OPERATIONS RESEARCH

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TRANSPORTATION PROBLEMS

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TRANSPORTATION PROBLEMS

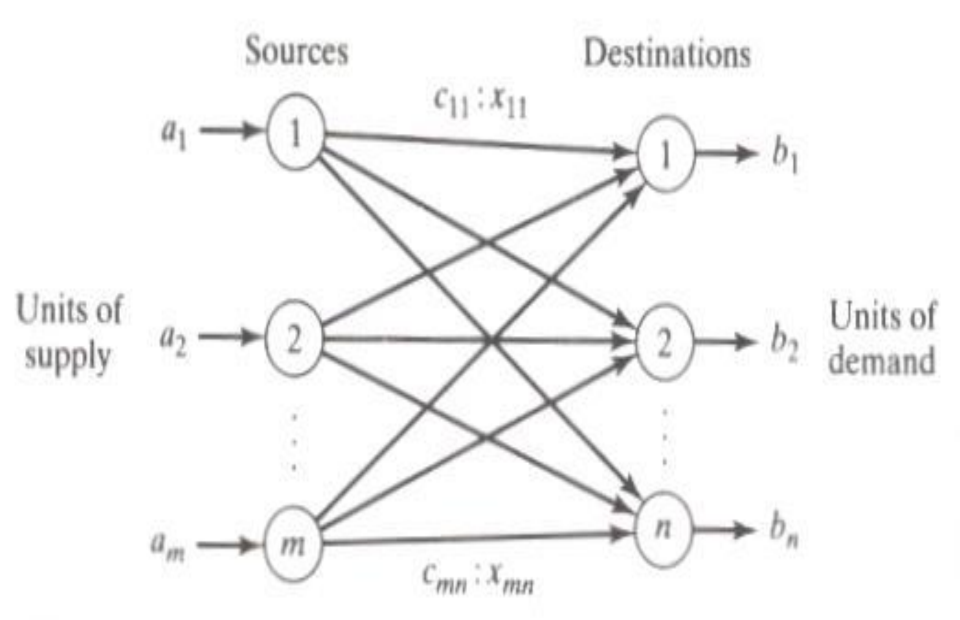
- The basic transportation problem was originally developed by F.L. Hitchcock in 1941 in his study entitled "The distribution of a product from several sources to numerous locations".
- Transportation problem is a special type of linear programming problem
  - in which products/goods are transported from a set of sources to a set of destinations subject to the supply and demand of the source and destination, respectively; This must be done in such a way as to minimize the cost.

SOURCE	DESTINATION	COMMODITY	OBJECTIVE
Plants	Markets	Finished goods / products	Minimizing total cost of shipping
Plants	Finished goods Warehouses	Finished goods / products	Minimizing total cost of shipping
Raw material warehouses	Plants	Raw materials	Minimizing total cost of shipping
Suppliers	Plants	Raw materials	Minimizing total cost of shipping

TRANSPORTATION PROBLEMS

- To illustrate a typical transportation model, suppose:
  - 'm' factories (Sources) supply certain products/goods to 'n' warehouses (Destination).
  - factory–i (i = 1,2, . . ., m) produce 'a_i' units and the warehouse– j (j = 1,2, . . ., n) requires 'b_i' units
  - the cost of transportation from factory–i to warehouse–j is 'C_{ij}'
  - the decision variables 'X_{ij}' being the amount transported from the factory–i to the warehouse–j
  - Our objective is to find the transportation pattern that will minimize the total transportation cost.

A GRAPHICAL REPRESENTATION OF TRANSPORTATION MODEL



LP FORMULATION OF THE TRANSPORTATION PROBLEMS

Let i index the sources, and j the destinations
m = # of sources, n = # destinations

Given:

S_i = quantity of goods available at source i

D_j = quantity of goods required at destination j

C_{ij} = unit cost of shipping goods from source i to destination j

Find:

X_{ij} = quantity of goods to be shipped from source i to destination j

Minimize $\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij}$

subject to

$$\sum_{j=1}^{n} X_{ij} \le S_i \quad \text{for } i=1, \dots m$$

no more is shipped from source i than is available

$$\sum_{i=1}^{m} X_{ij} \ge D_j, \quad j=1, \dots n$$

requirement at dstn. j is met

$$X_{ij} \ge 0, \quad \text{all } i \text{ \& } j$$

This is an LP with: $m \times n$ variables
$m + n$ constraints
(not including nonnegativity)

LP FORMULATION OF THE TRANSPORTATION PROBLEMS (Cont…)

The standard, "balanced", transportation problem has total supply = total demand

$$\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$$

so that all constraints will be "tight" at a feasible solution, i.e.,

$$\sum_{j=1}^{n} X_{ij} = S_i \quad \text{for } i=1, \ldots m$$

$$\sum_{i=1}^{m} X_{ij} = D_j \quad \text{for } j=1, \ldots n$$

TRANSPORTATION PROBLEMS

TRANSPORTATION PROBLEM IN THE TABULAR FORM:

		DESTINATIONS (j)					
		1	2	3	. . .	n	Supply
SOURCE (i)	1	C_{11}	C_{12}	C_{13}	. . .	C_{1n}	a_1
		X_{11}	X_{12}	X_{13}		X_{1n}	
	2	C_{21}	C_{12}	C_{12}	. . .	C_{12}	a_2
		X_{12}	X_{12}	X_{12}		X_{12}	
	. . .	. . .	. . .	. . .	. . .	. . .	. . .
	m	C_{m1}	C_{m2}	C_{m3}	. . .	C_{m3}	a_m
		X_{m1}	X_{m2}	X_{m3}		X_{m3}	
	Demand	b_1	b_2	b_3	. . .	b_n	$\sum a_i = \sum b_j$

