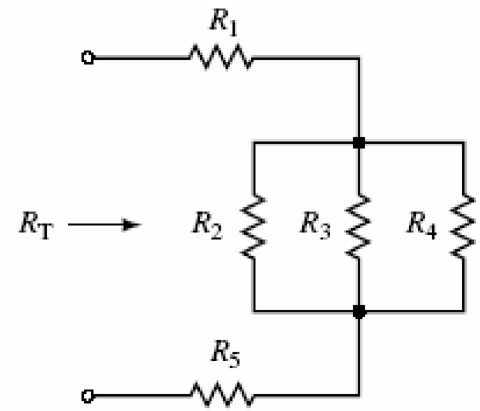
7 Series-Parallel Circuits

The Series-Parallel Network



In electric circuits, we define a **branch** as any portion of a circuit which can be simplified as having two terminals. The components between the two terminals may be any combination of resistors, voltage sources, or other elements. Many complex circuits may be separated into a combination of both series and/or parallel elements, while other circuits consist of even more elaborate combinations which are neither series nor parallel.

In order to analyze a complicated circuit, it is important to be able to recognize which elements are in series and which elements or branches are in parallel. Consider the network of resistors shown in Figure 7–1.

We immediately recognize that the resistors R_2, R_3, and R_4 are in parallel. This parallel combination is in series with the resistors R_1 and R_5. The total resistance may now be written as follows:

$$R_T = R_1 + (R_2 \| R_3 \| R_4) + R_5$$

EXAMPLE 7–1 For the network of Figure 7–2, determine which resistors and branches are in series and which are in parallel. Write an expression for the total equivalent resistance, R_T.

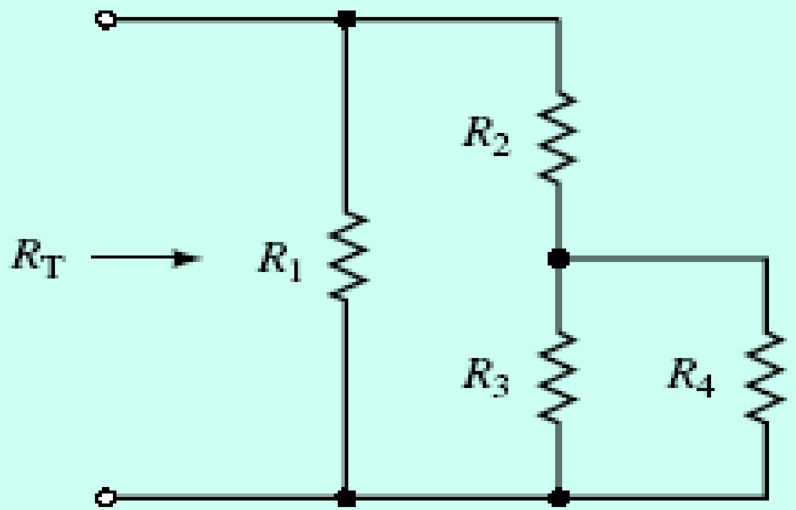


FIGURE 7–2

Solution First, we recognize that the resistors R_3 and R_4 are in parallel: $(R_3 \| R_4)$.

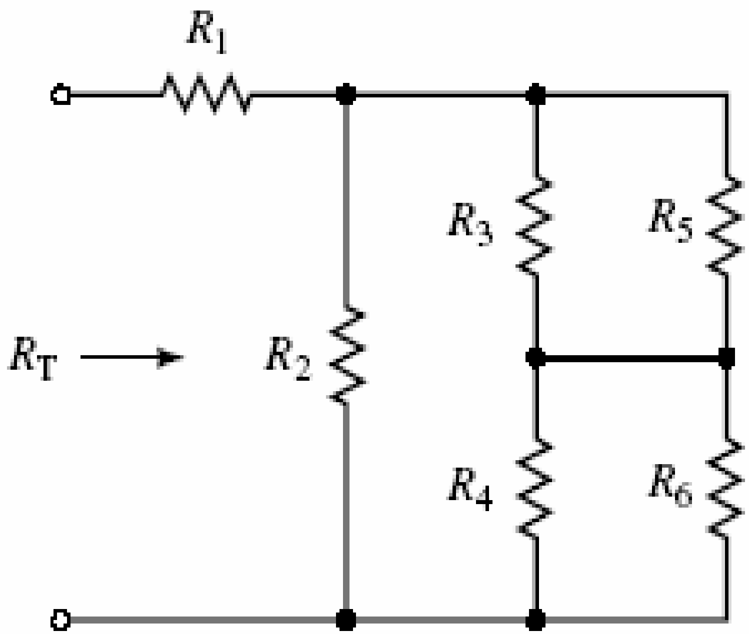
Next, we see that this combination is in series with the resistor R_2: $[R_2 + (R_3 \| R_4)]$.

Finally, the entire combination is in parallel with the resistor R_1. The total resistance of the circuit may now be written as follows:

$$R_T = R_1 \,\|\, [R_2 + (R_3 \| R_4)]$$

For the network of Figure 7–3, determine which resistors and branches are in series and which are in parallel. Write an expression for the total resistance, R_T.

FIGURE 7–3



Answer: $R_T = R_1 + R_2 \| [(R_3 \| R_5) + (R_4 \| R_6)]$

Analysis of Series-Parallel Circuits

Series-parallel networks are often difficult to analyze because they initially appear confusing. However, the analysis of even the most complex circuit is simplified by following some fairly basic steps. By practicing (not memorizing) the techniques outlined in this section, you will find that most circuits can be reduced to groupings of series and parallel combinations. In analyzing such circuits, it is imperative to remember that the rules for analyzing series and parallel elements still apply.

The same current occurs through all series elements.

The same voltage occurs across all parallel elements.

EXAMPLE 7–2 Consider the circuit of Figure 7–4.

