Military College of Signals

CS Department

Automata Theory and Formal Languages

Spring 2011 **BESE 14**

Assignment 02 – NFA to DFA

Due date: *15th Mar, 2011*

SOLUTION

Q1: Convert the NFA given in Table 1 to DFA. The start state is q_0 and the final state is q_3.

Table 1

	0	**1**	**λ**
q_0	q_0	q_0, q_1	-
q_1	q_2	-	q_2
q_2	-	q_3	-
q_3	q_3	q_3	-

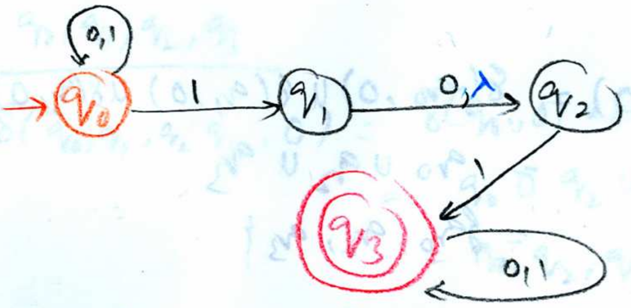
QUESTION 1

1. Step 1. Drawing NFA from table.

	0	1	λ
q_0	q_0	q_0, q_1	–
q_1	q_2	–	q_2
q_2	–	q_3	–
q_3	q_3	q_3	–

$S = q_0$

$q_3 \in F$



Accepts

$(0+1)^* 11 (0+1)^*$

$(0+1)^* 101 (0+1)^*$

2. Start at q_0

$\delta(q_0, 0) = q_0$, $\delta(q_0, 1) = q_0, q_1$

Since $\delta(q_1, \lambda) = q_2$ $\therefore$ q_1 & q_2 are same state

Hence $\delta(q_0, 1) = q_0, q_1, q_2$

3. At q_0, q_1, q_2

$\delta((q_0, q_1, q_2), 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0)$
$= q_0 \cup q_2 \cup \phi = q_0, q_2$

$\delta((q_0, q_1, q_2), 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1)$
$= q_0, q_1 \cup \phi \cup q_3 = q_0, q_1, q_2, q_3$

4. At (q_0, q_2)

$\delta((q_0, q_2), 0) = \delta(q_0, 0) \cup \delta(q_2, 0)$

$= q_0 \cup \phi = q_0$

$\delta((q_0, q_2), 1) = \delta(q_0, 1) \cup \delta(q_2, 1)$

$= q_0, q_1, q_3$

5. At q_0, q_1, q_3

$\delta((q_0, q_1, q_3), 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_3, 0)$

$= q_0 \cup q_2 \cup q_3$

$= \{q_0, q_2, q_3\}$

$\delta((q_0, q_1, q_3), 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_3, 1)$

$= q_0, q_1 \cup q_2, q_3$

$= q_0, q_1, q_2, q_3$

6. At q_0, q_2, q_3

$\delta((q_0, q_2, q_3), 0) = \delta(q_0, 0) \cup \delta(q_2, 0) \cup \delta(q_3, 0)$

$= q_0 \cup \phi \cup q_3 = q_0, q_3$

$\delta((q_0, q_2, q_3), 1) = \delta(q_0, 1) \cup \delta(q_2, 1) \cup \delta(q_3, 1)$

$= q_0, q_1, q_3 \cup q_3$

$= q_0, q_1, q_3$

7. At q_0, q_3

$$\delta((q_0, q_3), 0) = \delta(q_0, 0) \cup \delta(q_3, 0)$$
$$= q_0 \cup q_3 = q_0, q_3$$

