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Pumping Lamma for Regular Language

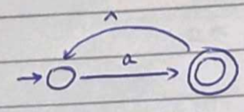
Languages
- ↳ RL → RE
  - ↳ FA
    - DFA
    - NFA
    - NFA ^
- ↳ Context free
- ↳ Context Sensitive

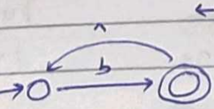
Q. $L = \{ x \mid x \in \{a,b\}^*;\ a^n b^m;\ n,m \geq 0 \}$

$a^n b^m \rightarrow a^* b^*$

Q. $L = \{ x \mid x \in \{a,b\}^*;\ a^n b^n;\ n \geq 0 \}$

$= \{ \wedge, ab, aabb, aaabbb, \dots \}$ ← FA not possible.





Pumping Lamma : used to prove that language is not RL

$x = u\ v\ w$

v ↓ pumping element

① $|uv| \leq p$ → # of states

② $|v| \geq 1$

③ $u\, v^i\, w \in L$ — looping component

$i \geq 0$

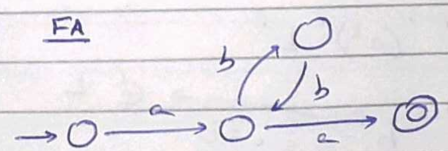
$i = 0$ pump down

$i \geq 1$ pump up

Q. $L = \{ x \mid x \in \{a,b\}^* \mid x$ starts with 'a' & ends with 'a' & it contains even # of b's & atmost 2 a's $\}$

$RE = a(bb)^* a + a$

FA



for exactly 2 a's,

$RE = a(bb)^* a$

① $|uv| \le p$ — # of states in FA

② $|v| \ge 1$

Q. abba

$u = a$

$v = bb$

$w =$ remaining

$p = 4$

$|uv| = 3 \le 4$

$|v| = 2 \ge 1$

is it valid combination? ✓

Q. abba

$u = ab$

$v = b$

$w =$ remaining

$p = 4$

$|uv| = 3 \le 4$

$|v| = 1 \ge 1$

$(ab)(b)^* a$ Is it valid combination? ✗

if $i = 2$,

$(ab)(b)^2 a$

abbba ✗

Q.

$a^n b^n$ $\boxed{a^p b^p}$ here, p is arbitrary (any #)

$u = a^{p-s}$

$|s| \ge 1$ — $v = a^s$

$w = b^p$

uv^iw — $i = 0$ pump down; $i = 1$ no pumping; $i > 1$ pump up

$x = a^{p-s} (a^s)^i b^p$

$= a^{p-s} b^p$; $i = 0$ $\rightarrow \notin L$ as $|s| \ge 1$

$p - s = p$? ✗

$x = a^{p-s} (a^s)^2 b^p$; $i = 2$

$= a^{p-s} . a^{2s} . b^p$

$= a^{p+s} . b^p$ $\notin L$

$p + s = p$? ✗

↓

$|s| \ge 1$

dont select number.

work with assumptions.

$a^i b^j$; $i > j$

$a^{p+m} b^p$; $|m| \ge 1$

min: p , max: $p+m$

$a^p a^m b^p$

$u\ (v)^i\ w$

$a^p b^p$; $i = 0$ pump down

$\notin L$

$a^i b^j$; $i < j$

$a^{p-m} b^p$; $|m| \ge 1$

$a^p a^{-m} b^p$

$u\ (v)^i\ w$

$a^p b^p$; $i = 0$ pump down

$\in L$

Pumping Lemma.

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$L = \{0^n 1 0^n ; L \in RL \text{ or not}\}$

$0^p 1 0^p$

$u = 0^{p-s}$

$v = 0^s$ — $|v| \geq 1$

$w = 10^p$

as $|v| \geq 1$

$\boxed{uv^iw \in L \quad i \geq 0}$

$x = 0^{p-s} (0^s)^i 10^p$

$= 0^{p-s} 10^p \quad ; i=0$ (pump down)

↓

$\notin L$ as $|s| \geq 1$

$p - s \neq p$

DFA Minimization. (using Myhill's Theorem)

Every transition has 2 possibilities :-

Not final ← $\boxed{NF \mid F}$ → Final state

e.g.,

$A \xrightarrow{0} F$, $A \xrightarrow{1} NF$

$B \xrightarrow{1} NF$, $B \xrightarrow{0} F$

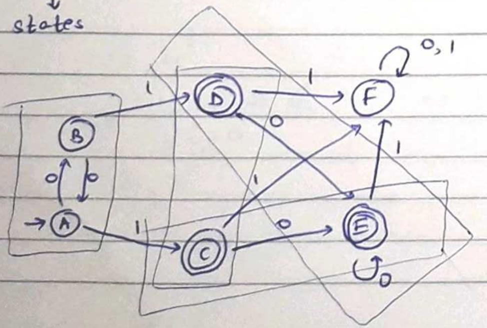
both have same transitions

Q. $L = \{x$ over $\{0,1\} \mid x$ accepts only one 1 in a valid string$\}$.

Steps (for DFA Minimization)

① Draw table for all pair of states (P, Q)

n x n dimension table
↓
states



← Given DFA whose minimization is required.

AB, CDE, F

Marking (✓) all pairs (P, Q) where P belongs to final state & Q doesn't belong to final state & vise versa.

②

	A	B	C	D	E	F
A	×	×	×	×	×	×
B	.	×	×	×	×	×
C	✓	✓	×	×	×	×
D	✓	✓	.	×	×	×
E	✓	✓	.	.	×	×
F	✓	✓	✓	✓	✓	×

→ Valid Cells

i) As AB = BA
Hence ignoring upper half.

ii) diagonal is excluded too.

