MID-1 - FORMULAE

$$p = \frac{x - x_0}{h}$$

x = required value

h = interval diff

EQUAL-INTERVALS

NFD:

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

NBD:

$$y_x = y_0 + p\nabla y_0 + \frac{p(p+1)}{2!}\nabla^2 y_0 + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_0$$

GAUSS-FORWARD:

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1}$$

$$+ \frac{p(p-1)(p+1)(p-2)}{4!}\Delta^4 y_{-2}$$

GAUSS-BACKWARD:

$$y_x = y_0 + p\Delta y_{-1} + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 y_{-2}$$

$$+\frac{p(p+1)(p-1)(p+2)}{4!}\Delta^4 y_{-2}$$

BESSEL'S:-

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\left(\frac{\Delta^2 y_{-1} + \Delta^2 y_0}{2}\right)$$

$$+\frac{p(p-1)(p-0.5)}{3!}\Delta^3 y_{-1}$$

STIRLING'S:-

$$y_x = y_0 + p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2!}\Delta^2 y_{-1} +$$

$$\frac{p(p-1)(p+1)}{3!}\left[\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right] +$$

$$\frac{p^2(p-1)(p+1)}{4!}\left(\Delta^4 y_{-2}\right)$$

LAPLACE-EVERETT'S:

$$y_x = \left[p y_1 + \frac{p(p^2-1^2)}{3!}\Delta^2 y_0 + \frac{p(p^2-1^2)(p^2-2^2)}{5!}\Delta^4 y_{-1}\right] +$$

$$\left[q y_0 + \frac{q(q^2-1^2)}{3!}\Delta^2 y_{-1} + \frac{q(q^2-1^2)(q^2-2^2)}{5!}\Delta^4 y_{-2}\right]$$

$$\boxed{q = 1-p}$$

UNEQUAL INTERVALS

LANGRANGE'S:-

$$y_x = \frac{(x-x_1)(x-x_2)\ldots\ldots(x-x_n)}{(x_0-x_1)(x_0-x_2)\ldots\ldots(x_0-x_n)} y_0 +$$

(ignore 1) $\dfrac{(x-x_0)(x-x_2)\ldots\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots\ldots(x_1-x_n)}\, y_1 +$

(ignore 2) $\dfrac{(x-x_0)(x-x_1)\ldots\ldots(x-x_n)}{(x_2-x_0)(x_2-x_1)\ldots\ldots(x_2-x_n)}\, y_2 + \ldots\ldots$

n-terms

NEWTON DIVIDED DIFFERENCE:-

$$y_x = y_0 + (x-x_0)\Delta y_0 + (x-x_0)(x-x_1)\Delta^2 y_0 + (x-x_0)(x-x_1)(x-x_2)\Delta^3 y_0 \ldots\ldots$$

NUMERICAL DIFFERENTIATION

EQUAL

NFD:-

$$\frac{dy}{dx} = \frac{1}{h}\left(\Delta y_0 + \frac{2p-1}{2!}\Delta^2 y_0 + \left(\frac{3p^2-6p+2}{3!}\right)\Delta^3 y_0\right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\Delta^2 y_0 + (p-1)\,\Delta^3 y_0 \right)$$

NBD:-

$$\frac{dy}{dx} = \frac{1}{h}\left(\nabla y_0 + \frac{2p+1}{2!}\nabla^2 y_0 + \left(\frac{3p^2+6p+2}{3!}\right)\nabla^3 y_0 \right)$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left(\nabla^2 y_0 + (p+1)\,\nabla^3 y_0 \right)$$

STIRLINGS:-

$$\frac{dy}{dx} = \frac{1}{h}\left[\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + p\,\Delta^2 y_{-1} + \left(\frac{3p^2-1}{3!}\right)\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) \right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_{-1} + p \frac{\left(\Delta^3 y_{-1} + \Delta^3 y_{-2} \right)}{2} \right]$$

UNEQUAL

NEWTON-DIVIDED DIFF:-

$$\frac{dy}{dx} = \Delta y_0 + (2x - x_0 - x_1)\, \Delta^2 y_0 \ldots\ldots\ldots$$

NUMERICAL INTEGRATION

TRAPEZOIDAL:-

$$\int_0^n f(x)\,dx = \frac{h}{2}\left[y_0 + y_n + 2\left(y_1 + y_2 + y_3 + \dots y_{n-1}\right)\right]$$

$$h = \frac{b-a}{n}$$

$\left(\begin{array}{l} b = \text{upper limit} \\ a = \text{lower limit} \end{array}\right)$

$$y_k = f(k)$$

SIMPSONS:-

$$\int_0^n y\,dx = \frac{h}{3}\left[\left(y_0 + y_n\right) + 4\left(y_1 + y_3 + y_5 + \dots y_{n-1}\right) + 2\left(y_2 + y_4 + y_6 \dots y_{n-2}\right)\right]$$

MAGIK!

$\Delta^2 y_0$:

$$\Delta^2 y_0 = y_2 - 2y_1 + y_0$$

$$\Delta^2 y_{-1} = y_1 - 2y_0 + y_{-1}$$

$$\Delta^2 y_{-2} = y_0 - 2y_{-1} + y_{-2}$$

$$\Delta^2 y_1 = y_3 - 2y_2 + y_1$$

$$\Delta^2 y_2 = y_4 - 2y_3 + y_2$$

$\Delta^3 y_0$:

$$\Delta^3 y_0 = y_3 - 3y_2 + 3y_1 - y_0$$

$$\Delta^3 y_{-1} = y_2 - 3y_1 + 3y_0 - y_{-1}$$

$$\Delta^3 y_{-2} = y_1 - 3y_0 + 3y_{-1} - y_{-2}$$

$$\Delta^3 y_1 = y_4 - 3y_3 + 3y_2 - y_1$$

$$\Delta^3 y_2 = y_5 - 3y_4 + 3y_3 - y_2$$