OPERATIONS RESEARCH

Network Analysis: CPM & PERT

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PROJECT NETWORKS

- Two types of project networks
  - Activity-on-Arc (AOA)
    - On this diagram, the activity is represented on an arc, while a node is used to separate an activity from its immediate predecessors.
  - Activity-on-Node (AON)
    - On this diagram, the activity is represented by the node, while the arc is used to showed the precedence relationship between the activities.

CONSTRUCTING A PROJECT NETWORK

- **EVENT**
  - A point in time when the activity is started or finished; does not consume time
- **ACTIVITY**
  - An element of the project that require time; may or may not required resources
  - Description of activities should use verb/noun format; develop project specifications
- **MERGE ACTIVITY**
  - More than one activity immediately preceding it (more than one arrow flowing to it)
- **PARALLEL ACTIVITIES**
  - Activities that can take place at the same time, if the manager wishes; may or may not occur simultaneously
- **PATH**
  - A sequence of connected, dependent activities
- **CRITICAL PATH**
  - Longest path (s) through the network; if an activity on the path is delayed, the project is delayed the same amount of time.
- **BURST ACTIVITY**
  - More than one activities immediately following it (more than one dependency arrow flowing from it)
- **DUMMY ACTIVITY**
  - A dummy activity, which is normally depicted by a dashed arrow, consumes no time or resources.

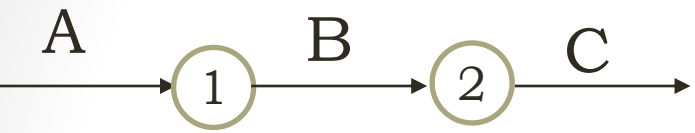
BASIC RULES: Developing a project network

EIGHT RULES APPLY:

1. Networks typically flow from left to right.
2. An Activity cannot begin until all preceding activities have been completed.
3. Arrows on networks indicate precedence and flow. Arrows can cross over each other.
4. Each activity should have a unique identification number.
5. An activity identification number must be larger than that of any activities preceding it.
6. Looping in not allowed (in other words recycling through a set of activities cannot take place).
7. Conditional statements are not allowed (i.e. this type of statement should not appear: if successful, do something; if not, do nothing)
8. When there are multiple starts, a common start node can be used to indicate a clear project beginning on the network. Similarly, a single project end, node can be used to indicate a clear ending.

Activity–On–Arc Network Fundamentals

Predecessor, Successor & Concurrent or Parallel activities



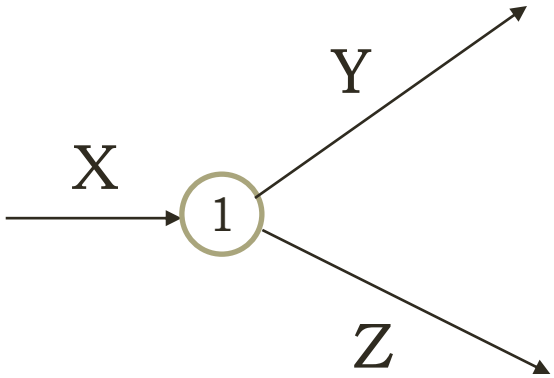
A is preceded by nothing

B is preceded by A

C is preceded by B

(A)

Y and Z are preceded by X

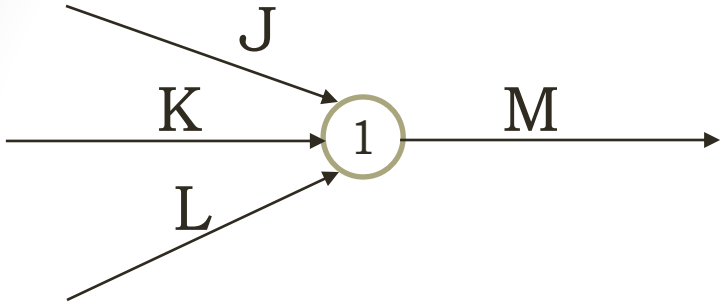


Y and Z can begin at the same time, if you wish

(B)

Activity – On – Arc Network Fundamentals

Predecessor, Successor & Concurrent or Parallel activities

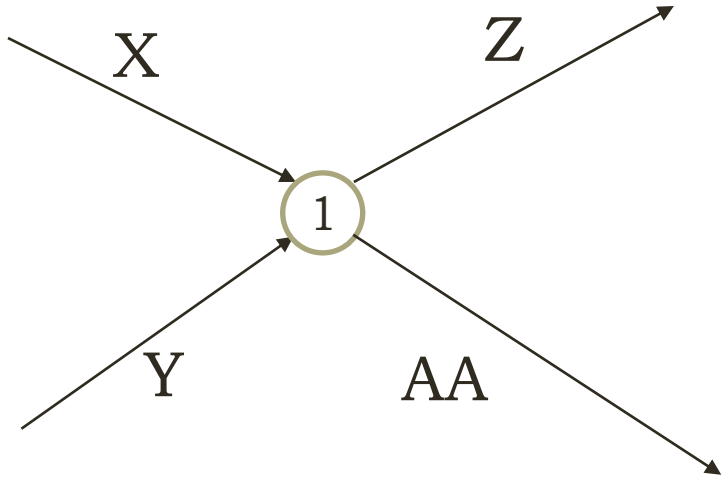


J,K & L can begin at the same time, if you wish (they need not occur simultaneously)

All (J,K,L) must be completed before M can begin

(C)

Z is preceded by X and Y



'AA' is preceded by X and Y

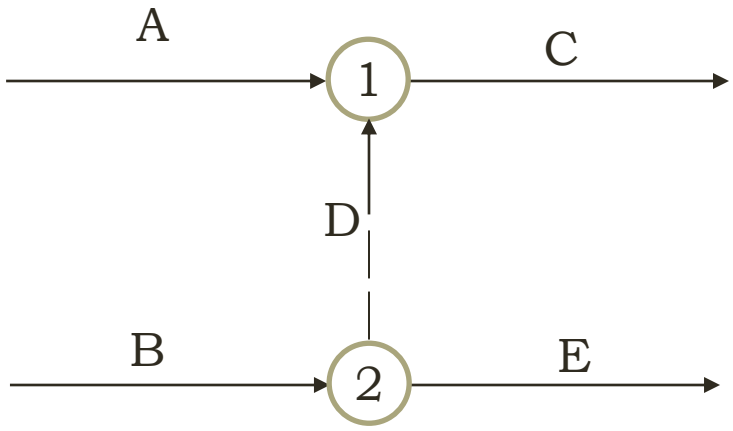
(D)

Activity – On – Arc Network Fundamentals

Predecessor, Successor & Concurrent or Parallel activities

Use of Dummy activity:

1. Activity 'C' can start immediately after 'A' and 'B' are completed.
2. Activity 'E' can start immediately after only 'B' is completed.



Here, 'D' is a Dummy Activity.

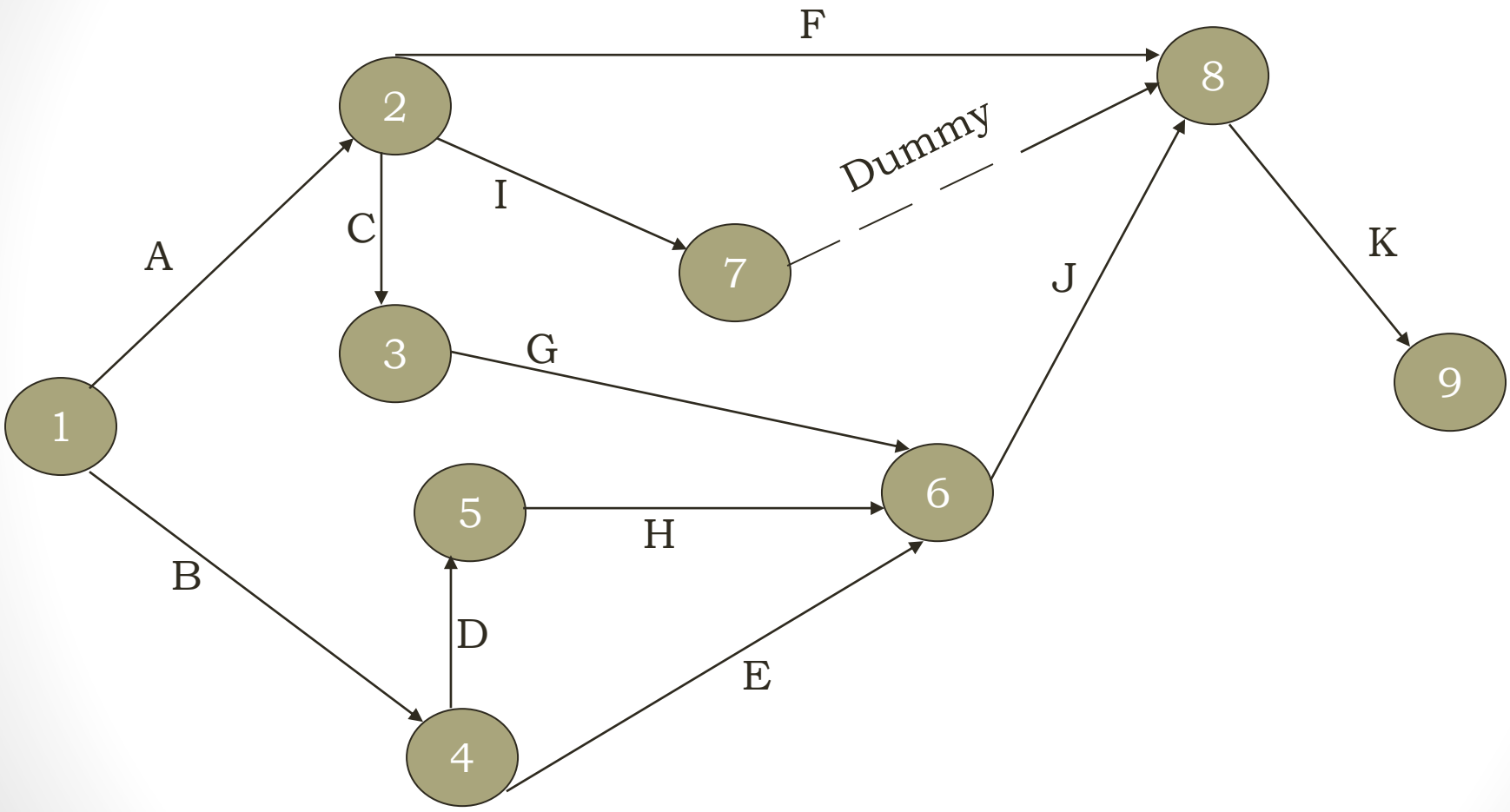
Example: (AOA Network)

