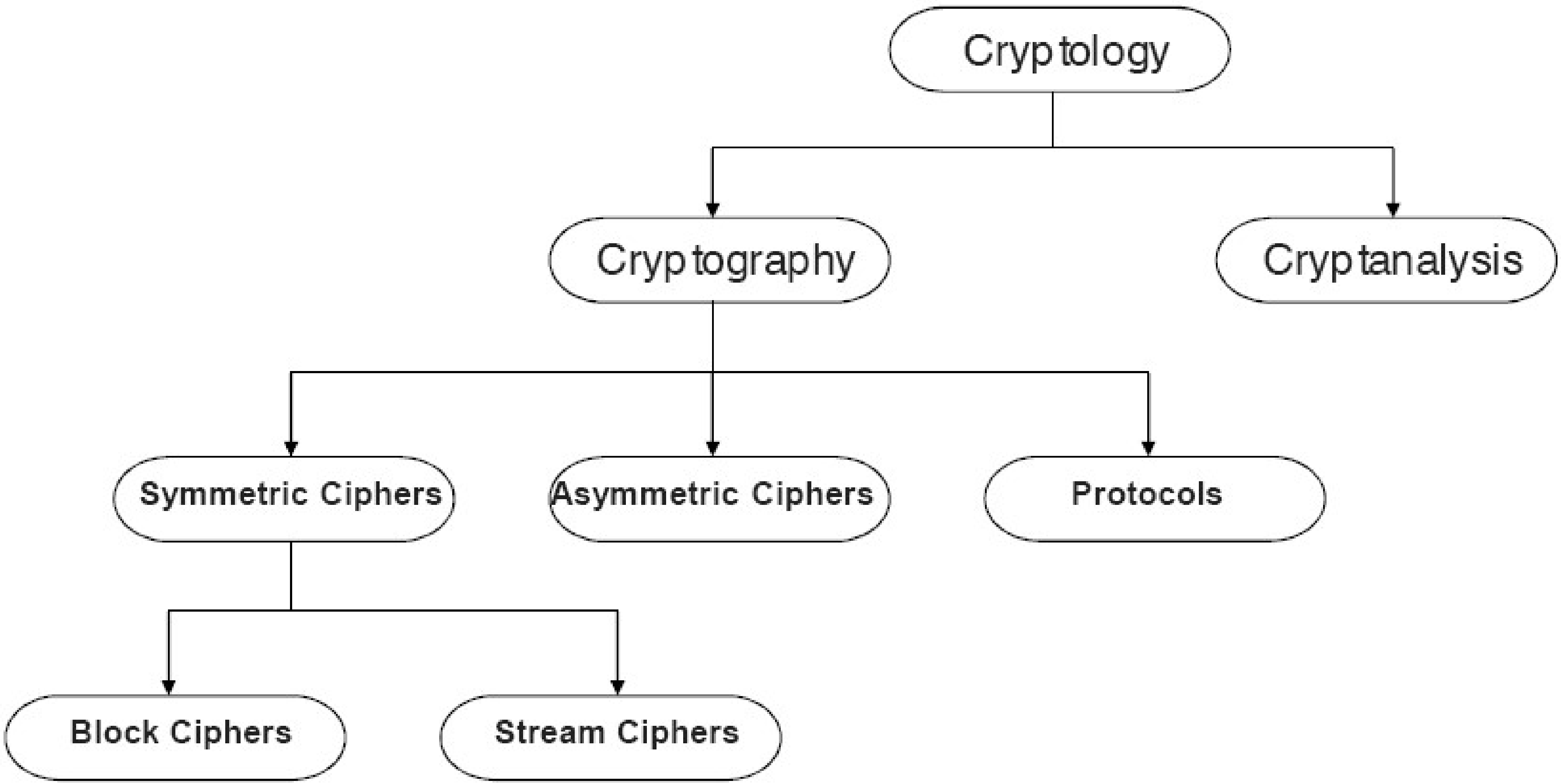
Information Security
CS3002

Lecture 7
9th September 2024

Dr. Rana Asif Rehman
Email: r.asif@lhr.nu.edu.pk

Classification of the Field of Cryptology



Adopted with thanks from: Chapter 1 of Understanding Cryptography by Christof Paar and Jan Pelzl

Private-Key Cryptography

- Traditional **private/secret/single key** cryptography uses **one** key.
- Shared by both sender and receiver.
- If this key is disclosed communications are compromised.
- Also is **symmetric**, parties are equal.
  - Hence does not protect sender from receiver forging a message & claiming is sent by sender.

Public-Key Cryptography

- **Public-key/two-key/asymmetric** cryptography involves the use of **two** keys:
  - A **public-key**, which may be known by anybody, and can be used to **encrypt messages**, and **verify signatures.**
  - A **private-key**, known only to the recipient, used to **decrypt messages**, and **sign** (create) **signatures.**
- Is **asymmetric** because
  - Those who encrypt messages or verify signatures **cannot** decrypt messages or create signatures.

