# Checking Tail Recursion in PicoML

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December 16, 2015

### 1 Overview

#### 1.1 Recursion

Allowing procedures to be recursive allows the programmer to write more readable and intuitive/natural programs. Recursive programs, at times, tend to be more efficient than a naive program with loops and no recursive calls. Recursion, thus is a handy tool for programmers.

### 1.2 Checking Tail Reclusion: Motivation

The convenience offered comes at a cost. Recursive programs are generally modelled by the use of stack frames. This means that recursive programs tend to consume extra space (stack) for every recursive call they make.

However, the extra space consumed can be overcome when the recursive call is the last thing the function does. In this case, the contents of the stack can be replaced by the new frame, and there is no need to push an additional frame.

The idea behind tail call optimization is essentially the same. Informally, a recursive function is tail recursive when the recursive call is the last thing executed by the function. Thus, if the compiler can detect if a function is tail-recursive, it can convert the function to an equivalent while-loop, thus avoiding an additional call that consumes extra stack space by virtue of the new frame added.

#### 1.3 Goal of the Project

In this project, we implement a tool that checks if an procedure is tail recursive or not. Specifically, we wish to analyze code written in PicoML PicoML is a restricted form of OCaml. As part of the assignments in the course, we have built an interpreter for this language. We aim to integrate the tool with the interpreter. That is, we would use the parsing and the type checking functionality written in older assignments. This would enable use to directly use the functionality for implementing our tool, and would save some effort, as compared to the scenario where we had to re-invent the wheel.

# 2 Implementation

We built two tail recursive checking programs. One for normal functions is based on MP6 while the other for CPS functions is based on MP7.

### 2.1 Tail Recursive Checking Based on MP6

Here is our proposal:

## 2.2 Tail Recursive Checking for CPS Based on MP7

Here is our proposal:

### 2.3 Code Structure

Describe the code structure. Justify that the code is modular

## 3 Tests

[TODO]We put our test cases into grader of MP6 and MP7. So to test our programs, just run './grader' after 'make', as in what we have to do in the assignments.