Consider the following data:

Activity	Description	Immediate Predecessor(s)
A	Select administrative and medical staff	-
B	Select site and do site survey	-
C	Select equipment	A
D	Prepare final construction plans and layout	B
E	Bring utilities to the site	B
F	Interview applicants and fill positions in nursing, support staff, maintenance, & security	A
G	Purchase and take delivery of equipment	C
H	Construct the hospital	D
I	Develop an information system	A
J	Install the equipment	E, G, H
K	Train nurses and support staff	F, I, J

Draw the AOA network Diagram.

AOA Network:



PRACTICE QUESTION # 1

Develop the network by yourself.

Activity	Description	Immediate Predecessor(s)
A	Procurement of parts for sub – assembly '1'	None
B	Procurement of parts for sub – assembly '2'	None
C	Procurement of parts for sub – assembly '3'	None
D	Building sub – assembly '1'	A
E	Building sub – assembly '2'	B
F	Building sub – assembly '4'	D,E
G	Building sub – assembly '3'	B,C
H	Building the final product	F,G
I	Final Test	H

PRACTICE QUESTION # 2

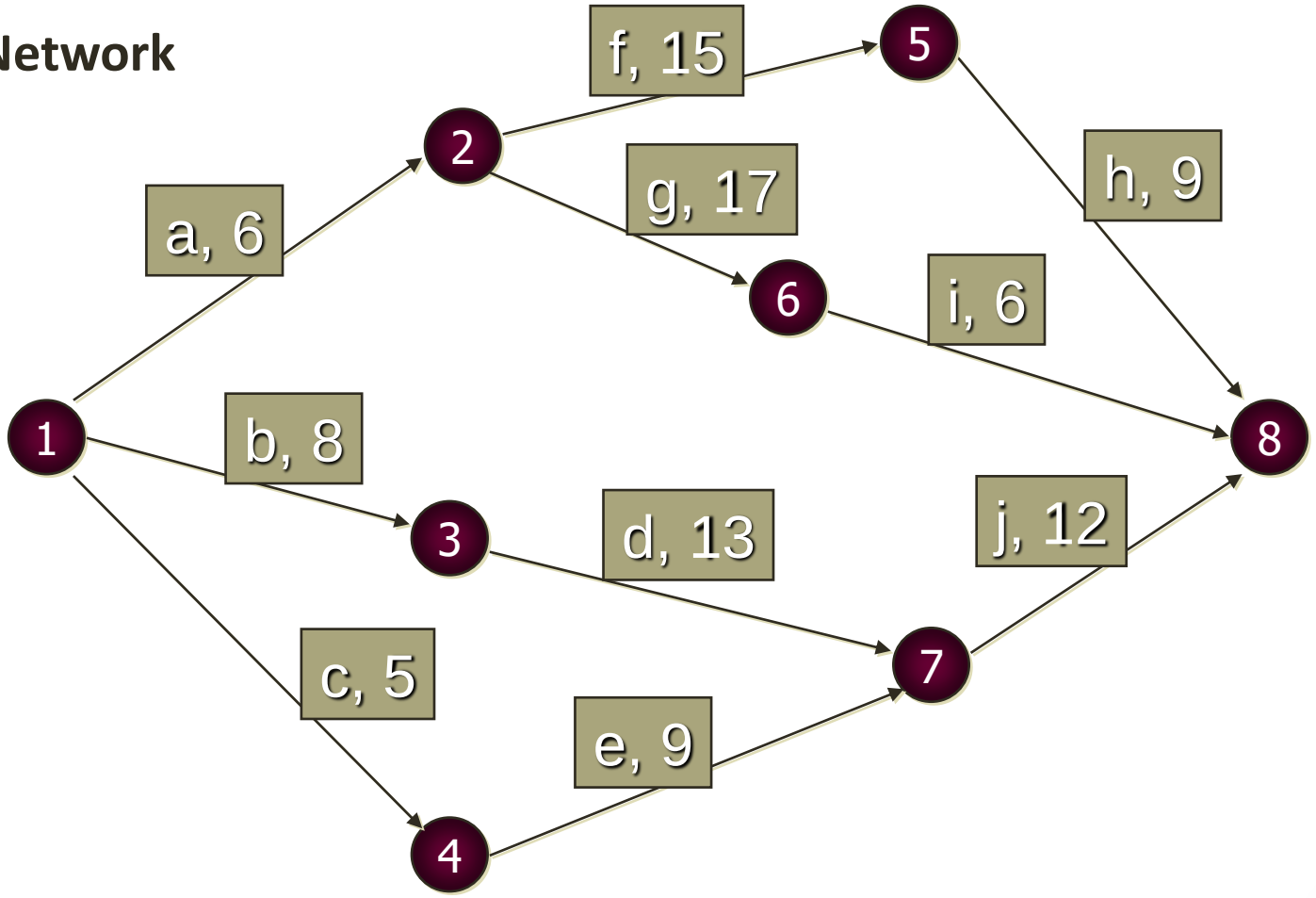
Network Information		
Country Engineers Design Department		
ACTIVITY	**DESCRIPTION**	**PROCEDING ACTIVITY**
A	Application Approval	None
B	Construction Plans	A
C	Traffic Study	A
D	Service Availability Check	A
E	Staff Report	B,C
F	Commission Approval	B,C,D
G	Wait for Construction	F
H	Occupancy	E,G

PROJECT NETWORK ANALYSIS: CRITICAL PATH METHOD (CPM)

- In 1957 the Critical Path Method (CPM) was developed as a network model for project management. CPM is a deterministic method that uses a fixed time estimate for each activity. While CPM is easy to understand and use, it does not consider the time variations that can have a great impact on the completion time of a complex project.