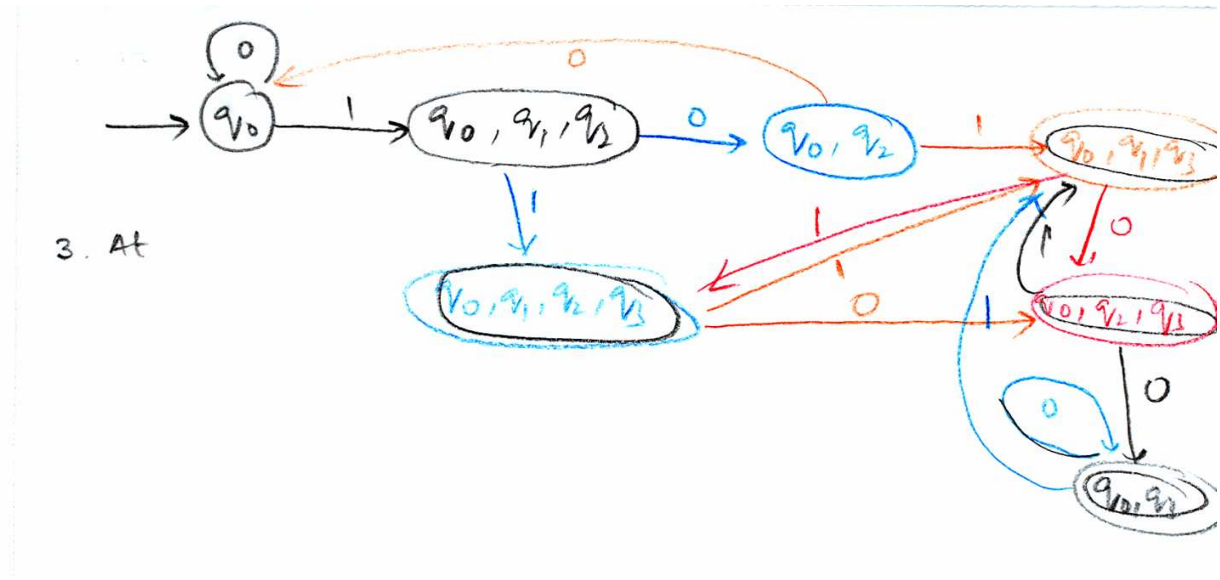
$$\delta((q_0, q_3), 1) = \delta(q_0, 1) \cup \delta(q_3, 1)$$
$$= q_0, q_1 \cup q_3$$
$$= q_0, q_1, q_3$$

8. At q_0, q_1, q_2, q_3

$$\delta(q_0, q_1, q_2, q_3, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0) \cup \delta(q_3, 0)$$
$$= q_0 \cup q_2 \cup \phi, q_3$$
$$= q_0, q_2, q_3$$

$$\delta(q_0, q_1, q_2, q_3, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) \cup \delta(q_3, 1)$$
$$= q_0, q_1 \cup \phi \cup q_3 \cup q_3$$
$$= q_0, q_1, q_3$$

9. Marking Final states. All with q_3


3. At

Q2: Convert the NFA given in Table 2 to DFA. The start state is A and the final state is D.

Table 2

	0	**1**
A	A, C	A, B
B	C	B
C	B, C	D
D	D	D

QUESTION 2

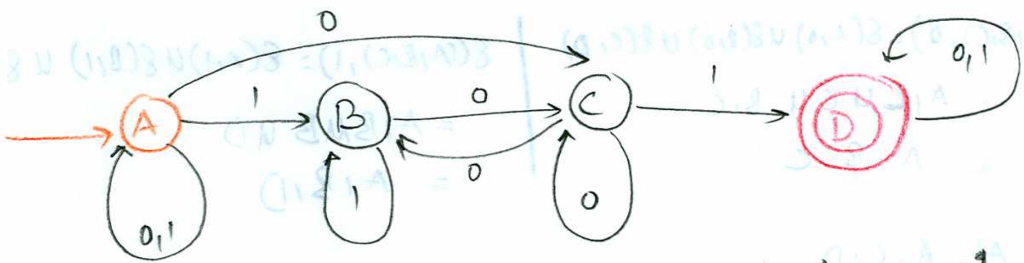
Convert the NFA to DFA.

	0	1
A	A, C	A, B
B	C	B
C	B, C	D
D	D	D

$S = A$

$D \in F$

1. Drawing NFA



Accepts $(0+1)^* 101 (0+1)^*$

$(0+1)^* 1(1^*01 + \cancel{0}0^*00)) + (0+1)^*$

2. Start at A

$\delta(A, 0) = \underline{A, C}$

$\delta(A, 1) = A, B$

3. At $\underline{A, C}$

$\delta((A,C), 0) = \delta(A, 0) \cup \delta(C, 0)$
$= A, C \cup B, C$
$= A, B, C$

$\delta((A,C), 1) = \delta(A, 1) \cup \delta(C,$
$= A, B \cup D$
$= A, B, D$

4. At A,B

$\delta((A,B),0) = \delta(A,0) \cup \delta(B,0)$
$= A,C \cup C$
$= A,C$

$\delta((A,B),1) = \delta(A,1) \cup \delta(B,1)$
$= A,B \cup B$
$= A,B$

5. At A,B,D

$\delta((A,B,D),0) = \delta(A,0) \cup \delta(B,0) \cup \delta(D,0)$
$= A,C \cup C \cup D$
$= A,C,D$

$\delta((A,B,D),1) = \delta(A,1) \cup \delta(B,1) \cup \delta(D,1)$
$= A,B \cup B,D$
$= A,B,D$

