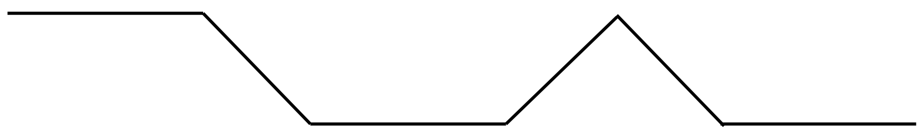
MILITARY COLLEGE OF SIGNALS
FINAL EXAM
BESE 14 (A & B)
EE 481 Digital Image Processing

Instructor: A/P Dr. Imran Siddiqi

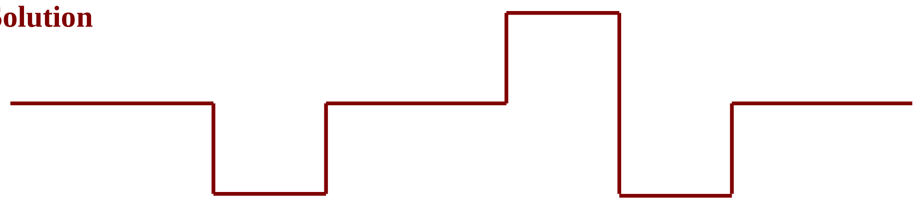
Time: 2.5 Hours
Max Marks: 50

(3+3+3)

1. **a.** The intensity profile of a single line in an image is shown in the following. Show the first derivative of the profile.



Solution



b. Consider the 5x5 image in the square with its *x* and *y* coordinates indicated on the top and left of the square respectively.

y \ x	0	1	2	3	4
0	50	100	100	100	50
1	100	50	200	150	100
2	100	200	30	200	100
3	100	150	200	50	100
4	50	150	150	100	50

Using a neighborhood of 3x3, compute the local threshold for the pixel at position (2,2), using the formula:

$$T = m - min(I)$$

m= Mean value of the neighborhood, *min(I)* = minimum gray level in the image

Solution

The region of interest is

50	200	150
200	30	200
150	200	50

m = Average gray value in the region of interest =137 min(I) = Minimum gray level in the image = 30

T= m –min(I) = 137-30 = 107

c. Applying the four anisotropic line detection masks at pixel position (2,2) in the image shown in part '*b*' (above), identify the direction to which this pixel is most likely to belong to.

Solution
The four line detection masks are:

$$\begin{matrix} -1 & -1 & -1 \\ 2 & 2 & 2 \\ -1 & -1 & -1 \end{matrix} \qquad \begin{matrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{matrix} \qquad \begin{matrix} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{matrix} \qquad \begin{matrix} -1 & -1 & 2 \\ -1 & 2 & -1 \\ 2 & -1 & -1 \end{matrix}$$

R1-Horizontal R2 -45 R3 - Vertical R4 45

Applying each of these on the region of interest

50	200	150
200	30	200
150	200	50

For R1
-1x50 + -1x200 + -1x150 + 2x200 + 2x30 + 2x200 + -1x150 + -1x200 + -1x50
= 60

For R2
2x50 + -1x200 + -1x150 + -1x200 + 2x30 + -1x200 + -1x150 + -1x200 + 2x50
=-840

For R3
-1x50 + 2x200 + -1x150 + -1x200 + 2x30 + -1x200 + -1x150 + 2x200 + -1x50
= 60

For R4
-1x50 + -1x200 + 2x150 + -1x200 + 2x30 + -1x200 + 2x150 + -1x200 + -1x50
=-240

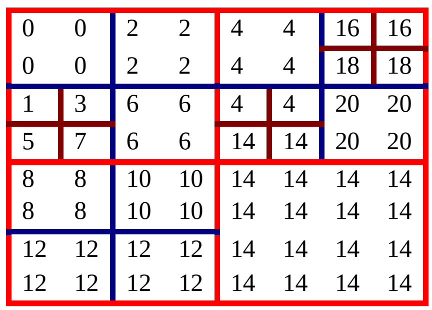
The maximum response (absolute) is that of R2 so the point is most likely to belong to a line at an angle of -45°.