CPM: EXAMPLE

CPM Network

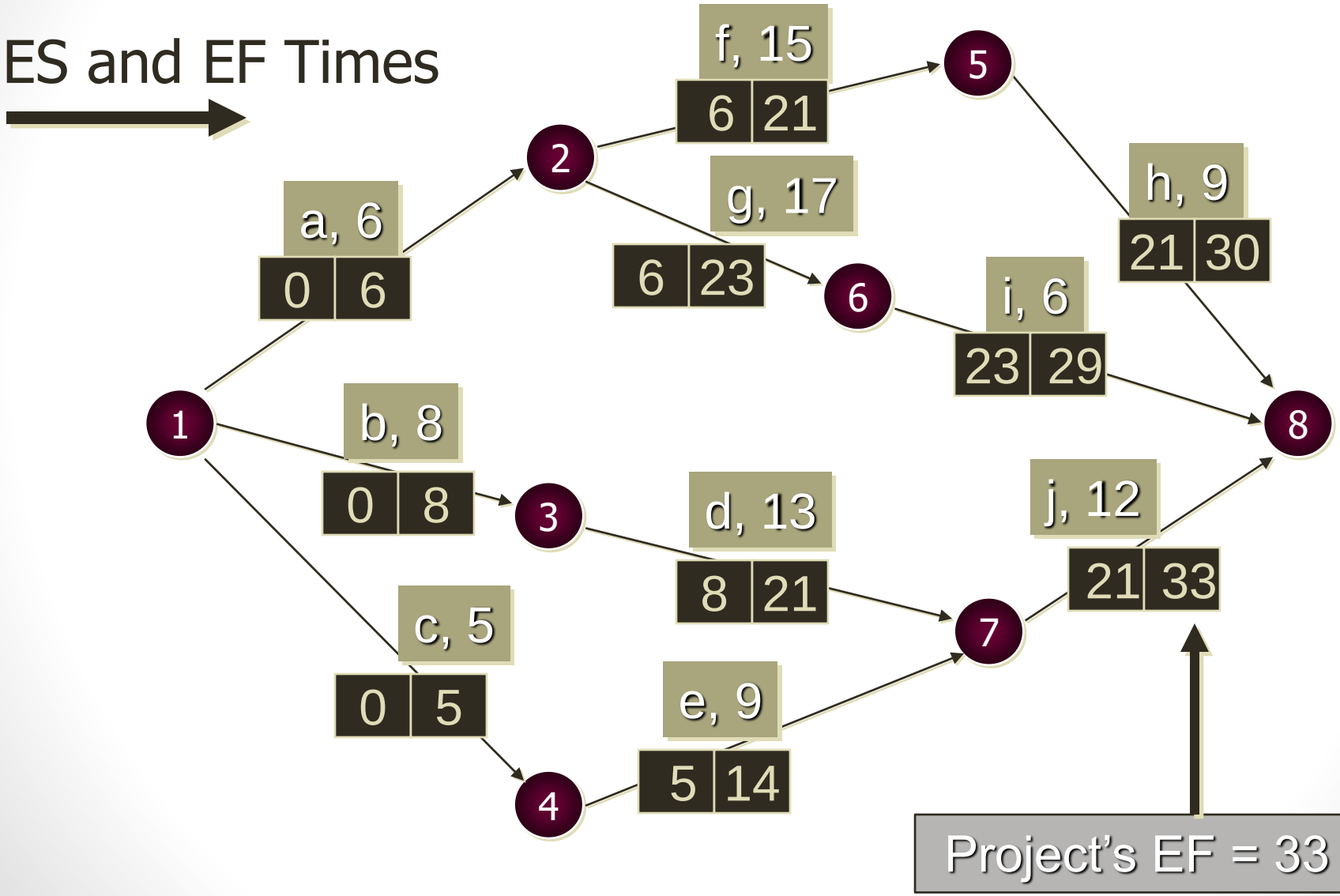


CPM / PERT TECHNIQUES: FUNDAMENTALS

Performing few simple computations allows PM to complete a process know as *Forward and Backward Pass*

- **FORWARD PASS – EARLIEST TIMES**
  1. How soon can the activity start? (early start – ES)
  2. How soon can the activity finish? (early finish – EF)
  3. How soon can the project be finished? (expected time – ET)
- **BACKWARD PASS – LATEST TIMES**
  1. How late can the activity start? (late start – LS)
  2. How late can the activity finish? (late finish – LF)
  3. Which activities represent critical path (CP)? This is the longest path in the network which, when delayed, will delay the project
  4. How long can the activity be delayed? (slack or float – SL)

CPM: EXAMPLE


ES and EF Times
a, 6
0 6
b, 8
0 8
c, 5
0 5
f, 15
6 21
g, 17
6 23
h, 9
21 30
i, 6
23 29
d, 13
8 21
e, 9
5 14
j, 12
21 33
1
2
3
4
5
6
7
8
Project's EF = 33

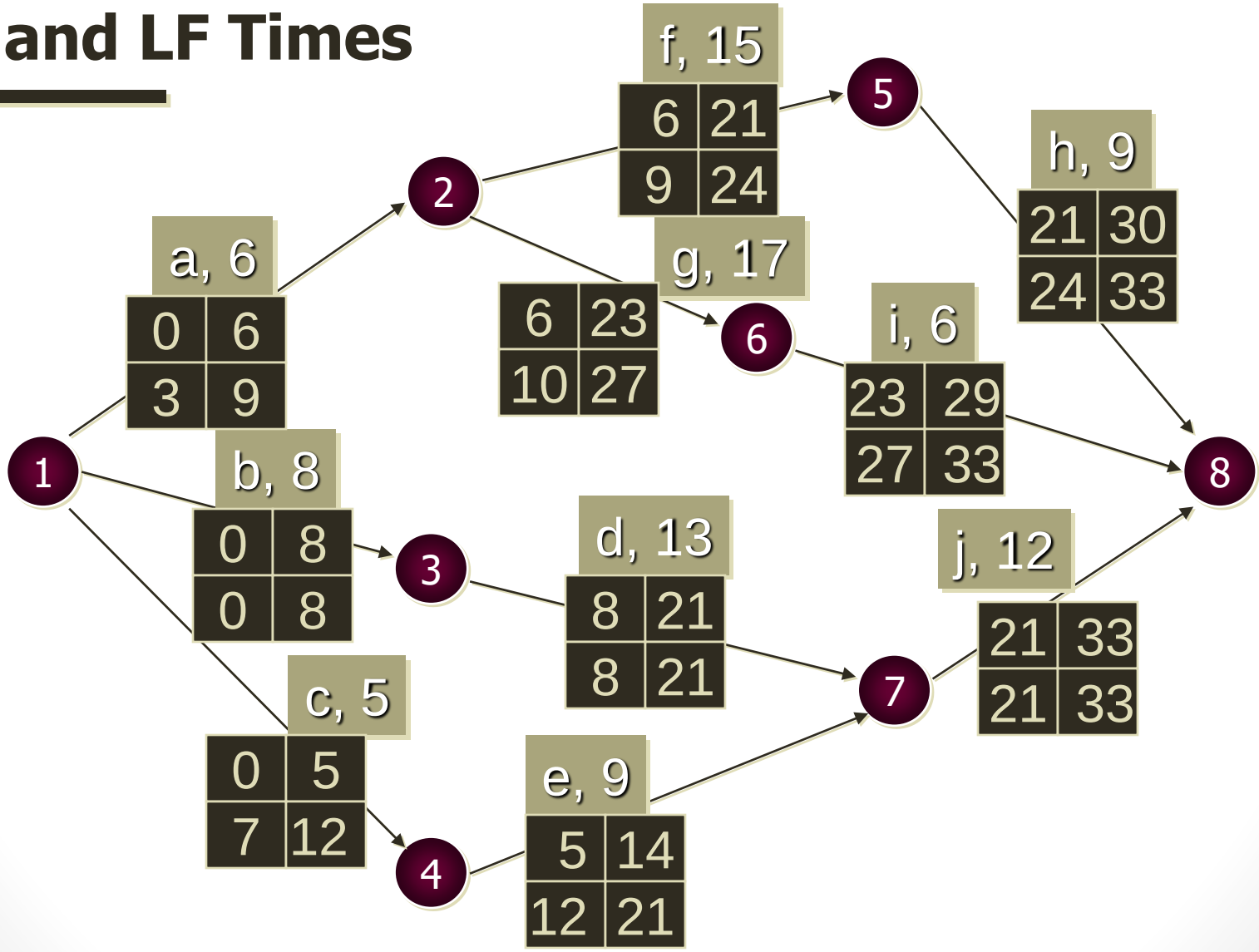
CPM / PERT TECHNIQUES: FUNDAMENTALS …

- **Backward Pass** – Latest Times; similar to Forward Pass; remember three things:

1. *Subtract* activity times along each path starting with the project end activity (LF – Dur. = LS)
2. *Carry* the LS to the next preceding activity to establish its LF, *unless*
3. Next preceding activity is a *burst* activity; select the smallest LS of all its immediate successor activities to establish its LF

CPM: EXAMPLE ...

