

Lecture 1-4 :- ⚡

Physical Security Personal Security Operations Security

Security: free from danger

Well-Secured Organization

Communications Security Network Security

Information Security: protection of info
of its critical elements, including systems
hardware that use, store, and transmit that info.

C.I.A triangle: Confidentiality, integrity, availability

①

Critical Characteristics of Info:

- 1.) Timeliness: no value if too late
- 2.) Availability: no interference/obstruction, required format
- 3.) Accuracy: free from mistakes
- 4.) Authenticity: state of being genuine
- 5.) Confidentiality: No exposure to unauthorized individuals/system
- 6.) Integrity: uncorrupted
- 7.) Possession: ownership (breach of confidentiality results in breach of possession)

Information Security is entire set of software, hardware, data, people, procedures, and networks necessary to use info as a resource in the organization.

Software

- difficult to secure
- easy target
- most attacks are on info

Hardware

- physical security policies
- secure physical location

Data

- most valuable asset
- main target of attacks

People

- weakest link
- social eng
- must be well-trained & informed

Procedures

- threat to integrity of data

Networks

2

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=> Risk Estimation :-

1.) Assets : Objects, data, people

2.) Vulnerability : Weakness of Asset

3.) Threat : Loss of security due to vulnerability

4.) Attack : threat occurrence

Risk Estim is the process of identifying vulnerabilities 3 threats 3 their impact 3 probability of occurring an attack.

=> Data Protection :-

- most valuable asset is data
- without data, an org loses its record of transactions 3 its ability to deliver value to its customers
- effective information security program is essential to protect the integrity 3 value of organization's data
- organizations should have secure infrastructure services based on size 3 scope of org

=> Threats :-

- threat is an object / person / entity that represents constant danger to asset
- management should be well-informed about threats

=> Threat Modelling : Theoretical use cases considered to identify potential threats.

(Microsoft Stride)

S

T

R

I

D

E

spoofing of
identity

tempering w/
data

repudiation
info

info
disclosure

denial of
service

elevation of
privilege

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⇒ **Attack**: deliberate act that exploits vulnerability
Accomplished by a threat-agent to damage or steal an org's info or asset

Exploit: technique to compromise a system

→ attack is the use of an exploit to achieve the compromise of a controlled system



~ Classes of Attacks ~



① Phishing :-

- attempt to steal sensitive info (usernames, passwords, credit card numbers, bank acc info). Pretends to be a reputable source with enticing request.
- Occurs via email or instant message
- Send fraud emails/messages that appear to be from a trusted source such as bank or govt agency → required to enter login page where user is prompted to enter login credentials.

→ Spear Phishing : targeted attack that uses personalized messaging, esp emails.

→ Vishing : Voice phishing, attackers spoof the calling phone num to appear as if its coming from a bank/institute.

→ Smishing : SMS phishing

② Buckdoor Attack :-

- sidestepping normal authentication procedures to gain unauthorized access to a system. involves exploiting system weaknesses or installing malicious software that creates an entry point for the attacker.

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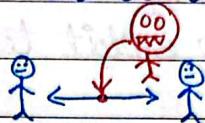
③ Password Crack: process of guessing passwords protecting a computer system

Brute force

try every possible combination

Dictionary

use list of commonly used passwords

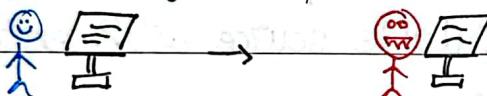


④ Packet-Sniffing / Eavesdropping

passwords/sensitive data are read by unauthorized 3rd parties

⑤ IP Spoofing / Masquerading

creation of ip packets with a false source ip address to impersonate another computer system. // someone takes the place of the legitimate host



⑥ Connection Hijacking / Data Spoofting / Alteration

data is modified during the transmission

⑦ Denial-of-Service (DoS)

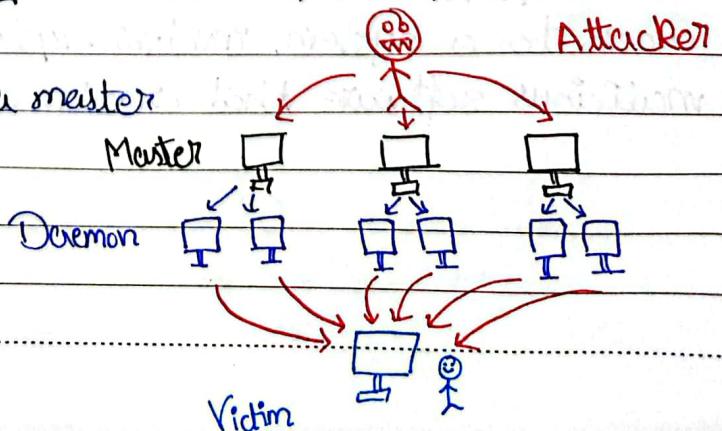
disrupt or shut down the normal functioning of a targeted server, service or network by overwhelming it with a flood of illegitimate requests that trigger a crash. // flood target machine with requests until normal traffic is unable to be processed e.g. Ping bombing

⑧ Distributed Denial of Service (DDoS)

① Software for DoS installed on many nodes (daemon, zombie, malbot) to create a Botnet.

② Daemons remotely controlled by a master

③ Effect of DoS attack x Daemons



⇒ Security Principles ←

- 1.) Least Privilege :- limit accessible data, resources, applications to only that which a user/entity requires to execute their specific task.
 - Without this, org create over-privileged users/entities that increase the potential for breaches 3 misuse of critical systems 3 data.
 - damage caused by compromised user/ app. is reduced.
- 2.) Separation of Privilege :- division of system privileges or excess rights to ensure that no single entity possesses all the necessary permissions to compromise the security of a system or access critical resources
 - Multiple conditions should be required to achieve excess to restricted resources
- 3.) Fail Safe Defaults :- access to an object or resource should be denied by default until explicitly granted. vital in minimizing security vulnerabilities 3 preventing unauthorized access to sensitive info.
e.g. firewall rules, access control lists
- 4.) Complete Medication :- every access to every object should be authorized. Access should be checked, e.g. not only when a file is opened, but also on each subsequent read/write to that file. Do double checking, whenever user wants to read file, check access.
Keep it simple

- 5.) Economy of Mechanism :- security mechanisms should be as simple as possible. If a design 3 implementation are simple, fewer possibilities exist for errors. The checking 3 testing process is less complex, cuz fewer components 3 codes need to be tested. fewer errors, less assumptions

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6.) Least common mechanism :- limits sharing , mechanisms used to access resources should not be shared . disallows the sharing of mechanisms that are common to more than one user or process if the users or processes are at different levels of privilege . Separate channel for users , separation of network resources .

