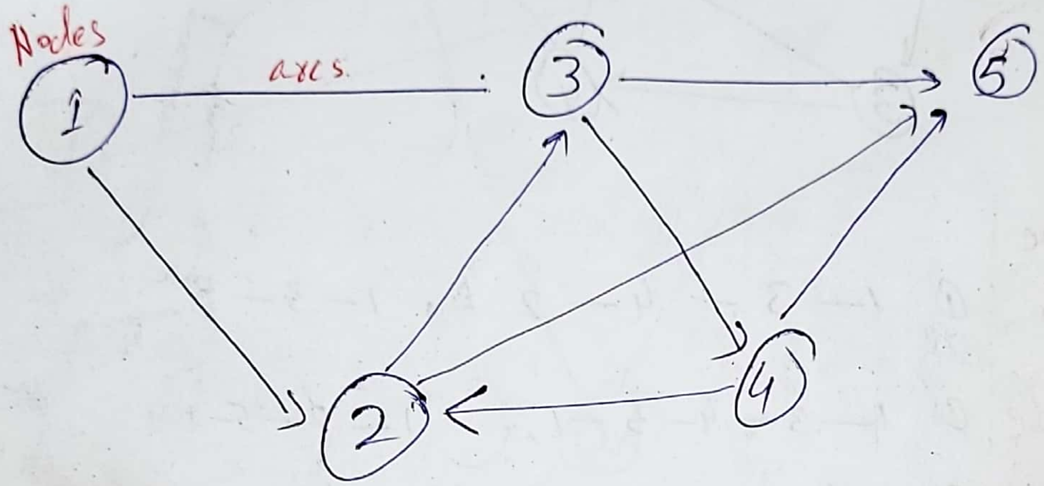
CH #6

Network Model

Sec 6.1 } Q1—12
Sec 6.2
Sec 6.3 — 6.3.1 → 13—17; 6.3.2 → 18—24; 6.3.3 → 25—27
Sec 6.3 + Floyd's Algorithm.
Sec 6.4 (Maximal Flow) 6.28—6.38

- Determine the Shortest root b/w two cities.



i) Directed Network → Directed Arcs.

ii) Path: set of arc Joining two distinct nodes

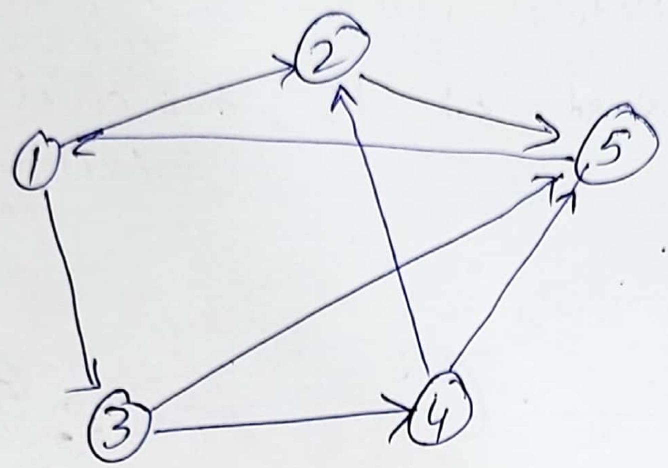
iii) Cycle or loop: if it Connects a node back to itself through other nodes.

iv) Connected: if Every Two distinct node are linked by at least one path

v) Tree: cycle-free connected network having subset of all nodes.

vi) Spanning Tree: Links all the nodes of the network $\boxed{n^{n-2}}$ Spanning Tree.

Problem 6-1



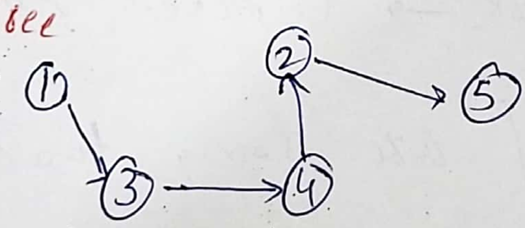
Determine

i) Path (i) 1 — 3 — 4 — 2 , 1 — 3 — 5.

ii) Cycle (i) 1 — 3 — 4 — 5 — 1, 1 — 2 — 5 — 1

iii) Tree (iii) 1 — 3 — 4 — 5 , 1 — 2 — 5

iv) Spanning Tree

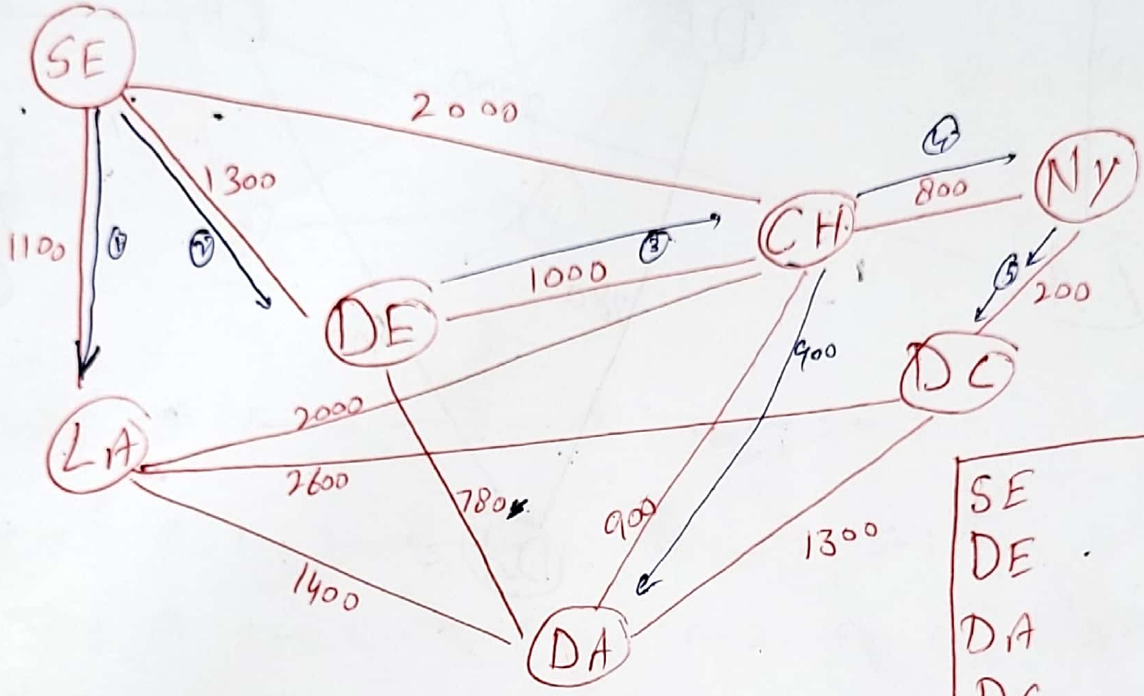


Sec 6.2

Minimal Spanning Tree Algorithm.

Read pg 250 sec 6.2

Problem 6-9 L.A must Link with Chicago (CH

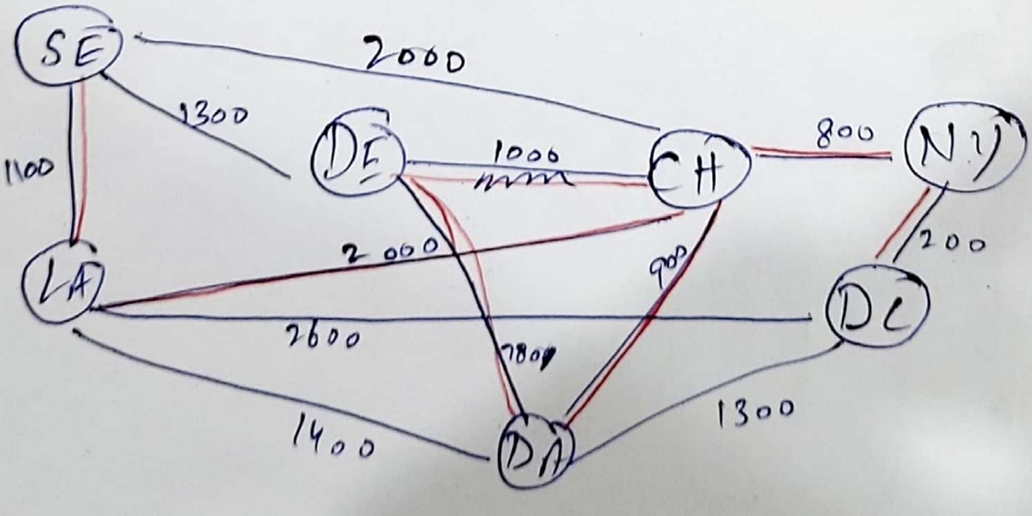


SE
DE
DA
DC
NY

1100 + 1300 + 1000 + 800 + 200 + 900

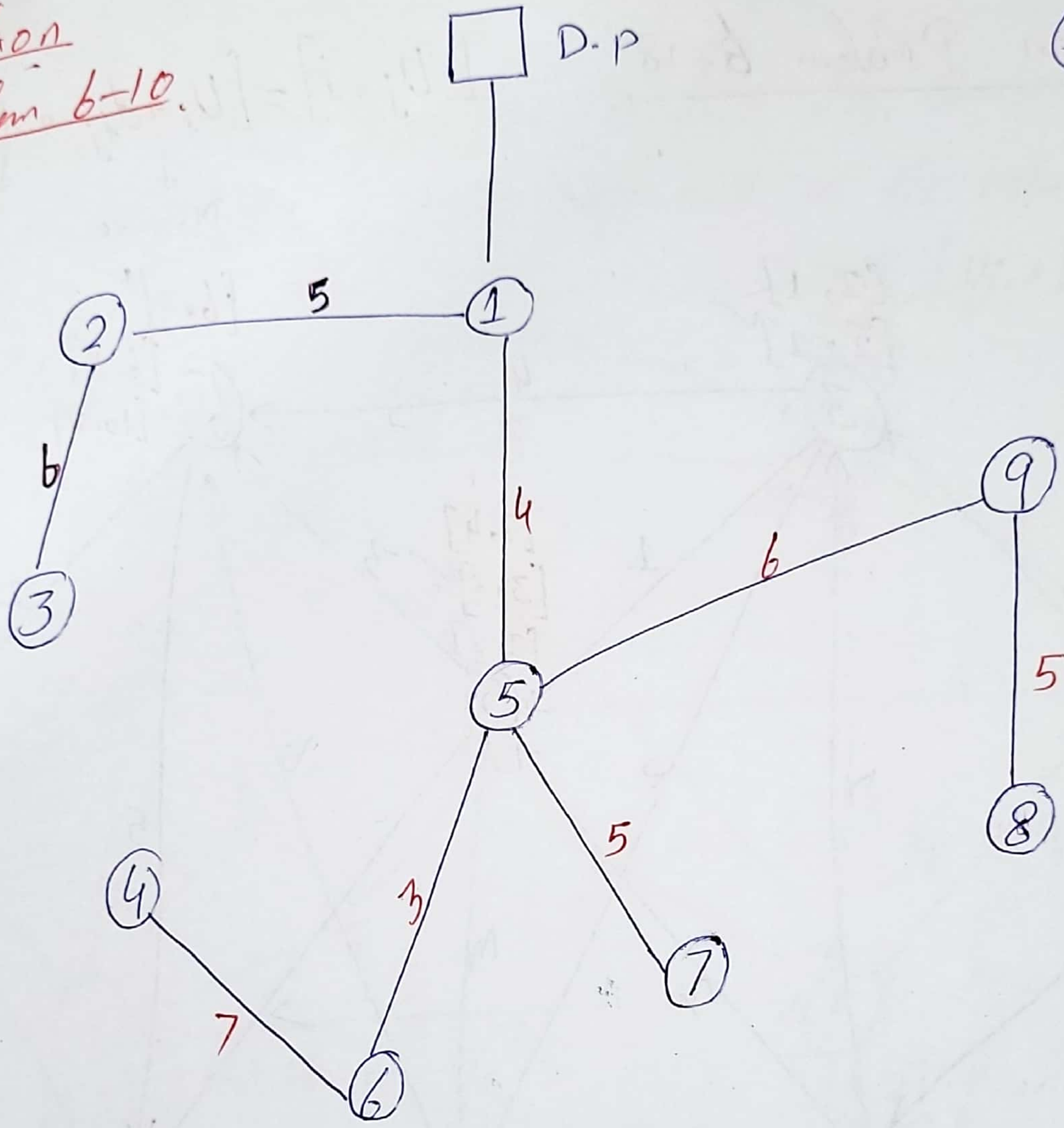
= 5300

L.A must Link with CH directly



Minimum Length = 1100 + 2000 + 900 + 780 + 800 + 200
= ~~580~~ 5780 miles

Solution
Problem 6-10.



