AUTOMATA THEORY & FORMAL LANGUAGES

SOLUTION - Spring 2011

MID-TERM EXAM

QUESTION 1 - INTRODUCTION

A. What is a finite language? Give two examples.

A language that has a finite number of members is a Finite language. e.g.

$\{a, ab\}$

$\{a, aa, aaa\}$

In contrast $\{a, aa, aaa, \ldots\}$ has INFINITE members

B. Differentiate among alphabet and string with examples.

ALPHABET	STRINGS
1. An alphabet is a set of symbols, or characters that defines a language	1. A string is a sequence of alphabets
2. Alphabets make strings	2. strings are composed of alphabets
$\Sigma = \{0,1\}$, $\Sigma = \{\alpha, \beta\}$	string $w = 010$, $w = 00$, $w = \alpha\beta\beta$

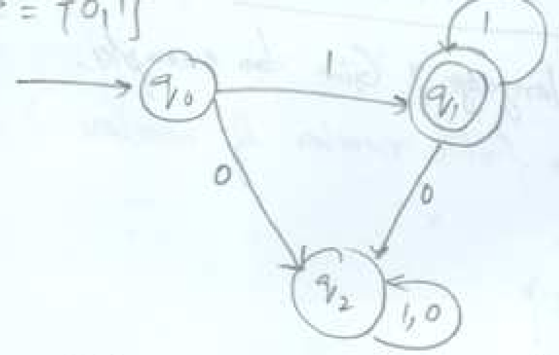
QUESTION 2 - FINITE AUTOMATA

A. what is a trap state? give example.

A state from where the automata cannot escape (for any gi string). A trap state is generally Non-accepting stat

For example, in the following DFA, q_2 is a trap state.

$\Sigma = \{0, 1\}$



$$\delta(q_2, 0) = q_2 \;\&\; \delta(q_2, 1) = q_2$$

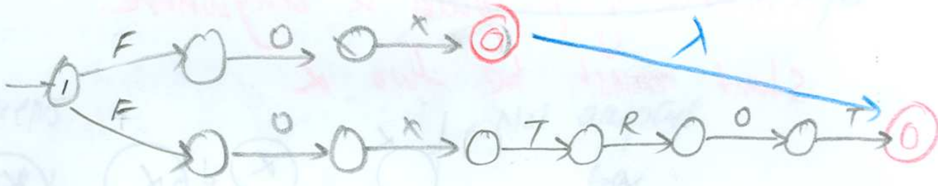
As is evident from the DFA as well as the transition, q_2 is a trap state

QUESTION 2 - FINITE AUTOMATA

B. Advantages of NFA over DFA

NFA is non-deterministic finite automata while DFA i[s] deterministic finite automata. NFA has certain adva[ntages] over DFA and these are -

i. **FLEXIBILITY**. NFA are more flexible than DFA. Flexibility allows an NFA to include choice or alternatives which are absent in DFA.



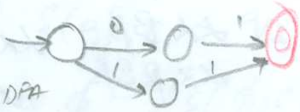
NFA

As is shown in figure at state 1 the choice of paths for F [is] available. These also include λ-transitions which are absent in [DFA]

ii. **EASY TO DRAW**. Humans are more suited to develop [NFA] as compared to developing DFA.

iii. **LESS STATES**. NFA have lesser states as compared [to] equivalent DFAs and hence are more simple as [...]

NFA



DFA

While NFA are advantageous they do suffer from non-determinism which is absent in DFA.

