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Q.1 find solution of the system at $x = 0.2$

$\frac{dy}{dx} = x + z$ $\quad \frac{dz}{dx} = x - y$ $\quad y(0) = 0$, $z(0) = 1$

$h = x_1 - x_0 = 0.2 - 0 = 0.2$

put $n = 0$

- $k_1 = hf(x_0, y_0, z_0)$
  $= 0.2(0 + 1)$
  $= 0.2$
- $l_1 = hg(x_0, y_0, z_0)$
  $= 0.2(0 - 0)$
  $= 0$

- $k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$
  $k_2 = 0.2\left(0 + \frac{0.2}{2}, 0 + \frac{0.2}{2}, 1\right)$
  $k_2 = 0.2(0.1 + 1) = 0.22$
- $l_2 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$
  $l_2 = 0.2\left(0.1, 0.1, \frac{1}{}\right)$
  $l_2 = (0.1 - 0.1) = 0$

- $k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$
  $k_3 = 0.2f\left(0.1, 0 + \frac{0.22}{2}, 1\right)$
  $k_3 = 0.2(0.1 + 1) = 0.22$
- $l_3 = hg\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$
  $l_3 = (0.1, 0.11, 1)(0.2) = 0.2(0.1 - 0.1)$
  $l_3 = 0.002$

- $k_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3)$
  $= 0.2(0.2, 0.22, 0.998)$
  $k_4 = 0.2396$
- $l_4 = hg(x_0 + h, y_0 + k_3, z_0 + l_3)$
  $= 0.2(0.2, 0.22, 0.998)$
  $l_4 = -0.0041$

$$\Delta y = \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) = \frac{1}{6}(0.2 + 2(0.22) + 2(0.22) + 0.2396)$$

$$\Delta y = 0.2199$$

$$y_1 = y_0 + \Delta y = 0 + 0.2199 = 0.2199$$

$$\Delta z = \frac{1}{6}(0 + 0 + 2(-0.002) + (-0.004)) = -0.00133$$

$$z_1 = z_0 + \Delta z = 1 - 0.0013 = 0.9986$$

Q.2 find solution of the system at $x = 0.1$

$\frac{dy}{dx} = x+y+z$, $\frac{dz}{dx} = x-y$

$y(0) = 0$

$z(0) = 1$, $h = 0.1$

put $n = 0$

- $k_1 = hf(x_0, y_0, z_0)$
  $= 0.1(0, 0, 1) = 0.1$

- $l_1 = hg(x_0, y_0, z_0)$
  $= 0.1(0, 0, 1) = 0$

- $k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}, z_0 + \frac{l_1}{2}\right)$
  $= 0.1(0.05, 0.05, 1)$
  $= 0.1(0.05 + 0.05 + 1) = 0.11$

- $l_2 = 0.1(0.05, 0.05, 1)$
  $l_2 = 0.1(0.05 - 0.05)$
  $l_2 = 0$

- $k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}, z_0 + \frac{l_2}{2}\right)$
  $= 0.1\left(0 + \frac{0.1}{2}, 0 + \frac{0.11}{2}, 1 + \frac{0}{2}\right)$
  $= 0.1(0.05 + 0.055 + 1)$
  $k_3 = 0.1105$

- $l_3 = 0.1(0.05, 0.055, 1)$
  $= 0.1(0.05 - 0.055)$
  $= -0.0005$

- $k_4 = hf(x_0 + h, y_0 + k_3, z_0 + l_3)$
  $= 0.1(0.1, 0.1105, 0.9995)$
  $k_4 = 0.121$