Minimum Pipeline network length = 5 + 6 + 4 + 6 + 5 + 5 + 3 + 7

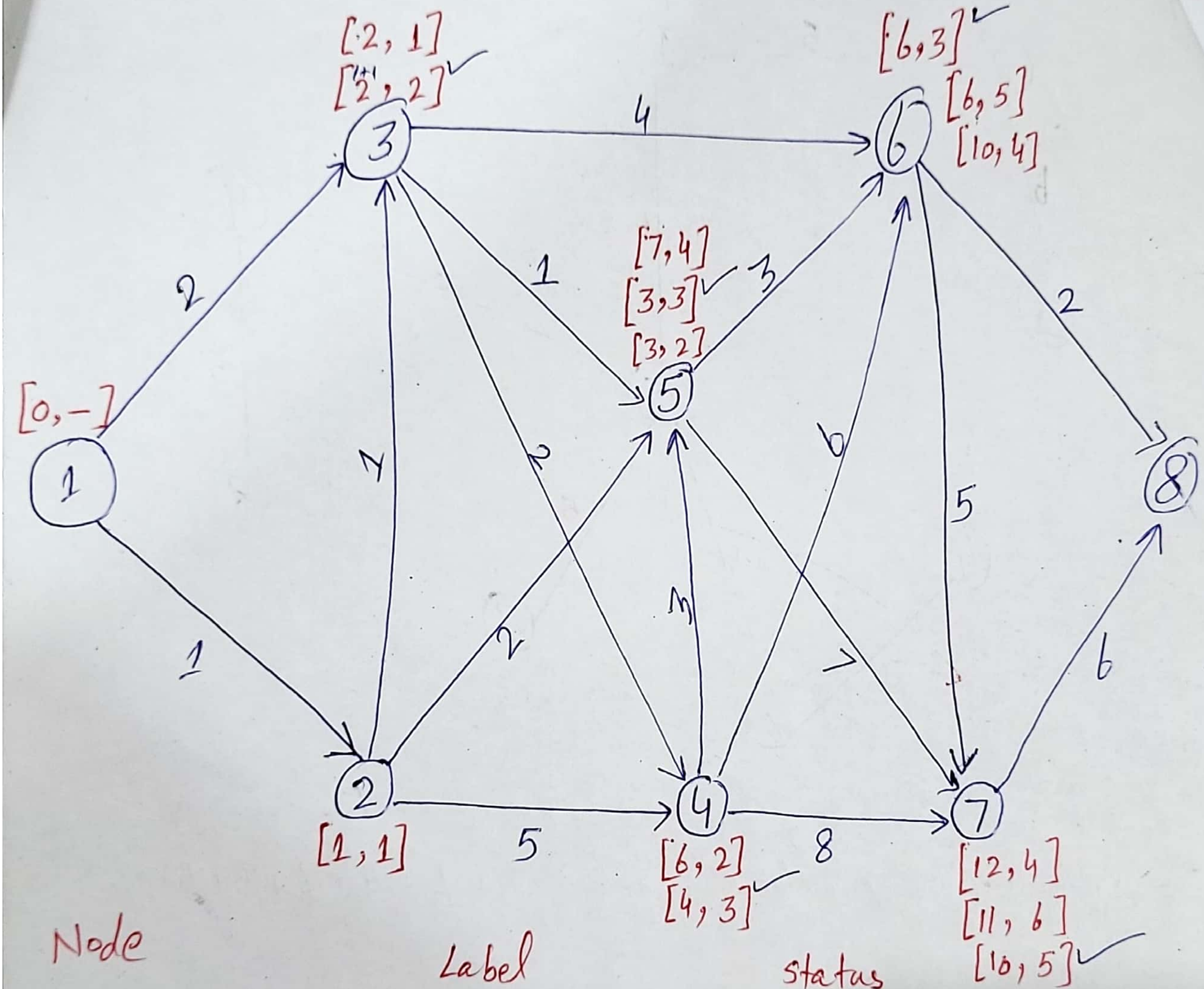
= 41 mile

Dijkstra's Algorithm.

Solution Problem 6-18

$$[U_j , i] = [U_i + d_{ij} , i]$$

$\downarrow$ Distance $\quad$ $\downarrow$ Node



Node	Label	Status
1	[0, –]	Permanent
2	[1, 1]	//
3	[1+1, 2], [2, 1]	[2, 2]
4	[6, 2], [4, 3]	[4, 3]
5	[3, 2], [3, 3], [7, 4]	[3, 3]
6	[6, 3], [6, 5], [10, 4]	[6, 3]
7	[12, 4], [11, 6], [10, 5]	[10, 5]

Total Distence = 10

Alternative routes

i) 1 → 3 → 5 → 7

ii) 1 → 2 → 3 → 5 → 7

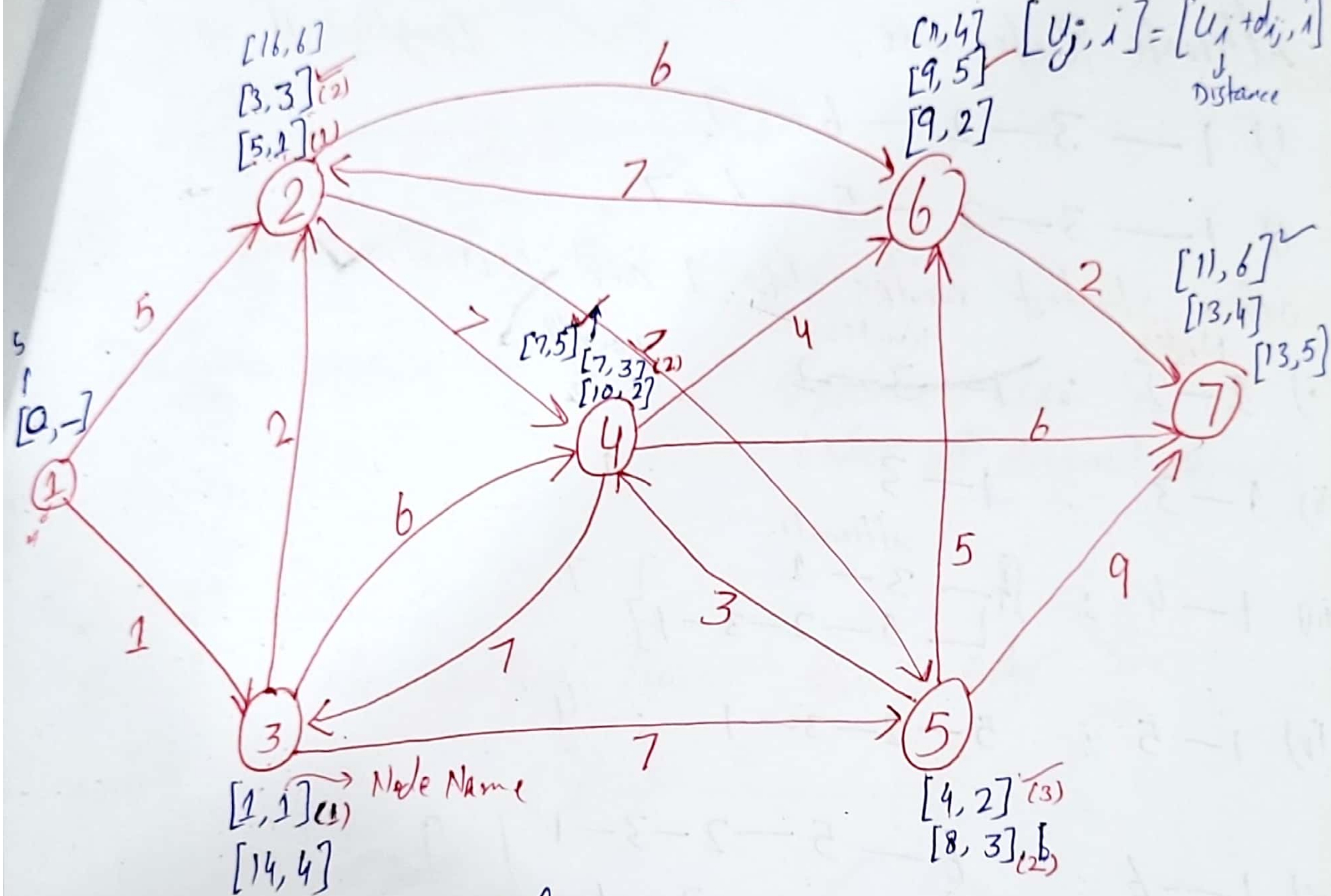
iii) 1 → 2 → 5 → 7

$O(|E| + |V|\log|V|)$

Problem 6-19 Dijkstra's Algorithm.

Use Dijkstra's Algorithm to find the shortest route between node 1 and every node in the network.

$[u_j, i] = [u_i + d_{ij}, i]$



Nodes	Label Distance	Permanent Distance
1	[0,—] → Starting Point, where you start.	[0,–]
2	[5,1], [3,3], [16,6]	[3,3]
3	[1,1], [14,4]	[1,1]
4	[10,2], [7,3], [7,5]	[7,5] . [7,3]
5	[4,2], [8,3]	[4,2]
6	[9,2], [9,5], [11,4]	[9,5], [9,2]
7	[13,5], [13,4], [11,6].	[11,6]

Route from 1 → 7

7 ⟷ 6 ⟷ 5 ⟷ 2 → 3 → 1
6 → 2 → 3 → 1

Alternate Routes are. Length = 11

i) 1 — 3 — 2 — 6 — 7

ii) 1 — 3 — 2 — 5 — 6 — 7

Other shortest routes b/w 1 and each other node

Nodes		Routes	Length
i) 1 — 2	:	~~1 — 3 — 2~~	3
ii) 1 — 3	:	1 — 3	1
iii) 1 — 4	:	4 — 3 — 1 (Alternate: 4 — 5 — 2 — 3 — 1)	7
iv) 1 — 5	:	5 — 2 — 3 — 1	4
v) 1 — 6	:	6 — 5 — 2 — 3 — 1 / 6 — 2 — 3 — 1	9

Floyds

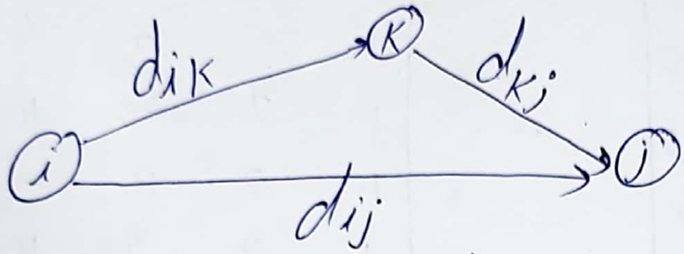
Determine

work

Floyd's Algorithm: Pg 258 (Please read it).

Determine shortest route b/w Any Two nodes in the work.

All Pair shortest Path.



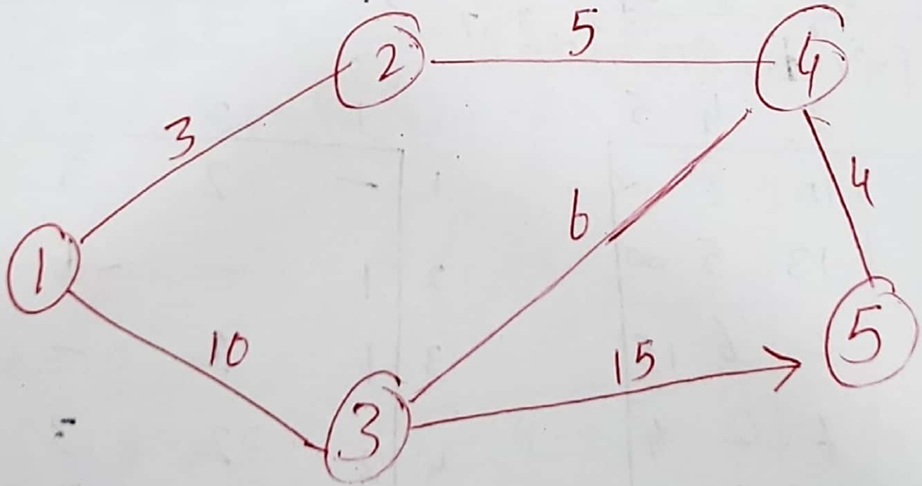
Such that $d_{ik} + d_{kj} < d_{ij}$

Distance Matrix D_K.