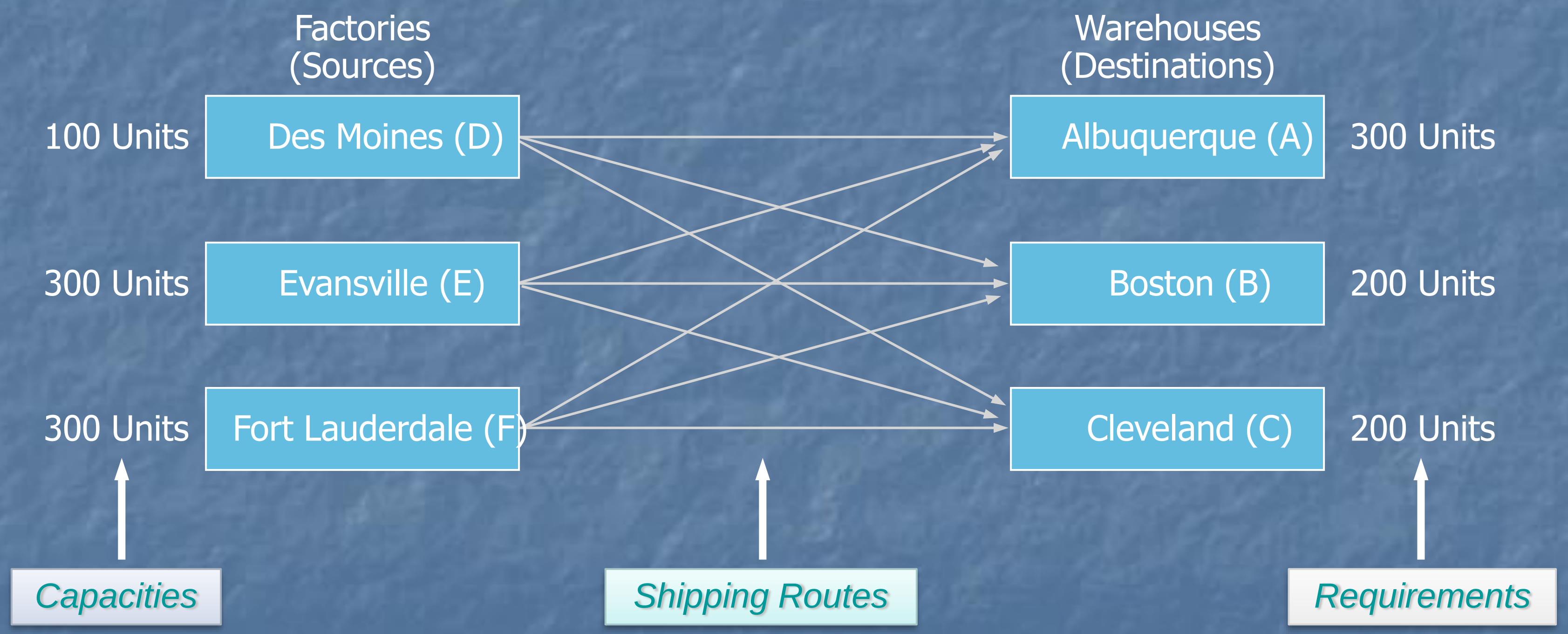
<u>TYPES OF THE TRANSPORTATION PROBLEM</u>:

There are two types of transportation problems.

1. Balanced Transportation Problem
   - ❖ the total supply at the sources is equal to the total demand at the destinations.
2. Unbalanced Transportation Problem
   - ❖ the total supply at the sources is not equal to the total demand at the destinations.

TRANSPORTATION PROBLEMS

- Example of a transportation problem in a network format



TRANSPORTATION PROBLEMS

- Transportation table for Executive Furniture

FROM \ TO	WAREHOUSE AT ALBUQUERQUE	WAREHOUSE AT BOSTON	WAREHOUSE AT CLEVELAND	FACTORY CAPACITY
DES MOINES FACTORY	\$5	\$4	\$3	100
EVANSVILLE FACTORY	\$8	\$4	\$3	300
FORT LAUDERDALE FACTORY	\$9	\$7	\$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Des Moines capacity constraint

Cost of shipping 1 unit from Fort Lauderdale factory to Boston warehouse

Cleveland warehouse demand

Total supply and demand

Cell representing a source-to-destination (Evansville to Cleveland) shipping assignment that could be made

TRANSPORTATION PROBLEMS

- **DEFINITIONS**
  - **FEASIBLE SOLUTION:**
    - Any set of non–negative allocations (X_{ij}>0) which satisfies the row and column sum is called a feasible solution.
  - **BASIC FEASIBLE SOLUTION(BFS):**
    - A feasible solution is called a BFS <u>if the number of non–negative allocations is equal to m+n–1</u> where 'm' is the number of rows, 'n' the number of columns in a transportation table.
      - <u>**DEGENERATE BASIC FEASIBLE SOLUTION:**</u> If a BFS contains less than m+n–1 non–negative allocations, it is said to be degenerate.
  - **OPTIMAL SOLUTION:**
    - A feasible solution is said to be optimal if it minimizes the total transportation cost. This is done through successive improvements to the initial basic feasible solution until no further decrease in transportation cost is possible.

