Name: Umang Mishra
BITS ID: 2020FC04103 (Section 1)

Q:1 A/Q Let the definition of variables be:

x_1 = number of blankets
x_2 = number of bedsheets
x_3 = number of pillows
x_4 = number of pillow covers

Products = 4 {Blankets, Bedsheets, Pillows, Pillow Covers}
Department = 4 {Cutting, Custioning, Fastening, Packaging}

	Blankets	Bedsheets	Pillow	Pillowcover	Capacity
Cutting	10	10	8	5	1500
Custioning	8	12	10	4	1500
Fastening	15	20	15	8	1500
Packaging	5	5	2	1	1500
Demand	1200	1000	800	750	
Unit Profit	40	50	20	10	

Since, we have 4 variables, & 4 processes, thus we will have only 4 constraints to care for having an optimized solution.

$$\left.\begin{aligned} 10x_1 + 10x_2 + 8x_3 + 5x_4 &\leq 1500 \\ 8x_1 + 12x_2 + 10x_3 + 4x_4 &\leq 1500 \\ 15x_1 + 20x_2 + 15x_3 + 8x_4 &\leq 1500 \\ 5x_1 + 5x_2 + 2x_3 + x_4 &\leq 1500 \end{aligned}\right\} \text{Constraints}$$

To maximize, $Z = 40x_1 + 50x_2 + 20x_3 + 10x_4$

where

$x_1\ x_2\ x_3\ x_4 \geq 0$

But $x_1 \leq 1200$ $\quad x_2 \leq 1000$ $\quad x_3 \leq 800$ $\quad x_4 \leq 750$

Also assumming the production will not go beyond the demand.

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Q:2 A/Q Given function $F(x,y,z) = \underbrace{3x^2+y+z^2}_{\to \text{eq (i)}}$ is subject to $\underbrace{x^2+z^2=16}_{\to \text{eq (ii)}}$

From eq^n (ii), we get $z^2 = 16-x^2$

On substituting this in eq (i) we get.

$$F(x,y,z) = 3x^2+y+16-x^2$$

or $F = 2x^2+y+16 \longrightarrow$ eq (iii)

Since Hessian Matrix of a multivariable function is given by -

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} & \cdots \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} & \cdots \\ \vdots & & \end{bmatrix}$$

In our case, Hessian matrix would be given by

$$H_f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$$

Now, lets find derivatives for eq (iii), we get $\frac{\partial^2 f}{\partial x^2} = 4$

$\frac{\partial^2 f}{\partial x \partial y} = 0$ $\quad \frac{\partial^2 f}{\partial y \partial x} = 0$ and $\frac{\partial^2 f}{\partial y^2} = 0$

So $H_f = \begin{bmatrix} 4 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow$ eq (iv)

Now for checking the convexity/concavity, we need to find the eigen value for eq (iv).

So, $\begin{vmatrix} 4-\lambda & 0 \\ 0 & 0-\lambda \end{vmatrix} = 0 \Rightarrow \lambda^2 - 4\lambda = 0$ or $\boxed{\lambda = 0, 4}$

∴ Since, one of the eigen value is a positive number, and the other one is 0. So, we can conclude that the given function is a Convex function.

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Q:3 SVD is a method for decomposing a matrix into three different matrices, such that

$$A = U \Sigma V^T$$

A/Q we have $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Calculating the SVD consists of finding the eigenvalues and eigenvectors of A^TA and AA^T.

Now we have $AA^T = \begin{bmatrix} 14 & 32 \\ 32 & 77 \end{bmatrix} \Rightarrow \begin{vmatrix} (14-\lambda) & 32 \\ 32 & (77-\lambda) \end{vmatrix} = 0$

So the eigenvalues for AA^T would be $\underline{0.597 \ \& \ 90.402}$

eigenvectors for AA^T would be $\begin{bmatrix} 0.418 & -2.387 \\ 1 & 1 \end{bmatrix} = EV_1$

on normalizing EV_1, we would get $EV_1' = \begin{bmatrix} 0.386 & -0.922 \\ 0.922 & 0.386 \end{bmatrix} = U$

Similarly for $A^TA = \begin{bmatrix} 17 & 22 & 27 \\ 22 & 27 & 36 \\ 27 & 36 & 45 \end{bmatrix} \Rightarrow \begin{vmatrix} (17-\lambda) & 22 & 27 \\ 22 & (27-\lambda) & 36 \\ 27 & 36 & (45-\lambda) \end{vmatrix} = 0$