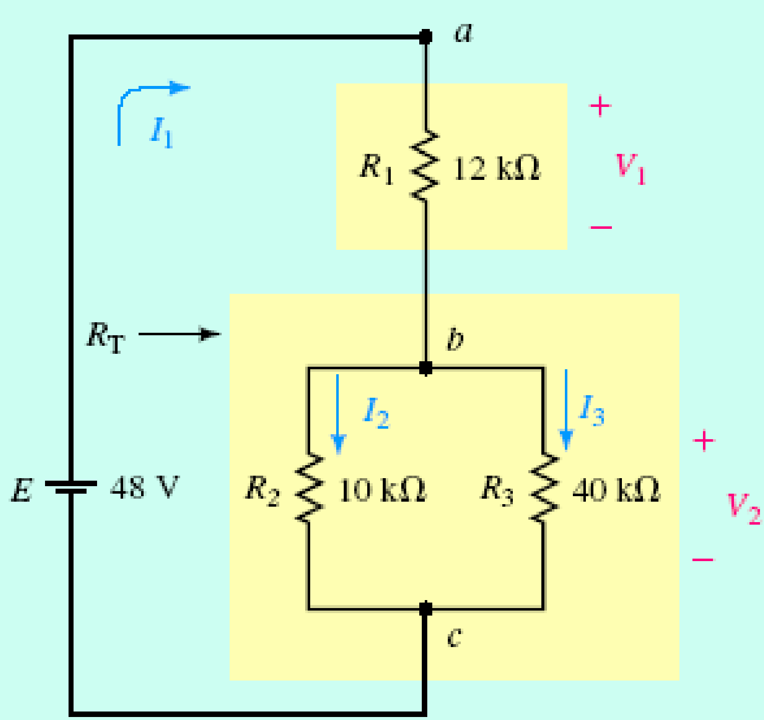
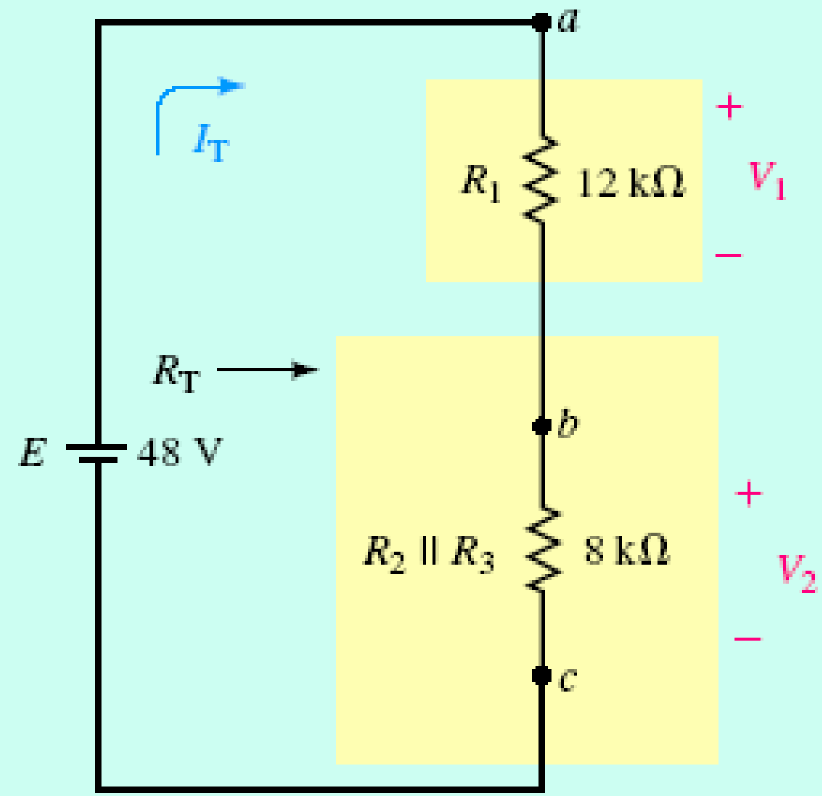


FIGURE 7–4



a. Find R_T.

b. Calculate I_1, I_2, and I_3.

c. Determine the voltages V_1 and V_2.

a. The total resistance of the circuit may be determined from the combination

$$R_T = R_1 + R_2 \| R_3$$

$$R_T = 12\ \text{k}\Omega + \frac{(10\ \text{k}\Omega)(40\ \text{k}\Omega)}{10\ \text{k}\Omega + 40\ \text{k}\Omega}$$

$$= 12\ \text{k}\Omega + 8\ \text{k}\Omega = 20\ \text{k}\Omega$$

b. From Ohm's law, the total current is

$$I_T = I_1 = \frac{48\ \text{V}}{20\ \text{k}\Omega} = 2.4\ \text{mA}$$

The current I_1 will enter node b and then split between the two resistors R_2 and R_3. This current divider may be simplified as shown in the partial circuit of Figure 7–6.

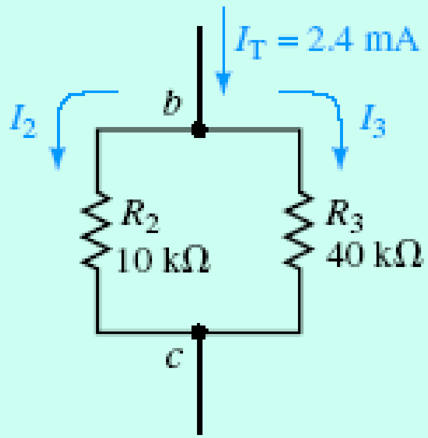
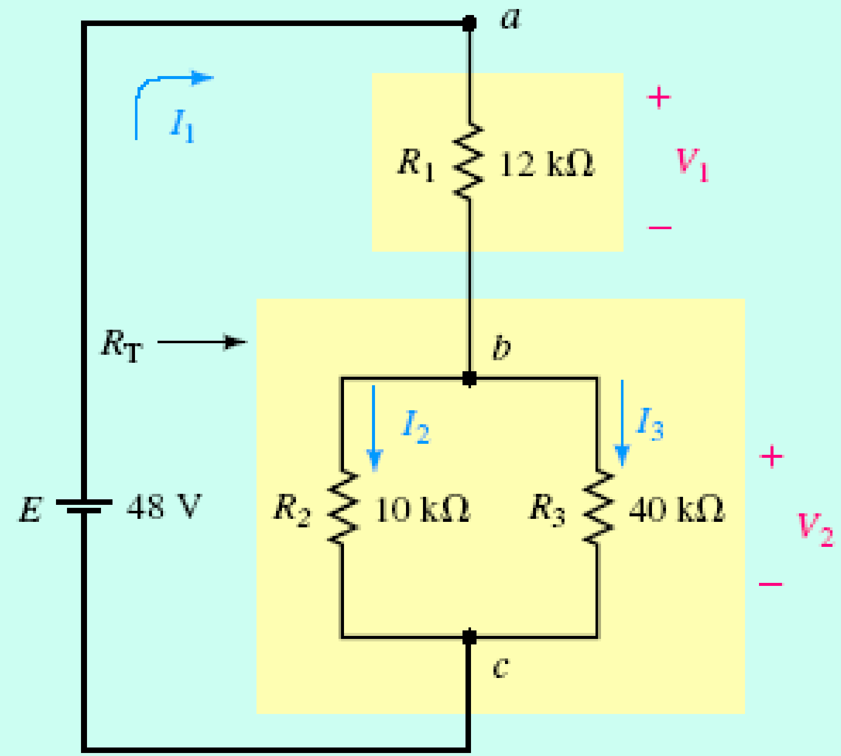


FIGURE 7–6



Applying the current divider rule to these two resistors gives

$$I_2 = \frac{(40\text{ k}\Omega)(2.4\text{ mA})}{10\text{ k}\Omega + 40\text{ k}\Omega} = 1.92\text{ mA}$$

$$I_3 = \frac{(10\text{ k}\Omega)(2.4\text{ mA})}{10\text{ k}\Omega + 40\text{ k}\Omega} = 0.48\text{ A}$$

c. Using the above currents and Ohm's law, we determine the voltages:

$$V_1 = (2.4\text{ mA})(12\text{ k}\Omega) = 28.8\text{ V}$$

$$V_3 = (0.48\text{ mA})(40\text{ k}\Omega) = 19.2\text{ V} = V_2$$

In order to check the answers, we may simply apply Kirchhoff's voltage law around any closed loop which includes the voltage source:

$$\begin{aligned}\sum V &= E - V_1 - V_3 \\ &= 48\text{ V} - 28.8\text{ V} - 19.2\text{ V} \\ &= 0\text{ V (checks!)}\end{aligned}$$

The solution may be verified by ensuring that the power delivered by the voltage source is equal to the summation of powers dissipated by the resistors.

