OPERATIONS RESEARCH

"OR:THE SCIENCE OF BETTER"

"Operations Research is the discipline of applying advanced analytical methods to help make better decisions."

Linear Programming: Model Formulation & Graphical Solution

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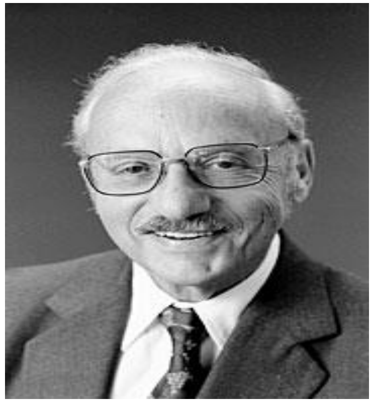
What is Linear Programming?

- First conceived by George B. Dantzig (1947); Dantzig's first paper was titled "Programming in Linear Structure"
- Koopmans Coined the term "Linear Programming" in 1948.
- Simplex Method was published in 1949 by Dantzig.

- A linear programming problem (LP) is a class of the mathematical programming problem, **a constrained optimization problem**, for which:
  - We attempt to maximize (or minimize) a linear function of the **decision variables**. **(Objective Function**)
  - The values of the decision variables must satisfy a set of **constraints**, each of which must be a linear inequality or linear equality.
  - A sign restriction on each variable. For each variable X_i the sign restriction can either say
    - $X_i \geq 0, X_i \leq 0, X_i$ unrestricted.



George B. Dantzig (1914–2005)

$$\text{Maximize or Minimize } Z = \sum_{i=1}^{n} C_i x_i$$

Subject to

$$\sum_{i=1}^{n} a_{ij} x_i \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b_j \text{ for } j = 1,2,\ldots,m$$

$$x_i \geq 0 \text{ for } i = 1,2,\ldots,n$$

LP Model Using Matrix Notation

$$\text{Maximize or Minimize } Z = c^T X$$

Subject to

$$AX \begin{Bmatrix} \leq \\ = \\ \geq \end{Bmatrix} b$$

$$X \geq 0$$

Where $A \in \mathbb{R}^{m\times n}, c \in \mathbb{R}^n, and\ b \in \mathbb{R}^m$

Assumption: $m \leq n, rank(A) = m$

Assumptions of the LP Model

- **LINEARITY OR PROPORTIONALITY:**
  - Proportionality means that the objective function and constraint coefficients are strictly proportional to the decision variable (e.g., If the first unit of production requires '2' hours of labor so it must the 50th and 100th unit also requires '2' hours of labor).

- **DIVISIBILITY:**
  - Divisibility means that non integer (fractional) values of the decision variables are acceptable.
  - ***Integer Programming*** is a special technique which can be used for finding non-fractional values of resource usage and decision variables).

- **CERTAINTY:**
  - Certainty means that the values of the parameters are known and constant

- **ADDITIVITY:**
  - Additivity means the total effect of each decision variable (Profit, Cost, etc.) must equal the sum of the effects contributed by each decision variable and terms of each constraint must be additive
  - (Total amount of resource consumed or provided) must equal the sum of the resources used (or provided) by each decision variable.

- **NON–NEGATIVITY:**
  - Non – negativity means that the decision variables are permitted to have only the values which are greater than or equal to zero.

Development (Formulation) of LP Model: Product Mix

A firm is engaged in producing two products ‘A’ and ‘B’. Each unit of product ‘A’ requires 3Kg of raw material and 5 labor hour for processing, where as each unit of product ‘B’ requires 6Kg of raw material and 4 labor hours of the same type. Every month the firm has the availability of 60Kg of raw material and 70 Labor hours. One unit of product ‘A’ sold earns profit Rs. 30 and one unit of product ‘B’ sold gives Rs. 40 as profit.

Formulate this problem as linear programming problem to determine as to how many units of each of the products should be produced per month so that the firm can earn maximum profit, assume all unit produced can be sold in the market.

Decision Variables: Let X_1 and X_2 be the number of products ‘A’ and ‘B’ respectively.

Objective Function:

Maximize $Z = 30X_1 + 40X_2$

CONSTRAINTS:

Material Constraint: $3X_1 + 6X_2 \leq 60$

Labor Constraint: $5X_1 + 4X_2 \leq 70$

Non–negativity Constraint: $X_1, X_2 \geq 0$.

The Complete LP problem model is:

Maximize: $Z = 30X_1 + 40X_2$

Subject to:

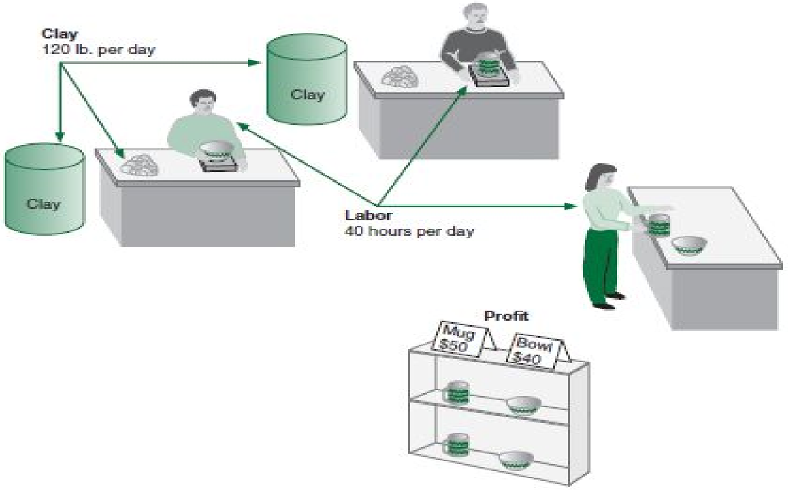
$$3X_1 + 6X_2 \leq 60$$

$$5X_1 + 4X_2 \leq 70$$

$$X_1, X_2 \geq 0$$

Development (Formulation) of LP Model: Product Mix

	Resource Requirements		
Product	Labor (Hr./Unit)	Clay (Lb./Unit)	Profit ($/Unit)
Bowl	1	4	40
Mug	2	3	50



- Product mix problem - Beaver Creek Pottery
- How many bowls and mugs should be produced to maximize profits given labor and materials constraints?
- **Resource Availability:** 40 hrs of labor per day (labor constraint)
  120 lbs of clay (material constraint)

Decision Variables: → Let X_1 & X_2 be the number of bowls and mugs produced, respectively.