(5+2)

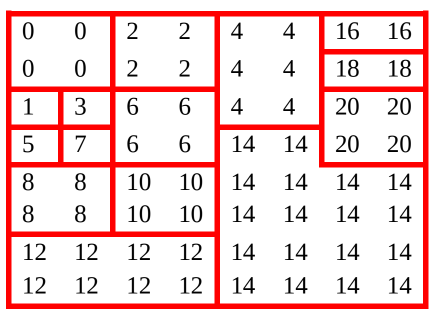
2. **a.** Apply region splitting and merging algorithm to the following image and show the resulting quad tree. Do not forget to indicate the nodes that need to be merged. The predicate to be used is: 'All pixels in a region should have the same gray value'.

0	0	2	2	4	4	16	16
0	0	2	2	4	4	18	18
1	3	6	6	4	4	20	20
5	7	6	6	14	14	20	20
8	8	10	10	14	14	14	14
8	8	10	10	14	14	14	14
12	12	12	12	14	14	14	14
12	12	12	12	14	14	14	14

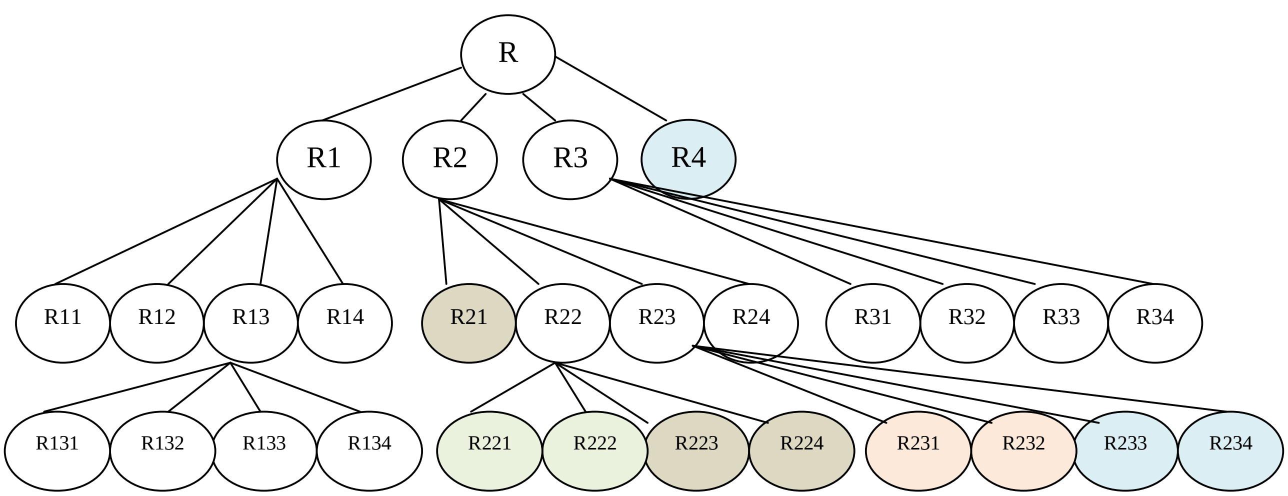
Solution



After Split



After Merge



b. Intensity slicing is applied to the following gray scale image by defining two planes at levels l_1=80 and l_2 = 160. The resulting partitions P_1, P_2 and P_3 are assigned red, green and blue colors respectively. You need to show the output image after this color assignment.

72	72	72	72	72
72	72	82	72	72
72	82	82	82	72
72	72	82	72	72
72	72	72	72	72

Solution

R	R	R	R	R
R	R	G	R	R
R	G	G	G	R
R	R	G	R	R
R	R	R	R	R

(5+5)

3. **a.** Using hit-and-miss transform, propose an algorithm to detect right angle convex corner points in images. You need to mention the structuring element(s) and the steps the algorithm should perform. Be precise and to the point. An example image and the corresponding output is shown in the following.

0	0	0	0	0	0	0	0	0	0
0	0	1	1	1	0	1	1	0	0
0	0	1	1	1	1	1	1	0	0
0	1	1	1	0	0	1	0	0	0
0	0	1	1	0	0	1	1	0	0
0	0	1	1	0	0	1	1	0	0
0	0	0	1	0	0	1	1	0	0
0	0	1	1	1	1	1	0	0	0
0	0	1	1	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Input Image

0	0	0	0	0	0	0	0	0	0
0	0	1	0	1	0	1	1	0	0
0	0	0	0	0	0	0	1	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	0	0
0	0	1	0	0	0	1	0	0	0
0	0	1	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Output of corner detector

Solution

For the four types of corners, the following four structuring elements can be used.