## 4 Listing

### 4.1 Direct stype PicoML expressions

```
open Definitions;;
  let rec check_let_in_meaningful x e =
      match e
      with ConstExp c -> false
       | VarExp v \rightarrow if (v = x) then true else false
       | MonOpAppExp (mon_op, e1) -> check_let_in_meaningful x e1
       | BinOpAppExp (bin_op, e1, e2) -> (check_let_in_meaningful x e1) || (check_let_in_meaningful
      x e2)
       | IfExp (e1, e2, e3) ->
          (check_let_in_meaningful x e1) || (check_let_in_meaningful x e2) || (
10
       check_let_in_meaningful x e3)
11
       \mid LetInExp (s, e1, e2) ->
           if (check_let_in_meaningful x e1)
12
               then (check_let_in_meaningful s e2)
13
               else (
14
                   if (x=s)
15
16
                       then false
17
                       else check_let_in_meaningful x e2
18
       | FunExp (s, e1) \rightarrow if (s=x) then false else (check_let_in_meaningful x e1)
19
       | AppExp (e1, e2) ->
20
           (check_let_in_meaningful x e1) \mid \mid (check_let_in_meaningful x e2)
21
22
       | LetRecInExp (g, y, e1, e2) \rightarrow
           if ((g=x) || (y=x))
23
               then false
24
25
               else if (check_let_in_meaningful x e1)
                   then (check_rec_f g e2)
26
27
                   else (check_let_in_meaningful x e2)
28
       | RaiseExp e1 -> (check_let_in_meaningful x e1)
       | TryWithExp (e0, nlopt, e1, nopt_e_lst) ->
29
           (check_let_in_meaningful x e0) || (check_let_in_meaningful_lst x ((nlopt,e1)::nopt_e_lst)
30
  and check_let_in_meaningful_lst x nopt_e_lst =
      match nopt_e_lst
32
      with [] -> false
33
34
       | (nopt, en)::rest -> (check_let_in_meaningful x en) || (check_let_in_meaningful_lst x rest)
35 and check_rec_f f e =
      match e
36
      with ConstExp c -> false
37
       | VarExp v -> false
38
      | MonOpAppExp (mon_op, e1) -> check_rec_f f e1
39
       | BinOpAppExp (bin_op, e1, e2) -> (check_rec_f f e1) || (check_rec_f f e2)
40
41
       | IfExp (e1, e2, e3) ->
           (check_rec_f f e1) || (check_rec_f f e2) || (check_rec_f f e3)
42
       | LetInExp (s, e1, e2) ->
43
          if (check_rec_f f e1)
44
               then (check_let_in_meaningful s e2)
45
46
               else
47
48
                   if (s=f) then false else ( (check_rec_f f e1) || (check_rec_f f e2) )
49
50
       | FunExp (s, e1) -> if (s=f) then false else (check_rec_f f e1)
51
       | AppExp (e1, e2) ->
52
          (
           match el
53
           with VarExp v \rightarrow if (v=f) then true else false
54
           | _ -> (check_rec_f f e1) || (check_rec_f f e2)
55
56
57
       | LetRecInExp (q, x, e1, e2) \rightarrow
    if ( (g=f) || (x=f))
```

```
then false
59
                else if (check_rec_f f e1)
60
                    then (check_rec_f g e2)
61
62
                    else (check_rec_f f e2)
       | RaiseExp e1 -> (check_rec_f f e1)
63
       | TryWithExp (e0, nlopt, e1, nopt_e_lst) ->
64
65
            (check_rec_f f e0) || (check_rec_f_lst f ((nlopt, e1) :: nopt_e_lst) )
66
   and check_rec_f_lst f nopt_e_lst =
67
68
       match nopt_e_lst
69
       with [] -> true
       ((nnopt, en)::rest) -> ((check_rec_f f en) || (check_rec_f_lst f rest));;
70
71
72 let rec check_tail_rec_f f e =
73
       match e
       with ConstExp c -> true
74
75
       | VarExp v -> true
       | MonOpAppExp (mon_op, e1) -> not (check_rec_f f e1)
76
       | BinOpAppExp (bin_op, e1, e2) -> (not (check_rec_f f e1)) && (not (check_rec_f f e2))
77
       | AppExp(e1, e2) -> if (check_rec_f f e2)
78
79
            then false
            else check_tail_rec_f f e1
80
       | IfExp (e1, e2, e3) ->
81
82
                (not (check_rec_f f e1)) &&
                (check_tail_rec_f f e2) &&
83
                (check_tail_rec_f f e3)
84
       | FunExp (x, e1) \rightarrow true
85
       | LetInExp (x, e1, e2) \rightarrow
86
87
            if (x = f)
                then (not (check_rec_f f el))
88
                else ( (not (check_rec_f f e1)) && (check_tail_rec_f f e2))
89
       | LetRecInExp (g, x, e1, e2) \rightarrow
90
            if (g = f) then true
91
92
            else if (not (g=f) && (x=f)) then (check_tail_rec_f f e2)
           else ( (check_tail_rec_f f e2))
93
94
        (* Before fix:
           if (g = f) then true
95
96
            else if (not (g=f) && (x=f)) then (check_tail_rec_f f e2)
            else ( (not (check_rec_f f e1)) && (check_tail_rec_f f e2))
97
98
       | TryWithExp (e', nlopt, el, nopt_e_lst) ->
99
100
            (
            if (check_rec_f f e')
101
                then false
102
                else let lst = ((n1opt, e1)::nopt_e_lst)
103
104
                    (List.fold_right (fun (intop, h) -> fun t -> (check_tail_rec_f f h) && t) lst
105
       true)
106
       | _ -> false ;;
107
108
109
110
   let check_tail_recursion dec =
       match dec
111
112
       with (Anon e) -> true
       | Let (s, e) -> true
113
       | LetRec (f, x, e) ->
114
115
           check_tail_rec_f f e ;;
```