Sequence Matrix S_K.
(Block all diagonal elements)

Example 6.3-5

Find the shortest Route b/w every Two nodes



Direct

Direct Distance Matrix

D_0

Sequence Matrix

$$D_0^0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 10 & \infty & \infty \\ 2 & 3 & 0 & \infty & 5 & \infty \\ 3 & 10 & \infty & 0 & 6 & 15 \\ 4 & \infty & 5 & 6 & 0 & 4 \\ 5 & \infty & \infty & \infty & 4 & 0 \end{array} \qquad S_0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 4 & 5 \\ 2 & 1 & - & 3 & 4 & 5 \\ 3 & 1 & 2 & - & 4 & 5 \\ 4 & 1 & 2 & 3 & - & 5 \\ 5 & 1 & 2 & 3 & 4 & - \end{array}$$

$D_1^1(2,3) = Min[D^0(2,3), D^0(2,1) + D^0(1,3)]$

$$D_1^1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 10 & \infty & \infty \\ 2 & 3 & 0 & 13 & 5 & \infty \\ 3 & 10 & 13 & 0 & 6 & 15 \\ 4 & \infty & 5 & 6 & 0 & 4 \\ 5 & \infty & \infty & \infty & 4 & 0 \end{array} \qquad S_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 4 & 5 \\ 2 & 1 & - & 1 & 4 & 5 \\ 3 & 1 & 1 & - & 4 & 5 \\ 4 & 1 & 2 & 3 & - & 5 \\ 5 & 1 & 2 & 3 & 4 & - \end{array}$$

$D^2(3,1) = \cancel{D^2(3,2)}\ min[\overset{10}{D^2(3,1)}, \overset{13}{D^2(3,2)} \overset{+}{+} \overset{3}{D^2(2,1)}]$

$$D_2^2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 10 & 8 & \infty \\ 2 & 3 & 0 & 13 & 5 & \infty \\ 3 & 10 & 13 & 0 & 6 & 15 \\ 4 & 8 & 5 & 6 & 0 & 4 \\ 5 & \infty & \infty & \infty & 4 & 0 \end{array} \qquad S_2 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 2 & 5 \\ 2 & 1 & - & 1 & 4 & 5 \\ 3 & 1 & 1 & - & 4 & 5 \\ 4 & 2 & 2 & 3 & - & 5 \\ 5 & 1 & 2 & 3 & 4 & - \end{array}$$

$$D_3^3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 10 & 8 & 25 \\ 2 & 3 & 0 & 13 & 5 & 28 \\ 3 & 10 & 13 & 0 & 6 & 15 \\ 4 & 8 & 5 & 6 & 0 & 4 \\ 5 & \infty & \infty & \infty & 4 & 0 \end{array} \qquad S_3 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 2 & 3 \\ 2 & 1 & - & 1 & 4 & 3 \\ 3 & 1 & 1 & - & 4 & 5 \\ 4 & 2 & 2 & 3 & - & 5 \\ 5 & 1 & 2 & 3 & 4 & - \end{array}$$

$$D_4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & 3 & 10 & 8 & 12 \\ 2 & 3 & 0 & 11 & 5 & 9 \\ 3 & 10 & 11 & 0 & 6 & 10 \\ 4 & 8 & 5 & 6 & 0 & 4 \\ 5 & 12 & 9 & 10 & 4 & 0 \end{array} \qquad S_4 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & - & 2 & 3 & 2 & 4 \\ 2 & 1 & - & 4 & 4 & 4 \\ 3 & 1 & 4 & - & 4 & 4 \\ 4 & 2 & 2 & 3 & - & 5 \\ 5 & 4 & 4 & 4 & 4 & - \end{array}$$

D_4 gives shortest path between all pair of vertices.

Find shortest route b/w Node 1 and 5 with path

Soln Shortest Distance is = 12.

Route: 1 — 4 — 5 ×

$\boxed{1 - 2 - 4 - 5}$ ✓

$S_{12} = 2,\ S_{24} = 4,\ S_{45} = 5$

$S_{15} \neq 5$

$S_{14} = 2 \neq 4$

$S_{12} = 2$

$S_{ij} = j$

Q6-21 From node 3 to 5.

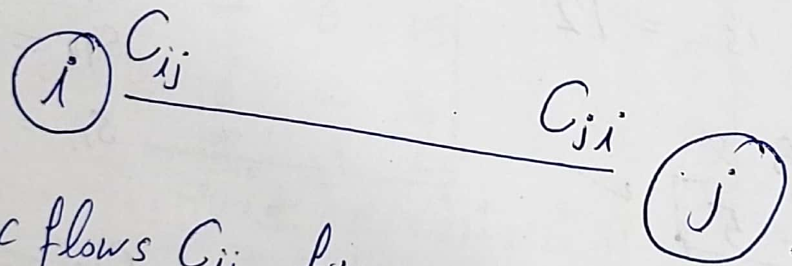
Distance = 10

Route: $S_{35} = 4 \neq 5$ $\boxed{3 - 4 - 5}$ ✓, $S_{34} = 4$

6.4 Maximal Flow Model.

Network model of Pipeline that transport crude oil from oil wells to refineries.

A pipe segment may be uni or Bidirectional depend on it's design



Arc flows C_{ij} from $i \longrightarrow j$.

Arc flow C_{ji} from $j \longrightarrow i$

6.4.1 Enum

flow

$\frac{3}{5}$ → capacity

6 T

6.4) Maximal Flow Model.

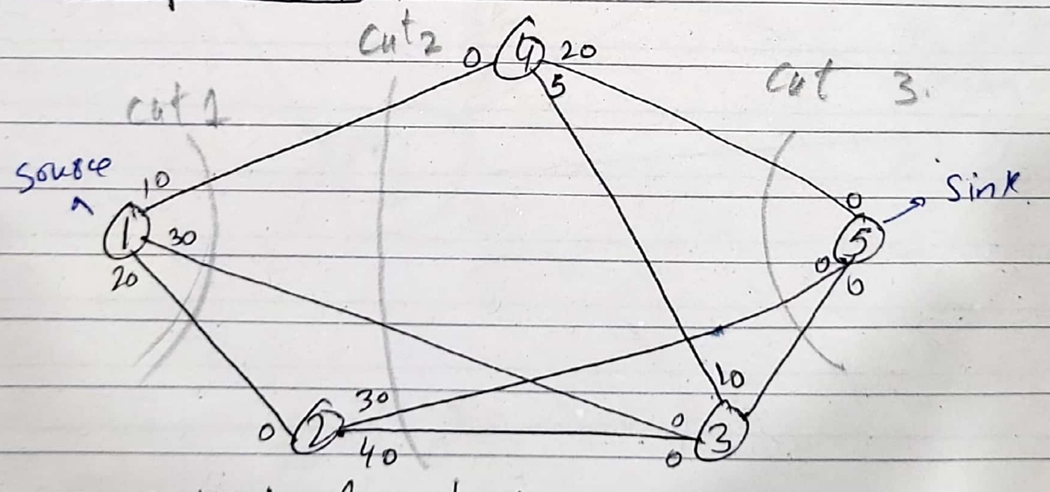
A Pipe segment may be uni or bidirectional depend on it's design.

6.4.1) Enumeration of Cuts.

A cut defines a set whose removal from the network distrupts flow between the source and sink nodes.

The Cut capacity equals the sum of the capacities of it's set of arc.

Example 6.4.1.



The Bidirectional capacities.

For example.

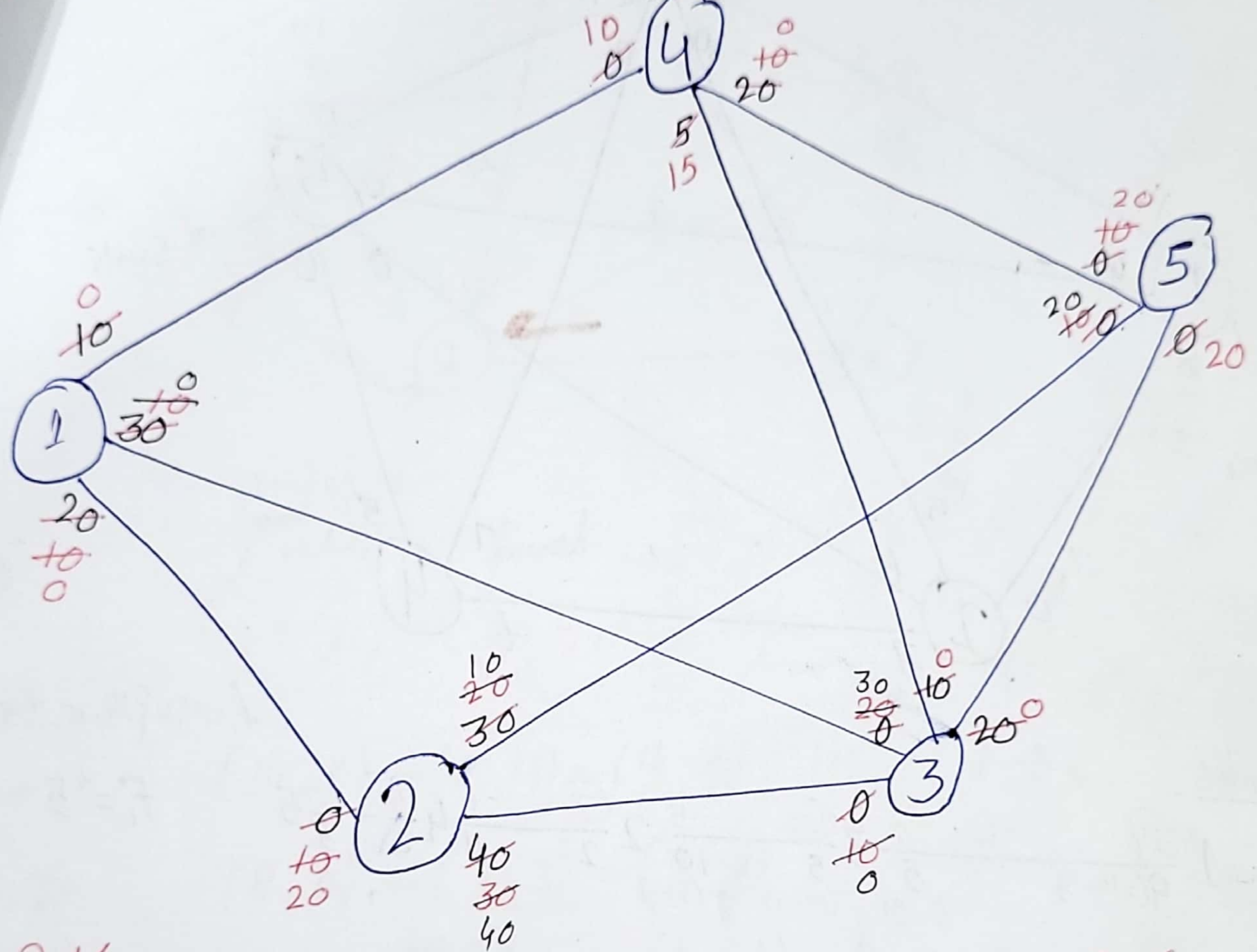
for arc (3,4), The flow limit is 10 from 3→4

for arc (4,3) The flow limit is 5 from 4→3

Cut	Associated Arc	capacity
1	(1,2), (1,3), (1,4)	20+30+10=60
2	(1,3), (1,4), (2,3), (2,5)	30+10+40+10=110
3	(2,5), (3,5), (4,5)	30+20+20=70

1 $\frac{10}{0}$ — $\frac{0}{10}$ 4 $\frac{10}{0}$ — $\frac{10}{20}$ 5 Total Maximal flow ... 15=

Example 6.4-2



Path:

1) $1 \frac{30}{10}$ ——— $\frac{0}{20} 3 \frac{20}{0}$ ——— $\frac{0}{20} 5$

$\therefore f_1 = \text{Min}\{30, 20\} = 20$ (c_{13}, c_{35})

Backtracking

$\{c_{ij} - f_p, c_{ji} + f_p$

(ii) $1 \frac{20}{10}$ ——— $\frac{0}{10} 2 \frac{40}{30}$ ——— $\frac{0}{10} 3 \frac{10}{0}$ ——— $\frac{5}{15} 4 \frac{20}{10}$ ——— $\frac{0}{10} 5$ $\quad f_2 = 10$

(iii) 1 ——— 2 ——— 3 ——— 1 So Ignore 3. Because only $C_{31} > 0$ and it create cycle.