TRANSPORTATION PROBLEMS:
METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

- North–West Corner / Upper – Left Corner Method
- Least Cost Method / Matrix Minima Method
- Vogel's Approximation Method / Penalty Method

- The three methods differ in the 'quality' of the starting basic solution they produce
- In general, the Vogel method yields the best starting basic solution, and the northwest-corner method yields the worst

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

NORTH–WEST CORNER METHOD

The method starts at the northwest-corner cell of the table

1. Start at the cell in the upper left-hand corner of the transportation table for a shipment, i.e., cell (1,1).
2. Compare available supply and demand for this cell. Allocate smaller of the two values in this cell. Encircle this allocation. Reduce the available supply and demand by this vale.
3. Move to the next cell according to the following scheme:
   1. If the supply exceeds the demand, the next cell is the adjacent cell in the row, i.e., $c(1,1) \longrightarrow c(1,2)$.
   2. If the demand exceeds supply, the next cell is the adjacent cell in the column, i.e., $c(1,1) \longrightarrow c(2,1)$.
   3. If the demand equals supply, (in other words there is a tie) the next cell is the adjacent cell diagonally, i.e., $c(1,1) \longrightarrow c(2,2)$.
4. Return to step 1.

- Repeat the process until all the supply and demand restrictions are satisfied

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

NORTH–WEST CORNER METHOD

1. Beginning in the upper left hand corner, we assign 100 units from Des Moines to Albuquerque. This exhaust the supply from Des Moines but leaves Albuquerque 200 desks short. We move to the second row in the same column.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 | $5	$4	$3	100
EVANSVILLE (*E*)	$8	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

NORTH–WEST CORNER METHOD

2. Assign 200 units from Evansville to Albuquerque. This meets Albuquerque's demand. Evansville has 100 units remaining so we move to the right to the next column of the second row.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	$5 100	$4	$3	100
EVANSVILLE (*E*)	$8 200	$4	$3	300
FORT LAUDERDALE (*F*)	$9	$7	$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

NORTH–WEST CORNER METHOD

3. Assign 100 units from Evansville to Boston. The Evansville supply has now been exhausted but Boston is still 100 units short. We move down vertically to the next row in the Boston column.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 | $5	| $4	| $3	100
EVANSVILLE (*E*)	200 | $8	100 | $4	| $3	300
FORT LAUDERDALE (*F*)	| $9	| $7	| $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

NORTH–WEST CORNER METHOD

4. Assign 100 units from Fort Lauderdale to Boston. This fulfills Boston's demand and Fort Lauderdale still has 200 units available.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 | $5	| $4	| $3	100
EVANSVILLE (*E*)	200 | $8	100 | $4	| $3	300
FORT LAUDERDALE (*F*)	| $9	100 | $7	| $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

NORTH–WEST CORNER METHOD

5. Assign 200 units from Fort Lauderdale to Cleveland. This exhausts Fort Lauderdale's supply and Cleveland's demand. The initial shipment schedule is now complete.

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 / $5	$4	$3	100
EVANSVILLE (*E*)	200 / $8	100 / $4	$3	300
FORT LAUDERDALE (*F*)	$9	100 / $7	200 / $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS:
METHODS FOR INITIAL BASIC FEASIBLE SOLUTION
NORTH–WEST CORNER METHOD

- We can easily compute the cost of this shipping assignment

ROUTE						
FROM	TO	UNITS SHIPPED	x	PER UNIT COST ($)	=	TOTAL COST ($)
D	*A*	100		5		500
E	*A*	200		8		1,600
E	*B*	100		4		400
F	*B*	100		7		700
F	*C*	200		5		1,000
						4,200

- This solution is feasible but we need to check to see if it is optimal

TRANSPORTATION PROBLEMS:

METHODS FOR INITIAL BASIC FEASIBLE SOLUTION

LEAST COST METHOD

1. Start at the cell with the least transportation cost. If there is a tie, choose a cell between the tied cells arbitrarily.
2. Compare the available supply and demand for this cell. Allocate the smaller of these two values to this cell. Encircle this allocation. Subtract this value from available supply and demand. If either the supply or demand remaining equals zero, no allocation is to be made. Go to next step.
3. Move to the next cell with the least transportation cost. If there is a tie, choose the next cell arbitrarily between the tied cells.
4. Go to step 2.

- Repeat the process until all the supply and demand restrictions are satisfied.

TRANSPORTATION PROBLEMS:

VOGEL'S APPROXIMATION METHOD

1. Calculate a penalty for each row (column) by subtracting the smallest cost element in the row (column) from the next smallest cost element in the same row (column).
2. Identify the row or column with the largest penalty. If there are ties, break ties arbitrarily. However, it is recommended to break the tie is to select the value that has the lowest cost entry in its row or column.
3. Allocate as many units as possible to the variable with the least cost in the selected row (column). The maximum amount that can be allocated is the smaller of the supply or demand.
4. Adjust the supply and demand to show the allocation made. Eliminate any row (column) that has just been completely satisfied by the allocation just made.
   1. If the supply is now zero, eliminate the source.
   2. If the demand is now zero, eliminate the destination.
   3. If both the supply and demand are zero, eliminate both the source & destination .
5. Compute the new penalties for each source & destination in the revised transportation tableau formed by step–4.
6. Repeat step–2 through 5 until all supply availability has been exhausted and all demand requirements have been met. It means that the initial feasible solution is obtained.