- $l_4 = 0.1(0.1, 0.1105, 0.9995)$
  $l_4 = -0.00105$

$y_1 = y_0 + \Delta y = 0 + 0.1103 = 0.1103$

$z_1 = z_0 + \Delta z = 1 - 0.00184 = 1.00184$

$\Delta y = \frac{1}{6}(0.1 + 2(0.11) + 2(0.1105) + 0.121)$

$\Delta y = 0.1103$

$\Delta z = \frac{1}{6}(0 + 0 + 2(-0.0005) - 0.00105)$

$\Delta z = -0.00184$

Q.1 $\frac{dy}{dx} = 3+xz$, $\frac{dz}{dx} = -xy+2$

$x=0, y=0, z=1, h=0.1$

for $x=0.2$

Put $n=0$

$y_1 = y_0 + hf(x_0, y_0, z_0) = 0+0.1(3+0(1)) = 0.3$

$z_1 = z_0 + hf(x_0, y_0, z_0) = 1+0.1(-(0)(0)+2) = 1.2$

Put $n=1$

$y_2 = y_1 + hf(x_1, y_1, z_1) = 0.3+0.1(3+(0.1)(1.2)) = 0.612$

$z_2 = z_1 + hf(x_1, y_1, z_1) = 1.2+0.1(-(0.1)(0.3)+2) = 1.397$

Put $n=2$

$y_3 = y_2 + hf(x_2, y_2, z_2) = 0.612+0.1(3+0.2(1.397)) = 0.939$

$z_3 = z_2 + hf(x_2, y_2, z_2) = 1.397+0.1(-(0.2)(0.612)+2) = 1.5847$

Q.2 $\frac{dy}{dx} = x+z$, $\frac{dz}{dx} = x+y+1$, $y(0)=1, z(0)=0$

for $x=0.1$

Put $n=0$

$y_1 = y_0 + hf(x_0, y_0, z_0) = 1+0.1(0.1+0) = 1.01$

$z_1 = z_0 + hf(x_0, y_0, z_0) = 0+0.1(0+1+1) = 0.2$

Q.1 $\frac{dy}{dx} = x - y^2$, $\frac{dz}{dx} = -x + z$

$x_0 = 0$	$y_0 = 0$	$z_0 = 1$	$y_0' = 0$	$z_0' = 1$
$x_1 = 0.2$	$y_1 = 0.02$	$z_1 = 1.1218$	$y_1' = 0.1996$	$z_1' = 1.081$
$x_2 = 0.4$	$y_2 = 0.0795$	$z_2 = 1.4682$	$y_2' = 0.393$	$z_2' = 1.308$
$x_3 = 0.6$	$y_3 = 0.1762$	$z_3 = 1.7379$	$y_3' = 0.5689$	$z_3' = 1.377$
$x_4 = 0.8$	$y_4 =$	$z_4 =$		

Milne's Method:

$$y_4^p = y_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3) = 0 + \frac{4(0.2)}{3}[2(0.1996) - 0.3937 + 2(0.568)]$$

$$y_4^p = 0.3049 \quad , \quad y_4^{p'} = 0.707$$

$$y_4^c = y_2 + \frac{h}{3}(f_2 + 4f_3 + f_4^{p'}) = 0.0795 + \frac{0.2}{3}(0.3937 + 4(0.5689) + 0.707)$$

$$y_4^c = 0.3046$$

$$z_4^p = z_0 + \frac{4h}{3}(2f_1 - f_2 + 2f_3) = 1 + \frac{4(0.2)}{3}(2(1.081) - 1.308 + 2(1.377))$$

$$z_4^p = 1.4620 \quad , \quad z_4^{p'} = 1.322$$

$$z_4^c = z_2 + \frac{h}{3}(f_2 + 4f_3 + f_4^{p'}) = 1.4682 + \frac{0.2}{3}(1.308 + 4(1.377) - 1.3435)$$

$$z_4^c = 2.0107$$

Adam's Method:

$$y_4^p = y_3 + \frac{h}{24}\left(55y_3' - 59y_2' + 37y_1' - 9y_0'\right)$$

$$= 0.1762 + \frac{0.2}{24}\left(55(0.5689) - 59(0.393) + 37(0.1996) - 9(0)\right)$$

