Assignment 2 :-

Q.1

a.) $AB'C(BD+CDE)+AC'$

$= AB'CBD + AB'CDE + AC'$ $\quad \because X.X' = 0$

$= 0 + A(B'CDE + C')$ $\quad \because X+YZ = (X+Y)(X+Z)$

$= A(C' + B'DE)(C+C')$ $\quad$ (distributive law)

$= A(C' + B'DE)(1) = AC' + AB'DE$ $\quad \because X + X' = 1$

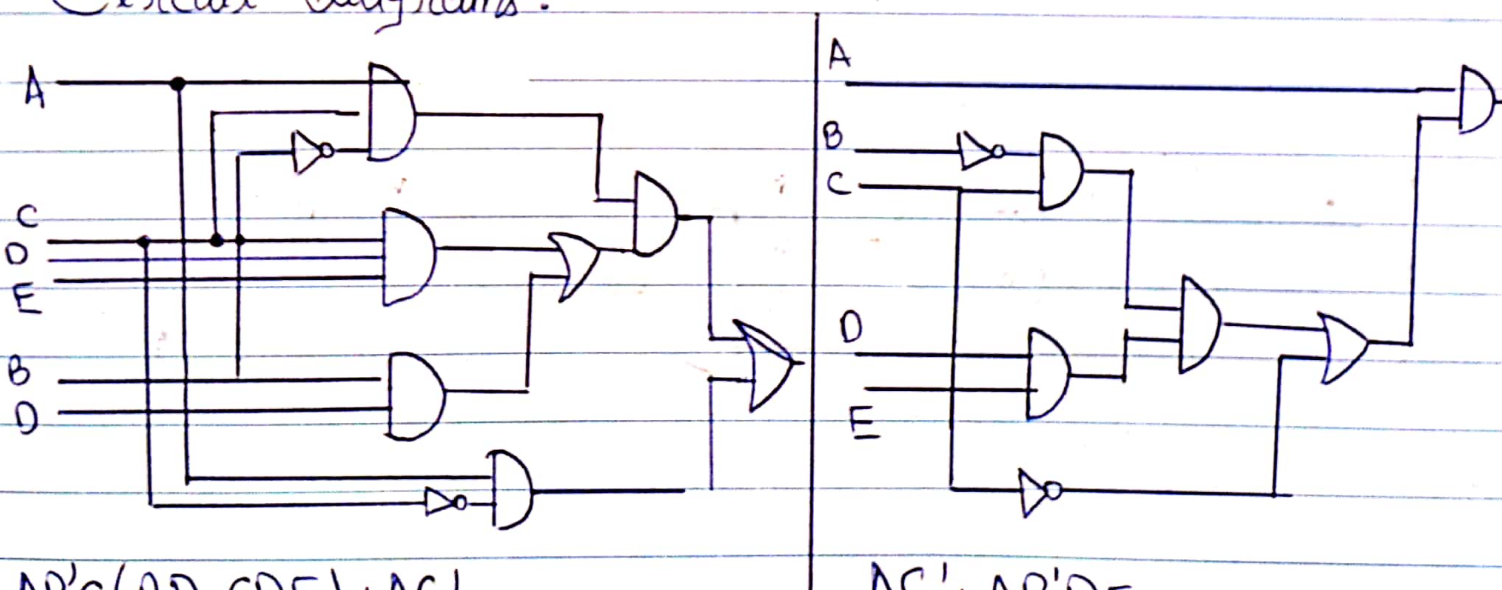
b.)

