

ROMBERG'S INTEGRATION

$$\int_0^1 \frac{dx}{1+x}$$

correct to 3 decimal places using _____.

$$\boxed{\log_e 2}$$

STEP-SIZE:-

$$h = \frac{1-0}{2} = 0.5$$

$$\frac{h}{2} = 0.25$$

$$\frac{h}{4} = 0.125$$

Using Trapezoidal :-

x

y

0

1

$n=1$

0.5

0.666

put n in $\frac{1}{1+x}$
i.e., $\left(\frac{1}{1+x}\right)$

1

0.5

$$I(h) = \frac{0.5}{2} (1 + 0.5 + 2(0.666)) \\ = 0.7803$$

when

$h = 0.25$

x

y

0

1

0.25

0.8

0.5

0.6666

0.75

0.5714

'I' 0.5

$$\begin{aligned} I\left(\frac{h}{2}\right) &= \frac{h}{2} (1 + 0.5 + 2(-\dots)) \\ &= \frac{0.25}{2} (-\dots) \\ &= 0.697 \end{aligned}$$

when $h = 0.125$

$$I\left(\frac{h}{4}\right) = 0.6941$$

using

Romberg's

$$h = I_1 = 0.7084$$

$$\frac{h}{2} = I_2 = 0.6970 \quad I_2' = I_2 + \frac{1}{3}(I_2 - I_1) = 0.6932 \quad I_1'' = I_2' + \frac{1}{3}(I_2' - I_1) = 0.6931$$

$$\frac{h}{4} = I_3 = 0.6941 \quad I_3' = I_3 + \frac{1}{3}(I_3 - I_2) = 0.6931$$

$$\begin{aligned} I(h, h/2) &= \frac{1}{3} \left(9I\left(\frac{h}{2}\right) - I(h) \right) \\ &= 0.6932 \end{aligned}$$

$I(h_2, h_1)$ = Using formulae :-

$$0.6931$$

$$I(h, h/2, h/4) = \frac{1}{3} \left[2I\left(\frac{h}{2}, \frac{h}{4}\right) - I(h, \frac{h}{2}) \right]$$

$$= 0.6931$$

$$\boxed{\log_e 2 = ?}$$

$$\int_0^1 \frac{1}{1+x} dx = \ln(1+x) \Big|_0^1$$

$$\begin{aligned} &= \ln 2 - \ln 1 \\ &= \boxed{0.6931} \end{aligned}$$

$$\int_0^1 \frac{dx}{1+x^2}$$

$$h = 0.5$$

$$\frac{h}{2} = 0.25$$

$$\frac{h}{4} = 0.125$$

when $h=0.5$

$$\begin{matrix} x & y \end{matrix}$$

$$0 \quad 1$$

$$0.5 \quad 0.8$$

$$1 \quad 0.5$$

$$I(h) = \frac{h}{2} (1 + 0.5 + 2(0.8))$$

$$= \frac{0.5}{2} (1.5 + 2(0.8))$$

$$\text{when } h=0.25 \quad \approx 0.775$$

$$\begin{matrix} x & y \end{matrix}$$

0

1

0.25

0.941

0.5

0.8

0.75

0.67

2

0.5

$$I(0.25) = \frac{0.25}{2} \left(1 + 0.5 + 2 \begin{pmatrix} 0.8 + 0.941 + \\ 0.67 \end{pmatrix} \right)$$

$$= 1.5655$$

when $h = 0.125$



[

$$I(h, \frac{h}{2}) = \frac{1}{3} \left(4 I\left(\frac{h}{2}\right) - I(h) \right)$$
$$= 1.829.$$

BISECTION METHOD

Q. Find root of $x^3 - x - 1$ using Bisection.

$$\rightarrow f(0) = -1 \quad (\text{Put } f(x) = 0)$$
$$f(1) = -1$$
$$f(2) = 5$$

Solution will lie in interval $[1, 2]$

We have to find the interval (x) where signs will be changed.

$$\rightarrow \text{Find mid point} = \frac{1+2}{2} = \boxed{1.5}$$

$$f(1.5) = 0.875$$

root will be in $[1, 1.5]$

No change in sign, find mid point

$$\xrightarrow{\hspace{1cm}} 1.25$$

$$f(1.25) = -0.2969$$

root in $[1.25, 1.5]$

$$\xrightarrow{\hspace{1cm}} 1.375$$

$$f(1.375) = 0.2296$$

root in $[1.25 - 1.375]$

$$\xrightarrow{\hspace{1cm}} 1.3125$$

$$f(1.3125) = -0.0515$$

root in $[1.25 - 1.3125]$

$$\xrightarrow{\hspace{1cm}} 1.3437$$

|

$$f(1.3437) = 0.08$$

$$\text{root} = [1.3125 - 1.3437]$$

$$\rightarrow 1.3281$$

$$f(1.3281) = 0.014$$

$$\text{root} = [1.3125 - 1.3281]$$

$$\rightarrow 1.3203$$

$$f(1.3203) = -0.01$$

$$\text{root} = [1.3281 - 1.3203]$$

$$\rightarrow 1.3242$$

$$f(1.3242) = 0.002$$

$$\text{root} = [1.3203 - 1.3243]$$

REGULA FALSI METHOD

- 1- Select interval where root lies.
- 2- Find root point x_2 , using:

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

- 3 Find $f(x)$ and decide interval where root lie:

$$x^3 - x - 1 = 0$$

$$\begin{aligned}x_0 &= -1 \\x_1 &= 1 \\x_2 &= 5\end{aligned}$$

$[-1, 5]$ is the interval.

ITERATIVE METHOD

Q# Find real root of equation
 $x^3 - x - 1 = 0$

$$f(x) = x^3 - x - 1$$

$$f(0) = -1$$

$$f(1) = -1$$

$$f(2) = 5$$

INTERVAL = /

$$x = Q(x)$$

1- $x^3 - x - 1 = 0$

$$x = x^3 - 1$$

$$x = Q(x)$$

$$\Rightarrow Q'(x) = 3x^2$$

$$Q'(1) = 3 \neq 1$$

$$Q'(2) = 12 \neq 1$$

2- $x^3 - x - 1 = 0$

$$x^3 - x = 1$$

$$x(x^2 - 1) = 1$$

$$x = \frac{1}{x^2 - 1}$$

$$x = Q(x)$$

$$Q(x) = (x^2 - 1)^{-1}$$

$$\theta'(x) = - (x^2 - 1)^{-2} (2x)$$

$$= \frac{-2x}{(x^2 - 1)^2}$$

Put $(1, 2)$

③ $x^3 - x - 1 = 0$

$$x^3 = 1 + x$$

$$x = (1 + x)^{1/3}$$

$$x = Q(x)$$

$$\theta'(x) = \frac{1}{3} (1 + x)^{-2/3}$$

$$\theta'(1) = 0.2 < 1$$

$$\theta'(2) = 0.16 < 1$$

$$\Theta(u) = ((+u))^{1/3}$$

Choose random value in
interval,

$$u_1 = \Theta(1.5) = 1.357$$

$$u_2 = \Theta(u_1) = 1.3308$$

$$u_3 = \Theta(u_2) = \dots \dots \dots$$

(until u repeats itself
as output).

NEWTON RAPHSON METHOD

- 1- Select interval where root lies
- 2- Find $x_0 = \frac{a+b}{2}$ and then find $f(x_0)$, $f'(x_0)$
- 3- $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$$f(x) = x^3 - x - 1, f'(x) = 3x^2 - 1$$

SECANT METHOD

$$f(x) = x e^x - \cos x$$

$$x_0 = 0 \quad f(0) = -1$$

$$x_1 = 1 \quad f(1) = 2.1779 \quad (\text{use calc in radian}).$$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)} = 0.31467$$

Now check whether it is root or not

$$f(x_2) = -0.5198 \quad (\text{repeat till it is nearly equal} = 0.000\ldots)$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = 0.4473$$

$$f(x) = -0.20200$$

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$$Q \Rightarrow x^3 = 2x + 5 \quad \text{using iterative}$$

$$x - e^x = 0 \quad \text{using secant}$$

NUMERICAL

COMPUTING

GAUSS ELIMINATION METHOD

$$\begin{aligned} 3x+y-z &= 3 \\ 2x-8y+2z &= -5 \\ x-2y+9z &= \end{aligned}$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array} \right]$$

PARTIAL PIVOTING:-

Look for max abs. value in 1st col.

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 0 & -26/3 & 5/3 & -7 \\ 0 & -\frac{7}{3} & \frac{23}{3} & 7 \end{array} \right] \quad R_2 - \frac{2}{3}R_1$$

$$R_3 - \frac{1}{3}R_1$$

$$\Leftarrow \left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 0 & \textcircled{-26/3} & 5/3 & -7 \\ 0 & 0 & \frac{231}{26} & \frac{231}{26} \end{array} \right] \quad (\text{Pivot})$$

$$\frac{231}{26} z = \frac{231}{26}$$

$$\Rightarrow \boxed{z = 1}$$

$$\Rightarrow \frac{-26}{3} y + \frac{5}{3} z = -7$$

$$\boxed{y = \frac{-26}{3}}$$

$$\Rightarrow \boxed{x = 1}$$

- 1-Take the largest no. in first col to the top of matrix.
- 2-Make all the numbers below pivot zero.