C. DFA

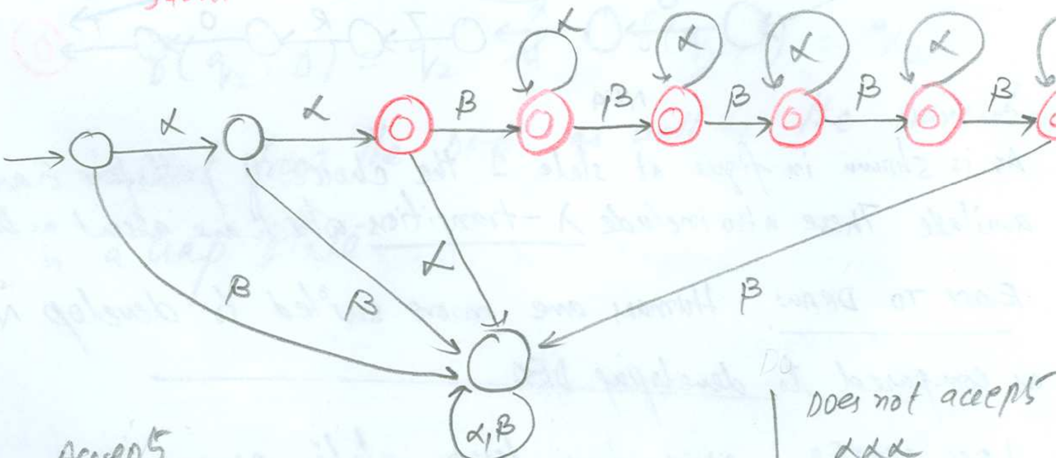
$\Sigma = \{\alpha, \beta\}$

Accepts all string that start with exactly two α; and does not have more than 5 β in the string

(could be zero or upto 5)

Location of β could be anywhere.

Start must be two α



Accepts

$\alpha\alpha$

$\alpha\alpha\beta$ $\alpha\alpha\beta\beta\beta\beta$

$\alpha\alpha\beta\beta$ $\alpha\alpha\beta\beta\beta\beta\beta$

$\alpha\alpha\beta\beta\beta$ $\alpha\alpha\beta(\alpha)^*\beta\cdot(\alpha^*)\beta(\alpha^*)\beta(\alpha)^*\beta(\alpha^*)$

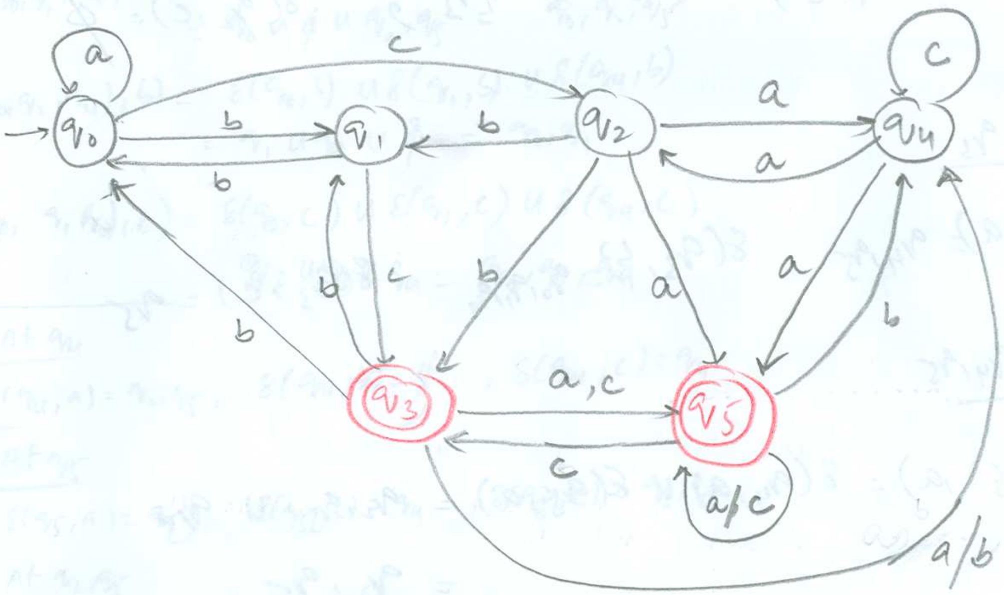
Does not accepts

$\alpha\alpha\alpha$

$\alpha\alpha\beta\beta\beta\beta\beta\beta$ —

β, $\alpha\beta$

1). ~~N~~FA $q_3, q_5 \in F$, $q_0 \in S$



It accepts

$a^* bc$ cb
$acbc$ ca etc

Not accepted
ba

CONVERSION TO DFA

i. start at q_0

$\delta(q_0, a) = q_0$ | $\delta(q_0, b) = q_1$ | $\delta(q_0, c) = q_2$

ii. At q_1

$\delta(q_1, a) = \phi$ | $\delta(q_1, b) = q_0$ | $\delta(q_1, c) = q_3$

iii. At ϕ = $\delta(\phi, a) = \phi$, | $\delta(\phi, b) = \phi$, $\delta(\phi, c) =$