EXAMPLE 7–3 Find the voltage V_{ab} for the circuit of

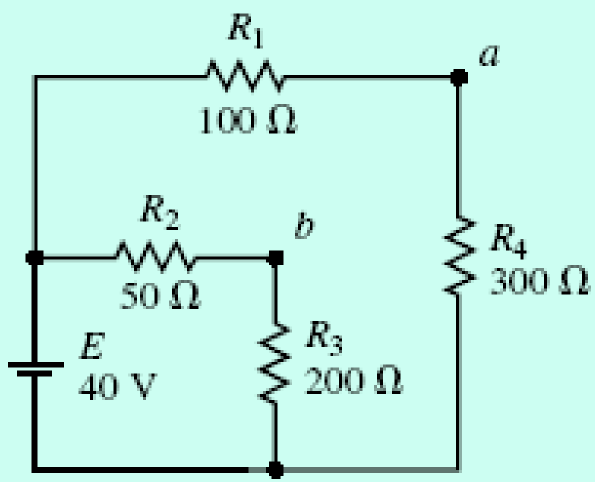
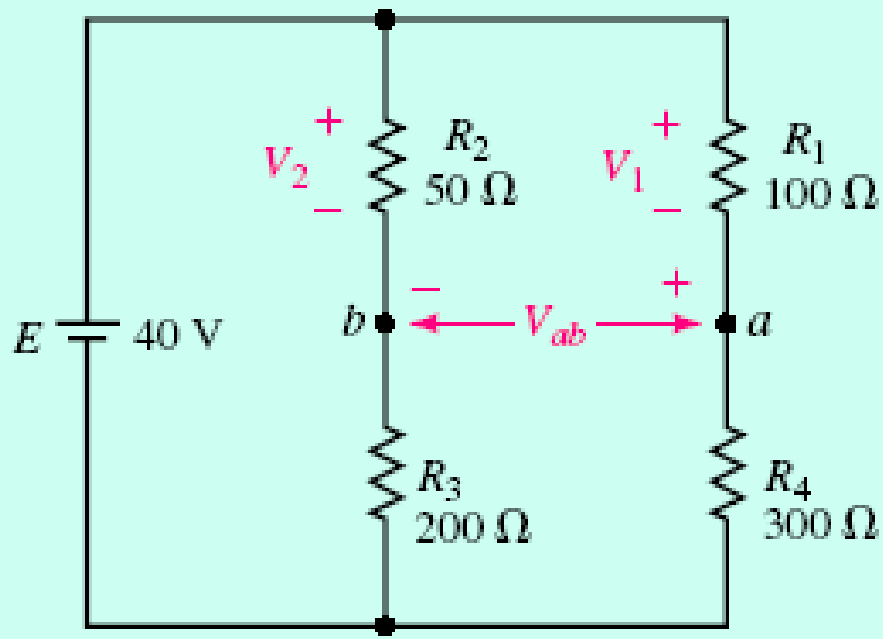


FIGURE 7–7



From Figure 7–8, we see that the original circuit consists of two parallel branches, where each branch is a series combination of two resistors.

If we take a moment to examine the circuit, we see that the voltage V_{ab} may be determined from the combination of voltages across R_1 and R_2. Alternatively, the voltage may be found from the combination of voltages across R_3 and R_4.

As usual, several methods of analysis are possible. Because the two branches are in parallel, the voltage across each branch must be 40 V. Using the voltage divider rules allows us to quickly calculate the voltage across each resistor. Although equally correct, other methods of calculating the voltages would be more lengthy.

$$V_2 = \frac{R_2}{R_2 + R_3}E$$

$$= \left(\frac{50\ \Omega}{50\ \Omega + 200\ \Omega}\right)(40\ \text{V}) = 8.0\ \text{V}$$

$$V_1 = \frac{R_1}{R_1 + R_4}E$$

$$= \left(\frac{100\ \Omega}{100\ \Omega + 300\ \Omega}\right)(40\ \text{V}) = 10.0\ \text{V}$$

As shown in Figure 7–9, we apply Kirchhoff's voltage law to determine the voltage between terminals *a* and *b*.

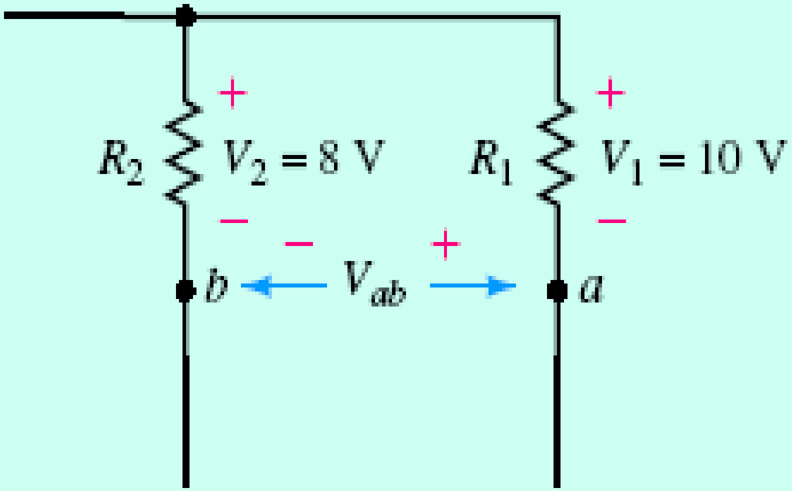


FIGURE 7–9

$$V_{ab} = -10.0\ \text{V} + 8.0\ \text{V} = -2.0\ \text{V}$$

EXAMPLE 7–4 Consider the circuit of Figure 7–10:

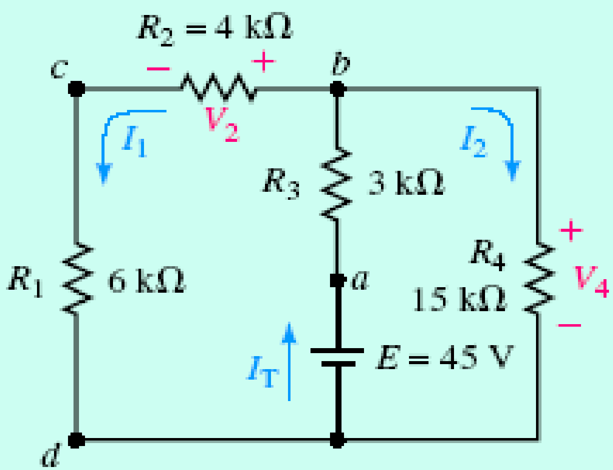
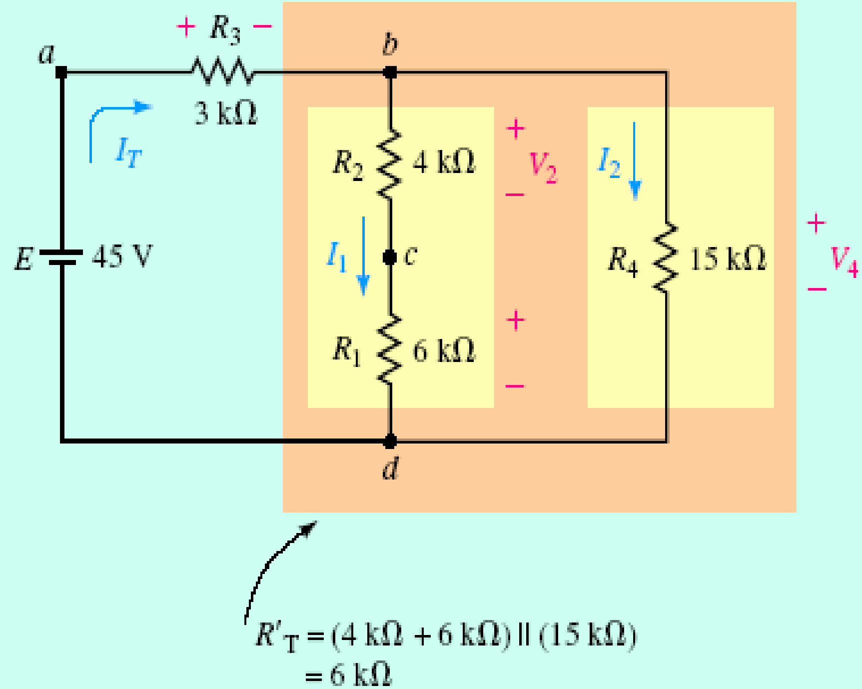