7.) Psychological Acceptability :- security mechanisms should not make the resource more difficult to access than if the security mechanisms were not present . Configuring & executing a program should be as easy & as intuitive as possible & any output should be clear , direct & useful . UI \rightarrow well designed & intuitive

8.) Open Design :- your system security should not rely on the secrecy of your implementation . Well-designed cryptography implementations are published publicly .

Date 6 Sep, 24

Lecture 3 :-



Plaintext : original msg

Ciphertext : coded msg

Encryption Algo : algo for transforming : plaintext → ciphertext

Key : info used in cipher, known only to sender/receiver

Encipher : convert plaintext → ciphertext

Decryption Algo : algo for ciphertext → plaintext

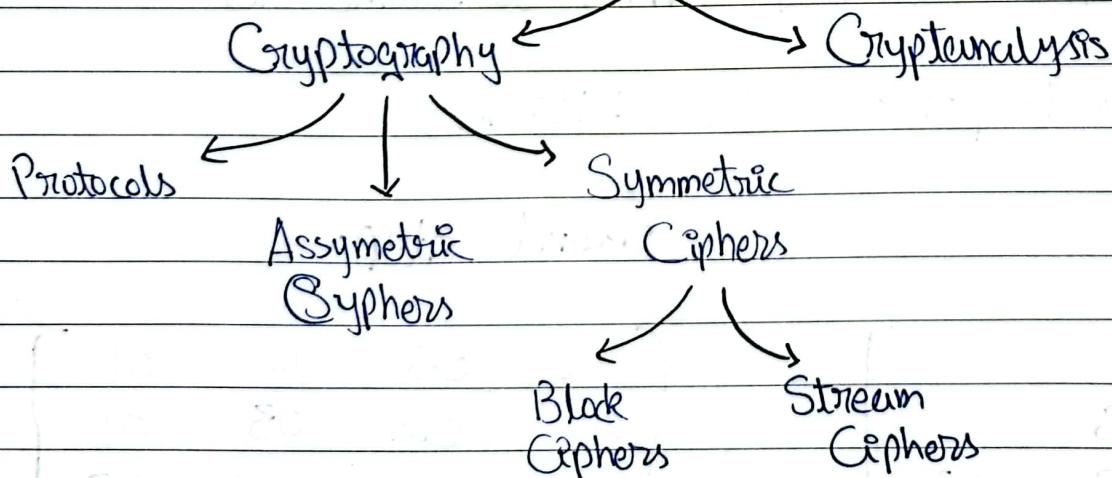
Decipher : convert ciphertext → plaintext

Cryptography : study of encryption principles / methods

Cryptanalysis : study of principles / methods of deciphering ciphertext without knowing key

Cryptology : field of both cryptography & cryptanalysis

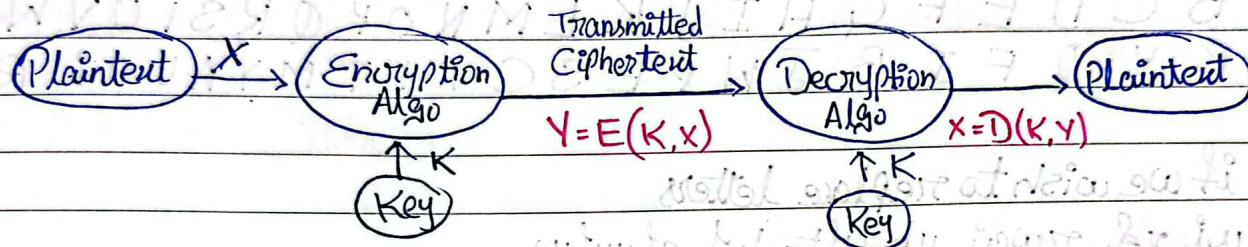
Cryptology



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Symmetric Cryptography :-

- sender/recipient share common key
- all classical encryption algo are private-key



2 Requirements :-

- 1.) strong encryption algo
- 2.) secret key known only to sender/receiver

1) Classical Substitution :-

1) Caesar Cipher :-

- earliest known substitution cipher
- first recorded use in military ciphers

a b c d e f g h i j k l m n o p q r s t u v
w x y z

m e e t . m e e t t e r t h e t o g a . p a r i t y
P h h w . ph diwhu wkh wrjd sdwub

- only 25 keys possible
- brute force search is easy

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2) Monoalphabetic Cipher :-

- each plaintext letter maps to a diff random ciphertext letter
- Key is 26 letters long

A Z N
X Y Z

Plain : A B C D E F G H I J K L M N O P Q R S T U V W

Key : D K V Q F I B J W P E S C X H T M Y A U O L R

Plaintext: if we wish to replace letters

w i n f r w c j u h y f t s d v t s f u u f y q

→ Keys possible: $26! = 4 \times 10^{26}$

human lang are redundant

→ problem: language characteristics

eng letter frequencies :-

- 1.) e
- 2.) t
- 3.) a
- 4.) o
- 5.) i
- 6.) s
- 7.) h

→ monoalphabetic substitution ciphers don't change relative letter frequencies

→ calculate letter freq for ciphertext

→ compare counts/plots against known values

→ for monoalpha must identify each letter

3) Vigenère Cipher :-

a b c d e f g h i j k l m n o p q r s t u v w x y z
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

	d	e	c	e	p	t	i	v	e	d	e	c	e	p	t	i	v	e	d	e	c	e	p	t	i	v
3	4	2	4	15	19	8	21	4	3	4	2	4	15	19	8	21	4	3	4	2	4	15	19	8	21	
22	4	0	17	4	3	8	18	2	14	21	4	17	4	3	18	0	21	4	24	14	20	17	18	4	11	
w	e	a	r	e	d	i	s	c	o	v	e	r	e	d	s	a	r	e	y	o	u	j	s	e	l	
z	i	c	v	t	w	q	n	g	r	z	q	v	t	w	a	v	z	h	c	q	y	g	l	m	q	o
25	8	2	21	19	22	16	39	6	17	25	6	21	19	22	26	21	25	7	28	16	24	32	37	12	32	

