

# Information Security

## CS3002

Lecture 6  
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Dr. Rana Asif Rehman  
Email: [r.asif@lhr.nu.edu.pk](mailto:r.asif@lhr.nu.edu.pk)

# Advanced Encryption Standard (AES)

# Origins

- Clear a replacement for DES was needed.
  - Have theoretical attacks that can break it.
  - Have demonstrated exhaustive key search attacks.
- US NIST issued call for ciphers in 1997.
- 15 candidates accepted in Jun 98.
- 5 were shortlisted in Aug-99.
- Rijndael was selected as the AES in Oct-2000.
- Issued as FIPS PUB 197 standard in Nov-2001 .

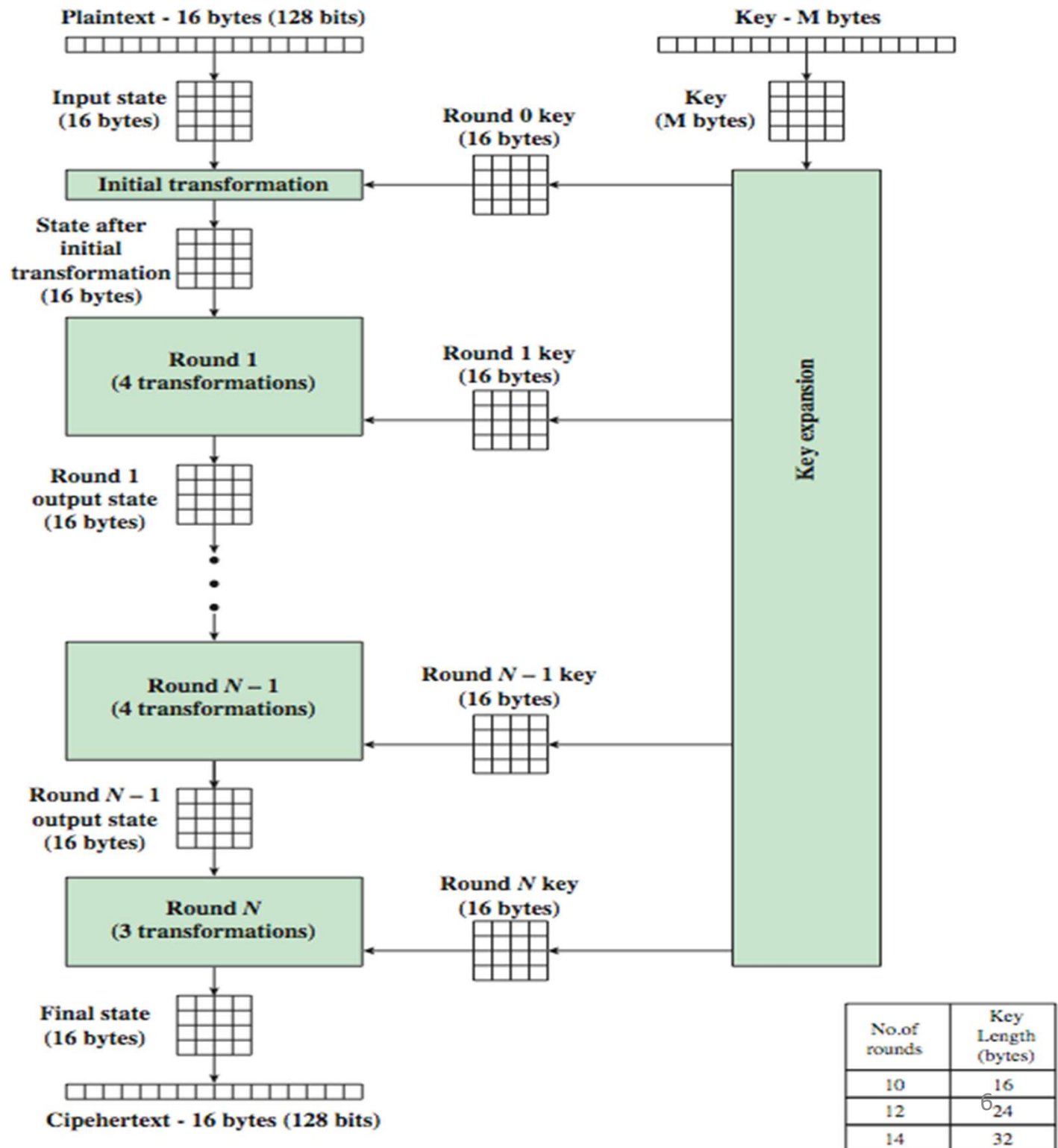
# The AES Cipher

- Allows *128, 192, and 256*-bit key sizes .
- Variable block length of *128, 192, or 256* bits. All nine combinations of key/block length possible.
  - A block is the smallest data size the algorithm will encrypt
- Vast speed improvement over DES in both hardware and software implementations
  - with fast XOR & table lookup implementation

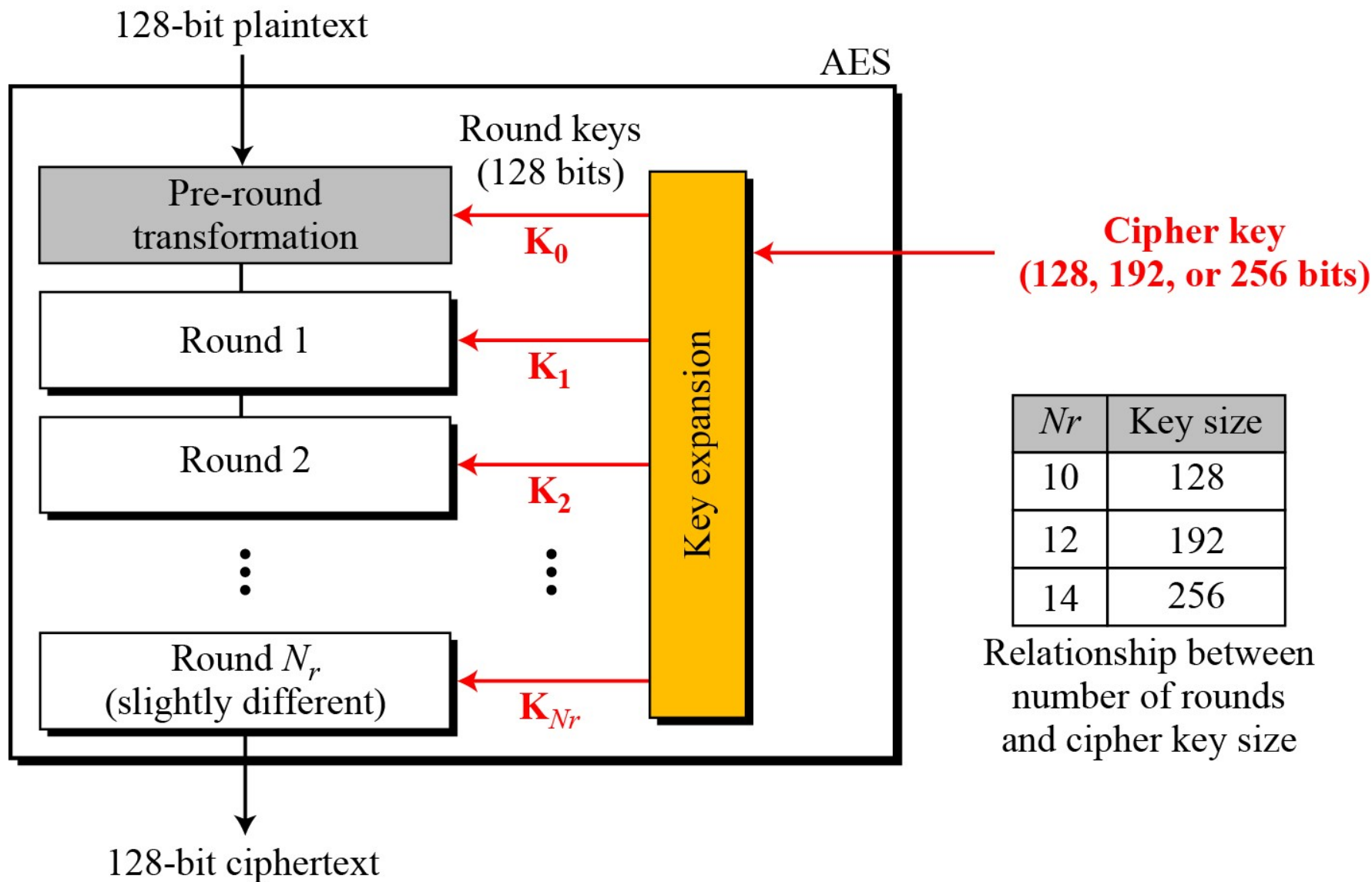
# The AES Cipher - Rijndael

- An **iterative** rather than **feistel** cipher
  - Processes data as block of 4 columns of 4 bytes.
  - Operates on entire data block in every round.
- Designed to be:
  - Resistant against known attacks.
  - Speed and code compactness on many CPUs.
  - Design simplicity.