6. At A,B,C

$\delta((A,B,C),0) = \delta(A,0) \cup \delta(B,0) \cup \delta(C,0)$
$= A,C \cup C \cup B,C$
$= A,B,C$

$\delta((A,B,C),1) = \delta(A,1) \cup \delta(B,1) \cup \delta(C,1)$
$= A,B \cup B \cup D$
$= A,B,D$

7. At A,C,D

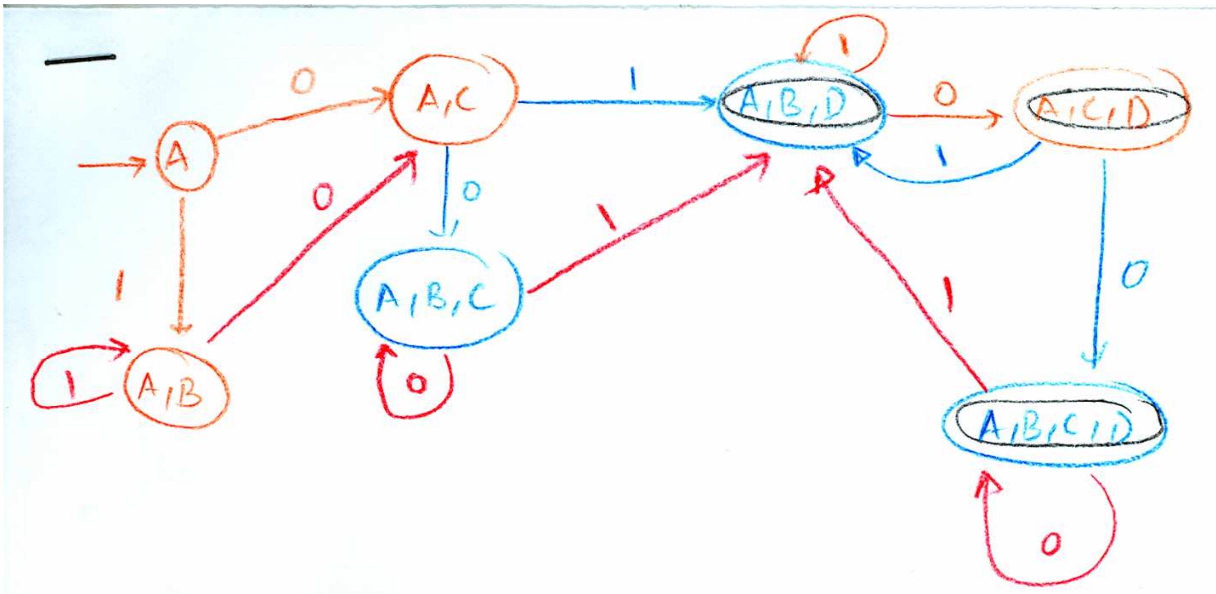
$\delta((A,C,D),0) = \delta(A,0) \cup \delta(C,0) \cup \delta(D,0)$
$= A,C \cup B,C \cup D$
$= A,B,C,D$

$\delta((A,C,D),1) = \delta(A,1) \cup \delta(C,1) \cup \delta(D,1)$
$= A,B \cup D \cup D$
$= A,B,D$

8. At A,B,C,D

$\delta((A,B,C,D),0) = \delta(A,0) \cup \delta(B,0) \cup \delta(C,0) \cup \delta(D,0)$
$= A,C \cup C \cup B,C \cup D$
$= A,B,C,D$

$\delta((A,B,C,D),1) = \delta(A,1) \cup \delta(B,1) \cup \delta(C,1) \cup \delta(D,1)$
$= A,B \cup B \cup D \cup D$
$= A,B,D$



9. Marking Final states. All having D

Q3: Convert the NFA given in Figure 1 to DFA.