$$\left[\begin{array}{ccc|c} 9 & 3 & 4 & 7 \\ 4 & 3 & 4 & 8 \\ 1 & 1 & 1 & 3 \end{array} \right]$$

$$R_2 \rightarrow R_2 - 4R_3, R_3 \rightarrow R_3 - 9R_3$$

$$\left[\begin{array}{ccc|c} 9 & 3 & 4 & 7 \\ 0 & -1 & 0 & -4 \\ 0 & -6 & -5 & -20 \end{array} \right] \quad |-1| \neq [-6]$$

$$\left[\begin{array}{ccc|c} 9 & 3 & 4 & 7 \\ 0 & -6 & -5 & -20 \\ 0 & -1 & 0 & -4 \end{array} \right] \quad \text{swap}$$

$$\left[\begin{array}{ccc|c} 9 & 3 & 4 & 7 \\ 0 & -6 & -5 & -20 \\ 0 & -1 & 0 & -4 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 9 & 3 & 4 & 7 \\ 0 & -6 & -5 & -20 \\ 0 & 0 & -5 & +4 \end{array} \right] \quad R_3 \rightarrow -6R_3 + R_2$$

$$\Rightarrow z = -\frac{4}{5}$$

$$\Rightarrow -6y = -20 + 5z = -20 + 5\left(-\frac{4}{5}\right)$$

$$= -24 = +4$$

$$\Rightarrow 9x = 7 - 42 - 3y = 7 - 4\left(-\frac{4}{5}\right) - 3(4) \Bigg/ 9$$

$$= -0.2$$

COMPLETE PIVOTING

$$\left[\begin{array}{ccc|c} 3 & 1 & -1 & 3 \\ 2 & -8 & 1 & -5 \\ 1 & -2 & 9 & 8 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 1 & -2 & 9 & 8 \\ 2 & -8 & 1 & -5 \\ 3 & 1 & -1 & 3 \end{array} \right] \quad R_3 \leftrightarrow R_1$$

$$= \left[\begin{array}{ccc|c} 9 & -2 & 1 & 8 \\ 1 & -8 & 2 & -5 \\ -1 & 1 & 3 & 3 \end{array} \right] \quad C_1 \leftrightarrow C_3$$

$$= \left[\begin{array}{ccc|c} 9 & -2 & 1 & 8 \\ 0 & -7 & 5 & -2 \\ 0 & \frac{7}{7} & \frac{28}{7} & \frac{35}{7} \end{array} \right] \quad \begin{aligned} R_2 &\rightarrow R_2 + R_3 \\ R_3 &\rightarrow 9R_3 + R_1 \\ R_3 &\rightarrow \frac{R_3}{7} \end{aligned}$$

$$= \left[\begin{array}{ccc|c} 9 & -2 & 1 & 8 \\ 0 & \textcircled{-7} & 5 & -2 \\ 0 & 1 & 4 & 5 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 9 & -2 & 1 & 8 \\ 0 & -7 & 5 & -2 \\ 0 & 0 & 33 & 33 \end{array} \right] R_3 \rightarrow +7R_3 + R_2$$

$$\Rightarrow n = 1 \quad (C_1 \text{ was swapped with } C_3)$$

$$\Rightarrow y = \frac{-2 - 5}{(-7)} = \frac{-7}{7} = +1$$

$$\Rightarrow z = (8 - 1 + 2(+1)) / 9$$

$$= 1$$

- 1- Take the biggest number from the whole matrix and mark it as pivot.
 2- Make all entries below pivot 0.

$$\left[\begin{array}{ccc|c} 28 & 4 & -1 & 32 \\ 1 & 3 & 10 & 24 \\ 2 & 17 & 4 & 35 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 28 & 4 & -1 & 32 \\ 0 & 80 & 281 & 640 \\ 0 & 11 & -16 & -13 \end{array} \right] \begin{matrix} R_3 \rightarrow R_3 - 2R_1 \\ R_2 \rightarrow 28R_2 - R_1 \end{matrix}$$

$$= \left[\begin{array}{ccc|c} 28 & -1 & 4 & 32 \\ 0 & 281 & 80 & 640 \\ 0 & -16 & 11 & -13 \end{array} \right]$$

$$= \left[\begin{array}{ccc|c} 28 & -1 & 4 & 32 \\ 0 & 281 & 80 & 640 \\ 0 & 0 & 273.18 & 411.6875 \end{array} \right] \begin{matrix} R_3 \rightarrow R_3 + 16R_2 \\ \hline 18 \end{matrix}$$

$$= \left[\begin{array}{ccc|c} 28 & -1 & 4 & 32 \\ 0 & 281 & 80 & 640 \\ 0 & 0 & 273.18 & 411.6875 \end{array} \right]$$

$$\Rightarrow z = \frac{411.6875}{273.18} = 1.507019$$

$$\Rightarrow 28ly + 80z = 640$$

$$y = \frac{640 - 80z}{281} = 1.8485$$

$$\Rightarrow 28x - ly + 4z = 32$$

$$x = \frac{32 + y - 4z}{28} = 0.9935$$

DO-LITTLE:

$$\begin{bmatrix} 2 & 5 & -3 \\ 4 & 7 & 1 \\ 3 & -5 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

$$\Rightarrow u_{11} = 2$$

$$\Rightarrow u_{12} = 5$$

$$\Rightarrow u_{13} = -3$$

$$\Rightarrow l_{21}u_{11} = 4$$

$$\Rightarrow l_{21}u_{12} + u_{22} = 7$$

$$l_{21} = 2$$

$$u_{22} = -3$$

$$\Rightarrow l_{21}u_{13} + u_{23} = 1$$

$$u_{23} = 7$$

$$\Rightarrow l_{31}u_{11} = 3$$

$$\Rightarrow l_{31}u_{12} + l_{32}u_{22} = -5$$

$$l_{31} = 1.5$$

$$l_{32} = 4.1667$$

$$\Rightarrow l_{31}u_{13} + l_{32}u_{23} + u_{33} = -2$$

$$u_{33} = -26.6667$$

L⁻¹ = B

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 25/6 & 1 \end{array} \right] \left[\begin{array}{c} y_1 \\ y_2 \\ y_3 \end{array} \right] = \left[\begin{array}{c} 7 \\ -3 \\ 2 \end{array} \right]$$

$$\Rightarrow y_1 = 7 \quad \Rightarrow \quad y_2 = -17$$

$$\Rightarrow y_3 = \frac{187}{3}$$

$$UX = 7$$

$$\left[\begin{array}{ccc|c} 2 & 5 & -3 & 7 \\ 0 & -3 & 7 & -17 \\ 0 & 0 & -80/3 & 187/3 \end{array} \right] \left[\begin{array}{c} x \\ y \\ z \end{array} \right] = \left[\begin{array}{c} 7 \\ -17 \\ 187/3 \end{array} \right]$$

$$\Rightarrow z = -187/80 \quad \Rightarrow \quad y = 17/80$$

$$\Rightarrow x = -43/80$$

CROUT'S:

$$\begin{bmatrix} 2 & 5 & -3 \\ 4 & 7 & 1 \\ 3 & -5 & -2 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow l_{11} = 2$$

$$\Rightarrow l_{11}u_{12} = 5$$

$$\Rightarrow l_{11}u_{13} = -3$$

$$u_{12} = 5/2$$

$$u_{13} = -3/2$$

$$\Rightarrow l_{21} = 4$$

$$\Rightarrow l_{21}u_{12} + l_{22} = 7$$

$$\Rightarrow l_{21}u_{13} + l_{22}u_{23} = 1$$

$$l_{22} = -3$$

$$u_{23} = -7/3$$

$$\Rightarrow l_{31} = 3$$

$$\Rightarrow l_{31}u_{12} + l_{32} = -5$$

$$l_{32} = -25/2$$

$$\Rightarrow l_{31}u_{13} + l_{32}u_{23} +$$

$$l_{33} = -2$$

$$l_{33} = -80/3$$

$$\text{L } Y = B$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 4 & -3 & 0 \\ 3 & -25/2 & -80/3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 7 \\ -3 \\ 2 \end{bmatrix}$$

$$\Rightarrow y_1 = 7/2$$

$$\Rightarrow y_2 = 17/3$$

$$\Rightarrow y_3 = \frac{-187}{80}$$

$$UX = Y$$

$$\begin{bmatrix} 1 & 5/2 & -3/2 \\ 0 & 1 & -7/3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7/2 \\ 17/3 \\ -187/80 \end{bmatrix}$$

$$\Rightarrow z = -187/80$$

$$\Rightarrow y = 17/80$$

$$\Rightarrow x = -43/80$$

CHOLESKY:

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}$$

Applicable only iff a matrix is symmetric and positive definite.

POSITIVE DEFINITE: if det of any 1×1 , 2×2 or 3×3 highlighted matrix > 0 its positive definite

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix}$$

$$\det |4| > 0$$

$$\det \begin{vmatrix} 4 & 2 \\ 2 & 17 \end{vmatrix} > 0$$

$$\det \begin{vmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{vmatrix} > 0$$

Now we can use Cholesky's method.