# AES Encryption Process

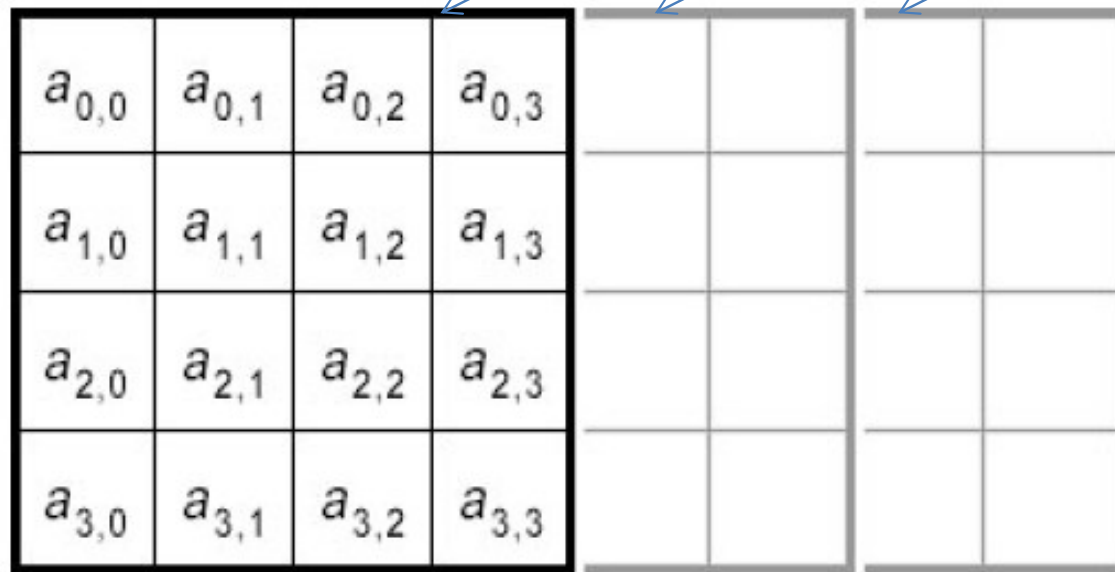


# AES Encryption Process



# The AES Cipher- Input Text

- Possible block sizes: 128, 192, 256 bit

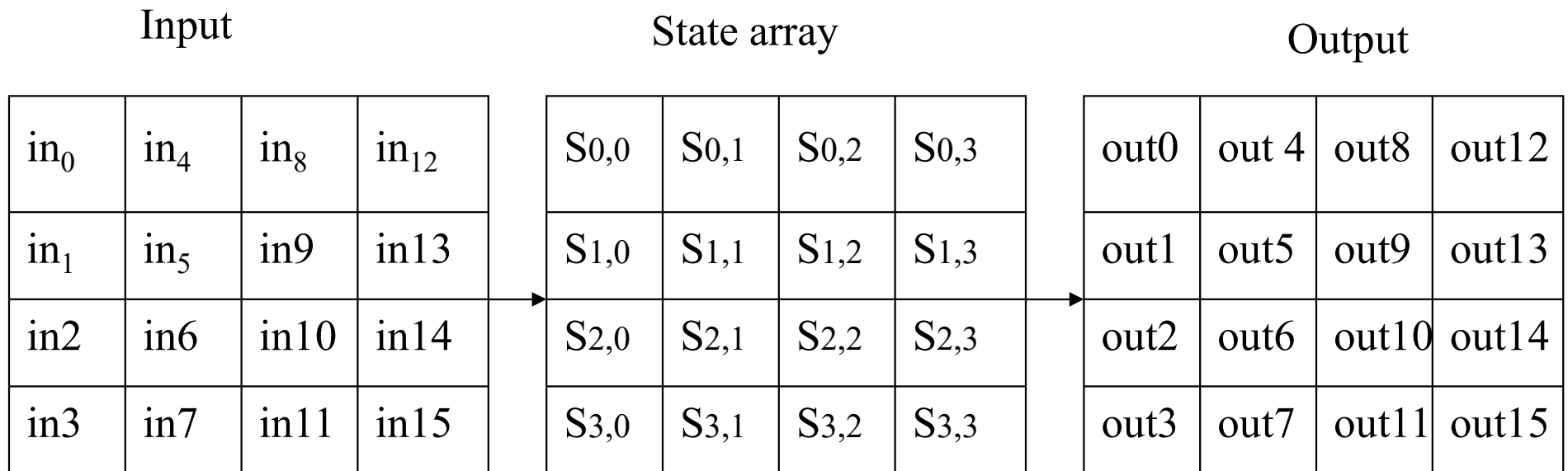


The state



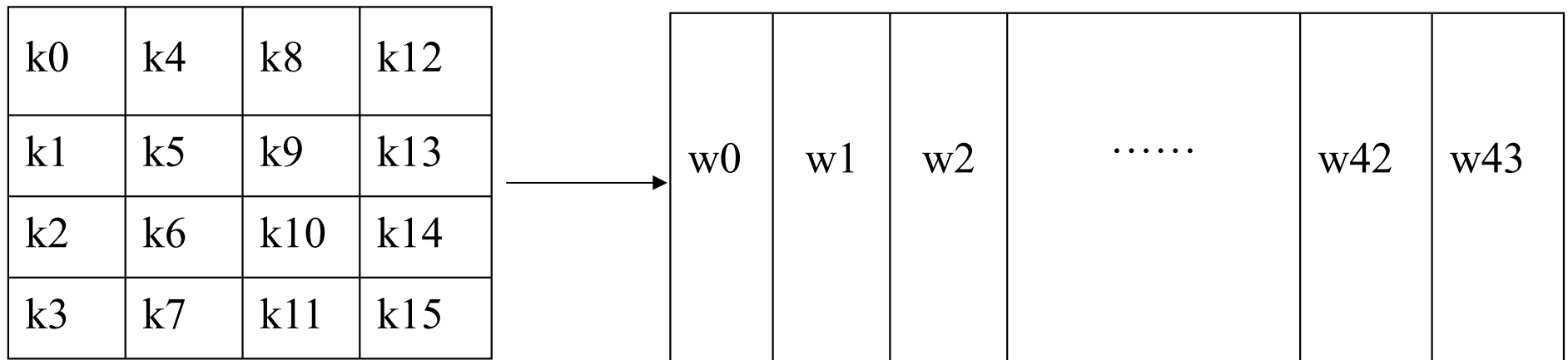
# The AES Cipher

- Assume 128 bit block as input
- Input blocks represented as states at intermediates stages.



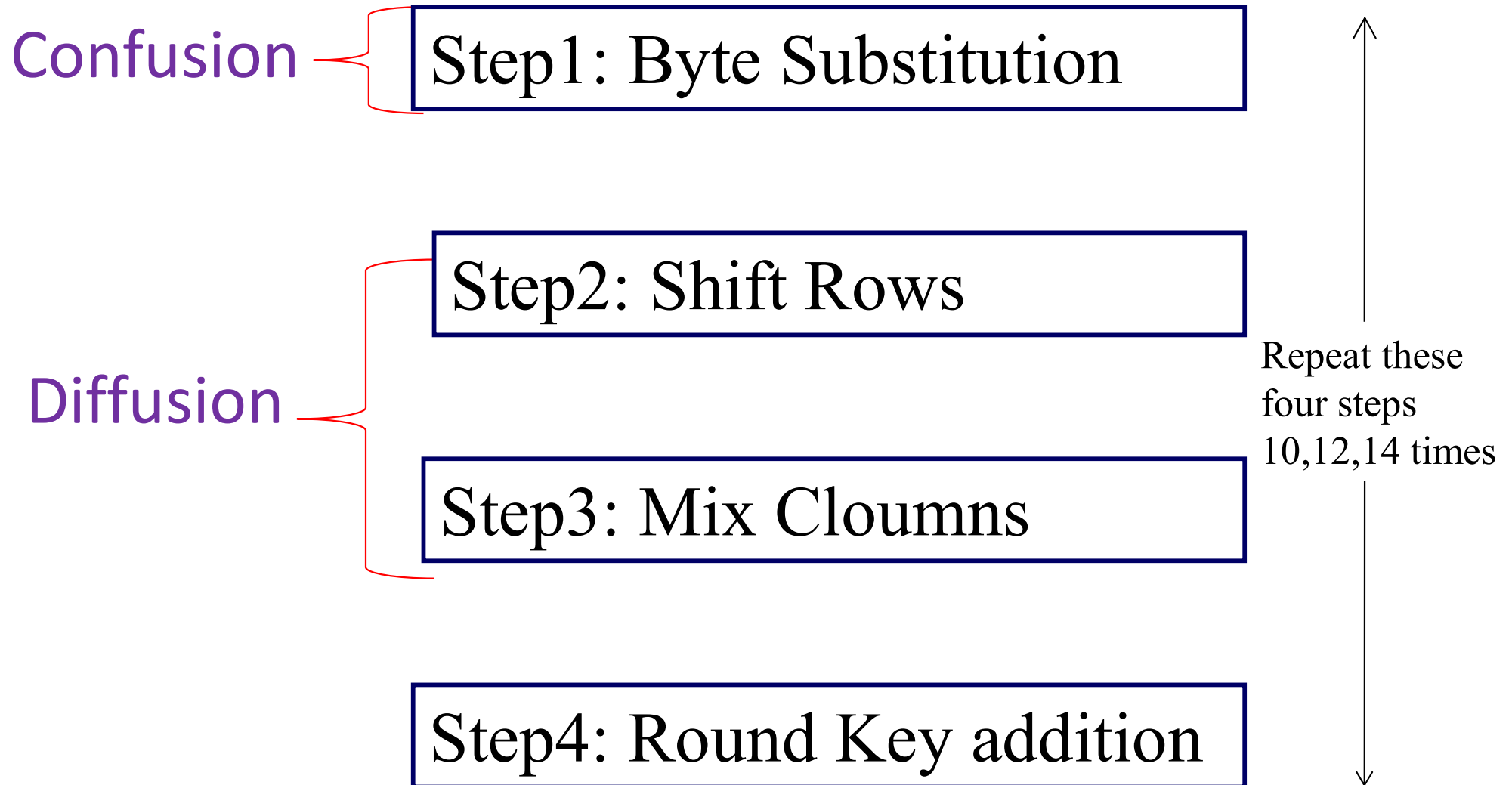
# The AES Cipher

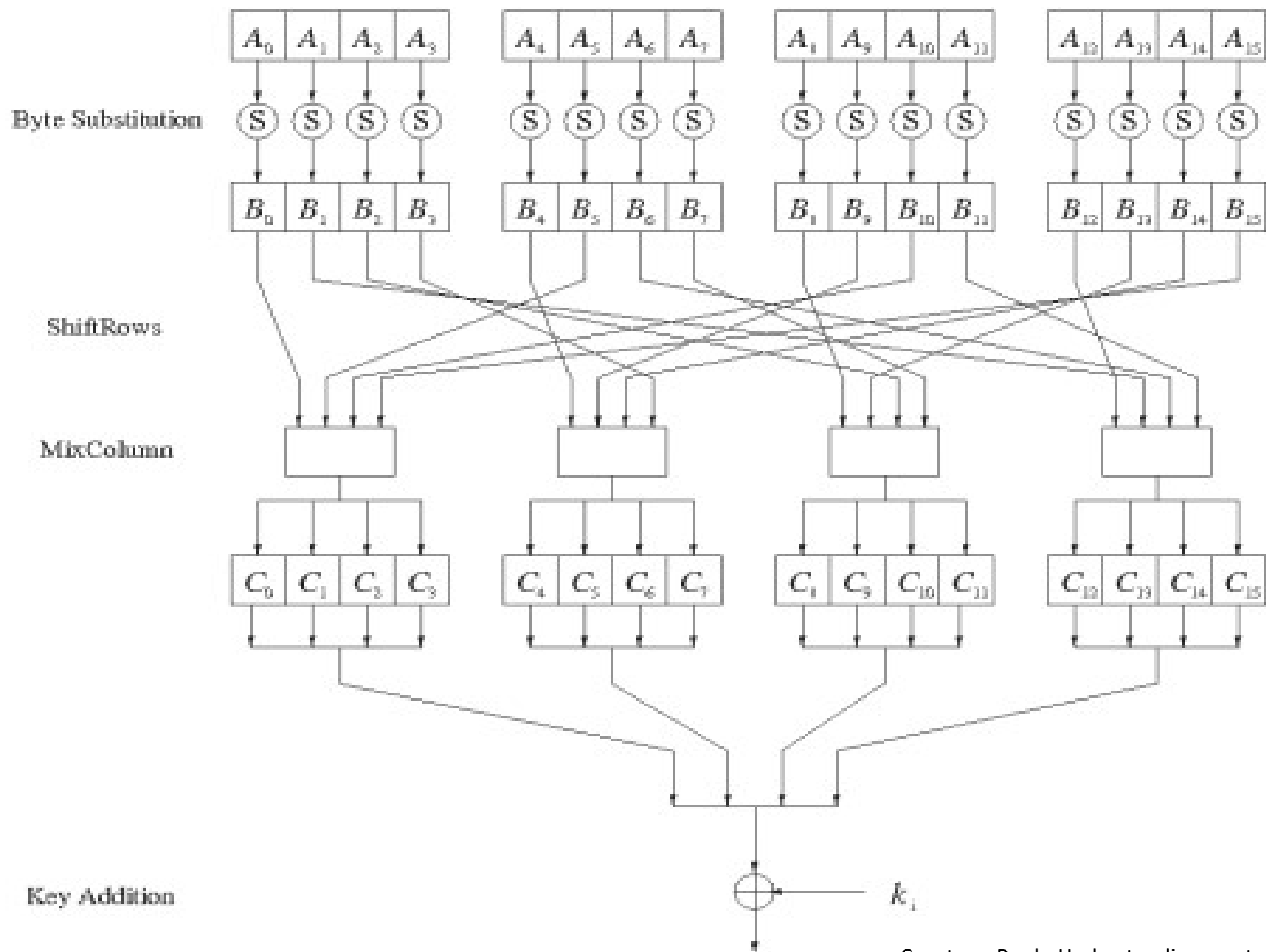
- Key received as input array of 4 rows and  $N_k$  columns.
- $N_k = 4, 6$ , or  $8$ , parameter which depends key size 128, 192 or 256.
- Input key is expanded into an array of 44/52/60 words of 32 bits each depending upon key size.
- 4 different words serve as a key for each round.