	1	
0	1	1
0	0	

	1	
1	1	0
	0	0

	0	0
1	1	0
	1	

0	0	
0	1	1
	1	

The vacant position represents don't cares.

1. Apply hit and miss transform on image I with structuring element B_i. Let the result be D_i.
2. Take the union of all D_is to get the corner points.

b. Let X be an image, Φ be an operator (dilation, erosion, opening or closing), B be a structuring element and Y be the result of repeatedly applying Φ on X. The operation is applied until convergence.

$$Y= ((((X \,\Phi\, B)\, \Phi\, B)\, \Phi\, B)\, \Phi\, B)\, \Phi \ldots\ldots$$

Given the output images Y_1, Y_2, Y_3, Y_4 and Y_5 find the operator Φ_i and structuring element B_i so that Y_i is the result of repeatedly applying Φ_i on X.

□	0	0	0	0	0	0	0
0	1	1	0	0	1	1	0
0	1	1	1	1	1	1	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

X

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0

Y_1

0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0
0	1	1	1	1	1	1	0

Y_2

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

Y_3

0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	1	1	1	1	0	0
0	0	0	0	0	0	0	0

Y_4

1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1
1	1	1	1	1	1	1	1

Y_5

Solution

Solution is NOT unique, a possible solution set follows.

B1= horizontal structuring element, [1 1 1]
Φ1 = Dilation

B2= vertical structuring element, [1 1 1]T
Φ2 = Dilation

B3 = [1 1 1]
Φ3 = Erosion

B4= [1 1 1]
Φ4 = Closing

B5= [1 1 1; 1 1 1;1 1 1]
Φ5 = Dilation

(6+3+2)

4. **a.** A simple algorithm for LZW Decoding is outlined in the following.

Initialize the dictionary
Output: Symbol for the first codeWord

While(there are more CodeWords)
{
CurrentCodeWord = Read the next codeWord
if(CurrentCodeWord is in the Dictionary)
    Output: Symbol for the CurrentCodeWord
else
    Output: PreviousOutput + first symbol of PreviousOutput

Insert in the Dictionary: PreviousOutput + first symbol of CurrentOutput
}

Assuming the message can have only two basic symbols '*a*' and '*b*', decode the following:

i.	0	1	2	4	3	6
ii.	1	0	2	3	0	6

Solution

i.

0 1 2 4 3 6

Dictionary	
Index	**Symbol**
0	a
1	b
2	ab
3	ba
4	aba
5	abab
6	bab

0	1	2	4	3	6
a	b	ab	aba	ba	bab

ii.

1 0 2 3 0 6

Dictionary	
Index	**Symbol**
0	a
1	b
2	ba
3	ab
4	baa
5	aba
6	aa

1	0	2	3	0	6
b	a	ba	ab	a	aa

b. The MPEG 1 encoder has produced the following bitstream for a '*block*' in the video.

50	-2	6	-3	10	2	1	0	0	-1	3	EOB

Show the matrix that results after the application of DCT to the block. (Assume that the quantization matrix comprises all 1s).

Solution

50	-2	2	1	0	0	0	0
6	10	0	0	0	0	0	0
-3	0	0	0	0	0	0	0
-1	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0

c. Assuming that each row in a binary image starts with a black run, the following is received as the *Run Length Encoded* string of an image. Decoding it and generate the image.

1 1 1 1 2
0 6
6
3 3
1 2 1 2

Solution

0	1	0	1	0	0
1	1	1	1	1	1
0	0	0	0	0	0
0	0	0	1	1	1
0	1	1	0	1	1

(5+6+2)

5. **a.** A packing plant (dealing with the packaging of chickens and ducks) has automated the sorting of the incoming slaughtered animals on a conveyer belt. The system measures the *length* and *weight* of each incoming object on the belt and classifies it as a chicken or a duck using the k-nearest neighbor algorithm (with k=3). The following ground truth data is available:

Length	6	3	6	9	8	1	3	2	8	7	5	5
Weight	4	2	6	6	6	3	4	3	7	5	7	5
Category	D	C	D	D	D	C	C	C	D	D	C	D