FIGURE 7–10



a. Find the total resistance R_T "seen" by the source E.

b. Calculate I_T, I_1, and I_2.

c. Determine the voltages V_2 and V_4.

Solution We begin the analysis by redrawing the circuit. Since we generally like to see the source on the left-hand side, one possible way of redrawing the resultant circuit is shown in Figure 7–11. Notice that the polarities of voltages across all resistors have been shown.

a. From the redrawn circuit, the total resistance of the circuit is

$$R_T = R_3 + [(R_1 + R_2)\|R_4]$$

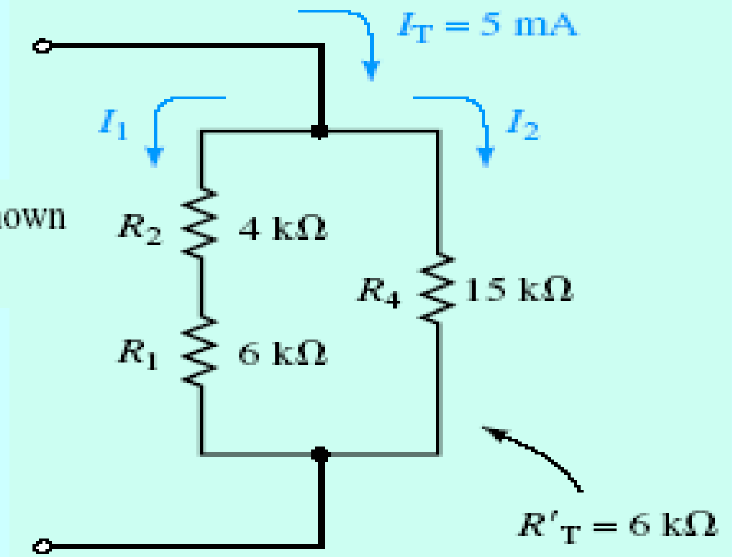
$$= 3\ \text{k}\Omega + \frac{(4\ \text{k}\Omega + 6\ \text{k}\Omega)(15\ \text{k}\Omega)}{(4\ \text{k}\Omega + 6\ \text{k}\Omega) + 15\ \text{k}\Omega}$$

$$= 3\ \text{k}\Omega + 6\ \text{k}\Omega = 9.00\ \text{k}\Omega$$

b. The current supplied by the voltage source is

$$I_T = \frac{E}{R_T} = \frac{45\ \text{V}}{9\ \text{k}\Omega} = 5.00\ \text{mA}$$

We see that the supply current divides between the parallel branches as shown in Figure 7–12.



Applying the current divider rule, we calculate the branch currents as

$$I_1 = I_T\frac{R'_T}{(R_1 + R_2)} = \frac{(5\ \text{mA})(6\ \text{k}\Omega)}{4\ \text{k}\Omega + 6\ \text{k}\Omega} = 3.00\ \text{mA}$$

$$I_2 = I_T\frac{R'_T}{R_4} = \frac{(5\ \text{mA})(6\ \text{k}\Omega)}{15\ \text{k}\Omega} = 2.00\ \text{mA}$$

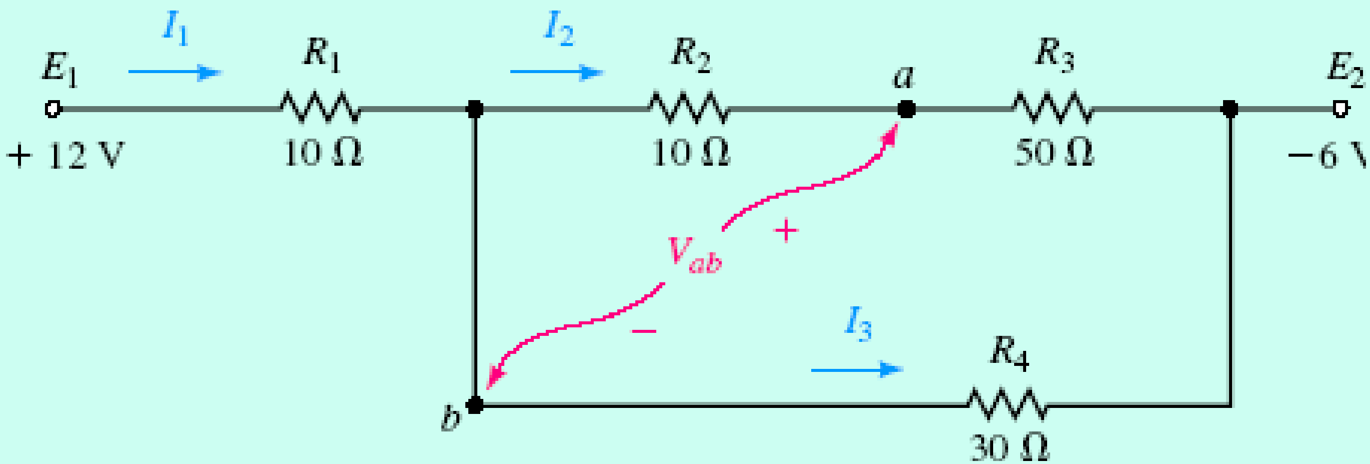
Notice: When determining the branch currents, the resistance R'_T is used in the calculations rather the the total circuit resistance. This is because the current $I_T = 5$ mA splits between the two branches of R'_T and the split is not affected by the value of R_3.

c. The voltages V_2 and V_4 are now easily calculated by using Ohm's law:

$$V_2 = I_1R_2 = (3\ \text{mA})(4\ \text{k}\Omega) = 12.0\ \text{V}$$

$$V_4 = I_2R_4 = (2\ \text{mA})(15\ \text{k}\Omega) = 30.0\ \text{V}$$

EXAMPLE 7–5 For the circuit of Figure 7–13, find the indicated currents and voltages.



Solution Because the above circuit contains voltage point sources, it is easier to analyze if we redraw the circuit to help visualize the operation.

The point sources are voltages with respect to ground, and so we begin by drawing a circuit with the reference point as shown in Figure 7–14.