$A \xrightarrow{0} NF$, $A \xrightarrow{1} F$

$B \xrightarrow{0} NF$, $B \xrightarrow{1} F$

$C \xrightarrow{0} F$, $C \xrightarrow{1} NF$

$D \xrightarrow{0} F$, $D \xrightarrow{1} NF$

$E \xrightarrow{0} F$, $E \xrightarrow{1} NF$

$F \xrightarrow{0} NF$, $F \xrightarrow{1} NF$

~~Step 2 is focused in finding~~ Tick conflicts.

(1 transition takes 1 state to F while same transition takes 2nd state to NF)

⇓

P belongs to F/NF & Q belongs to NF/F respectively. (Ticked).

If there are any ~~unmarked pairs~~ (P,Q) such that their alphabets' transitions maps to a marked cell, will also be marked.

③ See vacant boxes now. (BA, DC, EC, ED)

i) $A \xrightarrow{0} B$, $A \xrightarrow{1} C$; $B \xrightarrow{0} A$, $B \xrightarrow{1} D$ → $\boxed{BA}$ already unmarked, $\boxed{DC}$ already unmarked

ii) $C \xrightarrow{0} E$, $C \xrightarrow{1} F$; $D \xrightarrow{0} E$, $D \xrightarrow{1} F$ → $\boxed{EE}$, $\boxed{FF}$ discard

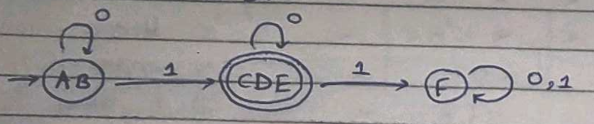
if both unmarked, leave. otherwise mark the pair (P,Q).
marked states are distinguishable. Unmarked are not.

iii) $C \xrightarrow{0} E$, $C \xrightarrow{1} F$; $E \xrightarrow{0} E$, $E \xrightarrow{1} F$ → $\boxed{EE}$, $\boxed{FF}$ discard

iv) $D \xrightarrow{0} E$, $D \xrightarrow{1} F$; $E \xrightarrow{0} E$, $E \xrightarrow{1} F$ → $\boxed{EE}$, $\boxed{FF}$ discard.

If any state gets marked, restart whole operation.

(4) Combine all unmarked pairs & make them a single state in the minimized DFA.



6/10/22 CFL : Context Free Language

RL : words
CFL : sentences

CFG

Context Free Grammar

SVO

Subject Verb Object

C++

;

CFG

$L = \{a^n b^n ; n \geq 0\}$

$= \{\wedge, ab, aabb, \dots\}$

```
void S (int n) {
  if (n ==0) return;
  cout << 'a';
  S(n-1);
  cout << 'b';
}
```

CFG needs 4 tuples

Σ = Terminals

V = Variables

S_v = Start variable

$P \rightarrow \alpha \in (\Sigma \cup V)^*$ $\quad P \rightarrow \wedge$ $\quad (\Sigma \cup V)^0$

Productions

$P \in V$

$\boxed{S \rightarrow aSb \mid \wedge}$ (1) (2)

Q. $\boxed{\begin{array}{l} S \rightarrow \wedge \\ S \rightarrow aSb \end{array}}$

$V = \{S\}$
$\Sigma = \{a, b\}$
$S_v = \{S\}$

Q.

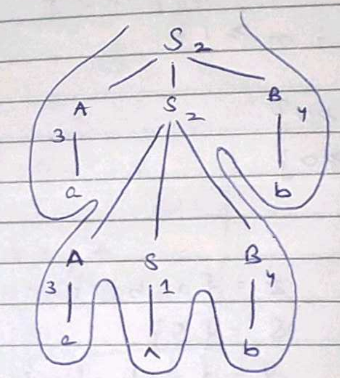
$S \rightarrow \wedge$	(1)
$S \rightarrow ASB$	(2)
$A \rightarrow a$	(3)
$B \rightarrow b$	(4)

4 Productions!

$V = \{S, A, B\}$
$\Sigma = \{a, b\}$
$S_v = \{S\}$

Q. aabb, How to derivate it? You need <u>Parse tree</u>.

Q. $aabb \in L$?



Valid when all leaves belong to terminal (or is $\wedge$).

aabb

You read parse tree from top to bottom, left to right.

Q. $L = \{a^n b^n ; \boxed{n \geq 1}\}$

$\boxed{S \rightarrow aSb \mid aabb}$ (1) (2)

$P \rightarrow \alpha$ CFG
$P ::= \alpha$ BNF
CFG $\neq$ BNF

Q. $L = \{a^n b^{2n} ; n \geq 1\}$ $\quad a^n b^n b^n$

$\boxed{S \rightarrow aSbb \mid abb}$ (1) (2)

$V = \{S\}$
$\Sigma = \{a, b\}$
$S_v = \{S\}$

Q. $L = \{a^n b^m ; n > m ; m >= 0\}$

$a^1 b^0 = a$ basecase

$\boxed{p \geq 1}$ $\underbrace{a^p}_{L_1}\underbrace{a^m b^m}_{L_2}$

$S \rightarrow L1L2$
$L1 \rightarrow aL1 \mid a$
$L2 \rightarrow aL2b \mid \wedge$

$S_v = \{S\}$
$\Sigma = \{a, b\}$
$S = \{S, L1, L2\}$

Homework

Q. $L = \{a^n b^m c^p ; m = n+p ; n, p \geq 0\}$

Q. $L = \{a^n b^m ; n \geq m ; m \geq 1\}$

$L = \{a^n b^{n+p} c^p ; n, p \geq 0\}$
$= \{\underbrace{a^n b^n}_{L_1}\underbrace{b^p c^p}_{L_2} ; n, p \geq 0\}$

$S \rightarrow L_1L_2$
$L_1 \rightarrow aL_1b \mid \wedge$
$L_2 \rightarrow bL_2c \mid \wedge$

