•. Assignment 3 (Question 1)

(a)- R(ABCD)

F ⇒ AB → C
CD → B
AD → B
AC → D

$ABCD^+ = (ABCD)$

$ABC^+ = (ABCD)$

S.K
$AB^+ = (ABCD)$ → minimal.

$A^+ = (A)$

$\Rightarrow (AB)^+ \Rightarrow A^+ = (A)$
$B^+ = (B)$

So, (AB) is a C.K

Prime attributes are = {AB}

i) Replace B with CD because (CD → B)

AB
CD ⇒ ACD

$(ACD)^+ = A^+ = A$
$C^+ = C$
$D^+ = D$

$AC^+ = ACDB$
$AD^+ = ADBC$
$CD^+ = CDB$ ⇒ Because of this ACD is not a C.K

ii) Replace B with AD (AD → B)

AB
AD ⇒ AD

$AD^+ = \{ADBC\}$
$\Rightarrow A^+ = \{A\}$
$\Rightarrow B^+ = \{D\}$

∵ AD is a C.K.

iii)- Replace D with AC (AC → D)

AD
A AC ⇒ AC

$AC^+ = A^+ \Rightarrow A$

$C^+ \Rightarrow C$

Yes, AC is a C.K

Hence, keys are {AC, AD, AB}

Q1) (b) - R(A, B, C, D, E)

F = {A → C, C → BD, D → A}

$(ABCDE)^+$ = {ABCDE} ← S.K

$(ABCD)^+$ = {ABCD}

~~(ABCE)~~ ~~≠~~ ~~A~~

S.K ⇒ AE, CE, DE

AE^+ = {ABCDE}

CE^+ = {CBDEA}

DE^+ = {DEACB}

C.K ⇒ {AE, DE, CE}.

Question NO 1 (c):

R(ABCDE)

C → AB

A → E

D → E

B D → C

CD → B

$(ABD)^+$ = ABCDE ✓

~~$(AD)^+$ = ABCDE~~ ?

$(BD)^+$ = ABCDE] checking proper subsets of ABD

$(CD)^+$ = ABCDE — checking replacements i.e ABD is on right side of C → AB, so replace and check.

Super keys :- ABD, CBD, ACD, BD, CD

C.keys = BD, CD.

Question no. 2 : Minimal Cover

a) R(A, B, C, D, E G)

$D \rightarrow E$

$ABC \rightarrow BDE$

$B \rightarrow G$

$A \rightarrow C$

$ABC \rightarrow G$

Step 1 : Decomposition !

D → E ✓

ABC → B X

ABC → D ✓

ABC → E X

B → G ✓

A → C ✓

ABC → G X

Step 2: Remove Duplication

D⁺ = { D, E }

ABC⁺ = { A, B, C, E, D }

ABC⁺ = { A, B, C, E, G }

ABC⁺ = { A, B, C, D, E, G }

B⁺ = { B, G }

A⁺ = { A, C }

ABC⁺ = { A, B, C, G, D }

D → E

ABC → D

B → G

A → C

~~ABC → CG~~

Remove Extraneous Attributes

Step 3: D → E

AB → D

B → G

A → C

~~ABC → CG~~

Step 4 = D → E

AB → D

B → G

A → C

Super Key = ABD, AB

CK = AB

b) R(A, B, C, D, E, I)

A → C

AB → C

C → DI

CD → I

EC → AB

EI → C

① A → C ✓

AB → C ×

C → D ✓

C → I ✓

EC → A ✓

EC → B ✓

EI → C ✓

② $A^+ = \{A\}$ ✓

$AB^+ = \{A, B, C, D, I\}$ ×

$C^+ = \{C, D, I\}$ ✓

$C^+ = \{C, D\}$ ✓

$EC^+ = \{E, C, D, I, B\}$ ✓

$EC^+ = \{E, C, A, I, D\}$ ✓

$EI^+ = \{E, I\}$ ✓

③ A → C

C → D

C → I

EC → A

EC → B

EI → C

④ A → C

C → DI

EC → AB

EI → C

S. key = ACEI, EI, AEI, ECI, AEC, EC

C.K = EI, EC as (AB → C so check ABE)

AE

$ABE^+ = \{ABECDI\}$

Check proper subset na

So, $AB^+ = \{ABCDI\}$?

S.K → $AE^+ = \{AECBDI\}$

$BE^+ = \{BE\}$?