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letter frequencies are obscured but not totally lost

4.) Kasiki Method :-

repetitions in ciphertext give clues to period, find same plaintext an exact period apart which results in same ciphertext then attack each monoalphabetic cipher individually using same techniques as b4

5.) Autokey Cipher :-

- ideally want a key as long as message
- Vigenere proposed the autokey cipher with keyword is prefixed to message as key knowing keyword can recover the first few letters
- still have frequency characteristics

Key: deceptive we are discovered say

Plaintext: we are discovered save yourself

the key & the plaintext may share the same frequency dist of letters, a statistical tech can be applied for cryptanalysis

6.) One-time Pad :-

- if a truly random key as long as the message is used, the cipher will be secured a one time pad.
- unbreakable since ciphertext bears no statistical relationship to the plaintext
- key can be used only once

NOT Practical :-

- 1.) large number random key formation
- 2.) distribution & protection among parties

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Classical Transposition :-

- hide the message by rearranging the letter order without altering the actual letters used
- recognize these since have the same frequency distribution as the original text

1) Rail-Fence Cipher :-

plaintext: meet me after the toga party 23 letters so 23 dots

m e o m o u o t o n o h o t o g p ? e o a o a o t o y
e t o e f e t e t e o a c i t

ciphertext: mematrhgprnyetefeteoact

ciphertext: sasoeairnaiyungetrcafar 22 dots

key: 3

s i o n o a o s o o e o c i o t
i o n o a o i o y o l o r o g o e o n o t
o n o a o f o y o l o r o g o e o n o t
o n o a o f o y o l o r o g o e o n o t

plaintext: Sir Reina esif you were greet

2) Row Transposition Ciphers

write letters of message out in rows over a specified number of columns
then reorder the columns according to some key

4 3 1 2 5 6 7

Plaintext: a t t a c k p Key: 4312567
o s t p o n e tknreuptm tsuo codw
d u n t i l t coin knly petz
w o a m u y z

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- recognizable cuz same letter frequencies as original text
- made secure by performing more than one stage of transposition

Bit-Oriented Ciphers :-

- numbers, graphics, audio & video data

1.) Stream Ciphers :-

- process a bit/byte at a time when encrypting/decrypting : Vernam Cipher
- if cryptographic keystream is random, cipher is unbreakable
- Bit-stream generator is a key controlled algo & must produce a bit stream that is cryptographically strong

Data :	0	0	1	0	1	1	0	1	1	1
Key :	1	0	0	1	1	0	0	0	1	XOR
Cipher :	1	0	1	1	0	1	0	1	1	0

Cipher : 1 0 1 1 0 1 0 1 1 0

Key : 1 0 0 1 1 0 0 0 0 1

Data : 0 0 1 0 1 1 0 1 0 1

XOR

2.) Block Ciphers :-

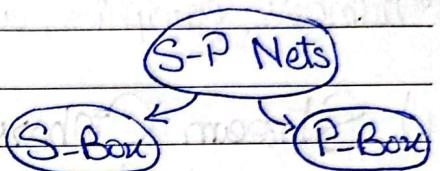
- process message in blocks of bits, like a substitution on v big characters 64-bits / 128
- many current ciphers are block ciphers.

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Lecture 5 :-

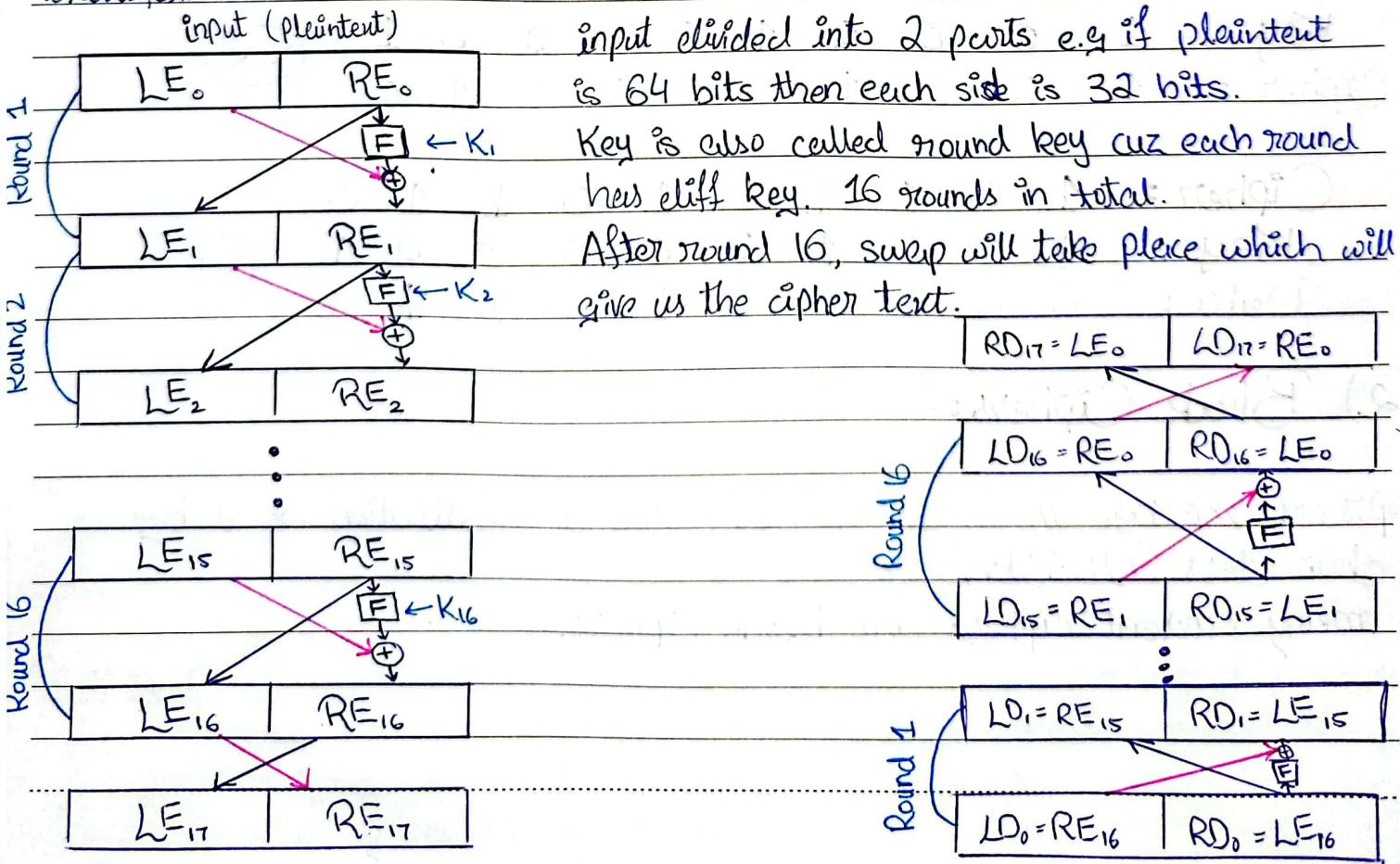
1) Claude Shannon & Substitution-Permutation Ciphers :-