Need for Both

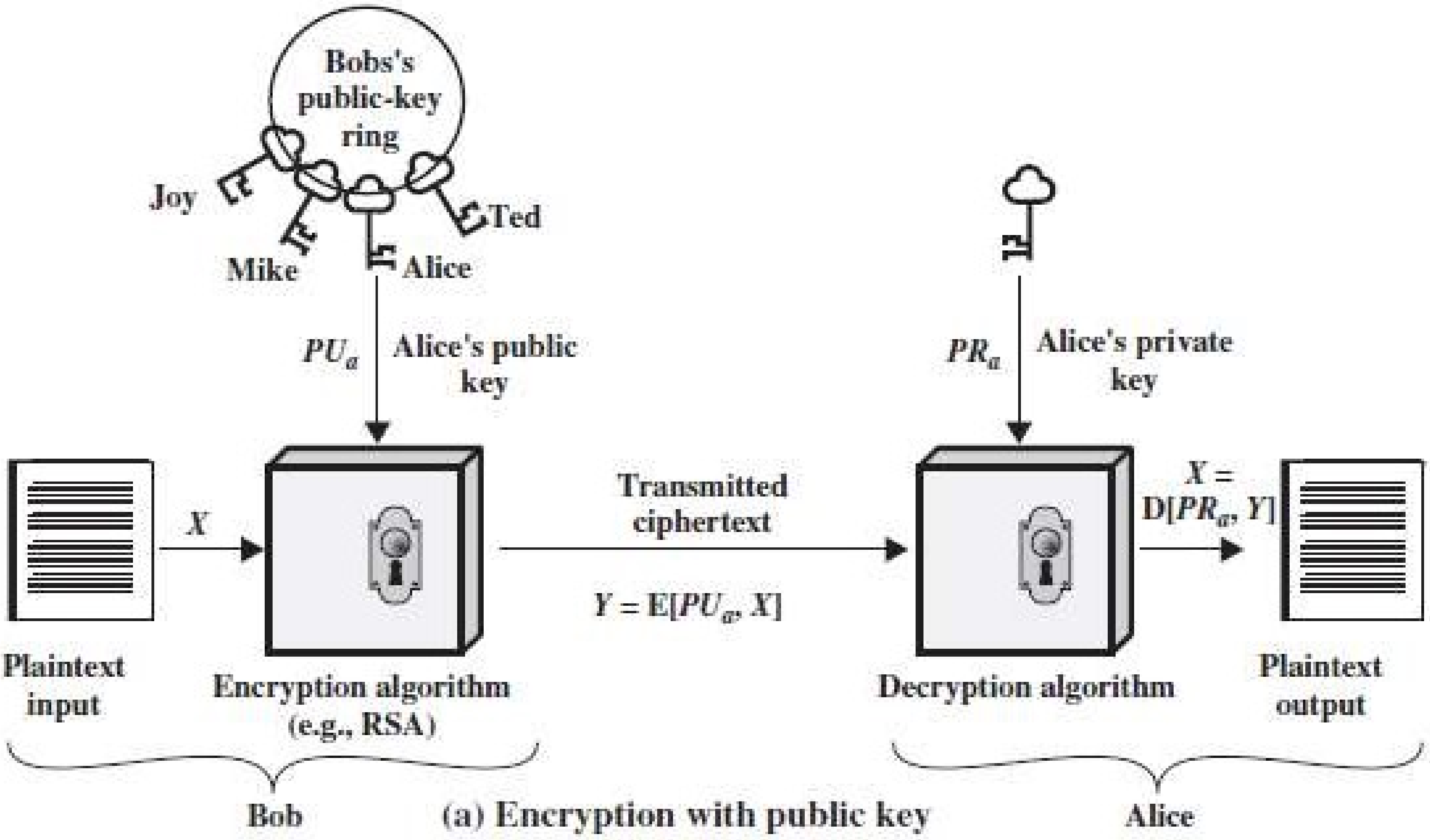
There is a very important fact that is sometimes misunderstood: The advent of asymmetric-key cryptography does not eliminate the need for symmetric-key cryptography.