SOLUTION:

$$\begin{bmatrix} 4 & 2 & 14 \\ 2 & 17 & -5 \\ 14 & -5 & 83 \end{bmatrix} = \begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} l_{11} & u_{12} & u_{13} \\ 0 & l_{22} & u_{23} \\ 0 & 0 & l_{33} \end{bmatrix}$$

$$\Rightarrow l_{11} = 2 \quad \Rightarrow l_{11}u_{12} = 2 \quad \Rightarrow l_{11}u_{13} = 14$$

$$u_{12} = 1 \quad u_{13} = 7$$

$$\Rightarrow l_{21}l_{41} = 2 \quad \Rightarrow l_{21}u_{12} + l_{22}^2 = 17 \quad \Rightarrow l_{21}u_{13} + l_{22}u_{23} = -5$$

$$l_{21} = 1 \quad \quad \quad l_{22} = 4 \quad \quad \quad u_{23} = -5$$

$$\Rightarrow l_{11}l_{31} = 14 \quad \Rightarrow l_{31}u_{12} + l_{32}l_{22} = -5$$

$$l_{31} = 7 \quad \quad \quad l_{32} = -3$$

$$\Rightarrow l_{31}u_{13} + l_{32}u_{23} + l_{33}^2 = 83$$

$$l_{33} = 5$$

$L\gamma = B$

$$\begin{bmatrix} 2 & 0 & 0 \\ 1 & 4 & 0 \\ 7 & -3 & 5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 14 \\ -101 \\ 155 \end{bmatrix}$$

$$\Rightarrow y_1 = 7$$

$$\Rightarrow y_2 = -27$$

$$\Rightarrow y_3 = 5$$

$UX = Y$

$$\begin{bmatrix} 2 & 1 & 7 \\ 0 & 4 & -3 \\ 0 & 0 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ -27 \\ 5 \end{bmatrix}$$

$$\Rightarrow x=3$$

$$\Rightarrow y=-6$$

$$\Rightarrow z=1$$

GAUSS JACOBI METHOD

$$a_1x + b_1y + c_1z = d_1 \Rightarrow x = \frac{1}{a_1} (d_1 - b_1y - c_1z)$$

$$a_2x + b_2y + c_2z = d_2 \Rightarrow y = \frac{1}{b_2} (d_2 - a_2x - c_2z)$$

$$a_3x + b_3y + c_3z = d_3 \Rightarrow z = \frac{1}{c_3} (d_3 - a_3x - b_3y)$$

CONDITION:

$$|a_{11}| \geq |b_1| + |c_1|$$

$$|b_2| \geq |a_{21}| + |c_2|$$

$$|c_3| \geq |a_{31}| + |b_3|$$

1st iteration:-

(put all other variables = 0)

$$x_1' = \frac{d_1}{a_1}$$

$$y_1' = \frac{d_2}{b_2}$$

$$z_1' = \frac{d_3}{c_3}$$

2nd iteration:

(put x_1' , y_1' , z_1' in equations).



$$\begin{aligned}8x + 2y - 3z &= 5 \\x + 5y + 2z &= 3 \\2x + 4y + 7z &= 4\end{aligned}$$

$$x = \frac{1}{8} (5 - 2y + 3z)$$

$$y = \frac{1}{5} (3 - x - 2z)$$

$$z = \frac{1}{7} (4 - 2x - 4y)$$

$$x' = \frac{5}{8}$$

$$y' = \frac{3}{5}$$

$$z' = \frac{4}{7}$$

2nd iteration:

$$x = \frac{1}{8} \left(5 - 2 \left(\frac{3}{5} \right) + 3 \left(\frac{4}{7} \right) \right) = 0.6892$$

$$y = 0.2467$$

$$z = 0.05$$

Repeat till x, y, z repeat their values.

GAUSS SEIDAL METHOD

Same as Jacobie.

In first iteration put only $y, z = 0$ to find x .
For y , put the ^{found} value of x and $z = 0$.

For z , put x, y

Now repeat and update x, y, z .

Order of Convergence.

BISECTION METHOD:

In general,

$$x_{n+1} = \frac{x_{n-1} + x_n}{2} \quad \text{--- A}$$

Let $f(x)=0$ be an equation and α be the exact root of $f(x)$
 $\Rightarrow f(\alpha)=0$

x_n may differ from α by error e_n

$$x_n = e_n + \alpha$$

$$x_{n-1} = e_{n-1} + \alpha$$

$$x_{n+1} = e_{n+1} + \alpha$$

Substituting in A

$$\alpha + e_{n+1} = \frac{\alpha + e_n + \alpha + e_{n-1}}{2}$$

$$e_{n+1} = 1 + \frac{e_{n-1} - e_n}{2}$$

$$e_{n+1} = \frac{e_n}{2} \left[1 + \frac{e_{n-1}}{e_n} \right]$$

$$e_{n+1} \approx \frac{e_n}{2} \quad \left(\text{neglect } \frac{e_{n-1}}{e_n} \right)$$

$$e_{n+1} \approx Ae_n \quad (A = 1/2)$$

$$e_{n+1} \approx Ae_n \quad (\text{where } A=1)$$

Rate of convergence is linear.