LS and LF Times

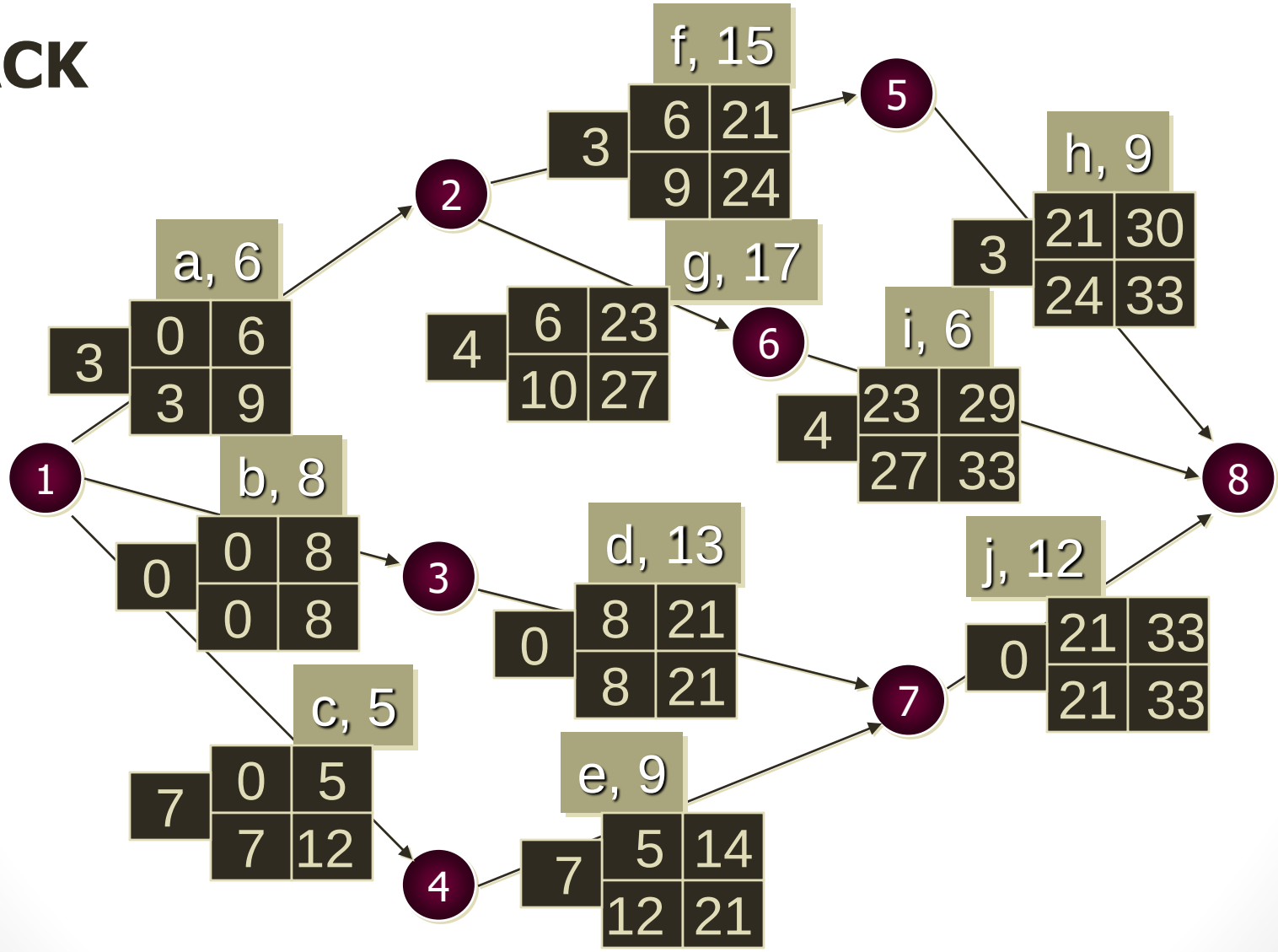


CPM / PERT TECHNIQUES: FUNDAMENTALS …

- **Slack or Float – SL**
  - Forward & Backward Passes Computed
  - Possible to determine which activities can be delayed by computing "Slack" or "Float"
  - LS – ES = SL
  - LF – EF = SL
  - Total Slack: tells us the amount of time an activity can be delayed; not delay project

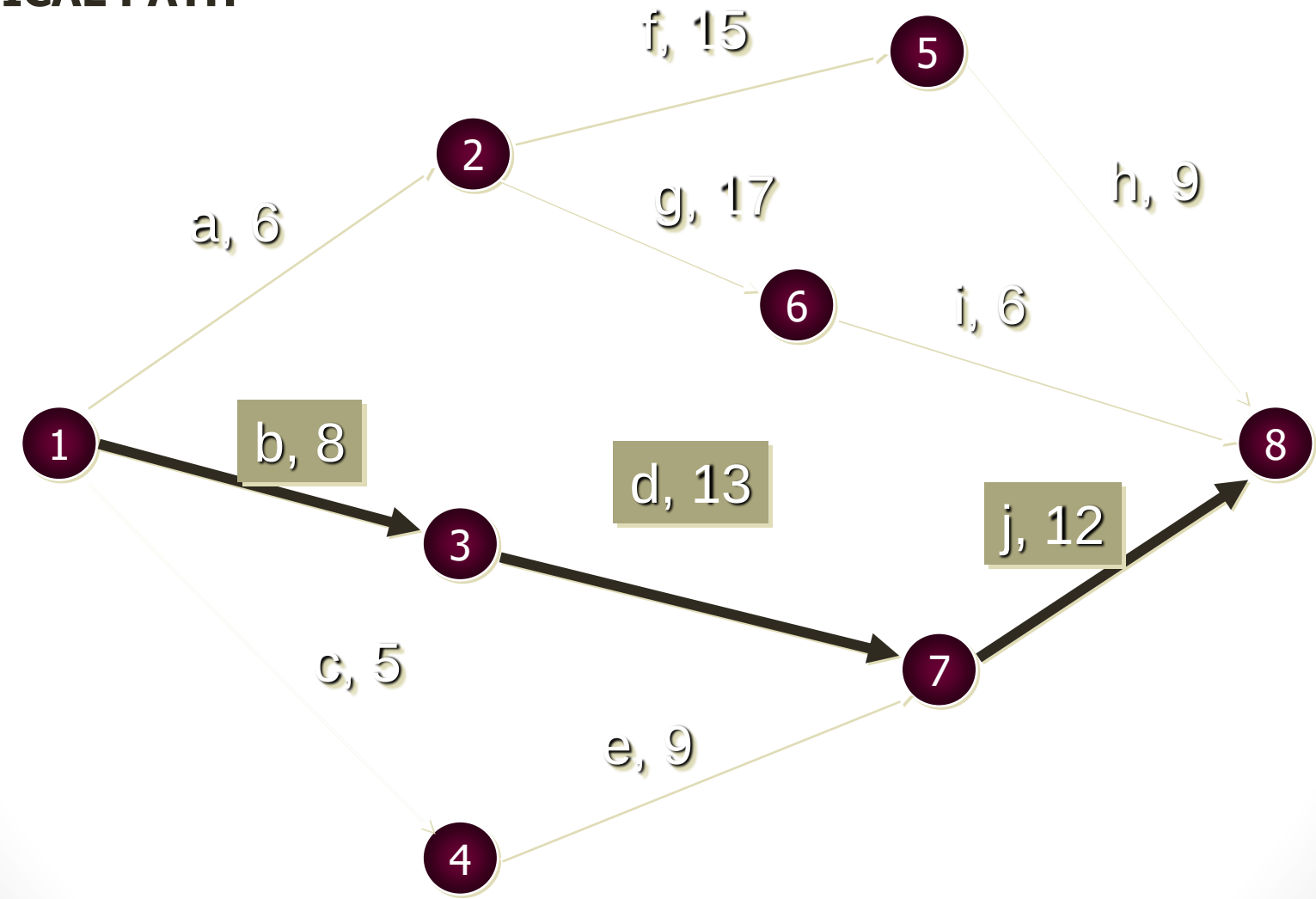
CPM EXAMPLE ...

SLACK



CPM EXAMPLE …

CRITICAL PATH



CPM EXAMPLE …

Critical Path:

1 → 3 → 7 → 8

Activities on the Critical Path:

b→ d → j

Total Project Time:

8+13+12 = **33**

Program/Project Evaluation & Review Technique (PERT)

- The technique is based on the assumption that an activity's duration follows a probability distribution instead of being a single value.

- The probabilistic information about the activities is translated into probabilistic information about the project.

