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Analytical Analysis - Science II - Class Assgn-1

Q1. 2 Drunk men start at origin & take N steps (left or right equally probable). Find probability of them meeting after N steps.

Ans: At any point, displacement from origin, $D = N_{Right} - N_{Left}$

Let x be no. of steps right

$\therefore D = x - (N-x) = 2x - N \Rightarrow x = \frac{N+D}{2}$

Using Binomial,
P(taken x steps right after N steps) :-

$$P_N(x) = \frac{N!}{x!(N-x)!}\left(\frac{1}{2}\right)^N$$

Since they meet $D = 0$ $\therefore x = \frac{N}{2} \times 2$ (for each drunk)
$\Rightarrow$ Total steps $= 2N$

$$\therefore P(\text{meeting again}) = \frac{(2N)!}{N!\,N!} \times \left(\frac{1}{2}\right)^{2N}$$

$$= \frac{(2N)!}{(N!)^2\, 2^{2N}}$$

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Related questions:

a) P(Meeting at origin after N steps)

To meet at origin or to reach origin, N must be even

$\Rightarrow \quad P = 0 \quad$, when N is odd

$\therefore \quad P(\text{origin} \mid N = \text{even}) = {}^{N}C_{\frac{N}{2}} \left(\frac{1}{2}\right)^{N}$

$\therefore P(\text{both meeting at origin}) = \begin{cases} \dfrac{({}^{N}C_{N/2})^2}{2^{2N}}, & N \text{ is even} \\ 0, & N \text{ is odd} \end{cases}$

b) Mean displacement

let X be RV representing the displacement of drunk man from origin

$X = X_1 + X_2 + X_3 + X_4 \cdots \cdots X_N$ where X_i = each step.

Since all steps are independent

$\therefore \quad E[X] = E[X_1 + X_2 \cdots X_N]$

$= E[X_1] + E[X_2] + \cdots\cdots$

$= \sum_{i=1}^{N} E[X_i]$

Now, $\quad E[X_i] = \frac{1}{2} \times (1) + \frac{1}{2} \times (-1) = 0.$

right   left

$\therefore \quad E[X] = \sum_{i=1}^{N} E[X_i] = \underline{0}$

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c) Mean square displacement.

Let D denote the displacement of drunk from origin

$D = x_1 + x_2 + x_3 \cdots x_N$ where x_i = each step.

$D^2 = (x_1 + x_2 \cdots x_N)^2$

$\langle D^2 \rangle = \langle (x_1 + x_2 + \cdots x_N)^2 \rangle$

$= \langle x_1^2 + x_2^2 \cdots x_N^2 + 2(x_1 x_2 + x_2 x_3 \cdots \cdots x_{N-1} x_N) \rangle$

Since all steps are independent.

$\therefore \langle D^2 \rangle = \langle x_1^2 \rangle + \langle x_2^2 \rangle \cdots \langle x_N^2 \rangle + 2(\langle x_1 x_2 \rangle + \langle x_2 x_3 \rangle + \cdots \langle x_{N-1} x_N \rangle)$

Now, $\langle x_i^2 \rangle = \underbrace{\frac{1}{2} \times (1)^2}_{\text{right}} + \underbrace{\frac{1}{2} \times (-1)^2}_{\text{left}} = 1$

$\langle x_i x_j \rangle = \frac{1}{4} \times (-1) + \frac{1}{4} \times (1) + \frac{1}{4} \times (-1) + \frac{1}{4} \times (1)$

$= 0$

$\therefore \langle D^2 \rangle = \sum_{i=1}^{N} \langle x_i^2 \rangle + 0$

$= N$