(iii) $1 \frac{10}{0}$ ——— $\frac{10}{20} 2 \frac{30}{20}$ ——— $\frac{0}{10} 5$ $\quad f_3 = 10$

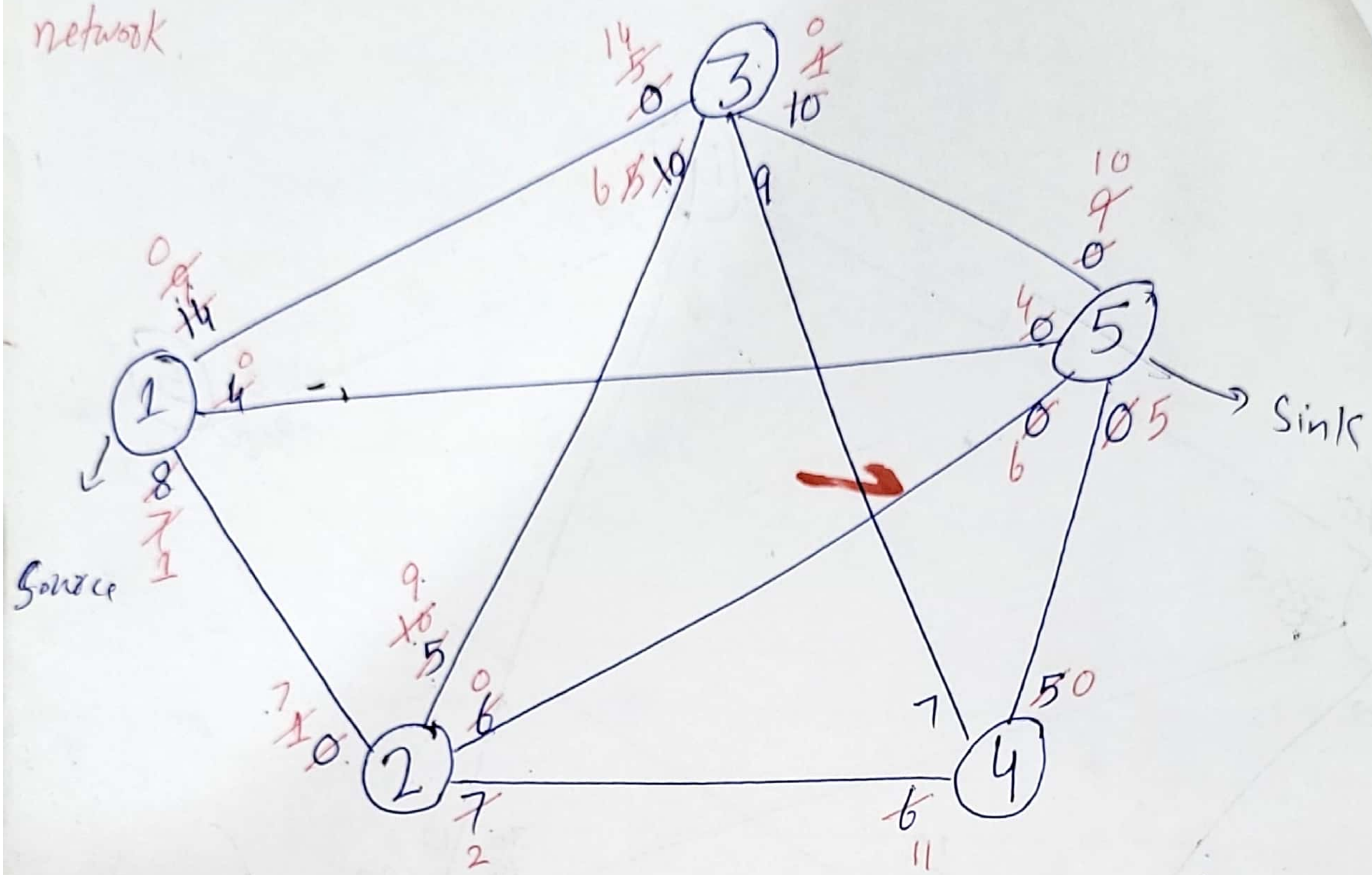
(iv) $1 \frac{10}{0}$ ——— $\frac{20}{30} 3 \frac{10}{0}$ ——— $\frac{30}{40} 2 \frac{20}{10}$ ——— $\frac{10}{20} 5$ $\quad f_4 = 10$

(v) $1 \frac{10}{0}$ ——— $\frac{0}{10} 4 \frac{10}{0}$ ——— $\frac{10}{20} 5$ $\quad f_5 = 10$

$\boxed{\text{Total Maximal flow} = 60}$

Problem 6-30

Determine the maximal flow in each arc for the network



Routes:

① $1 \frac{14}{9(=14-5)} \text{———} \frac{0}{5 \, (0+5)} 3 \frac{10}{5 \, (10-5)} \text{———} \frac{5}{10 \, (5+5)} 2 \frac{7}{2} \text{———} \frac{6}{11} 4 \frac{5}{0} \text{———} \frac{0}{5} 5$

$f_1 = \min\{14, 10, 7, 5\}$

$f_1 = 5$

② $1 \frac{9}{0} \text{———} \frac{5}{14} 3 \frac{10}{1} \text{———} \frac{0}{9} 5$ $\quad f_2 = \min\{9, 10\} = 9$

③ 1 ——— 2 ——— 3 ——— 4 → Not possible.

③ $1 \frac{8}{7} \text{———} \frac{0}{1} 2 \frac{10}{9} \text{———} \frac{5}{6} 3 \frac{1}{0} \text{———} \frac{9}{10} 5$ $\quad f_3 = \min\{8, 10, 1\} = 1$

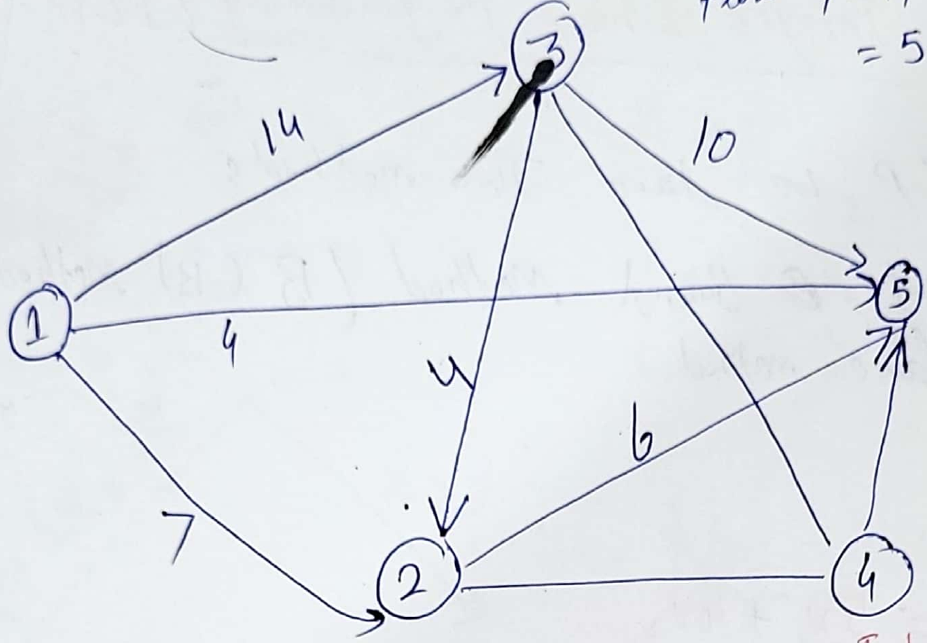
④ $1 \frac{7}{1} \text{———} \frac{1}{7} 2 \frac{6}{0} \text{———} \frac{0}{6} 5$ $\quad f_4 = 6$

⑤ $1 \frac{4}{0} \text{———} \frac{0}{4} 5$ $\quad f_5 = 4$

⑥ 1 — 2 — 3 — 4 — Not Possible. So No Break through.

Optimal soln

Total flow $f = f_1 + f_2 + f_3 + f_4 + f_5$
$= 5 + 9 + 1 + 8 + 4 = 25$



Initial. Last Residue

$(C_{ij}, C_{ji}) - (\bar{C}_{ij}, \bar{C}_{ji})$

$(8, 0) - (1, 7) = (7, -7)$ flow

Arc	Initial values (C_{ij}, C_{ji}) – final $(\bar{C}_{ij}, \bar{C}_{ji})$	Flow Amount	Direction
(1, 3)	$(14, 0) - (0, 14) = (14, -14)$	14	$1 \to 3$
(1, 2)	$(8, 0) - (1, 7) = (7, -7)$	7	$1 \to 2$
(1, 5)	$(4, 0) - (0, 4) = (4, -4)$	4	$1 \to 5$
(2, 3)	$(5, 10) - (9, 6) = (-4, 4)$	—	—
(2, 4)			
(2, 5)	$(6, 0) - (0, 6) = (6, -6)$	6	$2 \to 5$
(3, 2)	$(10, 5) - (6, 9) = (4, -4)$	4	$3 \to 2$
(3, 4)	$(9, 7) - (9, 7) = (0, 0)$	0	—
(3, 5)	$(10, 0) - (0, 10) = (10, -10)$	10	$3 \to 5$
(4, 3)	$(7, 9) - (7, 9) = (0, 0)$	0	—
(4, 2)	$(6, 7) - (11, 2) = (-5, 5)$	—	—
(4, 5)	$(5, 0) - (0, 5) = (5, -5)$	5	$4 \to 5$

CH # 9

Integer Linear Programming (ILP

To solve ILP we have Two methods

i) Branch and Bound Method (B & B) Method
ii) Cutting plane method.

Q 9-56 (d)

Min $Z = 5x_1 + 4x_2$

s.t

$$3x_1 + 2x_2 \geq 5$$

$$2x_1 + 3x_2 \geq 7$$

$x_1, x_2 \geq 0$ and integer.

Sol

We solve it Algebraically.