Objective Function: → Maximize: $Z = 40X_1 + 50X_2$

Constraints: →

$X_1 + 2X_2 \leq 40$ (Labor Constraint)

$4X_1 + 3X_2 \leq 120$ (Clay Constraint)

$X_1, X_2 \geq 0$ (Non–Negativity)

LP MODEL:→ Maximize: $Z = 40X_1 + 50X_2$ (Profit Function)

SUBJECT TO:

$X_1 + 2X_2 \leq 40$ (Labor Constraint)

$4X_1 + 3X_2 \leq 120$ (Clay Constraint)

$X_1, X_2 \geq 0$ (Non–Negativity)

Development (Formulation) of LP Model

A marketing manager wishes to allocate his annual advertising budget of Rs. 20,000 in two media 'A' and 'B'. The unit cost of message in media 'A' is Rs. 1000 and in media 'B' is Rs. 1500. Media 'A' is the monthly magazine and is desired in one issue. At least 5 messages should appear in media 'B'. The expected effective audience for one message in media 'A' is 40,000 and for media 'B' is 50,000. Formulate the problem as a LP model.

Decision Variables: →

Let X_1, X_2 be the number of messages in media 'A' and 'B', respectively.

Objective Function: Maximize: $Z = 40{,}000X_1 + 50{,}000X_2$ (Effective Audience Function)

Constraints: →

$1000X_1 + 1500X_2 \leq 20{,}000$ (Cost Constraint)

$X_1 \leq 1$ (Program Constraint – i)

$X_2 \geq 5$ (Program Constraint – ii)

$X_1, X_2 \geq 0$ (Non–Negativity Constraint)

LP MODEL:→ Maximize: $Z = 40{,}000X_1 + 50{,}000X_2$ (Effective Audience Function)

SUBJECT TO:

$1000X_1 + 1500X_2 \leq 20{,}000$ (Cost Constraint)

$X_1 \leq 1$ (Program Constraint – i)

$X_2 \geq 5$ (Program Constraint – ii)

$X_1, X_2 \geq 0$ (Non–Negativity Constraint)

Development (Formulation) of LP Model

The Apex Television Company has to decide on the number of 27- and 20-inch sets to be produced at one of its factories. Market research indicates that at most 40 of the 27-inch sets and 10 of the 20-inch sets can be sold per month. The maximum number of work-hours available is 500 per month. A 27-inch set requires 20 work-hours and a 20-inch set requires 10 work-hours. Each 27-inch set sold produces a profit of $120 and each 20-inch set produces a profit of $80. A wholesaler has agreed to purchase all the television sets produced if the numbers do not exceed the maxima indicated by the market research.

Decision Variables: →

Let x_1 = number of 27-inch TV sets to be produced per month,

x_2 = number of 20-inch TV sets to be produced per month

Objective Function: Maximize: $Z = 120\ x_1 + 80\ x_2$ (Total Profit Per Month)

Constraints: →

$x_1 \leq 40$ (Number of 27-inch sets sold per month)

$x_2 \leq 10$ (Number of 20-inch sets sold per month)

$20\ x_1 + 10\ x_2 \leq 500$ (Work-hours availability)

$X_1, X_2 \geq 0$ (Non–Negativity Constraint)

LP MODEL:→ Maximize $Z = 120\ x_1 + 80\ x_2$,

SUBJECT TO:

$$
\begin{aligned}
x_1 \qquad\qquad &\leq 40 \\
x_2 &\leq 10 \\
20\ x_1 + 10\ x_2 &\leq 500
\end{aligned}
$$

$X_1, X_2 \geq 0$ (Non–Negativity Constraint)

Development (Formulation) of LP Model: Product Mix

Four varieties of ties produced: i) one is an expensive, all-silk tie, ii) one is an all-polyester tie, and iii) two are blends of polyester and cotton. The table on the following slide illustrates the cost and availability (per monthly production planning period) of the three materials used in the production process

MATERIAL	COST PER YARD ($)	MATERIAL AVAILABLE PER MONTH (YARDS)
Silk	21	800
Polyester	6	3,000
Cotton	9	1,600

- The following table summarizes the contract demand for each of
  - the four styles of ties,
  - the selling price per tie, and
  - the fabric requirements of each variety.

VARIETY OF TIE	SELLING PRICE PER TIE ($)	MONTHLY CONTRACT MINIMUM	MONTHLY DEMAND	MATERIAL REQUIRED PER TIE (YARDS)	MATERIAL REQUIREMENTS
All silk	6.70	6,000	7,000	0.125	100% silk
All polyester	3.55	10,000	14,000	0.08	100% polyester
Poly–cotton blend 1	4.31	13,000	16,000	0.10	50% polyester–50% cotton
Poly–cotton blend 2	4.81	6,000	8,500	0.10	30% polyester–70% cotton

Fifth Avenue's goal is to maximize its monthly profit. It must decide upon a policy for product mix.

DECISION VARIABLES: Let
X_1 = # of all-silk ties;
X_2 = # polyester ties;
X_3 = # of blend 1 poly-cotton ties;
X_4 = # of blend 2 poly-cotton ties produced per month

Calculate profit for each tie: *Profit* = Sales price – (Cost per yard * Yards per tie)

Silk ties = \$6.70 – \$21 x 0.125 = \$4.08
Polyester = \$3.55 – \$6 x 0.08 = \$3.07
Poly-blend 1 = \$4.31 – (\$6 x 0.05 + \$9 x 0.05) = \$3.56
Poly-blend 2 = \$4.81 – (\$6 x 0.03 + \$9 x 0.07) = \$4.00

Objective function: maximize profit = $\$4.08X_1 + \$3.07X_2 + \$3.56X_3 + \$4.00X_4$

Subject to:

$0.125X_1 \le 800$ (Total Silk Availability Constraint)
$0.08X_2 + 0.05X_3 + 0.03X_4 \le 3{,}000$ (Total Polyester Availability Constraint)
$0.05X_3 + 0.07X_4 \le 1{,}600$ (Total Cotton Availability Constraint)
$X_1 \ge 6{,}000$ (Contract Constraint – 1)
$X_2 \ge 10{,}000$ (Contract Constraint – 2)
$X_3 \ge 13{,}000$ (Contract Constraint – 3)
$X_4 \ge 6{,}000$ (Contract Constraint – 4)
$X_1 \le 7{,}000$ (Demand Constraint – 1)
$X_2 \le 14{,}000$ (Demand Constraint – 2)
$X_3 \le 16{,}000$ (Demand Constraint – 3)
$X_4 \le 8{,}500$ (Demand Constraint – 4)
$X_1, X_2, X_3, X_4 \ge 0$ (Non-Negativity Constraints)

Development of LP Model: Portfolio Selection Problem

Mr. Ali has Rs. 70, 000 to investment in several alternatives. The alternative investments are national certificates with an 8.5% return, Defense Savings Certificates with a 10% return, NIT with a 6.5% return, and khas deposit with a return of 13%. Each alternative has the same time until maturity. In addition, each investment alternative has a different perceived risk thus creating a desire to diversify. Ali wants to know how much to invest in each alternative in order to maximize the return.

The following guidelines have been established for diversifying the investments and lessening the risk;

- No more than 20% of the total investment should be in khas deposit.
- The amount invested in Defense Savings Certificates should not exceed the amount invested in the other three alternatives.
- At least 30% of the investment should be in NIT and Defense Savings Certificates.
- The ratio of the amount invested in national certificates to the amount invested in NIT should not exceed one to three.

