

Problem

Thursday, September 28, 2023 5:45 PM

Find the missing terms:-

$$\begin{aligned} x_0 &= 0.0 & f(x_0) &= ? & f(x_0, x_1) &= ? & f(x_0, x_1, x_2) &= \frac{50}{7} \\ x_1 &= 0.4 & f(x_1) &= ? & f(x_1, x_2) &= 10 & & \\ x_2 &= 0.7 & f(x_2) &= 6 & & & & \end{aligned}$$

Sol Using Newton Divided differences.
we know that

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0} \quad \text{--- (1)}$$

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1} \quad \text{--- (2)}$$

$$f(x_0, x_1, x_2) = \frac{f(x_1, x_2) - f(x_0, x_1)}{x_2 - x_0} \quad \text{--- (3)}$$

Now putting values:-
(3) Becomes

$$\frac{50}{7} = \frac{10 - f(x_0, x_1)}{0.7 - 0}$$

$$\frac{0.7}{1} \times \frac{50}{7} = \frac{10 - f(x_0, x_1)}{1} \times 0.7$$

$$\frac{0.7}{10} \times \frac{10}{7} = \frac{10 - f(x_0, x_1)}{0.7} \times 0.1$$

$$5 = 10 - f(x_0, x_1)$$

$$\Rightarrow f(x_0, x_1) = 5$$

Now using eq (2)

$$f(x_1, x_2) = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

$$10 = \frac{6 - f(x_1)}{0.7 - 0.4}$$

$$0.3 \times 10 = \frac{6 - f(x_1)}{0.3} \times 0.3$$

$$3 = 6 - f(x_1)$$

$$\Rightarrow f(x_1) = 3$$

Now using eq ①

$$f(x_0, x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}$$

$$5 = \frac{3 - f(x_0)}{0.4 - 0.0}$$

$$0.4 \times 5 = \frac{3 - f(x_0)}{0.4} \times \cancel{0.4}$$

$$2 = 3 - f(x_0)$$

$$\Rightarrow \boxed{f(x_0) = 1}$$

Hence the required values