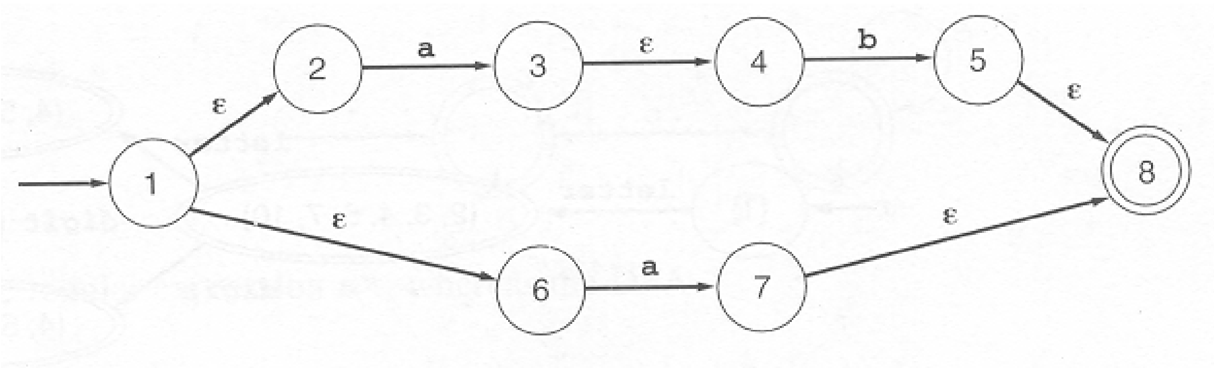
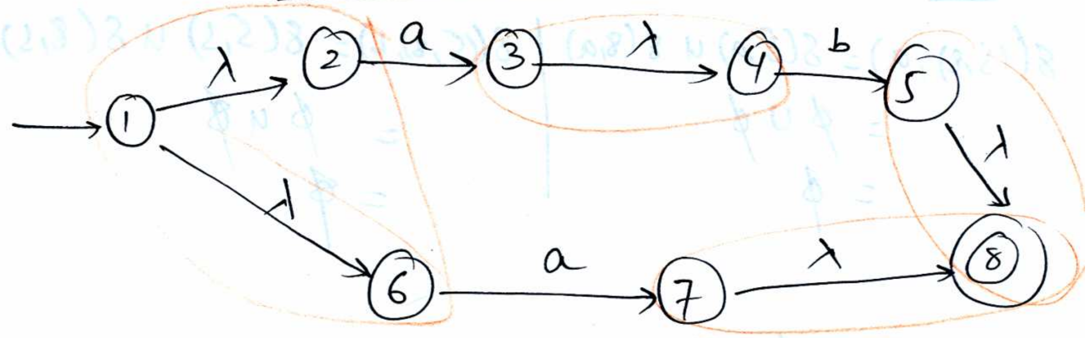


Figure 1

QUESTION 3



1. As highlighted 1,2,6 are one state
   3,4 " " "
   5,8 " " "
   7,8 " " "

2. Selecting 1 as start state
   but as 1,2,6 is a single state ∴
   1,2,6 is start state

$\delta((1,2,6),a) = \delta(1,a) \cup \delta(2,a) \cup \delta(6,a)$
$= \phi \cup 3,4 \cup 7,8$
$= 3,4,7,8$

$\delta((1,2,6),b) = \delta(1,b) \cup \delta(2,b) \cup \delta(6$
$= \phi \cup \phi \cup \phi$
$= \phi$

3. For ϕ $\delta(\phi,a) = \phi$ and $\delta(\phi,b) = \phi$

4. For 3,4,7,8

$\delta((3,4,7,8),a) = \delta(3,a) \cup \delta(4,a) \cup \delta(7,a) \cup \delta(8,a)$
$= \phi \cup \phi \cup \phi \cup \phi$
$= \phi$

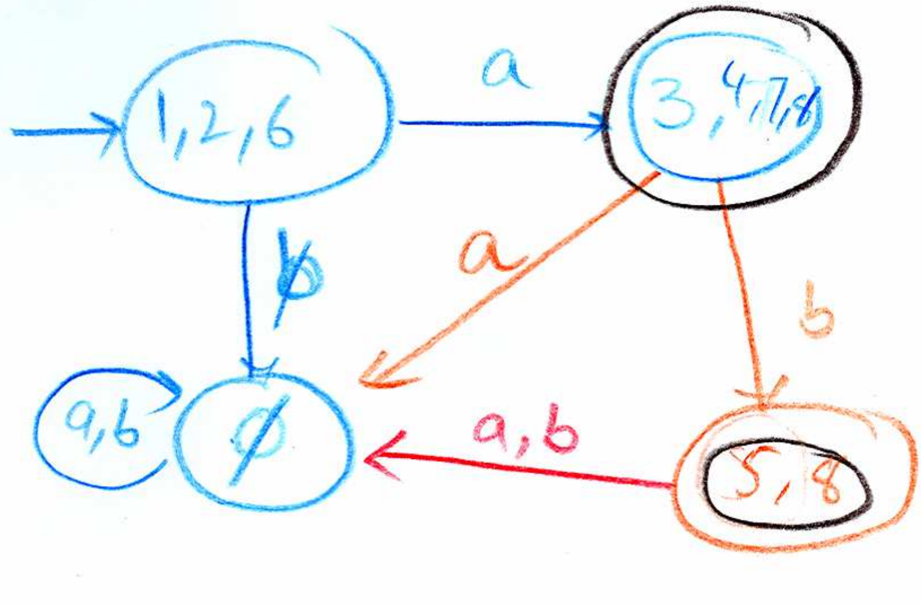
$\delta((3,4,7,8),b) = \delta(3,b) \cup \delta(4,b) \cup \delta(7,b) \cup \delta(8,b)$
$= \phi \cup 5,8 \cup \phi \cup \phi$
$= 5,8$ since 5,8 is one

