ROUND - ROBIN TOURNAMENTS

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CONGRUENCES CAN BE USED TO SCHEDULE ROUND – ROBIN TOURNAMENTS. HERE, WE WILL SHOW HOW TO SCHEDULE A TOURNAMENT FOR *N* DIFFERENT TEAMS, SO THAT EACH TEAM PLAYS EVERY OTHER TEAM EXACTLY ONE.

FIRST NOTE THAT IF N IS ODD, NOT ALL TEAMS CAN BE SCHEDULED IN EACH ROUND, SINCE WHEN TEAMS ARE PAIRED, THE TOTAL NO OF TEAMS PLAYING IS EVEN. SO, IF N IS ODD, WE ADD A DUMMY TEAM, AND IF A TEAM IS PAIRED WITH DUMMY TEAM DURING A PARTICULAR ROUND, IT DRAWS A BYE IN THAT ROUND AND DOES NOT PLAY. HENCE, WE CAN ASSUME THAT WE ALWAYS HAVE AN EVEN NO OF TEAMS, WITH THE ADDITION OF DUMMY TEAM IF NECESSARY.

EXAMPLE

TO SCHEDULE A ROUND – ROBIN TOURNAMENT WITH 5 TEAMS, LABELED, 1, 2, 3, 4, AND 5, WE INCLUDE A DUMMY TEAM LABELED 6. IN ROUND 1, TEAM 1 PLAYS TEAM j WHERE $1 + j \equiv 1(mod5)$. THIS IS THE TEAM $j = 5$, SO THAT THE TEAM 1 PLAYS TEAM 5. TEAM 2 IS SCHEDULED IN ROUND 1 WITH TEAM 4, SINCE THE SOLUTION OF $2 + j \equiv 1(mod5)$ IS $J = 4$. SINCE $i = 3$ IS THE SOLUTION OF THE CONRUENCE $2i \equiv 1(mod5)$, TEAM 3 IS PAIRED WITH THE DUMMY TEAM 6, AND HENCE, DRAWS A BYE IN THE FIRST ROUND. IF WE CONTINUE THIS PROCEDURE AND FINISH TO SCHEDULE THE OTHER ROUNDS, WE END UP WITH THE PAIRING SHOWN IN THE TABLE BELOW WHERE THE OPPONENT OF TEAM *i* IN ROUND *k* IS GIVEN IN THE kth ROW AND jth COLUMNS.

ROUND/TEAM	1	2	3	4	5
1	5	4	BYE	2	1
2	BYE	5	4	3	2
3	2	1	5	BYE	3
4	3	BYE	1	5	4
5	4	3	2	1	BYE