$\Sigma = \{a, b, c\}$
$S_v = \{S\}$
$V = \{S, L_1, L_2\}$

Productions = 5

$L = \{\underbrace{a^p}_{L_1}\underbrace{a^m b^m}_{L_2} ; p \geq 0 ; m \geq 1\}$

$S \rightarrow L_1L_2$
$L_1 \rightarrow aL_1 \mid \wedge$
$L_2 \rightarrow aL_2b \mid ab$

$\Sigma = \{a, b\}$
$S_v = \{S\}$
$V = \{S, L_1, L_2\}$

Productions = 5

Extra Q. $L = \{a^n b^m ; m \geq n ; n \geq 2\}$
$= \{\underbrace{a^n b^n}_{L_1}\underbrace{b^p}_{L_2} ; p \geq 0 ; n \geq 2\}$

$L \rightarrow L_1L_2$
$L_1 \rightarrow aL_1b \mid aabb$
$L_2 \rightarrow bL_2 \mid \wedge$

5 Productions!

$\Sigma = \{a, b\}$
$V = \{L, L_1, L_2\}$
$S_v = \{L\}$

Q. $L = \{(a^i b^i)^n (c^j d^j)^m ; n > m ; m \geq 0 ; i \geq 1 ; j \geq 3\}$

A B

S

$A \rightarrow aAb \mid ab$
$B \rightarrow cBd \mid cccddd$
$G \rightarrow AG \mid A$
$M \rightarrow AMG \mid \wedge$
$S \rightarrow GM$

9 Productions!

$L = \{A^n B^m ; n > m ; m \geq 0$
$\underbrace{A^p}_{G}\underbrace{A^m B^m}_{M} ; p \geq 1 ; m \geq 0\}$

$\Sigma = \{a, b, c, d\}$
$V = \{S, G, M, A, B\}$
$S_v = \{S\}$

Q. $L = \{a^i b^j c^j d^i ; i, j \geq 0\}$

$S \rightarrow aSd \mid G$
$G \rightarrow bGc \mid \wedge$

4 Productions!

$\Sigma = \{a, b, c, d\}$
$V = \{S, G\}$
$S_v = \{S\}$

Q. $L = \{a^i b^j c^j d^i ; i >= 0 ; j >= 2\}$

$S \rightarrow aSd \mid G$
$G \rightarrow bGc \mid bbcc$

Q. $L = \{a^i b^j c^j d^i \ ; \ i >= 2 \ ; \ j >= 0\}$

$$\boxed{\begin{array}{l} S \rightarrow aSd \mid aaGdd \\ G \rightarrow bGc \mid \wedge \end{array}}$$

Q. $L = \{a^i b^j c^j d^i \ ; \ i >= 2 \ ; \ j >= 1\}$

$$\boxed{\begin{array}{l} S \rightarrow aSd \mid aaGdd \\ G \rightarrow bGc \mid bc \end{array}}$$

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int a = 10;
float abc = 10.5;

$S \rightarrow T$ (type) id (identifier) = Num ;
$T \rightarrow$ int | float

$\longleftrightarrow$

Q. $L = \{a^n b^m c^m d^n \ ; \ n \geq 1 \ ; \ m \geq 2\}$

$$\boxed{\begin{array}{l} S \rightarrow aSd \mid aGd \\ G \rightarrow bGc \mid bbcc \end{array}}$$

(1) (2) (3) (4)

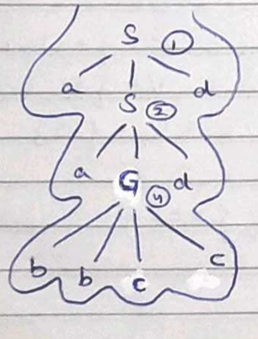
$\Sigma = \{a, b, c, d\}$
$V = \{S, G\}$
$S_v = \{S\}$

4 Productions !

$\longleftrightarrow$

aabbccdd

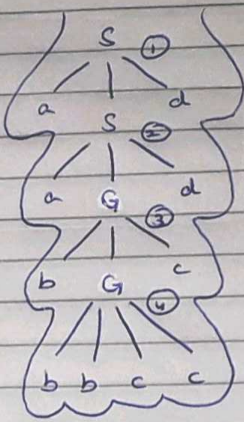
Using Parse Tree :-



$\longleftrightarrow$

aabbbccccdd

Using Parse Tree :-



$\longleftrightarrow$

Q. $L = \{a^n b^m \ ; \ n \leq m \ ; \ m \geq 0\}$

$= \{\underbrace{a^n b^n}_{L_1} \underbrace{b^p}_{L_2} \ ; \ p \geq 0 \ ; \ n \geq 0\}$

$$\boxed{\begin{array}{l} S \rightarrow L_1 L_2 \\ L_1 \rightarrow aL_1 b \mid \wedge \\ L_2 \rightarrow bL_2 \mid \wedge \end{array}}$$

$\Sigma = \{a, b\}$
$V = \{S, L_1, L_2\}$
$S_v = \{S\}$

5 productions.

$\longleftrightarrow$

Q. $L = \{a^n b^m \ ; \ n \geq m \ ; \ m \geq 0\}$

$= \{\underbrace{a^p}_{L_1} \underbrace{a^n b^n}_{L_2} \ ; \ p \geq 0 \ ; \ n \geq 0\}$

$$\boxed{\begin{array}{l} S \rightarrow L_1 L_2 \\ L_1 \rightarrow aL_1 \mid \wedge \\ L_2 \rightarrow aL_2 b \mid \wedge \end{array}}$$

$\Sigma = \{a, b\}$
$V = \{S, L_1, L_2\}$
$S_v = \{S\}$

5 productions !

$\longleftrightarrow$

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Q. Write a Palindrome for CFG language.