Gate literal cost :

$AB'C(BD+CDE)+AC' = 10+3+2 = 15$

$AC' + AB'DE = 6+2+2 = 10$

Circuit diagrams :



$AB'C(BD+CDE)+AC'$ $\qquad$ $AC'+AB'DE$

b.) $AB'C + A'BC + A'B'C$

$= AB'C + A'C(B+B')$ $\quad \therefore X + X' = 1$

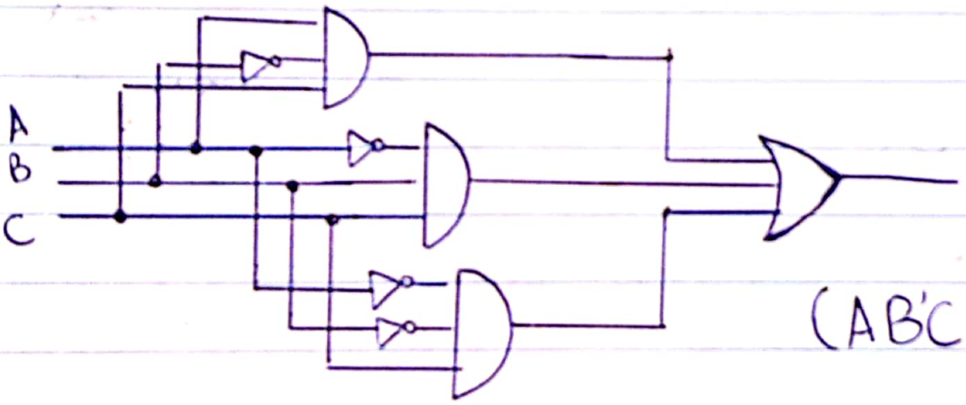
$= AB'C + A'C = C(AB' + A')$ $\quad \therefore X + YZ = (X+Y)(X+Z)$

$= C(A'+A)(A'+B')$

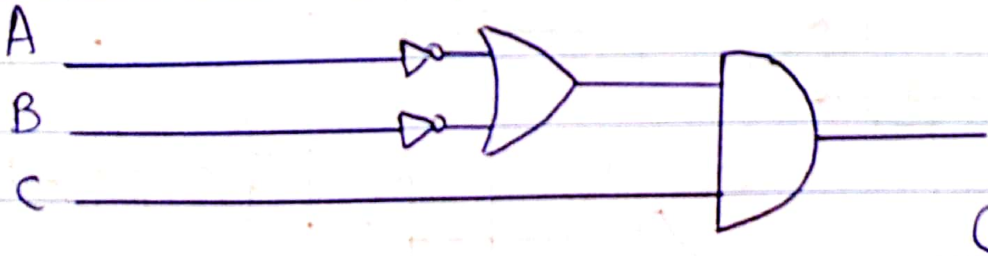
$= C(A'+B')$

$AB'C + A'BC + A'B'C = 9 + 3 + 2 = 14$

$C(A'+B') = 3 + 2 + 2 = 7$



$(AB'C + A'BC + A'B'C)$



$C(A'+B')$

c.) $BCDE + BC(DE)' + (BC)'DE$

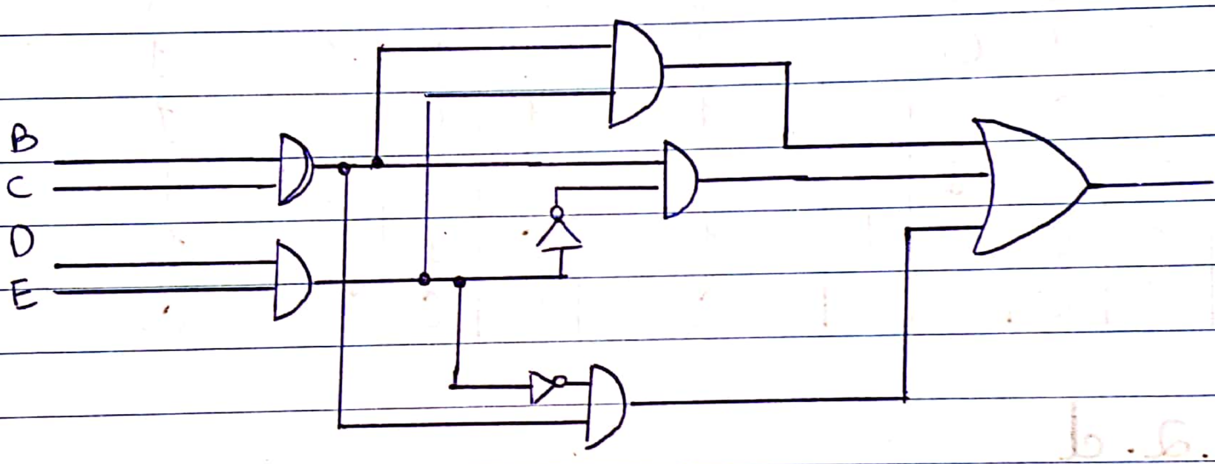
$= BC(DE + \overline{DE}) + \overline{BC}DE$ (distributive law)