(D=Duck, C=Chicken)

The distance measure used by the system to compare two objects is:

$$D(O_i,O_j) = |(\text{length})_i - (\text{length})_j| + |(\text{weight})_i - (\text{weight})_j|$$

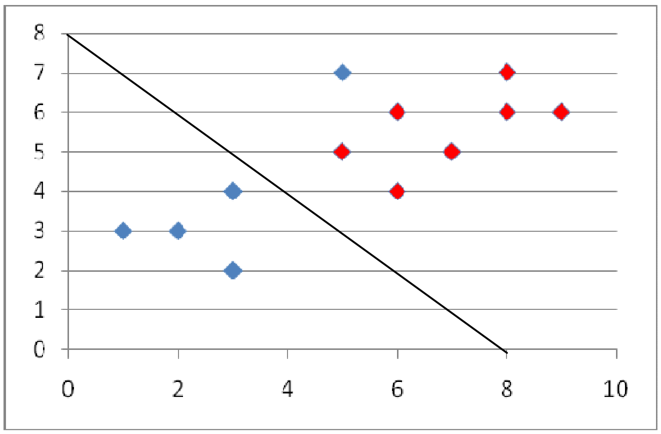
i. Given an object with length 5 units and weight 6 units, will the system classify it as a chicken or a duck?

ii. From the given ground truth data, suggest a linear decision boundary to separate chickens from ducks. You need to give an *equation* that describes the boundary.

Solution

i. The given trainin data can be ploted in two dimensional feature space as follows (Red=Duck, Blue=Chicken)

Weight



Length

The unknown objects is Q(5,6). We need to compute the distance d(Q,Ti) where Ti are the training examples.

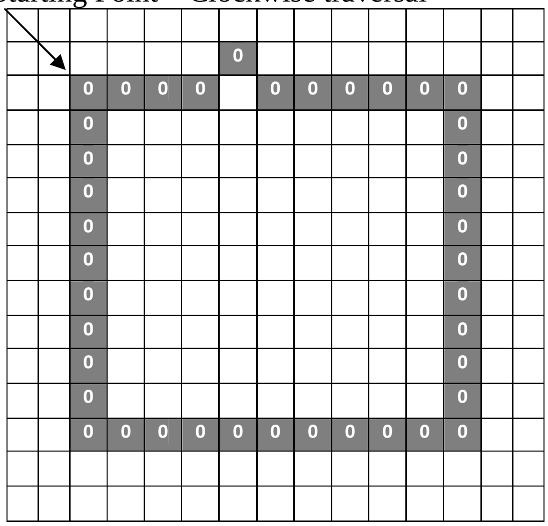
D(Q,T1) = abs(5-6) + abs(6-4) = 3.
D(Q,T2) = abs(5-3) + abs(6-2) = 6.
D(Q,T3) = abs(5-6) + abs(6-6) = 1.
D(Q,T4) = abs(5-9) + abs(6-6) = 4.
D(Q,T5) = abs(5-8) + abs(6-6) = 3.
D(Q,T6) = abs(5-1) + abs(6-3) = 7.
D(Q,T7) = abs(5-3) + abs(6-4) = 4.
D(Q,T8) = abs(5-2) + abs(6-3) = 6.
D(Q,T9) = abs(5-8) + abs(6-7) = 4.
D(Q,T10) = abs(5-7) + abs(6-5) = 3.
D(Q,T11) = abs(5-5) + abs(6-7) = 1.
D(Q,T12) = abs(5-5) + abs(6-5) = 1.

The three nearest neighbors (T3,T11,T12) have a distance of 1 from the query object. Out of these, two are ducks and one is chicken, so the query object will be classifed as duck by the system.

ii. The decision boundary is not unique. One possible boundary is shown in the plot above and can be represenetd by the equation: y=8-x.

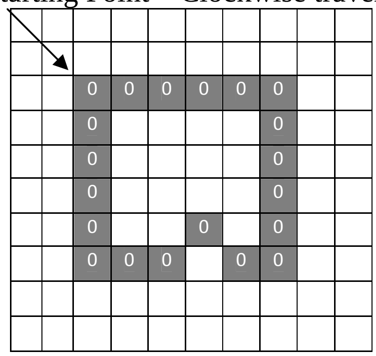
b. Consider two objects A and B, represented by their contours as illustrated in the following.

