

010001

National University of Computer and Emerging Sciences, Lahore Campus



Course Name:	Theory of Automata	Course Code:	CS-3005
Degree Program:	BS (CS)	Semester:	Fall 2023
Exam Duration:	60 Minutes	Total Marks:	30
Paper Date:	2-10-2023	Weight	17.5%
Section:	ALL	Page(s):	7
Exam Type:	Midterm-I		

Student Name: \_\_\_\_\_ Roll No. \_\_\_\_\_ Section: \_\_\_\_\_

Instruction/Notes: Answer in the space provided, showing complete working.  
**ROUGH SHEETS ARE NOT ALLOWED.**  
In case of confusion or ambiguity make a reasonable assumption.  
Good luck!

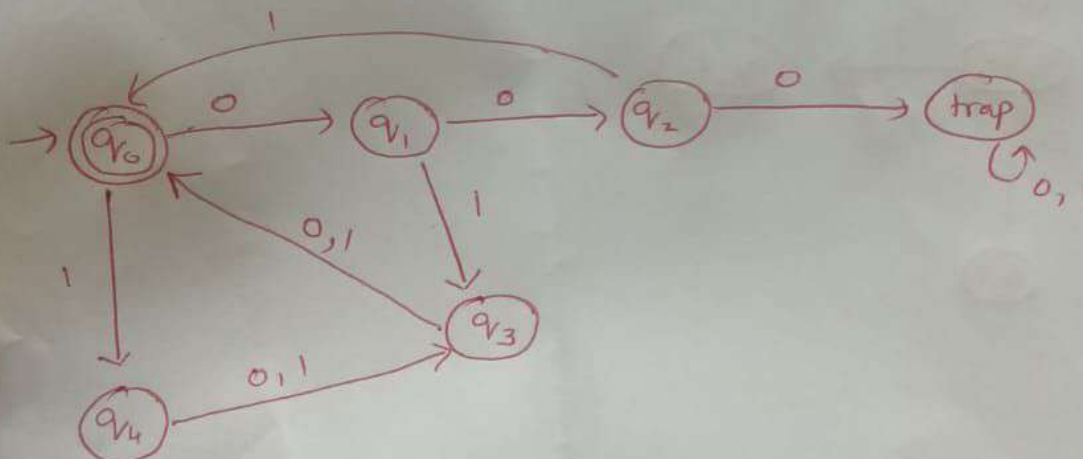
Question 1: (10 points):

Design deterministic finite automata (DFA) of the following language:

$$\Sigma = \{0,1\}$$

$L = \{x \mid x \in \Sigma^* \text{ and } |x| \text{ should be multiple of 3 and every three-length chunk of the string contains at most two occurrences of 0}\}$

010 and 001100 are two of the accepting strings  
0101 and 000010 are two of the rejecting strings

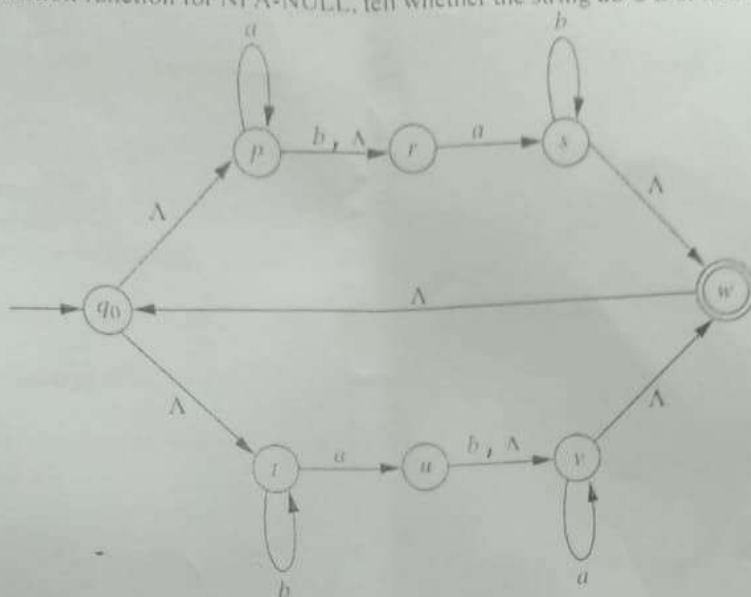


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Question 2: (5 points):

Using the extended transition function for NFA-NULL, tell whether the string  $ab \in L$  or not. Show full working.



$$\delta^*(q_0, ab)$$

$$\bigcup_{r \in \delta^*(q_0, a)} \delta(r, b) \rightarrow \textcircled{1}$$

$$\delta^*(q_0, \Lambda a) = \bigcup_{p \in \delta^*(q_0, \Lambda)} \delta(p, a) \rightarrow \textcircled{2}$$

$$\delta^*(q_0, \Lambda) = \bigcup \{q_0\} = \{q_0, p, t, r\}$$

Putting in  $\textcircled{2}$

$$\delta^*(q_0, a) = \bigcup_{p \in \{q_0, p, t, r\}} \delta(p, a)$$

$$= \bigcup \{\emptyset, \emptyset, \emptyset, \emptyset\}$$

$$= \bigcup \{p, u, s\}$$

$$= \{p, r, u, v, w, q_0, p, t, r, s, w\}$$

Putting in  $\textcircled{1}$

$$\delta^*(q_0, ab) = \bigcup_{r \in \{p, r, u, v, w, q_0, t, s, w\}} \delta(r, b)$$

$$= \bigcup \{r, v, t, s\}$$

$$= \{r, v, w, q_0, p, t, s\}$$

Since  $w \in \Lambda$

so

$ab \in L$ .