- Claude Shannon introduced idea of SP networks in 1949 paper
- form basis of modern block ciphers
- provide confusion & diffusion of msg & key
- substitution \leftarrow permutation

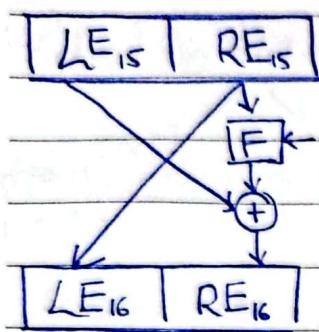


2.) Feistel Cipher :-

- utilize the concept of a product cipher
- alternates substitutions & permutations
- practical application of proposal by Claude-Shannon
- key length of k bits, block length of n bits, allow a total of 2^k transformations.

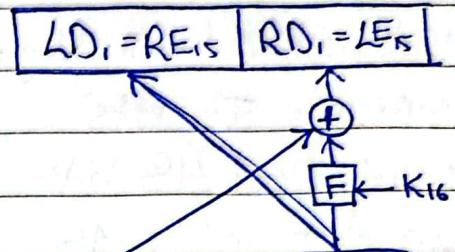


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$$LE_{16} = RE_{15}$$
$$RE_{16} = LE_{15} \oplus F(LE_{15}, K_{16})$$

$$RD_1 = LD_0 + F(RD_0, K_{16})$$
$$RE_{16} \oplus F(RE_{15}, K_{16})$$



Round 16

$$RD_1 = [LE_{15} \oplus F(RE_{15}, K_{16})] \oplus F(RE_{15}, K_{16})$$

$$= LE_{15} \oplus [F(RE_{15}, K_{16}) \oplus F(RE_{15}, K_{16})]$$

$$= (LE_{15} \oplus 0)$$

$$RD_1 = LE_{15}$$

Features:-

- block size
- key size
- num of rounds
- subkey generation algo
- round function
- fast encryption/decryption algo
- ease of analysis

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3.) Data Encryption Standard: DES

exact structure as feistel

- encrypts 64-bit data using 64-bit key

initial \leftrightarrow final
permutation permutation
of last round

input / output / main key = 64 bits

subkey: 56 bits

round key: 48 bits

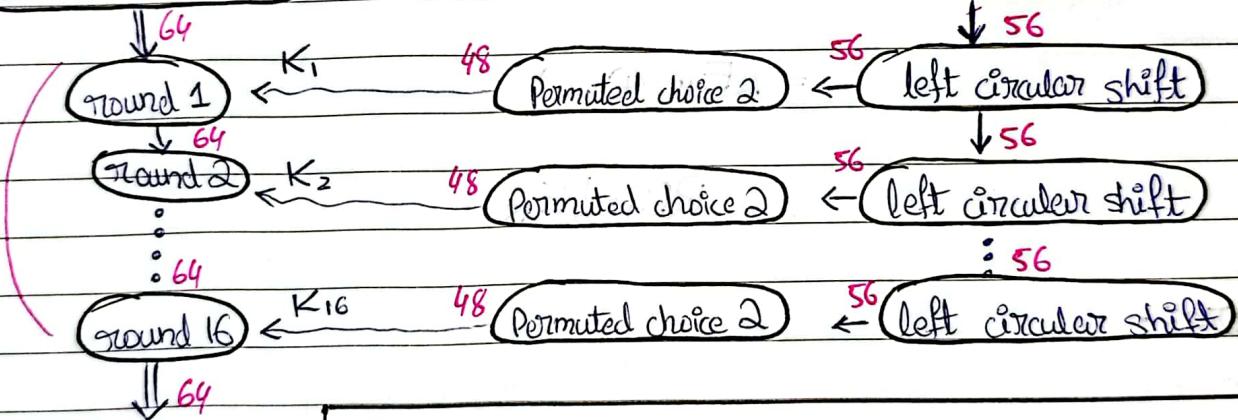
num of rounds: 16

Each round is given 64 bits into it

it outputs 64 bits

64 bit key

1st initial permutation



2nd

3rd

4th

32-bit Swap

inverse
initial
permutation

left half $i-1$

single
Round

right half $i-1$

Mixer Function

expansion permutation

\oplus Round Key

8 S-Boxes

transposition
P-Box

right half $i-1$

\oplus 32 XOR
right half i

left half i

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Roles of S-Boxes :-

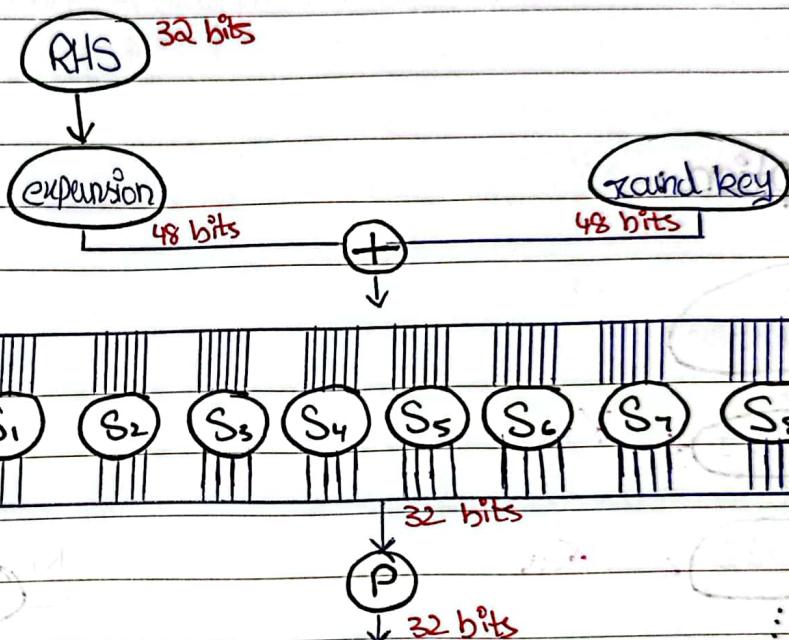
- 8 S-Boxes, each has 6-bit input, 4 bits out
- outer 2 bits (1, 6) used to select row
- inner 4 bits (2-5) used to select column