5. $\underline{5,8}$

$\delta((5,8),a) = \delta(5,a) \cup \delta(8,a)$
$= \phi \cup \phi$
$= \phi$

$\delta((5,8),b) = \delta(5,b) \cup \delta(8,b)$
$= \phi \cup \phi$
$= \phi$

6. Marking all states with 8 as final.



Q4: Convert the NFA given in Figure 2 to DFA.

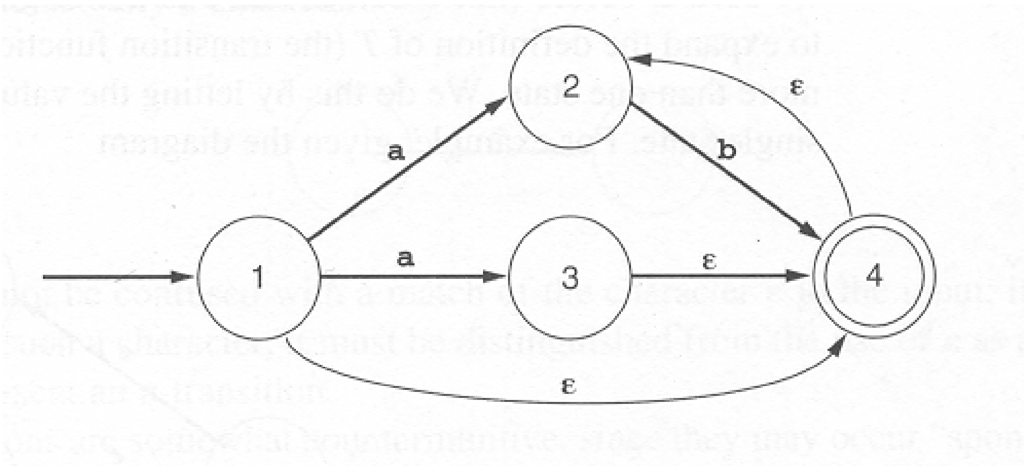
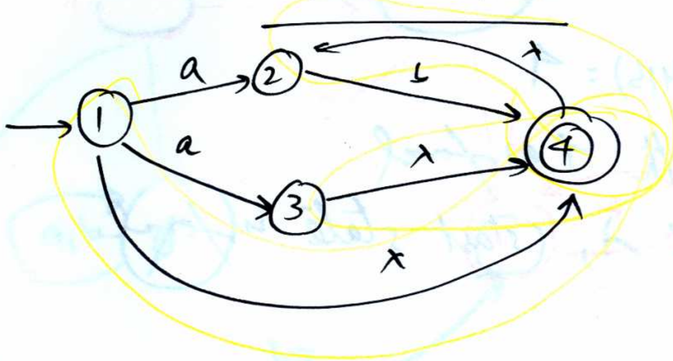


Figure 2

QUESTION 4



1. 1,4 and 3,4 are same. Also, 4,2 are same
2. The NFA accepts λ
3. At 1

   Since 1,4 is same

   $\therefore$ start state is 1,4

$\delta((1,4),a) = \delta(1,a) \cup \delta(4,a)$

$= 3,4 \cup \phi$

$= 2,3,4$

Since 3,4 is same

$\delta((1,4),b) = \delta(1,b) \cup \delta(4,b)$

$= 4 \cup 4$

$= 4$

1,4 and 4,2 are same.

4. At ϕ

   $\delta(\phi, a) = \phi \quad \delta(\phi, b) = \phi$

5. At 2,3,4

$\delta((2,3,4),a) = \delta(2,a) \cup \delta(3,a) \cup \delta(4,a)$

$= \phi \cup \phi \cup \phi$

$= \phi$

$\delta((2,3,4),b) = \delta(2,b) \cup \delta(3,b) \cup \delta(4,b)$

$= 4 \cup \phi \cup \phi$

$= 4$

6. At 4

4,2 is same

$\delta(4,a) = \phi$, $\delta(4,b) = 4$

7. Marking all states with 4 as final

8. Since NFA accepts λ, start state is final