$3x_1 + 2x_2 = 5$ $\quad$ $x_1 = 0 \Rightarrow x_2 = \frac{5}{2}$ $\left(0, \frac{5}{2}\right)$ (0, 2.5)

$x_2 = 0 \Rightarrow x_1 = \frac{5}{3}$ $\left(\frac{5}{3}, 0\right)$ (1.67, 0)

$2x_1 + 3x_2 = 7$ $\quad$ $x_1 = 0 \Rightarrow x_2 = \frac{7}{3}$, $x_1 = 0$ (2.3)

$x_2 = 0 \Rightarrow x_1 = \frac{7}{2}$ (3.5, 0)

Point of Intersection: solve eq (i) & (ii)

$$x_1 = 0.2, \quad x_2 = 2.2 \rightarrow Z = 9.8$$

LP 0

$x_1 = 0.2, \ x_2 = 2.2$
$Z = 9.8$

$x_1 \le 0$ $\qquad$ $x_1 \ge 1$

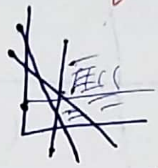
LP1

$x_1 = 0, \ x_2 = 2.5$
$Z = 10$

LP2

$x_1 = 1, \ x_2 = 1.67$
$Z = 11.67$

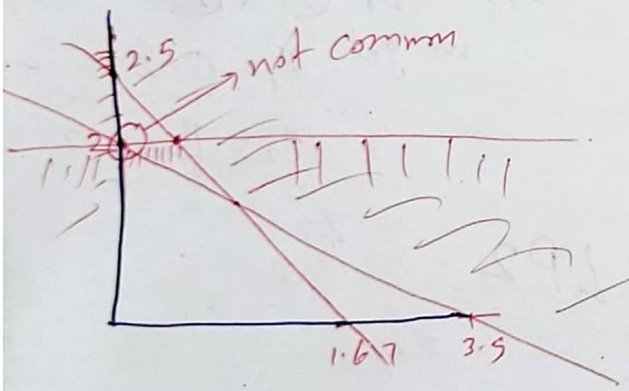
$x_2 \le 1$ $\qquad$ $x_2 \ge 2$



$x_2 \le 2$ $\qquad$ $x_2 \ge 3$

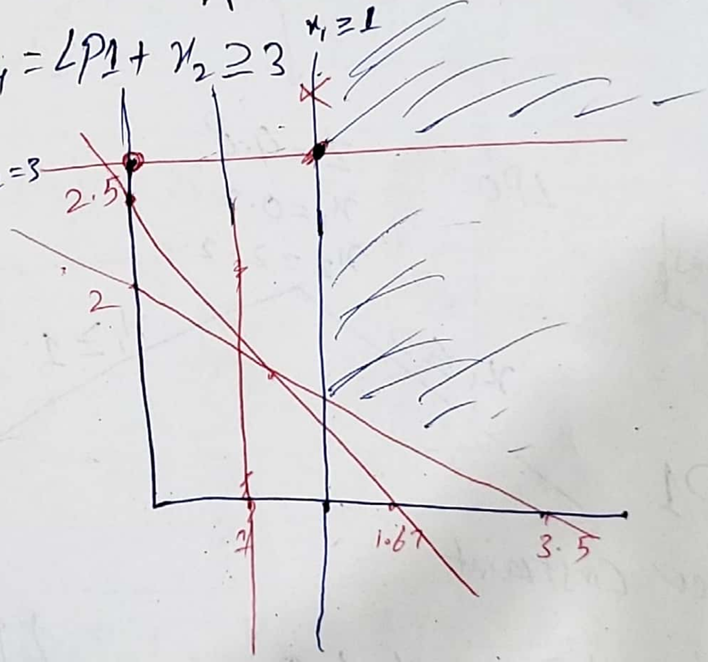
LP3

$LP3 = LP1 + x_2 \le 2$



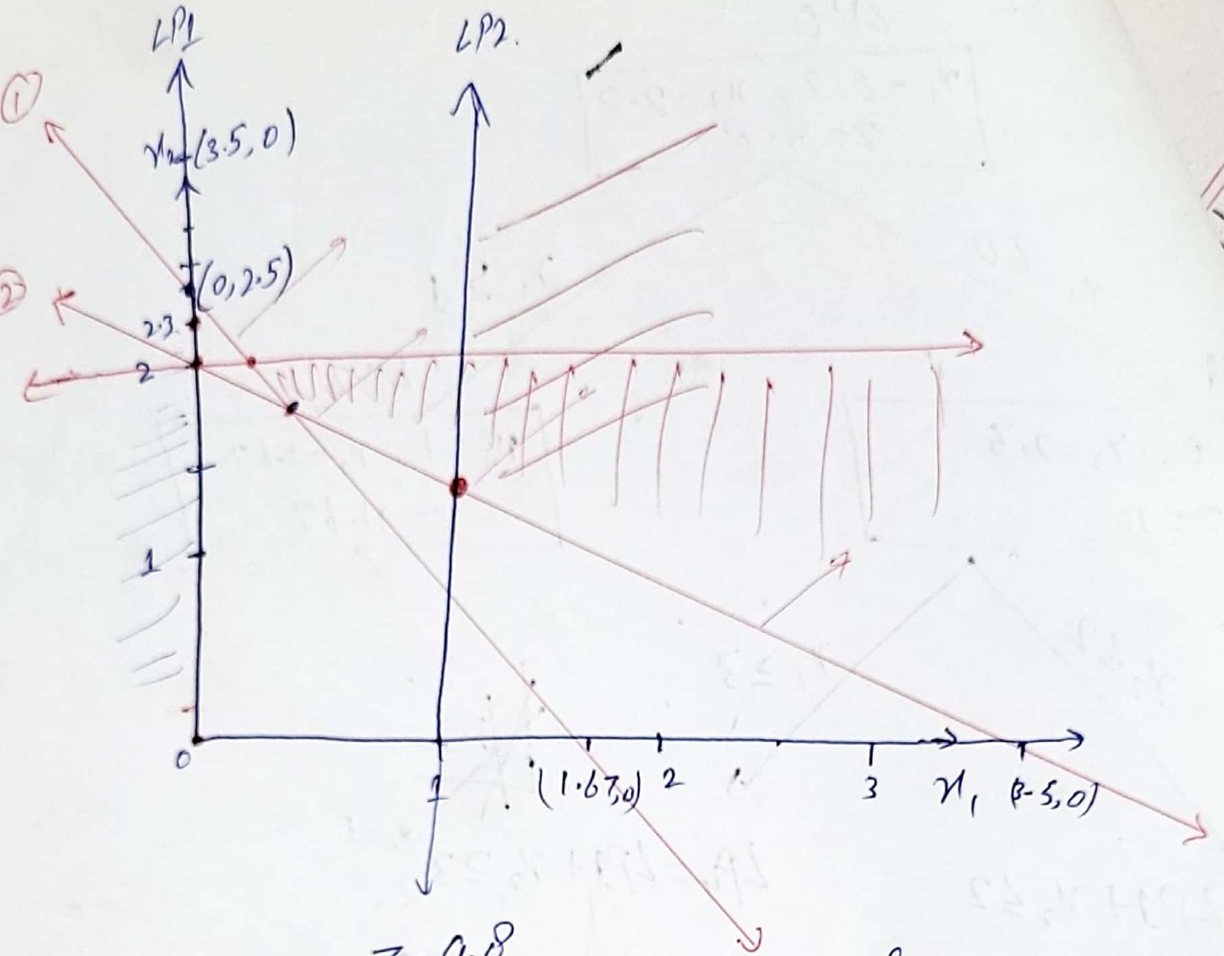
Infeasible.

$LP_4 = LP1 + x_2 \ge 3$



$x_2 = 3, \quad x_1 = 0$

$Z = 5(0) + 4(3) = 0 + 12 = 12$



LPO : $Z = 9.8$
$x_1 = 0.2$
$x_2 = 2.2$

Largest Decimal part

$x_1 \le 0$ — $x_1 \ge 1$

LP1 . LP2.

Solve Eq1 ① & ②
we get $x_1 = 0.2$
$x_2 = \frac{11}{5} = 2.2$
$Z =$

Add New Constraint

$LP1 = LPO + x_1 \le 0$

In this case $x_2 = 0$, $x_2 = 2.5$

New solution.

$x_1 = 0,\ x_2 = 2.5$. $Z = 5(0) + 4(2.5)$
$\boxed{Z = 10}$

$LP2 = LPO + x_1 \ge 1$

Solve Eq 2 and $x_1 = 1$

$2x_1 + 3x_2 = 7,\ x_1 = 1$

$x_2 = \frac{7-2}{3}$

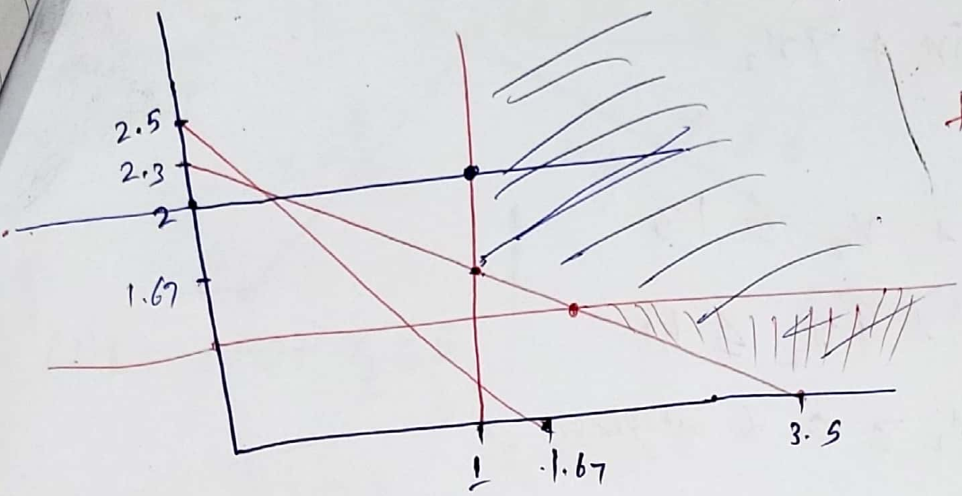
$\boxed{x_2 = \frac{5}{3} = 1.67}$

$x_1 = 1,\ x_2 = \frac{5}{3} = 1.67,$

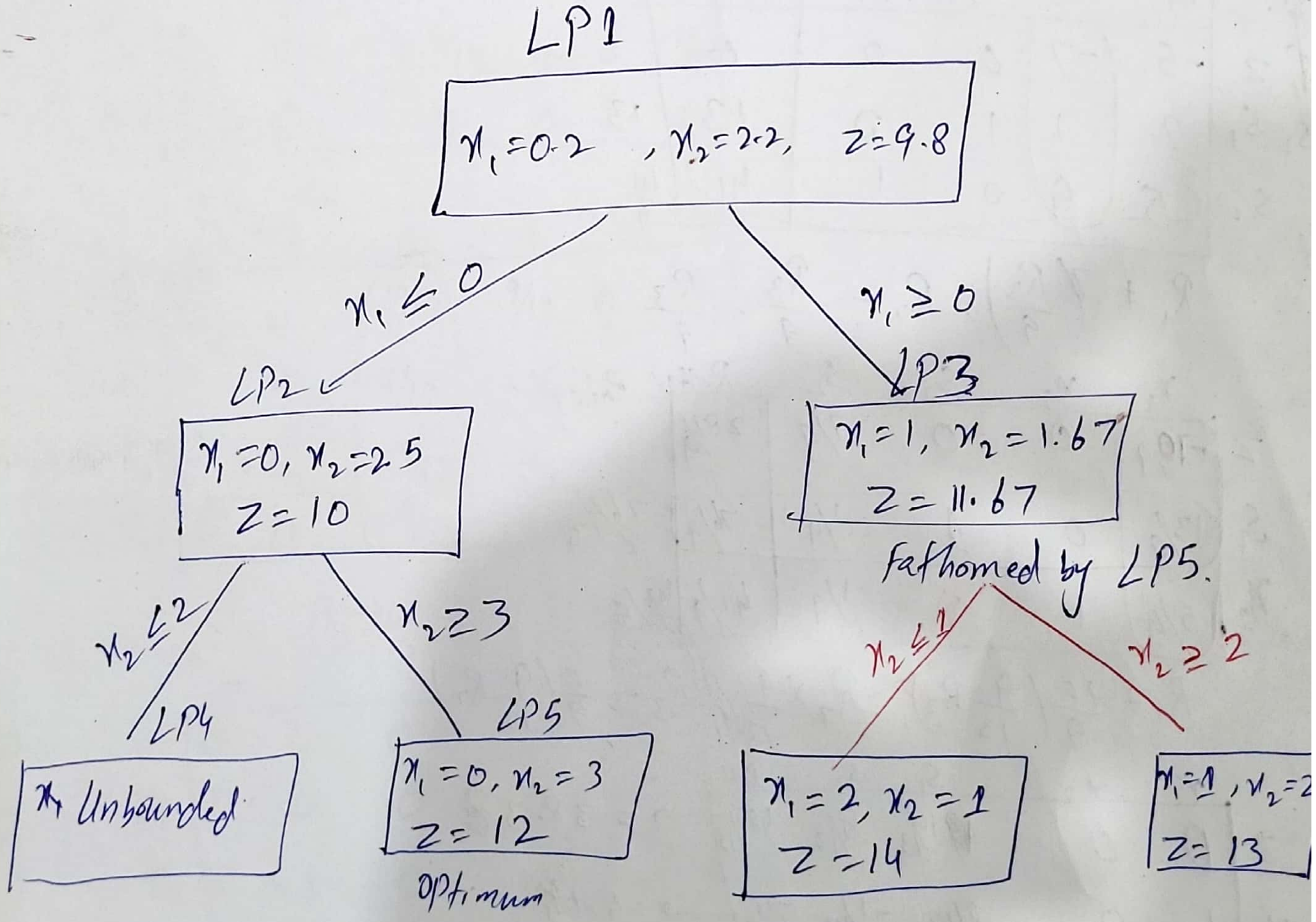
$Z = \frac{35}{3} = 11.67$

For LP2

$+ \; x_2 \geq 2$



$x_2 = 2, \; x_1 = 1, \; Z = 14$



Q9-56 (e) Develop B&B tree for

Max $Z = 5x_1 + 7x_2$ Algebraically

s.t

$2x_1 + x_2 \le 13$

$5x_1 + 9x_2 \le 41$

$x_1, x_2 \ge 0$ & integers.

