

# Assignment 4:

Ummah

Hussein

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Q.1

a.)  $R(A, B, C, D)$

$$F = \{ AB \rightarrow C, CD \rightarrow B, AD \rightarrow B, AC \rightarrow D \}$$

$$ABCD = R$$

$$ABC = R$$

$$AB = R$$

S.K: AB

b.)  $R(A, B, C, D, E)$

$$F = \{ A \rightarrow C, C \rightarrow BD, D \rightarrow A \}$$

$$ABCDE = R$$

$$ABCE = R$$

$$ACE = R$$

$$CE = R$$

Keys :  $\{ CE, AE, DE \}$

c.)  $R(A, B, C, D, E)$

$$F = \{ C \rightarrow AB, A \rightarrow E, D \rightarrow E, BD \rightarrow C, CD \rightarrow B \}$$

$$ABCDF = R$$

$$ABCD = R$$

$$ABD = R$$

$$BD = R$$

Keys :  $\{ BD, CD \}$

Q.2

a.)  $R(A, B, C, D, E, G)$

$$D \rightarrow E$$

$$ABC \rightarrow BDE$$

$$B \rightarrow C$$

$$A \rightarrow C$$

$$ABC \rightarrow G$$

$$D \rightarrow E$$

$$ABC \rightarrow B^*$$

$$ABC \rightarrow D$$

$$ABC \rightarrow E^*$$

$$B \rightarrow G$$

$$A \rightarrow C$$

$$ABC \rightarrow C^*$$

$$D \rightarrow E$$

$$ABC \rightarrow D$$

$$B \rightarrow G$$

$$A \rightarrow C$$

$$B \rightarrow G$$

$$A \rightarrow C$$

$$A \rightarrow C$$

$$ABCDEG = R$$

$$ABCDE = R$$

$$ABC = R$$

$$AB = R$$

b.)  $R(A, B, C, D, E, I)$

$$A \rightarrow C$$

$$AB \rightarrow C$$

$$C \rightarrow DI$$

$$CD \rightarrow I$$

$$EC \rightarrow AB$$

$$EI \rightarrow C$$

$$A \rightarrow C$$

$$AB \rightarrow C^*$$

$$C \rightarrow D$$

$$C \rightarrow I$$

$$CD \rightarrow I^*$$

$$EC \rightarrow A$$

$$EC \rightarrow B$$

$$EI \rightarrow C$$

$$A \rightarrow C$$

$$C \rightarrow D$$

$$C \rightarrow I$$

$$EC \rightarrow A$$

$$EC \rightarrow B$$

$$EI \rightarrow C$$

$$A \rightarrow C$$

$$C \rightarrow DI$$

$$EC \rightarrow AB$$

$$EI \rightarrow C$$

$$ABCDEI = R$$

$$ABCE = R$$

$$ACE = R$$

$$CE = R$$

$$\text{Keys: } \{CE, AE\}$$

C.)	$R(A, B, C, D, E, G)$	(d)
	$ABC \rightarrow CDEG$	$ABC \rightarrow C^*$
	$C \rightarrow E$	$ABC \rightarrow D$
	$A \rightarrow B$	$C \rightarrow E^*$
	$D \rightarrow G$	$A \rightarrow B^*$
		$ABC \rightarrow G^*$
		$D \rightarrow G$
		$C \rightarrow E$
		$A \rightarrow B$
		$D \rightarrow G$

$$ABCDEG = R \quad \text{Keys: } \{AC\}$$

$$ACDE = R$$

$$AC = R$$

Q.3

$$a) F_1 = \{A \rightarrow C, AB \rightarrow C, C \rightarrow DG, CD \rightarrow C_1, EC \rightarrow AB, EG \rightarrow C\}$$

$$F_2 = \{A \rightarrow C, C \rightarrow D, C \rightarrow C_1, EC \rightarrow A, EC \rightarrow B, EG \rightarrow C\}$$

$F_1$  covers  $F_2$

$F_1$ : closure  
 $F_2$ : LHS

- $A^+ : \{A, C, D, C_1\}$  so true ( $A \rightarrow C$ )
- $C^+ : \{C, D, C_1\}$  so true ( $C \rightarrow D$ ), ( $C \rightarrow C_1$ )
- $EC^+ : \{E, C, A, B, D, C_1\}$   
so true ( $EC \rightarrow A$ ), ( $EC \rightarrow B$ )
- $EG^+ : \{E, G, C, D, A, B\}$   
so true ( $EG \rightarrow C$ )