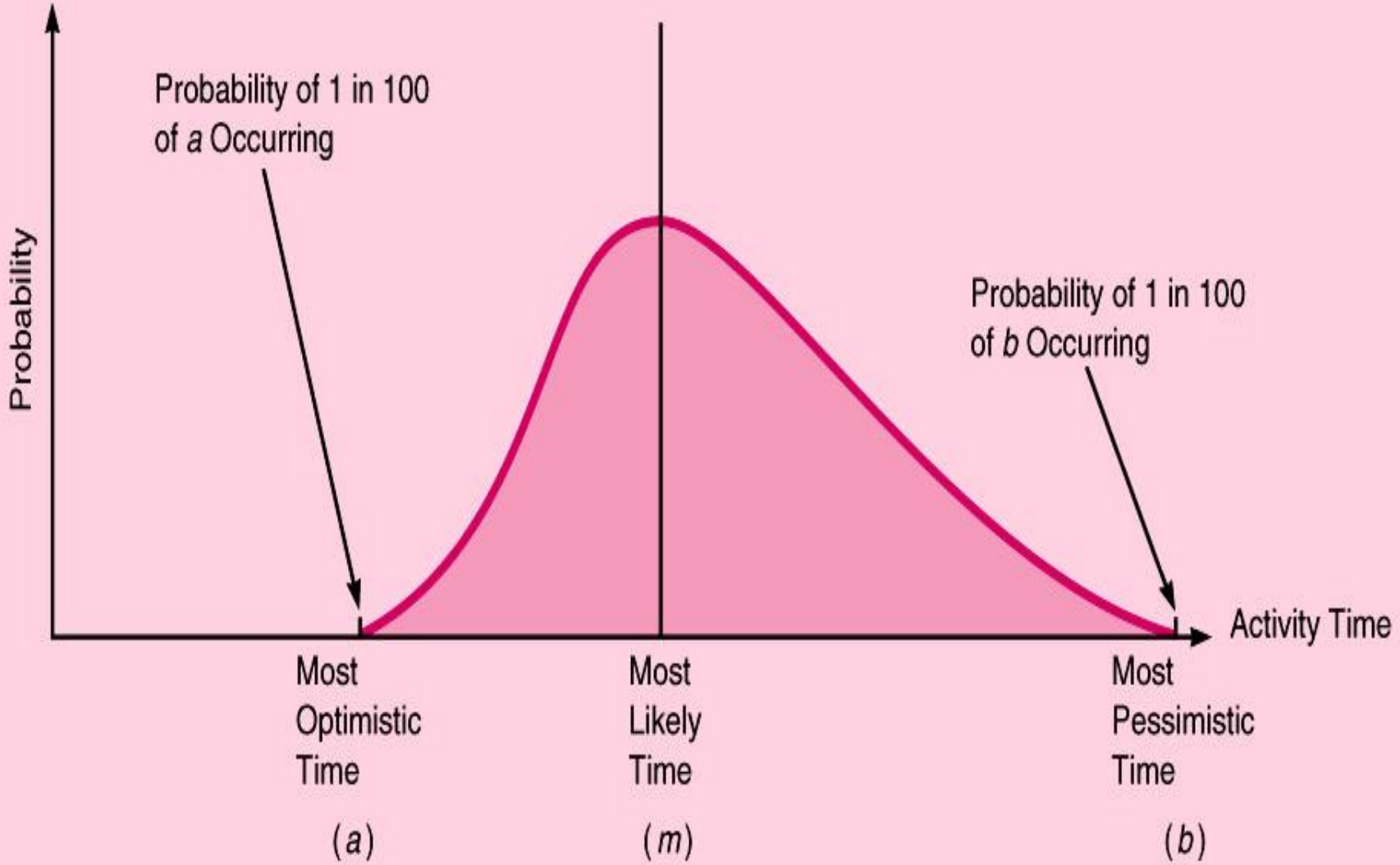
PERT

Three time estimates are required to compute the parameters of an activity's duration distribution:

- **Pessimistic Time (t_p)** - the longest time that an activity might require. Three standard deviations from the mean is commonly used for the pessimistic time.
- **Most Likely Time (t_m) -** the completion time having the highest probability. Note that this time is different from the expected time

- **Optimistic Time (t_o)** - generally the shortest time in which the activity can be completed. It is common practice to specify optimistic times to be three standard deviations from the mean so that there is approximately a 1% chance that the activity will be completed within the optimistic time.

BETA PROBABILITY DISTRIBUTION WITH THREE TIME ESTIMATES



PERT

PERT assumes a beta probability distribution for the time estimates. For a beta distribution, the expected time and variance for each activity can be:

Expected time = (Optimistic + 4 x Most likely + Pessimistic) / 6

$$t_e = (t_o + 4t_m + t_p) / 6$$

Variance = [(Pessimistic – Optimistic) / 6] ^{2}

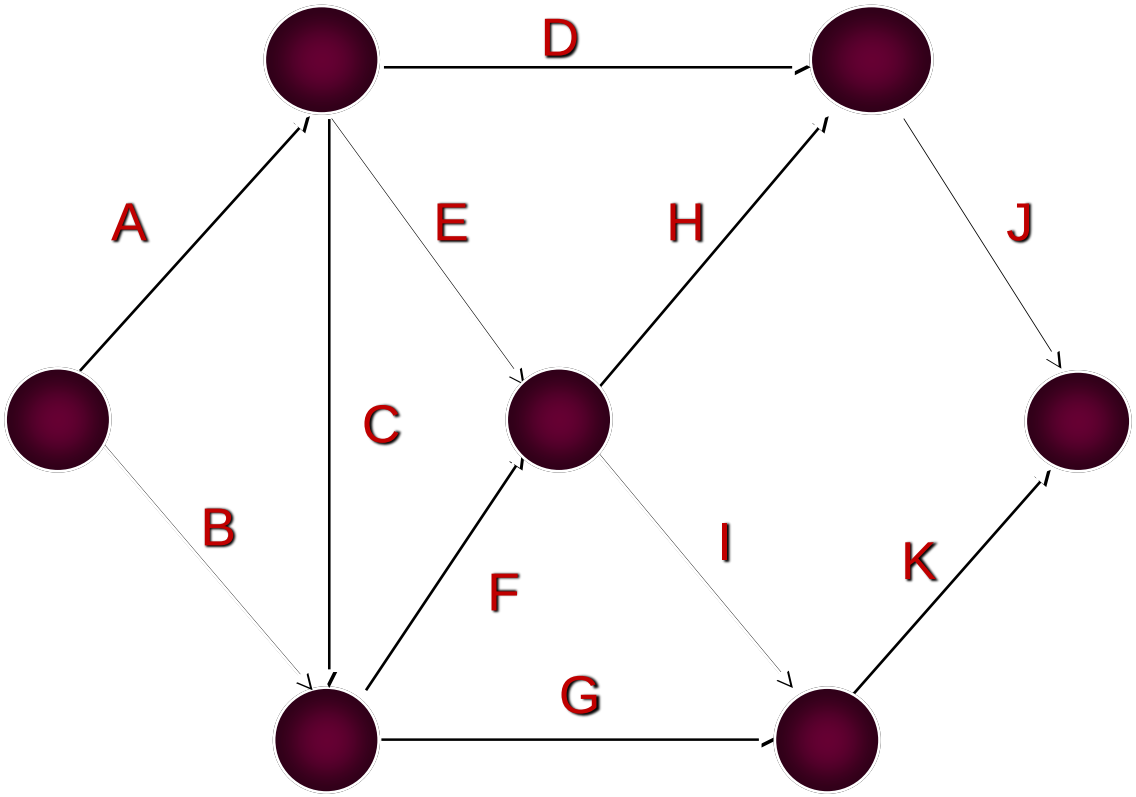
$$V_t = [(t_p - t_o) / 6]^2$$

PERT : EXAMPLE

Activity	Immed. Predec.	Optimistic Time (Hr.)	Most Likely Time (Hr.)	Pessimistic Time (Hr.)
A	--	4	6	8
B	--	1	4.5	5
C	A	3	3	3
D	A	4	5	6
E	A	0.5	1	1.5
F	B,C	3	4	5
G	B,C	1	1.5	5
H	E,F	5	6	7
I	E,F	2	5	8
J	D,H	2.5	2.75	4.5
K	G,I	3	5	7

PERT : EXAMPLE

PERT NETWORK



PERT : EXAMPLE

Activity	Expected Time	Variance
A	6	4/9
B	4	4/9
C	3	0
D	5	1/9
E	1	1/36
F	4	1/9
G	2	4/9
H	6	1/9
I	5	1
J	3	1/9
K	5	4/9

SOLUTION

EARLIEST/LATEST TIMES:

Activity	ES	EF	LS	LF	Slack
A	0	6	0	6	0 *critical
B	0	4	5	9	5
C	6	9	6	9	0 *
D	6	11	15	20	9
E	6	7	12	13	6
F	9	13	9	13	0 *
G	9	11	16	18	7
H	13	19	14	20	1
I	13	18	13	18	0 *
J	19	22	20	23	1
K	18	23	18	23	0 *

Activities on the Critical Path: A→ C → F → I → K

Total Project Time: 6+3+4+5+5 = **23**

PERT : EXAMPLE

Activity	Expected Time	Variance
A	6	4/9*
B	4	4/9
C	3	0*
D	5	1/9
E	1	1/36
F	4	1/9*
G	2	4/9
H	6	1/9
I	5	1*
J	3	1/9
K	5	4/9*