AE is also C.K

c) R(A, B, C, D, E, G)

ABC → CDEG

C → E

A → B

D → G

① ABC = C ✗

ABC = D ✓

ABC = E ✗

ABC = G ✗

C = E ✓

A = B ✓

D = G ✓

② $ABC^+ = \{A, B, C\}$

$ABC^+ = \{A, B, C, E, G\}$

$ABC^+ = \{A, B, C, E\}$

$ABC^+ = \{A, B, C, D, E, G\}$

$C^+ = \{C\}$

$A^+ = \{A\}$

$D^+ = \{D\}$.

③ $ABC \Rightarrow D$ → $AC \rightarrow D$

$C \rightarrow E$ $C \rightarrow E$

$A \rightarrow B$ $D \rightarrow G$

$D \rightarrow G$ $A \rightarrow B$

④ $AC \rightarrow D$

$C \rightarrow E$

$D \rightarrow G$

$A \rightarrow B$

$ABCDEG^+$.

$ABC^+ = \{ABCDEG\}$

Now check proper subsets

$AB = \{AB\}$

$AC = \{ACBDEG\}$

$BC = \{BC\}$.

S. keys = ACD

C.K = ~~ACD~~

So, AC is C.K.

Question no. 3 : Equivalent & Non-Equivalent

uestion 3:

OR

R(ABCDEG)

F1

$A \rightarrow C$

$AB \rightarrow C$

$C \rightarrow DG$

$CD \rightarrow G$

$EC \rightarrow AB$

$EG \rightarrow C$

$(A)^+ = \{A, C, D, G\}$ ✓

$(AB)^+ = \{A, B, C, D, G\}$ ✓

$(C)^+ = \{C, D, G\}$ ✓

$(CD)^+ = \{C, D, G\}$ ✓

$(EC)^+ = \{E, C, A, D, G, B\}$ ✓

$(EG)^+ = \{EGCDAB\}$ ✓

F2

$A \rightarrow C$

$C \rightarrow D$

$C \rightarrow G$

$EC \rightarrow A$

$EC \rightarrow B$

$EG \rightarrow C$

$(A)^+ = \{ACDG\}$ ✓

$(C)^+ = \{CDG\}$ ✓

$(EC)^+ = \{ABECDG\}$ ✓

$(EG)^+ = \{EGCDAB\}$ ✓

Both Functional Dependencies are equivalent.

$F1 \subseteq F2$ and $F2 \subseteq F1$.

11 A → 17.

(b) R(A B C D E I)

F1

A → C
AB → C
C → D I
CD → I
E I → AB
E I → C

F2

A → C
C → D
C → I
E C → A
E C → B
E I → C

(A)+ = {A C D I} ✓
(AB)+ = {A B C D I} ✗
(C)+ = {C D I} ✓
(EC)+ = {A C D I B E} ✓
(EI)+ = {E C I A B D} ✓

(A)+ = {A C D I} ✓
(C)+ = {C D I} ✓
(EC)+ = {A B E C D I} ✓
(EI)+ = {E I C D A B} ✓

Functional Dependencies not equivalent as AB is not present in F2.

F2 ⊆ F1 but F1 ⊄ F2.

estion 4:

1 (a):

$R(A,B,C,D,E)$

$F=\{A\to BC, C\to D, E\to D, BE\to A\}$

$R_1(A,E)$ $R_2(A,B,C)$ $R_3(D,E)$

Step 1:

$R_{12}(ABCE) \cup R_3(DE) = R$ ✓

Step 2:

$R_{12} \cap R_3 = E$ ✓

Step 3:

$(E)^+ = \{E, D\}$ As E is the common attribute and a super key in R_3, It is a lossless decomposition.

(b) $R_1(A,E)$ $R_2(A,B,C)$ $R_3(C,D)$

Step 1:

$R_{12}(ABCE) \cup R_3(CD) = R$

Step 2:

$R_{12} \cap R_3 = C$

Step 3:

$(C)^+ = \{C, D\}$ As C is the common attribute and a super key in R_3, It is a lossless decomposition.

Q2 (a) $R(A, B, C, D, E)$

$F = \{CD \rightarrow A, BE \rightarrow A, D \rightarrow B\}$

$R_1(ACD)$ $R_2(CDE)$ $R_3(BD)$

Step 1:

$R_{12}(ACDE) \cup R_3(BD) = R$

Step 2:

$R_{12} \cap R_3 = D$

Step 3:

$(D^+) = \{D, B\}$ As D is the common attribute and a superkey in R_3, it is a lossless decomposition.

(b) $R_1(ACD)$ $R_2(ABE)$ $R_3(BD)$

StepI:

$R_{12}(ABCDE) \cup R_3(BD) = R.$

Step2:

$R_{12} \cap R_3 = BD.$

step3:

$(BD)^+ = \{B, D\}$ As BD is the common attribute and a superkey in R_3, it is a lossless decomposition.

stion 3:

4(a) R (A B C D)

(a) F = { AB → C, C → D, D → A }
Keys: {AB}, {BC} and {BD}.

