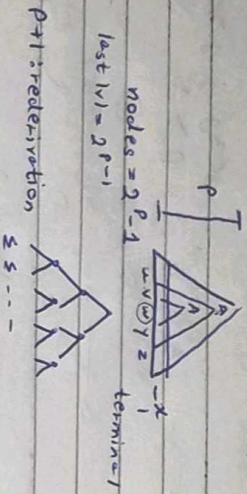


# 15/11/24 Pumping Lemma for CFL

$$L = a^n b^n c^n, n \geq 1$$

→ 2-stack PDA

$$\begin{cases} x = uv^i w \\ |y| \geq 0 \\ |uv| \leq p \end{cases}$$



CFG  
↓  
CNF  
 $A \rightarrow BC$   
 $B \rightarrow a$

$$\begin{aligned} x &= a^p b^p c^p \\ v &= a^r \\ w &= a^s \\ y &= a^t \\ z &= b^p c^p \end{aligned}$$

$$p = r+s+t$$

D1 start

D2 end

$$u = a^p b^p$$

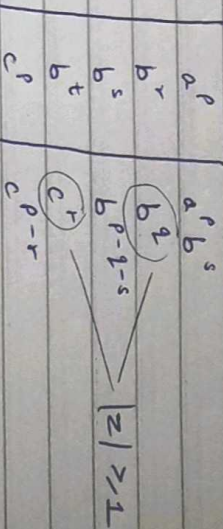
$$p = r+s+t$$

D3

$$\text{if } |u| \leq |z| > 0$$

at most 2 characters to pump.

$$\begin{aligned} u &= a^{p-s} \\ v &= a^s \\ w &= b^p \\ y &= b^t \\ z &= b^{p-q-t} c^p \end{aligned}$$



case I case II case III

$$x = a^p b^s (b^q)^i b^{p-q-s} (c^r)^i c^{p-r}$$

$$i = 2$$

$$\begin{aligned} &a^p b^s b^{2q} b^{p-q-s} c^{2r} c^{p-r} \\ &a^p b^{p+q} c^{p+r} \end{aligned}$$

$$|q^2| \geq 1$$

$$p \neq p+q$$

$$p \neq p+r$$

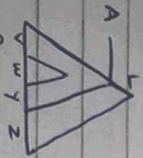
$$p \neq p+q+r$$

$$\begin{aligned} u &A z \\ &= uvAy z \\ &= uv^i A y^i z \end{aligned}$$

$$x = uv^i w y^i z$$

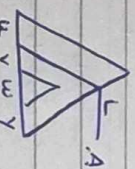
$$i \geq 0$$

$$\begin{aligned} |vy| &\geq 1 \\ |vwy| &\leq p \end{aligned}$$



Case I

Case II



Case III

'u' & 'z' can be alternatively zero.

$$|u| \neq |z| > 0$$



13/11/22

- Q.  $L_1 = \{a^i b^j; i < j\}$   
 $L_2 = \{a^i b^j; i \geq j\}$   
 $L_3 = \{a^i b^j; i \leq j\}$   
 $L_4 = \{a^i b^j; i \geq j\}$

$x = uv^i w$

pumping element

$i = 0, 2, 3, \dots$   
new  $x \in L$

$|v| \geq 1$   
 $|w| \leq p$

$L_1$   
 $x = a^{p-5} b^p$   $|s| \geq 1$

$u = a^{p-5} b^5 \Rightarrow p$   
 $v = b^5$   
 $w = b^{p-5}$

$x = a^{p-5} (b^5)^i b^{p-5}$

$i=0$  i pump down  
 $x = a^{p-5} b^{p-5} \in L_1$

$x = a^r (a^5)^i b^{p+5}$   
 $i=4$  pump up  
 $x = a^r a^{45} b^{p+5}$   
 $= a^{r+45} b^{p+5}$   
 $= a^{p+35} b^{p+5}$

$L_3$   
 $x = a^{p-5} b^p; |s| \geq 0 \rightarrow x = a^p b^p$  ; considering  $i=j$  first.