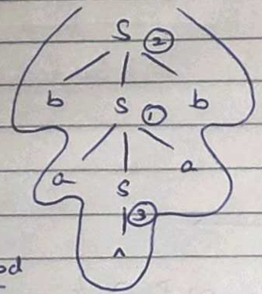
$\boxed{S \longrightarrow aSa \mid bSb \mid \Lambda \mid a \mid b}$ (1) (2) (3) (4) (5)

$\Sigma = \{a, b\}$
$V = \{S\}$
$S_v = \{S\}$

5 Productions!

Q. baab

Derivation/Parse Tree



Substitution Method

$\boxed{S \overset{*}{\Rightarrow} x \quad \hookrightarrow \in \Sigma^*}$

Q. aabaa

$S \overset{1}{\Rightarrow} aSa \overset{1}{\Rightarrow} aaSaa \overset{5}{\Rightarrow} \boxed{aabaa} \rightarrow \in \Sigma^*$

$aaSaa \downarrow x \notin \Sigma^*$

$\in V$

If $x \in \Sigma^*$ then $x \in L$ of CFG

Q. Write a CFG for NOT a Palindrome Language.

$$\boxed{\begin{array}{l} S \longrightarrow aSa \mid bSb \mid aAb \mid bAa \\ A \longrightarrow aA \mid bA \mid \Lambda \end{array}}$$

$\Sigma = \{a, b\}$
$V = \{S, A\}$
$S_v = \{S\}$

7 Productions!

conflict!

ab(b)bb(a)b a

Q. A == B

# of a = # of b

even length

$= \{\Lambda, ab, ba,$ 2a's 2b's
$\hookrightarrow abba, aabb, abab, baba, bbaa, baab, \ldots\}$

$$\boxed{\begin{array}{l} S \rightarrow aSb \mid bSa \mid aA \mid bB \mid \Lambda \\ A \rightarrow aAb \mid bAa \mid bS \mid aAA \\ B \rightarrow aBb \mid bBa \mid aS \mid bBB \end{array}}$$ ✗

$\Sigma = \{a, b\}$
$V = \{S, A, B\}$
$S_v = \{S\}$

← Right but has excessive productions.

✓ $\boxed{S \longrightarrow aSb \mid bSa \mid \Lambda \mid SS}$ (1) (2) (3) (4)

$\Sigma = \{a, b\}$
$V = \{S\}$
$S_v = \{S\}$

4 productions.

$$\boxed{\begin{array}{l} S \longrightarrow aB \mid bA \mid \Lambda \quad (1)\ (2)\ (3) \\ B \rightarrow \text{(4)}\ bS \mid \text{(5)}\ aBB \\ A \rightarrow \text{(6)}\ aS \mid \text{(7)}\ bAA \end{array}}$$ ✓

$\Sigma = \{a, b\}$
$V = \{S, A, B\}$
$S_v = \{S\}$

7 productions.

aabb

$S \overset{1}{\Rightarrow} aB \overset{5}{\Rightarrow} aaBB$
$\overset{4}{\Rightarrow} aabSB$
$\overset{3}{\Rightarrow} aabB$
$\overset{4}{\Rightarrow} aabbS$
$\overset{3}{\Rightarrow} aabb \quad \in \Sigma^*$

left-to-right : Left derivation

right to left : Right derivation

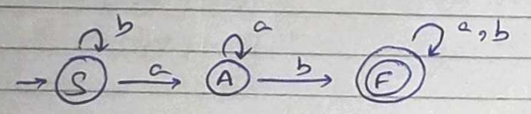
Q. $L = \{ (a^n b^n)^i (c^m d^m)^i ; n, m, i \geq 2 \}$

	①	②
$S \rightarrow$	ASC	$AACC$
	③	④
$A \rightarrow$	aAb	$aabb$
	⑤	⑥
$C \rightarrow$	cCd	$ccdd$

$\Sigma = \{a, b, c, d\}$
$V = \{S, A, C\}$
$S_v = \{S\}$

6 Productions!

DFA to CFG :-



Rules/Tips :-
State name = variable
transitions = productions
alphabets = terminal

$S \longrightarrow$	aA	bS	
$A \longrightarrow$	aA	bF	
$F \longrightarrow$	aF	bF	$\wedge$

$\Sigma = \{a, b\}$
$V = \{S, A, F\}$
$S_v = \{S\}$

all final states have null. transition.

RE to CFG:-

$\underbrace{\underbrace{\underbrace{(a+b)}_{A}{}^{*}}_{B}\ bba}_{S}$

$A \longrightarrow$	$a \mid b$
$B \longrightarrow$	$BA \mid \wedge$
$S \longrightarrow$	$Bbba$

$\Sigma = \{a, b\}$
$V = \{A, B, S\}$
$S_v = \{S\}$

Chomsky Normal form.
↑
CNF is CFG with modifications.

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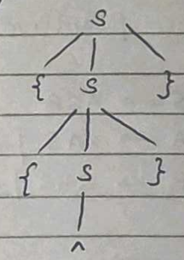
CNF (CFG → CFG)
CYK ↳ accepts
↳ parson

Q. $L = \{ \{^n \}^n, n \geq 0 \}$

Q. {{}}



CFG

	①	②
$S \rightarrow$	$\{S\}$	$\wedge$

CNF Conversion Steps

① Introduce a new start variable.

$S_0 \longrightarrow S$
$S \longrightarrow \{S\} \mid \wedge$

② Remove $\wedge$-productions.

Track where $\wedge$ came from : S
Substitute S with $\wedge$.

$S_0 \longrightarrow S \mid \wedge$
$S \longrightarrow \{S\} \mid \{\}$

③ Remove unit productions.

