Dynamic Bus Routing: A study on the viability of on-demand high-capacity ridesharing as an alternative to fixed-route buses in Singapore

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Abstract—Mobility-on-demand ridesharing transformed the transportation landscape by offering commuters the convenience of point-to-point transportation, while at the same time being more efficient than private vehicles. However, the benefits these services have to offer are fundamentally limited by their capacities, and are often seen as taxi equivalents. In this paper, we apply this concept to high-capacity buses in a population-dense environment. We first develop a simulator that uses a modified insertion algorithm to model the dynamic routing of such vehicles. Using Singapore as a case study, we demonstrate that dynamically routed buses are an efficient mode of mass transport, and in some cases may hold significant advantages over present-day fixed-route services. Finally, we hypothesize how this new transport mode may be integrated with the wider public transportation network, to create a more efficient commute for all.

Ridesharing, personalized public transit, simulation and modelling

I. INTRODUCTION

In the near future, as the world's urban population continues its growth and major cities increasingly encourage commuters to adopt public transport over private vehicles, ridership for the former will likely increase substantially. Indeed, more than 50% of residents in major population centers, such as Hong Kong, Singapore, Tokyo, and London, currently rely on public transport for their daily commute [1]. The challenge is to meet this burgeoning demand with a comprehensive and efficient public transport system, in order to ensure a high-quality living environment for urban residents in the years to come. While expanding the rail network and increasing bus fleet sizes are possible solutions, they are unsustainable, especially in major cities where land

is increasingly scarce. In recognition of this, there has been a significant focus on instead *optimizing* the existing network, especially in the case of bus routes [2] [3].

The advent of new technology may offer a third approach. Notably, the increasing ubiquity of smartphones and advances in location tracking and computing power in recent years have led to the rise of "mobility-on-demand" ridesharing services¹. These services offer commuters the convenience of point-to-point transportation from origin to destination, while simultaneously being a more efficient alternative to private vehicles due to their carpooling nature. However, the efficiency gains they offer are fundamentally limited by their capacity. With each vehicle carrying only up to four passengers, ridesharing today is relegated to a supporting role in public transportation, with mode share² significantly smaller than that of buses and trains [4].

In this paper, we propose applying mobility-on-demand (MoD) to high-capacity buses in dense, high-demand areas under the concept of "dynamic bus routing" (DBR). Like ridesharing services, such buses would not follow predefined routes nor schedules, but rather constantly generate their routes in real time to most efficiently serve commuters' travel patterns. In order to feasibly execute ridesharing on this scale, however, these buses sacrifice some degree of point-to-point connectivity (for example, ferrying commuters from bus-stop to bus-stop instead of doorstep to doorstep), and operate within a strictly defined catchment (to prevent excessive variance in travel and waiting times, and guarantee commuters a certain level of service).

¹ Such as Uber, Lyft (in the United States), Grab (in Southeast Asia), amongst many others.

² Due to a current lack of official data for ridesharing mode-share in Singapore, taxi mode-share was used as a proxy instead.

One of the earliest studies related to this topic [5] used a greedy algorithm to demonstrate that small capacity (6- to 8-seater) DBR-style buses were more efficient than (single-seater) taxis for serving demand in a bounded area. However, it only provided a comparison between the two services, and did not investigate the actual applicability of DBR in a real-world context. On the other hand, while many other studies [7] [8] have explored the potential of MoD services, such as "Dial-a-Ride", as a replacement for conventional fixed-route buses, the focus has always been on areas of low passenger demand, or to serve the transportation needs of the elderly or disabled. Due to cost and complexity [6], these services have traditionally been perceived as an inferior alternative to fixed-route conventional buses for high-demand areas.

In recent years, this perception has begun to shift, with commercial entities ³ striving to provide on-demand, multiple-passenger ridesharing services in population-dense cities. However, due to lack of integration with the existing transportation network, these services typically complement instead of replace existing public bus services, and/or run in areas under-served by public transport. As such, there is once again a lack of scale in operations to fully exploit efficiency gains. In this paper, we aim to maximize these potential gains by applying dynamic routing to high-capacity vehicles as a *replacement* for some existing public bus routes. The term "DBR" is specifically used to establish a clear distinction between this goal, and the traditional "Dial-a-Ride" concept.