kdc is key distribution channel

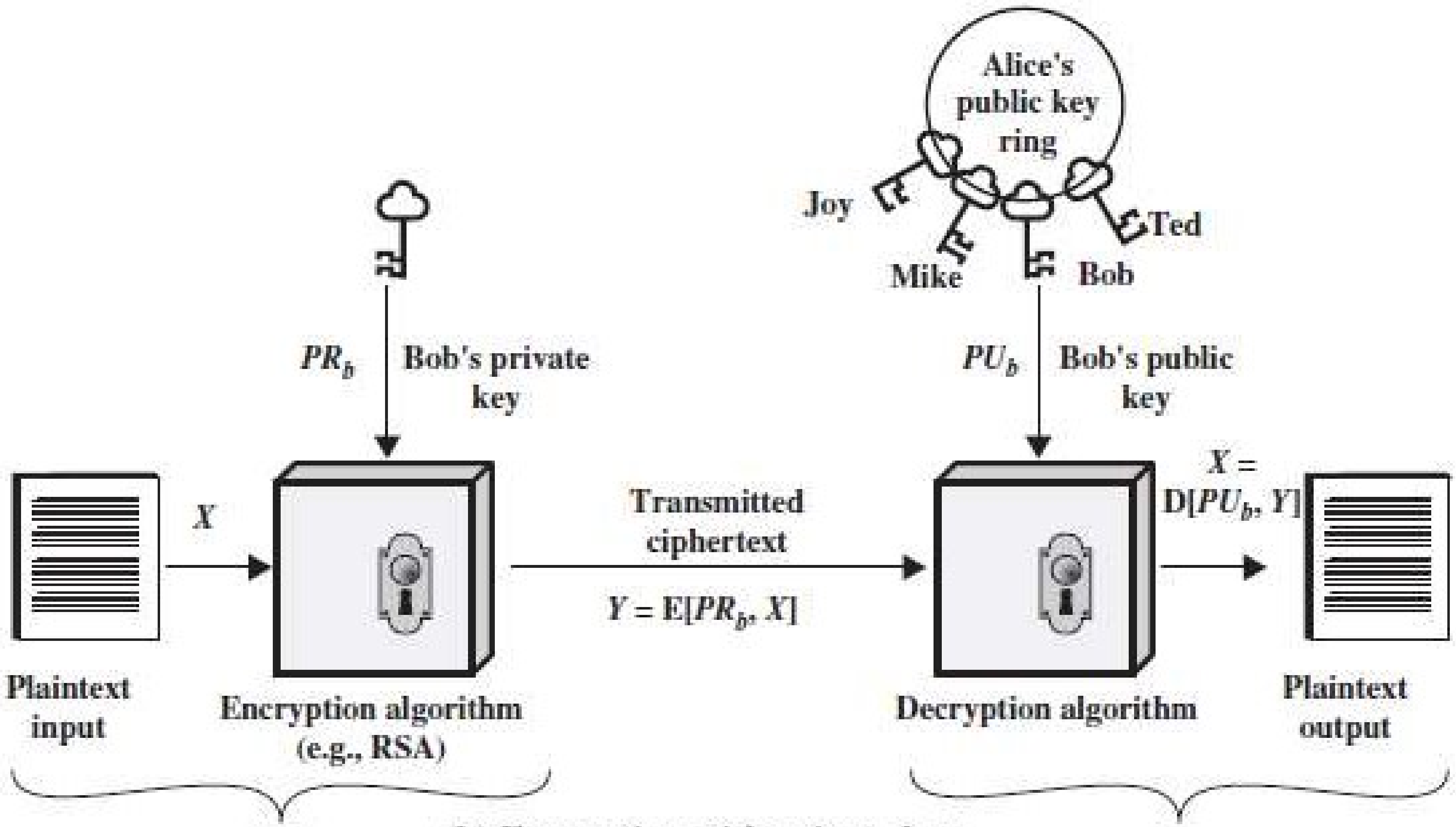
Why Public-Key Cryptography?

- Developed to address two key issues:
  - **Key distribution** – how to have secure communications in general without having to trust a KDC with your key.
  - **Digital signatures** – how to verify a message comes intact from the claimed sender.
- Public invention due to Whitfield Diffie & Martin Hellman at Stanford University in 1976
  - Known earlier in classified community.

Encryption


Bobs's public-key ring
Joy
Mike
Alice
Ted
PU_a
Alice's public key
PR_a
Alice's private key
X
Transmitted ciphertext
$Y = E[PU_a, X]$
$X = D[PR_a, Y]$
Plaintext input
Encryption algorithm (e.g., RSA)
Decryption algorithm
Plaintext output
Bob
(a) Encryption with public key
Alice