Formulate the problem as a LP model.

DECISION VARIABLES: Let X_1, X_2, X_3 & X_4 be the amount (Rs.) invested in national certificates, Defense savings certificates, NIT, and khas deposit, respectively.

OBJECTIVE FUNCTION: Maximize (the total return from all investment): $Z = 0.085X_1 + 0.100X_2 + 0.065X_3 + 0.130X_4$

CONSTRAINTS: →

$X_1 + X_2 + X_3 + X_4 = 70{,}000$ (Amount Availability Constraint)

$X_4 \leq (0.20 \text{ x } 70{,}000 = 14{,}000)$ (Investment Constraint – i)

$X_2 \leq (X_1 + X_3 + X_4) \rightarrow X_2 - X_1 - X_3 - X_4 \leq 0$ (Investment Constraint – ii)

$X_2 + X_3 \geq (0.30 \text{ x } 70{,}000 = 21{,}000)$ (Investment Constraint – iii)

$(X_1) / (X_3) \leq 1/3 \rightarrow 3X_1 - X_3 \leq 0$ (Investment Constraint – iv)

$X_1, X_2, X_3, X_4 \geq 0$ (Non–Negativity Constraint)

LP MODEL:→

Maximize $Z = 0.085X_1 + 0.100X_2 + 0.065X_3 + 0.130X_4$ (Investment return Function)

SUBJECT TO:

$X_1 + X_2 + X_3 + X_4 = 70{,}000$ (Amount Availability Constraint)

$X_4 \leq 14{,}000$ (Investment Constraint – i)

$X_2 - X_1 - X_3 - X_4 \leq 0$ (Investment Constraint – ii)

$X_2 + X_3 \geq 21{,}000$ (Investment Constraint – iii)

$3X_1 - X_3 \leq 0$ (Investment Constraint – iv)

$X_1, X_2, X_3, X_4 \geq 0$ (Non – Negativity Constraint)

Development of LP Model: Resource Allocation Problem

- All allocation models have in common that they attempt to allocate a scarce resource so as to optimize the consequence of that allocation.
  - The **"Product Mix Problem"** is a special case of the resource allocation problem
  - An agricultural allocation problem
    - Amount of farmland that is devoted to the i^{th} activity; maximize profit
  - A portfolio selection problem
    - the investor selecting the optimal portfolio from a set of possible portfolios

Indices:

- $i = 1 \dots n$ Activities (Products)
- $j = 1 \dots m$ Resources (Machines)

Parameters:

- p_i = Profit for activity 'i'
- b_j = Amount available of resource 'j'
- a_{ij} = Amount of resource 'j' used by a unit of activity 'i'

Decision Variables:

x_i = Amount of activity 'i' selected

$$\boldsymbol{Maximize}\ \text{Profit} = \sum_{i=1}^{n} p_i x_i$$

Subject to:

$$\sum_{i=1}^{n} a_{ij} x_i \leq b_j \ \ for \ \ j = 1,2,\dots,m$$

$$x_i \geq 0 \ \ for \ \ i = 1,2,\dots,n$$

Development (Formulation) of LP Model: Production Planning

Let us consider a company making a *single product*. The estimated demand for the product for the next four months are 1000, 800, 1200, 900 respectively. The company has a regular time capacity of 800 per month and an over time capacity of 200 per month. The cost of regular time production is Rs. 20 per unit and the cost of over time production is Rs. 25 per unit. The company can carry inventory to the next month and the holding cost is Rs. 3 per unit per month. *The demand has to be met every month*. Formulate linear programming problem for the above situation.

Decision Variables:

- Let R_t = Quantity Produced Using Regular time in month 't' ➔ t = 1, 2, 3, 4
- Let O_t = Quantity Produced Using Over time in month 't' ➔ t = 1, 2, 3, 4
- Let I_t = Quantity Carried at the end of month 't' to the next month ➔ t = 1, 2, 3

Objective Function:

Minimize (the total cost): $Z = 20\,(R_1+R_2+R_3+R_4) + 25(O_1+O_2+O_3+O_4) + 3\,(I_1+I_2+I_3)$

Constraints: →

Demand Constraints:

$R_1 + O_1 = 1000 + I_1$

$I_1 + R_2 + O_2 = 800 + I_2$

$I_2 + R_3 + O_3 = 1200 + I_3$

$I_3 + R_4 + O_4 = 900$

Capacity Constraints:

$R_t \leq 800$ Where t = 1, 2, 3, 4

$O_t \leq 200$ Where t = 1, 2, 3, 4

Non–Negativity Constraints:

$R_t\,,\, O_t\,,\, I_t \geq 0$

Development (Formulation) of LP Model: Production Planning

A Production Planning Problem (Single Product, Multi-period): Suppose a production manager is responsible for scheduling the monthly production levels of a certain product for a planning horizon of twelve months.

For planning purposes, the manager was given the following information:

- The total demand for the product in month t is d_t, for $t = 1, 2, \ldots, 12$. *These could either be targeted values or be based on forecasts.*
- The cost of producing each unit of the product in month t is c_t^p (Rs.), for $t = 1, 2, \ldots, 12$. There is no setup/fixed cost for production.
- The inventory holding cost per unit for month t is h_t (Rs.), for $t = 1, 2, \ldots, 12$. These are incurred at the end of each month.
- The production capacity for month t is C_t , for $t = 1, 2, \ldots, 12$.

The manager's task is to generate a production schedule that minimizes the total production and inventory-holding costs over this twelve-month planning horizon.

Assumption:

1. There is no initial inventory at the beginning of the first month.
2. Shortage of the product is not allowed at the end of any month.

Objective Function:

$$Minimize: \sum_{t=1}^{T} c_t^p Q_t + \sum_{t=1}^{T} h_t I_t$$

Constraints:

$$I_t = I_{t-1} + Q_t - d_t, \qquad t = 1,2, \ldots, T$$

$$Q_t \leq C_t, \qquad t = 1,2, \ldots, T$$

$$Q_t, I_t \geq 0$$

- ***Index:***
  - t: *Time period*
- ***Data & Parameters:***
  - c_t^p: *cost of production each unit in time period* t
  - h_t: *Inventory holding cost in time period t*
  - C_t: *Production capacity in time period t*
  - d_t: *demand of product in time period t*
- ***Decision Variable:***
  - Q_t: *Production in time period t*
  - I_t: *Inventory in time period t*

Development (Formulation) of LP Model

A Hotel remains open 24 hours a day. Waiters and busboys report for duty at 2 a.m., 6 a.m., 10 a.m., 2 p.m., 6 p.m. or 10 p.m. and each works an 8 hour shift. The following table shows the minimum number of workers needed during the six periods into which the day is divided. The hotel's scheduling problem is to determine how many waiters and busboys should report for work at the start of each period in order to minimize the total staff required for one day's operations.