$N_k = 4$

# Steps in Rijndael (AES)





Courtesy: Book: Understanding cryptography

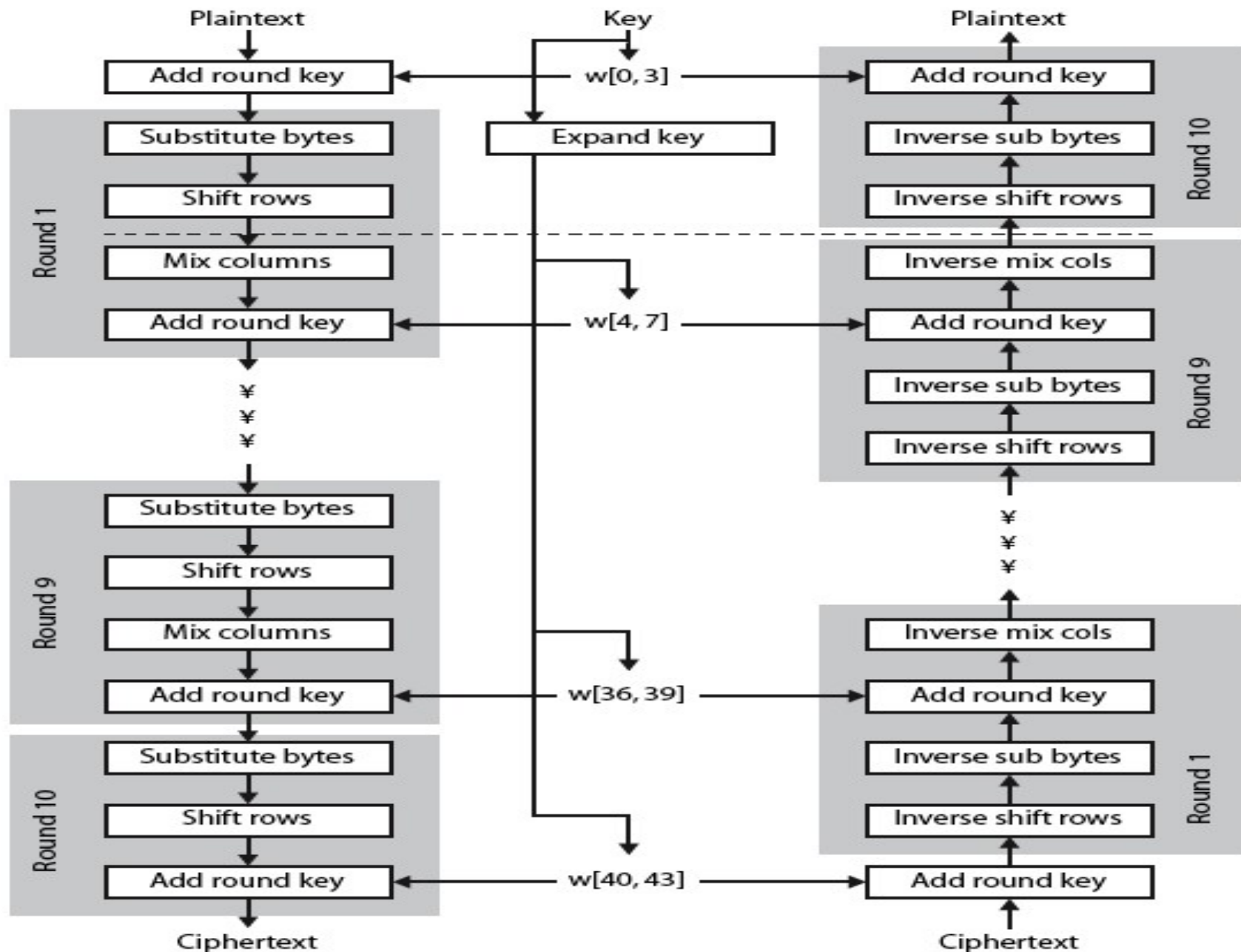
# Steps in Rijndael (AES)

- **SubBytes()** – uses S-box to perform a byte-by-byte substitution of State, making use of arithmetic over  $GF(2^8)$ .
- **ShiftRows()** – processes the State by cyclically shifting the last three rows of the State by different offsets.
- **MixColumns()** – takes all the columns of the State and mixes their data, independently of one another, making use of arithmetic over  $GF(2^8)$ .
- **AddRoundKey()** – round key is added to the State using XOR operation.

# Rijndael (AES)

- Data block of 4 columns of 4 bytes is state and key is expanded to array of words.
- Has 10/12/14 rounds in which state undergoes:
  - Byte substitution (1 S-box used on every byte)
  - Shift rows (permute bytes between groups/columns)
  - Mix columns (subs using matrix multiply of groups)
  - Add round key (XOR state with key material)
  - View as alternating XOR key & scramble data bytes
- Initial XOR key material (1op.) & incomplete last (3ops.) round (so 10 rounds each of 4 ops.)

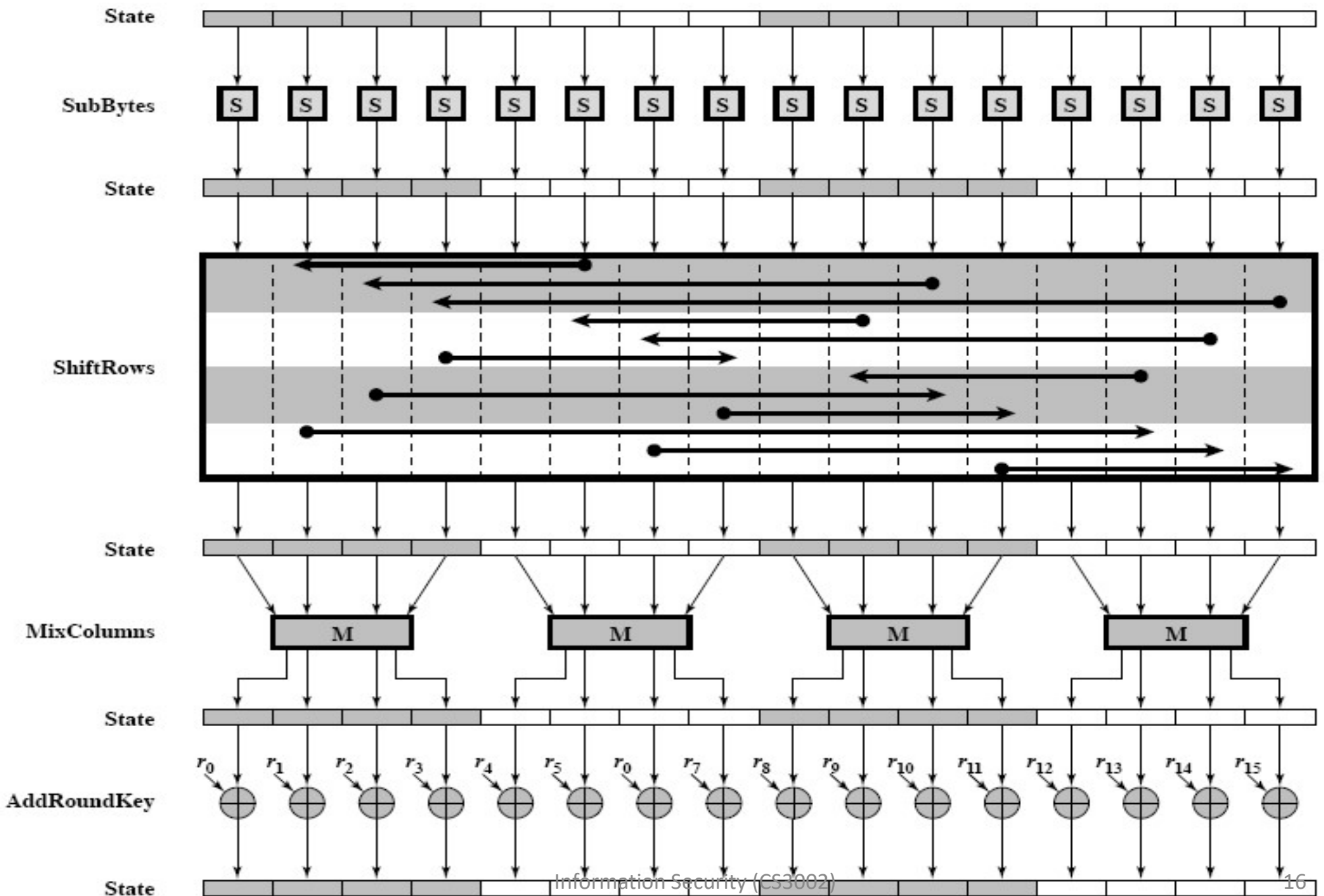
# Rijndael (AES)