As for technical considerations, although there have been many studies on MoD services, these studies often focus on fleet management. For example, [9] focuses on the problem of fleet *sizing* under different conditions, and not on the actual operation of the fleet, while [10] provides a comprehensive analysis of fleet *deployment* strategies accounting for demand fluctuations. The ridesharing problem, where one ride simultaneously serves multiple origin-destination (OD) pairs, is related to the dynamic vehicle routing problem [11]. More specifically, it is essentially a large-scale version of the dynamic multiple-vehicle pickup and delivery problem [12] [13]. Since these problems are NP-hard [14], they are often difficult to solve optimally; however, many heuristics [15] [16] have been developed that provide reasonably good solutions under certain contexts.

Work done on these problems typically focus on finding good optimization algorithms, and therefore often only consider vehicles of smaller, taxi-esque capacities, or very small demand [17] [18] [19] [20]. One recent study [21] developed a real-time algorithm for larger-scale ridesharing in up to ten-seater vehicles that could be run in real time, and applied it to a very large demand pool of all taxi trips in Manhattan. However, this still falls short of the expected 85-passenger capacity of a typical bus. As algorithm runtime scales exponentially with passengers-per-trip, a different approach is required. Given the capacity of a typical bus, it is no longer viable to search for the *optimal* solution to the

ridesharing problem. Instead, we focus on finding a heuristic that generates a reasonably good solution quickly enough to respond to commuters in real-time. Furthermore, as the main first-mile/last-mile mode in Singapore, public bus demand naturally exhibits different behavior from taxi or ridesharing demand, and therefore different challenges. For example, implementing DBR along high-demand corridors (such as arterial roads to MRT⁴ stations) would likely result in routes very similar to those plied by existing fixed-route buses, and therefore be meaningless. On the other hand, demand originating from relatively secluded (but still within the town) areas must still be served within a certain time frame in order to adhere to public service standards. As such, nuance is required in selecting both the bus-operation catchment, and the solution heuristics, in order to demonstrate the feasibility of DBR as an alternative to traditional fixed-route feeder buses in a practical setting.

In this paper, we design a dynamic bus routing Simulator that utilizes historical smart-card data to model the implementation of DBR, in areas of high demand. In the next section, we elaborate on the structure of the Simulator, with special emphasis on the optimization function chosen. We then conduct a case study on a residential town in Singapore and analyze the results. Finally, we summarize our findings and offer suggestions for future research.

II. THE DYNAMIC BUS ROUTING SIMULATOR

A. Overview

In order to investigate the potential benefits of dynamically routed buses, an event-based, agent-based simulator [Fig. 1] was developed in the R programming language. In the simulator, agents refer to buses, which are modelled microscopically with associated parameters such as capacities, dwell time behavior, etc., and also make decisions, such as route choice or passenger allocation (that is, deciding which bus a new passenger should be assigned to).

The Simulator takes as input a **Travel Demand** table consisting of passengers' origins, destinations, and times of entry into the system, as well as a **Supply Parameters** table consisting of bus parameters (capacities, etc.) and network parameters (bus stop locations, a cost matrix for travelling between stops, etc.) These inputs are summarized in [Table I]. In order to accurately represent real-life conditions, busagents in the Simulator are only aware of current demand—they do not know when or where a new passenger will enter the system, thus resulting in routes that constantly change in response to new information to best optimize the system. Upon completion, the Simulator returns metrics detailing both passenger experience (waiting time, travel time) and bus performance (distance covered, occupancies, etc.)

An event is deemed to have occurred when a new passenger i enters the system at time t_i . This triggers a series of responses: firstly, all bus locations and occupancies are

³ Such as GrabShuttle+, Chariot, Via, Citymapper, amongst many others.

⁴ Singapore's subway system.

updated based on their expected routes. For example, all passengers with alighting times between t_{i-1} and t_i are removed from their respective buses. The relevant times are then recorded in the output **Passenger Table**. Passenger i is then assigned to a bus j based on a generic choice function, which effectively defines the system-optimal conditions. Since there are multiple "reasonable" definitions of optimality which may contradict each other (for example, minimizing passenger travel time vs. minimizing bus travel distance), the decision of which function to use is left to the user. Finally, the expected route for bus j is updated in the **Bus Information Table** to account for the additional boarding and alighting stops associated with passenger i. These outputs are summarized in [Table II].