iv. At q_2

$\delta(q_2, a) = q_4, q_5$, $\delta(q_2, b) = q_1 q_3$, $\delta(q_2, c) = \phi$

v. At q_3

$\delta(q_3, a) = q_4, q_5$ $\delta(q_3, b) = q_0, q_1, q_4$ $\delta(q_3, c) = q_5$

vi. At q_4, q_5

$\delta((q_4, q_5), a) = \delta(q_4, a) \cup \delta(q_5, a) = q_2, q_5 \cup q_5$
$= q_2, q_5$

$\delta((q_4, q_5), b) = \delta(q_4, b) \cup \delta(q_5, b) = \phi \cup q_4$
$= q_4$

$\delta((q_4, q_5), c) = \delta(q_4, c) \cup \delta(q_5, c) = q_4 \cup q_3, q_5$
$= q_3, q_4, q_5$

vii At $q_1 q_3$

$\delta(q_1, q_3, a) = \delta(q_1, a) \cup \delta(q_3, a) = \phi \cup q_4, q_5$
$= q_4, q_5$

$\delta((q_1, q_3), b) = \delta(q_1, b) \cup \delta(q_3, b) = q_0 \cup q_0, q_1, q_4$
$= q_0, q_1, q_4$

$\delta((q_1, q_3), c) = \delta(q_1, c) \cup \delta(q_3, c) = q_3 \cup q_5 = q_3, q_5$

At q_0, q_1, q_4

$\delta((q_0, q_1, q_4), a) = \delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_4, a)$
$= q_0 \cup \phi \cup q_2, q_5 = q_0, q_2, q_5$

$\delta((q_0, q_1, q_4), b) = \delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_4, b)$
$= q_1 \cup q_0 \cup \phi = q_0, q_1$

$\delta((q_0, q_1, q_4), c) = \delta(q_0, c) \cup \delta(q_1, c) \cup \delta(q_4, c)$
$= q_2 \cup q_3 \cup q_4 = q_2, q_3, q_4$

ix. At q_4

$\delta(q_4, a) = q_2, q_5$, $\delta(q_4, b) = \phi$, $\delta(q_4, c) = q_4$

x. At q_5

$\delta(q_5, a) = q_5$, $\delta(q_5, b) = q_4$, $\delta(q_5, c) = q_3, q_5$

xi At $q_2 q_5$

$\delta((q_2, q_5), a) = \delta(q_2, a) \cup \delta(q_5, a)$
$= q_4, q_5 \cup q_5 = q_4, q_5$

$\delta((q_2, q_5), b) = \delta(q_2, b) \cup \delta(q_5, b)$
$= q_1, q_3 \cup q_4 = q_1, q_3, q_4$

$\delta((q_2, q_5), c) = \delta(q_2, c) \cup \delta(q_5, c)$
$= \phi \cup q_3, q_5 = q_3, q_5$

xii. At q_3, q_4, q_5

$\delta((q_3, q_4, q_5), a) = \delta(q_3, a) \cup \delta(q_4, a) \cup \delta(q_5, a)$
$= q_4, q_5 \cup q_2, q_5 \cup q_5 = q_2, q_4, q_5$

$\delta\{(q_3, q_4, q_5), b\} = \delta(q_3, b) \cup \delta(q_4, b) \cup \delta(q_5, b)$
$= q_0, q_1, q_4 \cup \phi \cup q_4 = q_0, q_1, q_4$