(a) Encryption

(b) Decryption

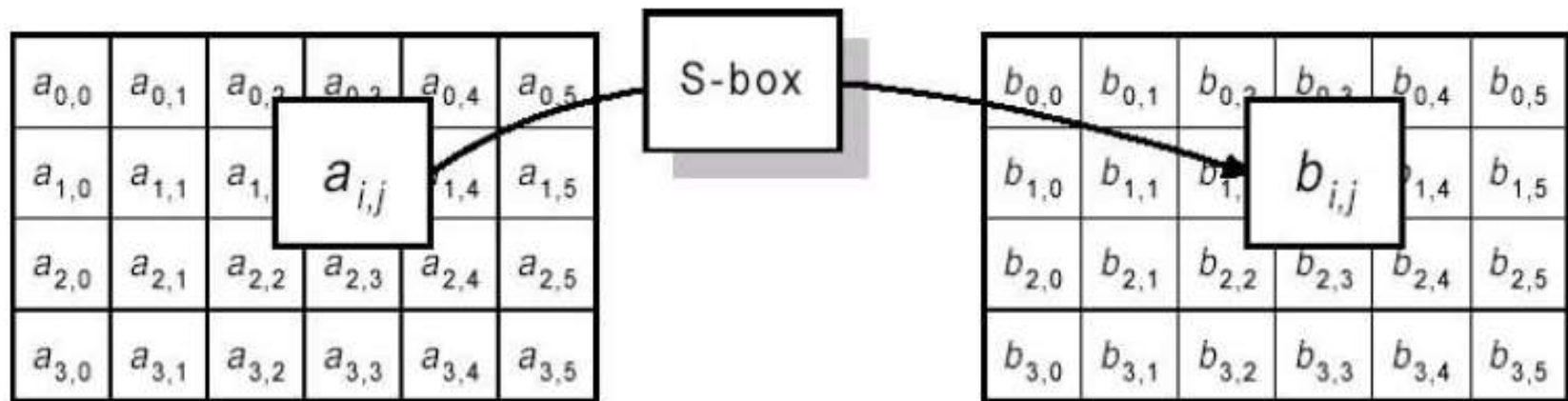
# AES Encryption Round





# 1. Byte Substitution

- Non linear byte substitution
  - Multiplicative inverse in  $GF(2^8)$



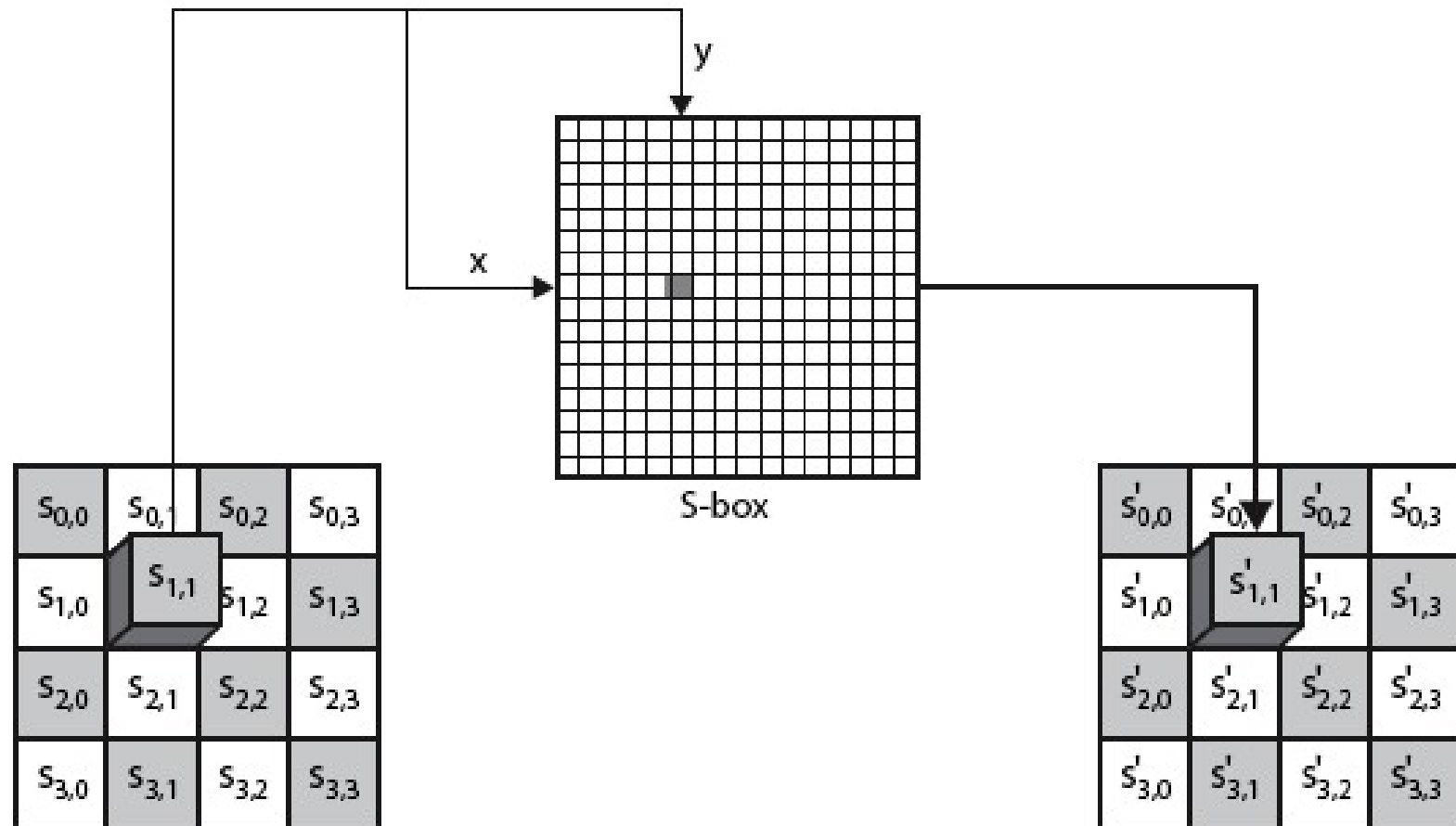
ByteSub acts on individual bytes of the state

**Table 5.2** AES S-Boxes

		<i>y</i>															
		0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
<i>x</i>	0	63	7C	77	7B	F2	6B	6F	C5	30	01	67	2B	FE	D7	AB	76
	1	CA	82	C9	7D	FA	59	47	F0	AD	D4	A2	AF	9C	A4	72	C0
	2	B7	FD	93	26	36	3F	F7	CC	34	A5	E5	F1	71	D8	31	15
	3	04	C7	23	C3	18	96	05	9A	07	12	80	E2	EB	27	B2	75
	4	09	83	2C	1A	1B	6E	5A	A0	52	3B	D6	B3	29	E3	2F	84
	5	53	D1	00	ED	20	FC	B1	5B	6A	CB	BE	39	4A	4C	58	CF
	6	D0	EF	AA	FB	43	4D	33	85	45	F9	02	7F	50	3C	9F	A8
	7	51	A3	40	8F	92	9D	38	F5	BC	B6	DA	21	10	FF	F3	D2
	8	CD	0C	13	EC	5F	97	44	17	C4	A7	7E	3D	64	5D	19	73
	9	60	81	4F	DC	22	2A	90	88	46	EE	B8	14	DE	5E	0B	DB
	A	E0	32	3A	0A	49	06	24	5C	C2	D3	AC	62	91	95	E4	79
	B	E7	C8	37	6D	8D	D5	4E	A9	6C	56	F4	EA	65	7A	AE	08
	C	BA	78	25	2E	1C	A6	B4	C6	E8	DD	74	1F	4B	BD	8B	8A
	D	70	3E	B5	66	48	03	F6	0E	61	35	57	B9	86	C1	1D	9E
	E	E1	F8	98	11	69	D9	8E	94	9B	1E	87	E9	CE	55	28	DF
	F	8C	A1	89	0D	BF	E6	42	68	41	99	2D	0F	B0	54	BB	16