Authentication



(b) Encryption with private key

Advantages

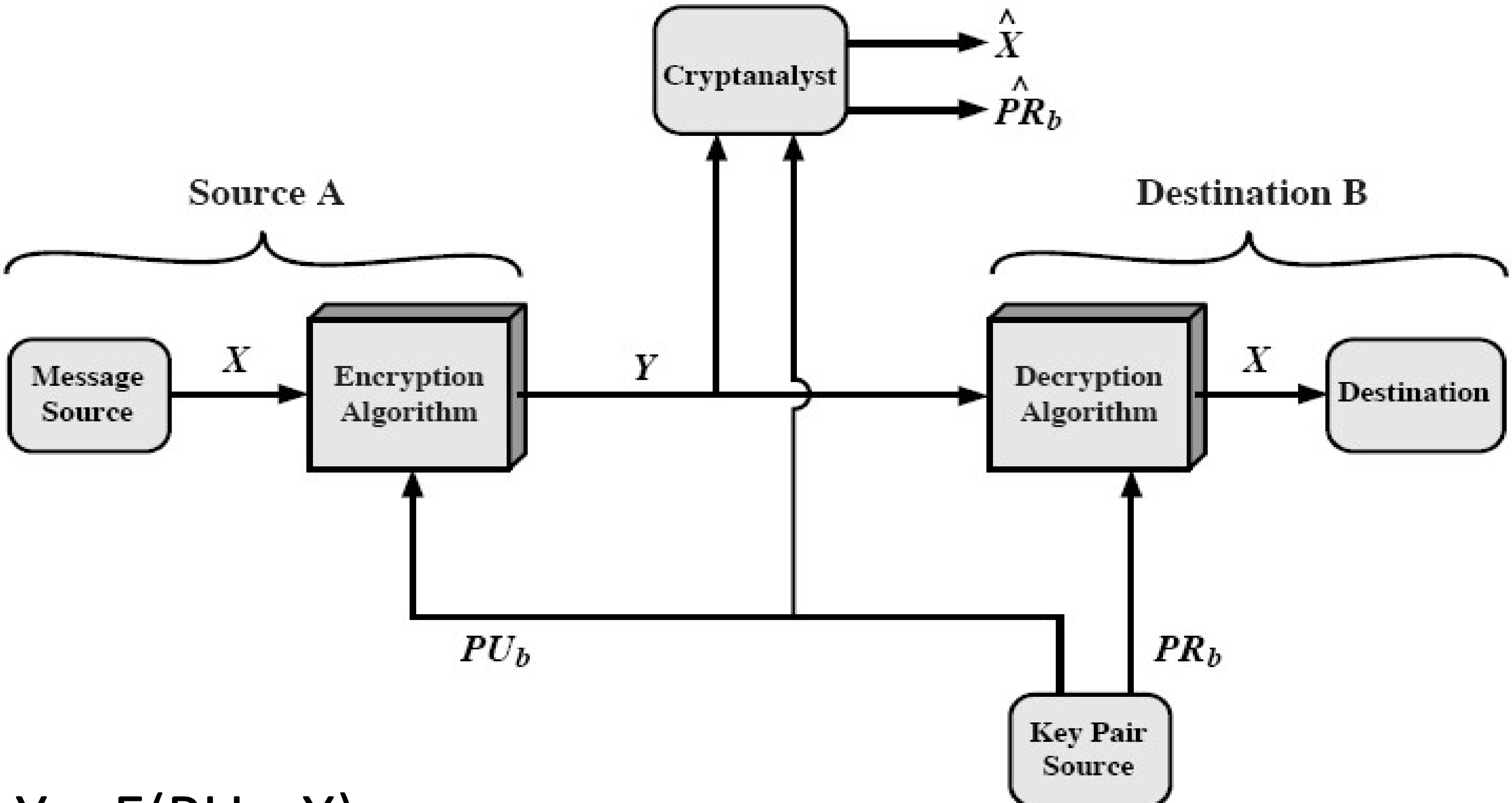
- Each user generates pair of keys.
- Place one of keys in public register or accessible file (public key).
- Private keys generated locally.
- Private key need not to be distributed.
- Keys can be changed at any time.
  - At any time, a system can change its private key and publish the companion public key to replace its old public key.

Public-Key Applications

- Can classify uses into 3 categories:
  - **Encryption/decryption** (provide secrecy)
  - **Digital signatures** (provide authentication)
  - **Key exchange** (of session keys)
- Some algorithms are suitable for all uses, others are specific to one.

Algorithm	Encryption/Decryption	Digital Signature	Key Exchange
RSA	Yes	Yes	Yes
Elliptic Curve	Yes	Yes	Yes
Diffie-Hellman	No	No	Yes
DSS	No	Yes	No