$u = a^r$   
 $v = a^5$   
 $w = b^p$  here  $|s| \geq 0$  but where  $|v| \geq 0$   
 $x = a^{p-5} (b^5)^i b^{p-5}$   
 $x = a^r (a^5)^4 b^p$  pump up  
 $= a^{r+45} b^p$   
 $= a^{r+45} b^{r+5}$   $\notin L$

$i=4$  pump up  
 $x = a^{p-5} b^{45} b^{p-5} = a^{p+35} b^{p+5} \notin L$

$w|w$   
 $01^*01 \in L$

Q.  $L = \{a^i b^j c^k; i < j < k\}$

$|u|=0$   $|z|=0$   $|w| \leq |z| > 0$   
 $u = a^n$   $v = a^m$   $w = a^s$   $z = b^p c^{p+s+r}$   
 $x = a^n a^m a^s b^p c^{p+s+r} b^p c^{p+s+r} b^p c^{p+s+r}$   
 $p = n+m+s$   $p = l+m+n$   $s = a+b+c$   $|b| \geq 1$   
 $(a^n)^3 a^m (a^s)^3 b^{p+s} c^{p+s+r}$  ;  $i=3$   
 $a^{3n} a^m a^3 b^{p+s} c^{p+s+r}$   
 $a^{p+2n+2s} b^{p+s} c^{p+s+r} \notin L$   $a > b$ , contradiction!

$|xy| \geq 2$   
 $|vwx| \leq p$

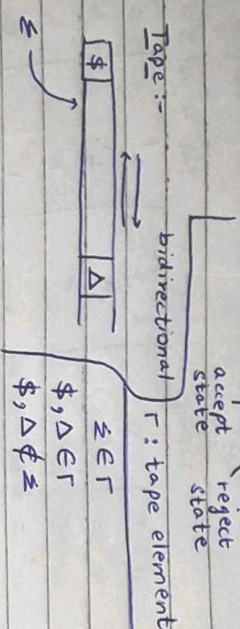
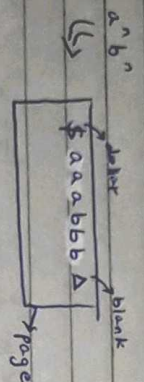
$x = a^p b^{p+s} c^{p+s+r}$   
 $|s| \leq |p| \geq 1$



22/01/2022

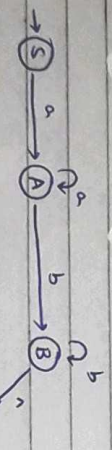
# Turing Machine

FA + Tape (Single/Multiple)  
 - language acceptor  
 - function computation ( $x+y$ )  
 $(x, y)$



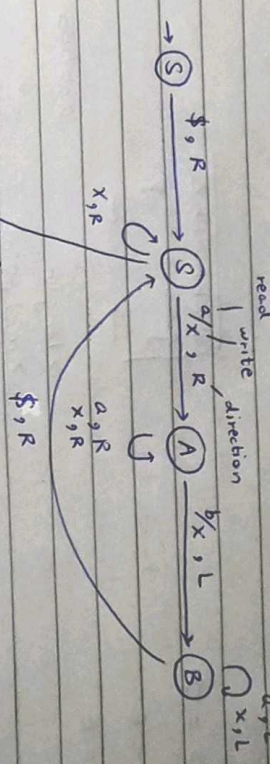
Q: set of states  
 $\Sigma$ : set of alphabets  
 $q_0$ : start state  
 $q_f$ : final state  
 $\Gamma$ : tape elements  
 $\delta$ : transition function  
 $\delta(q, a) = (q', \Delta, d)$   
 $q$ : current state  
 $a$ : current symbol  
 $q'$ : next state  
 $\Delta$ : symbol to write  
 $d$ : direction to move