(a) S-box

# Byte Substitution (cont.)



# Byte Substitution (cont.)

EA	04	65	85
83	45	5D	96
5C	33	98	B0
F0	2D	AD	C5

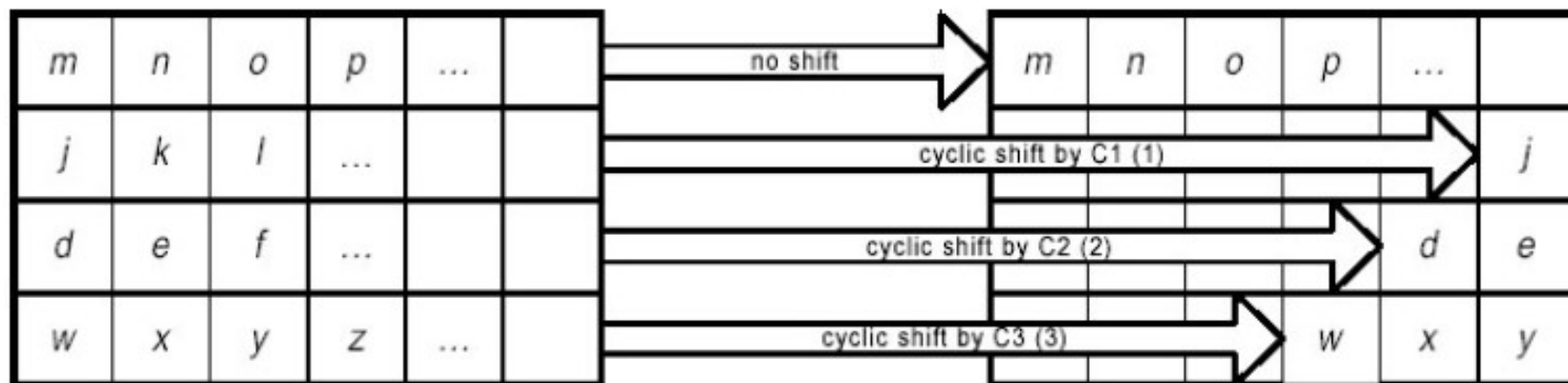
→

87	F2	4D	97
EC	6E	4C	90
4A	C3	46	E7
8C	D8	95	A6

## 2. Shift Rows

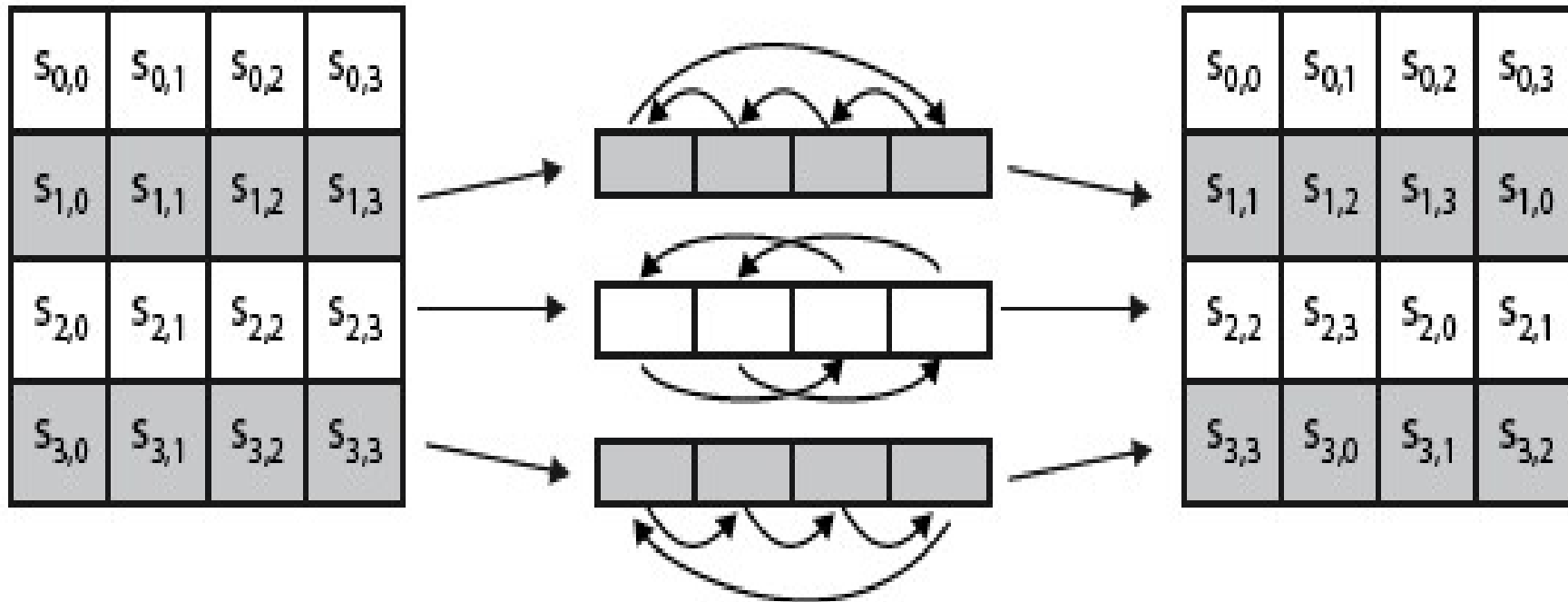
- rotating the rows with different offsets
  - Depending on block size
  - Cyclic Shift left

BlockSize	C1	C2	C3
128	1	2	3
196	1	2	3
256	1	3	4



ShiftRow operates on the rows of the state

# Shift Rows (cont.)



- Decrypt inverts using *shifts to right*
- Since state is processed by columns, this step permutes bytes between the columns

### 3. Mix Columns

- Each column is processed separately.
- Each byte is replaced by a value dependent on all 4 bytes in the column.
- Can **express each column** as 4 equations
  - to derive each new byte in column.
- Decryption requires use of inverse matrix
  - with larger coefficients, hence a little harder

# Mix Columns (cont.)

$$\begin{bmatrix} 02 & 03 & 01 & 01 \\ 01 & 02 & 03 & 01 \\ 01 & 01 & 02 & 03 \\ 03 & 01 & 01 & 02 \end{bmatrix} \begin{bmatrix} s_{0,0} & s_{0,1} & s_{0,2} & s_{0,3} \\ s_{1,0} & s_{1,1} & s_{1,2} & s_{1,3} \\ s_{2,0} & s_{2,1} & s_{2,2} & s_{2,3} \\ s_{3,0} & s_{3,1} & s_{3,2} & s_{3,3} \end{bmatrix} = \begin{bmatrix} s'_{0,0} & s'_{0,1} & s'_{0,2} & s'_{0,3} \\ s'_{1,0} & s'_{1,1} & s'_{1,2} & s'_{1,3} \\ s'_{2,0} & s'_{2,1} & s'_{2,2} & s'_{2,3} \\ s'_{3,0} & s'_{3,1} & s'_{3,2} & s'_{3,3} \end{bmatrix}$$

$$s'_{0,j} = (2 \cdot s_{0,j}) \oplus (3 \cdot s_{1,j}) \oplus s_{2,j} \oplus s_{3,j}$$

$$s'_{1,j} = s_{0,j} \oplus (2 \cdot s_{1,j}) \oplus (3 \cdot s_{2,j}) \oplus s_{3,j}$$

$$s'_{2,j} = s_{0,j} \oplus s_{1,j} \oplus (2 \cdot s_{2,j}) \oplus (3 \cdot s_{3,j})$$

$$s'_{3,j} = (3 \cdot s_{0,j}) \oplus s_{1,j} \oplus s_{2,j} \oplus (2 \cdot s_{3,j})$$

The following is an example of MixColumns:

87	F2	4D	97	→	47	40	A3	4C
6E	4C	90	EC		37	D4	70	9F
46	E7	4A	C3		94	E4	3A	42
A6	8C	D8	95		ED	A5	A6	BC

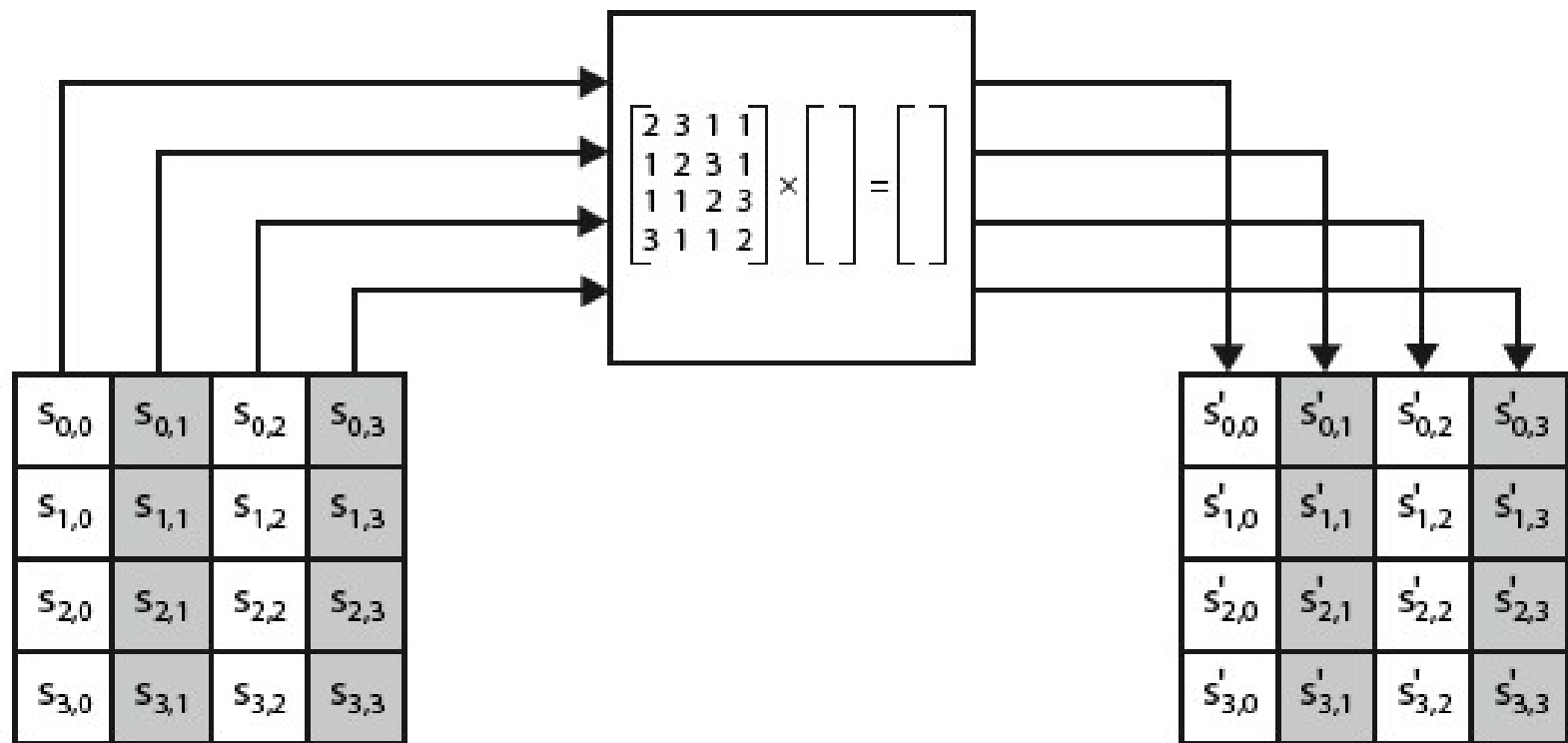


# Mix Columns (cont.)

$$\begin{aligned}
 &(\{02\} \cdot \{87\}) \oplus (\{03\} \cdot \{6E\}) \oplus \{46\} \oplus \{A6\} = \{47\} \\
 &\{87\} \oplus (\{02\} \cdot \{6E\}) \oplus (\{03\} \cdot \{46\}) \oplus \{A6\} = \{37\} \\
 &\{87\} \oplus \{6E\} \oplus (\{02\} \cdot \{46\}) \oplus (\{03\} \cdot \{A6\}) = \{94\} \\
 &(\{03\} \cdot \{87\}) \oplus \{6E\} \oplus \{46\} \oplus (\{02\} \cdot \{A6\}) = \{ED\}
 \end{aligned}$$

Multiplication and Addition is in  $GF(2^8)$ .

# Mix Columns (cont.)



**Fig.5.5 (b)**

## 4. Add Round Key

- Apply the roundkey with a bitwise XOR.
- Size of roundkey equals block size.

$a_{0,0}$	$a_{0,1}$	$a_{0,2}$	$a_{0,3}$	$a_{0,4}$	$a_{0,5}$
$a_{1,0}$	$a_{1,1}$	$a_{1,2}$	$a_{1,3}$	$a_{1,4}$	$a_{1,5}$
$a_{2,0}$	$a_{2,1}$	$a_{2,2}$	$a_{2,3}$	$a_{2,4}$	$a_{2,5}$
$a_{3,0}$	$a_{3,1}$	$a_{3,2}$	$a_{3,3}$	$a_{3,4}$	$a_{3,5}$