TRANSPORTATION PROBLEMS:
METHODS FOR OPTIMAL SOLUTION

- **Stepping–Stone Method**
  - The *stepping-stone method* is an iterative technique for moving from an initial feasible solution to an optimal feasible solution
- **MODI (modified distribution) Method**
  - The MODI (*modified distribution*) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths
  - Because of this, it can often provide considerable time savings over the stepping-stone method for solving transportation problems
  - If there is a negative improvement index, then only one stepping-stone path must be found
  - This is used in the same manner as before to obtain an improved solution

TRANSPORTATION PROBLEMS:
METHODS FOR OPTIMAL SOLUTION

- **STEPPING–STONE METHOD**
  - The stepping-stone method works by testing each unused square (Non – Basic Variable) in the transportation table to see what would happen to total shipping costs if one unit of the product were tentatively shipped on an unused route

1. Select an unused square (Non – Basic Variable) to evaluate
2. Beginning at this square, trace a closed path back to the original square via squares that are currently being used with only horizontal or vertical moves allowed
3. Beginning with a plus (+) sign at the unused square, place alternate minus (–) signs and plus signs on each corner square of the closed path just traced
4. Calculate an *improvement index* by adding together the unit cost figures found in each square containing a plus sign and then subtracting the unit costs in each square containing a minus sign
5. Repeat steps 1 to 4 until an improvement index has been calculated for all unused squares. If all indices computed are greater than or equal to zero, an optimal solution has been reached. If not, it is possible to improve the current solution and decrease total shipping costs.

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

FROM \ TO	ALBUQUERQUE (*A*)	BOSTON (*B*)	CLEVELAND (*C*)	FACTORY CAPACITY
DES MOINES (*D*)	100 \$5	\$4	\$3	100
EVANSVILLE (*E*)	200 \$8	100 \$4	\$3	300
FORT LAUDERDALE (*F*)	\$9	100 \$7	200 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- For the Executive Furniture Corporation data

Steps 1 and 2. Beginning with Des Moines–Boston route we trace a closed path using only currently occupied squares, alternately placing plus and minus signs in the corners of the path

- In a *closed path*, only squares currently used (Basic Variables) for shipping can be used in turning corners
- *Only one* closed route is possible for each square we wish to test

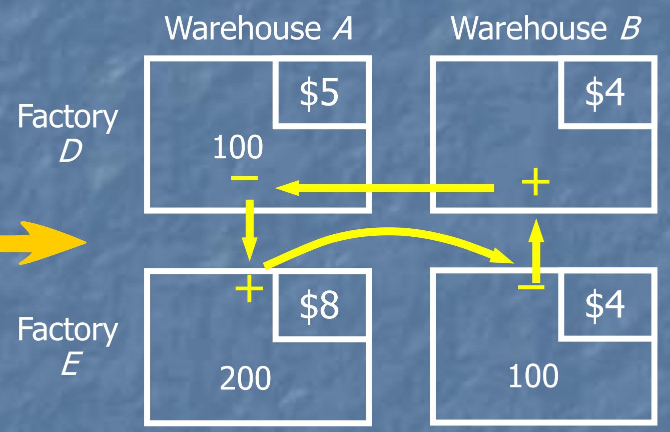
TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

Step 3. We want to test the cost-effectiveness of the Des Moines–Boston shipping route so we pretend we are shipping one desk from Des Moines to Boston and put a plus in that box

- But if we ship one *more* unit out of Des Moines we will be sending out 101 units
- Since the Des Moines factory capacity is only 100, we must ship *fewer* desks from Des Moines to Albuquerque so we place a minus sign in that box
- But that leaves Albuquerque one unit short so we must increase the shipment from Evansville to Albuquerque by one unit and so on until we complete the entire closed path

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

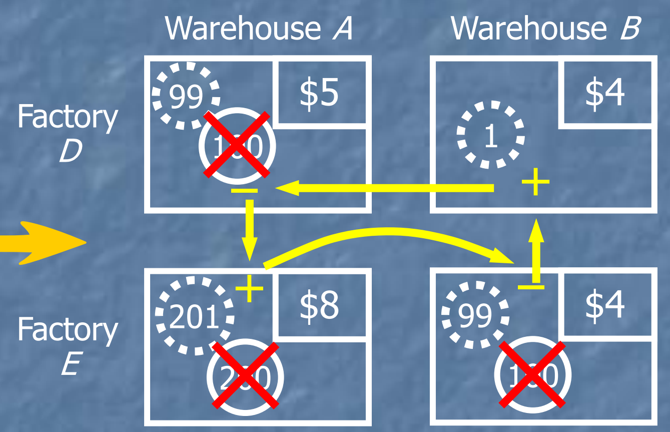
- Evaluating the unused Des Moines–Boston shipping route



TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 ($5)	($4)	($3)	100
EVANSVILLE	200 ($8)	100 ($4)	($3)	300
FORT LAUDERDALE	($9)	100 ($7)	200 ($5)	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

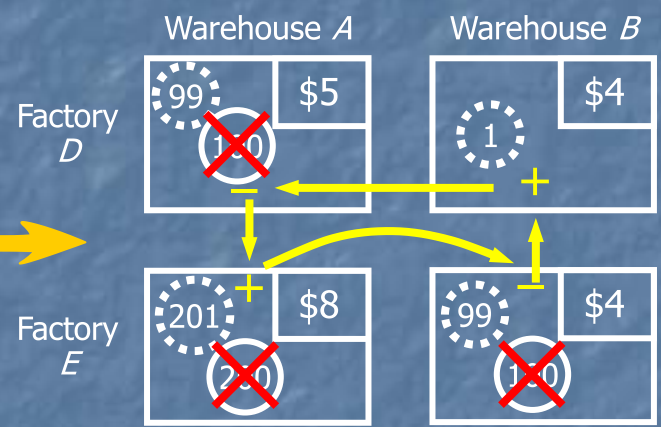
- Evaluating the unused Des Moines–Boston shipping route



TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	100 \$5	\$4	\$3	100
EVANSVILLE	200 \$8	100 \$4	\$3	300
FORT LAUDERDALE	\$9	100 \$7	200 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

- Evaluating the unused Des Moines–Boston shipping route



TO / FROM	ALBUQUERQUE	BOSTON	CLEVELAND	
DES MOINES	100 $5	$4	$3	
EVANSVILLE	200 $8	100 $4	$3	
FORT LAUDERDALE	$9	100 $7	200 $5	
WAREHOUSE REQUIREMENTS	300	200	200	700

Result of Proposed Shift in Allocation

= 1 x $4
− 1 x $5
+ 1 x $8
− 1 x $4 = +$3

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

Step 4. We can now compute an *improvement index* (I_{ij}) for the Des Moines–Boston route

- We add the costs in the squares with plus signs and subtract the costs in the squares with minus signs

Des Moines–Boston index $= I_{DB} = +\$4 - \$5 + \$5 - \$4 = +\$3$

- This means for every desk shipped via the Des Moines–Boston route, total transportation cost will *increase* by \$3 over their current level

TRANSPORTATION PROBLEMS

STEPPING–STONE METHOD

Step 5. We can now examine the Des Moines–Cleveland unused route which is slightly more difficult to draw

- Again we can only turn corners at squares that represent existing routes
- We must pass through the Evansville–Cleveland square but we can not turn there or put a + or – sign
- The closed path we will use is: + *DC* – *DA* + *EA* – *EB* + *FB* – *FC*

- Evaluating the Des Moines–Cleveland shipping route

FROM \ TO	ALBUQUERQUE	BOSTON	CLEVELAND	FACTORY CAPACITY
DES MOINES	$5 100 −	$4	Start $3 +	100
EVANSVILLE	$8 200 +	$4 100 −	$3	300
FORT LAUDERDALE	$9	$7 100 +	$5 200 −	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Des Moines–Cleveland improvement index $= I_{DC} = + \$3 - \$5 + \$8 - \$4 + \$7 - \$5 = + \$4$

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

- Opening the Des Moines–Cleveland route will not lower our total shipping costs
- Evaluating the other two routes we find

Evansville-Cleveland index $= I_{EC} = + \$3 - \$4 + \$7 - \$5 = + \$1$

- The closed path is + *EC* – *EB* + *FB* – *FC*

Fort Lauderdale–Albuquerque index $= I_{FA} = + \$9 - \$7 + \$4 - \$8 = - \$2$

- The closed path is + *FA* – *FB* + *EB* – *EA*

- ***So opening the <u>Fort Lauderdale-Albuquerque</u> route will lower our total transportation costs***

TRANSPORTATION PROBLEMS
STEPPING–STONE METHOD

- In the Executive Furniture problem there is only one unused route with a negative index (Fort Lauderdale-Albuquerque)
- If there was more than one route with a negative index, we would choose the one with the largest improvement
- We now want to ship the maximum allowable number of units on the new route
- The quantity to ship is found by referring to the closed path of plus and minus signs for the new route and selecting the *smallest number* found in those squares containing minus signs
- To obtain a new solution, that number is added to all squares on the closed path with plus signs and subtracted from all squares the closed path with minus signs
- All other squares are unchanged
- In this case, the maximum number that can be shipped is 100 desks as this is the smallest value in a box with a negative sign (*FB* route)
- We add 100 units to the *FA* and *EB* routes and subtract 100 from *FB* and *EA* routes
- This leaves balanced rows and columns and an improved solution

STEPPING–STONE METHOD

- Stepping-stone path used to evaluate route *FA*

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	200 $8 −	100 $4 +	$3	300
F	+ $9	100 $7 −	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- Second solution to the Executive Furniture problem

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 $5	$4	$3	100
E	100 $8	200 $4	$3	300
F	100 $9	$7	200 $5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

Total shipping costs have been reduced by (100 units) x ($2 saved per unit) and now equals $4,000

STEPPING–STONE METHOD

- This second solution may or may not be optimal
- To determine whether further improvement is possible, we return to the first five steps to test each square that is *now* unused
- The four new improvement indices are

D to *B* = I_{DB} = + \$4 – \$5 + \$8 – \$4 = + \$3
(closed path: + *DB* – *DA* + *EA* – *EB*)

D to *C* = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2
(closed path: + *DC* – *DA* + *FA* – *FC*)

E to *C* = I_{EC} = + \$3 – \$8 + \$9 – \$5 = – \$1
(closed path: + *EC* – *EA* + *FA* – *FC*)

F to *B* = I_{FB} = + \$7 – \$4 + \$8 – \$9 = + \$2
(closed path: + *FB* – *EB* + *EA* – *FA*)

STEPPING–STONE METHOD

- Path to evaluate for the *EC* route

FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
D	100 \$5	\$4	\$3	100
E	100 \$8 −	200 \$4	Start \$3 +	300
F	100 \$9 +	\$7	200 \$5 −	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- An improvement can be made by shipping the maximum allowable number of units from *E* to *C*

- **Total cost of third solution**

ROUTE						
FROM	TO	DESKS SHIPPED	x	PER UNIT COST (\$)	=	TOTAL COST (\$)
D	*A*	100		5		500
E	*B*	200		4		800
E	*C*	100		3		300
F	*A*	200		9		1,800
F	*C*	100		5		500
						3,900

STEPPING–STONE METHOD

- Third and optimal solution

FROM \ TO	A	B	C	FACTORY CAPACITY
D	100 \$5	\$4	\$3	100
E	\$8	200 \$4	100 \$3	300
F	200 \$9	\$7	100 \$5	300
WAREHOUSE REQUIREMENTS	300	200	200	700