Sol

$2x_1 + x_2 + S_1 = 13$

$5x_1 + 9x_2 + S_2 = 41$

	x_1	x_2	S_1	S_2	R.H.V	
Z	-5	-7	0	0	0	
S_1	2	1	1	0	13	13
S_2	5	9	0	1	41	$\frac{41}{9}$

$R_1 + 7\left(\frac{R_3}{9}\right)$, $R_2 - \frac{R_3}{9}$, $\frac{R_3}{9}$

	x_1	x_2	S_1	S_2	R.H.V	Ratio
Z	$-10/9$	0	0	7/9	287/9	
S_1	13/9	0	1	-1/9	76/9	76/13
x_2	5/9	1	0	1/9	41/9	41/5

$R_1 + \frac{10}{9}\left(\frac{9}{13}R_2\right)$, $R_2 \times \frac{9}{13}$, $R_3 - \frac{5}{9}\left(\frac{9}{13}R_2\right)$

	x_1	x_2	S_1	S_2	R.H.V	
Z	0	0	10/13	9/13	499/13	$\to = 38.38$
x_1	1	0	9/13	-1/13	76/13	$\to = 5.85$
x_2	0	1	-5/13	2/13	17/13	$\to = 1.31$

LP0

$x_1 = 5.85,\ x_2 = 1.31$

$Z = 38.38$

greater decimal part

$x_1 \leq 5$ | $x_1 \geq 6$

$LP1 = LP0 + x_1 \leq 5$

$LP2 = LP0 + x_1 \geq 6$

$x_1 + S_3 = 5$

Now solve LP1

	x_1	x_2	S_1	S_2	S_3	R.H.V
Z	0	0	10/13	9/13	0	499/13
x_1	1	0	9/13	-1/13	0	76/13
x_2	0	1	-5/13	2/13	0	17/13
S_3	$\boxed{1}$	0	0	0	1	5

$R_4 - R_2$

	x_1	x_2	S_1	S_2	S_3	R.H.V
Z	0	0	10/13	9/13	0	499/13
x_1	1	0	9/13	-1/13	0	76/13
x_2	0	1	-5/13	2/13	0	17/13
S_3	0	0	-9/13	1/13	1	-11/13

Apply Dual Simplex

$\min\left\{\left|\frac{c_j}{\alpha_{rj}}\right|,\ \alpha_{rj} < 0\right\}$

$\frac{13}{-9}R_4,\ R_1 - \frac{10}{13}\left(\frac{-13}{9}R_4\right),\ R_2 + R_4,\ R_3 + \frac{5}{13}\left(\frac{-13}{9}R_4\right)$

Z	0	0	0	7/9	10/9	337/9	→ 37.44
x_1	1	0	0	0	1	5	
x_2	0	1	0	1/9	-5/9	16/9	→ 1.78
S_1	0	0	1	-1/9	-13/9	11/9	→ 1.22

New model

LP0

$x_1 = 5.85,\ x_2 = 1.31$
$Z = 38.38$

Largest Decimal Part $= 0.85$

$x_1 \le 5$ → **LP1**

$x_1 \ge 6$ → $LP2 = LP0 + x_1 \ge 6$

LP1

$x_1 = 5,\ x_2 = 1.78$
$z = 37.44$

$x_2 \le 1$ → $LP3 = LP1 + x_2 \le 1$

$x_2 \ge 2$ → $LP4 = LP1 + x_2 \ge 2$ → DIY

Now solve LP3, $x_2 + S_4 = 1$

	x_1	x_2	S_1	S_2	S_3	S_4	RHV
Z	0	0	0	7/9	10/9	0	337/9
x_1	1	0	0	0	1	0	5
x_2	0	1	0	1/9	-5/9	0	16/9
S_1	0	0	1	-1/9	-13/9	0	11/9
S_4	0	$\boxed{1}$	0	0	0	1	1

$R_5 - R_3$

	x_1	x_2	S_1	S_2	S_3	S_4	RHV
Z	0	0	0	7/9	10/9	0	337/9
x_1	1	0	0	0	1	0	5
x_2	0	1	0	1/9	-5/9	0	16/9
S_1	0	0	1	-1/9	-13/9	0	11/9
S_4	0	0	0	-1/9	5/9	1	-7/9

↓ DIY

w solve LP2

$$x_1 \geq 6 \rightarrow -x_1 + S_3 = -6$$

	x_1	x_2	S_1	S_2	S_3	R.H.V
Z	0	0	10/13	9/13	0	499/13
x_1	1	0	9/13	-1/13	0	76/13
x_2	0	1	-5/13	2/13	0	17/13
S_3	$\boxed{-1}$	0	0	0	1	-6

$R_4 + R_2$

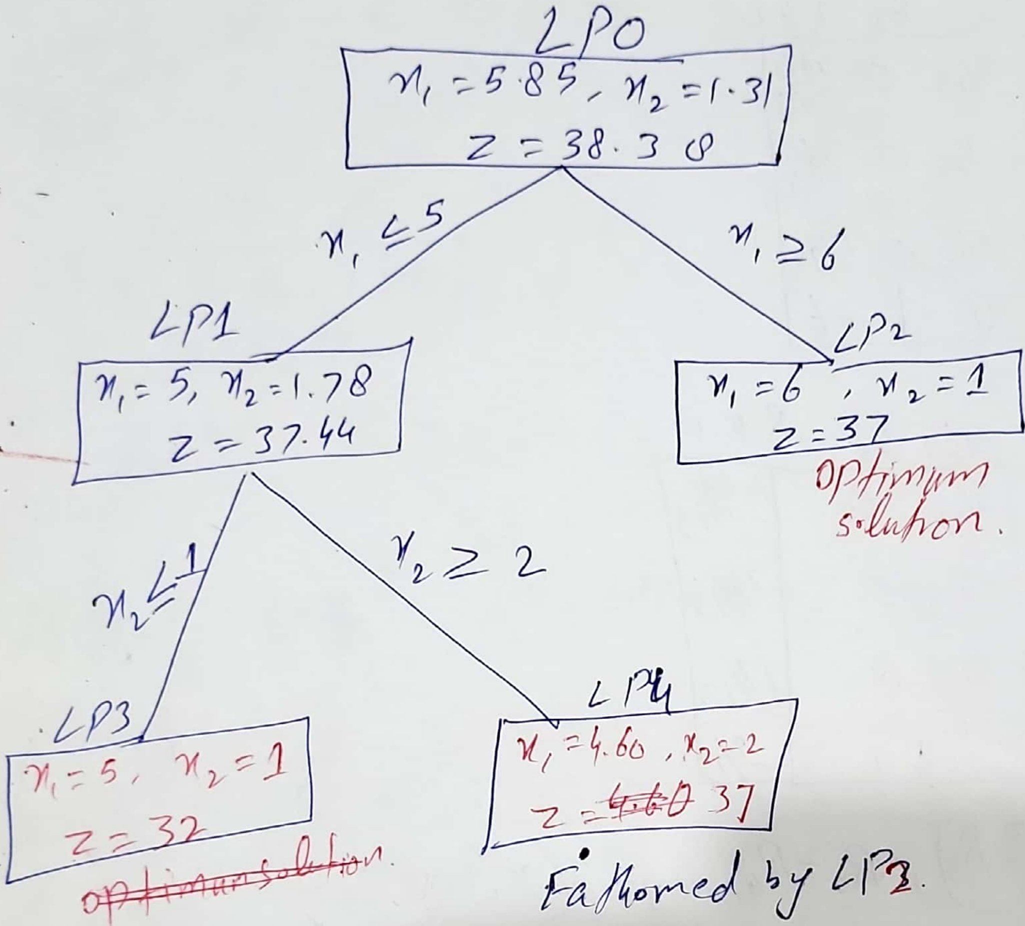
	x_1	x_2	S_1	S_2	S_3	R.H.V
Z	0	0	10/13	9/13	0	499/13
x_1	1	0	9/13	-1/13	0	76/13
x_2	0	1	-5/13	2/13	0	17/13
S_3	0	0	9/13	-1/13	1	-2/13

$\frac{13}{-1}R_4$, $R_1 + \frac{9}{13}\left(\frac{13}{1}R_4\right)$, $R_2 - R_4$, $R_3 + 2(R_4)$

	x_1	x_2	S_1	S_2	S_3	R.H.V
Z	0	0	7	0	9	37
x_1	1	0	0	0	-1	6
x_2	0	1	1	0	2	4
S_2	0	0	-9	1	-13	2

Ft

Final Model is



9.2) Cutting plane Algorithm.

Special constraint (called Cuts) are Added to the solution Space in a manner that Render an integ

The added cut donot eliminate any of the feasible integer point but must pass through at least one feasible or infeasible integer point. These are basic requirment for any cut.

Scanned with CamScanner

$-1+(x) = \frac{-3}{5}$

$x = -\frac{3}{5} + 1 \Rightarrow x = \frac{2}{5}$

5

6

Ex 9.2 Q No 2(d)

1

Q) Solve By fractional Cut

$$\text{Min } z = 5x_1 + 4x_2$$

$$3x_1 + 2x_2 \geq 5$$

$$2x_1 + 3x_2 \geq 7$$

$$x_1, x_2 \geq 0, \text{ And integer}$$

Ex (9.56 Part (d))

soln

Already solve

The optimal tableau is

	x_1	x_2	x_3	x_4	R.H.V
Min z	0	0	$-7/5$	$-2/5$	$49/5$
x_1	1	0	$-3/5$	$2/5$	$1/5$
x_2	0	1	$2/5$	$-3/5$	$11/5$

The first cut is at x_1.

$$x_1 - \frac{3}{5}x_3 + \frac{2}{5}x_4 = \frac{1}{5}$$

$$x_1 + \left(-1+\frac{2}{5}\right)x_3 + \frac{2}{5}x_4 = \frac{1}{5}$$

And cut is. $\frac{2}{5}x_3 + \frac{2}{5}x_4 \geq \frac{1}{5}$

$$-\frac{2}{5}x_3 - \frac{2}{5}x_4 + S_1 = -\frac{1}{5}$$

$x_1 - x_3 = -\frac{2}{5}x_3 - \frac{2}{5}x_4 + \frac{1}{5} \leq \frac{1}{5}$

And for integers

$-\frac{2}{5}x_3 - \frac{2}{5}x_4 + \frac{1}{5} \leq 0$

$-\frac{2}{5}x_3 - \frac{2}{5}x_4 \leq -\frac{1}{5}$

	x_1	x_2	x_3	x_4	S_1	R.H.V
min z	0	0	$-7/5$	$-2/5$	0	$49/5$
x_1	1	0	$-3/5$	$2/5$	0	$1/5$
x_2	0	1	$2/5$	$-3/5$	0	$11/5$
S_1	0	0	$-2/5$	$-2/5$	1	$-1/5$
			3.5	1		

$\min\left\{\left|\frac{-7/5}{-2/5}\right|, \left|\frac{-2/5}{-2/5}\right|\right\}$

$-\frac{5}{2}R_4,\ R_1 - R_4,\ R_2 + R_4,\ R_3 + \frac{3}{5}\left(-\frac{5}{2}R_4\right)$

$-1+\frac{1}{2}=-\frac{1}{2}$ $\quad x=-\frac{3}{2}+1 \Rightarrow -\frac{1}{2}$ $\quad 1-x=-\frac{3}{2}$

$-\frac{3}{2}=-1+\frac{1}{4}=-\frac{3}{4}=-\frac{3}{2}$ $\quad -x=-\frac{3}{2}-1$ $\quad -x=-\frac{5}{2}$ $\quad x=\frac{5}{2}$

$-1+x=-\frac{3}{2}$ $\quad 1-\frac{5}{2}=-\frac{3}{2}$ $\quad x=-\frac{1}{2}$ $\quad -5/2+1=-\frac{3}{2}$ $\quad -1-\frac{3}{2}$

(6)

$$
\begin{array}{c|cccccc}
 & x_1 & x_2 & x_3 & x_4 & S_1 & R.H.V \\
\hline
\text{Min } z & 0 & 0 & -1 & 0 & -1 & 10 \\
x_1 & 1 & 0 & -1 & 0 & 1 & 5 \\
x_2 & 0 & 1 & 1 & 0 & -3/2 & 5/2 \\
x_4 & 0 & 0 & 1 & 1 & -5/2 & 1/2
\end{array}
$$

Made cut At x_2.