 $\oplus$ 

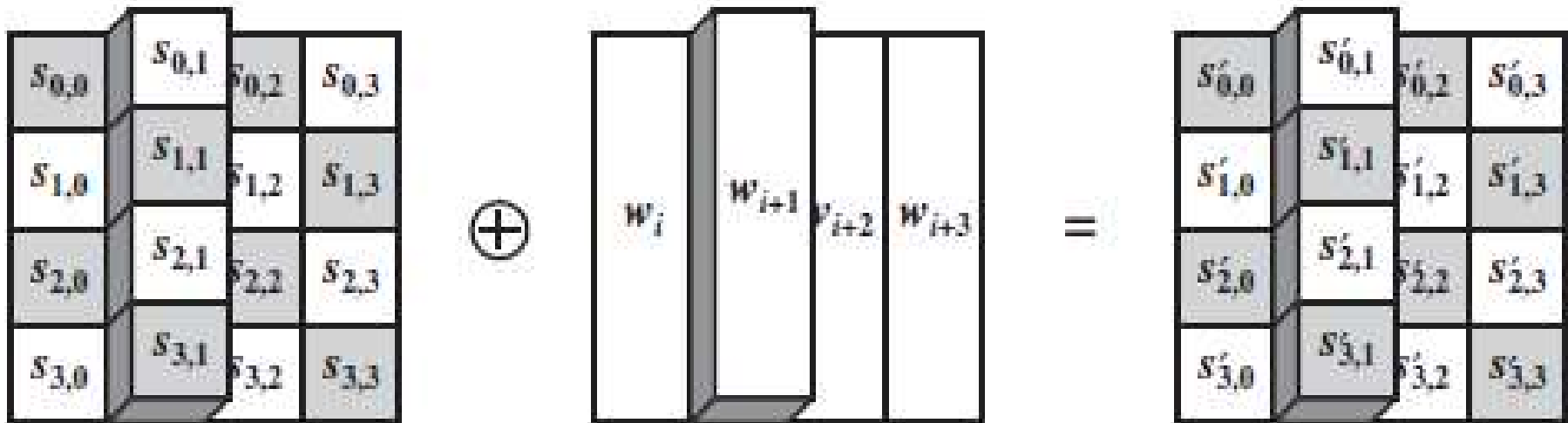
$k_{0,0}$	$k_{0,1}$	$k_{0,2}$	$k_{0,3}$	$k_{0,4}$	$k_{0,5}$
$k_{1,0}$	$k_{1,1}$	$k_{1,2}$	$k_{1,3}$	$k_{1,4}$	$k_{1,5}$
$k_{2,0}$	$k_{2,1}$	$k_{2,2}$	$k_{2,3}$	$k_{2,4}$	$k_{2,5}$
$k_{3,0}$	$k_{3,1}$	$k_{3,2}$	$k_{3,3}$	$k_{3,4}$	$k_{3,5}$

 $=$ 

$b_{0,0}$	$b_{0,1}$	$b_{0,2}$	$b_{0,3}$	$b_{0,4}$	$b_{0,5}$
$b_{1,0}$	$b_{1,1}$	$b_{1,2}$	$b_{1,3}$	$b_{1,4}$	$b_{1,5}$
$b_{2,0}$	$b_{2,1}$	$b_{2,2}$	$b_{2,3}$	$b_{2,4}$	$b_{2,5}$
$b_{3,0}$	$b_{3,1}$	$b_{3,2}$	$b_{3,3}$	$b_{3,4}$	$b_{3,5}$

Bitwise round key addition

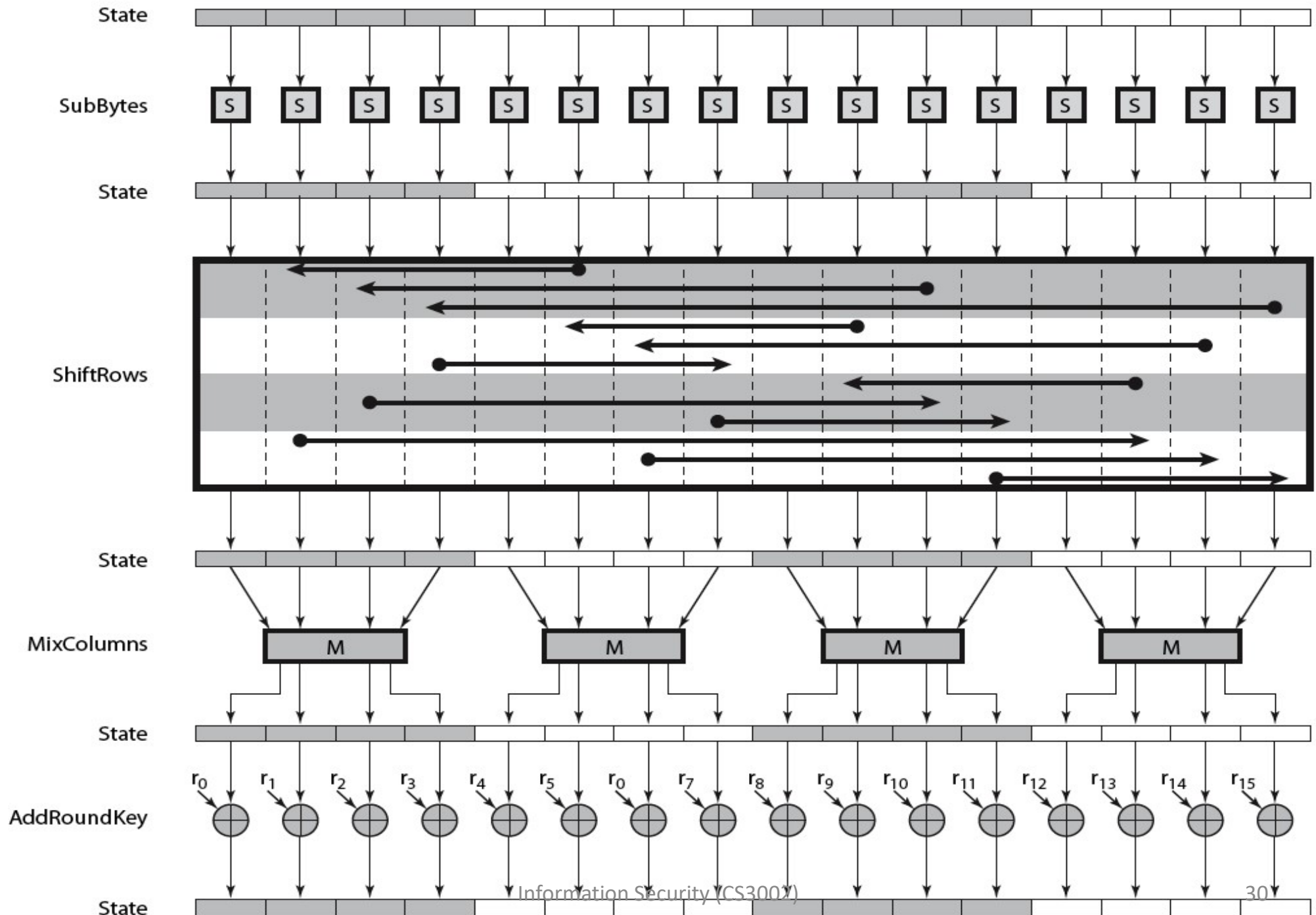
# Add Round Key (cont.)



# Add Round Key (cont.)

- Inverse for decryption identical.
  - Since XOR own inverse, with reversed keys.
- Designed to be as simple as possible.
  - A form of **Vernam cipher** on expanded key.
  - Requires other stages for complexity / security.

# AES Round



# AES Key Expansion

$N_k=4,6,8$

$N_r=10,12,14$

# AES Key Expansion

- Takes as input a  $N_k$  word key and produces a linear array of  $N_k * (N_r+1)$  words.
- Expanded key provide a  $N_k$  word round key for the initial AddRoundKey() stage and for each of the  $N_r$  rounds of the cipher.
- The key is first copied into the first  $N_k$  words, the remainder of the expanded key is filled  $N_k$  words at a time.



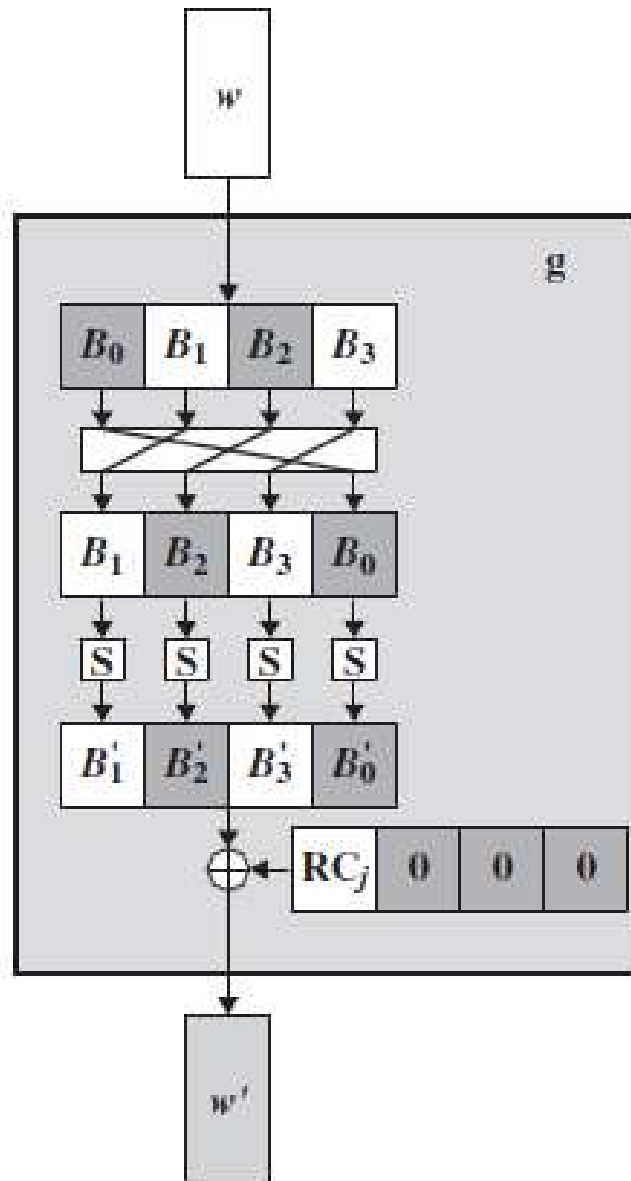
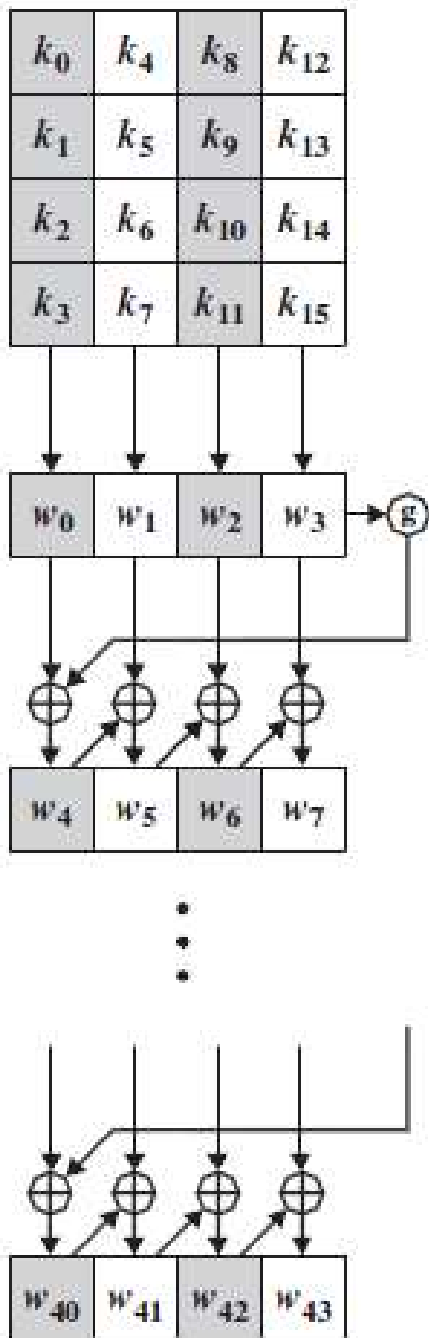
# Key Expansion (cont.)