Period	Time	Number of Waiters and Busboys required
1	2 a.m. – 6 a.m.	4
2	6 a.m. – 10 a.m.	13
3	10 a.m. – 2 p.m.	15
4	2 p.m. – 6 p.m.	8
5	6 p.m. – 10 p.m.	12
6	10 p.m. – 2 a.m.	5

Formulate the problem as a LP model.

DECISION VARIABLES: Let X_1, X_2, X_3, X_4, X_5, X_6 be the number of waiters and Busboys in time periods.

THE COMPLETE LP MODEL IS:

Minimize: $Z = X_1 + X_2 + X_3 + X_4 + X_5 + X_6$ (Waiters & Busboys Function)

Subject to :

$X_1 + X_2 \geq 13$ (Waiters & Busboys requirement for periods 1 & 2 Constraint)

$X_2 + X_3 \geq 15$ (Waiters & Busboys requirement for periods 2 & 3 Constraint)

$X_3 + X_4 \geq 8$ (Waiters & Busboys requirement for periods 3 & 4 Constraint)

$X_4 + X_5 \geq 12$ (Waiters & Busboys requirement for periods 4 & 5 Constraint)

$X_5 + X_6 \geq 5$ (Waiters & Busboys requirement for periods 5 & 6 Constraint)

$X_6 + X_1 \geq 4$ (Waiters & Busboys requirement for periods 6 & 1 Constraint)

$X_1, X_2, X_3, X_4, X_5, X_6 \geq 0$ (Non – Negativity Constraint)

PERIODS	MINIMUM REQUIREMENTS
1 and 2	13
2 and 3	15
3 and 4	8
4 and 5	12
5 and 6	5
6 and 1	4

Convex & Concave Functions

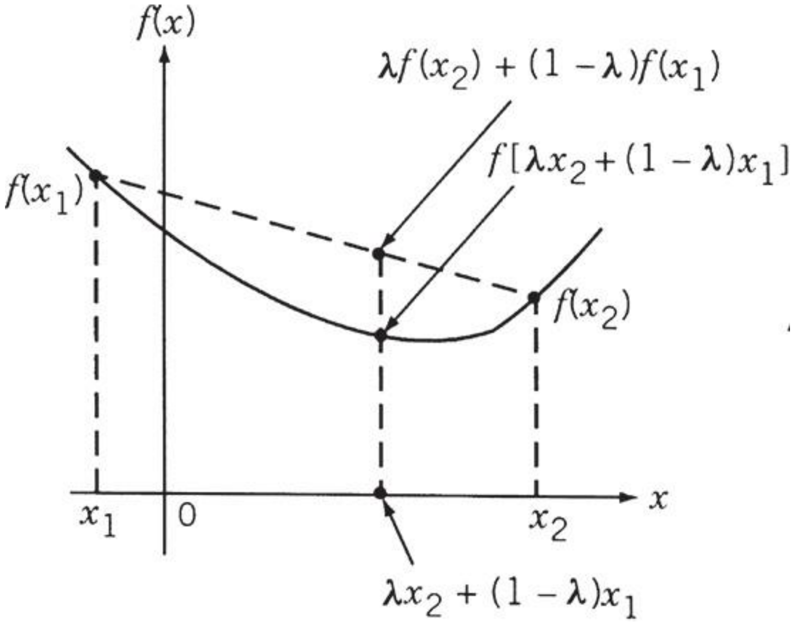
Convex & Concave functions play a central role in optimization

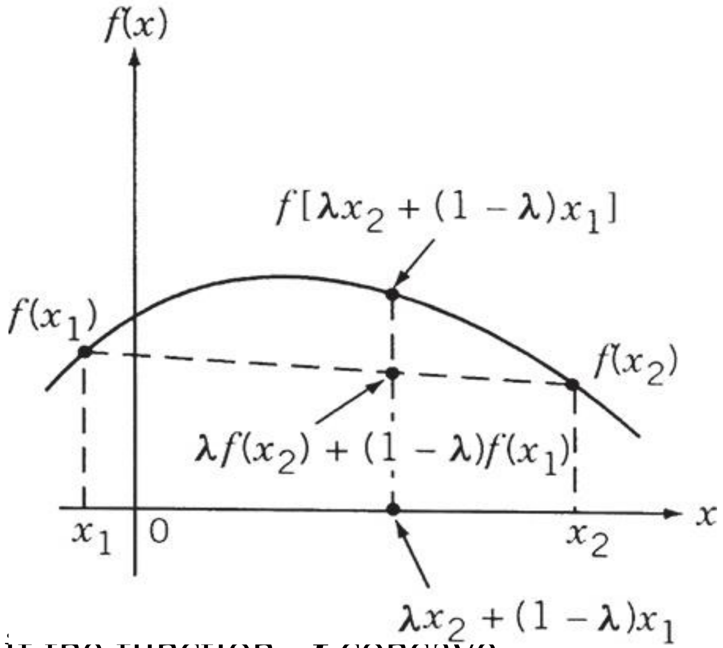
A function $f : \Re^n \mapsto \Re$ *is called* **convex** *if for every* $\mathbf{x}, \mathbf{y} \in \Re^n$, *and every* $\lambda \in [0, 1]$, *we have*

$$f(\lambda \mathbf{x} + (1 - \lambda)\mathbf{y}) \leq \lambda f(\mathbf{x}) + (1 - \lambda) f(\mathbf{y}).$$

A function $f : \Re^n \mapsto \Re$ *is called* **concave** *if for every* $\mathbf{x}, \mathbf{y} \in \Re^n$, *and every* $\lambda \in [0, 1]$, *we have*