$= BC + \overline{BC}DE = (BC + \overline{BC})(BC + DE)$ $(x + x' = 1)$

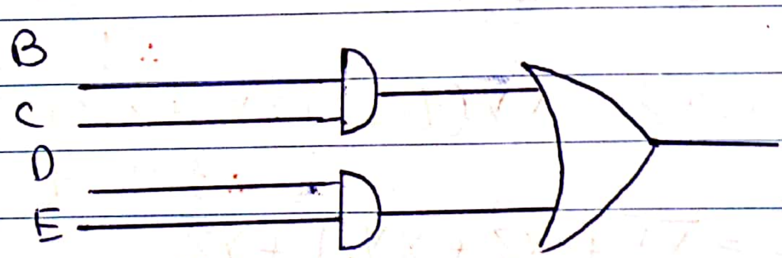
$= BC + DE$ $(x.1) = x$

$BCDE + BC(\overline{DE}) + (\overline{BC})DE = 12 + 3 + 4 = 17$

$BC + DE = 4 + 2 + 0 = 6$



$BCDE + BC(DE)' + (BC)'DE$



$BC + DE$

Q 2.1.c

$$\bar{X}Y + \bar{Y}Z + X\bar{Z} = X\bar{Y} + Y\bar{Z} + \bar{X}Z$$

X	Y	Z	$\bar{X}Y$	$\bar{Y}Z$	$X\bar{Z}$	$\bar{X}Y+\bar{Y}Z+X\bar{Z}$	$X\bar{Y}$	$Y\bar{Z}$	$\bar{X}Z$	$X\bar{Y}+Y\bar{Z}+\bar{X}Z$
0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	0	0	1	1
0	1	0	1	0	0	1	0	1	0	1
0	1	1	1	0	0	1	0	0	1	1
1	0	0	0	0	1	1	1	0	0	1
1	0	1	0	1	0	1	1	0	0	1
1	1	0	0	0	1	1	0	1	0	1
1	1	1	0	0	0	0	0	0	0	0

Q 2.2.d

$$\bar{X}\bar{Y} + \bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y} + XZ + Y\bar{Z}$$

$\therefore\ 1 = X + \bar{X}$

$$\bar{X}\bar{Y} + \bar{Y}Z \cdot 1 + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y} + \bar{Y}Z(X+\bar{X}) + XZ + XY + Y\bar{Z}$$

$\therefore\ 1 + X = 1$

$$= \bar{X}\bar{Y} + X\bar{Y}Z + \bar{X}\bar{Y}Z + XZ + XY + Y\bar{Z} = \bar{X}\bar{Y}(1+Z) + X\bar{Y}Z + XZ + XY + Y\bar{Z}$$

$\therefore\ 1 + \bar{Y} = 1$

$$= \bar{X}\bar{Y} + XZ(1+\bar{Y}) + XY + Y\bar{Z} = \bar{X}\bar{Y} + XZ + XY \cdot 1 + Y\bar{Z}$$

$$= \bar{X}\bar{Y} + XZ + XY(Z+\bar{Z}) + Y\bar{Z} = \bar{X}\bar{Y} + XZ + XYZ + XY\bar{Z} + Y\bar{Z}$$

$$= \bar{X}\bar{Y} + XZ(1+Y) + XY\bar{Z} + Y\bar{Z} = \bar{X}\bar{Y} + XZ + XY\bar{Z} + Y\bar{Z} = \bar{X}\bar{Y} + XZ + Y\bar{Z}(1+X)$$

