SOLUTION OF NUMERICAL ANALYSIS FINAL PAPER

SOL TO Q#1 $\frac{1}{10} = (0.1)_{10} = (0.000110011\,0011 - - - -)_2$

$= (1.10011001 - - -) \times 2^{-4}$, Exponent $= -4$, Mantissa $= 1.10011 ..$

Biased Exponent $= -4 + 127 = 123 = (1111011)_2$

Representation by machine (4)

$\underbrace{0}_{S}\ \underbrace{01111011}_{E}\ \underbrace{0011001 - - -}_{M}$ (1)

$-\infty = \underbrace{1}_{S}\ \underbrace{11111111}_{E}\ \underbrace{00000 - -}_{M}$

SOL TO Q#2

$f(x) = (x-1)^{0.5}$ is NOT a real valued function, e.g (2)

$f(0) = (0-1)^{1/2} = \sqrt{-1}$ is an imaginary number

$f(x) = e^{x}$, $E \leq \frac{f^{(n+1)}(c)(x-x_0)^{n+1}}{(n+1)!}$, $c \in (0, .1)$

$f^{(n+1)}(c) = e^{c}$, Max $f^{n+1}(.1) = 1.10517$

$\frac{1}{2} \times 10^{-12} \leq \frac{(1.10517)(0.1-0)^{n+1}}{(n+1)!}$ This relation is true when $\boxed{n=7}$

SoL To Q#3 $f(x) = \int_{-1}^{x} \cos t^2\, dt$, $x_0 = 0$ & $\int_{-1}^{0} \cos t^2 dt = .904$

$f(0) = 0.90425$, $f'(x) = \cos x^2$, $f'(0) = 1$

$f''(x) = -2\sin x^2 \cdot (2x)$, $f''(0) = 0$, Similarly $f'''(0) = f^{IV}(0) = 0$

So expansion of $\boxed{f(x) = 0.90425 + x}$.

SOL TO Q4 If data does not exhibit polynomial nature the resulting curve may exhibit large oscillation, that is why higher degree polynomial is not recommended.

$$P(x) = l_0 y_0 + l_1 y_1 + l_2 y_2 = \frac{(x-1)(x-3)}{3} \times 0 + \frac{(x-0)(x-3)}{-2} \times 1 +$$

$$\frac{(x-0)(x-1)}{6} \times 2 = \frac{x(x-1)}{3} - \frac{x(x-3)}{2}, \quad P(3.5) \approx 2.0416$$

SOL TO Q#5 $P(t) = \frac{L}{1+Ce^{At}}$ linearized form of the expression is

$$\ln\left(\frac{L}{P} - 1\right) = At + \ln C.$$

$$Y = Ax + B, \quad Y = \ln\left(\frac{L}{P} - 1\right)$$

$x = t$

Normal equations are $B = \ln C$

$$150A + (-10)B = -197.63253$$

$$-10A + 4B = 67.8108$$

$$A = -0.224784$$

$$B = 16.39074$$

$$C = e^B \approx 13134330.$$

SOL TO Q#6 $f(x) = 3x^2 - e^x$

$f(0) = -1$, $f(1) = 0.2817$ So ONE root is lying in the interval $[0,1]$. Initial guess may be taken as 0.5. Apply Newton method we get a ~~...~~ sequence like 0.9141553341, 0.910027869, 0.910011475, --- And relative error is less than 5×10^{-5}. $\left|\frac{p_{i+1} - p_i}{p_{i+1}}\right| = 7.56 \times 10^{-6}$.

SOL TO Q#7, $h = \frac{\pi}{10}$

$$\text{Integral} \approx \frac{1}{\pi}\left(\frac{h}{3}\right)\left[f(0) + 4f\left(\frac{\pi}{10}\right) + 2f\left(\frac{\pi}{5}\right) + \cdots + f(\pi)\right]$$

$$\approx 4.216888$$

SOL TO Q#8 $\int_0^2 x e^{-x} dx$, $M = ?$, $h = ?$, $a = 0$, $b = 2$.

$f(x) = x e^{-x}$, $f'(x) = -x e^{-x} + e^{-x} = e^{-x}(-x+1)$

$f''(x) = e^{-x}(-1) + (-x+1)(-e^{-x}) = -e^{-x} + x e^{-x} - e^{-x}$

$f''(x) = e^{-x}(x-2)$, $\max f''(c) = 2$ with $x = 0$

$E = \dfrac{-(b-a) f''(c) \cdot h^2}{12}$, $E \leq \frac{1}{2} \times 10^{-12}$

$h = 0.1224744423$, $M = 1632993$.

SOL TO Q#9

(a) $y_1 = y_2 = y_3 = 0 \,-\,-\,-\,-$ Euler's Method is not applicable.

(b) $y_1 = y_0 + \frac{h}{2}\left(f(t_0, y_0) + p_1 \right)$

$p_1 = y_0 + h f(t_0, y_0) = 1$

$y_1 \simeq -0.43233$ Similarly

$y_2 = -0.490842$.