$X \rightarrow Y$
$Y \in V$ &
$|Y| = 1$

$S_0 \longrightarrow \wedge \mid \{S\} \mid \{\}$
$S \longrightarrow \{S\} \mid \{\}$

→ $\Sigma V \Sigma$ → $\Sigma\Sigma$

$\Sigma = \{ \{, \} \}$
$V = \{S, S_0\}$

④ $\boxed{V \rightarrow VV}$ $\boxed{V \rightarrow \Sigma}$

$A \rightarrow \{$
$B \rightarrow \}$

$P \rightarrow AB$

$S_0 \longrightarrow \wedge \mid ASB \mid AB$
$S \longrightarrow ASB \mid AB$

$P \rightarrow AS$

$S_0 \to \wedge \mid PB \mid AB$ $\quad A \to \{$
$S \to PB \mid AB$ $\quad B \to \}$
$P \to AS$

Terminals = Leafs
Other = Intermediate

Q. $S \to ASB$ $\quad$ Convert to CNF.
$A \to aAS \mid a \mid \wedge$
$B \to SbS \mid A \mid bb$

① $S_0 \to S$
$S \to ASB$
$A \to aAS \mid a \mid \wedge$
$B \to SbS \mid A \mid bb$

② $S_0 \to S$
$S \to ASB \mid SB$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid A \mid bb \mid \wedge$

$S_0 \to S$
$S \to ASB \mid SB \mid AS \mid S$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid A \mid bb$

③ $S \to \cancel{S}$

$S_0 \to ASB \mid SB \mid AS$
$S \to ASB \mid SB \mid AS$
$A \to aAS \mid a \mid aS$
$B \to SbS \mid bb \mid aAS \mid a \mid aS$

④ $X \to a$
$Y \to b$

$S_0 \to ASB \mid SB \mid AS$
$S \to ASB \mid SB \mid AS$
$A \to XAS \mid X \mid XS$
$B \to SYS \mid YY \mid XAS \mid X \mid XS$

$S_0 \to ASB \mid SB \mid AS$
$S \to ASB \mid SB \mid AS$
$A \to XAS \mid a \mid XS$
$B \to SYS \mid YY \mid XAS \mid a \mid XS$

$S_0 \to PB \mid SB \mid AS$ $\quad P \to AS$
$S \to PB \mid SB \mid AS$ $\quad Q \to YS$
$A \to XP \mid a \mid XS$
$B \to SQ \mid YY \mid XP \mid a \mid XS$

20/10/22

Cocke Younger Kasami
CYK Parser → used to check membership of any string to a given language.

Q. $L = \{ (^n)^n ; n \geq 0 \}$

CFG $\quad S \to (S) \mid \wedge$

CNF
① $S_0 \to S$
$S \to (S) \mid \wedge$

② $S \to S \mid \wedge$
$S \to (S) \mid ()$

③ $S_0 \to (S) \mid () \mid \wedge$
$S \to (S) \mid ()$

④ $A \to ($ $\quad B \to)$
$S_0 \to ASB \mid AB \mid \wedge$
$S \to ASB \mid AB$

$A \to ($ $\quad B \to)$
$C \to AS$
$S_0 \to CB \mid AB \mid \wedge$
$S \to CB \mid AB$

CYK → using CNF on previous page.

$(()) = x$ ↓ $|x| = 4$ ✓

Table = $|x| \times |x|$
= 4×4

index	subset
Variable(s)	

bottom up parser

"substring length on y-axis."

j=4	4-1 (()) S_0, S			
j=3	3-1 (() C	3-2 ()) ϕ		
j=2	2-1 ((ϕ	2-2 () S_0, S	2-3)) ϕ	
j=1	1-1 (A	1-2 (A	1-3) B	1-4) B
	(	(	)	)

→ if this cell has start varicble, then the string is valid input.

① Input along horizontal axis.

4x4

② تک length کی substring اس سے بن سکتی ہے۔
(on vertical axis)

③ which variable(s) are producing required terminal. (inside box)

④ if there is no valid production in CNF, write ϕ.

j = 1
→ "(" A
→ ")" B

→ j = 2
"((" AA = ϕ → "))" BB = ϕ

"()" AB = S_0, S

"(()" — '(' A, '()' S_0, S ⟹ AS_0, AS (Cartesian Product)
↓ ↓
ϕ U C

j=3

'((' — ϕ, ')' — B ⟹ ϕ → C

"())" — '(' — A, '))' — ϕ ⟹ ϕ

'()' — S_0, S, ')' — B ⟹ S_0B, SB
↓ ↓
ϕ U ϕ

→ ϕ

j=4
"(())"   3 levels
divide in pair of 2.

1,3	2,2	3,1
(,())	((,))	((),)
1,1 − 3,2	2,1 − 2,3	3,1 − 1,4
A, ϕ ×	U ϕ ×	U C, B

{ S_0, S }

H.W

Q. (()))

j=5	(())) ϕ				
j=4	(()) S_0, S	())) ϕ			
j=3	(() C	()) ϕ	))) ϕ		
j=2	((ϕ	() S_0, S	)) ϕ	)) ϕ	
j=1	(A	(A	) B	) B	) B
	(	(	)	)	)

"(()))"

1,4	2,3	3,2	4,1
(,()))	((,)))	((),))	((()),)
1,1 − 4,2	2,1 − 3,3	3,1 − 2,3	4,1 − 1,5
A, ϕ ×	U ϕ ×	U C − ϕ ×	U S_0, S − B

{ S_0B, SB } doesnt exist so ϕ

25/10/22 $L = \{a^n b^n ; n \geq 0\}$

$(^n)^n \Rightarrow ()\ \ (())\ \ ((()))$

Push Down Automata (PDA) 7 tuples

↓

FA + Stack (Memory)

↓

5 tuples

1) q_0
2) f_n
3) Σ
4) $\delta = Q \times \Sigma$
5) Q
6) S_0 → top element of stack.
7) Γ = {set of stack elements}

$\boxed{\delta = Q \times \Sigma \times \Gamma \rightarrow Q, \Gamma}$

↓

new transition function.