Ans: $\overline{X}\,\overline{Y} + XZ + Y\overline{Z}$ (Hence proved)

Q.2.3.b.)

$$WY + \overline{W}Y\overline{Z} + WXZ + \overline{W}X\overline{Y} = WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z$$

$$WY(1+\overline{X}\,\overline{Z}) + \overline{W}Y\overline{Z}(X+\overline{X}) + WXZ(Y+\overline{Y}) + \overline{W}X\overline{Y}(Z+\overline{Z})$$

$$= WY + W\overline{X}Y\overline{Z} + \overline{W}XY\overline{Z} + \overline{W}\,\overline{X}Y\overline{Z} + WXYZ + WX\overline{Y}Z + \overline{W}X\overline{Y}Z + \overline{W}X\overline{Y}\,\overline{Z}$$

$$= WY + WXYZ + \overline{W}XY\overline{Z} + \overline{W}X\overline{Y}\,\overline{Z} + \overline{W}\,\overline{X}Y\overline{Z} + W\overline{X}Y\overline{Z} + WX\overline{Y}Z + \overline{W}X\overline{Y}Z$$

$$= WY(1+YZ) + \overline{W}X\overline{Z}(Y+\overline{Y}) + \overline{X}Y\overline{Z}(W+\overline{W}) + X\overline{Y}Z(W+\overline{W})$$

$$= WY(1) + \overline{W}X\overline{Z}(1) + \overline{X}Y\overline{Z}(1) + X\overline{Y}Z(1)$$

$$= WY + \overline{W}X\overline{Z} + \overline{X}Y\overline{Z} + X\overline{Y}Z = RHS$$

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Q.2.3.c)

$$A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C = (\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D)$$

Let $y = A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C$

$$\bar{y} = \overline{A\bar{D} + \bar{A}B + \bar{C}D + \bar{B}C} = (\overline{A\bar{D}}).(\overline{\bar{A}B}).(\overline{\bar{C}D}).(\overline{\bar{B}C})$$

$$\bar{y} = (\bar{A}+D)(A+\bar{B})(C+\bar{D})(B+\bar{C}) = (A\bar{A} + \bar{A}\bar{B} + AD + \bar{B}D)(BC + C\bar{C} + B\bar{D} + \bar{C}\bar{D})$$

$$\bar{y} = (0 + \bar{A}\bar{B} + AD + \bar{B}D)(BC + 0 + B\bar{D} + \bar{C}\bar{D})$$

$$\bar{y} = \bar{A}\bar{B}BC + \bar{A}\bar{B}B\bar{D} + \bar{A}\bar{B}\bar{C}\bar{D} + ABCD + ABD\bar{D} + A\bar{C}D\bar{D} + B\bar{B}CD + B\bar{B}D\bar{D} + \bar{B}\bar{C}D\bar{D}$$

$$\bar{y} = 0 + 0 + \bar{A}\bar{B}\bar{C}\bar{D} + ABCD + 0 + 0 + 0 + 0 + 0$$

$$\bar{y} = \bar{A}\bar{B}\bar{C}\bar{D} + ABCD$$

$$\bar{\bar{y}} = \overline{\bar{A}\bar{B}\bar{C}\bar{D} + ABCD} = (ABCD).(\overline{ABCD})$$

$$y = (\bar{A}+\bar{B}+\bar{C}+\bar{D})(A+B+C+D) = RHS$$

Q.2.6.b $(\overline{A+B+C}).\overline{ABC}$

Applying deMorgans law

$(\bar{A}\bar{B}\bar{C}).(\bar{A}+\bar{B}+\bar{C}) = \bar{A}\bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C}\bar{C}$

