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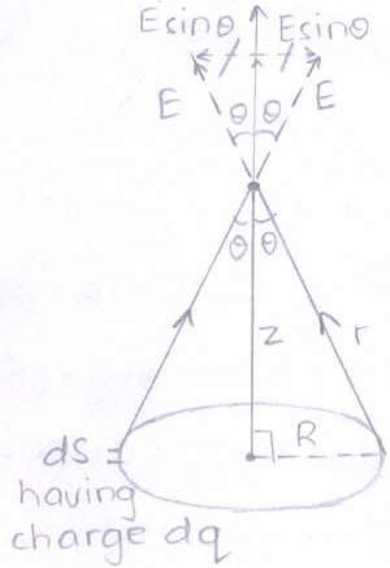
Question #2:-

Part a:-

Electric Field due to ring of charge:-



Consider a ring of charge, as shown above. The circumference of the ring is "S", the total charge on it is "q" and has a field "E". In order to calculat E at point P, as marked, we consider a differentiated arbitary segment of the ring, having length "ds" and a charge ~~density~~ "dq".

we know, $\lambda = \frac{q}{l}$

here for the differentiated segment

$$\lambda = \frac{dq}{ds} \quad \text{—(1)}$$

The differentiated field due to this dq will be according to Coulomb's law;

eq(2) becomes;

$$dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{dq}{(z^2+R^2)}$$

From (1); $dq = \lambda dS$

$$\Rightarrow dE = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dS}{z^2+R^2} \quad \text{— (3)}$$

If we notice, another dE acts on the point p from exactly the opposite end of the ring; Now, the resultant field on point p can be calculated by resolving each dE into the rectangular components. From figure, the vertical components get cancelled out. Therefore, we are left with;

$$dE_x = 2\, dE \cos\theta \quad \text{— (4)}$$

From figure;

$$\cos\theta = \frac{z}{\sqrt{z^2+R^2}} \quad \text{— (5)}$$

Using (5) and (3) in eq (4);

$$dE_x = 2 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda dS}{z^2+R^2} \cdot \frac{z}{\sqrt{z^2+R^2}}$$

$$dE_x = \frac{1}{2\pi\epsilon_0} \cdot \frac{\lambda z}{(z^2+R^2)^{3/2}} \cdot dS$$

Integrating b/s, we get the total field.

$$\int dE_x = \frac{1}{2\pi\epsilon_0} \frac{\lambda z}{(z^2+R^2)^{3/2}} \int dS$$

$$E = \frac{\lambda z}{\quad} \cdot S$$

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we know that; $\lambda = \frac{q}{S}$ for ring

$$\Rightarrow \lambda S = q \qquad \because S = 2\pi R$$

eq(1) becomes;

$$\boxed{E = \frac{1}{4\pi\epsilon_0} \cdot \frac{qz}{(R^2+z^2)^{3/2}}}$$

Field intensity due to ring of charge!

Special Case:-

if $z \gg R$, i.e. point p goes away from the ring, the R can be neglected.

$\therefore$ the eq. becomes;

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{z^2}$$

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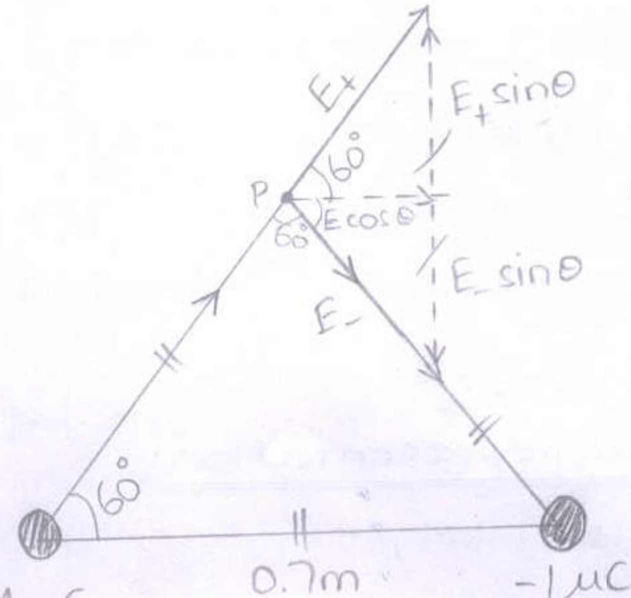
i.e. field due to a point charge.

$\therefore$ at large distance, the ring appears to be a point charge..

Part b:-



$E_+ = E_- = E$

$F = k\frac{qq}{r^2}$

Solution:-

From the figure, we see that;

$$E_p = 2E\cos\theta$$

Applying coulombs law;

we know; $|q_1| = |q_2| = 1\mu C = q$

$$E_p = 2 \cdot \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\cos\theta$$

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putting the given values;

$$E_p = \frac{2(9\times10^9 Nm^2/C^2)(1\times10^{-6}C)\cos 60^\circ}{(0.7m)^2}$$

$$E_p = \frac{9000}{0.49} N/C$$

$$\boxed{E_p = 1.836\times10^4 N/C}$$

Question #3:-

Gauss's Law:-

Statement:-

This law can be defined as;

Diagram?