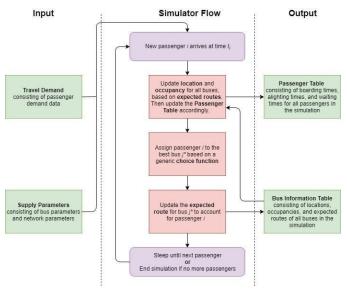


Fig. 1. Flow of the dynamic routing simulator

TABLE I. INPUTS TO THE DYNAMIC ROUTING SIMULATOR

Travel Demand	Supply Parameters	
Time of entry into	Fleet size	
system	 Bus capacities 	
 Origin 	 Bus stop locations 	
 Destination 	 Dwell time parameters 	
	 Cost matrix 	

TABLE II. OUTPUTS FROM THE DYNAMIC ROUTING SIMULATOR

Passenger Table	Bus Information Table	
Boarding time	Route taken	
 Alighting time 	 Distance covered 	
 Waiting time 	 Occupancy over 	
 Waiting time reliability 	time	
Travel time reliability		

In the interests of simplicity, buses in the Simulator are assumed to be uniform (with the same speeds, capacities, etc.) and constantly in operation without the need for breaks. Passengers were assumed to behave ideally and always arrive at their requested pick-up point before the bus. They were also assumed to have a minor degree of flexibility in their origins and destinations – that is, buses may alight passengers

at nearby stops (e.g. the stop directly across the road) instead of the requested destination itself, if it is more efficient to do so. Note that the design of the Simulator does not preclude these assumptions from being modified if necessary.

B. Choice Function

In order to decide on an appropriate choice function, we note that any reasonable option will necessarily have to of a routing function that sequences boarding/alighting stops, and a cost function that determines the assignment of passengers to vehicles [22]. The bus routing problem for any given bus i in our Simulator is dynamic and deterministic as described in [11] and can be modelled as a Travelling Salesman Problem (TSP) on a directed graph, with the origin and destination of each passenger on that bus the nodes, and the cost matrix the edges. The primary differences are that, firstly, the bus does not need to return to its initial node after visiting all nodes, and secondly, that the destination node for any given passenger cannot be visited before the origin node for that passenger (i.e. a passenger cannot be dropped off before he is picked up.)

While many good heuristics have been developed to solve the TSP [23], it is crucial to choose an algorithm that works quickly, even at the expense of some optimality, since it will have to be run every time a passenger enters the system both in the Simulator as well as in any real-world implementation. In this paper, we modify the efficient insertion heuristic developed in [16] to cater to dynamic demand. This modified heuristic took approximately 2 seconds to process each new passenger, which we consider an acceptably fast computation time. The goal of the choice function here is to return the best route, and the cost of this route, for bus j to serve the new passenger i. By iterating across all buses in the system, we can then optimally assign a bus to serve this passenger.

We define:

- WT_{max} and TT_{max} as the maximum allowable waiting and travel times for passengers, respectively;
- M_j as the set of m passengers (P) to be served by bus j (that is, those waiting-to-board or already on-board bus j, as well as the new passenger P_i);
- t_k^{entry} as the system entry time, t_k^{board} as the boarding time and t_k^{alight} as the alighting time of passenger P_k , $k \in (M_j)$;
- L_k^{board} as the boarding location and L_k^{alight} as the alighting location of passenger P_k , $k \in (M_j)$;
- R_j as the current route of bus j, a series of nodes $\{L_0, L_1, L_2, ... L_n\}$, $n \le 2m$, where L_0 is the current location of the bus and $L_{1...n}$ are boarding and alighting locations of passengers $P_{k^*}, k^* \in (M_i), k^* \ne i$

• C_j as the current route cost of bus j, here defined as the sum of total in-system travel times $C_j = \sum_{k^*=1}^{m-1} t_{k^*}^{alight} - t_{k^*}^{entry} \ \forall \ k^* \in M_j, k^* \neq i$

The boarding location of the new passenger, L_i^{board} , is then taken and an eligible edge e^* in R_j found such that the costs (as defined in the cost matrix) of inserting node L_i^{board} between the nodes of e^* is minimal. The arrival times at all nodes that lie after the L_i^{board} insertion point on the route are recalculated, and the imposed waiting time (1) and travel time (2) limits are checked:

$$t_k^{board} - t_k^{entry} \le WT_{max} \ \forall \ k \in M_i$$
 (1)

$$t_k^{alight} - t_k^{board} \le TT_{max} \,\forall \, k \in M_i$$
 (2)