$\delta((q_3, q_4, q_5), c) = \delta(q_3, c) \cup \delta(q_4, c) \cup \delta(q_5, c)$
$= q_5 \cup q_4 \cup q_3, q_5 = q_3, q_4, q_5$

xiii At q_0, q_1

$\delta\{(q_0, q_1), a\} = \delta(q_0, a) \cup \delta(q_1, a)$
$= q_0 \cup \phi = q_0$

$\delta\{(q_0, q_1), b\} = \delta(q_0, b) \cup \delta(q_1, b)$
$= q_1 \cup q_0$
$= q_0, q_1$

$\delta((q_0, q_1),$
$= \delta(q_0, c$
$= q_2 \cup$

xiv At q_1, q_3, q_4

$\delta((q_1, q_3, q_4), a) = \delta(q_1, a) \cup \delta(q_3, a) \cup \delta(q_4, a)$
$= \phi \cup q_4, q_5 \cup q_2, q_5 = q_2, q_4, q_5$

$\delta((q_1, q_3, q_4), b) = \delta(q_1, b) \cup \delta(q_3, b) \cup \delta(q_4, b)$
$= q_0 \cup, q_0, q_1, q_4 \cup \phi = q_0, q_1, q_4$

$\delta((q_1, q_3, q_4, c) = \delta(q_1, c) \cup \delta(q_3, c) \cup \delta(q_4, c)$
$= q_3 \cup q_5 \cup q_4 = q_3, q_4, q_5$

xv. At q_3, q_5

$\delta((q_3, q_5), a) = \delta(q_3, a) \cup \delta(q_5, a) = q_4, q_5 \cup q_5 = q_4, q_5$

$\delta((q_3, q_5), b) = \delta(q_3, b) \cup \delta(q_5, b) = q_0, q_1, q_4 \cup q_4 = q_0, q_1, q_4$

$\delta((q_3, q_5) c) = \delta(q_3, c) \cup \delta(q_5, c) = q_5 \cup q_3, q_5 = q_3, q_5$