- This solution is optimal as the improvement indices that can be computed are all greater than or equal to zero

D to $B = I_{DB} = + \$4 – \$5 + \$9 – \$5 + \$3 – \$4 = + \$2$

(closed path: + *DB* – *DA* + *FA* – *FC* + *EC* – *EB*)

D to $C = I_{DC} = + \$3 – \$5 + \$9 – \$5 = + \$2$

(closed path: + *DC* – *DA* + *FA* – *FC*)

E to $A = I_{EA} = + \$8 – \$9 + \$5 – \$3 = + \$1$

(closed path: + *EA* – *FA* + *FC* – *EC*)

F to $B = I_{FB} = + \$7 – \$5 + \$3 – \$4 = + \$1$

(closed path: + *FB* – *FC* + *EC* – *EB*)

SUMMARY OF STEPS IN TRANSPORTATION ALGORITHM (MINIMIZATION)

1. Set up a balanced transportation table
2. Develop initial solution using either the northwest corner method, Least Cost Method or Vogel's approximation method
3. Calculate an improvement index for each empty cell using either the stepping-stone method or the MODI method. If improvement indices are all nonnegative, stop as the optimal solution has been found. If any index is negative, continue to step 4.
4. Select the cell with the improvement index indicating the greatest decrease in cost. Fill this cell using the stepping-stone path and go to step 3.

MODI (modified distribution) Method

- The MODI (*modified distribution*) method allows us to compute improvement indices quickly for each unused square without drawing all of the closed paths
- Because of this, it can often provide considerable time savings over the stepping-stone method for solving transportation problems
- If there is a negative improvement index, then only one stepping-stone path must be found
- This is used in the same manner as before to obtain an improved solution

How to Use the MODI Approach?

- In applying the MODI method, we begin with an initial solution obtained by using the northwest corner rule
- We now compute a value for each row (call the values R_1, R_2, R_3 if there are three rows) and for each column (K_1, K_2, K_3) in the transportation table
- In general we let

R_i = value for assigned row i

K_j = value for assigned column j

C_{ij} = cost in square ij (cost of shipping from source i to destination j)

Five Steps in the MODI Method

1. Compute the values for each row and column, set

   $$R_i + K_j = C_{ij}$$

   but only for those squares that are currently used or occupied
2. After all equations have been written, set $R_1 = 0$
3. Solve the system of equations for R and K values
4. Compute the improvement index for each unused square by the formula

   $$\text{Improvement Index } (I_{ij}) = C_{ij} - R_i - K_j$$
5. Select the best negative index and proceed to solve the problem as you did using the stepping-stone method

Solving the Executive Furniture Corporation Problem with MODI

- The initial northwest corner solution is repeated in below Table
- Note that to use the MODI method we have added the R_is (rows) and K_js (columns)

	K_j	K_1	K_2	K_3	
R_i	FROM \ TO	*A*	*B*	*C*	FACTORY CAPACITY
R_1	*D*	100 | $5	| $4	| $3	100
R_2	*E*	200 | $8	100 | $4	| $3	300
R_3	*F*	| $9	100 | $7	200 | $5	300
	WAREHOUSE REQUIREMENTS	300	200	200	700

Solving the Executive Furniture Corporation Problem with MODI

- The first step is to set up an equation for each occupied square (Basic Variables)
- By setting $R_1 = 0$ we can easily solve for K_1, R_2, K_2, R_3, and K_3

(1) $R_1 + K_1 = 5$	$0 + K_1 = 5$	$K_1 = 5$
(2) $R_2 + K_1 = 8$	$R_2 + 5 = 8$	$R_2 = 3$
(3) $R_2 + K_2 = 4$	$3 + K_2 = 4$	$K_2 = 1$
(4) $R_3 + K_2 = 7$	$R_3 + 1 = 7$	$R_3 = 6$
(5) $R_3 + K_3 = 5$	$6 + K_3 = 5$	$K_3 = -1$

Solving the Executive Furniture Corporation Problem with MODI

- The next step is to compute the improvement index for each unused cell (Non–Basic Variables) using the formula

$$\text{Improvement index } (I_{ij}) = C_{ij} - R_i - K_j$$

- We have

Des Moines-Boston index	$I_{DB} = C_{12} - R_1 - K_2 = 4 - 0 - 1 = +\$3$
Des Moines-Cleveland index	$I_{DC} = C_{13} - R_1 - K_3 = 3 - 0 - (-1) = +\$4$
Evansville-Cleveland index	$I_{EC} = C_{23} - R_2 - K_3 = 3 - 3 - (-1) = +\$1$
Fort Lauderdale-Albuquerque index	$I_{FA} = C_{31} - R_3 - K_1 = 9 - 6 - 5 = -\$2$

Solving the Executive Furniture Corporation Problem with MODI

- The steps we follow to develop an improved solution after the improvement indices have been computed are
  1. Beginning at the square with the best improvement index, trace a closed path back to the original square via squares that are currently being used
  2. Beginning with a plus sign at the unused square, place alternate minus signs and plus signs on each corner square of the closed path just traced
  3. Select the smallest quantity found in those squares containing the minus signs and add that number to all squares on the closed path with plus signs; subtract the number from squares with minus signs
  4. Compute new improvement indices for this new solution using the MODI method
     - Note that new R_i and K_j values must be calculated
- Follow this procedure for the second and third solutions

PRACTICE QUESTION # 1

- Punjab Flour Mill has four branches A, B, C & D and four warehouses 1, 2, 3, and 4. Production, demand and transportation costs are given below:

PRODUCTION (TONES)	DEMAND (TONES)
A – 35	1 – 70
B – 50	2 – 30
C – 80	3 – 75
D – 65	4 – 55

Transportation Costs (in Rs):

From:	A	A	A	A	B	B	B	B	C
To:	1	2	3	4	1	2	3	4	1
Cost:	10	7	6	4	8	8	5	7	4

From:	C	C	C	D	D	D	D
To:	2	3	4	1	2	3	4
Cost:	3	6	9	7	5	4	3

Use North–West Corner, Least Cost, & Vogel's Approximation Methods to find the initial feasible solution

PRACTICE QUESTION # 2

Suppose that England, France, and Spain produce all the wheat, barley, and oats in the world. The world demand for wheat requires 125 million acres of land devoted to wheat production. Similarly, 60 million acres of land are required for barley and 75 million acres of land for oats. The total amount of land available for these purposes in England, France, and Spain is 70 million acres, 110 million acres, and 80 million acres, respectively. The number of hours of labor needed in England, France and Spain to produce an acre of wheat is 18, 13, and 16, respectively. The number of hours of labor needed in England, France, and Spain to produce an acre of barley is 15, 12, and 12, respectively. The number of hours of labor needed in England, France, and Spain to produce an acre of oats is 12, 10, and 16, respectively. The labor cost per hour in producing wheat is \$9.00, \$7.20, and \$9.90 in England, France, and Spain, respectively. The labor cost per hour in producing barley is \$8.10, \$9.00, and \$8.40 in England, France, and Spain respectively. The labor cost per hour in producing oats is \$6.90, \$7.50, and \$6.30 in England, France, and Spain, respectively. The problem is to allocate land use in each country so as to meet the world food requirement and minimize the total labor cost.

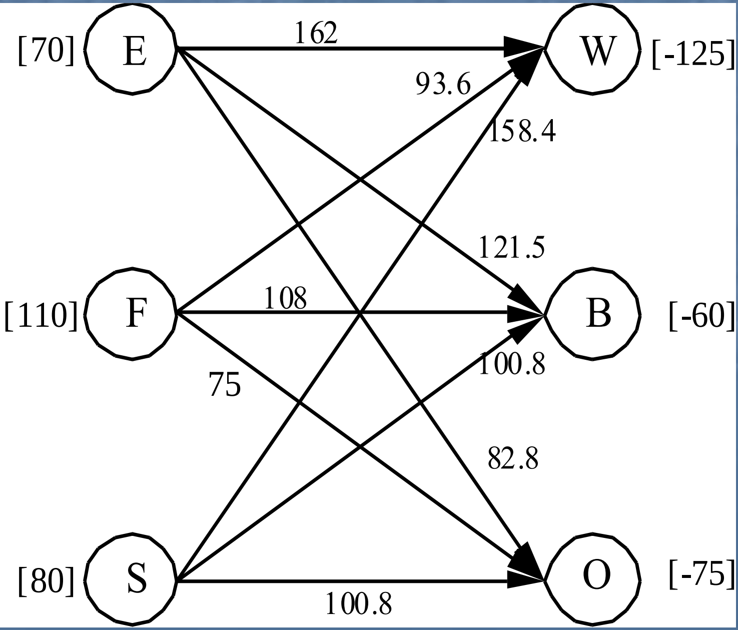
PRACTICE QUESTION (Cont...)

Formulation the problem is:

Unit Cost ($ million)

		Destination			
		Wheat	Barley	Oats	Supply
Source	England	162	121.5	82.8	70
	France	93.6	108	75	110
	Spain	158.4	100.8	100.8	80
Demand		125	60	75	

The network presentation of the Problem:



UNBALANCED TRANSPORTATION PROBLEMS

- In real-life problems, total demand is frequently not equal to total supply
- These *unbalanced problems* can be handled easily by introducing *dummy sources* or *dummy destinations*
- If total supply is greater than total demand, a dummy destination (warehouse), with demand exactly equal to the surplus, is created
- If total demand is greater than total supply, we introduce a dummy source (factory) with a supply equal to the excess of demand over supply
- In either case, shipping cost coefficients of zero are assigned to each dummy location or route as no goods will actually be shipped
- Any units assigned to a dummy destination represent excess capacity
- Any units assigned to a dummy source represent unmet demand

DEMAND LESS THAN SUPPLY

- Suppose that the Des Moines factory increases its rate of production from 100 to 250 desks
- The firm is now able to supply a total of 850 desks each period
- Warehouse requirements remain the same (700) so the row and column totals do not balance
- We add a dummy column that will represent a fake warehouse requiring 150 desks
- This is somewhat analogous to adding a slack variable
- We use the northwest corner rule and either stepping-stone or MODI to find the optimal solution

Initial solution to an unbalanced problem where demand is less than supply

FROM \ TO	A	B	C	DUMMY WAREHOUSE	TOTAL AVAILABLE
D	250 ($5)	($4)	($3)	(0)	250
E	50 ($8)	200 ($4)	50 ($3)	(0)	300
F	($9)	($7)	150 ($5)	150 (0)	300
WAREHOUSE REQUIREMENTS	300	200	200	150	850

Total cost = 250($5) + 50($8) + 200($4) + 50($3) + 150($5) + 150(0) = $3,350

New Des Moines capacity

DEMAND GREATER THAN SUPPLY

- The second type of unbalanced condition occurs when total demand is greater than total supply
- In this case we need to add a dummy row representing a fake factory
- The new factory will have a supply exactly equal to the difference between total demand and total real supply
- The shipping costs from the dummy factory to each destination will be zero