REGULA-FALSI :

In general,

$$x_{n+1} = \frac{x_{n-1} f(x_n) - f(x_{n-1}) x_n}{f(x_n) - f(x_{n-1})}$$

$$x_{n+2} = \frac{x_n f(x_{n+1}) - f(x_n) x_{n+1}}{f(x_{n+2}) - f(x_n)} \quad \text{— (A)}$$

Let $f(x)=0$ be an equation and α be the exact root of $f(x)$
 $\Rightarrow f(\alpha)=0$

x_n may differ from α by error e_n

$$x_n = c_n + \alpha$$

$$x_{n-1} = c_{n-1} + \alpha$$

$$x_{n+1} = c_{n+1} + \alpha$$

Substituting in (A)

$$\frac{c_{n+1} + \alpha = \frac{(\alpha + e_n) f(\alpha + c_{n+1}) - (\alpha + e_{n+1}) f(\alpha + e_n)}{f(\alpha + c_{n+1}) - f(\alpha + e_n)}}{e_{n+1} + \alpha = \frac{f(\alpha + e_{n+1}) \alpha + f(\alpha + c_{n+1}) e_n - f(\alpha + e_n)(\alpha) - f(\alpha + e_n)}{(c_{n+1})}}$$

$$\frac{e_{n+1} + \alpha = \frac{\alpha (f(\alpha + e_{n+1}) - f(\alpha + e_n)) + f(\alpha + e_{n+1}) e_n - f(\alpha + e_n)}{f(\alpha + e_{n+1}) - f(\alpha + e_n)}}{(c_{n+1})}$$

$$e_{n+2} + \alpha = \alpha + \frac{f(\alpha + e_{n+1})e_n - f(\alpha + e_n)(e_{n+1})}{f(\alpha + e_{n+1}) - f(\alpha + e_n)}$$

(TAYLOR SERIES)

$$e_{n+2} = \left(e_n \left[f(\alpha) + (e_{n+1}) f'(\alpha) + \frac{(e_{n+1})^2}{2!} f''(\alpha) \dots \dots \right] - \right) \div$$

$$\left(e_{n+1} \left[f(\alpha) + e_n f'(\alpha) + \frac{e_n^2}{2!} f''(\alpha) \dots \dots \right] \right)$$

$$\left(\left[f(\alpha) + (e_{n+1}) f'(\alpha) + \frac{(e_{n+1})^2}{2!} f''(\alpha) \dots \dots \right] - \right)$$

$$\left(\left[f(\alpha) + e_n f'(\alpha) + \frac{e_n^2}{2!} f''(\alpha) \dots \dots \right] \right)$$

$$f(\alpha = 0)$$

$$e_{n+2} = \left(\cancel{\left(e_n \right)} \cancel{\left(e_{n+1} \right)} f'(\alpha) + \frac{e_n (e_{n+1})^2}{2!} f''(\alpha) \dots \dots - \right) \div$$

$$\cancel{\left((e_{n+1}) (e_n) f'(\alpha) + (e_{n+1}) \cancel{\left(e_n^2 \right)} f''(\alpha) \dots \dots \right)}$$

$$\left(e_{n+1} f'(\alpha) - e_n f'(\alpha) + \frac{(e_{n+1})^2}{2!} f''(\alpha) \right)$$

$$\left(- \frac{(e_n)^2}{2!} f''(\alpha) \right)$$

Neglecting higher powers,

$$e_{n+2} = \frac{e_n e_{n+1}}{2} f''(\alpha) (e_{n+1} - e_n)$$

$$\overline{f'(\alpha)(e_{n+2} - e_n) + f''(\alpha)(e_{n+2}^2 - e_n^2)}$$

$$e_{n+2} = \frac{e_n e_{n+1}}{2} f''(\alpha) (e_{n+2} - e_n)$$

$$\overline{[f'(\alpha) + f''(\alpha)(e_{n+1} + e_n)] (e_{n+1} - e_n)}$$

$$e_{n+2} = \frac{e_n e_{n+1}}{2} \frac{f''(\alpha)}{f'(\alpha)} \left[1 + \frac{f''(\alpha)(e_{n+1} + e_n)}{2f'(\alpha)} \right]^{-1}$$