S_0 → either \$ or @

FA

$\rightarrow (Q_0) \xrightarrow{\Sigma} (Q_1)$

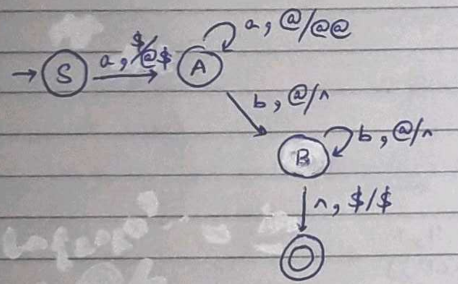
PDA

$\rightarrow (Q_0) \xrightarrow{\Sigma, \Gamma/\Gamma} (Q_1)$

alphabet, pop/push = read/write

$L = \{a^n b^m ; n, m \geq 1\}$

$n == m$

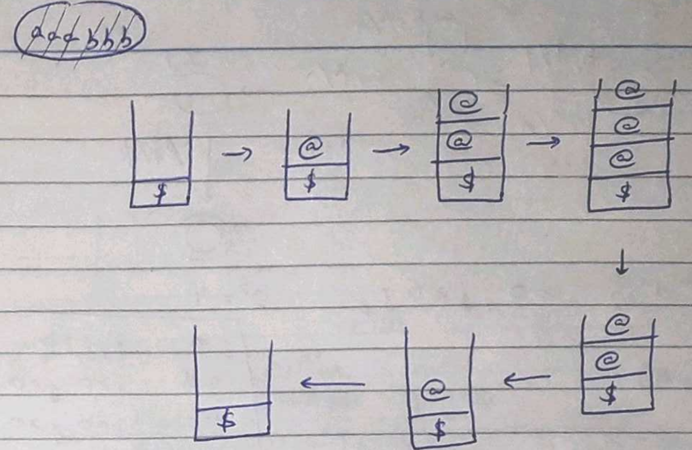


Stack Operations

1) pop ⇒ $\Sigma, \Gamma_a/\wedge$
2) replace ⇒ $\Sigma, \Gamma_a/\Gamma_b$
3) No Op. ⇒ $\Sigma, \Gamma_a/\Gamma_a$
4) push ⇒ $\Sigma, \Gamma_a/\Gamma_b\Gamma_a$

$\ldots\Gamma_c\Gamma_b\Gamma_a$ ⟵

pushing is right to left



27/10/22 Construct a PDA for the following lenguage.

$L = \{a^i b^j c^i ; i \geq 0 ; j \geq 2\}$

$\Sigma, \Gamma/\Gamma$ → write (push) ; read (pop)

$\Sigma, \Gamma/\wedge$ → pop

$\Sigma, \Gamma_a/\Gamma_a$ → no operation

$\Sigma, \Gamma_a/\Gamma_b$ → replace

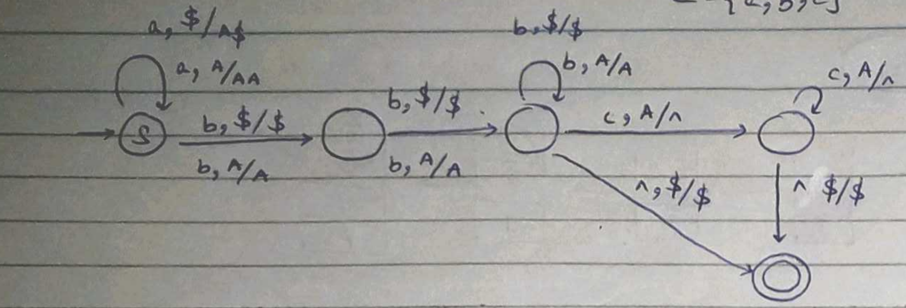
$\Sigma, \Gamma_a/\Gamma_b\Gamma_a$ → push

$\Sigma, \Gamma_a/\Gamma_b\Gamma_b\Gamma_a$ → multiple push (not allowed in exam)

$\Gamma = \{\$, A\}$

$Q \times (\Sigma \times \Gamma)$

$\Sigma = \{a, b, c\}$

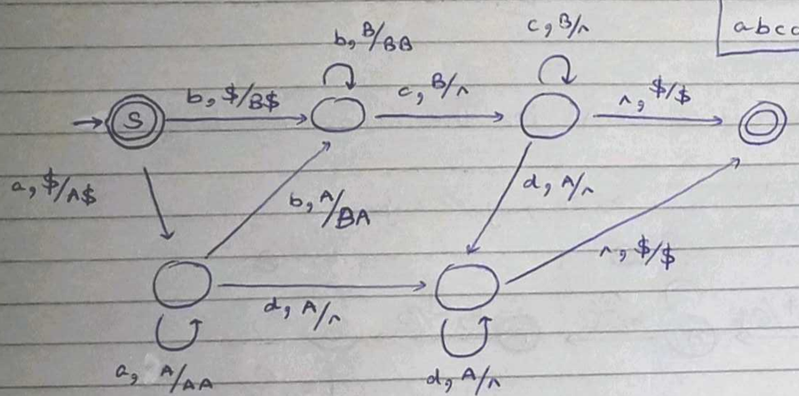


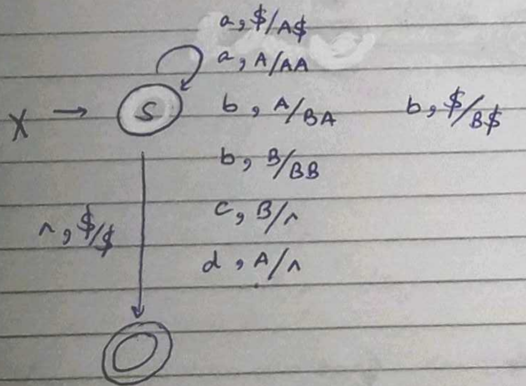
Q. $L = \{a^i b^j c^j d^i ;\ i \geq 0, j \geq 0\}$