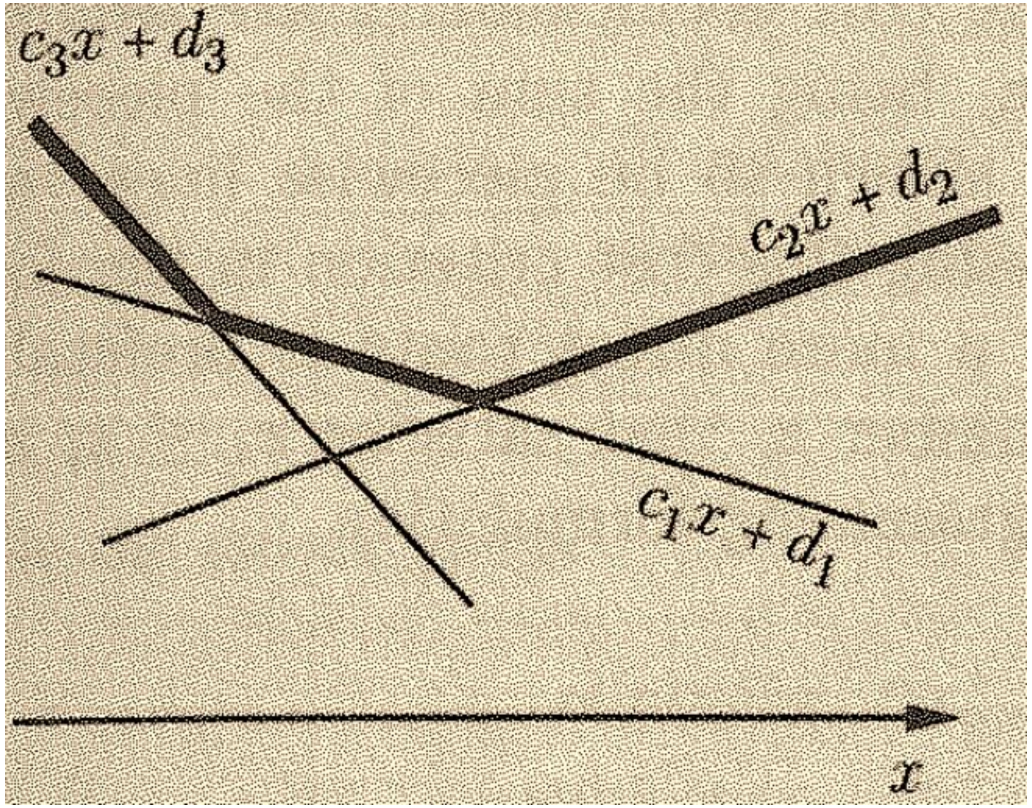
$$f(\lambda \mathbf{x} + (1 - \lambda)\mathbf{y}) \geq \lambda f(\mathbf{x}) + (1 - \lambda) f(\mathbf{y}).$$



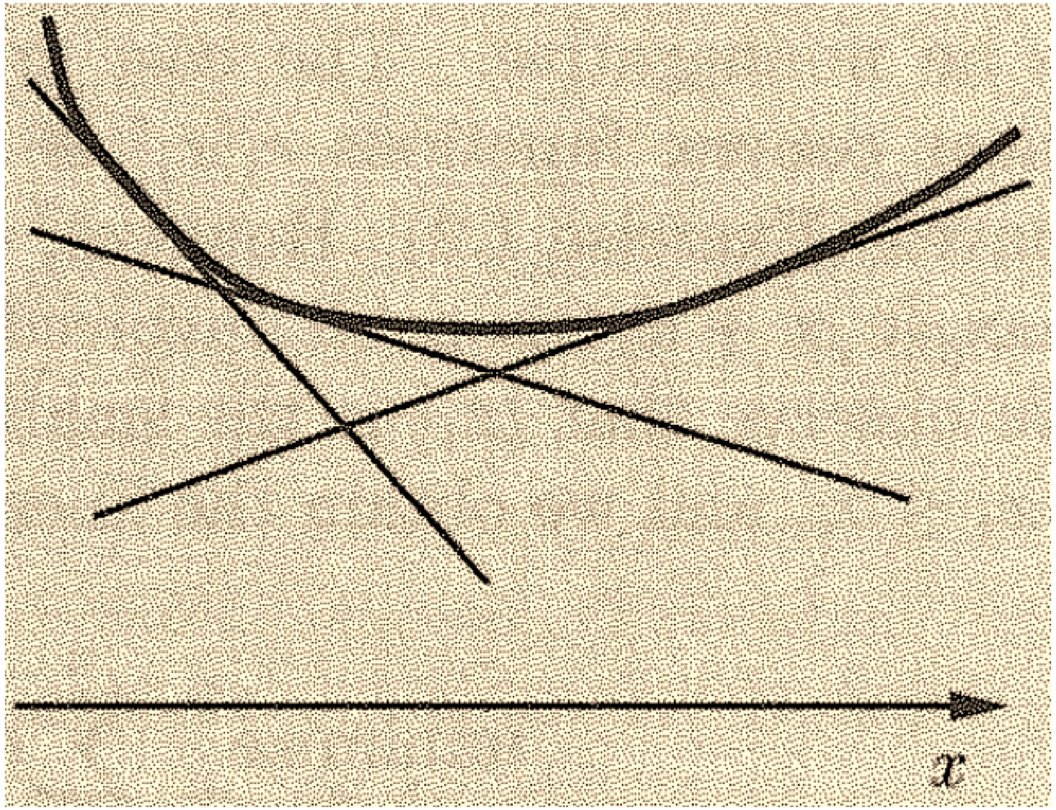


- A function f is convex if and only if the function $-f$ concave.
- A function $f(x) = a_0 + \sum_{i=1}^{n} a_i x_i$ (where $a_0, a_1 \ldots a_n$ are scalars), called a **Affine function** → It is both convex & concave.

Piecewise Linear Convex Objective Functions



A piecewise linear convex function of a single variable



An approximation of a convex function by a piecewise linear convex function

- An important class of Optimization problems with nonlinear objective function can be cast as Linear programming problems

Data Fitting Problem

We are given $\boldsymbol{n}$ data points of the form $(\boldsymbol{x_i}, \boldsymbol{y_i})$, $\boldsymbol{i = 1, 2, \dots, n}$, and wish to build a model that predicts the value of the variable $\boldsymbol{y}$ from knowledge of the vector $\boldsymbol{X}$.

- In such a situation, one often uses a linear model of the form $\boldsymbol{y = X'a}$, where $\boldsymbol{a}$ is a parameter vector to be determined.
- Given a particular parameter vector $\boldsymbol{a}$, the *residual, or prediction error*, at the $\boldsymbol{i^{th}}$ data point is defined as $|y_i - X_i'a|$. Choose a model that results in small residuals.
- $|y_i - X_i'a| = e_i$

LP Model:

Minimize:

$$\sum_{i=1}^{n} \boldsymbol{e_i}$$

Subject to:

$$\boldsymbol{y_i - X_i'a \le e_i} \qquad \boldsymbol{i = 1, 2, \dots, n}$$

$$\boldsymbol{-y_i + X_i'a \le e_i} \qquad \boldsymbol{i = 1, 2, \dots, n}$$

Quadratic Model:

$$\sum_{i=1}^{n} \boldsymbol{e_i^2} = \sum_{i=1}^{n} (\boldsymbol{y_i - X_i'a})^2$$

in order to obtain a “best least squares fit“; can be solved using Calculus methods.