Listing 1: Tail recursion for PicoML expressions

### 4.2 CPS transformed PicoML expressions

```
2 open Definitions
4
  let rec convert_f_exp e name_of_f =
      match e
6
       with ConstCPS (k, c) ->
           print_string "\nConstCPS \n";
9
10
          convert_f_cont k name_of_f
11
       \mid VarCPS (k, g) ->
12
13
          (*
           print_string "\nVarCPS \n";
14
15
           *)
               (
16
17
               {\tt match}\ {\tt k}
               with FnContCPS (number_of_f, e') ->
18
                   let rec_list = convert_f_exp e' name_of_f
19
20
                   in
                   if (g = name_of_f)
21
22
                            then (number_of_f :: rec_list)
                            else rec_list
23
24
25
       | MonOpAppCPS (k, _, _, _) ->
26
27
           print_string "\nMonOpAppCPS \n";
28
29
           *)
           convert_f_cont k name_of_f
30
31
       | BinOpAppCPS (k , _, _, _, _) ->
32
          (*
           print_string "\nBinOpAppCPS \n";
33
34
           convert_f_cont k name_of_f
35
       | IfCPS (b, e1, e2) ->
36
37
          (*
           print_string "\nIfCPS \n";
38
39
           *)
           (convert_f_exp e1 name_of_f)@(convert_f_exp e2 name_of_f)
40
       | AppCPS (k, _, _, _) ->
41
42
           (*
          print_string "\nAppCPS \n";
43
44
          convert_f_cont k name_of_f
45
46
      | FunCPS (k, _, _, _, _) ->
47
          print_string "\nFunCPS \n";
48
49
           *)
50
           convert_f_cont k name_of_f
       | FixCPS (k, \_, \_, \_, \_, \_) ->
51
          (*
52
          print_string "\nFixCPS \n";
53
54
           convert_f_cont k name_of_f
55
56
57
58 and
59
60 convert_f_cont k name_of_f =
61
      (
      match k
62
  with FnContCPS (_, e') ->
```

```
convert_f_exp e' name_of_f
64
65
       | _ -> []
       ) ;;
66
67
68
69 let rec check_cps_tail_rec_f flist original_k x k e =
70
       match e
       with ConstCPS (k', c) ->
71
72
           cont_tail_recursive original_k k' flist
        \mid VarCPS (k', v) ->
73
74
           (*
           print_string "\nVarCPS\n";
75
76
           *)
77
            cont_tail_recursive original_k k' flist
        | MonOpAppCPS (k', mono_op, o1, exk) ->
78
79
           (*
           print_string "\nMonOpAppCPS\n";
80
           *)
81
82
            cont_tail_recursive original_k k' flist
        | BinOpAppCPS (k', bin_op, o1, o2, exk) ->
83
84
            (*
           print_string "BinOp\n";
85
86
            *)
87
            cont_tail_recursive original_k {\tt k'} flist
        | IfCPS (b, e1, e2) ->
88
            (check_cps_tail_rec_f flist original_k x k el) && (check_cps_tail_rec_f flist original_k
89
        x k e2)
        \mid AppCPS (k', e1, e2, exk) ->
90
91
            (
92
            (*
           print_string ("AppCPS:\n"^e1^", "^e2^", f: "^f^"\n");
93
94
           if (List.exists (fun x \rightarrow x = e1) flist)
95
                then
96
97
                    if (k'=original_k)
98
                        then
99
100
                             (*
                             print_string "true\n";
101
102
                             *)
103
                             true
                         else
104
                              (*
105
                              print_string "false\n";
106
107
                              false
108
                    )
109
110
                else
                    cont_tail_recursive original_k k' flist
111
112
113
        \mid FunCPS (kappa, x, k, ek, e) -> true
        | FixCPS (kappa, f, x, k, ek, e) -> true
114
115
and cont_tail_recursive original_k k flist =
117
       match k
       with ContVarCPS i -> true
118
       | External -> true
119
120
        \mid FnContCPS (x, e) ->
           check_cps_tail_rec_f flist original_k x k e (*TODO x ?*)
121
122
        | ExnMatch ek -> true ;;
123
125 let check_tail_recursion dec =
match dec
127
       with Anon e -> true
```

```
| Let (x,e) \rightarrow true
128
129
        | LetRec (f, x, e) \rightarrow
            let (i,j) = (next_index(),next_index())
130
131
                let ecps2 = cps_exp e (ContVarCPS i) (ExnContVarCPS j)
132
133
134
                print_string ((string_of_exp_cps ecps2)^"\n");
135
136
                let flist = convert_f_exp ecps2 f
137
138
                     in
                     check_cps_tail_rec_f flist (ContVarCPS i) x (ContVarCPS i) ecps2 ;;
139
```

Listing 2: Tail recursion for CPS transformed expressions

# References

[1] Leslie Lamport, LaTeX: a document preparation system, Addison Wesley, Massachusetts, 2nd edition, 1994.