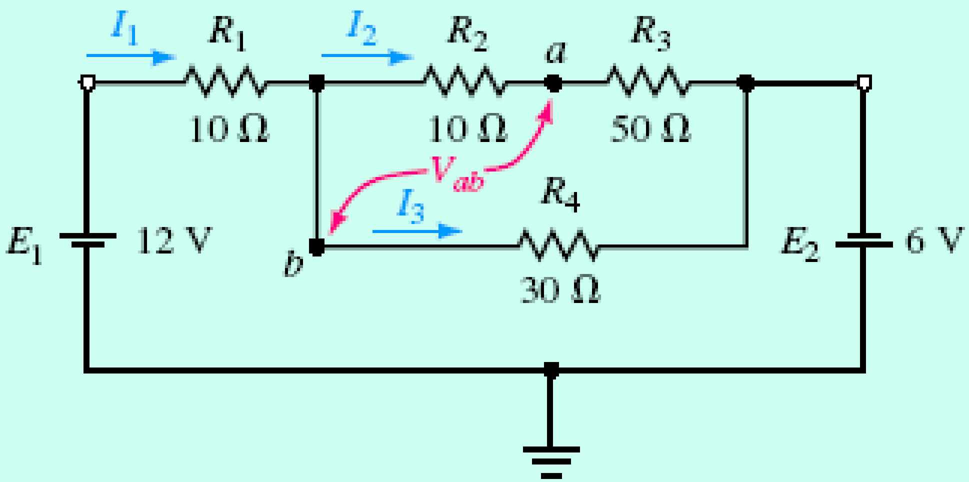


FIGURE 7–14

Now, we can see that the circuit may be further simplified by combining the voltage sources ($E = E_1 + E_2$) and by showing the resistors in a more suitable location. The simplified circuit is shown in Figure 7–15.

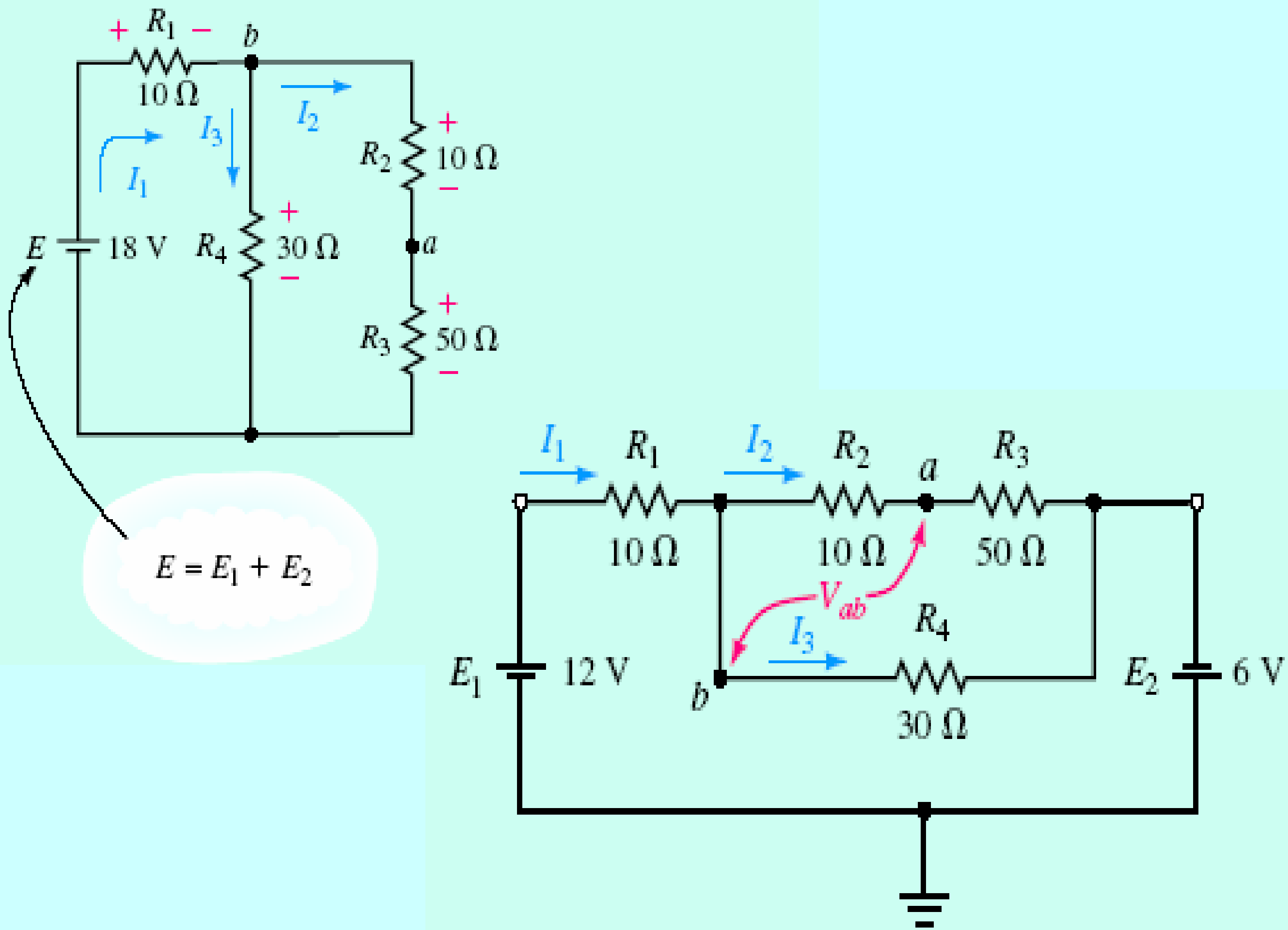
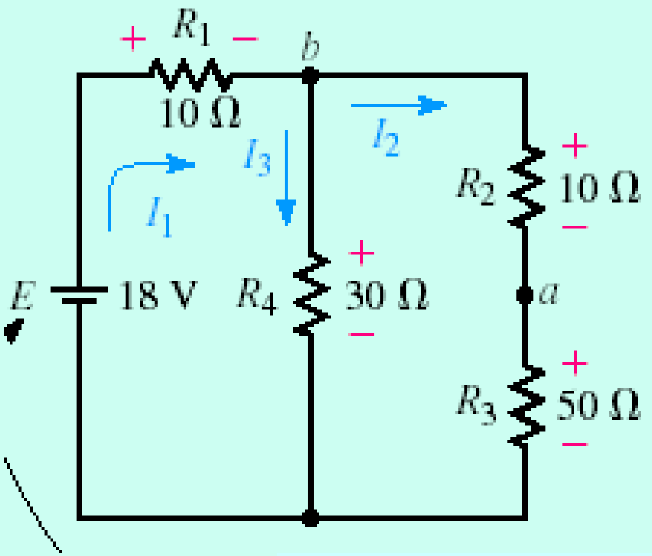


FIGURE 7–15

The total resistance "seen" by the equivalent voltage source is



$$R_T = R_1 + [R_4 \| (R_2 + R_3)]$$

$$= 10\ \Omega + \frac{(30\ \Omega)(10\ \Omega + 50\ \Omega)}{30\ \Omega + (10\ \Omega + 50\ \Omega)} = 30.0\ \Omega$$

And so the total current provided into the circuit is

$$I_1 = \frac{E}{R_T} = \frac{18\ \text{V}}{30\ \Omega} = 0.600\ \text{A}$$

At node b this current divides between the two branches as follows:

$$I_3 = \frac{(R_2 + R_3)I_1}{R_4 + R_2 + R_3} = \frac{(60\ \Omega)(0.600\ \text{A})}{30\ \Omega + 10\ \Omega + 50\ \Omega} = 0.400\ \text{A}$$

$$I_2 = \frac{R_4 I_1}{R_4 + R_2 + R_3} = \frac{(30\ \Omega)(0.600\ \text{A})}{30\ \Omega + 10\ \Omega + 50\ \Omega} = 0.200\ \text{A}$$

The voltage V_{ab} has the same magnitude as the voltage across the resistor R_2, but with a negative polarity (since b is at a higher potential than a):

$$V_{ab} = -I_2 R_2 = -(0.200\ \text{A})(10\ \Omega) = -2.0\ \text{V}$$

EXAMPLE 7–6 The circuit of Figure 7–17 is referred to as a *bridge circuit* and is used extensively in electronic and scientific instruments.