"The electric flux through a closed figure is $1/\epsilon_0$ times the total charge enclosed in it."

Mathematical Representation:-

It is written as;

Φ_E

$$\varphi_E = \frac{1}{\epsilon_0} \cdot q$$

total charged enclosed in a closed figure.

Applications of Gauss's law:-

1. Flux/Field by an infinite line of charges:-

To solve a problem using Gauss's law, we first need to decide an appropriate Gaussian surface, then we find the flux throught that Gaussian surface. In case of infinite line of charges, we select ~~a~~ cylinder ~~as~~, enclosing the line of charges, having length/height "h" and radius "r". The field lines due to the charges ~~is~~ are outwards, perpendicular to the curved surface of the cylinder. Flux i.e. $\varphi_E = \bar{E}.\bar{A}$. will be maximum through the curved surface, There will be no flux passing through the upper and lower side of the cylinder, as the angle b/w $\bar{E}$ and $\bar{A}$ is 90° and $\cos 90^\circ = 0$.

Therefore; Flux can be calculated as;

$$\varphi_E = \bar{E}.\vec{A}$$

$$\varphi_E = EA\cos\theta \qquad \theta = 0^\circ$$

$$\varphi_E = EA$$

we do not know q; therefore we can write it in terms of λ;

$$\lambda = \frac{q}{h}$$

$$\Rightarrow q = \lambda h$$

eq① becomes;

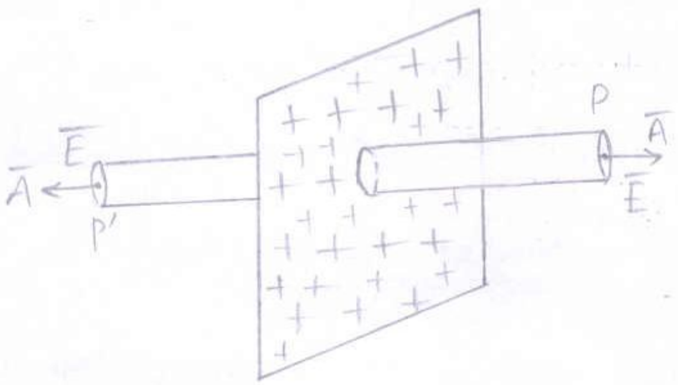
$$\lambda \cancel{h} = E(2\pi \epsilon_0 r \cancel{h})$$

$$\lambda = E(2\pi \epsilon_0 r)$$

$$\Rightarrow \boxed{E = \frac{1}{2\pi \epsilon_0 r} \cdot \lambda}$$

field due to infinite line of charge.

2. Field due to continuous sheet of charge:-



$5\frac{1}{2}$

The appropriate figure, we chosed is a long cylinderical pipe. we can see that from the figure that flux throught the curved portion is zero, as $\vec{A}$ and $\vec{E}$ make an angle of 90°.

Flux is only passing through the flat plates of the cylinder. Therefore total flux will be;

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Rearranging the eq:

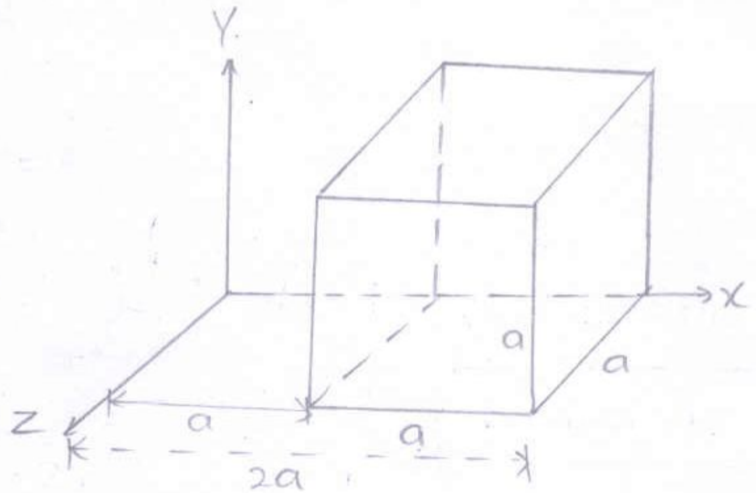
$$2E = \frac{q}{A} \cdot \frac{1}{\epsilon_0}$$

we know that $\sigma = \frac{q}{A}$ (charge density)

$$\Rightarrow 2E = \frac{\sigma}{\epsilon_0}$$

$$\boxed{E = \frac{\sigma}{2\epsilon_0}}$$

Part b:-



The Given data:-

$E_x = bx^{1/2}$

$E_y = E_z = 0$

$b = 8830\ N/cm^{1/2}$, $a = 13\,cm = 0.13\,m$

i) $\varphi_E = ?$

ii) $q = ?$

Solution:-

Flux will be only through right and left faces of cube:

putting the values we get;