Ignore $\left[1 + \frac{f''(\alpha)(e_{n+1} + e_n)}{2f'(\alpha)} \right]^{-1}$

$$e_{n+2} \approx \frac{e_n e_{n+1}}{2} \frac{f''(\alpha)}{f'(\alpha)}$$

$$e_{n+2} \approx M e_{n+1}$$
B

We know,

$$e_{n+1} = A e_n^k$$

$$e_{n+2} = A e_{n+1}^k$$

$$e_n = \left(\frac{e_{n+1}}{A} \right)^{1/k}$$

From (B)

$$A e_{n+1}^k = M e_n e_{n+1}$$

$$A e_{n+1}^k = M \left(\frac{e_{n+1}}{A} \right)^{1/k} \cdot e_{n+1}$$

$$A e_{n+1}^k = M (e_{n+1})^{1/k} \cdot e_{n+1} \cdot A^{-1/k}$$

$$A e_{n+1}^{\boxed{k}} = M \cdot A^{-1/k} \cdot e_{n+1}^{\boxed{1/k+1}}$$

$$\frac{1}{k} + 1 = k$$

$$k^2 = k + 1$$

$$k^2 - k - 1 = 0$$

$$k = \frac{1 \pm \sqrt{5}}{2}$$

$$k = \frac{1 + \sqrt{5}}{2} \quad (\text{Taking } +)$$

$$k = 1.618$$

SECANT:-

In general,

$$x_{n+1} = x_n - \frac{f(x_n)(x_n - x_{n-1})}{f(x_n) - f(x_{n-1})} \quad \text{--- (A)}$$

Let $f(x)=0$ be an equation and α be the exact root of $f(x)$
 $\Rightarrow f(\alpha)=0$

x_n may differ from α by error e_n

$$x_n = e_n + \alpha$$

$$x_{n-1} = e_{n-1} + \alpha$$

$$x_{n+1} = e_{n+1} + \alpha$$

Substituting in (A)

$$\alpha + e_{n+1} = \alpha + e_n - \frac{f(\alpha + e_n)(\alpha + e_n - \alpha - e_{n-1})}{f(\alpha + e_n) - f(\alpha + e_{n-1})}$$

$$\alpha + e_{n+1} = \alpha + e_n - \frac{f(\alpha + e_n)(e_n - e_{n-1})}{f(\alpha + e_n) - f(\alpha + e_{n-1})} \quad \text{--- (B)}$$

TAYLOR - SERIES EXPANSION

$$f(\alpha + e_n) = f(\alpha) + e_n f'(\alpha) + \frac{e_n^2}{2!} f''(\alpha) \dots \dots$$

$$f(\alpha + e_{n-1}) = f(\alpha) + e_{n-1} f'(\alpha) + \frac{e_{n-1}^2}{2!} f''(\alpha) \dots \dots$$

$$f(\alpha + e_n) - f(\alpha + e_{n-1})$$

$$= (e_n - e_{n-1}) f'(\alpha) + \left(\frac{e_n^2 - e_{n-1}^2}{2!} \right) f''(\alpha)$$

$$= (e_n - e_{n-1}) \left[f'(\alpha) + \frac{f''(\alpha)}{2} (e_n + e_{n-1}) \right]$$

Substituting in (B)

$$\cancel{\alpha + e_{n+1}} = \cancel{\alpha + e_n - (e_n - e_{n-1})} \left[f(\alpha) + e_n f'(\alpha) + \frac{e_n^2}{2} f''(\alpha) \dots \right] \\ \cancel{(e_n - e_{n-1})} \left[f'(\alpha) + \frac{f''(\alpha)}{2} (e_n + e_{n-1}) \right]$$

$$f(\alpha) = 0$$

$$e_{n+1} = e_n - \left[\frac{e_n f'(\alpha) + \frac{e_n^2}{2} f''(\alpha) \dots}{f'(\alpha) \left[1 + \left(\frac{e_n + e_{n-1}}{2} \right) \frac{f''(\alpha)}{f'(\alpha)} \right]} \right]$$

$$e_{n+1} = e_n - \frac{f'(\alpha)}{\cancel{f'(\alpha)}} \left[e_n + \frac{e_n^2}{2!} \cdot \frac{f''(\alpha)}{f'(\alpha)} \right]$$

$$\cancel{f'(\alpha)} \left[1 + \frac{(e_n + e_{n-1})}{2} \frac{f''(\alpha)}{f'(\alpha)} \right]$$

$$= e_n - \left[e_n + \frac{e_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} \right] \left[1 + \frac{(e_n + e_{n-1}) f''(\alpha)}{2 f'(\alpha)} \right]^{-1}$$

Binomial theorem full entry marty hope

$$= e_n - \left[e_n + \frac{e_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} \right] \left[1 - \frac{(e_n + e_{n-1}) f''(\alpha)}{2 f'(\alpha)} \right]$$

$$= e_n - \left[e_n - \frac{e_n(e_n + e_{n-1})}{2} \frac{f''(\alpha)}{f'(\alpha)} + \frac{e_n^2}{2} \frac{f''(\alpha)}{f'(\alpha)} \dots \right]$$

