

(1) A card is drawn at random from a 52 deck cards so that each card has an equal probability of being drawn. Find the probability of drawing:

$$\left. \begin{array}{l} \text{(a) red card} = \frac{26}{52} \text{ possible} \\ \text{Jack} = \frac{4}{52} \text{ possible} \end{array} \right\} \text{overlap} = \frac{1}{52}$$

$$\begin{aligned} P(\text{not Red and not Jack}) &= 1 - \left(\frac{26}{52} + \frac{4}{52} - \frac{2}{52} \right) \\ &= 1 - \frac{28}{52} = \frac{24}{52} \end{aligned}$$

(b) A face card or a red card

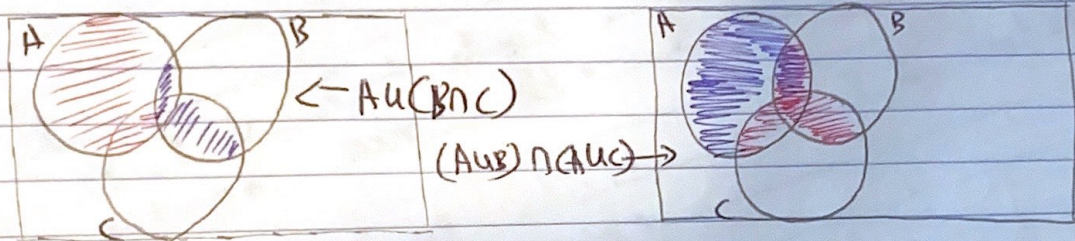
$$\left. \begin{array}{l} P(\text{Face}) = \frac{12}{52} \text{ (6 are red)} \\ P(\text{red}) = \frac{26}{52} \text{ (6 are face)} \end{array} \right\} \text{overlap} = \frac{6}{52}$$

$$P(\text{Face or red}) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52}$$

$$= \frac{32}{52}$$

(2) Use Venn Diagram to verify the following statement

$$\underbrace{A}_{\text{Red}} \cup \underbrace{(B \cap C)}_{\text{Blue}} = \underbrace{(A \cup B)}_{\text{Blue}} \cap \underbrace{(A \cup C)}_{\text{Red}}$$



$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

(3) 7 2 4 9

less than 7000
~~even~~ odd

less than 7000 means first digit is {2} or {4}
possible outcome = ~~2!~~ 2! + 2! = 4

let x be less than 7000 and odd

$P(x < 7000) \Rightarrow 2$ or 4 will be first digit
 $P(x \text{ is odd}) \rightarrow$ last number can't be 2 or 4

* if 2 is first digit, possible outcome are $3! = 3 \times 2 \times 1 = \underline{6}$

* if 4 is first digit, possible outcome are $3! = 3 \times 2 \times 1 = \underline{6}$

* if 2 is first digit, and 4 is last digit, outcomes are $2! = 2 \times 1 = \underline{2}$

* if 4 is first digit and 2 is last digit, outcomes = $2! = 2 \times 1 = 2$

$P(\text{being less than 700} \cap \text{odd number})$

$$= \frac{6}{4!} + \frac{6}{4!} - \frac{2}{4!} - \frac{2}{4!}$$

$$= \frac{6}{24} + \frac{6}{24} - \frac{2}{24} - \frac{2}{24} =$$

$$\frac{8}{24}$$

$$\text{or } \frac{1}{3}$$

(4) In how many ways can there be a first, second, and 4th place runner with 8 runners

$$\begin{aligned} \text{Ex } P(8) &= \frac{8!}{8!(5)!} = \frac{8 \times 7 \times 6}{3 \times 2} = 86 \\ \frac{8!}{4!6} &= \frac{8 \times 7 \times 6 \times 5}{6} = 1680 \end{aligned}$$

$$\frac{8!}{4!(6)} = \frac{8 \times 7 \times 6 \times 5 \times 4!}{4!(6)} = \frac{8 \times 7 \times 6 \times 5}{6} = 280$$

(5) Define a sample space for following
for 6-sided die, 2-sided coin

~~P(die)~~ ~~1/6~~ die = 6 outcomes,
~~1/2~~ coin = 2 outcomes

Possible outcomes = $6 \times 2 = 12$