$= \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} + \bar{A}\bar{B}\bar{C} = \bar{A}\bar{B}\bar{C}(1+1+1) = \bar{A}\bar{B}\bar{C}$

d.) $\bar{A}\bar{B}D + \bar{A}\bar{C}D + BD$

$(\bar{A}\bar{B} + \bar{A}\bar{C} + B)D = (\bar{A}\bar{B} + B + \bar{A}\bar{C})D$

$= ((\bar{A}+B)(\bar{B}+B) + \bar{A}\bar{C})D = ((\bar{A}+B)(1) + \bar{A}\bar{C})D$

$= (\bar{A}+B+\bar{A}\bar{C})D = (\bar{A}(1+C)+B)D = (\bar{A}+B)D$

Q.2.7

a.) $\bar{X}\bar{Y} + XYZ + \bar{X}Y$ to 3 literals

$= \bar{X}(Y+\bar{Y}) + XYZ = \bar{X} + XYZ = (\bar{X}+X)(\bar{X}+YZ)$

$\therefore Y+\bar{Y} = 1$

$= \bar{X} + YZ = 3$ literals

$X+YZ = (X+Y)(X+Z)$

b.) $X + Y(Z + \overline{X+Z})$ to 2 literals

$= X + Y(Z + \bar{X}.\bar{Z}) = X + Y(Z+\bar{Z})(Z+\bar{X}) = X + Y(Z+\bar{X})$

$= X + YZ + Y\bar{X} = YZ + X + Y\bar{X} = YZ + (X+Y)(X+\bar{X}) = YZ + X + Y$

$= Y(1+Z) + X = X + Y$

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c.) $\overline{W}X(\overline{Z}+\overline{Y}Z)+X(W+\overline{W}YZ)$ to one literal

$= \overline{W}X\overline{Z}+\overline{W}X\overline{Y}Z+WX+\overline{W}XYZ = WX+\overline{W}X\overline{Z}+\overline{W}X\overline{Y}Z+\overline{W}XYZ$

$= WX+\overline{W}X\overline{Z}+\overline{W}XZ(\overline{Y}+Y) = WX+\overline{W}X\overline{Z}+\overline{W}XZ$

$= WX+\overline{W}X(Z+\overline{Z}) = WX+\overline{W}X = X(W+\overline{W}) = X$

d.) $(AB+\overline{A}\overline{B})(\overline{C}\overline{D}+CD)+\overline{AC}$

$= AB\overline{C}\overline{D}+ABCD+\overline{A}\overline{B}\overline{C}\overline{D}+\overline{A}\overline{B}CD+\overline{A}+\overline{C}$

$(\overline{B}\overline{C}\overline{D}+\overline{B}CD+1)$

$= ABCD+AB\overline{C}\overline{D}+\overline{C}+\overline{A}\overline{B}\overline{C}\overline{D}+\overline{A}\overline{B}CD+\overline{A} = ABCD+\overline{C}(AB\overline{D}+1)+\overline{A}($ ^

$= ABCD+\overline{C}+\overline{A}(\overline{B}\overline{C}\overline{D}+1) = ABCD+\overline{C}+\overline{A} = \overline{A}+A(BCD)+\overline{C}$

$= (\overline{A}+A)(\overline{A}+BCD)+\overline{C} = \overline{A}+BCD+\overline{C} = \overline{A}+C(BD)+\overline{C}$

$= \overline{A}+(C+\overline{C})(\overline{C}+BD) = \overline{A}+\overline{C}+BD$

Q.12

a.) $(AB+C)(B+\overline{C}D)$

$= AB+AB\overline{C}D+CB+C.\overline{C}D = AB+AB\overline{C}D+BC+0.D$

$= AB(1+\overline{C}D)+BC = AB+BC$ (SOP form)

$= B(A+C)$

$= (B+0)(A+C)$ POS form

b.) $\bar{X} + X(X+\bar{Y})(Y+\bar{Z})$

$= \bar{X} + X(XY + X\bar{Z} + Y.\bar{Y} + \bar{Y}\bar{Z}) = \bar{X} + X(XY + X\bar{Z} + \bar{Y}\bar{Z})$