Step 1: All functional dependencies are in minimal cover

AB → C / satisfies all forms.
C → D / BCNF violation
D → A / BCNF violation.

3rd Normal Form is satisfied.

Decomposition into BCNF:

Move FD that violates BCNF in one relation, and others in the other relation.

R_1 (C D) R_2 (D A) R_3 (B C)

Checking Lossless Decomposition:

① $R_{12} \cup R_3 = R$. ✓

② $R_{12} \cap R_3 = C$ ✓

③ $(C)^+ = \{C, D, A\}$ As C is the common attribute, it is the superkey of R_{12}, and hence it is a lossless decomposition.

Checking Preserving Dependency:

① For R_1: $(C)^+ = \{C, D\}$ $\{C \rightarrow D\} = F_1$

② For R_2: $(D)^+ = \{D, A\}$ $\{D \rightarrow A\} = F_2$

③ For R_3: $(B)^+ = \{B\}$ $(C)^+ = \{C\}$ $F_3 = \{\}$

As $F_1 \cup F_2 \cup F_3 \equiv F$. It is not Preserving Dependency, AB → C is lost.

(b)

$F = \{ A \to B,\ B \to C,\ C \to D,\ D \to A \}$

Keys: A, B, C, D

All functional Dependencies satisfy minimal cover.

$A \to B$ / satisfies all forms.
$B \to C$ / satisfies all forms
$C \to D$ / " " "
$D \to A$ / " " "

BCNF normal form is the highest normal form achieved.

Q4(B):

R (A, B, C, D, E, F)

$FDs = \{ CD \to A,\ BD \to C,\ AC \to B,\ D \to E,\ E \to F \}$

Keys: BD and CD.

All functional dependencies satisfy minimal cover.

$CD \to A$ / Satisfies all normal forms.
$BD \to C$ / " " " "
$AC \to B$ / BCNF violation
$D \to E$ / Partial Dependency
$E \to F$ / Transitive Dependency

Best Normal form is first

① Decomposition of Partial Dependency:

$(D^+) = \{ D, E, F \}$

R_1 (D E F)
$D \to E$ ✓
$E \to F$ (T.D)

R_2 (A B C D)
$CD \to A$ ✓ Keys: CD and BD
$BD \to C$ ✓
$AC \to B$ BCNF violation.

» Decomposition of Transitive Dependency:

$(E)^+ = \{E, F\}$

$R_{11}(\underline{E}\ F)$ | $R_{12}(\underline{D}\ E)$

$E \to F$ ✓ | $D \to E$ ✓

③ Decomposition in BCNF:

$R_{21}(\underline{AC}B)$ | $R_{22}(\underline{D}A\underline{C})$

$AC \to B$ ✓ | $CD \to A$ ✓

Complete list of BCNF schema relations:

$R_1(\underline{E}\ F)$ $R_{12}(\underline{D}\ E)$ $R_3(\underline{AC}B)$

$E \to F$ $D \to E$ $AC \to B$

$R_4(A\underline{DC})$

$CD \to A$

It is not preserving dependency, as $\underline{BD \to C \text{ is lost.}}$

Q4 ©:

$R(A\ B\ C\ D)$ $F = \{B \to D, D \to A\}$

All f.ds satisfy minimal cover.

Finding all possible keys:

$\underline{ABCD}$ Mandatory $(ABCD)^+ = \{ABCD\}$

$(ABC)^+ = \{ABCD\}$

$(BC)^+ = \{ABCD\}$

Key = $\{BC\}$

$B \to D$ / Partial Dependency

$D \to A$ / Transitive Dependency.

) $F = \{ A \to B, B \to$ -- A

) Removing Partial Dependency:

$(B)^+ = \{B, D, A\}$

$R_1 (\underline{B}\ DA)$

$B \to D$ ✓

$D \to A$ / Transitive

| $R_2 (\underline{CB})$

2) Removing Transitive Dependency

$(D)^+ = \{D, A\}$

$R_{11} (\underline{D}, A)$

$D \to A$ ✓.

| $R_{12} (\underline{BD})$

$B \to D$ ✓

List of BCNF schema lists:

$R_1 (\underline{D}A)$ $R_2 (\underline{B}D)$ $R_3 (\underline{CB})$

$F_1 = \{D \to A\}$ $F_2 = \{B \to D\}$ $F_3 = \{\}$

It is preserving Dependency.