**PART A**

Consider a language  $L$  defined over the alphabet set  $\Sigma$ . Suppose  $D_1$  is a deterministic finite automata (DFA) with 5 tuples  $(Q, \Sigma, q_0, A, T)$  where

- $Q$  = finite set of states
- $\Sigma$  = finite set of alphabets
- $q_0$  = initial state
- $A$  = set of final states
- $T$  = set of transition functions.

Construct finite automata  $F_1$  ((DFA or NFA or NFA-NULL but clearly mention which FA you have developed)) for  $L^R$  where

$L^R$  = Reverse of  $L$ .

You have to define all the 5 tuples of  $F_1 (Q_1, \Sigma_1, p_0, A_1, T_1)$

$Q_1 = \{ Q \cup p_0 \}$

$\Sigma_1 = \{ \Sigma \}$

$p_0 = p_0$

$A_1 = \{ q_0 \}$

$T_1 = \{ T_1(p_0, \Lambda) \rightarrow A \}$   
 invert all transition

if  $T(q, a) \rightarrow r$  in  $D_1$   
 then for  $F_1$

$T_1(r, a) \rightarrow q$

FA = NFA -  $\Lambda$

**Hint:**

Construct FA for  $L^R$  and then fill the tuples.

If  $L$  accepts the string  $x = x_0 x_1 \dots x_n$  { where  $x_0 x_1 \dots x_n \in \Sigma$  } then  $L^R$  will accept  $y = x_n x_{n-1} \dots x_1 x_0$ .

**For example**

Example #1

$L = \{x \mid x \in \{a,b\}^* \text{ and } x = abbb\}$

Then

$L^R = \{x \mid x \in \{a,b\}^* \text{ and } x = bbba\}$

Example #2

$L = \{x \mid x \in \{a,b\}^* \text{ and } x \text{ ends with } ab\}$

Then

$L^R = \{x \mid x \in \{a,b\}^* \text{ and } x \text{ starts with } ba\}$

Number: \_\_\_\_\_  
Question 3 (5+2+2+2+4 = 15 points): Short answers

Section: \_\_\_\_\_

### PART A

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$Q$  = finite set of states

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$L^R$  = Reverse of  $L$ .

You have to define all the 5 tuples of  $F_1$  ( $Q_1, \Sigma_1, p_0, A_1, T_1$ )

$Q_1 = \{ \quad Q \cup P_0 \quad \}$

$\Sigma_1 = \{ \quad \Sigma \quad \}$

$p_0 = \quad P_0 \quad$

$A_1 = \{ \quad q_0 \quad \}$

$T_1 = \{ \quad T_1(P_0, a) \rightarrow A \quad \}$   
invert all transition

if  $T(q, a) \rightarrow r$  in  $D_1$   
then for  $F_1$

$FA = \quad NFA - A \quad$

$T_1(r, a) \rightarrow q$

Hint:

Construct FA for  $L^R$  and then fill the tuples.

If  $L$  accepts the string  $x = x_0 x_1 \dots x_n$  { where  $x_0 x_1 \dots x_n \in \Sigma$  } then  $L^R$  will accept  $y = x_n x_{n-1} \dots x_1 x_0$ .

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Example #1

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Example #2

$L = \{x \mid x \in \{a,b\}^* \text{ and } x \text{ ends with } ab\}$

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### PART B

True/ False with justification (no marks without justification)

Every DFA is also a NFA-NULL

True

IN NFA-NULL transition function is defined as  $Q \times \{ \epsilon, 0, 1 \} \rightarrow 2^Q$   
DFA:  $Q \times \Sigma \rightarrow Q$  &  $Q \in 2^Q$ . It is not necessary to have a Null-transition for every state

### PART C

Language is regular if it has FA and R.E.

### PART D

Give regular expression for the following language.

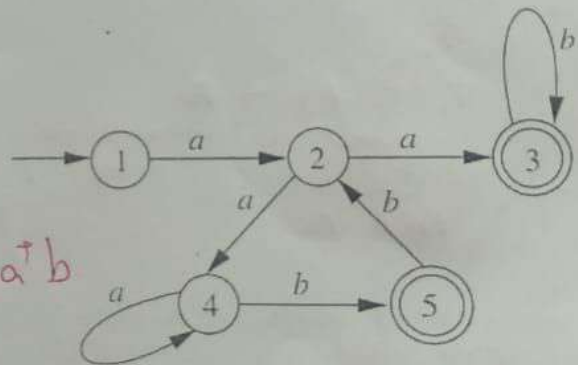
$L = \{x \mid x \in \{a,b\}^* \text{ and } x \text{ starts with } ab \text{ and ends with } ba\}$

Ans:  $ab(a+b)^*ba + aba$   
① ①

### PART E

NFA for the Language L is given below.

- a) Write regular expression for the language accepted by this FA? [Hint: No need to apply state elimination method]



$a(a^+bb)^*ab^*$  +  $a(a^+bb)^*aa^+b$   
① ①

$a(a^+bb)^* [ab^* + aa^+b]$

- b) Enumerate the language  $L^*$  (complement of L) [at least 10 elements in increasing order of length]

$\rightarrow \Lambda$   
 $\rightarrow ab^+(a+b)^*$   
 $\rightarrow b^+(a+b)^*$   
 $\rightarrow aaa^+ + a$

$\{ \Lambda, b, ba, ab, bb, aaa, aba, abb, bab, bbb, \dots \}$