8 4-bit groups :
32 bit output

input 011001

row 01 : 1

column 1100 : 12



Mangler Function :-

- 1.) expansion
- 2.) XOR with key
- 3.) S-Box
- 4.) P-Box

1.) Expansion :-

4.) P-Box :-

A table is given 3 it just changes the position of the bits. Used for diffusion

32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1
8	32	8			

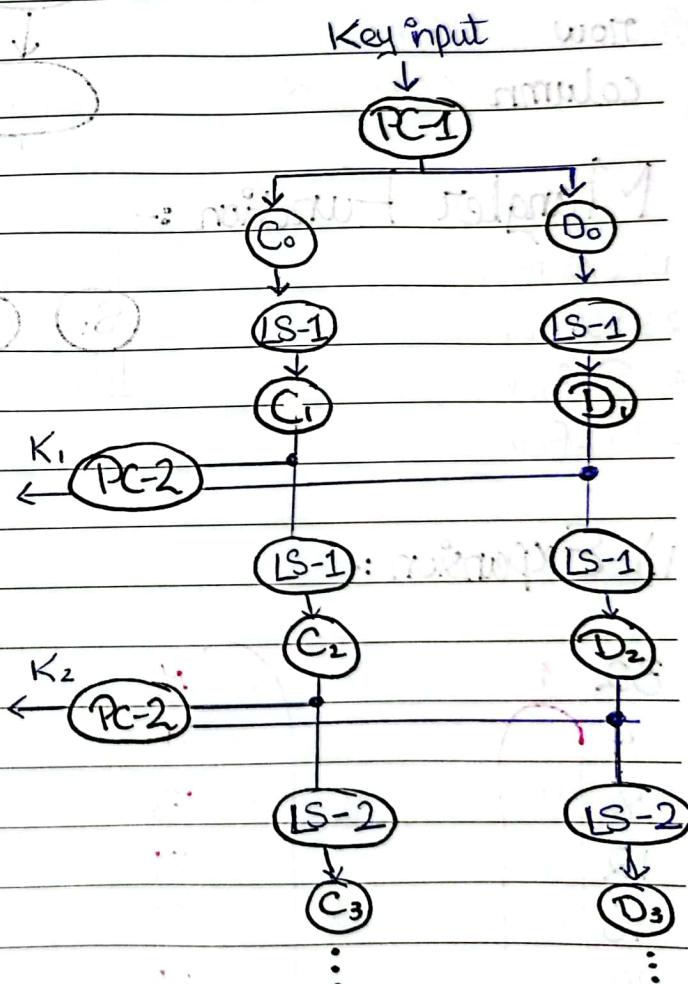
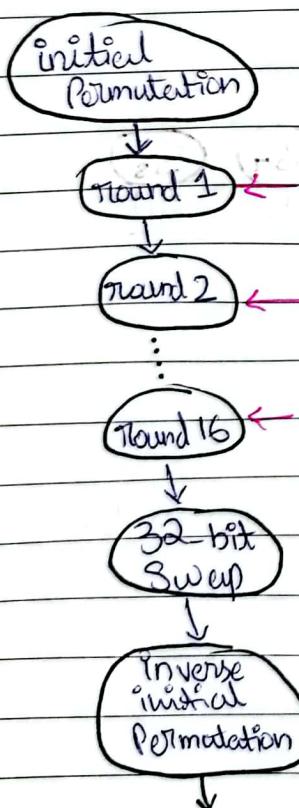
= 48 bits

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1 Key Scheduling :-

- 64 bit input is given to permuted choice 1 \Rightarrow it gives 56 bits
 \hookrightarrow the bits dropped are : 8, 16, 24, 32, 40, 48, 56, 64
- The 56 bit key is called the Effective Key.
- Left Shift : 1 shift for rounds 1, 2, 9, 16
2 shift for all other rounds
- Permutated Choice 2 gets 56 bit input, 8 bits are dropped \Rightarrow 48 bits are permuted

Decryption :-



just order of the round keys is changed.