Unbalanced transportation table for Happy Sound Stereo Company

FROM \ TO	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	$6	$4	$9	200
PLANT *X*	$10	$5	$8	175
PLANT *Y*	$12	$7	$6	75
WAREHOUSE DEMAND	250	100	150	500 \ 450

Totals do not balance

DEMAND GREATER THAN SUPPLY

Initial solution to an unbalanced problem in which demand is greater than supply

TO / FROM	WAREHOUSE *A*	WAREHOUSE *B*	WAREHOUSE *C*	PLANT SUPPLY
PLANT *W*	200 | $6	| $4	| $9	200
PLANT *X*	50 | $10	100 | $5	25 | $8	175
PLANT *Y*	| $12	| $7	75 | $6	75
PLANT *Y*	| 0	| 0	50 | 0	50
WAREHOUSE DEMAND	250	100	150	500

Total cost of initial solution = 200($6) + 50($10) + 100($5) + 25($8) + 75($6) + $50(0) = $2,850

DEGENERACY IN TRANSPORTATION PROBLEMS

- *Degeneracy* occurs when the number of occupied squares or routes in a transportation table solution is less than the number of rows plus the number of columns minus 1
- Such a situation may arise in the initial solution or in any subsequent solution
- Degeneracy requires a special procedure to correct the problem since there are not enough occupied squares to trace a closed path for each unused route and it would be impossible to apply the stepping-stone method or to calculate the *R* and *K* values needed for the MODI technique
- To handle degenerate problems, create an artificially occupied cell
- That is, place a zero (representing a fake shipment) in one of the unused squares and then treat that square as if it were occupied
- The square chosen must be in such a position as to allow all stepping-stone paths to be closed
- There is usually a good deal of flexibility in selecting the unused square that will receive the zero

DEGENERACY IN AN INITIAL SOLUTION

- The Martin Shipping Company example illustrates degeneracy in an initial solution
- They have three warehouses which supply three major retail customers
- Applying the northwest corner rule the initial solution has only four occupied squares
- This is less than the amount required to use either the stepping-stone or MODI method to improve the solution (3 rows + 3 columns – 1 = 5)
- To correct this problem, place a zero in an unused square, typically one adjacent to the last filled cell

DEGENERACY IN AN INITIAL SOLUTION

- Initial solution of a degenerate problem

FROM \ TO	CUSTOMER 1	CUSTOMER 2	CUSTOMER 3	WAREHOUSE SUPPLY
WAREHOUSE 1	100 / $8	0 / $2	$6	100
WAREHOUSE 2	0 / $10	100 / $9	20 / $9	120
WAREHOUSE 3	$7	$10	80 / $7	80
CUSTOMER DEMAND	100	100	100	300

Possible choices of cells to address the degenerate solution

DEGENERACY DURING LATER SOLUTION STAGES

- A transportation problem can become degenerate after the initial solution stage if the filling of an empty square results in two or more cells becoming empty simultaneously
- This problem can occur when two or more cells with minus signs tie for the lowest quantity
- To correct this problem, place a zero in one of the previously filled cells so that only one cell becomes empty
- **<u>Bagwell Paint Example</u>**
  - After one iteration, the cost analysis at Bagwell Paint produced a transportation table that was not degenerate but was not optimal
  - The improvement indices are

factory *A* – warehouse 2 index = +2
factory *A* – warehouse 3 index = +1
factory *B* – warehouse 3 index = –15
factory *C* – warehouse 2 index = +11

Only route with a negative index

DEGENERACY DURING LATER SOLUTION STAGES

- Bagwell Paint transportation table

FROM \ TO	WAREHOUSE 1	WAREHOUSE 2	WAREHOUSE 3	FACTORY CAPACITY
FACTORY *A*	70 | $8	| $5	| $16	70
FACTORY *B*	50 | $15	80 | $10	| $7	130
FACTORY *C*	30 | $3	| $9	50 | $10	80
WAREHOUSE REQUIREMENT	150	80	50	280

DEGENERACY DURING LATER SOLUTION STAGES

- Tracing a closed path for the factory B – warehouse 3 route

FROM \ TO	WAREHOUSE 1	WAREHOUSE 3
FACTORY *B*	50 (\$15) −	+ (\$7)
FACTORY *C*	30 (\$3) +	50 (\$10) −

- This would cause two cells to drop to zero
- We need to place an artificial zero in one of these cells to avoid degeneracy

MORE THAN ONE OPTIMAL SOLUTION

- It is possible for a transportation problem to have multiple optimal solutions
- This happens when one or more of the improvement indices zero in the optimal solution
- This means that it is possible to design alternative shipping routes with the same total shipping cost
- The alternate optimal solution can be found by shipping the most to this unused square using a stepping-stone path
- In the real world, alternate optimal solutions provide management with greater flexibility in selecting and using resources

MAXIMIZATION TRANSPORTATION PROBLEMS

- If the objective in a transportation problem is to maximize profit, a minor change is required in the transportation algorithm
- Now the optimal solution is reached when all the improvement indices are negative or zero
- The cell with the largest positive improvement index is selected to be filled using a stepping-stone path
- This new solution is evaluated and the process continues until there are no positive improvement indices

UNACCEPTABLE OR PROHIBITED ROUTES

- At times there are transportation problems in which one of the sources is unable to ship to one or more of the destinations
- When this occurs, the problem is said to have an *unacceptable* or *prohibited route*
- In a minimization problem, such a prohibited route is assigned a very high cost to prevent this route from ever being used in the optimal solution
- In a maximization problem, the very high cost used in minimization problems is given a negative sign, turning it into a very bad profit

QUESTIONS