Starting Point – Clockwise traversal

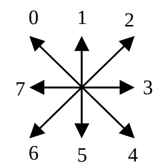


Object A

Starting Point – Clockwise traversal



Object B



Compare the two objects by computing the distance between the histograms of their chain codes. Use the following metric to compare histograms h^1 and h^2:

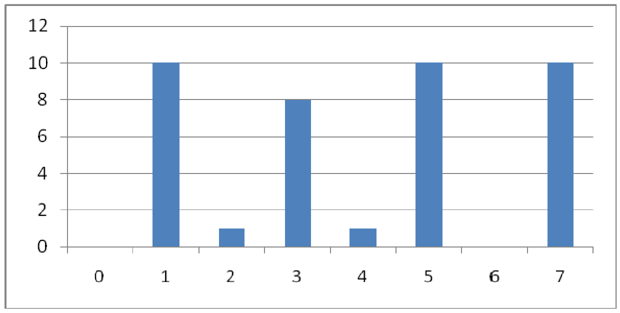
$$d(h^1, h^2) = \sum_{j=1}^{NBins} \frac{\left|h_j^1 - h_j^2\right|}{h_j^1 + h_j^2}$$

h^i_j = Value in the *jth* bin of the *ith* histogram
NBins = the number of bins in each histogram

Solution

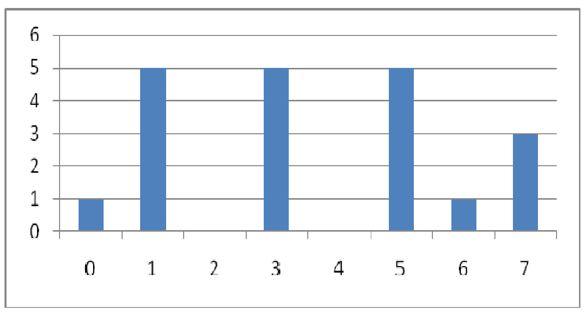
Chain code of A: 3 3 3 2 4 3 3 3 3 3 5 5 5 5 5 5 5 5 5 5 7 7 7 7 7 7 7 7 7 7 1 1 1 1 1 1 1 1 1 1
Histogram of chain code:

Code	Freq	Normalized Freq
0	0	0
1	10	0.25
2	1	0.025
3	8	0.2
4	1	0.025
5	10	0.25
6	0	0
7	10	0.25



Chain code of B: 3 3 3 3 3 5 5 5 5 5 7 0 6 7 7 1 1 1 1 1

Code	Freq	Normalized Freq
0	1	0.05
1	5	0.25
2	0	0
3	5	0.25
4	0	0
5	5	0.25
6	1	0.05
7	3	0.15



In order to compare the two, we need to find the distance between the normalized histograms.

$$\sum_{j=1}^{NBins} \frac{\left|h_j^1 - h_j^2\right|}{h_j^1 + h_j^2} = \frac{|0-0.05|}{(0+0.05)} + \frac{|0.25-0.25|}{(0.25+0.25)} + \frac{|0.025-0|}{(0.025+0)} + \frac{|0.2-0.25|}{(0.2+0.25)} +$$

$$\frac{|0.025-0|}{(0.025+0)} + \frac{|0.25-0.25|}{(0.25+0.25)} + \frac{|0-0.05|}{(0-0.05)} + \frac{|0.25-0.15|}{(0.25+0.15)}$$

$$= 1 + 0 + 1 + 0.1111 + 1 + 0 + 1 + 0.25 = 4.36666$$

c. The chain code computed from the boundary of an object is dependent upon the starting point. Can you suggest a chain code representation that is invariant to the starting point? You may take the example of object *A* above. Please note that you need to propose a representation of the chain code and NOT its histogram.

Solution

Consider the chain code of object B above (starting point=top left)

3 3 3 3 3 5 5 5 5 5 7 0 6 7 7 1 1 1 1 1

If starting point is top right pixel, the chain code is

The chain code can be considered as a number and arranged in order

0 1 1 1 1 1 3 3 3 3 3 5 5 5 5 5 6 7 7 7

Which will be independent of the starting point.

++++++++++++++ Bon Courage ++++++++++++