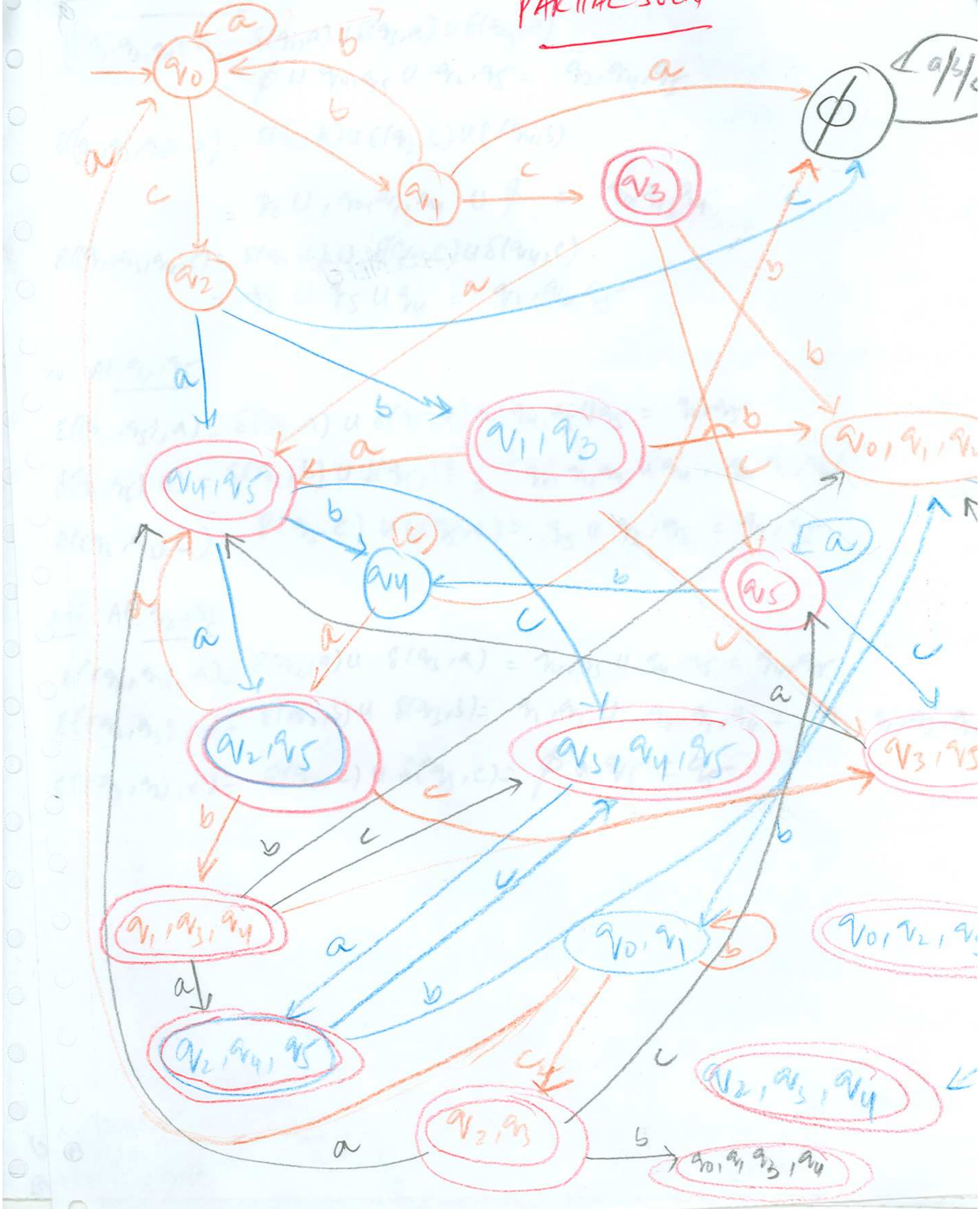
xvi At q_2, q_3

$\delta((q_2, q_3), a) = \delta(q_2, a) \cup \delta(q_3, a) = q_4, q_5 \cup q_4, q_5 = q_4, q_5$

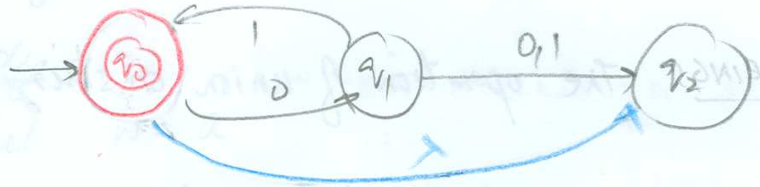
$\delta((q_2, q_3), b) = \delta(q_2, b) \cup \delta(q_3, b) = q_1, q_3 \cup, q_0, q_1, q_4 = q_0, q_1, q_3,$

$\delta((q_2, q_3), c) = \delta(q_2, c) \cup \delta(q_3, c) = \phi \cup q_5 = q_5$

PARTIAL SOLN



E. Language generated by NFA



Accepted strings	Not-Accepted strings
λ since $q_0 \in F$	00
01	01
0101	λ
010101	010
$(01)^n$	011
	$0101\lambda = 0101$
	01010101λ . . .

Hence, the languages is

$$L = \{ (01)^n \; ; \; n \geq 0 \}$$

ANSWER 3. REGULAR EXPRESSIONS

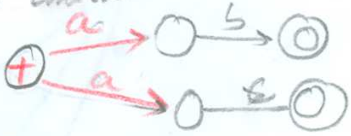
A. UNION on STRINGS The operation of union on strings
defined as,

Let w and s be any two strings then the express
w+s is the union of both. This means that the u
either belongs to w, or s or both.

The symbol for union is '+'

Example.
Let w=abc and s=cba be two strings on Σ = {
Then if the union is
r = abc this means that r is either b
to w or s. In expression form w+s => abc+cba

Union is both commutative and associative and op
on two values. The following FA shows union

(ab)+(ac) · 

KLEENE STAR ON STRINGS.

