N^2}{16^2} + \frac{9N^2}{16^2} + \frac{9N^2}{16^2} + \frac{81N^2}{16^2} = \left(\frac{10N}{16}\right)^2$

$\vdots$

(1)

$\vdots$

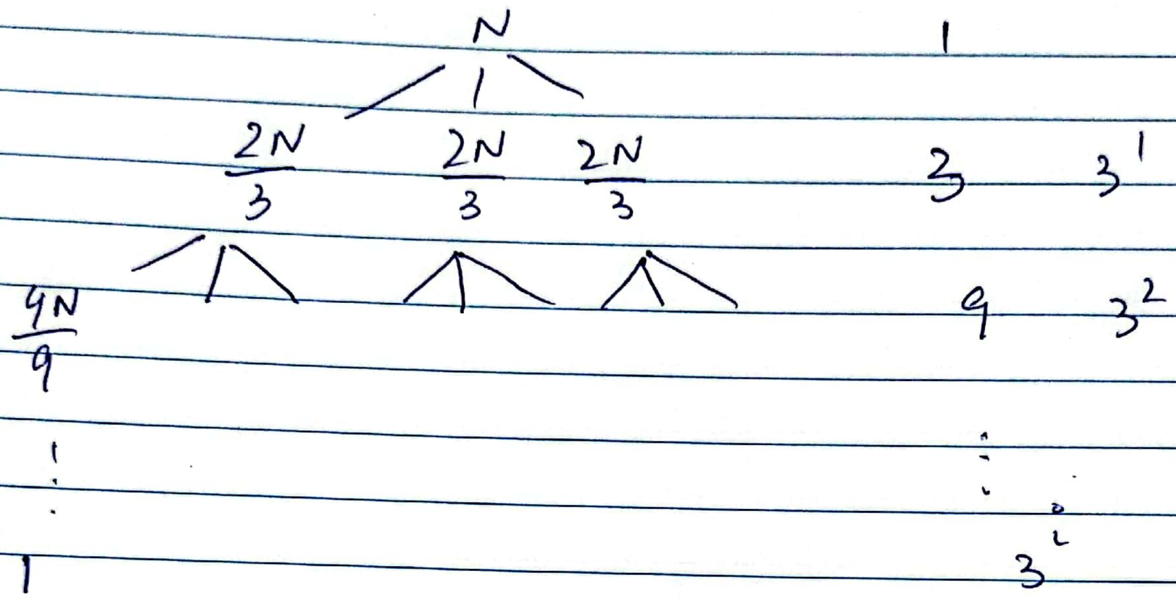
$\left(\frac{10}{16}\right)^{\log_{4/3} N} N^2$

$$N^2 \left[1 + \frac{10}{16} + \left(\frac{10}{16}\right)^2 + \dots + \left(\frac{10}{16}\right)^{\log_{4/3} N}\right]$$

$$\text{ratio} = \frac{10}{16} < 1$$

$$N^2 \left[\frac{1}{1 - \frac{10}{16}}\right] = \boxed{N^2}$$

$$T(n) = 3T\left(\frac{2N}{3}\right) + O(1)$$



$$i = \log_{3/2} N$$

$$1 + 3 + 3^2 + 3^3 + \cdots + 3^{\log_{3/2} N}$$

ratio $= 3$

$$\frac{3^{\log_{3/2} N} - 1}{3 - 1} = \boxed{N^{\log_{3/2} 3}}$$