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Activity 2 :-

Q.1 RSA algo is used to encrypt info 3 public key to decrypt msg is $\{7, 187\}$. Retrieve original msgs.

$$C_1 = 16, C_2 = 24$$

$$PU = \{e, n\} = \{7, 187\}$$

$$e = 7, n = 187$$

$$d = \frac{(\phi(n) \times x_i) + 1}{e}$$

$$n = p \times q$$

$$187 = 17 \times 11$$

$$d = \frac{(160 \times 1) + 1}{7} = 23$$

so

7

$$p = 17, q = 11$$

$$\text{so } d = 23$$

$$\phi(n) = (17-1) \times (11-1)$$

$$= 16 \times 10 = 160$$

$$PR = \{d, n\} = \{23, 187\}$$

$$M_1 = C_1^d \bmod n = 16^{23} \bmod 187$$

$$= [(16^8 \bmod 187) \times (16^8 \bmod 187) \times (16^4 \bmod 187) \times (16^2 \bmod 187) \times (16 \bmod 187)] \bmod 187$$

=

$$= 103$$

$$\begin{array}{c} \downarrow \\ 86 \times 86 \\ = 103 \end{array}$$

$$\begin{array}{c} \downarrow \\ 69 \times 69 \\ = 86 \end{array}$$

$$\begin{array}{c} \downarrow \\ 16 \times 16 \\ = 69 \end{array}$$

$$\downarrow \\ 16$$

$$= (103 \times 103 \times 86 \times 69 \times 16) \bmod 187$$

$$= 1007260896 \bmod 187 = 169$$

$$M_2 = C_2^d \bmod n = 24^{23} \bmod 187 = 63$$

b.) Diffie-Hellman key exchange using prime 47 & generator 11.

Alice chooses secret 9 & Bob chooses secret 16.

$$q=47, \alpha=11, X_A=9, X_B=16$$

Generate public key:- $Y_A = \alpha^{X_A} \bmod q$

$$= 11^9 \bmod 47$$

$$= [11^2 \bmod 47 \times 11^4 \bmod 47 \times (11^2 \bmod 47) \times (11 \bmod 47)] \bmod 47$$

$$27$$

$$37$$

$$27$$

$$11$$

$$= (37 \times 27 \times 27 \times 11) \bmod 47$$

$$= 39$$

$$Y_B = \alpha^{X_B} \bmod q = 11^{16} \bmod 47 = 3$$

Generate Secret key:-

$$K = Y_B^{X_A} \bmod q = 3^9 \bmod 47 = 37$$

$$K = Y_A^{X_B} \bmod q = 39^{16} \bmod 47 = 37$$

$$K = 37$$

Q.2 $p=7, q=11, e=7$

$$n = p \times q = 77$$

$$\phi(n) = (p-1) \times (q-1) = 60$$

$$d = \frac{(60 \times 5) + 1}{7} = \frac{301}{7} = 43$$

$$PU = \{e, n\} = \{7, 77\}$$

Assume $M=2$

$$C = M^e \bmod n = 2^7 \bmod 77 = 51$$

$$M = C^d \bmod n = 51^{43} \bmod 77$$

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$$\begin{aligned} & [(S1^{16} \bmod 77) \times (S1^8 \bmod 77) \times (S1^8 \bmod 77) \times (S1^4 \bmod 77) \times (S1^4 \bmod 77) \times (S1^2 \bmod 77) \times (S1 \bmod 77)] \\ & \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow \quad \quad \downarrow \bmod 77 \\ & \quad 37 \quad \quad 53 \quad \quad 53 \quad \quad 58 \quad \quad 58 \quad \quad 60 \quad \quad 51 \end{aligned}$$

$$= (37 \times 37 \times 53 \times 60 \times 51) \bmod 77 = 2$$

Q.3 $p=61, q=53, m=10$; use RSA

$$\begin{aligned} n &= p \times q = 61 \times 53 = 3233 \\ \phi(n) &= 60 \times 52 = 3120 \\ e &= 7 \\ d &= \frac{(\phi(n) \times i) + 1}{e} = \frac{(3120 \times 4) + 1}{7} = 1783 \end{aligned}$$

$$PU = \{e, n\} = \{7, 3233\}$$

$$C = M^e \bmod n = 10^7 \bmod 3233 =$$

$$\begin{aligned} & [(10^4 \bmod 3233) \times (10^2 \bmod 3233) \times (10 \bmod 3233)] \bmod 3233 \\ & \quad 301 \quad \quad 100 \quad \quad 10 \\ & = (301000) \bmod 3233 = 331 \end{aligned}$$

$$C = 331$$

$$M = C^d \bmod n = 331^{1783} \bmod 3233 = 10$$

b.) $p=11, q=13, e=7, m=9$; use RSA

$$\begin{aligned} n &= p \times q = 143 \\ \phi(n) &= 120 \\ d &= \frac{(120 \times 6) + 1}{7} = 103 \\ C &= M^e \bmod n = 9^7 \bmod 143 = 48 \\ M &= C^d \bmod n = 48^{103} \bmod 143 = 9 \end{aligned}$$

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Q.4.a $p=23, q=5$

$p=23, \alpha=5, \underbrace{x_A=2, x_B=3}_{\text{randomly generate}}$

public key:-

$$Y_A = \alpha^{x_A} \bmod p = 5^2 \bmod 23 = 2 \quad Y_B = \alpha^{x_B} \bmod p = 5^3 \bmod 23 = 10$$

secret key:-

$$K = Y_B^{x_A} \bmod p = 10^2 \bmod 23 = 8$$

$$K = Y_A^{x_B} \bmod p = 2^3 \bmod 23 = 8$$

$$K = 8$$

b.) $p=11, \alpha=2, x_A=5, x_B=12$

public key:-

$$Y_A = \alpha^{x_A} \bmod p = 2^5 \bmod 11 = 10 \quad Y_B = \alpha^{x_B} \bmod p = 2^{12} \bmod 11 = 4$$

secret key:-

$$K = Y_B^{x_A} \bmod p = 4^5 \bmod 11 = 1$$

$$K = Y_A^{x_B} \bmod p = 10^{12} \bmod 11 = 1$$

$$K = 1$$