If these conditions are not satisfied, the current edge e^* is declared ineligible, a new eligible edge is chosen, and the process repeated. If no more eligible edges can be found, then bus j is declared an ineligible candidate to serve P_i . Otherwise, L_i^{board} is added to R_j between the nodes of edge e^* . This process is then repeated for the alighting location of the new passenger, L_i^{alight} . After both nodes are inserted, the new cost (3) of the route is calculated:

$$C_j^{new} = \sum_{k=1}^m t_k^{alight} - t_k^{entry} \ \forall \ k \in M_j$$
 (3)

Finally, P_i is assigned to the bus j that incurs the least additional cost (4) from serving P_i :

$$j^* = \arg\min(C_j^{new} - C_j) \ \forall \ j \in (all \ buses)$$
 (4)

III. CASE STUDY

A. Overview

In this section, we apply the Simulator to a real-world example by studying the effects of implementing DBR in an unspecified (due to confidentiality concerns) residential Town X (TX) in Singapore. TX is a population-dense, predominantly residential town, linked by a nearby expressway and the MRT to major industrial and commercial districts. As a result, the critical transport nodes in TX are the Transfer Hub – a bus stop located directly on the expressway that provides a key connection to inter-town express buses – and the MRT station, which both experience significant levels of commuter traffic at all times of the day. The latter is fed by a comprehensive short-distance "people mover system" (PMS), that runs on elevated guideways and provides a high-frequency service along major demand corridors, connecting the majority of residents to their destinations.



Fig. 2. Feeder bus routes in Town X

The PMS (shown in grey) is complemented by four feeder routes (Services A, B, C and D), with a total fleet size of 24 single-deck buses with capacities of approximately 90 passengers each [Fig. 2]. These services are primarily designed to cater to isolated locations not served by the PMS, and hence ply convoluted routes. Furthermore, due to routing difficulties, none of the four services serve the Transfer Hub, instead feeding commuters back to the MRT station. Due to this, commuters travelling from isolated areas to the Transfer Hub are often required to walk, or make multiple transfers, during the first-mile leg of their journeys. These factors together result in inefficient travel patterns and long travel times for these isolated commuters.

Given this natural segregation of commuters, TX is an ideal testbed for the implementation of DBR, which can address the major weaknesses of current fixed-route services by efficiently serving areas of sporadic demand and providing point-to-point connectivity between uncommon yet closely interspersed OD pairs, while simultaneously delegating high level-of-service corridors to the PMS system. These benefits are compounded by TX's high population density and more than 60% public transport mode share [24], resulting in many opportunities for efficiency gains. In TX, DBR has the potential to be a considerably superior option to current feeder services, serving not only commuters from the isolated regions, but also those who currently have to make multiple transfers within the town to reach their destination (such as those accessing the Transfer Hub).

B. Simulator Parameters

In this case study, we investigate the viability of replacing all four feeder services in TX with DBR. While there are many approaches to practically implement DBR, we assumed a model similar to that of ride-sharing services, where users book their rides via an app, track the progress of their assigned bus in real time, and hence always arrive at their pick-up location on time.

In order to estimate demand, we used historical Automatic Fare Collection smart-card data to obtain passenger boarding and alighting information at all bus stops in TX over the course of a day⁵, and selected trips that were made on a feeder service, or that were part of a journey with multiple legs within TX. This resulted in approximately 17,000 trips, out of the 55,000 total intra-TX trips made daily. [Fig. 3] illustrates the distribution of these 17,000 trips, with the thickness of each line representing the demand for that specific OD. As expected, we observe both the presence of many thin lines – representing the many sporadic ODs – as well as a significant number of lines spanning the isolated residences and the Transfer Hub, reflecting the limitations of existing fixed-route services in connecting commuters to their destinations.



Fig. 3. DBR Demand Distribution in Town X

For simplicity, dwell times were estimated, based on studies of bus dwell times in Singapore [25] and other cities [26], via a linear approximation, with an intercept of 10 seconds⁶ and a coefficient of 1 second per boarding/alighting passenger. The cost matrix was obtained from Google Maps car-travel time estimates, while maximum waiting time and travel time limits were set at 15 minutes and 30 minutes respectively, in line with the regulatory standards currently imposed on bus operators in Singapore [27].