Areas of Application of Linear Programming

INDUSTRIAL APPLICATIONS

- Product mix problems
- Blending Problems
- Productions Scheduling Problems
- Trim Loss Problems
- Assembly Line Balancing
- Make–or–Buy (Sub–Contracting) Problems

MANAGEMENT APPLICATIONS

- Media Selection Problems
- Portfolio Selection Problems
- Profit Planning Problems
- Man–Power Scheduling Problem
- Transportation Problems
- Assignment Problems
- Production Planning

MISCELLANEOUS APPLICATIONS

- Diet Problems
- Agriculture Problems
- Flight Scheduling Problems
- Environment Protection
- Facilities Location
- Data Envelopment Analysis

LP SOLUTION: GRAPHICAL METHOD

A Carpenter wants to maximize his profit; the LP model shows his objective and constraints:

Maximize: $5\ X_1 + 3\ X_2$

(Where X_1 = Number of Chairs, X_2 = Number of Tables)

Subject to:

$$2\ X_1 + X_2 \leq 40$$
$$X_1 + 2\ X_2 \leq 50$$
$$X_1,\ X_2 \geq 0$$

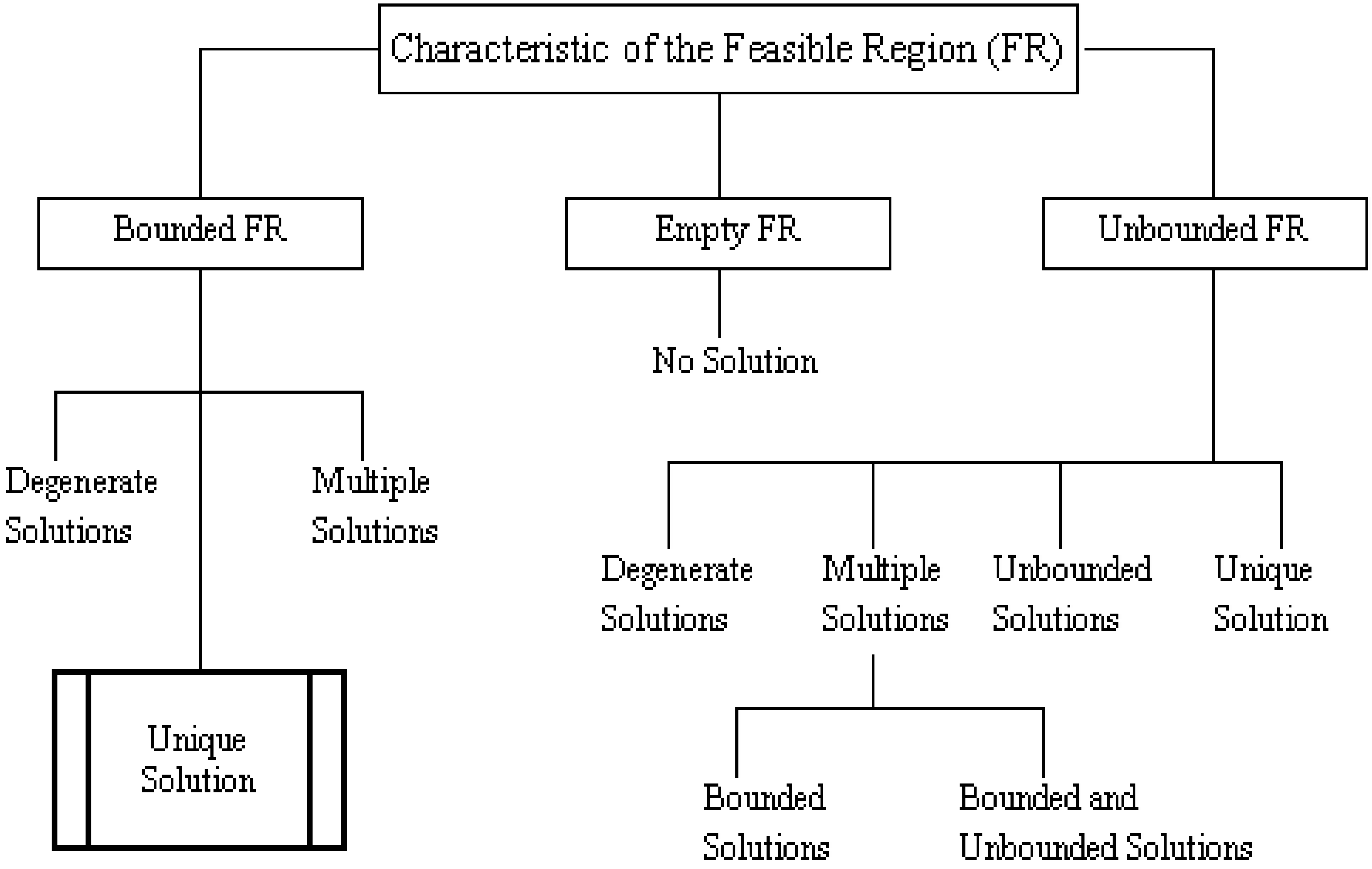
Solve the following LP problem graphically:

Minimize: $6\ X_1 + 14\ X_2$

Subject to:

$$5X_1 + 4X_2 \geq 60$$
$$3X_1 + 7X_2 \leq 84$$
$$X_1 + 2X_2 \geq 18$$
$$X_1,\ X_2 \geq 0$$

CLASSIFICATION OF LP SOLUTION



CLASSIFICATION OF LP SOLUTION…

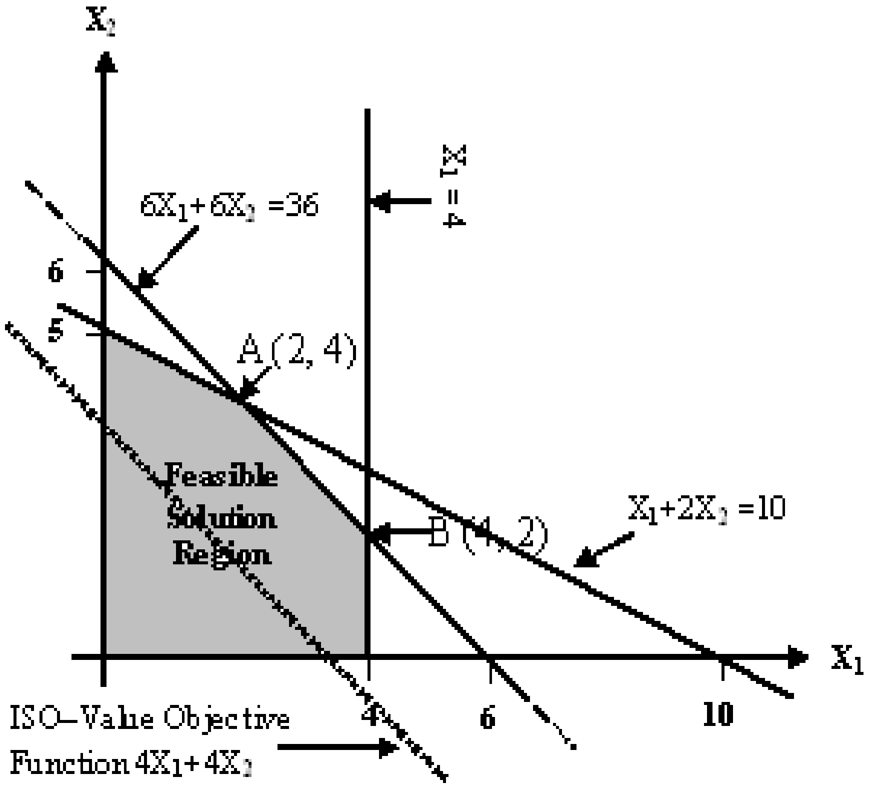
- **Multiple / Alternate Optimum Solution:**
  - More than One Optimal Solution:
    - Two or more optimal solutions may exist, and
    - This actually allows management great flexibility in deciding which combination to select.