PERT : EXAMPLE

Probability the project will be completed within 24 hours

$$V_{path} = V_A + V_C + V_F + V_I + V_K$$
$$= 4/9 + 0 + 1/9 + 1 + 4/9$$
$$= 2$$

$$\sigma_{path} = 1.414$$

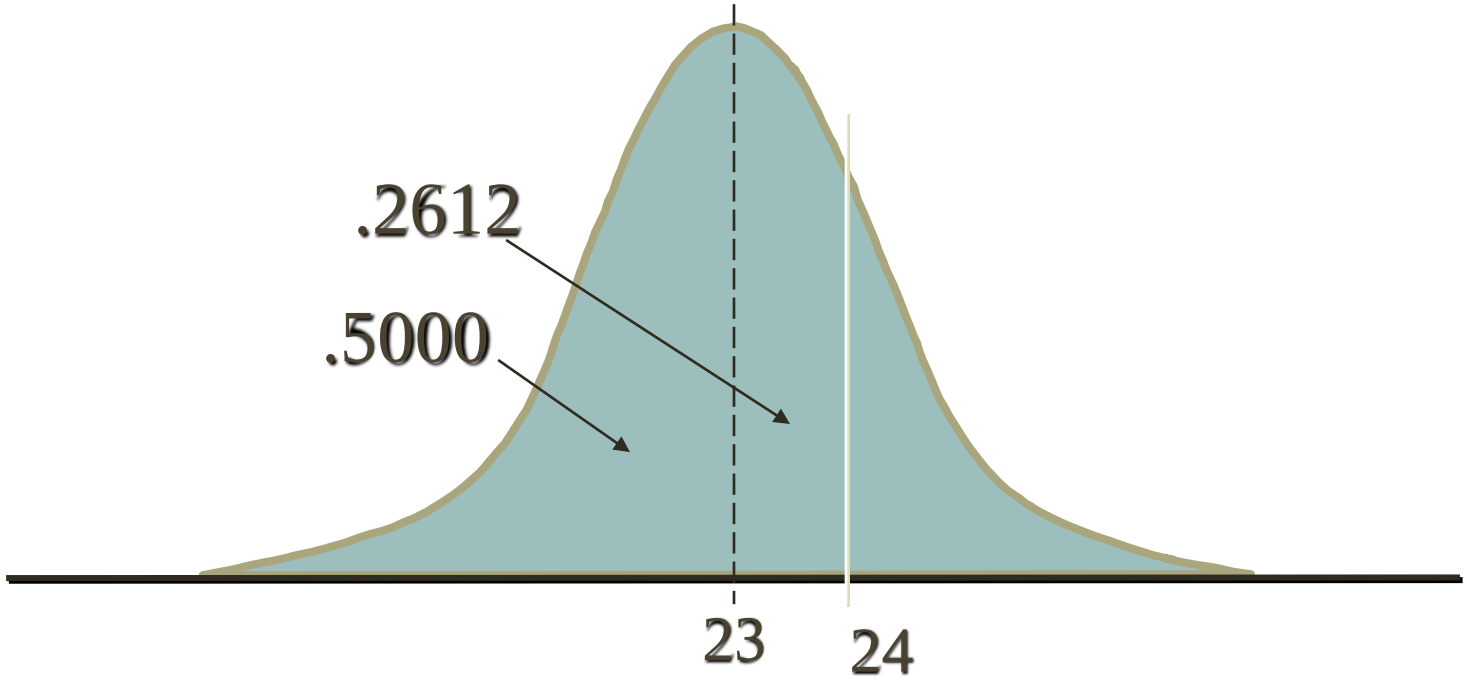
$$\text{Expected Time} = E_A + E_C + E_F + E_I + E_K$$
$$= 6 + 3 + 4 + 5 + 5$$
$$= 23$$

What is the probability that the project will be completed within the 24 hours?

$$z = (24 - 23)/\sigma = (24\text{-}23)/1.414 = .71$$

PERT : EXAMPLE

- Probability the project will be completed within 24 hours



From the Standard Normal Distribution table:

$P(z \leq .71) = .5 + .2612 =$ **.7612**

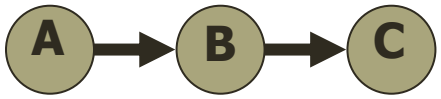
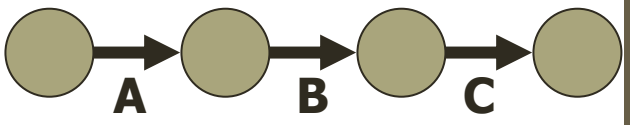
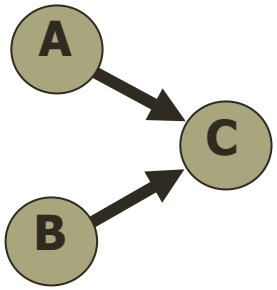
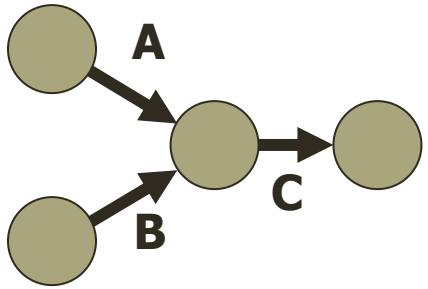
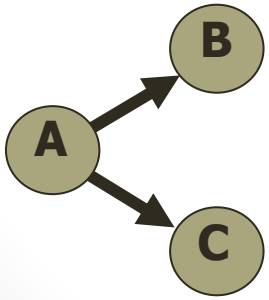
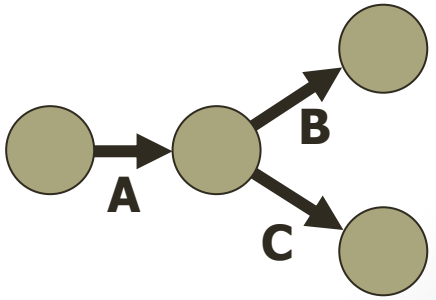
PRACTICE QUESTION

A medical institute is planning to hold an annual conference on eradication of cancer. In order to coordinate the project, it was decided to use PERT network. The major activities and time estimates a, m, and b for each activity are carefully computed and gives as follows.

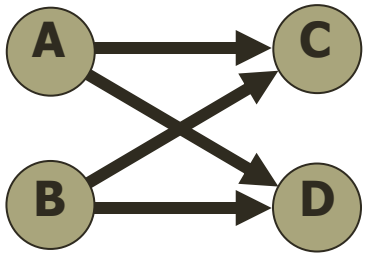
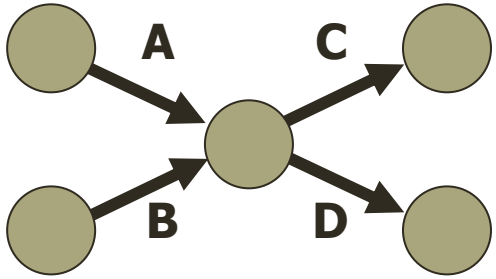
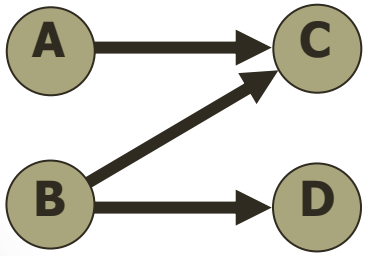
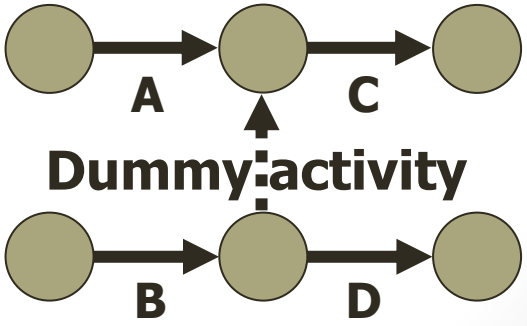
Activities	Predecessors	Estimates		
		a	m	b
A	-	2	4	12
B	-	10	12	26
C	A	8	9	10
D	A	10	15	20
E	A	7	7.5	11
F	B, C	9	9	9
G	D	3	3.5	7
H	E, F, G	5	5	5

1) Draw the PERT diagram for the project. **2)** Compute Expected Time and Variance for each activity. **3)** Compute ES, EF, LS, LF and Slack time. **4)** Identify the critical path and find its length and variance. **5)** What is the probability that the project will be completed within 30 days?

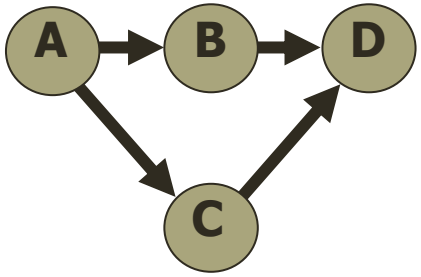
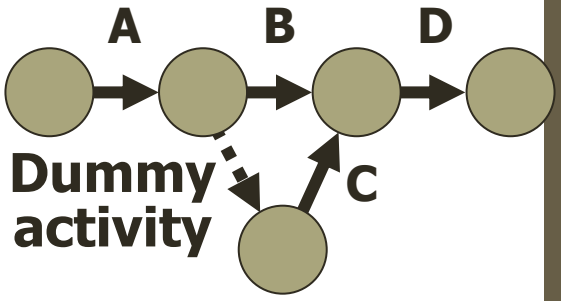
A Comparison of AON and AOA Network Conventions

Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(a) 	A comes before B, which comes before C.	
(b) 	A and B must both be completed before C can start.	
(c) 	B and C cannot begin until A is completed.	