Finally, it should be noted that although the choice function of the Simulator aims to minimize total passenger travel time, the overall benefits are also contingent on the initial parameters chosen, with trade-offs between different benefits. For example, bus fleet size is naturally inversely correlated with commuter travel and waiting times; as such, parameters that minimize the former will find less improvements in the latter. In this paper, we analyze two scenarios – one with a fleet size equivalent to the present-day feeder fleet size in TX, and one where fleet size is minimized while still maintaining the aforementioned regulated travel and waiting time service limits.

C. Results

Due to fluctuations in demand patterns and volume, DBR performance will naturally vary by time-of-day. In view of brevity, this paper only focuses on the AM Peak, with the simulation run from 0600 to 0900 and the first 30 minutes discarded to allow the system to reach steady-state. During the AM Peak, demand volume, and hence stress on the dynamic routing system, is highest; as such, it represents a lower bound for DBR performance compared to fixed-route buses, which are less affected due to their scheduled routes.

TABLE III. FIXED-ROUTE VS. DBR BENEFITS

AM Peak: 0630 – 0900	Base Case	S1. Max Fleet	S2. Min Fleet		
Service Provider Metrics					
Served Demand	3693	3693	3693		
Fleet Size	24	24	21		
Max. Capacity	90	30	30		
Average Occupancy	8.3	6.1	8.1		
Commuter Metrics					
Travel Time ⁷ (min)	7.7	5.3	5.6		
Wait Time (min)	4.2	4.0	5.9		
Std. Dev of Estimated vs. Actual Travel Time (min) ⁸	NA	1.2	1.8		
% of Improved Travel Times	NA	78.5%	67.6%		

In the first scenario, where fleet size is maintained, the implementation of DBR allowed the same fleet of 24 buses to serve commuters more efficiently, with a reduction of almost 30% in average travel times and 5% in waiting times [Table III]. We also observe that the extended "tail" of long travel times is noticeably absent in the DBR case [Fig. 4], likely due to the elimination of transfers (and hence significant improvements for trips with originally long travel times). All in all, the majority of commuters enjoyed consistent or improved service levels, with 79% of commuters enjoying an absolute decrease in travel time and 95% of commuters being no more than 3 minutes worse off, in addition to the added convenience of transfer-less trips within TX. This increased efficiency reduced the average occupancies and maximum required capacities of buses, allowing smaller-capacity (30-seater) minibuses to be used in lieu of the full-size (90-seater) omnibuses used today. These minibuses are likely to enable the utilization of certain road features (such as U-turns and small roads) that the omnibuses cannot use, thus providing more direct journeys and compounding efficiency gains.

In the second scenario, where fleet size was minimized, DBR provided considerable benefits to service providers, with an almost 12.5% reduction in fleet size, and a similar 66% reduction in vehicle capacity. At the same time, although waiting times increased, this was compensated by shorter travel times, resulting in similar intra-TX journey times of about 11.5 minutes overall. Even under this reduced

⁵ 14 May 2018.

⁶ Note that 10 seconds is much higher than the intercepts estimated in [13a] and [13b], in order to account for time taken to exit the bus bays (a common feature of most bus stops in Singapore).

⁷ The time taken for the portion of a commuter's journey that lies within TX, inclusive of transfer times (if the journey has multiple legs within TX).

⁸ This measures the expected uncertainty in travelling time (i.e. the arrival time at destination which was promised to the commuter when booking a bus, vs. the actual arrival time at destination), which stems from the spontaneous rerouting present in DBR.

fleet size implementation, many more commuters enjoyed short-duration (<6 min) trips [Fig. 4], with the "extended tail" of travel times once again absent, as well as better travel times in general, testifying to the inherent efficiency of DBR over the current fixed-route feeder bus network. With fleet sizes being the primary contributor to costs – due to capital expenditure of almost \$500,000 [28] per vehicle, in addition to subsequent operation and maintenance costs – DBR represents an avenue towards significant savings, without degrading existing service quality.