$$y_4^p = 0.3052 \quad , \quad y_4^{p'} = 0.7068$$

$$y_4^c = y_3 + \frac{h}{24}\left(9y_4' + 19y_3' - 5y_2' + y_1'\right)$$

$$= 0.1762 + \frac{0.2}{24}\left(9(0.7068) + 19(0.5689) - 5(0.393) + 0.1996\right)$$

$$y_4^c = 0.3045$$

$$z_4^p = z_3 + \frac{h}{24}\left(55z_3' - 59z_2' + 37z_1' - 9z_0'\right)$$

$$= 1.7379 + \frac{0.2}{24}\left(55(1.377) - 59(1.308) + 37(1.081) - 9(1)\right)$$

$$z_4' = 1.9842 \quad , \quad z_4^{p'} = 1.3442$$

$$z_4^c = z_3 + \frac{h}{24}\left(9z_4' + 19z_3' - 5z_2' + z_1'\right)$$

$$= 1.7379 + \frac{0.2}{24}\left(9(1.3442) + 19(1.377) - 5(1.308) + 1.081\right)$$

$$= 2.0112$$

Q.2 $\frac{dy}{dt} = -0.2y + \cos t$; $\frac{dx}{dt} = 0.3x + \sin t$

$t_0 = 0$	$x_0 = 2$	$y_0 = -1$	$y_0' = 1.2$	$x_0' = 0.6$
$t_1 = 0.1$	$x_1 = 2.06$	$y_1 = -0.88$	$y_1' = 1.1710$	$x_1' = 0.7178$
$t_2 = 0.2$	$x_2 = 2.1317$	$y_2 = -0.7648$	$y_2' = 1.1338$	$x_2' = 0.83381$
$t_3 = 0.3$	$x_3 = 2.215$	$y_3 = -0.6539$	$y_3' = 1.0861$	$x_3' = 0.96017$

Milne's Method:

$$y_4^p = y_0 + \frac{4h}{3}(2y_1' - y_2' + 2y_3') = -1 + \frac{4(0.1)}{3}\left(2(1.1710) - 1.1330 + 2(1.08611)\right)$$

$$y_4^p = -0.54917 \; ; \; y_4' = 1.0308$$

$$y_4^c = y_2 + \frac{h}{3}(y_2' + 4y_3' + y_4') = -0.7648 + \frac{0.1}{3}\left(1.1330 + 4(1.0861) + 1.0308\right)$$

$$y_4^c = -0.54785$$

$$x_4^p = 2 + \frac{4(0.1)}{3}\left(2(0.6) - 0.83817 + 2(0.96017)\right) = 2.30429$$

$$x_4' = 1.0807$$

$$x_4^c = 2.1317 + \frac{0.1}{3}\left(0.83817 + 4(0.9601) + 1.0807\right) = 2.3236$$

Adam's Method :-

$$y_4^p = y_3 + \frac{h}{24}(55y_3' - 59y_2' + 37y_1' - 9y_0')$$

$$= -0.6539 + \frac{0.1}{24}(55(1.0861) - 59(1.1338) + 37(1.171) - 9(1.2))$$

$$y_4^p = -0.548 \quad ; \quad y_4' = 1.031$$

$$y_4^c = y_3 + \frac{h}{24}(9y_4' + 19y_3' - 5y_2' + y_1')$$

$$= -0.6539 + \frac{0.1}{24}(9(1.031) + 19(1.0861) - 5(1.1338) + 1.171)$$

$$= -0.5471$$

$$x_4^p = x_3 + \frac{h}{24}(55x_3' - 59x_2' + 37x_1' - 9x_0')$$

$$= 2.215 + \frac{0.1}{24}(55(0.96017) - 59(0.838) + 37(0.717) - 9(0.6))$$

$$x_4^p = 2.3031 \qquad x_4' = 1.091$$

$$x_4^c = x_3 + \frac{h}{24}(9x_4' + 19x_3' - 5x_2' + x_1')$$

$$= 2.215 + \frac{0.1}{24}(9(1.091) + 19(0.9601) - 5(0.838) + 0.717)$$

$$x_4^c = 2.313$$