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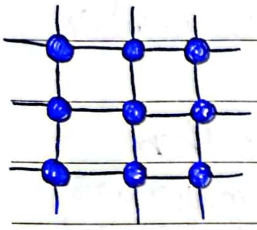
21L-1858
6-L

Date

Assignment 1

Q.1

i.) 2D Mesh without wraparound



rows : $\sqrt{p}$
col : $\sqrt{p}$
Num of Nodes : $\sqrt{p} \times \sqrt{p}$
$= p$

Num of links in each row : $\sqrt{p}-1$
Num of links in each col : $\sqrt{p}-1$

Cost : rows(Num of links in each row) + col(Num of links in each col) $= \sqrt{p}(\sqrt{p}-1)+\sqrt{p}(\sqrt{p}-1)$

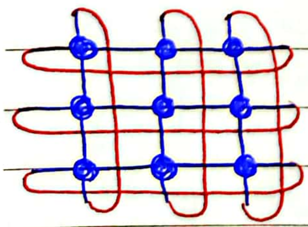
Cost : $p-\sqrt{p}+p-\sqrt{p} = 2p-2\sqrt{p} = 2(p-\sqrt{p}) = 2(9-3) = 2(6) = 12$

Diameter : (Num of links in each row) + (Num of links in each col) $= \sqrt{p}-1+\sqrt{p}-1 = 2\sqrt{p}-2$
$= 2(\sqrt{p}-1)$
$2(\sqrt{9}-1) = 2(2) = 4$

Bisection Width : Num of rows / Num of col $= \sqrt{p} = \sqrt{9} = 3$

Arc-Connectivity : 2

ii.) 2D Mesh with wraparound



Num of Nodes $= p$
rows : $\sqrt{p}$
col : $\sqrt{p}$

Diameter : $2\left\lfloor \frac{\text{No. of rows/col}}{2} \right\rfloor$

Diameter : $2\left\lfloor \frac{\sqrt{p}}{2} \right\rfloor = 2\left\lfloor \frac{3}{2} \right\rfloor$
$2\lfloor 1.5 \rfloor = 2(1)$
$= 2$

Cost : 2(Num of Nodes) $= 2p = 2(9) = 18$
Bisection Width : 2(rows/col) $= 2\sqrt{p} = 2\sqrt{9} = 2.3 = 6$
Arc Connectivity : 4

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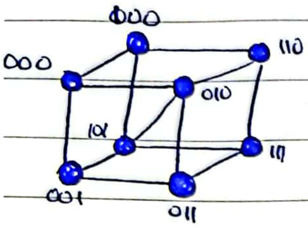
iii.) 3-D Cube



Num of nodes = p

Cost: every node has 3 connections so $p \times 3$

To avoid double counting it becomes $\frac{p \times 3}{2} = \frac{8 \times 3}{2} = 12$

This will hold true for all cube topologies with 3 connections

Diameter: Each node is directly connected to its adjacent nodes in three dimensions (x, y and z). The farthest two nodes in the network are those placed at the opposite corners of the cube so

Length of shortest path using edges = $1+1+1 = 3$

Bisection Width = $\frac{\text{num of nodes}}{2} = \frac{8}{2} = 4$, we can cut along both x or y-axis, in both cases, answer will be the same

Arc-Connectivity = 3

^ Since each node is connected to three adjacent nodes

iv.) Linear Array without wraparound

Num of nodes = p

Cost: Num of nodes $- 1 = p - 1 = 5 - 1 = 4$

Diameter $= p - 1 = 5 - 1 = 4$

Bisection Width = 1

Arc-Connectivity = 1

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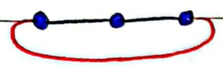
v.) Linear Array with Wrap Around



Cost : p Bisection Width : 2

Diameter : 1 Arc-Connectivity : 2

vi.) HyperCube :

An a hypercube network with p nodes, p is a power of 2, denoted as 2^k where k is the number of dimensions

Each node is connected to $\log_2 p$ other nodes

Cost: Since there are p nodes in total $= p\log_2 p$

To avoid double counting it becomes : $\frac{p\log_2 p}{2}$

Diameter: The farthest two nodes are those placed on opposite corners so the formula in a hypercube network becomes

$p = 2^k$, the max dist b/w any two nodes is k

$\Leftrightarrow k = \log_2 p$

Diameter $= \log_2 p$

Arc-Connectivity : Since each node is connected to $\log_2 p$ other nodes so that is it's arc-connectivity.

Bisection Width : Since the num of nodes is p, we need to cut through $\frac{p}{2}$ edges to divide the hypercube in half.

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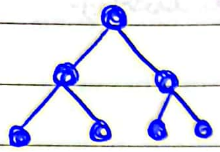
vii) Complete Binary Tree :-



Each node (except the root) has exactly one parent node & can have at most two children nodes

Cost: $p-1$ (every node except root has exactly one incoming edge).
$7-1=6$

Diameter: The depth of a binary tree with p nodes is $\log_2 p$
To accomodate the case where p is not a power of 2 we add 1:
so it becomes: $\log(p+1)$

The longest path in a binary tree goes from deepest leaf node to another deepest leaf node, we count both upward & downward traversals.

$$\text{diameter: } 2\log\left\lfloor \frac{(p+1)}{2} \right\rfloor$$

upward & downward traversal

floor to get integer value for height

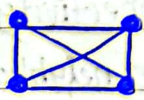
divide by 2 to get height

$$2\log\left\lfloor \frac{(7+1)}{2} \right\rfloor$$

$$2\log\left\lfloor \frac{8}{2} \right\rfloor = 2\log(4) = 4$$

Bisection Width: 1   **Arc-Connectivity:** 1

viii.) Completely Connected :



Diameter: 1 (Since each node is connected to every other node)

Arc-Connectivity: $p-1$

Bisection Width: Divide the network into two equal parts, each part will contain $\frac{p}{2}$ nodes, and each node in one group needs to be connected to each node in the other group so

$$\frac{p}{2} \times \frac{p}{2} = \frac{p^2}{4} = \text{Bisection Width}$$

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Cost: Num of nodes : p

Num of edges : $p-1$

each node is connected to every other node except itself

To avoid double counting : $\frac{p(p-1)}{2}$

Q.2

	Completely Connected	Mesh
Cost:	$\frac{p(p-1)}{2}$ each node connected to all other nodes Quadratic Complexity : p^2 Gets expensive when num of nodes increase	$2(p-\sqrt{p})$ each node connected to immediate neighbours only Linear Complexity : p
Arc Connectivity:	Max ac : $p-1$	2 ; lower ac in comparison to completely connected
Diameter:	1 ; highest level of connectivity possible	$2(\sqrt{p}-1)$; Moderate Connectivity, directly connected to immediate neighbour
Bisection Width:	$\frac{p^2}{4}$, relatively high compared to Mesh Network	$\sqrt{p}$, low compared to completely connected

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Completely Connected offers low latency as the diameter is 1 and there is no need to traverse multiple hops BUT as the num of nodes grow, the num of connections increase rapidly which leads to scalability issues.
Mesh Networks are more scalable than completely connected networks, as the increase in num of nodes does not lead to significant increase in num of edges BUT the diameter of nodes located far apart diagonally can result in longer paths.

Completely Connected Interconnection is suitable for scenarios where max connectivity b/w nodes is required such as high-performing computing applications.
Mesh interconnection networks are suitable for scenarios where moderate connectivity is sufficient such as in distributed systems or small to medium sized networks & a balance b/w cost and connectivity is desired.