$F_2$  covers  $F_1$

LHS:  $\{F_2\}$   
closure:  $F_2$

- $A^+ : \{A, C, D, C_1\}$  so true ( $A \rightarrow C$ )
- $AB^+ : \{A, B, C, D, C_1\}$  so true ( $AB \rightarrow C$ )
- $C^+ : \{C, D, C_1\}$  so true ( $C \rightarrow DG$ )
- $CD^+ : \{C, D, C_1\}$  so true ( $CD \rightarrow C_1$ )
- $EC^+ : \{E, C, A, B, D, C_1\}$  so true ( $EC \rightarrow AB$ )
- $EG^+ : \{E, G, C, D, C_1, A, B\}$   
so true ( $EG \rightarrow C$ )

equivalent fds

b.)  $F_1 = \{ A \rightarrow C, AB \rightarrow C, C \rightarrow DI, CD \rightarrow I, EC \rightarrow AB, EI \rightarrow C \}$

$$F_2 = \{ A \rightarrow C, C \rightarrow D, C \rightarrow I, EC \rightarrow A, EC \rightarrow B, EI \rightarrow C \}$$

①  $F_1$  covers  $F_2$   $F_1$ : closure  
 $F_2$ : LHS

$A^+ = \{ A, C, D, I \}$  so yes  $F_2(A \rightarrow C)$  is true

$C^+ = \{ C, D, I \}$  so yes  $F_2(C \rightarrow D)$  is true,  $F_2(C \rightarrow I)$  is true

$EC^+ = \{ E, C, A, B, D, I \}$  so yes  $F_2(EC \rightarrow A), (EC \rightarrow B)$  is true

$EI^+ = \{ E, I, C \}$  so yes true  $F_2(EI \rightarrow C)$

②  $F_2$  covers  $F_1$   $F_2$ : closure  
 $F_1$ : LHS

$A^+ = \{ A, C, D, I \}$  so yes  $F_1(A \rightarrow C)$  is true

$AB^+ = \{ A, B, C, D, I \}$  so yes  $F_1(AB \rightarrow C)$  is true

$C^+ = \{ C, D, I \}$  so  $F_1(C \rightarrow DI)$  is true

$CD^+ = \{ C, D, I \}$  so  $F_1(CD \rightarrow I)$  is true

$EC^+ = \{ E, C, A, B, D, I \}$  so yes  $F_1(EC \rightarrow AB)$  is true

$EI^+ = \{ E, I, C, D, A, B \}$  so yes  $F_1(EI \rightarrow C)$  is true

equivalent fds

**Q.4**

1.)  $R(A, B, C, D, E)$

$$F : \{ A \rightarrow BC, C \rightarrow D, E \rightarrow D, BE \rightarrow A \}$$

a.)  $R_1(A, E), R_2(A, B, C), R_3(D, E)$

$R_2(A, B, C), R_1(A, E)$

$R_{23}(A, B, C, E), R_3(D, E)$

$$1) R_2 \cup R_1 : \{ A, B, C, E \}$$

$$R_{23} \cup R_3 : \{ A, B, C, E, D \}$$

$$2) R_2 \cap R_3 : \emptyset$$

$$R_{23} \cap R_3 : \{ E \}$$

3) A is S.Key in  $R_2$

E is SK in  $R_3$  **lossless**

b.)  $R_1(A, E), R_2(A, B, C), R_3(C, D)$

$R_2(A, B, C), R_3(C, D)$

$R_{23}(A, B, C, D), R_1(A, E)$

$$1) R_1 \cup R_3 : \{ A, B, C, D \}$$

$$R_{23} \cup R_1 : \{ A, B, C, D, E \}$$
 **lossless**

$$2) R_1 \cap R_3 : \emptyset$$

$$R_{23} \cap R_1 : \{ A \}$$

3) A is S.Key in  $R_1$

A is SK in  $R_{23}$

2.)  $R(A, B, C, D, E)$  ;  $F : \{ C \rightarrow A, BE \rightarrow A, D \rightarrow B \}$  ; Key :  $\{ CDE \}$