Kleene star represents a repetition of zero or more t
This operation is defined as
w^* where w is repeated zero or more tim

The symbol is '*'

Example

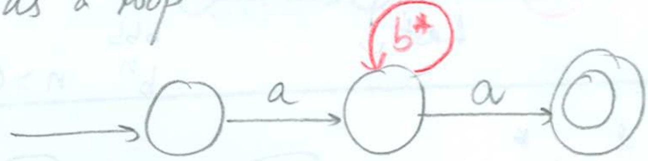
Let $w = a$

then $w^* = \lambda, a, aa, aaa, aaa, \ldots$

$$w^* = \{a^n : n \geq 0\}$$

Kleene star is a unary operator. The following FA

Kleene star as a loop

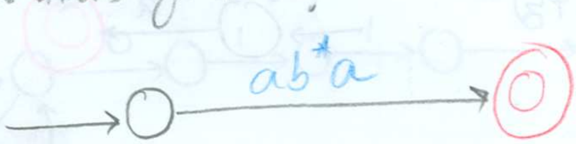


The regular expression is ab^*a

B. GENERALIZED NFA

A generalized NFA is a special NFA on which all h
are marked with regular expressions. Also

i. It has only one final state other than the start state
ii. No arrows come out of the accept state
iii. No arrows go in to the start state.



is a generalized NFA

(2) All strings of a's and b's ending in atleast one b

$\underbrace{\text{All strings of a's and b's}}_{\text{anything}}$ ending → Last

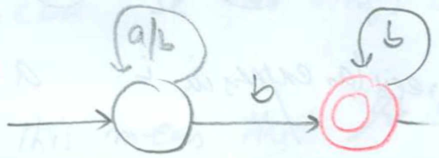
$\underbrace{\text{atleast one b}}$

b
bb
bbb
$b^n \quad n > 0$

$$(a+b)^* \, bb^*$$

or

$$(a+b)^* \, b^+$$



(3) All strings of a's and b's containing atleast one a

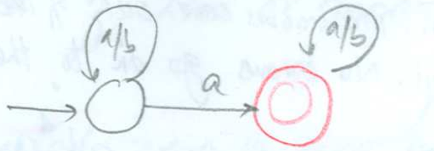
$\underbrace{\text{All strings of a's and b's}}_{\text{anything}}$ containing → No location is fixed

$\underbrace{\text{atleast one a}}$

a
aaa
aaa

λ Not accepted
b^n Not accepted

$$(a+b)^* \, a \, (a+b)^*$$



(4) All strings of a's and b's that contain atleast two b's

anything

any location

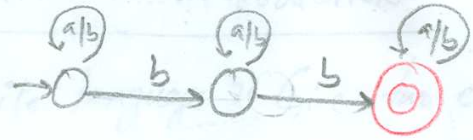
bb ✓ b ✗

bbb ✓ λ ✗

b^n $n>1$

consecutive info. No

$$(a+b)^* \, b \, (a+b)^* \, b \, (a+b)^*$$



D. FA $\Sigma = \{0, 1\}$

$r = (11)^* \, 0110 \, (00)^*$

writing FA

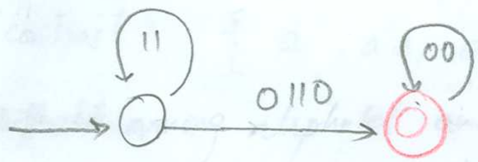
accepts

0110

0110|00

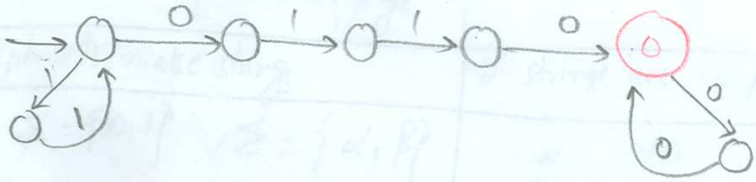
11|0110|00

11|0110



Regular exp

Resolving concatenation



Resolving concatenation in loop