$$\varphi_R = (8830\, N/Cm^{1/2})(2\times 0.13)^{1/2}(0.13)^2$$

$$\varphi_R = 76.09\ Nm^2/c \quad (+ve;\ outwards)$$

Taking inward flux as -ve)

$$\varphi_L = -E_x A$$

$$\varphi_L = -b x^{1/2} A$$

$$x = a$$

$$\Rightarrow \varphi_L = -(8830)(0.13)^{1/2}(0.13)^2$$

$$\varphi_L = -53.80\ Nm^2/c$$

Total flux will be;

$$\varphi_E = \varphi_R + \varphi_L$$

$$\varphi_E = (76.09 - 53.80)\ Nm^2/c$$

i) $\boxed{\varphi_E = 22.29\ Nm^2/c}$ ✓

From Gauss's law;

$$\varphi_E = \frac{1}{\epsilon_0} q$$

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$$\Rightarrow q = \varphi_E \epsilon_0$$

$$\epsilon_0 = 8.85\times 10^{-12}\ C^2/Nm^2$$

$$\Rightarrow q = (22.29\ Nm^2/c)(8.85\times 10^{-12}\ C^2/Nm^2)$$

ii) $\boxed{q = 1.97\times 10^{-10}\ C}$ ✓

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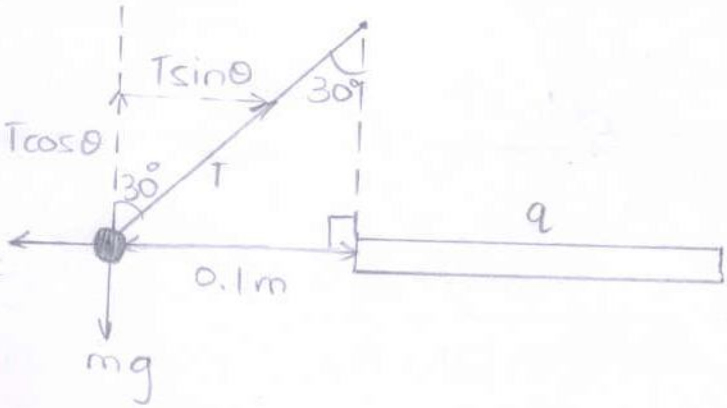
Question #1:-

Part b:-



Given data:-

$m = 1\text{ gm} = 1\times10^{-3}\text{ kg}$

charge on rod $= q$

charge on ball $= 0.01q = q'$

$d = 0.1\text{m}$

$T = ?$

$q' = ?$

Solution:-

The system is in equilibrium;

$\Sigma F_x = 0$ and $\Sigma F_y = 0$

i) $\Sigma F_x = 0$

$F - T\sin\theta = 0$

$F = T\sin\theta$ — (1)

ii) $\Sigma F_y = 0$

$mg = T\cos\theta$

$T = \dfrac{mg}{\cos\theta}$ — (2)

Using (2) in (1);

$\boxed{F = mg\tan\theta}$ — (3)

From (2);

ii) The rod is applying force on the ball.
From coulomb's law;

$$F' = \frac{1}{4\pi\epsilon_0} \cdot \frac{q(0.01q)}{(0.1m)^2} \quad \text{— (4)}$$

$F' = F$ from figure;
Equating (4) and (3)

$$mg \tan\theta = \frac{0.01 q^2 (K)}{(0.1m)^2} \quad \text{— (5)}$$

$$\Rightarrow q^2 = \frac{(0.1m)^2 mg \tan\theta}{0.01 \times 9 \times 10^9}$$

putting the values;

$$q^2 = \frac{(0.1)^2 (1\times 10^{-3})(9.8) \tan 30^\circ}{0.01 \times 9 \times 10^9}$$

$$q^2 = \frac{5.65 \times 10^{-5}}{9 \times 10^7} \ C^2$$

$$q^2 = 6.277 \times 10^{-13} \ C^2$$

$4\frac{1}{2}$

$$\Rightarrow q = 7.9 \times 10^{-7} \ C$$

charge on rod.

$$\Rightarrow \boxed{q' = 0.01q = 7.9 \times 10^{-9} \ C}$$

charge on moth ball.

Q No 1

Part a:-

"Electrostatic Force"

Definition:-

experimented over the electrostatic forces b/w charges and formulated a law known as Coulomb's law.*

Coulomb's law:-

Statement:-

The law is stated as;

"The force of attraction or repulsion between two charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

Mathematical Expression:-

$$F \propto \frac{q_1 q_2}{r^2}$$

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$$F = k\frac{q_1 q_2}{r^2} \quad \text{--- (1)}$$

where k is the constant of proportionality and its experimental value is;

$$k = \frac{1}{4\pi\epsilon_0}$$

$$\text{or} \quad k = 9\times10^9\ Nm^2/c^2$$

Some properties:-

The force always act along the line that joins the two charges.

3) Similarity with Gravitational law:-

Like gravitational law it also follows the inverse square law. ~~the~~

It involves magnitude of charges in places of masses in gravitational law.

4) Difference from Gravitational law:-

Gravitational ~~law~~ force is only attractive while this force is attractive as well as repulsive.

End.