Asymptotic Analysis

Asymptotic analysis is a technique used to describe the growth rate of an algorithm as the input size increases. Three common notations used in asymptotic analysis are:

- Big-oh (O), big omega (Ω) and big theta (Θ)

Big-oh (O) provides upper bound.

Big omega (Ω) provides lower bound.

Big theta (Θ) provides tight bound.

Upper bound:

$f(N) = O(g(N))\ if\ \boldsymbol{f(N) \leq c * g(n)}\ for\ all\ values\ of\ n \geq\ n_0\ where\ c\ and\ n_0\ are + ve\ constants$

Lower bound:

$f(N) = \Omega(g(N))\ if\ \boldsymbol{f(N) \geq c * g(n)}\ for\ all\ values\ of\ n \geq\ n_0\ where\ c\ and\ n_0\ are + ve\ constants$

Tight bound:

$f(N) = \Theta(g(N))\ if\ \boldsymbol{c_1 * g(N) \leq f(N) \leq c_2 * g(n)}\ for\ all\ n \geq\ n_0\ where\ c\ and\ n_0\ are$

$+ve\ constants.$

Practice Questions

Q#1: $f(N) = \frac{1}{2}N^2 + 3N$ Prove that $f(N) = \Theta(N^2)$

Q#2: $f(N) = 2N^3 - 7N + 1$ Prove that $f(N) = \Omega(N^3)$

Q#3: $f(N) = 3N^2 + 2 * 4^N + 3N^3 logN$ find closest upper bound.

Q#4: which of the following is true about $f(N) = 2^{N+10}$

- $f(N) = \Theta(2^N)$
- $f(N) = O(2^N)$
- $f(N) = \Omega(2^N)$

Q#5: which of the following is true about $f(N) = 2^{10N}$

- $f(N) = \Theta(2^N)$
- $f(N) = O(2^N)$
- $f(N) = \Omega(2^N)$

Q#6: which of the following is true about $f(N) = \log_2 N$

- $f(N) = O(\log_8 N)$
- $f(N) = \Omega(\log_8 N)$
- $f(N) = \Theta(\log_8 N)$

Q#7: which of the following is true about $f(N) = N^3 \log_2 N$

- $f(N) = O(N \log_2 N)$
- $f(N) = \Omega(N \log_2 N)$
- $f(N) = \Theta(\mathrm{N} \log_2 N)$

Q#8: which of the following is true about $f(N) = 8^N$

- $f(N) = O(4^N)$
- $f(N) = \Omega(2^{4N})$
- $f(N) = \Theta(2^{3N})$