Q.  $a^n b^m$  FA

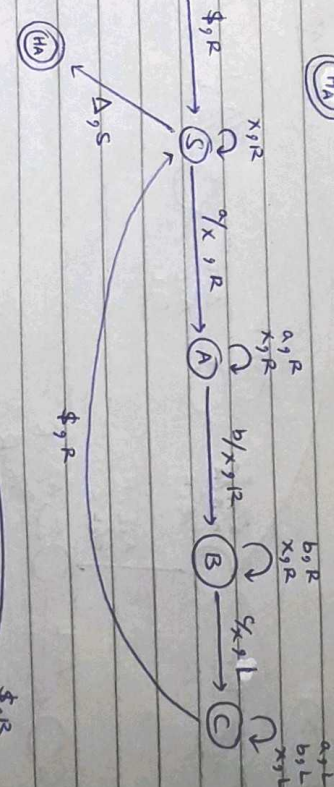


Q.  $x^r \rightarrow (Q \cup \{h_a, h_r\})x^r$   
 $\Sigma \cup \{u, \Delta\}$   
 right left static  
 direction

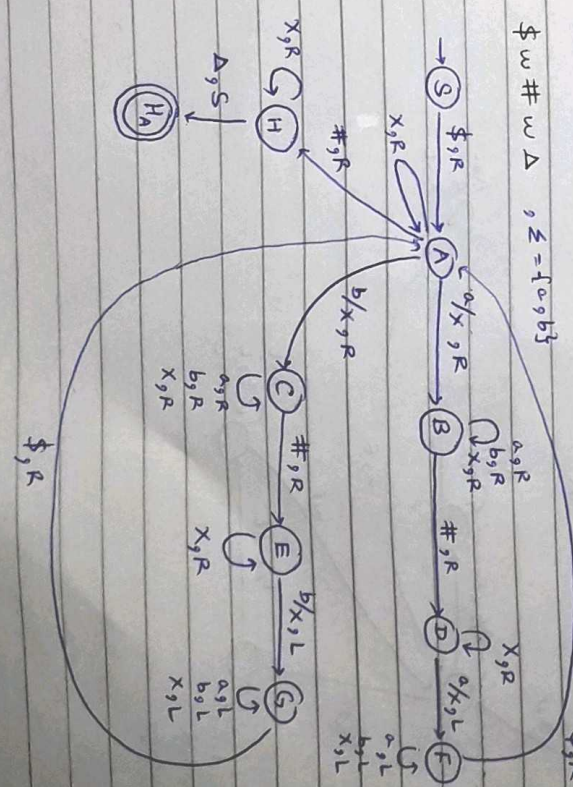
Q.  $a^n b^m$  Turing Machine.