So the eigenvalues would be $\underline{0, \ 0.597 \text{ and } 90.402}$

eigenvectors would be $EV_2 = \begin{bmatrix} 0.608 & -1.386 & 1 \\ 0.804 & -0.193 & -2 \\ 1 & 1 & 1 \end{bmatrix}$

on normalizing EV_2 we would get $V = \begin{bmatrix} 0.428 & -0.805 & 0.408 \\ 0.566 & -0.112 & -0.816 \\ 0.704 & 0.581 & 0.408 \end{bmatrix}$

Σ is given by the square-root of the most important (higher) eigenvalues.

So $\Sigma = \begin{bmatrix} \sqrt{90.402} & 0 & 0 \\ 0 & \sqrt{0.593} & 0 \end{bmatrix} = \begin{bmatrix} 9.508 & 0 & 0 \\ 0 & 0.772 & 0 \end{bmatrix}$

So, we can say that the SVD for the given A would be $\boxed{U \Sigma V^T}$

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Also, the maximum value for $||A||x$ would be obtained at the eigenvector corresponding to the highest eigenvalue, which is 90.402 (here)

So $$A \times \begin{bmatrix} 0.608 \\ 0.804 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ \cancel{4} & 5 & 6 \end{bmatrix}_{2\times3} \begin{bmatrix} 0.608 \\ 0.804 \\ 1 \end{bmatrix}_{3\times2}$$

or $$Ax = \begin{bmatrix} 5.217 \\ 12.456 \end{bmatrix}_{2\times2}$$

Now $$||Ax|| = \sqrt{(5.217)^2 + (12.456)^2} = \sqrt{27.185 + 155.151}$$

$$||Ax|| = \sqrt{182.336} = \underline{13.503 \text{ units}}$$

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Q:4

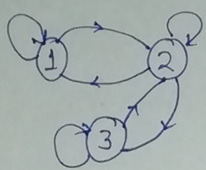
(a) A relation which is Reflexive and Symmetric, But not Transitive.

For R: $f(x,x)\ \forall x \in R$

R is reflexive here as $f(x,x) \rightarrow R$.

R is symmetric here as $(x,y) \in R$ & $(y,x) \in R\ \forall x,y \in R$.

R is not-transitive as $(x,y) \in R$ & $(y,z) \in R$, but $(x,z) \notin R$.



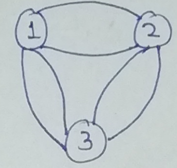
R = {(1,1) (2,2) (3,3)
(1,2) (2,1) (2,3) (3,2)}

	1	2	3
1	1	1	0
2	1	1	1
3	0	1	1

(b) A relation which is ~~neither~~ not reflexive, ~~nor~~ symmetric but transitive.

R = {(1,2) (2,1) (1,3) (3,1)
(2,3) (3,2) }

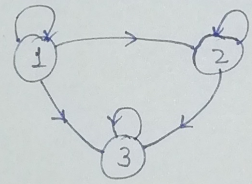
Here R is not reflexive but is both Symmetric and Transitive



	1	2	3
1	0	1	1
2	1	0	1
3	1	1	0

(c) A relation which is reflexive and transitive, but not symmetric.

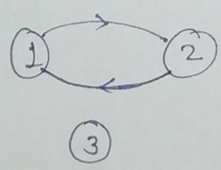
R = {(1,1) (2,2) (2,3)
(3,3) (1,2) (1,3) }



	1	2	3
1	1	1	1
2	0	1	1
3	0	0	1

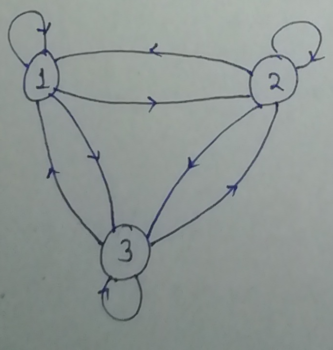
(d) A relation which is not reflexive and transitive but only symmetric

R = {(1,2) (2,1)}



	1	2	3
1	0	1	0
2	1	0	0
3	0	0	0

(e) An equivalence relation. R = { (1,1) (2,2) (3,3) (1,2) (2,1) (1,3)
(3,1) (2,3) (3,2) }



	1	2	3
1	1	1	1
2	1	1	1
3	1	1	1