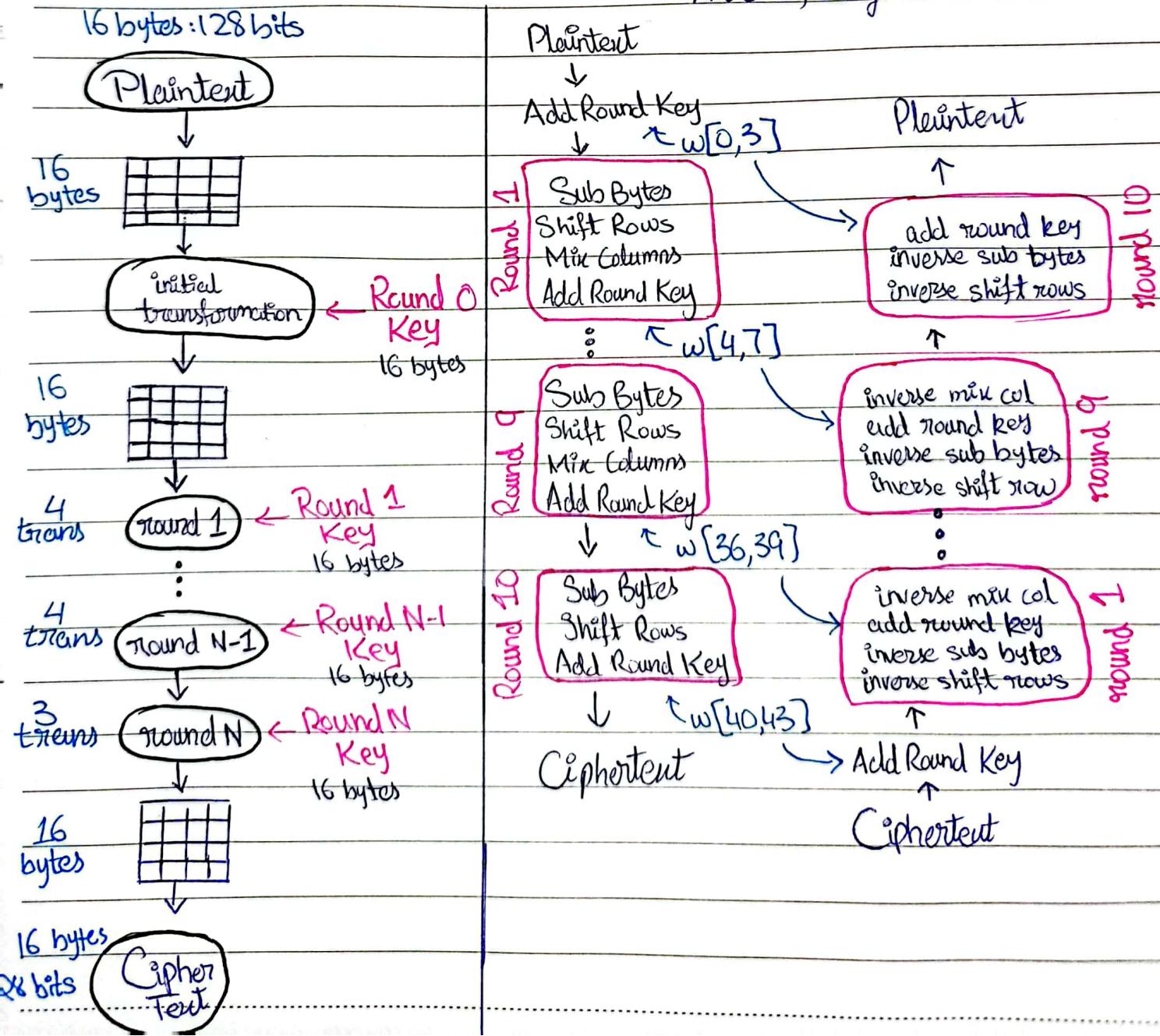


Lecture 6 :-

Advanced Encryption Standard (AES) :-

	AES-128	AES-192	AES-256
Key Size	128	192	256
Plaintext Size	128	128	128
Num of Rounds	10	12	14
Round Key Size	128	128	128

44 words for key



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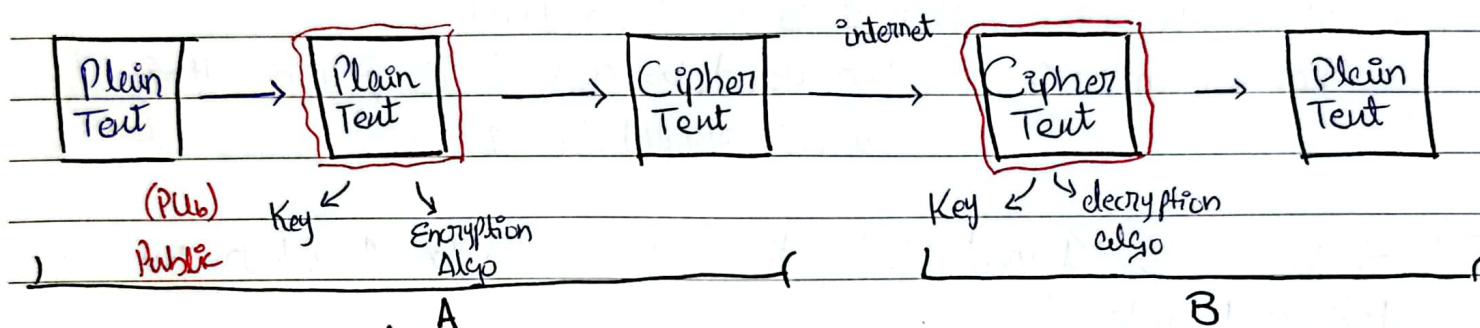
Lecture 7 : RSA

Asymmetric Encryption :-

- public key / two keys

Private

(PR_B)



Use the public key ^{of B} at source to encrypt it & use the private key at destination to decrypt it

If you use private key to encrypt then it will create a signature

=> RSA :-

- ① Take 2 prime numbers: $p \neq q$ ($p \neq q$)
- ② Calculate n , $n = p \times q$
- ③ find ϕ

e.g. $\phi(5) = ?$ so we need to choose the gcd

$$\gcd(1, 5) = 1$$

$$\gcd(2, 5) = 1$$

$$\gcd(3, 5) = 1$$

$$\gcd(4, 5) = 1$$

$$\phi(5) = 4$$

(count number of 1s)

$$\phi(8) = ?$$

$$\gcd(1, 8) = 1$$

$$\gcd(2, 8) = 2$$

$$\gcd(3, 8) = 1$$

$$\gcd(4, 8) = 4$$

$$\gcd(5, 8) = 1$$

$$\gcd(6, 8) = 2$$

$$\gcd(7, 8) = 1$$

~~cancel~~

$$\phi(8) = 4$$

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if n is a prime num $\phi(n) = n - 1$

$$\phi(5) = 4$$

$$\phi(7) = 6$$

if n is in the form of $p \times q$

$$\phi(n) = \phi(p \times q)$$

$$= \phi(p) \times \phi(q)$$

$$= (p-1) \times (q-1)$$

4.) Select encryption key e where:

e should be greater than 1 & less than $\phi(n)$

$\text{gcd}(e, \phi(n)) = 1$ when you take gcd of $e \geq \phi(n)$ then answer should be 1

5.) Solve $e \cdot d = 1 \pmod{\phi(n)}$, the ans should be 1 when you take mod, take d value