a.)  $R_1(A, C, D), R_2(C, D, E) \not\beta R_3(B, D)$

$R_1(A, C, D), R_2(C, D, E)$

$R_{12}(A, C, D, E), R_3(B, D)$

$$1) R_1 \cup R_2 : \{ A, C, D, E \}$$

$$R_{12} \cup R_3 : \{ A, B, C, D, E \}$$

$$2) R_1 \cap R_2 : \{ C, D \}$$

$$R_{12} \cap R_3 : \{ D \}$$

3) CD is SK in  $R_1$

D is SK in  $R_3$

**lossless**

b.)  $R_1(A, C, D), R_2(A, B, E) \not\rightarrow R_3(B, D)$

$R_1(A, C, D), R_2(A, B, E)$

$R_1 \cup R_2 = \{A, B, C, D, E\}$

$R_1 \cap R_2 = \{A\}$

A is not a SK in  $R_1$

$R_1(A, C, D), R_3(B, D)$

$R_1 \cup R_3 = \{A, B, C, D\}$

$R_1 \cap R_3 = \{D\}$

D is SK in  $R_3$

$R_3(A, B, C, D), R_2(A, B, E)$

$R_3 \cup R_2 = \{A, B, C, D, E\}$

$R_3 \cap R_2 = \{AB\}$

AB is not a SK in both  $R_2$   
 $\not\rightarrow R_3$

Q.5

a.)  $F = \{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$  Keys:  $\{AB\}, \{BC\}, \{BD\}$

$AB \rightarrow C$

$C^+ = \{C, D, A\}$

$C \rightarrow D$  (BCNF)

$R_1(\underline{ABC}), R_2(\underline{CDA})$

$D \rightarrow A$  (BCNF)

$C \rightarrow D$

Ans:  $R_1(\underline{BC}), R_2(\underline{DA}), R_3(\underline{CD})$

$D \rightarrow A$  (TO)

$\hookrightarrow R_3(\underline{D}, A)$

lost  $AB \rightarrow C$  so not preserved

b.)  $F = \{ A \rightarrow B, B \rightarrow C, C \rightarrow D, D \rightarrow A \}$  Keys:  $\{A\}, \{B\}$   
 $\{C\}, \{D\}$

$A \rightarrow B$   
 $B \rightarrow C$  - No BCNF violation exists  
 $C \rightarrow D$   
 $D \rightarrow A$  HNF: BCNF ; dependency preserving

#### 4.b

$R(A, B, C, D, E, F)$ , Keys:  $\{BD\}, \{CD\}$

$F: \{$

a.) HNF: 1NF

$CD \rightarrow A$

$BD \rightarrow C$

$AC \rightarrow B$  BCNF

b.)  $D^+: \{D, E, F\}$

$R_1(D, EF), R_2(ABCD)$

$D \rightarrow E$  Pd

$D \rightarrow E$

$E \rightarrow F$   $\nsubseteq$  Td

$E \rightarrow F$  (td)

↓

d.)  $AC^+ = \{A, C, B\}$

$R_3(\underline{ACB})$

↓  
 $AC \rightarrow B$

c.)  $R_{12}(\underline{E}F)$

$E \rightarrow F$

$R_1(\underline{E}F), R_2(\underline{D}, E), R_3(\underline{A}, \underline{C}, B), R_4(\underline{C}, \underline{D}, A)$

not dep preserving, lost:  $BD \rightarrow C$

4.c)  $\{A \leftarrow D, D \leftarrow C, C \leftarrow B, B \leftarrow A\} = \{D \leftarrow A, A \leftarrow B, B \leftarrow C, C \leftarrow D\}$

$R(A, B, C, D)$

first find the key : ABCD

$B \rightarrow D$

$D \rightarrow A$

a.) HNF : 1NF

b.)  $R_1(B, D, A), R_2(C, B)$

$B \rightarrow D$

$D \rightarrow A$

c.)  $R_{11}(D, A), R_{12}(B, D)$

$D \rightarrow A$

$B \rightarrow D$

d.) No BCNF violation

$R_1(\underline{D}, A)$

$R_2(\underline{B}, D)$

$R_3(\underline{B}, C)$

dependency

preserving