$x_2 + x_3 - \frac{3}{2}S_1 = 5/2$

~~$x_2 + x_3 + (-1 - \frac{1}{2})S_1 = 2 + \frac{1}{2}$~~

~~The cut is~~

~~$\frac{1}{2}S_1 + S_2 = -\frac{1}{2}$~~

~~$x_2 + x_3 + (1 - \frac{5}{2})S =$~~

$x_2 + x_3 - \frac{3}{2}S_1 = 5/2$

$x_2 + x_3 + (-2 + \frac{1}{2})S_1 = 2 + \frac{1}{2}$

$\frac{1}{2} + \frac{1}{2}S_1 \le 0$

$S_1 \le -1$

$\frac{1}{2}S_1 + S_2 = -\frac{1}{2}$

$-2 + \frac{1}{2} = -\frac{3}{2}$

So cut is $-\frac{1}{2}S_1 + \frac{1}{2} \le 0$.

$-\frac{1}{2}S_1 + S_2 = -\frac{1}{2}$

so Add this constrain.

$$
\begin{array}{c|ccccccc}
 & x_1 & x_2 & x_3 & x_4 & S_1 & S_2 & R.H.V \\
\hline
\text{Min } z & 0 & 0 & -1 & 0 & -1 & 0 & 10 \\
x_1 & 1 & 0 & -1 & 0 & 1 & 0 & 0 \\
x_2 & 0 & 1 & 1 & 0 & -3/2 & 0 & 5/2 \\
x_4 & 0 & 0 & 1 & 1 & -5/2 & 0 & 1/2 \\
S_2 & 0 & 0 & 0 & 0 & -1/2 & 1 & -1/2
\end{array}
$$

$-2R_5, \; R_1 + (-2R_5), \; R_2 - (-2R_5), \; R_3 + \frac{3}{2}(-2R_5)$

$R_4 + \frac{5}{2}(-2R_5)$

	x_1	x_2	x_3	x_4	S_1	S_2	R.H.V
Min z	0	0	-1	0	0	-2	11
x_1	1	0	-1	0	0	2	-7
x_2	0	1	1	0	0	-3	4
x_4	0	0	1	1	0	-5	3
S_1	0	0	0	0	1	-2	1

$-R_2$, $R_1 - R_2$, $R_3 + R_2$, $R_4 + R_2$.

	x_1	x_2	x_3	x_4	S_1	S_2	R.H.V
Min z	-1	0	0	0	0	-4	12
x_3	-1	0	1	0	0	-2	1
x_2	1	1	0	0	0	-1	3
x_4	1	0	0	1	0	-3	2
S_1	0	0	0	0	1	-2	1

This is the optimal integer tableau where

$z = 12$, $x_1 = 0$, $x_2 = 3$.

① ④

~~Ex 9.2B~~

Question No 6. Problem 9–70 (a)

Solve the following problem by fractional cut.

a) Max $z = 4x_1 + 6x_2 + 2x_3$

s.t

$$4x_1 - 4x_2 \le 5$$
$$-x_1 + 6x_2 \le 5$$
$$-x_1 + x_2 + x_3 \le 5$$
$$\cancel{x_1, x_2, x_3 \le 5}$$
$$x_1, x_2, x_3 \ge 0 \text{ and integers}$$

soln

First find the optimum of this tableau

	x_1	x_2	x_3	x_4	x_5	x_6	R.H.V	
max z	-4	-6	-2	0	0	0	0	
x_4	4	-4	0	1	0	0	5	∞
x_5	-1	6	0	0	1	0	5	0.83
x_6	-1	1	1	0	0	1	5	5

$\frac{R_3}{6}$, $R_1 + R_3$, $R_2 + 4\left(\frac{R_3}{6}\right)$; $R_4 - \frac{R_3}{6}$

	x_1	x_2	x_3	x_4	x_5	x_6	R.H.V	
max z	-5	0	-2	0	1	0	5	
x_4	10/3	0	0	1	2/3	0	25/3	5/2
x_2	-1/6	1	0	0	1/6	0	5/6	∞
x_6	-7/6	0	1	0	-1/6	1	25/6	∞

$\frac{3}{10}R_2$, $R_1 + 5\left(\frac{3}{10}R_2\right)$, $R_3 + \frac{1}{6}\left(\frac{3}{10}R_2\right)$, $R_4 + \frac{7}{6}\left(\frac{3}{10}R_2\right)$

	x_1	x_2	x_3	x_4	x_5	x_6	R.H.V
max z	0	0	-2	3/2	2	0	35/2
x_1	1	0	0	3/10	1/5	0	5/2
x_2	0	1	0	1/20	1/5	0	5/4
x_6	0	0	1	1/4	$\cancel{7/5}$ 0	1	25/4

(2) (5)

$R_1 + 2R_4$

	x_1	x_2	x_3	x_4	x_5	x_6	R.H.V
max z	0	0	0	2	2	2	30 ~~95/3~~
x_1	1	0	0	3/10	1/5	0	5/2
x_2	0	1	0	1/20	1/5	0	5/4
x_3	0	0	1	1/20	0	1	25/4 ~~85/12~~

This is optimal tableau.
From the x_1 - row
we make first cut.

$$x_1 + \frac{3}{10} x_4 + \frac{1}{5} x_5 = 5/2$$

min fraction of form

$$x_1 + \left(\frac{3}{10}\right) x_4 + \left(\frac{1}{5}\right) x_5 = 2 + \frac{1}{2}.$$

$\frac{3}{10} = 0 + \frac{3}{10}$

$2\frac{1}{2} = 2 + \frac{1}{2}$

Ans cut is. $\left(0 + \frac{3}{10}\right)$

$$-\frac{3}{10} x_4 - \frac{1}{5} x_5 \le -\frac{1}{2}.$$

OR

$$-\frac{3}{10} x_4 - \frac{1}{5} x_5 + S_1 = -\frac{1}{2}.$$

So

Add this constrain into optimal tableau

	x_1	x_2	x_3	x_4	x_5	x_6	S_1	R.H.V
max z	0	0	0	2	2	2	0	30
x_1	1	0	0	3/10	1/5	0	0	5/2
x_2	0	1	0	1/20	1/5	0	0	5/4
x_3	0	0	1	1/20	0	1	0	25/4
S_1	0	0	0	-3/10	-1/5	0	1	-1/2

20/3   10

$3\frac{5}{6} = 5 + \frac{5}{6}$, $\frac{7}{6} = 1 + \frac{1}{6}$

3

6

$-\frac{10}{3}R_5$, $R_1 - 2\left(-\frac{10}{3}R_5\right)$, $R_2 + R_5$, $R_3 - \frac{1}{20}\left(-\frac{70}{3}R_5\right)$, $R_4 - \frac{1}{4}\left(-\frac{10}{3}R_5\right)$

$$
\begin{array}{c|cccccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & S_1 & R.H.V \\
\hline
\max z & 0 & 0 & 0 & 0 & 2/3 & 2 & 20/3 & 80/3 \\
x_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 2 \\
x_2 & 0 & 1 & 0 & 0 & 1/6 & 0 & 1/6 & 7/6 \\
x_3 & 0 & 0 & 1 & 0 & -1/6 & 1 & 5/6 & 35/6 \\
x_4 & 0 & 0 & 0 & 1 & 2/3 & 0 & -10/3 & 5/2
\end{array}
$$

Making cut at x_2.

$$x_2 + \frac{1}{6}x_5 + \frac{1}{6}S_1 = 7/6$$

$$x_2 + \left(0 + \frac{1}{6}\right)x_5 + \left(0 + \frac{1}{6}\right)S_1 = 1 + \frac{1}{6}$$

The cut is.

$$-\frac{1}{6}x_5 - \frac{1}{6}S_1 \leq -\frac{1}{6}$$

And

$$-\frac{1}{6}x_5 - \frac{1}{6}S_1 + S_2 = -\frac{1}{6}$$

Add constrain into optimal tableau.

$$
\begin{array}{c|ccccccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & S_1 & S_2 & R.H.V \\
\hline
\max z & 0 & 0 & 0 & 0 & 2/3 & 2 & 20/3 & 0 & 80/3 \\
x_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\
x_2 & 0 & 1 & 0 & 0 & 1/6 & 0 & 1/6 & 0 & 7/6 \\
x_3 & 0 & 0 & 1 & 0 & -1/6 & 1 & 5/6 & 0 & 35/6 \\
x_4 & 0 & 0 & 0 & 1 & 2/3 & 0 & -10/3 & 0 & 5/2 \\
S_2 & 0 & 0 & 0 & 0 & -1/6 & 0 & -1/6 & 1 & -1/6 \\
\hline
 & & & & & |-4| & & |-40| & &
\end{array}
$$

$-6R_6$, $R_1 - \frac{2}{3}(-6R_6)$, $R_3 + R_6$, $R_4 - R_6$, $R_5 - \frac{2}{3}(-6R_6)$

(4)

(7)

$$\begin{array}{c|ccccccccc}
 & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & S_1 & S_2 & R.H.V \\
\hline
\text{Max } Z & 0 & 0 & 0 & 0 & 0 & 2 & 6 & 4 & 26 \\
x_1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \\
x_2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
x_3 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & -1 & 6 \\
x_4 & 0 & 0 & 0 & 1 & 0 & 0 & -4 & -4 & 1 \\
x_5 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & -6 & 1
\end{array}$$

so

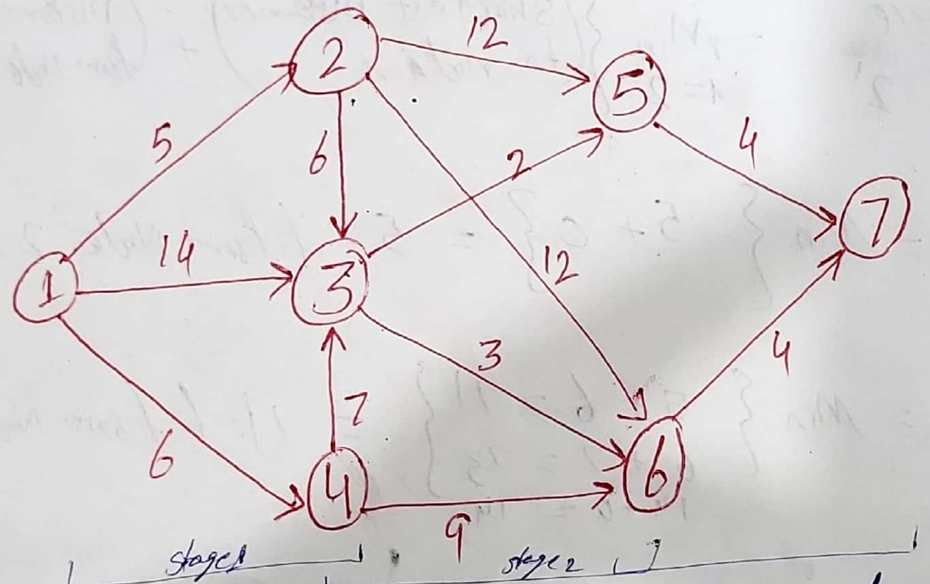
The solution is optimal and all are integers.