Applications: Confidentiality

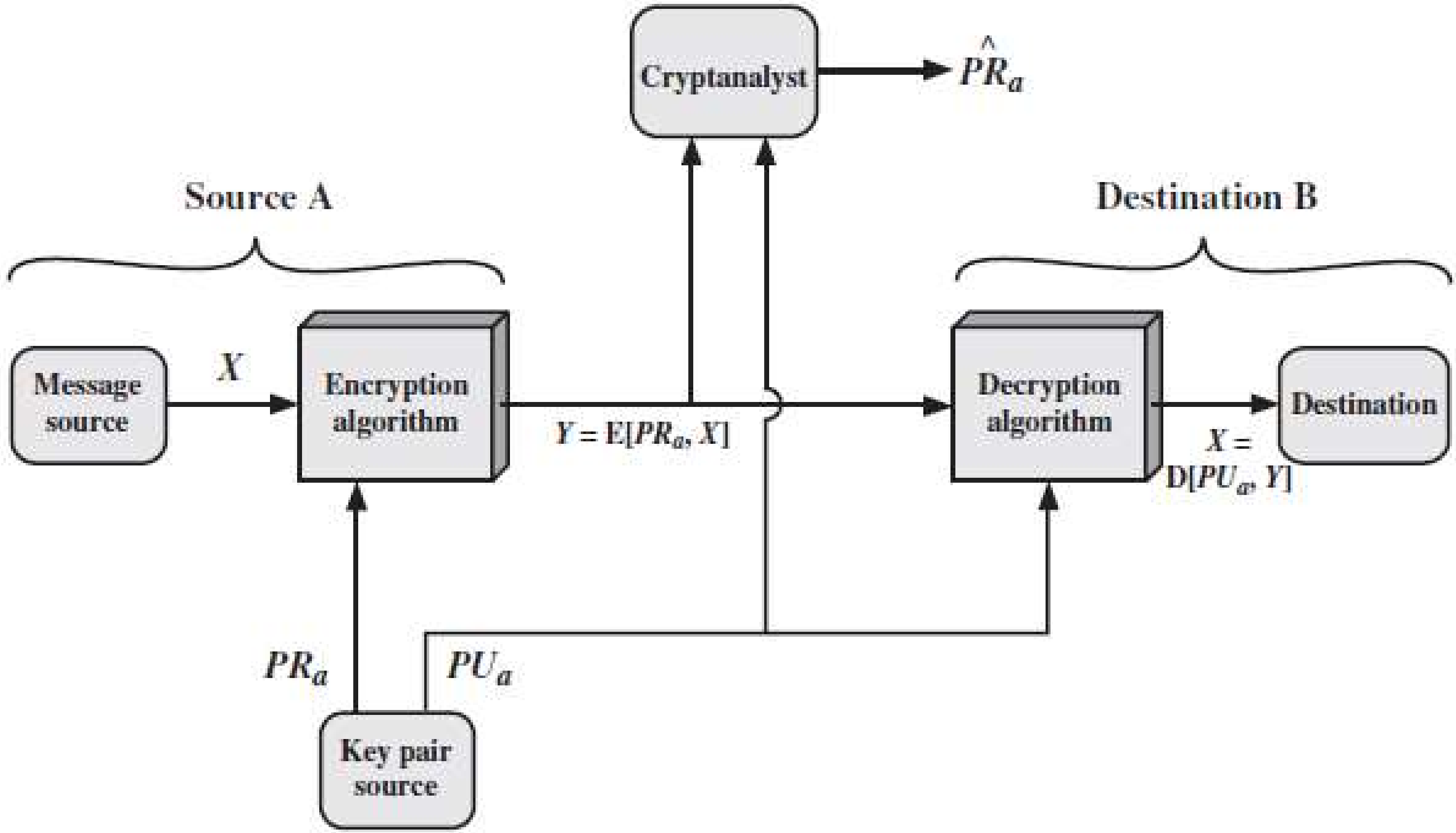


$Y = E(PU_b, X)$

$X = D(PR_b, Y)$

11

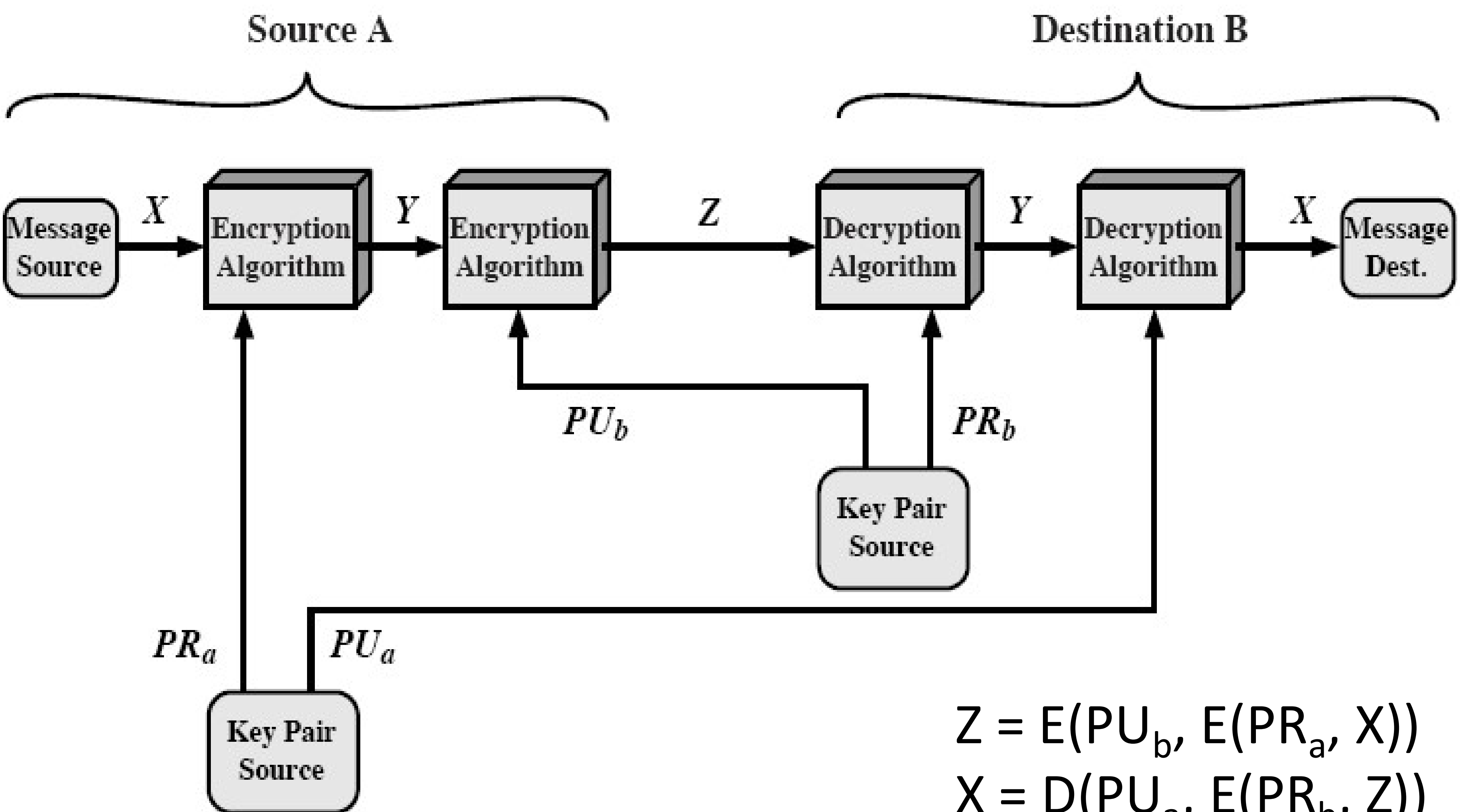
Applications: Authentication



$Y = E(PR_a, X)$

$X = D(PU_a, Y)$

Applications: Confidentiality + Authentication



$$Z = E(PU_b, E(PR_a, X))$$
$$X = D(PU_a, E(PR_b, Z))$$

Requirements for Public-Key Cryptography (1/2)

1. Computationally easy for a party B to generate a pair (public key PU_b, private key PR_b).
2. Easy for sender to generate ciphertext:

$$C = E(PU_b, M)$$

3. Easy for the receiver to decrypt ciphertext using private key:

$$M = D(PR_b, C) = D[PR_b, E(PU_b, M))]$$

Requirements for Public-Key Cryptography (2/2)

4. Computationally infeasible to determine private key (PR_b) knowing public key (PU_b).
5. Computationally infeasible to recover message M, knowing PU_b and ciphertext C.
6. Either of the two keys can be used for encryption, with the other used for decryption:

$$M = D[PU_b, E(PR_b, M)] = D[PR_b, E(PU_b, M)]$$

RSA

RSA

- By Rivest, Shamir & Adleman of MIT in 1977.
- Best known & widely used public-key scheme.
- Based on exponentiation in a finite (Galois) field over integers modulo a prime.