$2^2 = 4$

λ	i, j = 0
ad	i > 0 j = 0
bc	i = 0 j > 0
abcd	i, j > 0

$\Gamma = \{\$, A, B\}$





# of a's = # of d's

# of b's = # of c's

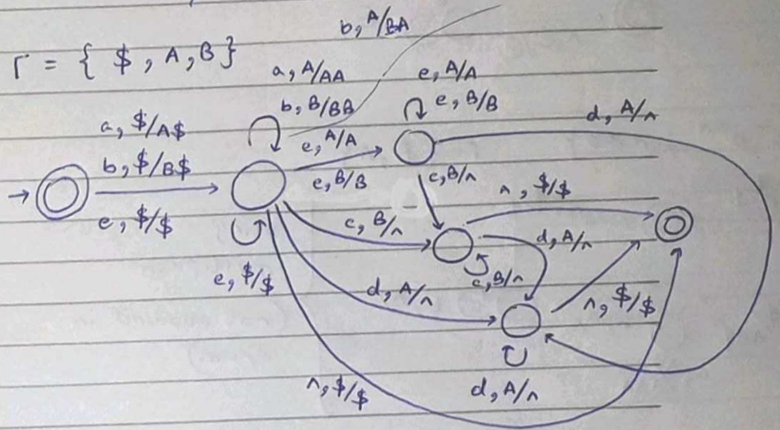
longest pattern length + 1

Q. $L = \{a^i b^j e^k c^j d^i \quad i \geq 0 ; j \geq 0 ; k \geq 0\}$

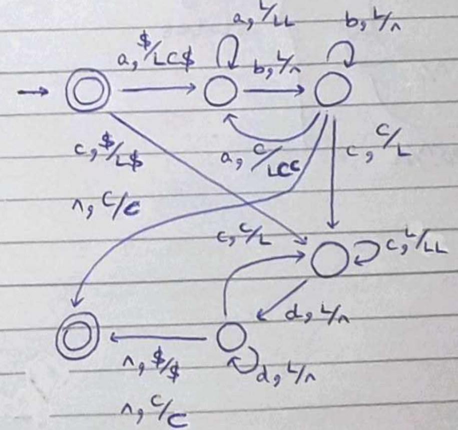
$2^3 = 8$ combinations.

k=0	k>0
λ	e
ad	aed
bc	bec
abcd	abecd

$\Gamma = \{\$, A, B\}$



Q. $L = \{(a^i b^i)^n (c^j d^j)^m ;\ n \geq m, i, j, m \geq 0\}$

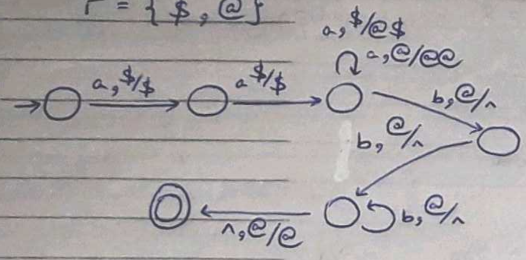


m = 0
λ
m = 1, n = 1
ab i > 0
cd j > 0
abcd i, j > 0
m = 1, n = 2
abab
cd
ababcd

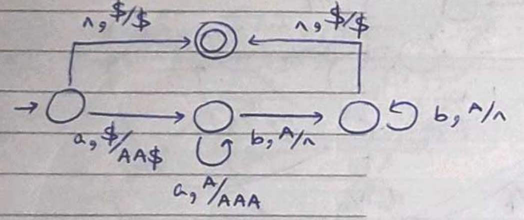
$\Gamma = \{\$, C, L\}$

Extra Q. $L=\{a^n b^m ; n > m , m \geq 2\}$

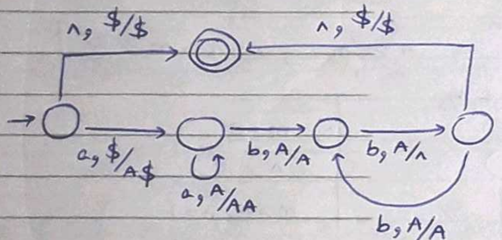
$\Gamma = \{\$, @\}$



Q. $L=\{a^n b^{2n} ; n \geq 0\}$ $\Gamma = \{\$, A\}$



using multipush (not allowed in exam)



using states

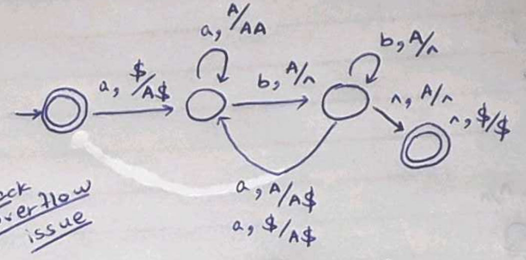
Quiz 4 (Prank)

1/11/22

Design PDA :