Q.  $a^n b^m$



Q.  $\$ w \# w \Delta$ ,  $\Sigma = \{a, b\}$



$\$ a^n b^m \Delta$



24/1/22

Example 7.5 TM book → abbaab  
Pg 230 John Martin

Q. Design a STM that will recognize a language 'ww' over {a,b}

Www)  $\cdot 2 = 0$

$0 + 0 = 0$

$E + E = E$

> length  
> mid point

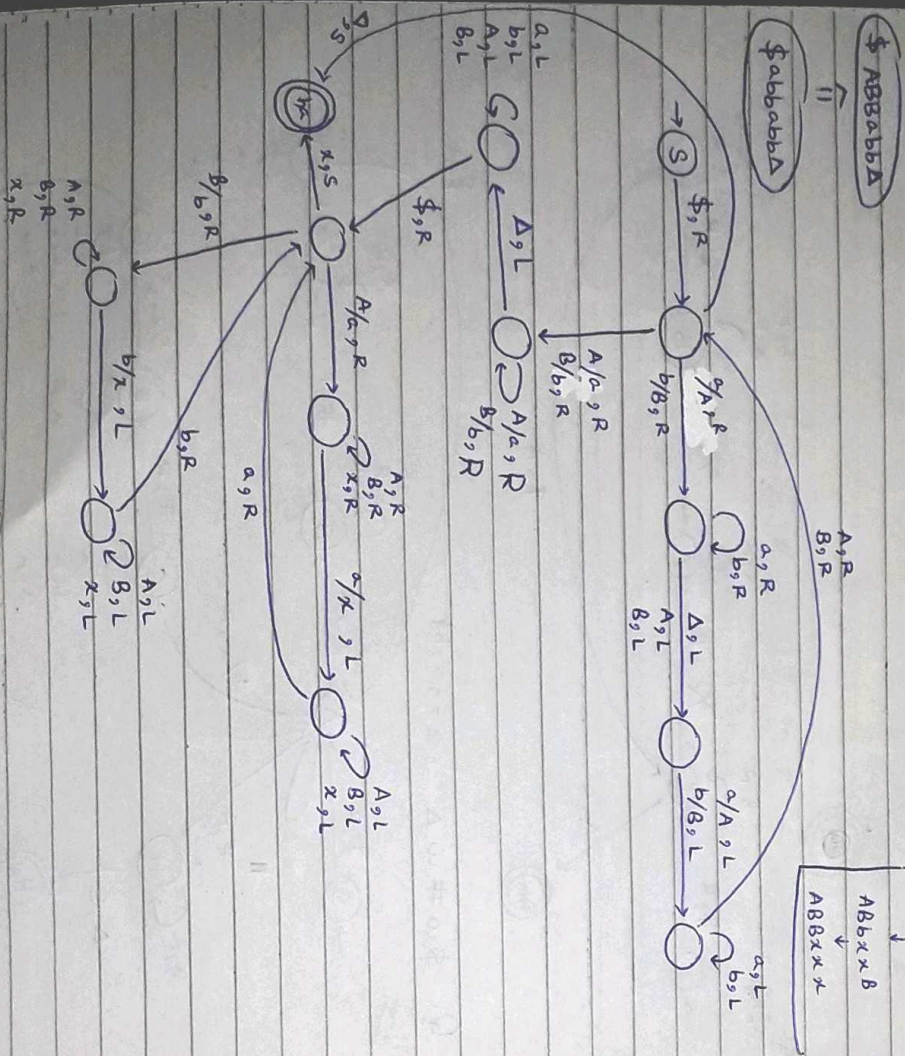
abbABB

AbbxBB

ABbxXB

ABBBxxx

\$abb|abbΔ  
↑  
ABBB



Pg 235

Example 7.10 John C. Martin

Q. Reverse of a String

Www

i) > Place Capital A at end

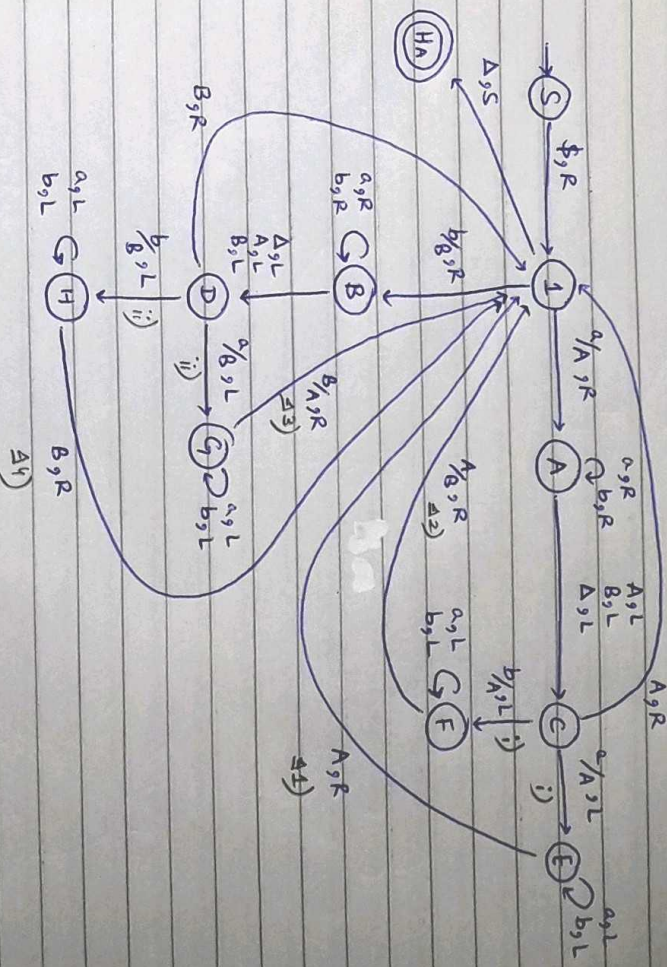
A1) > Place Capital A at start

A2) > Place Capital B at start

ii) > Place Capital B at end

A3) > Place Capital A at start

A4) > Place Capital B at start



\$abbΔ  
↓  
\$abba  
↓  
\$bbaa  
↓  
\$baab  
↓  
\$aabb



## Content

Recursive language / Turing Decidable language  $\supset$  hr

↳ Recursively Enumerable Language  $\Rightarrow$  has