A Comparison of AON and AOA Network Conventions

	Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(d)		C and D cannot begin until both A and B are completed.	
(e)		C cannot begin until both A and B are completed; D cannot begin until B is completed. A dummy activity is introduced in AOA.	

A Comparison of AON and AOA Network Conventions

Activity on Node (AON)	Activity Meaning	Activity on Arrow (AOA)
(f) 	B and C cannot begin until A is completed. D cannot begin until both B and C are completed. A dummy activity is again introduced in AOA.	

QUESTION: (AON Network)

Consider the following data:

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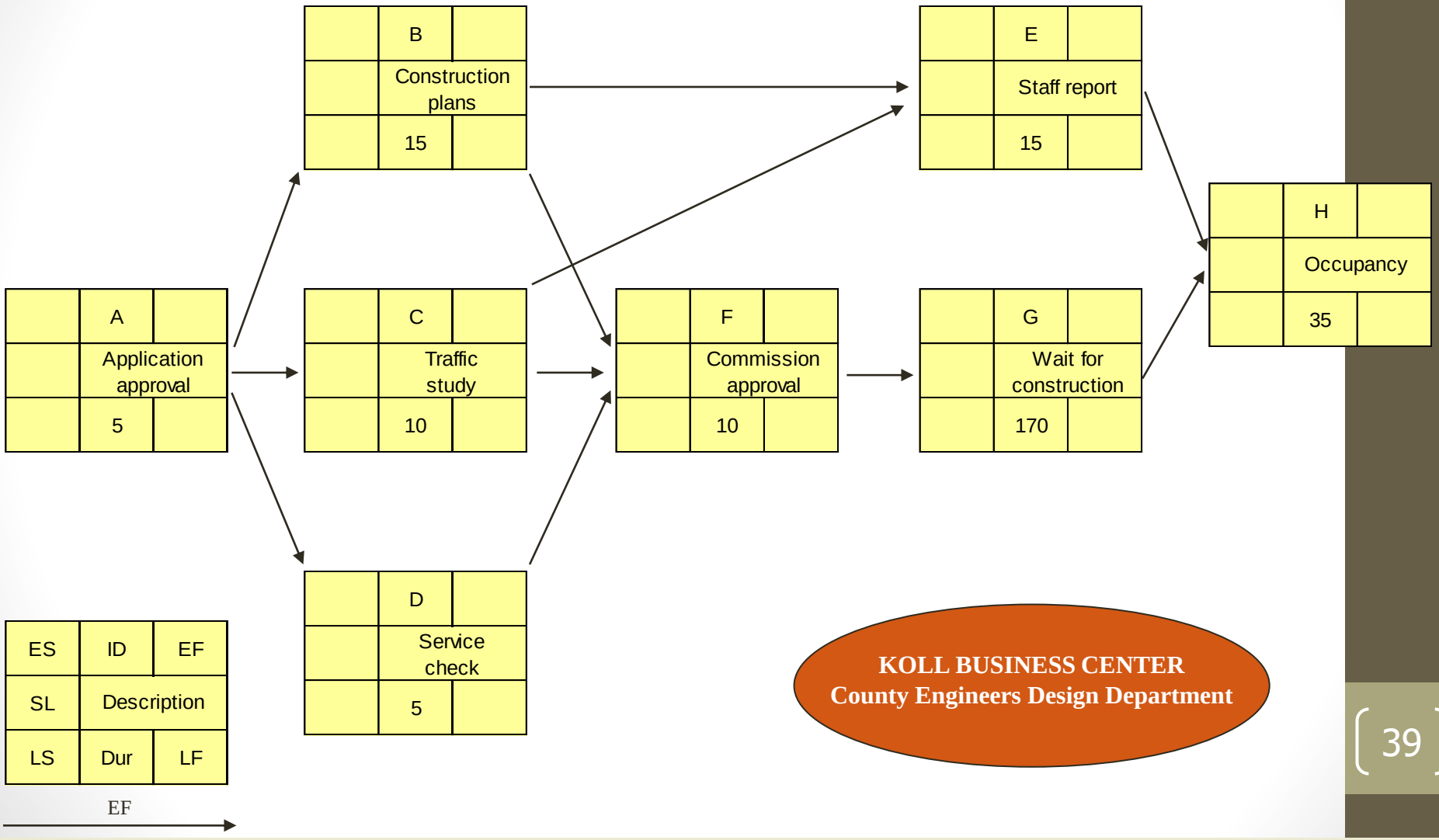
A	Select administrative and medical staff	-
B	Select site and do site survey	-
C	Select equipment	A
D	Prepare final construction plans and layout	B
E	Bring utilities to the site	B
F	Interview applicants and fill positions in nursing, support staff, maintenance, & security	A
G	Purchase and take delivery of equipment	C
H	Construct the hospital	D
I	Develop an information system	A
J	Install the equipment	E, G, H
K	Train nurses and support staff	F, I, J

Draw the AON network Diagram.

CRITICAL PATH METHOD (CPM)

Network Information			
Country Engineers Design Department			
ACTIVITY	**DESCRIPTION**	**PROCEDING ACTIVITY**	**ACTIVITY TIME (Duration)**
A	Application Approval	None	5
B	Construction Plans	A	15
C	Traffic Study	A	10
D	Service Availability Check	A	5
E	Staff Report	B,C	15
F	Commission Approval	B,C,D	10
G	Wait for Construction	F	170
H	Occupancy	E,G	35

CRITICAL PATH METHOD (CPM)