$= \bar{X} + XY + X\bar{Z} + X\bar{Y}\bar{Z} = \bar{X} + X(Y + \bar{Z} + \bar{Y}\bar{Z}) = \bar{X} + X(Y + \bar{Z}(1+\bar{Y}$

$= \bar{X} + X(Y+\bar{Z}) = (\bar{X}+X)(\bar{X}+Y+\bar{Z}) = \bar{X} + Y + \bar{Z}$ (SOP)

POS: $\bar{X} + X(X+\bar{Y})(Y+\bar{Z}) = (\bar{X}+X)(\bar{X}+X+\bar{Y})(\bar{X}+Y+\bar{Z})$

$= (\bar{X}+X+\bar{Y})(\bar{X}+Y+\bar{Z})$

c.) $(A+B\bar{C}+CD)(\bar{B}+EF)$

$= (A\bar{B} + AEF + B.\bar{B}C + B\bar{C}EF + CD\bar{B} + CDEF)$

$= A\bar{B} + AEF + B\bar{C}EF + \bar{B}CD + CDEF$ (SOP)

POS: $(A+B\bar{C}+CD)(\bar{B}+EF) = (A+B\bar{C}+C)(A+B\bar{C}+D)(\bar{B}+EF)$

$= (A+C+B)(A+C+\bar{C})(A+D+B)(A+D+\bar{C})(\bar{B}+E)(\bar{B}+F)$

$= (A+C+B)(A+B+D)(A+\bar{C}+D)(\bar{B}+E)(\bar{B}+F)$

Q.2.14.d

$$F(A,B,C) = \sum m(0,2,3,4,5,7)$$

A \ BC	00	01	11	10
0	1		1	1
1	1	1	1	

$$F(A,B,C) = \bar{B}\bar{C} + AC + \bar{A}B$$

Q.2.15.c $\bar{A}\bar{B} + A\bar{C} + \bar{B}C + \bar{A}B\bar{C}$

$$= \bar{A}\bar{B}(C+\bar{C}) + A\bar{C}(B+\bar{B}) + \bar{B}C(A+\bar{A}) + \bar{A}B\bar{C}$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + A\bar{C}B + A\bar{C}\bar{B} + A\bar{B}C + \bar{A}\bar{B}C + \bar{A}B\bar{C}$$

$$= \bar{A}\bar{B}C + \bar{A}\bar{B}\bar{C} + AB\bar{C} + A\bar{B}\bar{C} + A\bar{B}C + \bar{A}B\bar{C}$$

$$= 001 + 000 + 110 + 100 + 101 + 010$$

$$= m_1 + m_0 + m_6 + m_4 + m_5 + m_2$$

$$= \sum m(0,1,2,4,5,6)$$

A \ BC	00	01	11	10
0	1	1		1
1	1	1		1

$$= \bar{B} + \bar{C}$$

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Q. 2.17. a)

$$F(W,X,Y,Z) = \sum m(0,2,5,8,9,10,11,12,13)$$

WX \ YZ	00	01	11	10
00	1			1
01		1		
11	1	1		
10	1	1	1	1

$F(W,X,Y,Z) = W\overline{X} + W\overline{Y} + X\overline{Y}Z + \overline{X}\,\overline{Z}$

Q. 2.17. b

$$F(A,B,C,D) = \sum m(1,3,6,7,9,11,12,13,15)$$

AB \ CD	00	01	11	10
00		1	1	
01			1	1
11	1	1	1	
10		1	1	

$F(A,B,C,D) = AB\overline{C} + \overline{A}BC + CD + \overline{B}D$

Q. 2.18. c $\quad \overline{B}\,\overline{D} + ABD + \overline{A}BC$

AB \ CD	00	01	11	10
00	1			1
01			1	1
11		1	1	
10	1			1

$= \overline{B}\,\overline{D}(A+\overline{A})(C+\overline{C}) + ABD(C+\overline{C}) + \overline{A}BC(D+\overline{D})$