CH # 12

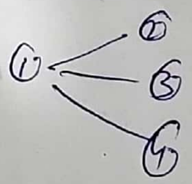
Deterministic Dynamic Programming (DDP)

The main idea of DP is to decompose the problem into subproblems. Optimum solution of one subproblem is used as an input to next subproblem.

Q 12-5 Determine the shortest route b/w city 1 to 7



To solve problem by DP, first decompose into stages.



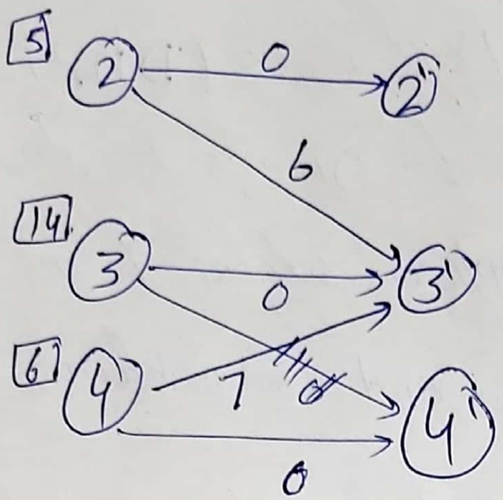
Stage 1:

shortest distance from Node 1 to 2 = 5

" " " " 1 to 3 = 14

1 to 4 = 6

stage 2:



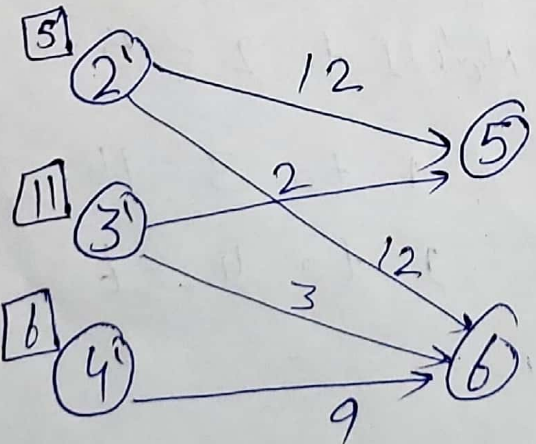
Shortest distance ~~fro~~ to Node 2' $= \min_{i=2} \left\{ \left(\text{Shortest Distance to node } i\right) + \left(\text{Distance from Node } i \text{ to } 2'\right) \right\}$

$= \min \{ 5 + 0 \} = 5$ (from Node 2)

Shortest Distance to Node 3' $= \min \left\{ \begin{array}{l} 5 + 6 = 11 \\ 6 + 7 = 13 \\ 14 + 0 = 14 \end{array} \right\} = 11$ (from Node 2)

Shortest Distance to Node 4' $= \min \{ 6 + 0 \} = 6$ (from Node 4)

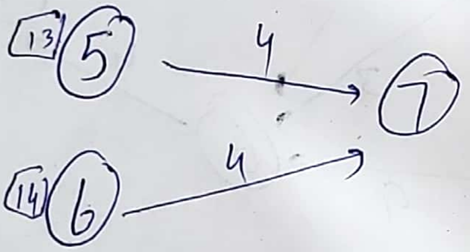
stage 3



Shortest Distance to Node 5 $= \text{Min}\left\{\begin{array}{l} 5+12 = 17 \\ 11+2 = 13 \\ \cancel{6+0} \end{array}\right\} = 13$ (from 3)

Shortest Distance to Node 6 $= \text{Min}\left\{\begin{array}{l} 5+12 = 17 \\ 11+3 = 14 \\ 6+9 = 15 \end{array}\right\} = 14$ from 3

stage 4



Shortest Distance to Node 7 $= \text{Min}\left\{\begin{array}{l} 13+4 = 17 \\ 14+4 = 18 \end{array}\right\} = 17$ (from node 5)

And shortest route is:

Reverse of

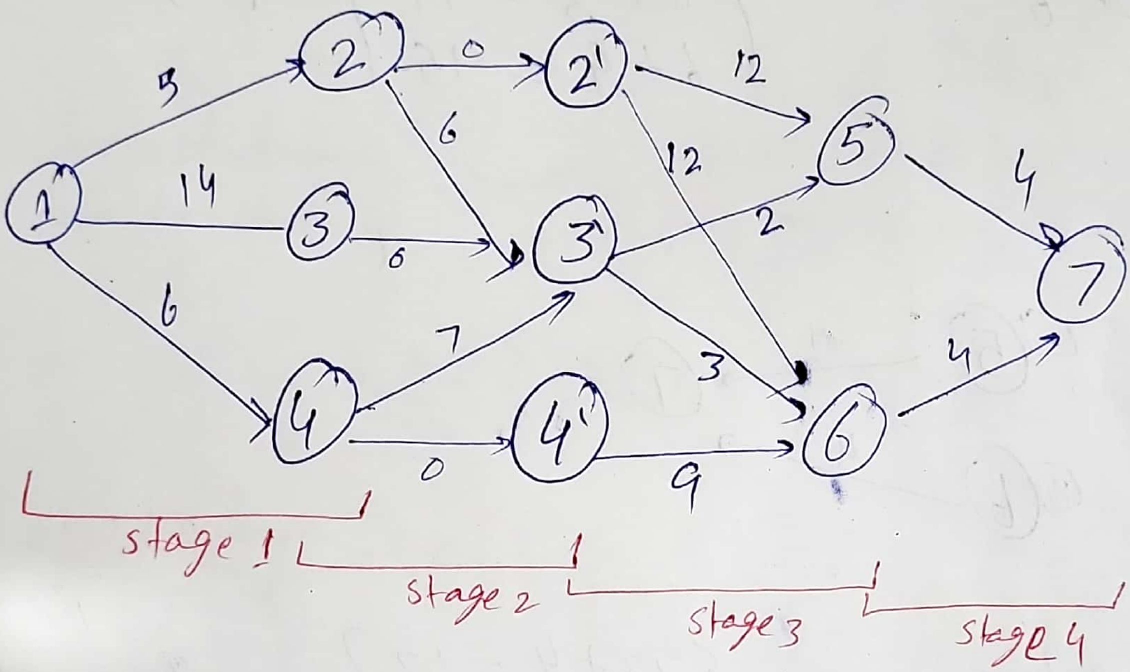
$7 \longrightarrow 5 \longrightarrow 3 \longrightarrow 2 \longrightarrow 1$

The shortest route is

$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 5 \longrightarrow 7$

Backward Recursion

$$f_i(x_i) = \underset{\substack{\text{all feasible} \\ (x_{i-1},\, x_i) \text{ routes}}}{Min} \left\{ d(x_{i-1}, x_i) + f_{i-1}(x_{i-1}) \right\}, \; i = 1, 2, 3$$



stage 4:

x_4	$d(x_4, x_5)$	optimum soln	
	$x_5 = 7$	$f_4(x_4)$	x_5^*
5	4	4	7
6	4	4	7

stage 3

stage 3

	$d(x_3, x_4) + f_4(x_4)$		optimum	
x_3	$x_4 = 5$	$x_4 = 6$	f_3	x_4^*
2'	12+4=16	12+4=16	16	5, 6
3'	2+4=6	3+4=7	6	5
4'	—	9+4=13	13	6

→ Node Name

Stage 2

	$d(x_2, x_3) + f_3$			optimum soln	
x_2	$x_3 = 2'$	$x_3 = 3'$	$x_3 = 4'$	f_2	x_3^*
2	0+16=16	6+6=12	~~0+13=13~~	12	3'
3	—	0+6=6	—	6	3'
4	—	7+6=13	0+13=13	13	3', 4'

stage 1

	$d(x_1, x_2) + f_2$			optimum soln	
x_1	$x_2 = 2$	$x_2 = 3$	$x_2 = 4$	f_1	x_2^*
1	5+12=17	14+6=20	13+6=19	17	2

Solution:

Shortest Distance = 17.

Route

1 — 2 — 3 — 5 — 7

CH #19

Simulation:

A representation of reality through the use of Model (Mathematical Model) or devices, which will react in the same manner as reality under a given set of conditions.

Types of Simulation

i) Analog Simulation

Simulating the reality in phsical form (e.g children's park, planetarium etc.

ii) Computer Simulation

For ~~problems of~~ Complex ~~rate~~ decision making problems, we formulate complex system into mathematical model for which computer programme is developed. Using high speed Computers then solve it.

Types of Simulation Models.

i) Continuous Model

it deals with systems whose behavior change continuously with time. These model use difference-differential Equation to describe the interaction among the different elements of the system.

For example world population dynamics.

ii) Discrete Models.

it deals study of waiting lines. These changes only when customer enters or leave the system.

The instant at which changes take place occurs at specific discrete points.

3) Classification of Simulation Models.

i) Simulation of Deterministic Model

The input & output are not random variables.

ii) Simulation of Probabilistic Models:

Random numbers/sampling is used.

The method used to solve these models is know as Monte-Carlo Technique/method.

iii) Stimulation of static Model

Do not take variable time.

iv) Simulation of Dynamic ~~Pro~~ Model.

Deals with time varying

Monte-Carlo Simulation

it is a Computerized mathematical technique that allows ~~the~~ to model the probability of different outcomes in a process that cannot be easily predicted due to random variables.

it depends on a sequence of random numbers which is generated during the simulation.

Here application of Monte-Carlo applications to solve multiple integrals, estimation of constant π. and Matrix inversion.

Example 19.1-1

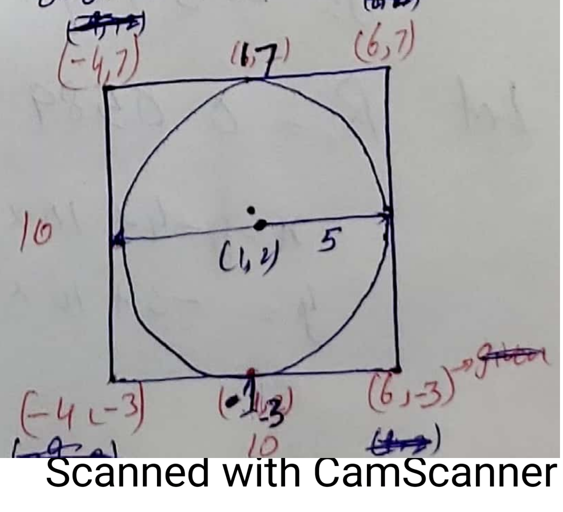
Use Monte-Carlo sampling to estimate area of the circle having eqn.

$$(x-1)^2 + (y-2)^2 = 25.$$

Soln

$$(x-h)^2 + (y-k)^2 = r^2$$

Then centre $= (h,k) = (1,2)$, radius $= r = 5$.



if m out of n sampled points fall within the circle then

$$\text{Approximate Area of circle} = \frac{m}{n}\left(\text{Area of the square}\right) = \frac{m}{n}(10 \times 10) = 100\frac{m}{n}$$

To ensure all points are equally divided in the square, we take

$$f_1(x) = \frac{1}{10}, \quad \overset{x_1}{-4} \le x \le \overset{x_2}{6}$$

$$f_2(y) = \frac{1}{10}, \quad \overset{y_1}{-3} \le y \le \overset{y_2}{7}$$

A pair of 0-1 Random numbers R_1, and R_2 can be used to generate random points (x, y) in the square as.

$$x = \overset{x_1 +}{-4} + \overset{x_2 - x_1}{[6-(-4)]}\, R_1$$

$$y = -3 + [7-(-3)]\, R_2$$

Let $R_1 = 0.0589$ & $R_2 = 0.6733$

$$x = -4 + 10 \times 0.0589 = -3.411$$

$$y = -3 + 10 \times 0.6733 = 3.733$$

Point fall inside the circle if

$$(-3.411-1)^2+(3.733-2)^2 = 22.46 < 25$$

Similarly count the points which lie inside the circle.

Then

$$\text{Area of circle} = \frac{m}{n}(10 \times 10) = \frac{100\,m}{n}$$

Let say if we test 50 random numbers then 39 may be lie in circle.

$$\text{Area of circle} = \frac{39}{50} \times 100 = 78.$$