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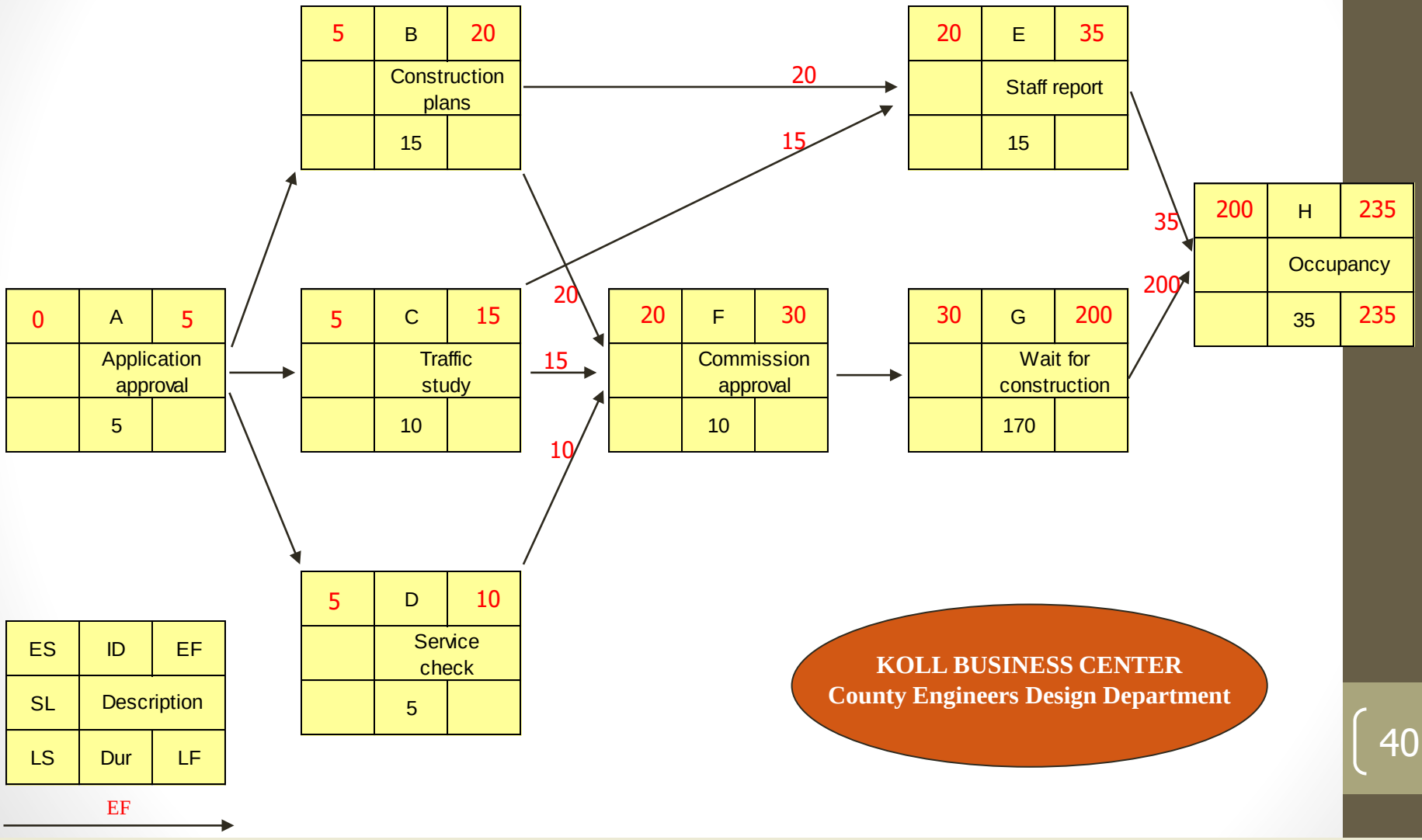
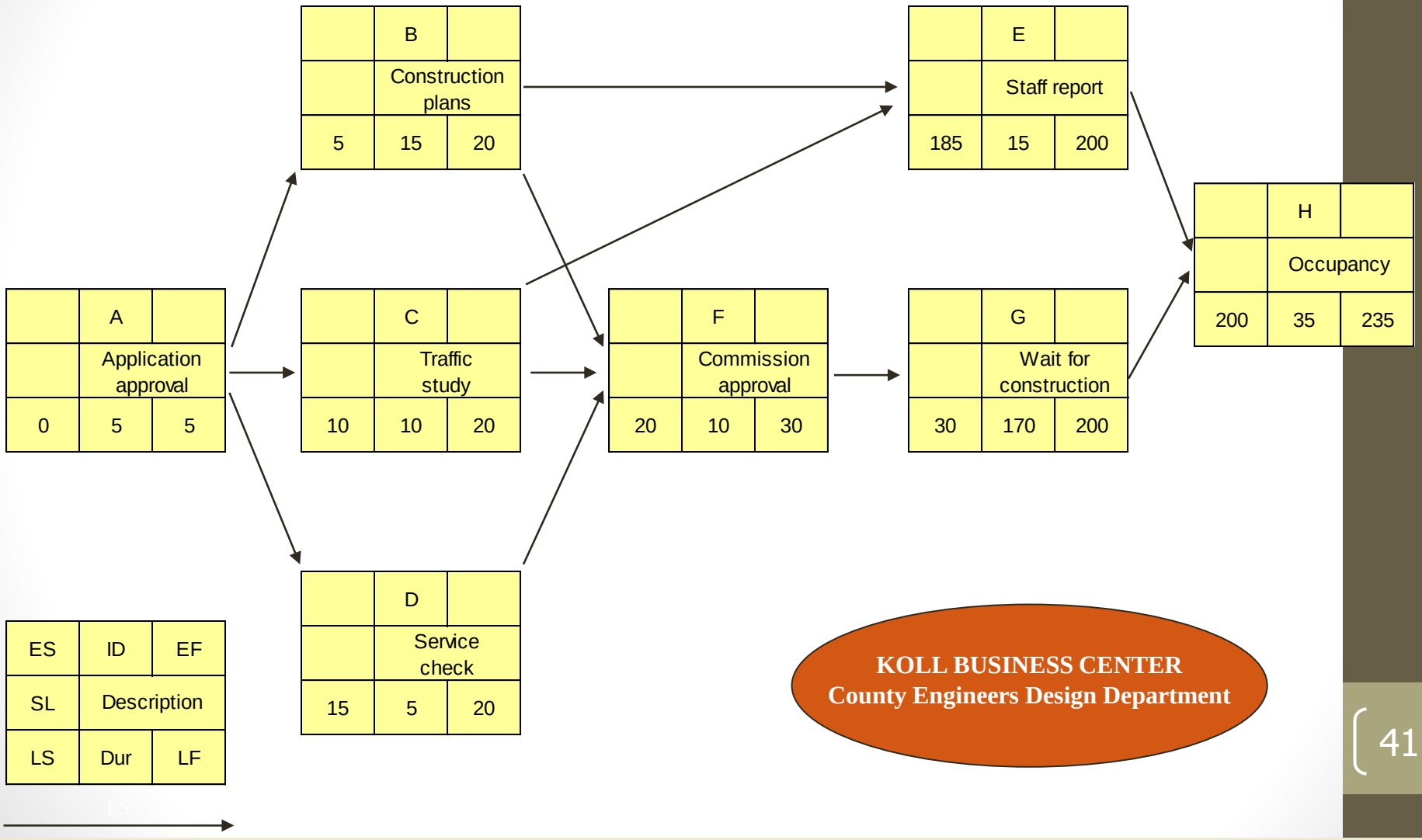


KOLL BUSINESS CENTER
County Engineers Design Department

CRITICAL PATH METHOD (CPM)



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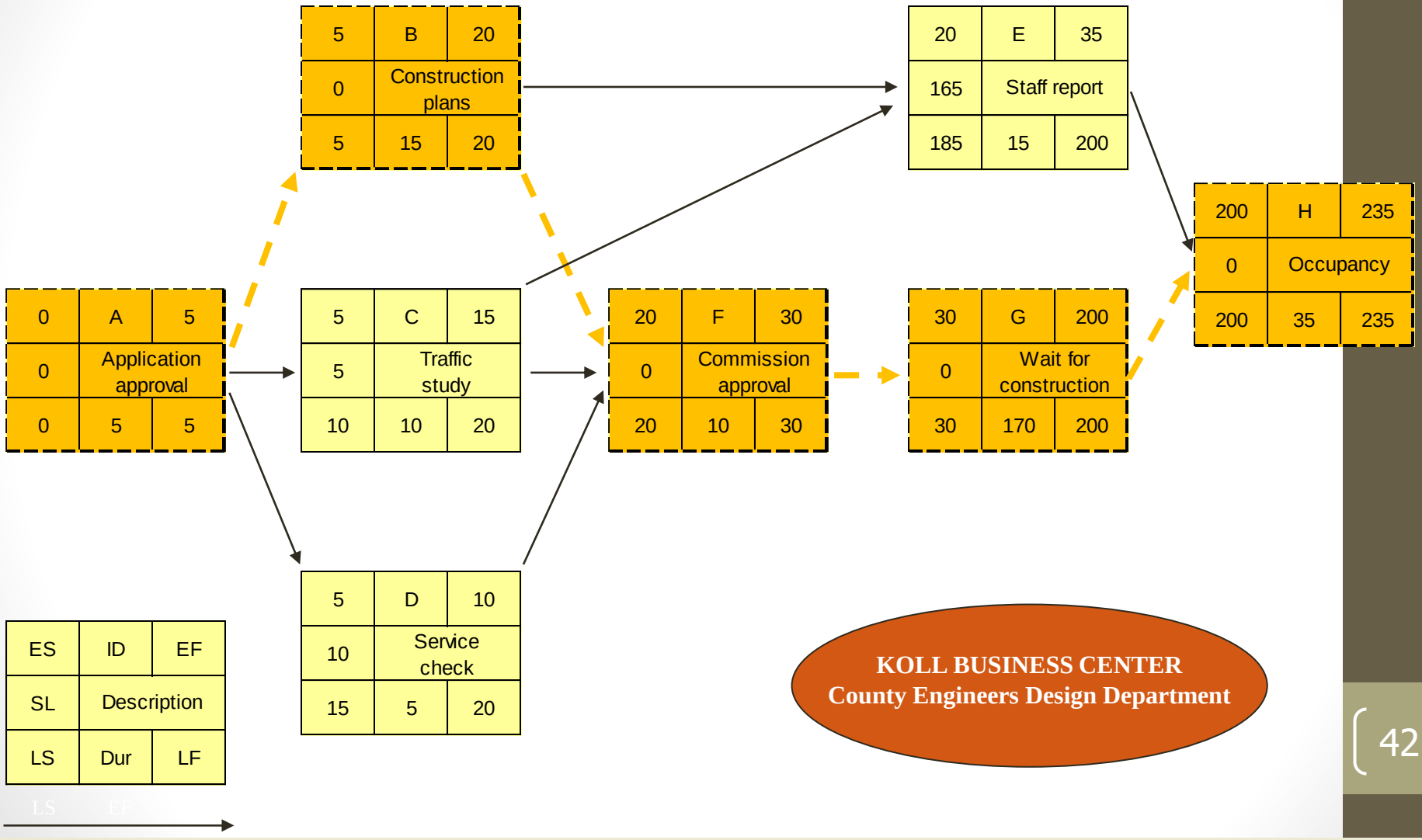

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County Engineers Design Department

PRACTICE QUESTION

Activities	Predecessors	Duration (Days)
A	-	4
B	-	12
C	A	9
D	A	15
E	A	7.5
F	B, C	9
G	D	3.5
H	E, F, G	5

Develop AON Network and find the Critical Path.

WHAT IS A FLOAT (SLACK)?

- A float shows time available for delaying an activity without delaying Finish Date of the Project.

- **TYPES OF FLOATS**
  - Total Float
  - Free Float

- **TOTAL FLOAT:**
  - The maximum free time available for an activity.
  - LF – EF (or) LS – ES

- **FREE FLOAT:**
  - The time an activity can be delayed without delaying any succeeding activity.
  - ES_j – ES_i – Activity time ➔ ES_j – (ES_i + Activity Time) ➔ ES_j – EF_i

QUESTIONS