Ignored higher powers above ^

$$= e_n - \left[e_n - \frac{f''(\alpha)}{f'(\alpha)} \left(\frac{e_n^2}{2} \right) - \frac{f''(\alpha)}{f'(\alpha)} \frac{e_n e_{n-1}}{2} + \frac{e_n^2}{2} \cdot \frac{f''(\alpha)}{f'(\alpha)} \right]$$

$$= e_n - e_n + \frac{f''(\alpha)(e_n e_{n-1})}{f'(\alpha) 2}$$

$$e_{n+1} = A(e_n e_{n-1})$$

$$A = \frac{f''(\alpha)}{2f'(\alpha)}$$

Generally,

$$e_{n+1} = A e_n^k$$

$$e_n = A e_{n-1}^k$$

$$\Rightarrow e_m = e_n^{1/k} A^{-1/k}$$

$$A e_n^k = A e_n e_{n-1}$$

$$A e_n^k = A e_n e_n^{1/k} A^{-1/k}$$

$$A e_n^k = A e_n^{1/k+1} A^{-1/k}$$

$$\Rightarrow k = \frac{1}{k} + 1$$

$$k^2 = 1 + k$$

$$k^2 - k - 1 = 0$$

$$k = \frac{1 + \sqrt{5}}{2}$$

$$k = 1.618$$

NEWTON-RAPHSON'S:

Let $f(x)=0$ be an equation and α be the exact root of $f(x)$
 $\Rightarrow f(\alpha)=0$

x_n may differ from α by error e_n

$$x_n = e_n + \alpha$$

$$x_{n+1} = e_{n+1} + \alpha$$

$$x_{n+1} = e_{n+1} + \alpha$$

In general,

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$e_{n+1} + \alpha = e_n + \alpha - \left[f(\alpha) + e_n f'(\alpha) + \frac{e_n^2}{2!} f''(\alpha) \right] \\ \frac{\left[f''(\alpha) + e_n f''(\alpha) + \frac{e_n^2}{2!} f'''(\alpha) \right]}{f'(\alpha) + e_n f''(\alpha) + \frac{e_n^2}{2!} f'''(\alpha)}$$

$$e_{n+1} = e_n - \frac{\left[f(\alpha) + e_n f'(\alpha) + e_n^2/2 f''(\alpha) \right]}{\left[f'(\alpha) + e_n f''(\alpha) + e_n^2/2 f'''(\alpha) \right]}$$

Since e_n is small, e_n^2, e_n^3 will be very small, neglect them.

$$e_{n+1} = e_n - \frac{e_n f'(\alpha)}{f'(\alpha) + e_n f''(\alpha)}$$

~~$$e_{n+1} = \frac{e_n f'(\alpha) + e_n^2 f''(\alpha) - e_n f'(\alpha)}{f'(\alpha) + e_n f''(\alpha)}$$~~

$$e_{n+1} = \frac{e_n^2 f''(\alpha)}{f'(\alpha) \left[1 + \frac{e_n f''(\alpha)}{f'(\alpha)} \right]}$$

$$\frac{e_{n+1}}{e_n^2} = \frac{f''(\alpha)}{f'(\alpha)} \left[1 + \frac{e_n f''(\alpha)}{f'(\alpha)} \right]^{-1}$$

BINOMIAL:-

$$\frac{e_{n+1}}{e_n^2} = \frac{f''(\alpha)}{f'(\alpha)} \left(1 - \frac{e_n f''(\alpha)}{f'(\alpha)} + \dots \right)$$

$$\frac{e_{n+1}}{e_n^2} = \frac{f''(\alpha)}{f'(\alpha)} - \frac{e_n f''(\alpha)}{\left(f'(\alpha) \right)^2} + \dots$$

$$e_{n+1} \approx \frac{f''(\alpha)}{f'(\alpha)} e_n^2$$

$$e_{n+1} \approx A e_n^2$$

$$A \boxed{k} = A e_n^2$$

$$\boxed{k=2}$$

Rate of convergence is quadratic

[ITERATIVE:-

In general,

$$x_{n+1} = f(x_n) \longrightarrow \textcircled{A}$$

Let $f(x)=0$ be an equation and α be the exact root of $f(x)$

$$\Rightarrow f(\alpha)=0$$

x_n may differ from α by error e_n

$$x_n = e_n + \alpha$$

$$\alpha = f(\alpha)$$

$$x_{n-1} = e_{n-1} + \alpha$$

$$x_{n+1} = e_{n+1} + \alpha$$

Substituting in A

$$\alpha + e_{n+1} = f(\alpha + e_n)$$

$$\cancel{\alpha + e_{n+1}} = \cancel{f(\alpha)} + e_n f'(\alpha) + \frac{e_n^2}{2!} f''(\alpha) \dots \dots$$

$$e_{n+1} = e_n f'(\alpha) \quad (\text{Ignore higher powers})$$

$$e_{n+1} \approx A e_n$$

$$A = f'(\alpha)$$

$$A \boxed{k}_{e_{n+1}} = A \boxed{1}_{e_n}$$

$$\Rightarrow k = 1$$

Rate of convergence of iterative method is linear.