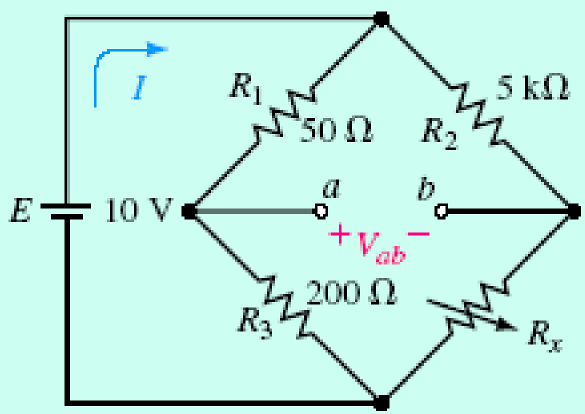


FIGURE 7–17

Calculate the current I and the voltage V_{ab} when

a. $R_x = 0\ \Omega$ (short circuit)

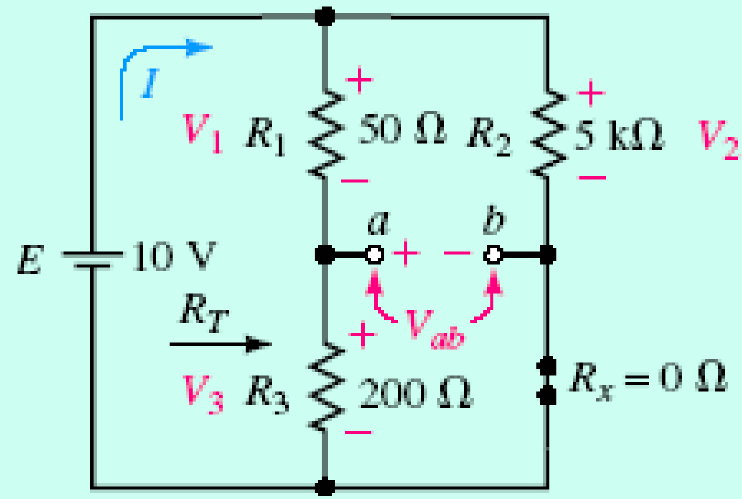
b. $R_x = 15\ \text{k}\Omega$

c. $R_x = \infty$ (open circuit)

Solution

a. $R_x = 0\ \Omega$:

The circuit is redrawn as shown in Figure 7–18.



The voltage source "sees" a total resistance of

$$R_T = (R_1 + R_3)\|R_2 = 250\ \Omega\|5000\ \Omega = 238\ \Omega$$

resulting in a source current of

$$I = \frac{10\ \text{V}}{238\ \Omega} = 0.042\ \text{A} = 42.2\ \text{mA}$$

Now, since the variable resistor is a short circuit, the entire source voltage will appear across the resistor R_2, giving

$$V_2 = 10.0\ \text{V}$$

The voltage V_{ab} may be determined by solving for voltage across R_1 and R_2.

The voltage across R_1 will be constant regardless of the value of the variable resistor R_x. Hence

$$V_1 = \left(\frac{50\ \Omega}{50\ \Omega + 200\ \Omega}\right)(10\ \text{V}) = 2.00\ \text{V}$$

$$V_{ab} = -V_1 + V_2 = -2.00\ \text{V} + 10.0\ \text{V} = +8.00\ \text{V}$$

b. $R_x = 15\ \text{k}\Omega$:

The circuit is redrawn in Figure 7–19.

$$R_T = (R_1 + R_3)\|(R_2 + R_x)$$
$$= 250\ \Omega\|20\ \text{k}\Omega = 247\ \Omega$$

$$I = \frac{10\ \text{V}}{247\ \Omega} = 0.0405\ \text{A} = 40.5\ \text{mA}$$

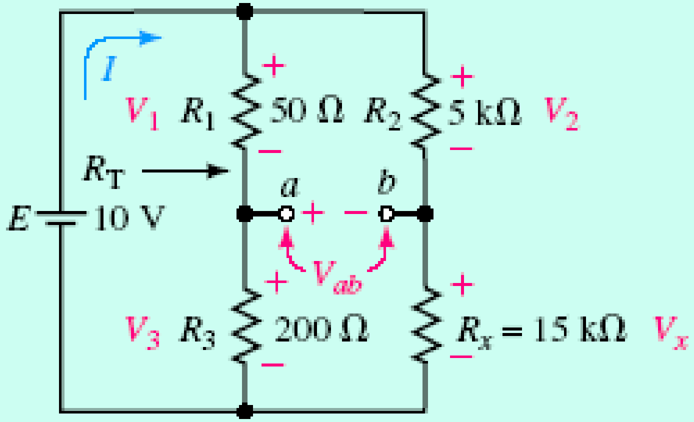


FIGURE 7–19

The voltages across R_1 and R_2 are

$$V_1 = 2.00\ \text{V} \quad \text{(as before)}$$

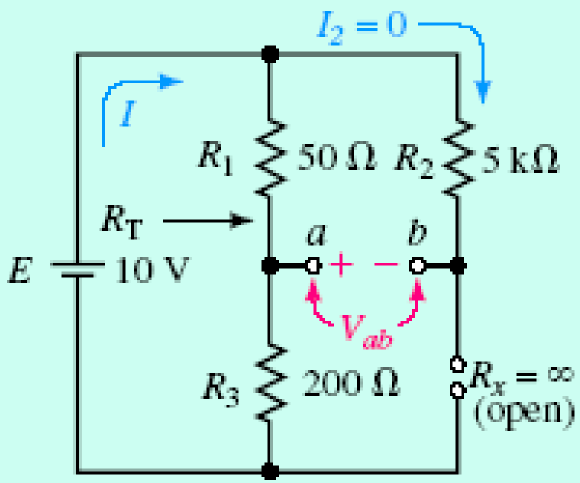
$$V_2 = \frac{R_2}{R_2 + R_x}E$$
$$= \left(\frac{5\ \text{k}\Omega}{5\ \text{k}\Omega + 15\ \text{k}\Omega}\right)(10\ \text{V}) = 2.50\ \text{V}$$

Now the voltage between terminals a and b is found as

$$V_{ab} = -V_1 + V_2$$
$$= -2.0\ \text{V} + 2.5\ \text{V} = +0.500\ \text{V}$$

c. $R_x = \infty$:

The circuit is redrawn in Figure 7–20.