6.) Encryption: $C = M^e \pmod{n}$

7.) Decryption: $M = C^d \pmod{n}$

$$M = 2$$

Example:-

1.) $p=3, q=5$

2.) $n = p \times q = 15$

3.) $\phi(n) = (3-1) \times (5-1) = 8$

Encryption:

$$C = 2^3 \pmod{15}$$

$$= 8 \pmod{15}$$

$$C = 8$$

4.) $d \Rightarrow e \cdot d = 1 \pmod{\phi(n)}$

$$3d = 1 \pmod{8}$$

$\therefore d=3$

$$3 \times 3 = 1 \pmod{8}$$

$$d = 3$$

$3 \times 8 = 1$

Decryption:

$$M = 8^3 \pmod{15}$$

$$= 512 \pmod{15}$$

$$M = 2$$

5.) PU = $\{e, n\} = \{3, 15\}$

PR = $\{d, n\} = \{3, 15\}$

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Example :-

$$1.) P=17, q=11$$

$$2.) n = 17 \times 11 = 187$$

$$3.) \phi(n) = (17-1) \times (11-1) = 16 \times 10 = 160$$

$$4.) e=7$$

$$5.) d=? \quad \text{formula: } d = \frac{(\phi(n) \times i) + 1}{e} = \frac{(160 \times 1) + 1}{7} = \frac{161}{7} = 23$$

the answer should not be in decimals, take whole number

$$5.) 7 \cdot 23 = 1 \bmod 160$$

$$d=23$$

$$6.) PU = \{e, n\} = \{7, 187\}$$

$$PR = \{d, n\} = \{23, 187\}$$

$$M=88 \quad (\text{Take by user})$$

decimal:

$$(88-88) \div (187 \times 187)$$

Encryption :

$$C = 88^7 \bmod 187$$

$$C = [(88^4 \bmod 187) \times (88^2 \bmod 187) \times (88 \bmod 187)] \bmod 187$$

$$\begin{array}{ccc} 77 \times 77 & \downarrow & \downarrow \\ \downarrow & 88 \times 88 & 88 \\ 132 & \downarrow & 77 \end{array}$$

$$C = (132 \times 77 \times 88) \bmod 187 = 11$$

Decryption : $M = 11^{23} \bmod 187$ (make groups like 1, 2, 4, 8, 16)

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Example :-

$$\textcircled{1} \quad p = 47, q = 59$$

$$\textcircled{2} \quad n = p \times q = 2773$$

$$\textcircled{3} \quad \phi(n) = (46) \times (58) = 2668$$

$$\textcircled{4} \quad e = 3$$

$$\textcircled{5} \quad d = \frac{(\phi(n) * i) + 1}{e} = \frac{2668 * 1 + 1}{3} = \frac{2669}{3} = 889.6$$

$$d = \frac{(2668 * 2) + 1}{3} = \frac{5336 + 1}{3} = 1779$$

$$d = 1779$$

$$6.) \quad PU = \{e, n\} = \{3, 2773\}$$

$$PR = \{d, n\} = \{1779, 2773\}$$

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Activity 2 :-

Q.1 RSA algo is used to encrypt into 3 public key to decrypt msg is $\{7, 187\}$. Retrieve original msgs.

$$C_1 = 16, C_2 = 24$$

$$PU = \{e, n\} = \{7, 187\}$$

$$e = 7, n = 187$$

$$d = \frac{\phi(n) + 1}{e}$$

$$n = p \times q$$

$$187 = 17 \times 11$$

so

$$p = 17, q = 11$$

$$\phi(n) = (17-1) \times (11-1)$$

$$= 16 \times 10 = 160$$

$$so d = 23$$

$$d = \frac{(160 \times 1) + 1}{7} = 23$$

$$PR = \{d, n\} = \{23, 187\}$$

$$M_1 = C_1^d \bmod n = 16^{23} \bmod 187$$

$$= [(16^8 \bmod 187) \times (16^8 \bmod 187) \times (16^4 \bmod 187) \times (16^2 \bmod 187) \times (16 \bmod 187)] \bmod 187$$

$$= \begin{matrix} & \downarrow \\ 103 & \end{matrix} \quad \begin{matrix} 86 \times 86 \\ \downarrow \\ 103 \end{matrix} \quad \begin{matrix} 69 \times 69 \\ \downarrow \\ 86 \end{matrix} \quad \begin{matrix} \downarrow \\ 16 \times 16 \\ = 69 \end{matrix} \quad \begin{matrix} \downarrow \\ 16 \end{matrix}$$

$$= (103 \times 103 \times 86 \times 69 \times 16) \bmod 187$$

$$= 1007260896 \bmod 187 = 169$$

$$M_2 = C_2^d \bmod n = 24^{23} \bmod 187 = 63$$

$$Q.2 \ p=7, q=11, e=7$$

$$n = p \times q = 77$$

$$\phi(n) = (p-1) \times (q-1) = 60$$

$$d = \frac{(60 \times 5) + 1}{7} = \frac{301}{7} = 43$$

$$PU = \{e, n\} = \{7, 77\}$$

Assume M = 2

$$C = M^e \text{ mod } n = 2^7 \text{ mod } 77 = 51$$

$$M = C^{d \text{ mod } \phi(n)} = 51^{43 \text{ mod } 60} \text{ mod } 77$$

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$$\begin{aligned} & \left[(S1^{16} \bmod 77) \times (S1^8 \bmod 77) \times (S1^5 \bmod 77) \times (S1^4 \bmod 77) \times (S1^4 \bmod 77) \times (S1^2 \bmod 77) \times (S1 \bmod 77) \right] \\ & \downarrow \quad \downarrow \bmod 77 \\ & 37 \quad 53 \quad 53 \quad 58 \quad 58 \quad 60 \quad 51 \end{aligned}$$

$$= (37 \times 37 \times 53 \times 60 \times 51) \bmod 222024400 \bmod 77 = 2$$

Q.3 $p=61, q=53, m=10$; use RSA

$$\begin{aligned} n &= p \times q = 61 \times 53 = 3233 \\ \phi(n) &= (60 \times 52) = 3120 \\ e &= 7 \end{aligned}$$

$$PU = \{e, n\} = \{7, 3233\}$$

$$C = M^e \bmod n = 10^7 \bmod 3233 =$$

$$\begin{aligned} & \left[(10^4 \bmod 3233) \times (10^2 \bmod 3233) \times (10 \bmod 3233) \right] \bmod 3233 \\ & \quad 301 \quad 100 \quad 10 \\ & = (301000) \bmod 3233 = 331 \end{aligned}$$

$$C = 331$$

$$M = C^d \bmod n = 331^{1783} \bmod 3233 = 10$$

b.) $p=11, q=13, e=7, m=9$; use RSA

$$n = p \times q = 143$$

$$\phi(n) = 120$$

$$d = (120 \times 6) + 1 = 103$$

$$C = M^e \bmod n = 9^7 \bmod 143 = 48$$

$$M = C^d \bmod 143 = 48^{103} \bmod 143 = 9$$

Date

Lecture 8 :-

Diffie-Hellman Key Exchange
not for encryption/decryption.