Q. $L=\{(a^i b^j)^n ; i \geq j ; n \geq 1 ; j \geq 0\}$

$\Gamma = \{\$, A\}$



n = 1

ʌ	i = 0, j = 0
a	i = 1, j = 0
ab	i = 1, j = 1

n = 2

aa	i = 1, j = 0
abab	i = 1, j = 1

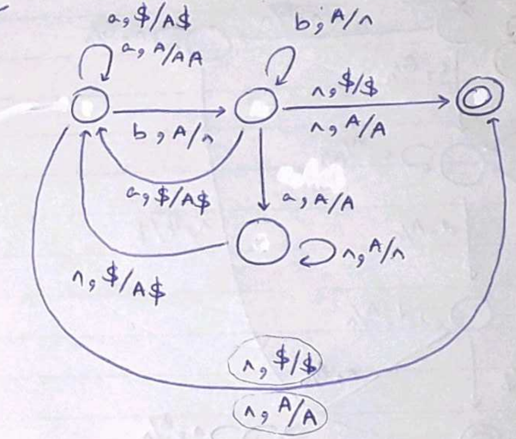
Stack overflow issue

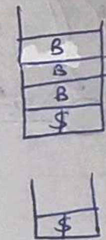
Q. $L=\{(a^i b^j)^n ; i \geq j ; n \geq 3 ; j \geq 0\}$

$\Gamma = \{\$, A, B\}$

Solution

n≥1

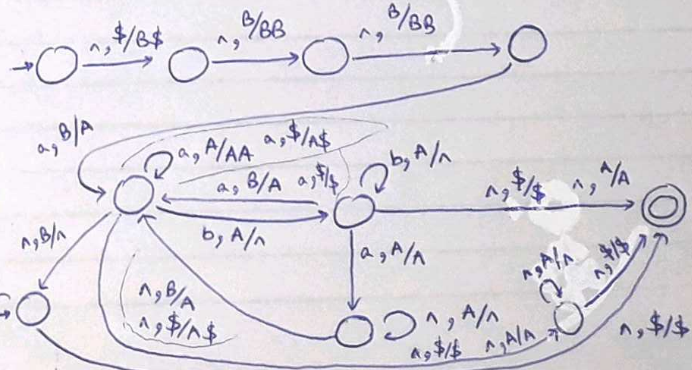




n ≥ 3

n ≥ 1

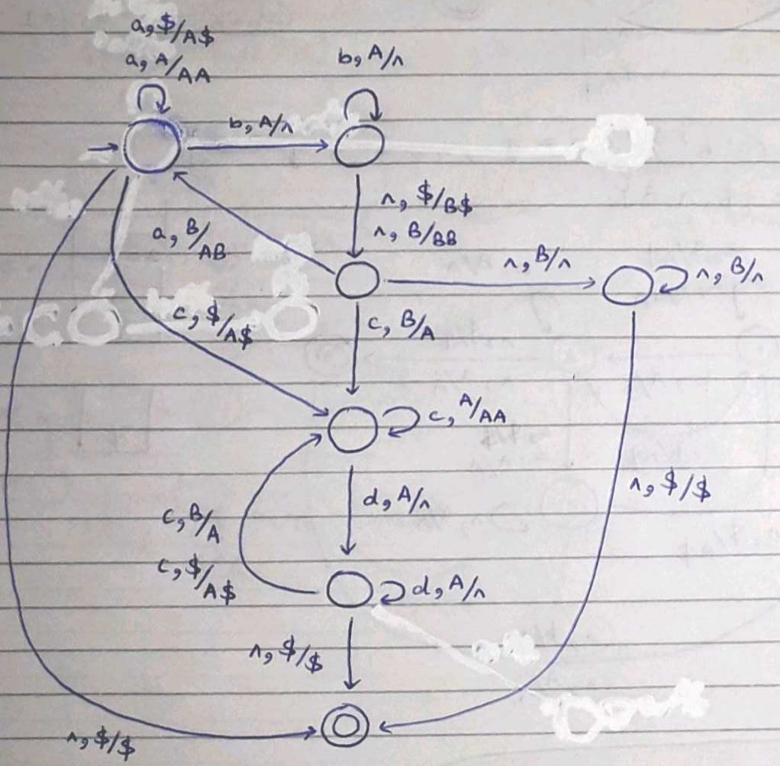
n≥3



Q. $L = \{ (a^i b^i)^n (c^j d^j)^n ; n, i, j \geq 0 \}$

λ
ab — $i=1$
abab — $n=2$
$n=1$
cd
cdcd — $j=1$
abcd $n=1, i, j=1$
ababcdcd $n=2, i, j=2$

$\Gamma = \{ \$, A, B \}$



2/11/22. <u>Practice</u>

Write a CFG for following lenguage.

Q. $L = \{ a^i b^j ; i < 2j ; j > 0 \}$

a	b
i	j
0,1	1 → ab, b
0,1,2,3	2 → bb, abb, aabb, aaabb
0,1,2,3,4,5	3 → bbb, abbb, ..., aaaaabbb
0,1,2,3,4,5,6,7	4 → bbbb, abbbb, ... aaaaaaabbbb

$i = 2j-1$?

$S \longrightarrow Sb \mid aSb \mid \cancel{aaSb} \mid$ (1) (2) (3)
aAb (4)
only when $i = 2j$

$A \longrightarrow aaAb \mid \lambda$ (5) (6)

$S \overset{4}{\Rightarrow} aAb \overset{5}{\Rightarrow} aaaAbb \overset{5}{\Rightarrow} aaaaaAbbb$
$\overset{5}{\Rightarrow} aaaaaaaAbbbb$
$\overset{6}{\Rightarrow} aaaaaaabbbb$

Q. $L = \{ a^i b^j ; i < 3j ; j > 0 \}$

$i = 3j - 1$?

$S \rightarrow Sb \mid aSb \mid aAb \mid aaSb \mid aaAb$ (1) (2) (3) (4) (5)
$A \rightarrow aaaAb \mid \lambda$ (6) (7)

i	j	
0,1,2	1	b, ab, aab
0,1,2,3,4,5	2	bb, abb, ... aaaaabb
0,1,2,3,5,..., 8	3	bbb, abbb, ..., aaaaaaaabbb
0,1,2,..., 11	4	bbbb, abbbb, ..., aaaaaaaaaaabbbb