## Pseudo Code 16 byte key

```
KeyExpansion(byte key[16], word w[44])
{
    word temp;
    for (i = 0; i < 4; i++) w[i] = (key[4*i], key[4*i+1],
                                     key[4*i+2], key[4*i+3]);
    for (i = 4; i < 44; i++)
    {
        temp = w[i-1];
        if ( i mod 4 = 0 )    temp = SubWord(RotWord(temp))
                               XOR Rcon[i/4];
        w[i] = w[i-4] XOR temp;
    }
}
```

## Key Expansion (cont.)

- **RotWord** performs a one byte circular left shift on a word. For example:  
$$\text{RotWord}[b_0, b_1, b_2, b_3] = [b_1, b_2, b_3, b_0]$$
- **SubWord** performs a byte substitution on each byte of input word using the S-box (similar to SubBytes() )
- **SubWord(RotWord(temp) )** is XORed with Rcon[j] – the round constant.



(b) Function  $g$

j	1	2	3	4	5	6	7	8	9	10
RC[j]	01	02	04	08	10	20	40	80	1B	36

Nk=4,6,8

Nr=10,12,14

# Pseudo Code for All Key Sizes

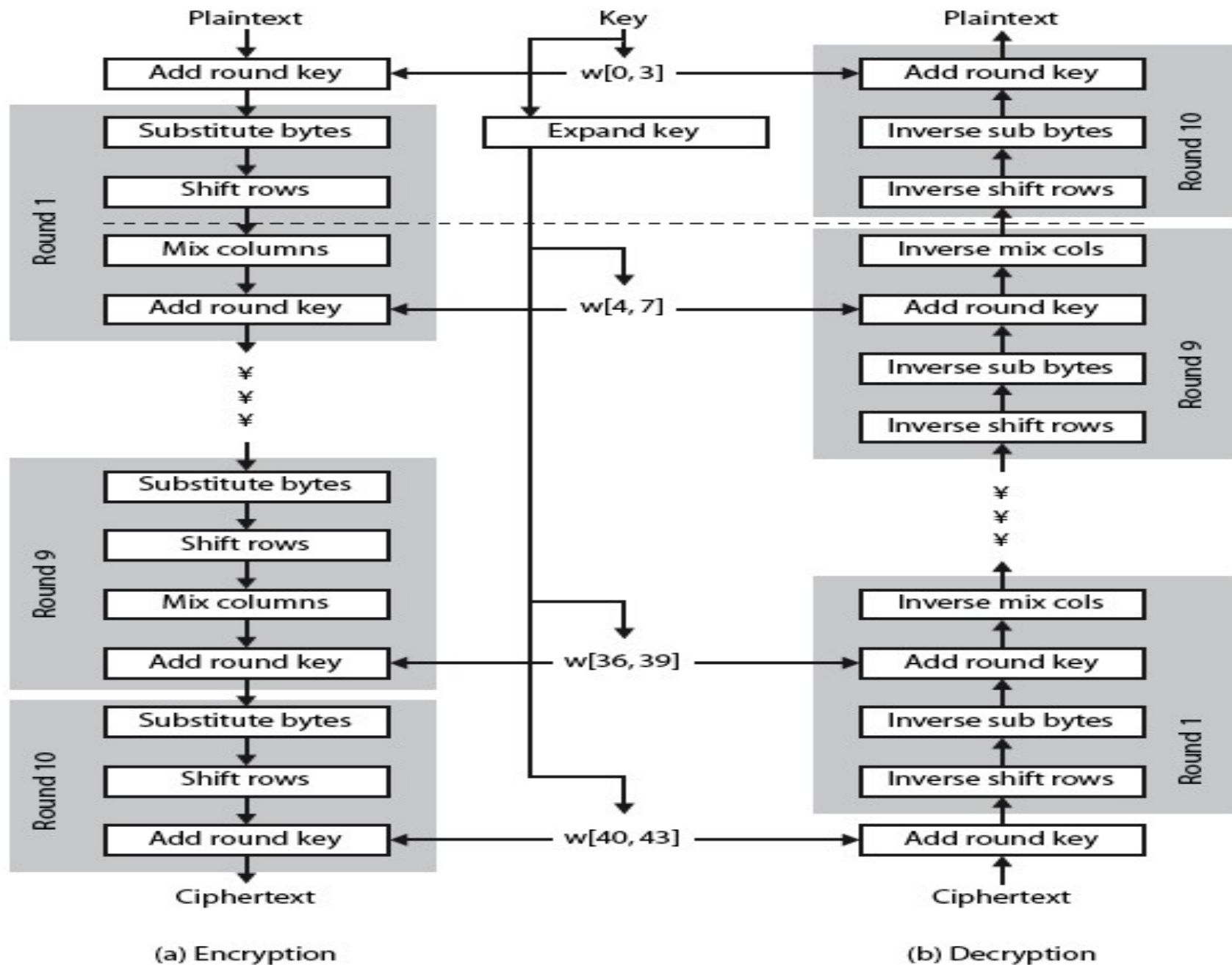
```
KeyExpansion(int* Key[4*Nk], int* EKey[Nb*(Nr+1)])
{
    for(i = 0; i < Nk; i++)
        EKey[i] = (Key[4*i], Key[4*i+1], Key[4*i+2], Key[4*i+3]);
    for(i = Nk; i < Nb * (Nr + 1); i++)
    {
        temp = EKey[i - 1];
        if (i % Nk == 0)
            temp = SubByte(RotByte(temp)) ^ Rcon[i / Nk];
        EKey[i] = EKey[i - Nk] ^ temp;
    }
}
```

# Decryption of AES (Self-Study)

# AES Decryption

- AES decryption is not identical to encryption since steps done in reverse.
- But can define an equivalent inverse cipher with steps as for encryption.
  - but using inverses of each step
  - with a different key schedule
- Works since result is unchanged when
  - swap byte substitution & shift rows
  - swap mix columns & add (tweaked) round key

# Rijndael



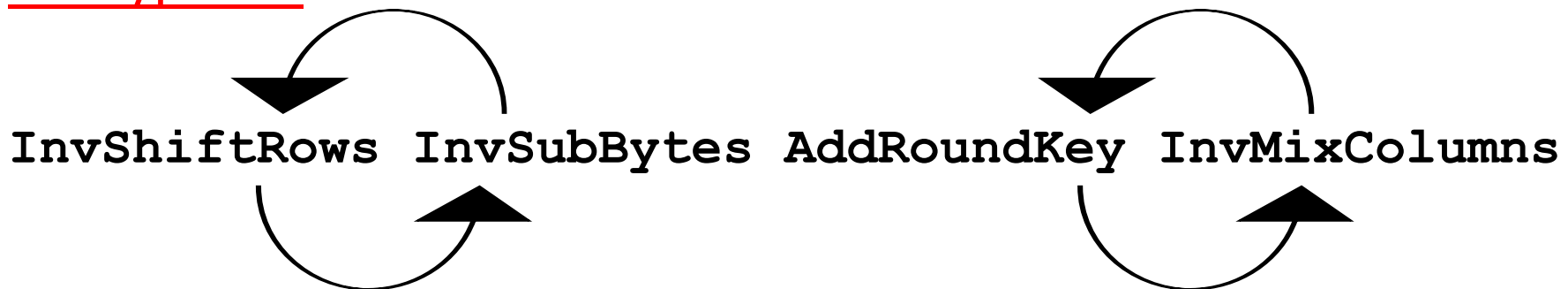
# Equivalent Inverse Cipher

- The original sequence is :

## Encryption:

SubBytes ShiftRows MixColumns AddRoundKey

## Decryption:



- Thus **InvShiftRows** needs to be interchanged **with** **InvSubBytes** **and** **AddRoundKey** **with** **InvMixColumns**.



# Equivalent Inverse Cipher

- **InvShiftRows** – Affects sequence of bytes but does not alter byte content and does not depend on the byte content to perform transformation.
- **InvSubBytes** – Affects content of bytes but does not alter byte sequence and does not depend on the byte sequence to perform transformation.
- Thus **InvShiftRows** and **InvSubBytes** can be interchanged. For given state  $S$ ,

**InvShiftRows (InvSubBytes (S) )**

**=**

**InvSubBytes (InvShiftRows (S) )**

# Equivalent Inverse Cipher

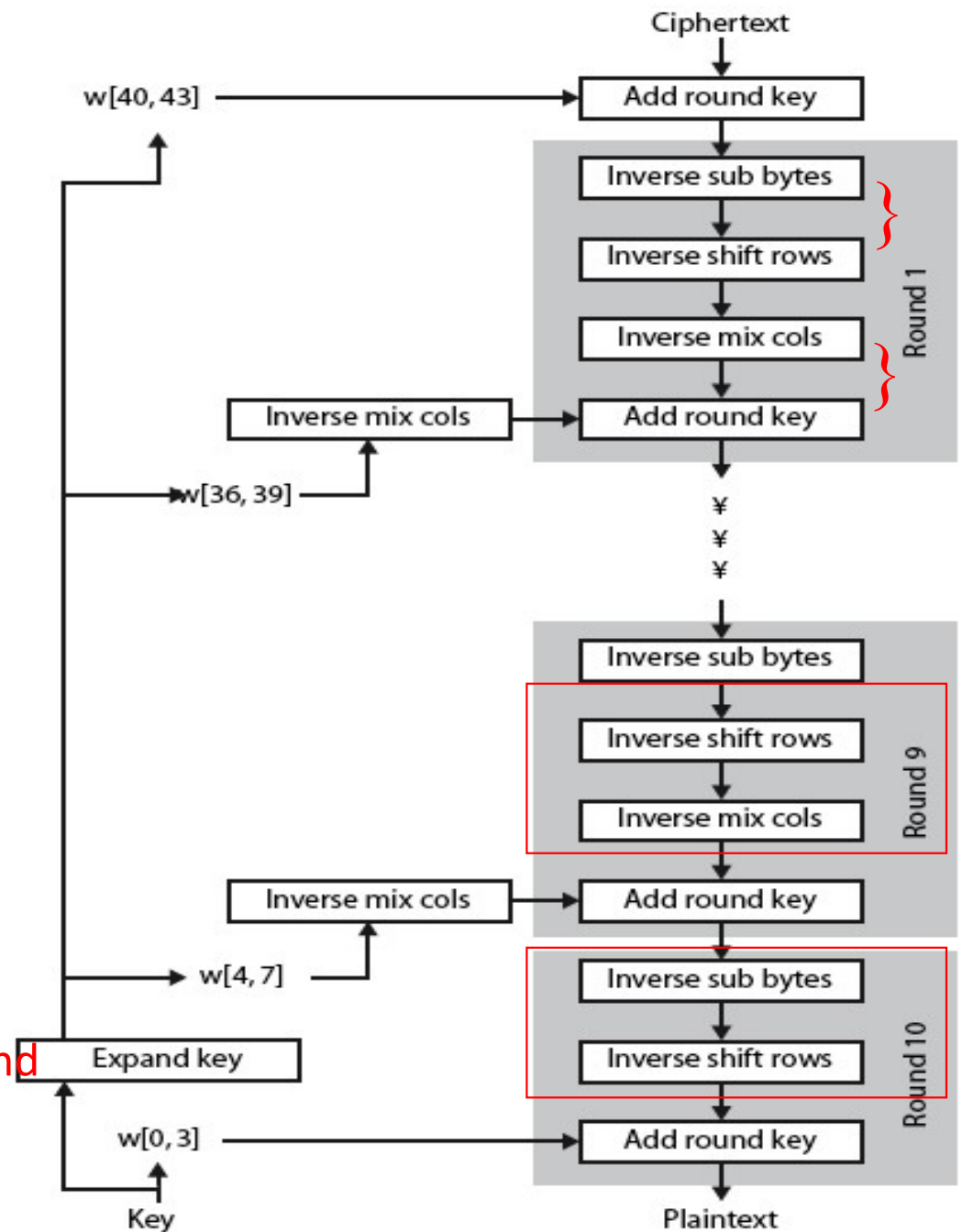
- If key is viewed as sequence of words then both **AddRoundKey** and **InvMixColumns** operate on state one column at a time.
- These operations are linear with respect to the column input: State – S and key - w
$$\text{InvMixColumns}(S \text{ XOR } w) = [\text{InvMixColumns}(S)] \text{ XOR } \text{InvMixColumns}(w)$$