Because the second branch is an open circuit due to the resistor R_x, the total resistance "seen" by the source is

$$R_T = R_1 + R_3 = 250\ \Omega$$

resulting in a source current of

$$I = \frac{10\text{ V}}{250\ \Omega} = 0.040\text{ A} = 40.0\text{ mA}$$

The voltages across R_1 and R_2 are

$$V_1 = 2.00\text{ V} \quad \text{(as before)}$$

$$V_2 = 0\text{ V} \quad \text{(since the branch is open)}$$

And so the resulting voltage between terminals a and b is

$$V_{ab} = -V_1 + V_2$$

$$= -2.0\text{ V} + 0\text{ V} = -2.00\text{ V}$$

The previous example illustrates how voltages and currents within a circuit are affected by changes elsewhere in the circuit. In the example, we saw that the voltage V_{ab} varied from -2 V to $+8$ V, while the total circuit current varied from a minimum value of 40 mA to a maximum value of 42 mA. These changes occurred even though the resistor R_x varied from 0 Ω to ∞.

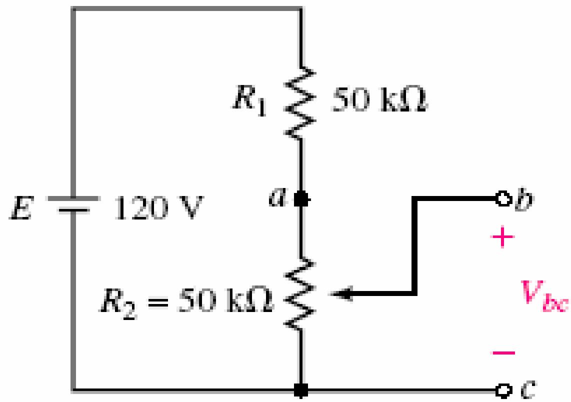
A **transistor** is a three-terminal device which may be used to amplify small signals. In order for the transistor to operate as an amplifier, however, certain dc conditions must be met. These conditions set the "bias point" of the transistor. The bias current of a transistor circuit is determined by a dc voltage source and several resistors. Although the operation of the transistor is outside the scope of this textbook, we can analyze the bias circuit of a transistor using elementary circuit theory.

Potentiometers

As mentioned in Chapter 3, variable resistors may be used as potentiometers as shown in Figure 7–28 to control voltage into another circuit.

The volume control on a receiver or amplifier is an example of a variable resistor used as a potentiometer. When the movable terminal is at the uppermost position, the voltage appearing between terminals b and c is simply calculated by using the voltage divider rule as

$$V_{bc} = \left(\frac{50\ \text{k}\Omega}{50\ \text{k}\Omega + 50\ \text{k}\Omega}\right)(120\ \text{V}) = 60\ \text{V}$$



Alternatively, when the movable terminal is at the lowermost position, the voltage between terminals b and c is $V_{bc} = 0$ V, since the two terminals are effectively shorted and the voltage across a short circuit is always zero.

The circuit of Figure 7–28 represents a potentiometer having an output voltage which is adjustable between 0 and 60 V. This output is referred to as the **unloaded output,** since there is no load resistance connected between the terminals b and c. If a load resistance were connected between these terminals, the output voltage, called the **loaded output,** would no longer be the same. The following example is an illustration of circuit loading.

EXAMPLE 7–10 For the circuit of Figure 7–29, determine the range of the voltage V_{bc} as the potentiometer varies between its minimum and maximum values.

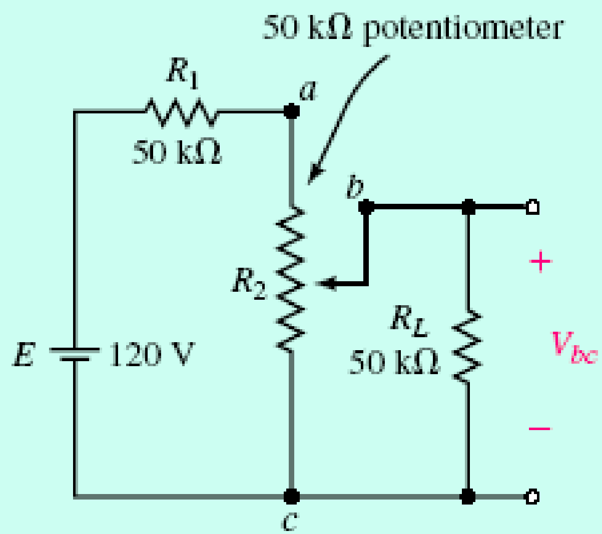
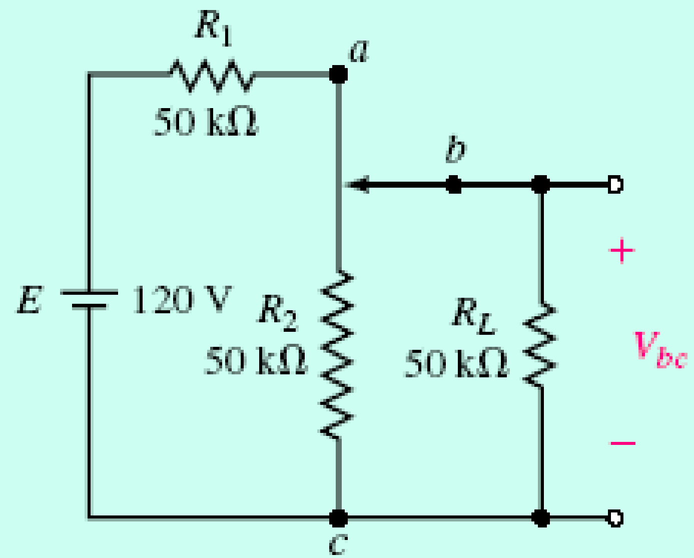


FIGURE 7–29



Solution The minimum voltage between terminals b and c will occur when the movable contact is at the lowermost contact of the variable resistor. In this position, the voltage $V_{bc} = 0$ V, since the terminals b and c are shorted.

The maximum voltage V_{bc} occurs when the movable contact is at t[he] uppermost contact of the variable resistor. In this position, the circuit may [be] represented as shown in Figure 7–30.

$$V_{bc} = \frac{R_2 \| R_L}{(R_2 \| R_L) + R_1} E$$

$$= \left(\frac{25\text{ k}\Omega}{25\text{ k}\Omega + 50\text{ k}\Omega} \right)(120\text{ V}) = 40\text{ V}$$

We conclude that the voltage at the output of the potentiometer is adjustable from 0 V to 40 V for a load resistance of $R_L = 50$ kΩ.

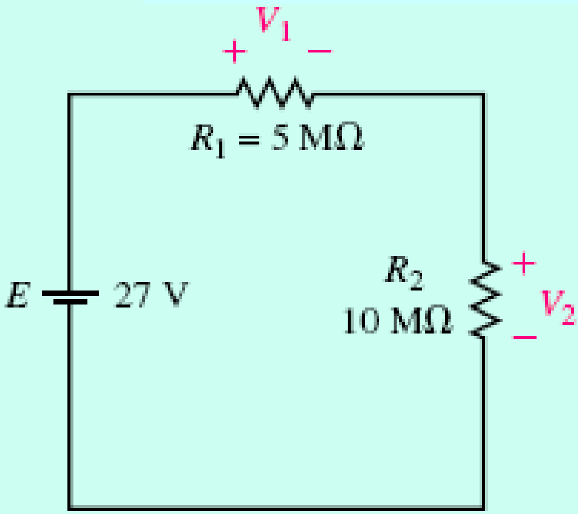
By inspection, we see that an unloaded potentiometer in the circuit of Figure 7–29 would have an output voltage of 0 V to 60 V.