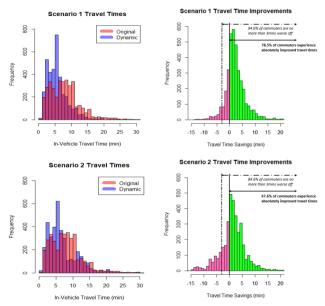


Fig. 4. Travel time comparisons across all scenarios

Travel Time Discrepancies

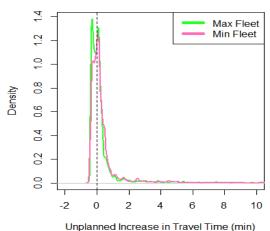


Fig. 5. Travel time uncertainties in both scenarios

Finally, we address travel time uncertainty – a common concern with DBR-style services, due to the constant spontaneous rerouting of buses, and one that is especially pertinent during the AM Peak hours, where consistency is key due to the requirements of non-discretionary travel. While our results [Table III] suggest that this is not a significant issue in the TX implementation, with the standard deviation well within 2 minutes in both scenarios and the vast

majority of commuters enjoying travel times in-line with their expectations, we note that there is still a long "tail" [Fig. 5] with uncertainties of up to 30 minutes. Although this encompasses less than 5% of commuters, it is nevertheless an important problem, as a key function of any successful mass transport system is to provide *reliability* in daily commutes. This will be further discussed in the conclusion.

IV. CONCLUSION AND FURTHER WORK

This paper illustrates how DBR can complement existing high-demand fixed routes to optimize travel patterns within towns, where demand is not easily served by traditional fixed-route feeder services. In the scenarios analyzed above, commuters benefited greatly from the introduction of DBR, both quantitatively in the form of shorter journey times, and qualitatively in the form of point-to-point trips, guaranteed boarding even during peak hours, and assurance in bus arrival times. Similarly, service providers enjoyed potential cost savings via reductions in fleet size and required capacity, which allowed for the introduction of smaller, cheaper, buses. At the same time, however, it should be stressed that DBR is not a panacea for all public transportation problems. It is illsuited to handle popular transportation corridors (which contain ODs with consistently large demand), and also ideally requires commuters with a high level of technological familiarity. Indeed, in this paper, we assumed a perfect adoption rate from commuters who previously took feeder services. In reality, this is unlikely to be the case (especially for young children and the elderly), which may impede efficiency gains.

While the simulator developed in this paper has been a useful tool to explore the benefits of DBR, further improvements are required before considering any real-world implementation. In order to guarantee reliability and eliminate the travel time uncertainty "tail", an additional constraint should be added to ensure that commuters' travel and waiting times do not differ significantly from the times initially promised to them (when booking via the app). "Hiccups" such as commuters occasionally being late, drivers taking bathroom breaks, etc. could also be implemented, to better reflect real-life conditions. This is especially important to determine DBR operating policy (such as whether to wait for a late passenger), as a single delay may have cascading effects that lead to delays for other passengers in the system. Another improvement could be the use of different choice functions. The focus of this paper was not on the choice function, and hence a fast insertion algorithm was chosen; however, a more nuanced choice function would likely be able to generate more optimal routing and passenger assignments, and yield better results. However, this would also require significantly more computational time. Ideally, for practical implementation, and as an area of future work, the simulation model should be redeveloped using an objectoriented language, for general efficiency and convenience.

There are also opportunities to expand the scope of the scenarios studied, both temporally and spatially. In this paper, the effects of DBR were only studied during the AM peak hours, when demand was highest. It would be interesting to extend the simulation across an entire day—perhaps including shift changes and mandatory breaksinvestigate how efficiency gains vary over time and evaluate the benefits of full-day vs. peak-hour DBR. As for the latter, a logical next step would be to model DBR in multiple towns, instead of TX in isolation. By developing an inter-region model to link these towns, each with their own DBR system, it would be possible to investigate how they interact with each other to obtain a large-scale picture of DBR-enabled passenger flows between regions and compare it to travel patterns today. Yet another topic that warrants further study could be the potential incidental benefits of DBR – that is, whether the additional flexibility that dynamic routing provides can be used to influence commuter behavior to generate positive outcomes for the public transport network as a whole. For example, DBR could be used to alleviate train loading along critical stretches of the MRT during peak hours by redirecting commuters to stations along less crowded lines, thus helping to smoothen demand and reduce crowding.

In conclusion, while there are still many obstacles to overcome, this paper has shown that DBR has very real benefits and, if carefully implemented and managed, has the potential to be a valuable addition to the public transport ecosystem, providing a seamless and efficient commute for many underserved commuters today.

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