Alice

Bob

1.) Suppose prime num q

1.) suppose q

2.) Suppose α

2.) suppose α

$$\text{i)} \alpha < q$$

$$\alpha \bmod q$$

ii) α is a primitive root of q

$$\alpha^2 \bmod q$$

$$\alpha^3 \bmod q \Rightarrow$$

$$\{1, 2, 3, \dots, q-1\}$$

$$\vdots$$

$$\alpha^{q-1} \bmod q$$

3.) Generate private key x_A

3.) Generate private key x_B

4.) Generate public key

4.) Generate public key

$$Y_A = \alpha^{x_A} \bmod q$$

$$Y_B = \alpha^{x_B} \bmod q$$

Share

5.) Generate Secret Key

5.) Generate Secret Key

$$K = Y_B^{x_A} \bmod q$$

$$K = Y_A^{x_B} \bmod q$$

Should be equal

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Mathematically :-

$$\begin{aligned}
 K' &= Y_B \xrightarrow{x_A} \text{Alice Secret Key} \\
 &= (\alpha^{x_B} \bmod q)^{x_A} \bmod q \\
 &= (\alpha^{x_B})^{x_A} \bmod q \quad > (=)
 \end{aligned}$$

Example: $(2^3 \bmod 3)^2 \bmod 3 = (2^3)^2 \bmod 3$
 $(8 \bmod 3)^2 \bmod 3 = 8^2 \bmod 3$
 $(2) \bmod 3 = 64 \bmod 3$
 $64 \bmod 3 = 1$

$$\begin{aligned}
 K &= \alpha^{x_B x_A} \bmod q \\
 &= (\alpha^{x_A})^{x_B} \bmod q \\
 &= (\alpha^{x_A} \bmod q)^{x_B} \bmod q \quad > (=)
 \end{aligned}$$

$$K = (Y_A)^{x_B} \quad \begin{matrix} \hookrightarrow \text{Bob Secret} \\ \text{Key} \end{matrix}$$

Example :-

$$q = 11, \alpha = 2$$

Randomly generate:-

$$X_A = 2 \quad X_B = 3$$

	1	2	3	4	5	6	7	8	9	10
1	1	1	1	1	1	1	1	1	1	1
2	2	4	8	5	10	9	7	3	6	1
3										
4										
5										
6										
7										
8										
9										
10										

choose a number that has no repetitions. e.g
if 2 had any repetition we would have
taken 3 and so on...

Public Key :-

$$Y_A = 2^2 \bmod 11 = 4 \bmod 11$$

$$Y_A = 4 \quad \begin{matrix} \downarrow \\ \text{shared} \end{matrix}$$

$$Y_B = 2^3 \bmod 11 = 8 \bmod 11$$

$$Y_B = 8$$

$$\begin{aligned}
 K &= 8^2 \bmod 11 = 64 \bmod 11 \\
 K &= 9
 \end{aligned}
 \quad \left\{
 \begin{array}{l}
 K = 4^3 \bmod 11 = 64 \bmod 11 \\
 K = 9
 \end{array}
 \right.$$

Alice Secret Key Bob Secret Key

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Example :-

$$q = 353, \alpha = 3$$

$$X_A = 97, X_B = 233$$

Public Key :-

$$Y_A = \alpha^{x_A} \pmod{q}$$

$$= 3^{97} \pmod{353}$$

$$Y_B = \alpha^{x_B} \pmod{q}$$

$$= 3^{233} \pmod{353}$$

Secret Key :-

$$K = Y_B \pmod{q}$$

$$K = Y_A \pmod{q}$$

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b.) Diffie-Hellman key exchange using prime 47 & generator 11.
Alice chooses secret 9 & Bob chooses secret 16.

$$q = 47, \alpha = 11, X_A = 9, X_B = 16$$

Generate public key:- $Y_A = \alpha^{X_A} \bmod q$
 $= 11^9 \bmod 47$

$$= [(11^2 \bmod 47) \times (11^4 \bmod 47) \times (11^2 \bmod 47) \times (11 \bmod 47)] \bmod 47$$

$$\begin{array}{cccc} 27 & 37 & 27 & 11 \\ \times & \times & \times & \times \\ \hline \end{array}$$

$$= (37 \times 27 \times 27 \times 11) \bmod 47$$

$$= 39$$

$$Y_B = \alpha^{X_B} \bmod q = 11^{16} \bmod 47 = 3$$

Generate Secret Key :-

$$K = Y_B^{X_A} \bmod q = 3^9 \bmod 47 = 37$$

$$K = Y_A^{X_B} \bmod q = 39^{16} \bmod 47 = 37$$

$$K = 37$$

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Q.4.a $p=23, q=5$

$p=23, \alpha=5, \underbrace{x_A=2, x_B=3}_{\text{randomly generate}}$

randomly generate

public key :-

$$Y_A = \alpha^{x_A} \bmod p = 5^2 \bmod 23 = 2 \quad Y_B = \alpha^{x_B} \bmod p = 5^3 \bmod 23 = 10$$

secret key :-

$$K = Y_B \bmod p = 10^2 \bmod 23 \\ = 8$$

$$K = Y_A \bmod p = 2^3 \bmod 23 \\ = 8$$

$$K = 8$$

b.) $p=11, \alpha=2, X_A=5, X_B=12$

public key :-

$$Y_A = \alpha^{x_A} \bmod p = 2^5 \bmod 11 = 10 \quad Y_B = \alpha^{x_B} \bmod p = 2^{12} \bmod 11 = 4$$

secret key :-

$$K = Y_B \bmod p = 4^5 \bmod 11 = 1$$

$$K = Y_A \bmod p = 10^5 \bmod 11 = 1$$

$$K = 1$$