Loading Effects of Instruments

The degree to which the circuits are affected is called the **loading effect** of the instrument.

for an instrument to provide an accurate indication of how a circuit operates, the loading effect should ideally be zero. In practice, it is impossible for any instrument to have zero loading effect, since all instruments absorb some energy from the circuit under test, thereby affecting circuit operation.

EXAMPLE 7–11 Calculate the loading effects if a digital multimeter, having an internal resistance of 10 MΩ, is used to measure V_1 and V_2 in the circuit of Figure 7–31.



Solution In order to determine the loading effect for a particular reading, we need to calculate both the unloaded voltage and the loaded voltage.

For the circuit given in Figure 7–31, the unloaded voltage across each resistor is

$$V_1 = \left(\frac{5\ \text{M}\Omega}{5\ \text{M}\Omega + 10\ \text{M}\Omega}\right)(27\ \text{V}) = 9.0\ \text{V}$$

$$V_2 = \left(\frac{10\ \text{M}\Omega}{5\ \text{M}\Omega + 10\ \text{M}\Omega}\right)(27\ \text{V}) = 18.0\ \text{V}$$

When the voltmeter is used to measure V_1, the result is equivalent to connecting a 10-MΩ resistor across resistor R_1, as shown in Figure 7–32.

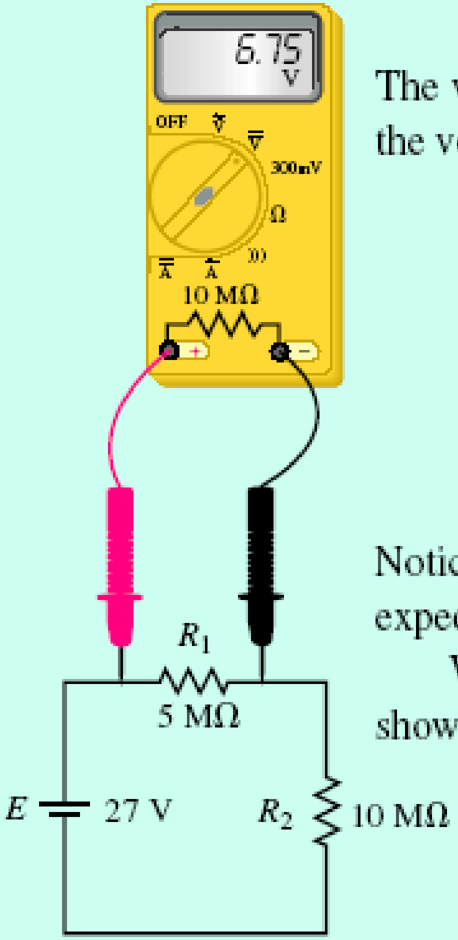


FIGURE 7–32

The voltage appearing across the parallel combination of R_1 and resistance of the voltmeter is calculated as

$$V_1 = \left(\frac{5\ \text{M}\Omega \| 10\ \text{M}\Omega}{(5\ \text{M}\Omega \| 10\ \text{M}\Omega) + 10\ \text{M}\Omega}\right)(27\ \text{V})$$

$$= \left(\frac{3.33\ \text{M}\Omega}{13.3\ \text{M}\Omega}\right)(27\ \text{V})$$

$$= 6.75\ \text{V}$$

Notice that the measured voltage is significantly less than the 9 V that we had expected to measure.

With the voltmeter connected across resistor R_2, the circuit appears as shown in Figure 7–33.

The voltage appearing across the parallel combination of R_2 and resistance of the voltmeter is calculated as

$$V_2 = \left(\frac{10\ \text{M}\Omega \| 10\ \text{M}\Omega}{5\ \text{M}\Omega + (10\ \text{M}\Omega \| 10\ \text{M}\Omega)}\right)(27\ \text{V})$$

$$= \left(\frac{5.0\ \text{M}\Omega}{10.0\ \text{M}\Omega}\right)(27\ \text{V})$$

$$= 13.5\ \text{V}$$

Again, we notice that the measured voltage is quite a bit less than the 18 V that we had expected.

Now the loading effects are calculated as follows.

When measuring V_1:

$$\text{loading effect} = \frac{9.0\ \text{V} - 6.75\ \text{V}}{9.0\ \text{V}} \times 100\%$$

$$= 25\%$$

When measuring V_2:

$$\text{loading effect} = \frac{18.0\ \text{V} - 13.5\ \text{V}}{18.0\ \text{V}} \times 100\%$$

$$= 25\%$$

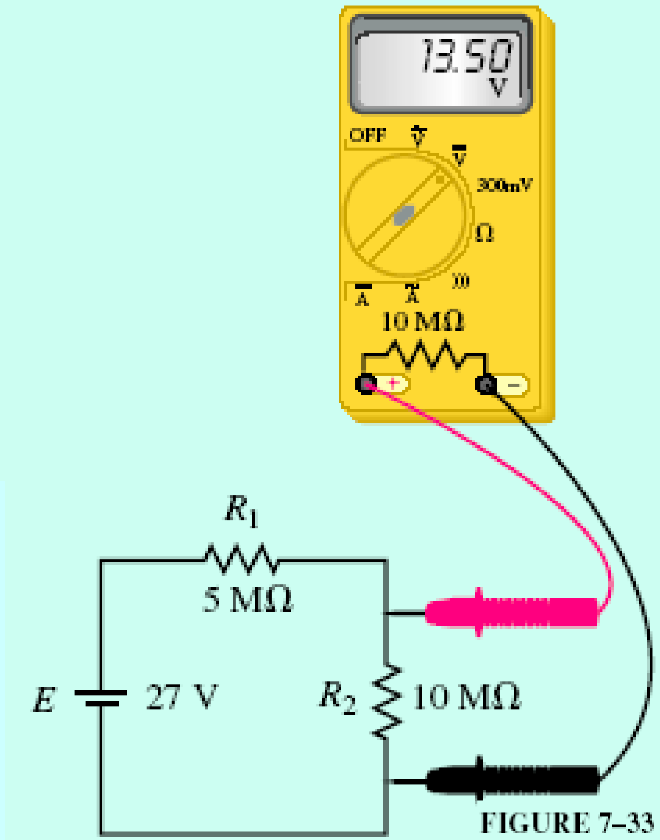


FIGURE 7–33

This example clearly illustrates a problem that novices often make when they are taking voltage measurements in high-resistance circuits. If the measured voltages $V_1 = 6.75$ V and $V_2 = 13.50$ V are used to verify Kirchhoff's voltage law, the novice would say that this represents a contradiction of the law (since $6.75\text{ V} + 13.50\text{ V} \neq 27.0\text{ V}$). In fact, we see that the circuit is behaving exactly as predicted by circuit theory. The problem occurs when instrument limitations are not considered.

PRACTICAL NOTES...

Whenever an instrument is used to measure a quantity, the operator must always consider the loading effects of the instrument.