  - nb. exponentiation takes $O((\log n)^3)$ operations (easy)
- Uses large integers (eg. 1024 bits).
- Security due to cost of factoring large numbers.
  - nb. factorization takes $O(e^{\log n \log \log n})$ operations (hard)

RSA Key Setup

- Each user generates a public/private key pair by:
- Selecting two large primes at random - p, q
- Computing their system modulus $n=p.q$
  - note $\varnothing(n)=(p-1)(q-1)$
  - >512 bits $->$ $1.340e^{154}$
- Selecting at random the encryption key e
  - where $1<e<\varnothing(n)$, $\gcd(e,\varnothing(n))=1$
- Solve following equation to find decryption key d
  - $e.d=1 \bmod \varnothing(n)$ and $0 \leq d \leq n$
- Publish their public encryption key: PU={e,n}
- Keep secret private decryption key: PR={d,n}

RSA Use

- To encrypt a message M the sender:
  - Obtains **public key** of recipient $PU=\{e,n\}$
  - Computes: $C = M^e \bmod n$, where $0 \leq M < n$
- To decrypt the ciphertext C the owner:
  - Uses their private key $PR=\{d,n\}$
  - Computes: $M = C^d \bmod n$
- Note that the message M must be smaller than the modulus n (block if needed) ??

RSA Example - Key Setup (cont.)