Turing Recognizable language.  $\boxed{\$}$  nonempty

## Algebraic ( $x+y, x-y,$

STM

 $(x, y)$ 

MTTm

nonempty

ΔΔxΔΔ

x Δ Δ Δ Δ

RECUR

②

STM

NTM

NTTM

$x+y$

 $2 + 3$ 

11111

\$	1	1	#	1	1	1	Δ	Δ	...
----	---	---	---	---	---	---	---	---	-----

Turing machine is initialized with blank characters  $\Delta$

$M_2$

\$	1	1	1	1	1	1	$\Delta$	$\Delta$
----	---	---	---	---	---	---	----------	----------

M2

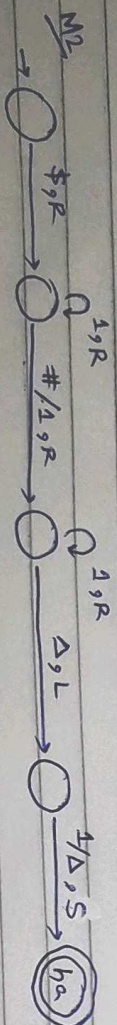
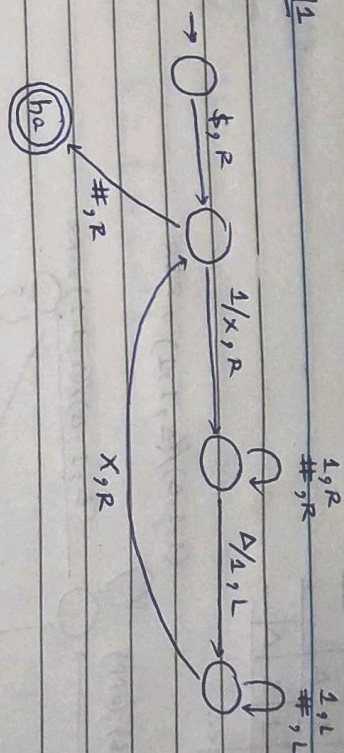
\$	1	1	1	1	1	Δ
----	---	---	---	---	---	---

\$	x	x	#	1	1	1	1	1	Δ	Δ	...
----	---	---	---	---	---	---	---	---	---	---	-----

Recommended

xxyy

y times add x.

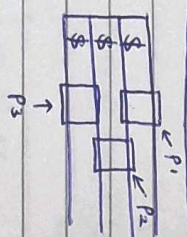

$$\frac{M_2}{E I}$$


## STM - Single Tape TM

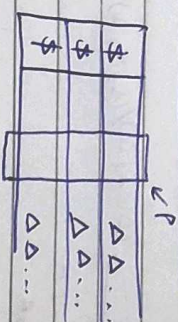
 $\bar{r}_a / \bar{r}_b, D$ 

6
Δ
Δ
:

MTM - Multi Tape TM


$$(\tau_a \tau_b \tau_c) / (\tau_a \tau_b \tau_c), \quad D_1, D_2, D_3$$

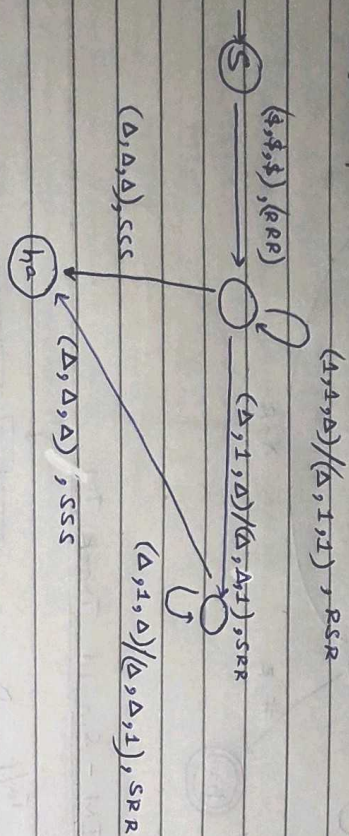
## MITM - Multi Track


$$\frac{[a]_b \Gamma_c}{[a]_b \Gamma_c}, \rightarrow$$

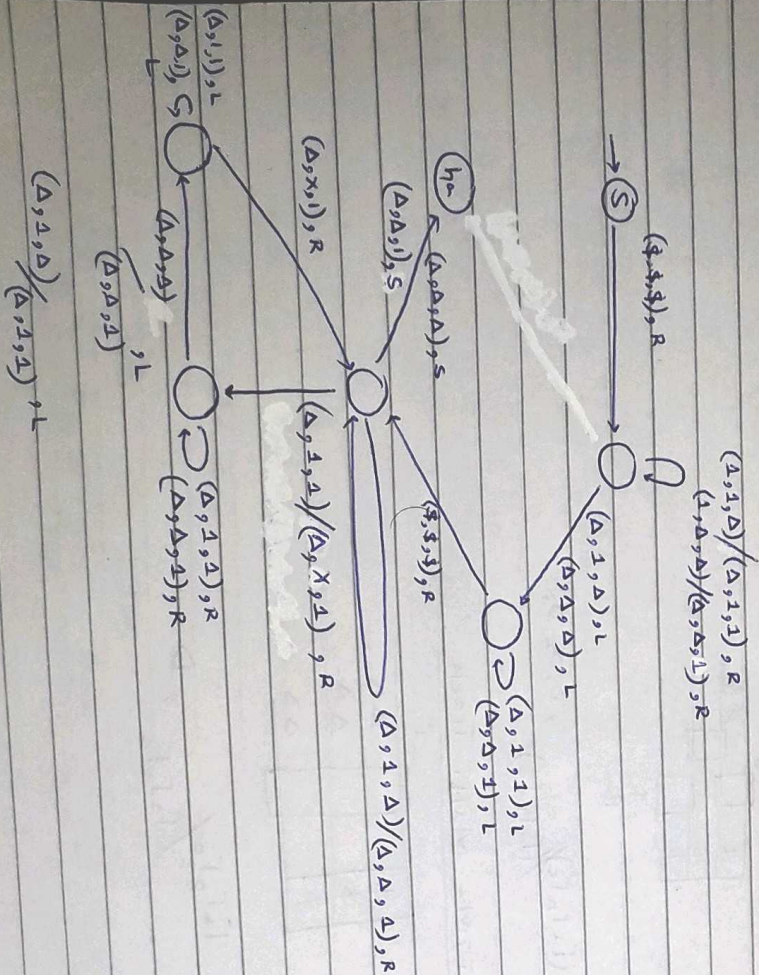


# MTM - Multi Tape

x	\$	1	1	Δ	...
y	\$	1	1	1	Δ
x+y	\$	Δ	...		



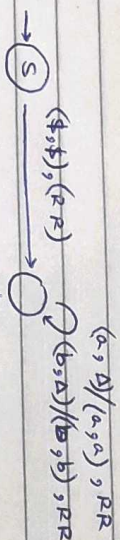
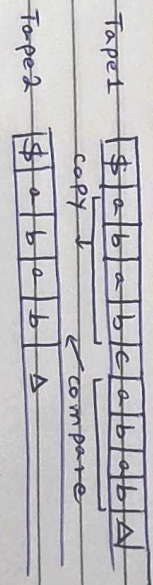
## MTM - Multi Track



## Extra

### Q. MTM

$$L = \{ w c w \} \quad w \in \Sigma^* ; \Sigma = \{ a, b \}$$



### Multiple Tapes on Single Tape:

<Tape 1> # <Tape 2> Δ

Efficiency ↑