Graph Plot:
1st constraint: x1=0, x2=5 (0,5)
x2=0, x1=10 (10,0)
2nd constraint: x1=0, x2=6 (0,6)
x2=0, x1=6 (6,0)

Maximize: $Z = 4X_1 + 4X_2$
Subject to:

$$X1 + 2X2 \leq 10$$
$$6X1 + 6X2 \leq 36$$
$$X1 \leq 4$$
$$X1, X2 \geq 0$$

for greater than shade the area above, for less than shade the area below



To find optimum solution plug the corner points of feasible region in the optimization function

As, at point 'A' (2,4) the value of the objective function is: $Z = 4(2) + 4(4) = 24$, and at point 'B' (4,2) the value of the objective function is: $Z = 4(2) + 4(4) = 24$.

At the solution there can be an overflow decision variable, slack decision variable (extra left), binding (exactly equal)

CLASSIFICATION OF LP SOLUTION…

- **Unbounded Solution:** In some LP models, the values of the variables may be increased indefinitely without violating any of the constraints in this result the solution space becomes unbounded so As a result the value of the objective function may increase (in Maximization Case) or decrease (in Minimization Case) indefinitely. But it is wrong to conclude that just because the solution space is unbounded then solution also unbounded. The solution space may be unbounded but the solution may be finite.

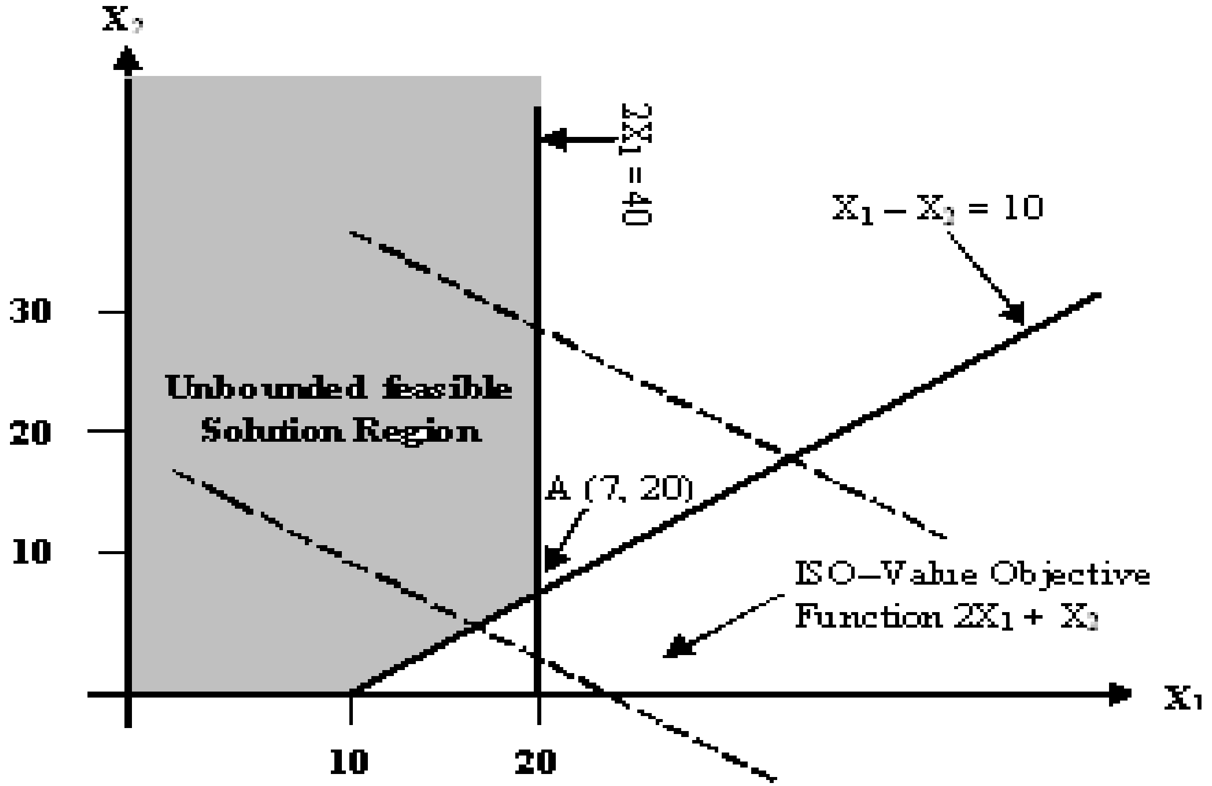
Maximize: $Z = 2X_1 + X_2$

Subject to:

$X_1 - X_2 \leq 10$

$2X_1 \leq 40$

$X_1, X_2 \geq 0$



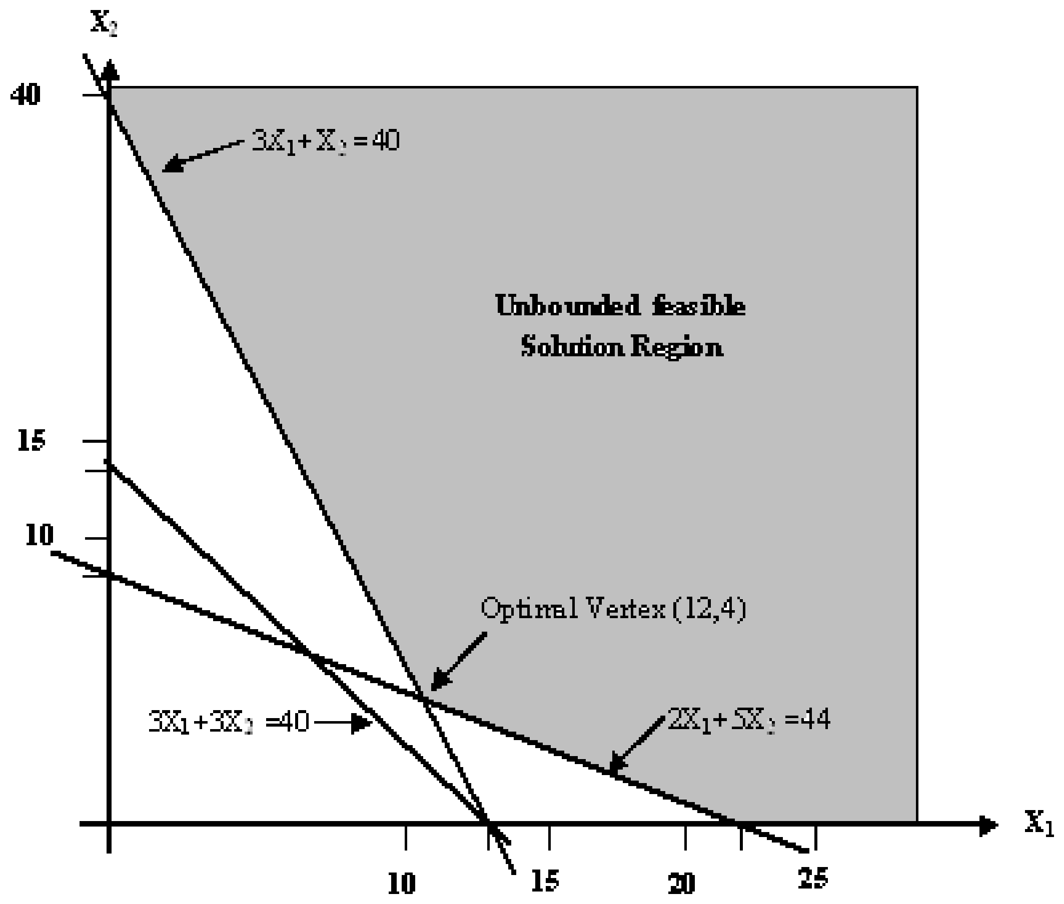
CLASSIFICATION OF LP SOLUTION…

Unbounded Solution: (Example # 2)

Minimize: $6X_1 + 4X_2$

Subject to:

$$3X_1 + 3X_2 \geq 40$$
$$3X_1 + X_2 \geq 40$$
$$2X_1 + 5X_2 \geq 44$$
$$X_1, X_2 \geq 0$$



Optimal value to be 88, at $X_1 = 12$, and $X_2 = 4$.

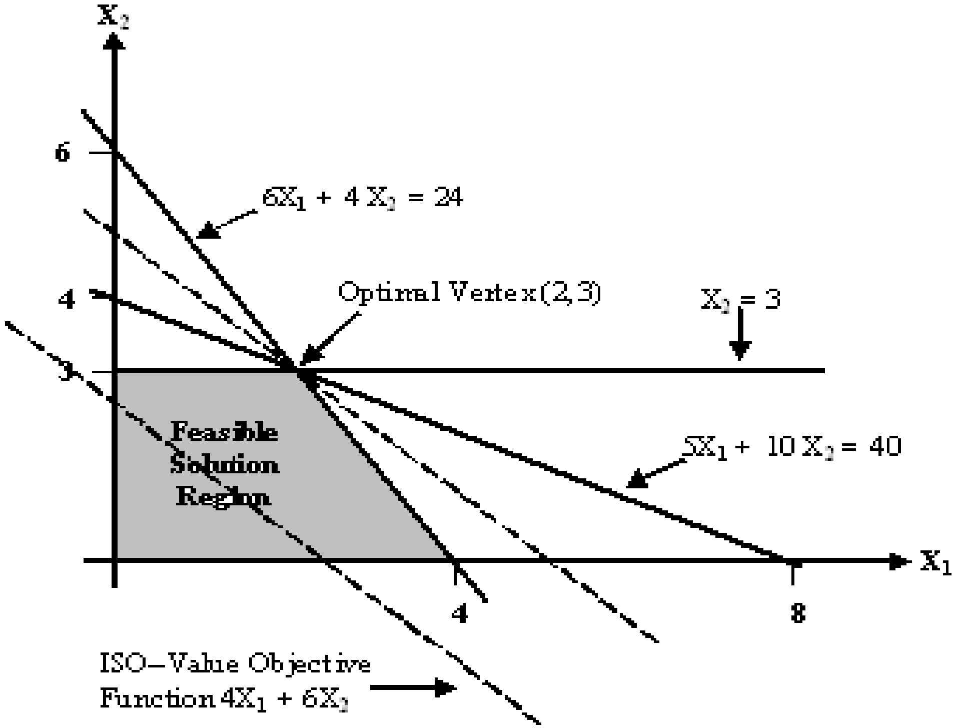
CLASSIFICATION OF LP SOLUTION…

- **Degeneracy:** In LP problems, intersection of two constraints will define a corner point of the feasible region. But if more than two constraints pass through any one of the corner points of the feasible region, excess constraints will not serve any purpose, and therefore they act as redundant constraints, so redundant constraints create the Degeneracy.

Maximize: $Z = 4X_1 + 6X_2$

Subject to:

$$6X_1 + 4X_2 \leq 24$$
$$X_2 \leq 3$$
$$5X_1 + 10X_2 \leq 40$$
$$X_1, X_2 \geq 0$$



Optimal value to be 26, at $X_1 = 2$, and $X_2 = 3$.

CLASSIFICATION OF LP SOLUTION…

- **Non–existing / Infeasible Solution:** A linear programming problem is infeasible if there is no solution exists that satisfies all of the constraints.

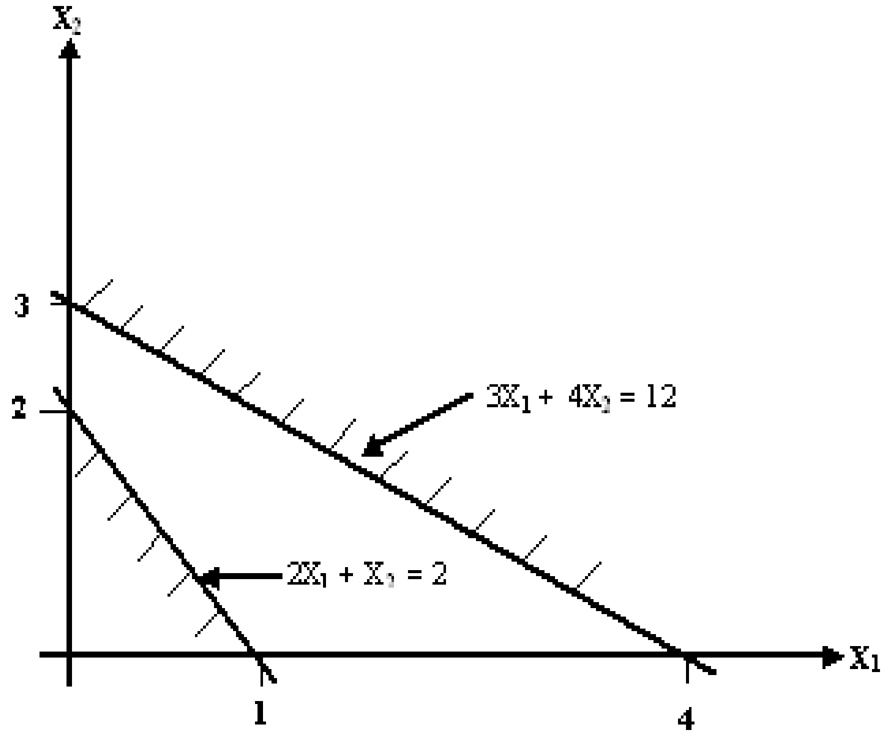
Maximize: $Z = 3X_1 + 2X_2$

Subject to:

$2X_1 + X_2 \leq 2$

$3X_1 + 4X_2 \geq 12$

$X_1, X_2 \geq 0$



QUESTIONS