Key Generation by Alice	
Select p, q	p and q both prime, $p \neq q$
Calculate $n = p \times q$	
Calcuate $\phi(n) = (p-1)(q-1)$	
Select integer e	$\gcd(\phi(n), e) = 1; 1 < e < \phi(n)$
Calculate d	$d \equiv e^{-1} \pmod{\phi(n)}$
Public key	$PU = \{e, n\}$
Private key	$PR = \{d, n\}$

Encryption by Bob with Alice's Public Key	
Plaintext:	$M < n$
Ciphertext:	$C = M^e \bmod n$

Decryption by Alice with Alice's Public Key	
Ciphertext:	C
Plaintext:	$M = C^d \bmod n$

Why RSA Works

- Because of Euler's Theorem
- $a^{\varnothing(n)} \bmod n = 1$ where $\gcd(a,n)=1$
- In RSA have:
  - $n=p.q$
  - $\varnothing(n)=(p-1)(q-1)$
  - Carefully chose e & d to be inverses $\bmod \varnothing(n)$
  - Hence $e.d=1+k.\varnothing(n)$ for some k

$$C^d = M^{e.d} = M^{1+k.\varnothing(n)} = M^1.(M^{\varnothing(n)})^k$$
$$= M^1.(1)^k = M^1 = M \bmod n$$

RSA Example - Key Setup

1. Select primes: p=17 & q=11
2. Compute $n = pq$ =17 x 11=187
3. Compute $\varnothing(n)=(p-1)(q-1)$=16 x 10=160
4. Select e: gcd(e,160)=1; choose e=7
5. Determine d: de=1 mod 160 and d < 160 Value is d=23 since 23x7=161= 10x160+1
   Extended Euclidean Method
6. Publish public key PU={7,187}
7. Keep secret private key PR={23,187}

RSA Example - En/Decryption (1/2)

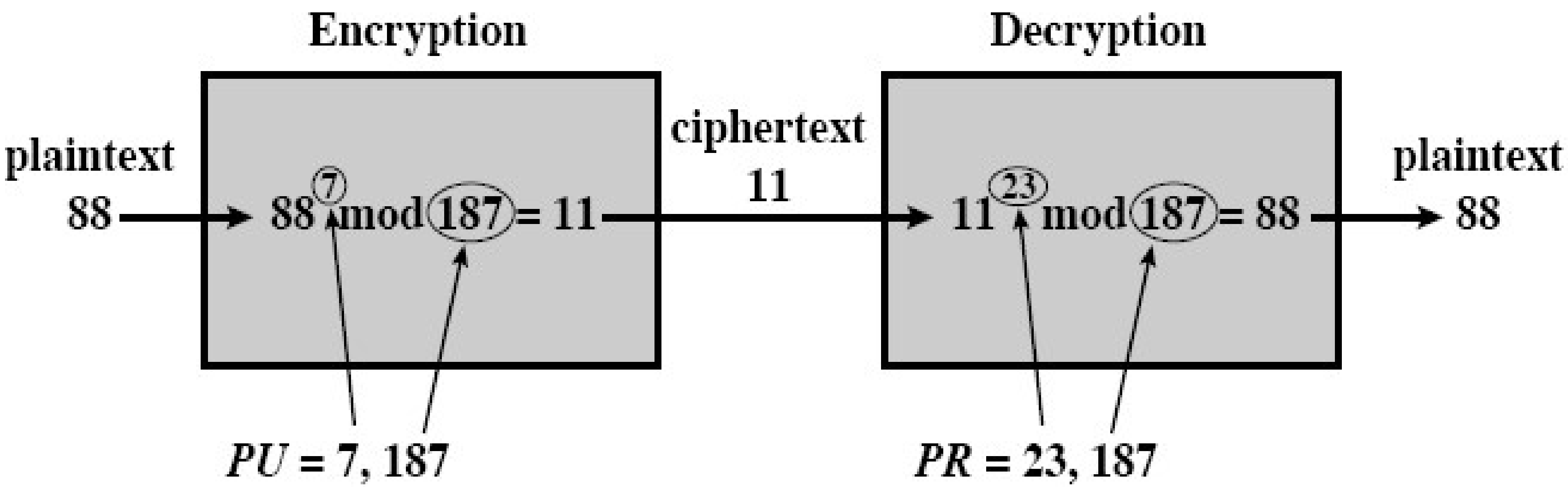
- Sample RSA encryption/decryption is:
- Given message $M = 88$ ($88<187$)
- Encryption:

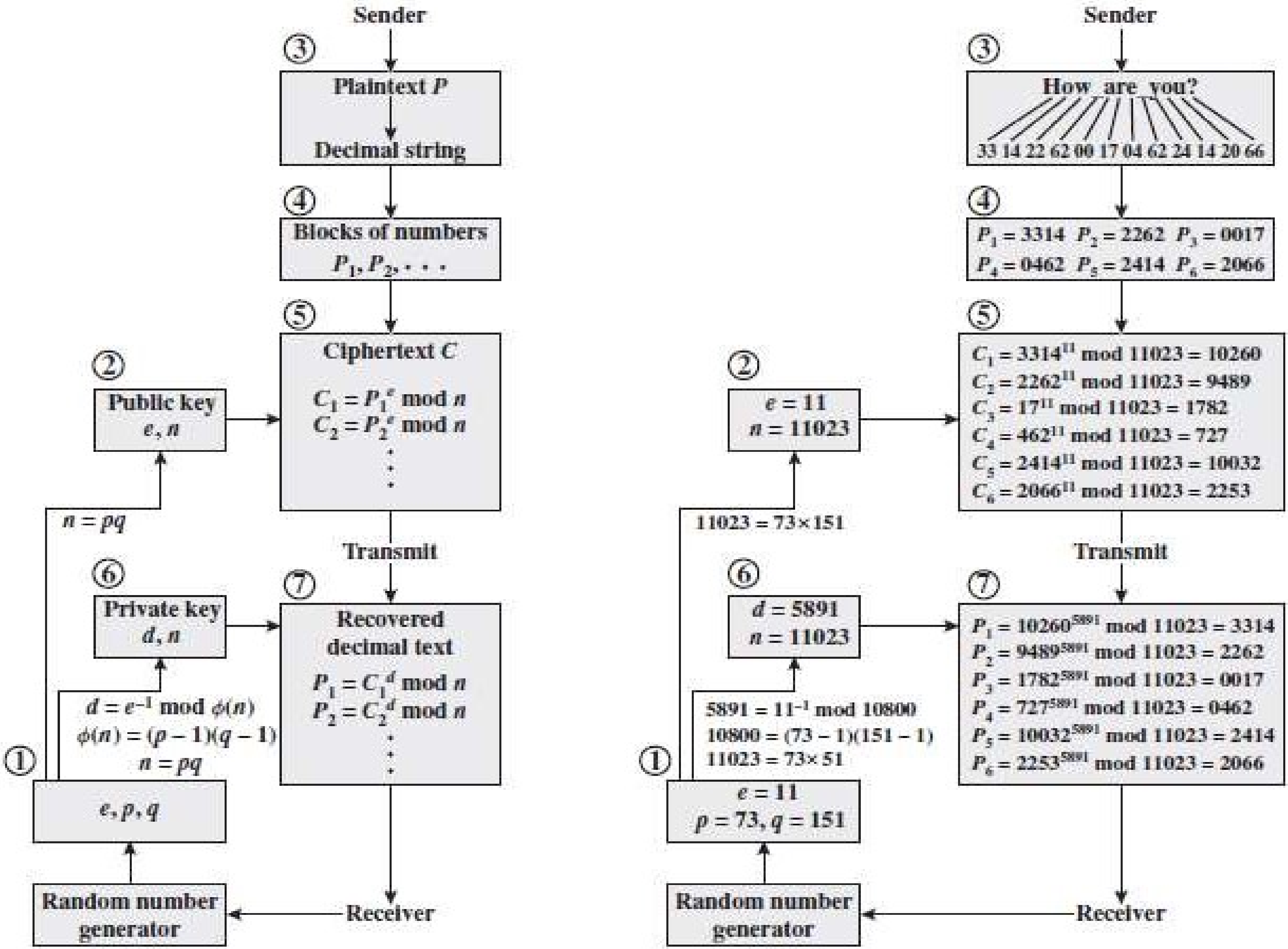
$$C = 88^7 \bmod 187 = 11$$

- Decryption:

$$M = 11^{23} \bmod 187 = 88$$

RSA Example - En/Decryption (2/2)

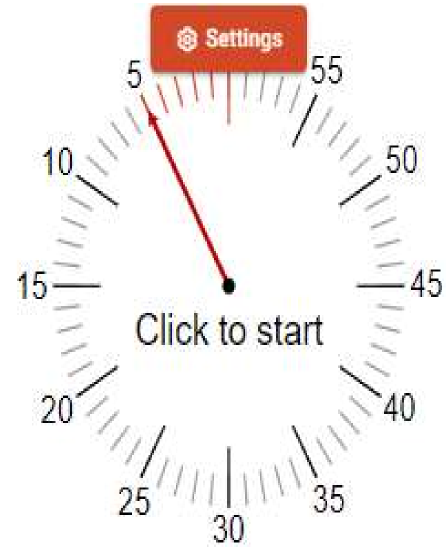




(a) General approach

(b) Example

Activity Time



Generate the public and private key pair by applying the RSA algorithm:

p = 47
q = 59

Answer:

Step 1: Let p = **47** and q = **59**. Thus n = **47** x **59** = **2773**

Step 2: Select e = **17**

Step 3: Publish (n,e) = (**2773**, **17**)

Step 4: (p-1) x (q-1) = **46** x **58** = **2668**

Use the Euclidean Algorithm to compute the modular inverse of **17** modulo **2668**. The result is d = **157**

<< Check: **17** x **157** = 2669 = 1(mod **2668**) >>

Public key is (**2773**,**17**)

Private key is **157**