On demand bus transport
Incorporating revenue management
Static Deterministic version
Mouna - 27/11/21

Reference: (Bertsimas and Popescu 2003)

→ The change concerns the selection step (last step) of the on-demand bus process:

Realization of demand at time t

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Calculate the shortest path for each available bus if it takes the new request

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Filter candidate buses, i.e. those that have a new route meeting the constraints of max alight time and max boarding time of all customers

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If candidate bus set is empty then reject the demand request
Otherwise, select the bus that leads to the highest additional profit

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Profit calculation for a given bus:
Let j the current route of the bus and j' the new route (after re-routing to take the new request

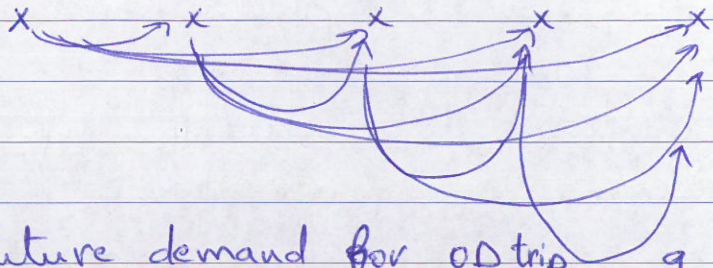
Additional profit $= \Delta P(j,t) = P(j',t) - P(j,t)$
$P(j,t)$ is the optimal objective value of the integer program maximizing profit given deterministic future demand for all possible trips and given current capacity on each leg.

Integer Program

Notations

Given route j of the bus

Q_j : Set of all OD trips q (possible) on route j



D_q^t : Future demand for OD trip q in the remaining time horizon starting t

L_j : Set of legs (arcs) l on route j

L_q : Set of legs on OD trip q

d_l : Length of leg l

n_l^t : Current capacity of leg l

a_{lq} : Binary equal to 1 if leg l is used on OD trip q

p_q : Profit from selling OD trip q calculated as:

$$p_q = \sum_{l \in L_q} D_q^t \, d_l$$

y_q : Integer allocation variable for OD trip q ($\approx$ Number of seats allocated)

Model

$$\text{Max } P(j,t) = \sum_{q \in Q_j} p_q \, y_q$$

$$\text{s.t.} \quad \sum_{q \in Q_j} a_{lq} \, y_q \leq n_l^t \qquad \forall l \in L_j \qquad \text{(capacity)}$$

$$0 \leq y_q \leq D_q^t \qquad \forall q \in Q_j \qquad \text{(demand)}$$

$$y \text{ integer}$$

Example for section 4.3 Model

Suppose a bus has the current route j:

$$1 \longrightarrow 2 \longrightarrow 3 \longrightarrow 4$$

- Q_j = Set of all (possible) OD trips $= \{[1,2], [1,3], [1,4], [2,3], [2,4], [3,4]\}$
- L_j = Set of legs on the route $= \{(1,2), (2,3), (3,4)\}$
- We define the length of each leg and the current capacity of each leg as follows:

Length: $d_{(1,2)} = 1 \quad d_{(2,3)} = 2 \quad d_{(3,4)} = 3$

Capacity: $n^t_{(1,2)} = 7 \quad n^t_{(2,3)} = 8 \quad n^t_{(3,4)} = 9$

- We define the demand and the set of legs on each OD trip as follows:

$[1,2] \to \begin{cases} \text{Demand } D^t_{[1,2]} = 7 \\ \text{Set of legs } L_{[1,2]} = \{(1,2)\} \end{cases}$

$[1,3] \to \begin{cases} \text{Demand } D^t_{[1,3]} = 6 \\ \text{Set of legs } L_{[1,3]} = \{(1,2), (2,3)\} \end{cases}$

$[1,4] \to \begin{cases} \text{Demand } D^t_{[1,4]} = 5 \\ \text{Set of legs } L_{[1,4]} = \{(1,2), (2,3), (3,4)\} \end{cases}$

$[2,3] \to \begin{cases} \text{Demand } D^t_{[2,3]} = 4 \\ \text{Set of legs } L_{[2,3]} = \{(2,3)\} \end{cases}$

$[2,4] \to \begin{cases} \text{Demand } D^t_{[2,4]} = 3 \\ \text{Set of legs } L_{[2,4]} = \{(2,3), (3,4)\} \end{cases}$

$[3,4] \to \begin{cases} \text{Demand } D^t_{[3,4]} = 2 \\ \text{Set of legs } L_{[3,4]} = \{(3,4)\} \end{cases}$

- Binary matrix A : (3×6)

$$\begin{array}{c} \\ (1,2) \\ (2,3) \\ (3,4) \end{array} \begin{array}{c} \begin{array}{cccccc} [1,2] & [1,3] & [1,4] & [2,3] & [2,4] & [3,4] \end{array} \\ \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{pmatrix} \end{array}$$

- Profit of each OD trip:

$P_{[1,2]} = 7 \times (1) = 7$

$P_{[1,3]} = 6 \times (1+3) = 18$

$P_{[1,4]} = 5 \times (1+2+3) = 30$

$P_{[2,3]} = 4 \times 2 = 8$

$P_{[2,4]} = 3 \times (2+3) = 15$

$P_{[3,4]} = 2 \times 3 = 6$

Integer Program

$$\text{Max} \quad 7\,Y_{[1,2]} + 6\,Y_{[1,3]} + 5\,Y_{[1,4]} + 4\,Y_{[2,3]} + 3\,Y_{[2,4]} + 2\,Y_{[3,4]}$$

s.t

$$\left.\begin{aligned} Y_{[1,2]} + Y_{[1,3]} + Y_{[1,4]} &\leq 7 \\ Y_{[1,3]} + Y_{[1,4]} + Y_{[2,3]} + Y_{[2,4]} &\leq 8 \\ Y_{[1,4]} + Y_{[2,4]} + Y_{[3,4]} &\leq 9 \end{aligned}\right\} \text{Capacity constraints on each leg.}$$

$$\left.\begin{aligned} Y_{[1,2]} &\leq 7 \\ Y_{[1,3]} &\leq 6 \\ Y_{[1,4]} &\leq 5 \\ Y_{[2,3]} &\leq 4 \\ Y_{[2,4]} &\leq 3 \\ Y_{[3,4]} &\leq 2 \end{aligned}\right\} \text{Demand constraints on each OD trip}$$

$Y_{[1,2]}, Y_{[1,3]}, Y_{[1,4]}, Y_{[2,3]}, Y_{[2,4]}, Y_{[3,4]}$ are integer