$= \overline{B}\,\overline{D}(AC + A\overline{C} + \overline{A}\,\overline{C} + \overline{A}C) + ABDC + ABD\overline{C} + \overline{A}BCD + \overline{A}BC\overline{D}$

$= A\overline{B}C\overline{D} + A\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}\,\overline{C}\,\overline{D} + \overline{A}\,\overline{B}C\overline{D} + ABCD + AB\overline{C}D + \overline{A}BCD + \overline{A}BC\overline{D}$

$= 1010 + 1000 + 0000 + 0010 + 1111 + 1101 + 0111 + 0110$

Q.2.19.a

$$F(W,X,Y,Z) = \sum m(0,2,5,7,8,10,12,13,14,15)$$

WX \ YZ	00	01	11	10
00	1			1
01		1	1	
11	1	1	1	1
10	1			1

Prime: $XZ, WX, \bar{X}\bar{Z}, W\bar{Z}$

Essential: $XZ, \bar{X}\bar{Z}$

Q.2.20.c

$$F(W,X,Y,Z) = \sum m(0,2,4,6,7,8,9,12,13,15)$$

WX \ YZ	00	01	11	10
00	1			1
01	1		1	1
11	1	1	1	
10	1	1		

Prime Implicants: $W\bar{Y}, XYZ, \bar{Y}\bar{Z}, \bar{W}\bar{Z}$

Essential: $W\bar{Y}, \bar{W}\bar{Z}$

$$F = W\bar{Y} + \bar{W}\bar{Z} + XYZ$$

Q.2.22.c) $(\bar{A}+\bar{B}+D)(\bar{A}+\bar{D})(A+B+\bar{D})(A+\bar{B}+C+D)$

$\bar{A}+\bar{B}+D$	$\bar{A}+\bar{D}$	$A+B+\bar{D}$	$A+\bar{B}+C+D$
1 1 - 0	1 - - 1	0 0 - 1	0 1 0 0
1 1 0 0	1 0 0 1	0 0 0 1	
1 1 1 0	1 0 1 1	0 0 1 1	
	1 1 0 1		
	1 1 1 1		

AB \ CD	00	01	11	10
00	1	0	0	1
01	0	1	1	1
11	0	0	0	0
10	1	0	0	1

AB \ CD	00	01	11	10
00	1			1
01		1	1	1
11				
10	1			1

$F=(B+\bar{D})(\bar{A}+\bar{B})(\bar{B}+C+D)$

$F=\bar{B}\bar{D}+\bar{A}BD+\bar{A}BC$

Q.2.23.b.) $F(W,X,Y,Z) = \prod M(3,11,13,15)$

WX \ YZ	00	01	11	10
00			0	
01				
11		0	0	
10			0	

WX \ YZ	00	01	11	10
00	1	1		1
01	1	1	1	1
11	1			1
10	1	1		1

$F = (\overline{W}+\overline{X}+\overline{Z})(X+\overline{Y}+\overline{Z})$

$F = \overline{Z} + \overline{W}X + \overline{X}\,\overline{Y}$

Q.2.24.C $F(A,B,C) = \sum m(1,2,4), d(A,B,C) = \sum m(0,3,6,7)$

A \ BC	00	01	11	10
0	X	1	X	1
1	1		X	X

$F = \overline{A} + \overline{C}$

Q.26.b.) $F(W,X,Y,Z) = \sum m(3,4,9,15), d(W,X,Y,Z) = \sum m(0,1,2,5,10,12,14)$

WX \ YZ	00	01	10	11
00	X	X	1	X
01	1	X		
10	X		1	X
11		1		X

WX \ YZ	00	01	11	10
00	X	X		X
01		X	0	0
11	X	0		X
10	0		0	X

$F = \overline{W}\,\overline{X} + \overline{W}\,\overline{Y} + W\overline{X}Y + \overline{X}\,\overline{Y}Z$

$F = (\overline{Y}+Z)(X+Z)(\overline{W}+\overline{X}+Y)(\overline{W}+X+\overline{Y})$
$(W+\overline{X}+\overline{Y})$