# Equivalent Inverse Cipher

$$\begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} y0 \text{ XOR } k0 \\ y1 \text{ XOR } k1 \\ y2 \text{ XOR } k2 \\ y3 \text{ XOR } k3 \end{bmatrix} = \begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} y0 \\ y1 \\ y2 \\ y3 \end{bmatrix} + \begin{bmatrix} 0E & 0B & 0D & 09 \\ 09 & 0E & 0B & 0D \\ 0D & 09 & 0E & 0B \\ 0B & 0D & 09 & 0E \end{bmatrix} \begin{bmatrix} y0 \\ y1 \\ y2 \\ y3 \end{bmatrix}$$

- Thus **InvMixColumns** and **AddRoundKey** can be interchanged.

# AES Decryption



- `InvShiftRows`  $\leftrightarrow$  `InvSubBytes` and
- `AddRoundKey`  $\leftrightarrow$  `InvMixColumns`.

# Two Decryptions compared

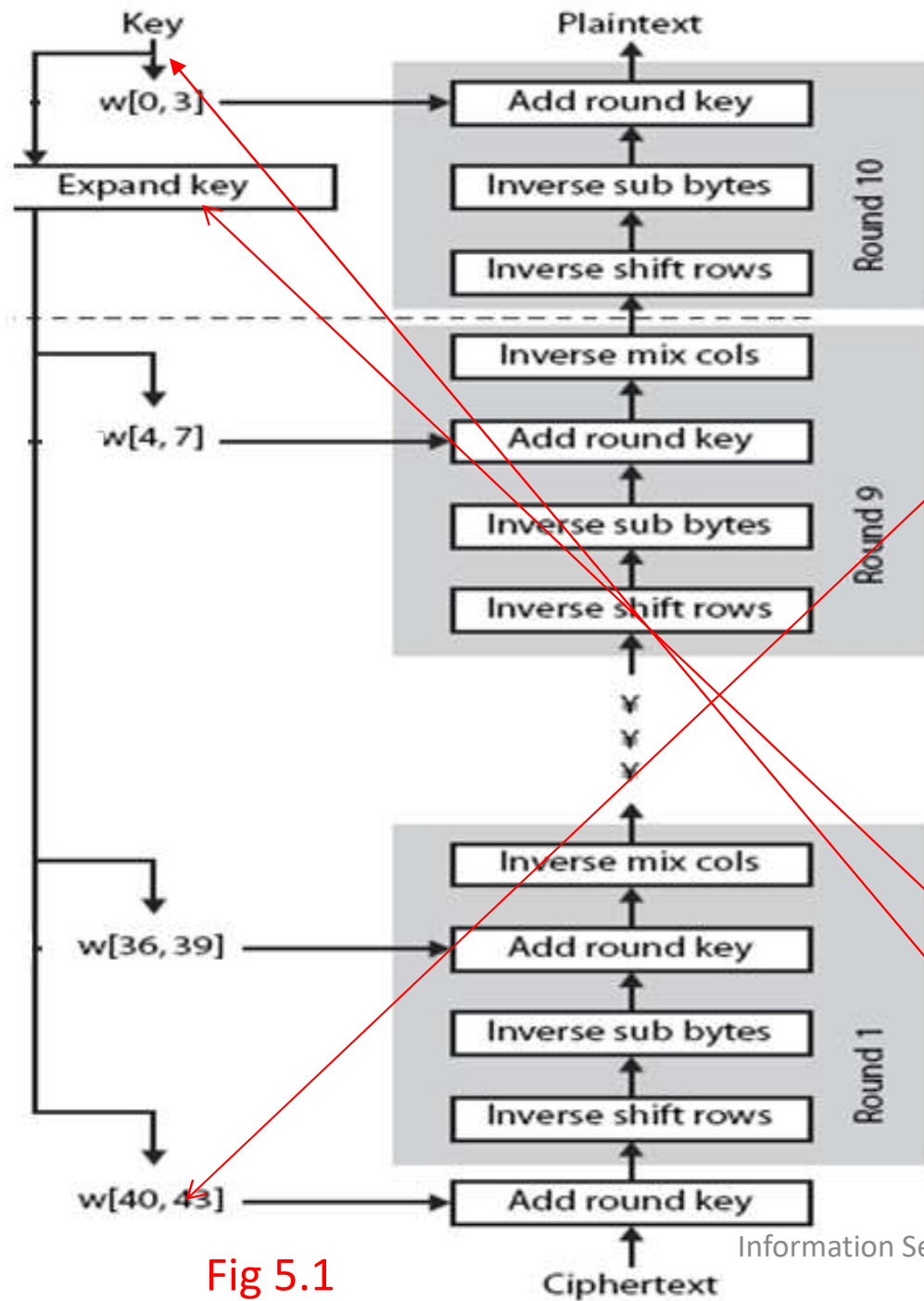


Fig 5.1

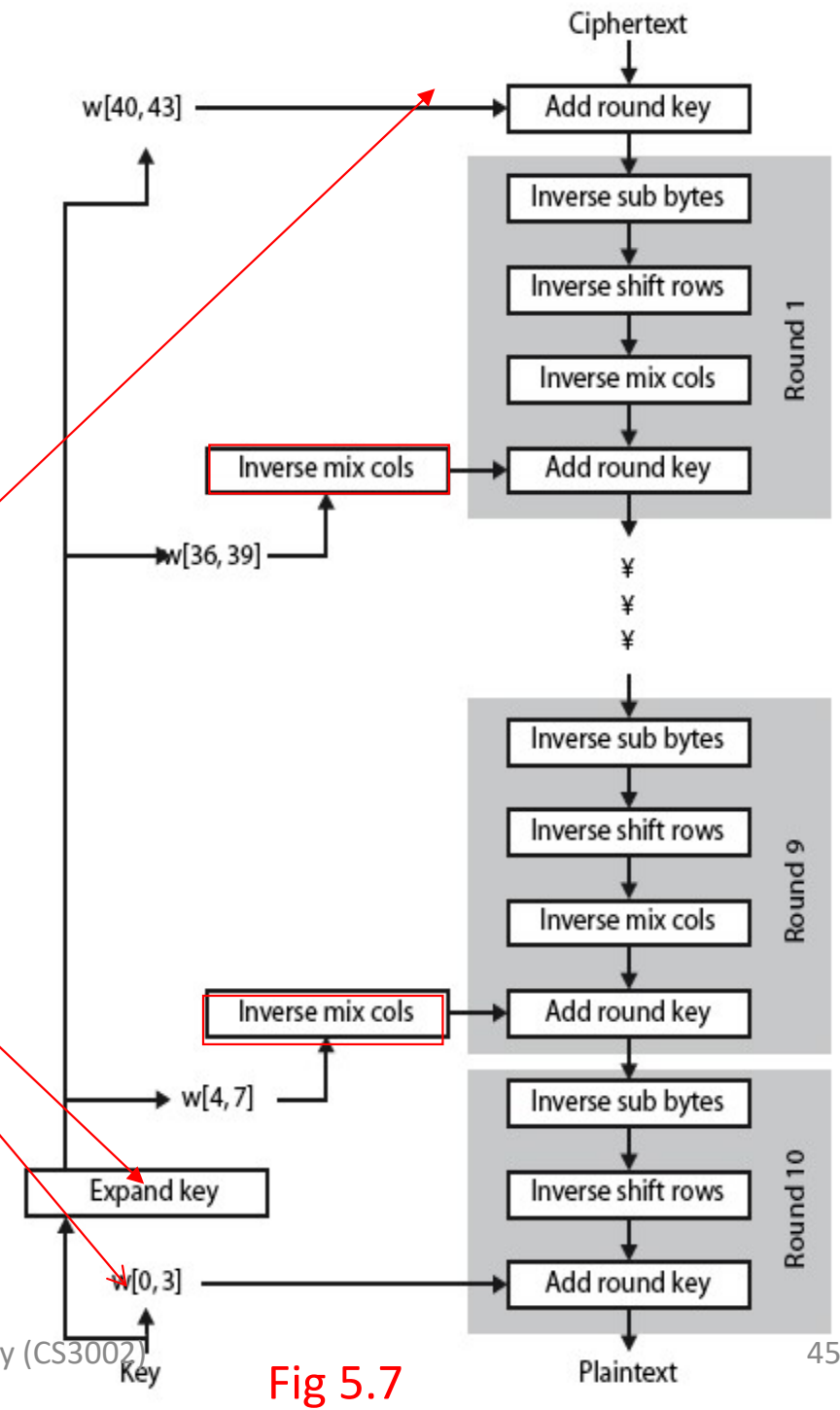


Fig 5.7

# Equivalent Inverse Cipher Advantages

- AS AES decryption cipher is not identical to the encryption cipher (Fig 5.1)
- Disadvantage – Two separate software or hardware modules are required if performing both encryption and decryption.
- So equivalent version of the decryption algorithm that has the same structure ( the same sequence of transformations) as the encryption algorithm, can use same software (Fig 5.7)