

# Lecture 23 – Absolute Extrema

## The Big Idea


Before, we talked about relative maxima and relative minima — the local hills and valleys of a graph.

But what if we zoom out and look at the **entire mountain range**? Now we want to find the **tallest peak** and the **deepest valley** — no matter where they are.

That's what we call **absolute extrema**.

- Absolute Maximum → the highest point of the whole graph.
- Absolute Minimum → the lowest point of the whole graph.

Think of it like this:

 The Earth's surface is a function.

- Mt. Everest → Absolute Maximum (highest point).
- Mariana Trench → Absolute Minimum (deepest point).

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## Step 1: The Concept

When we say a function has an **absolute maximum**, we mean:  
There is some point  $x_0$  such that

$f(x_0) \geq f(x)$  for every  $x$  in the domain.

And for absolute minimum:

$f(x_0) \leq f(x)$  for all  $x$  in the domain.

So,

- Relative extrema = local hills and valleys
  - Absolute extrema = the highest and lowest in the entire journey
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## Step 2: A Simple Example

Consider

$$f(x) = 2x + 1$$

on the interval  $[0, 3)$ .

At  $x = 0 \rightarrow f(0) = 1 \rightarrow$  this is the minimum value.

As  $x$  moves toward 3,  $f(x)$  keeps increasing.

But the interval is  $[0, 3)$ , **not**  $[0, 3]$ ,  
so  $x = 3$  is **not included**.

That means even though we can get as close to 7 as we want,  
we never actually reach 7.

✓ So there's **no absolute maximum**, only a **minimum at  $x = 0$** .

That's the power of understanding **intervals** —  
whether they're open or closed changes everything.

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## Step 3: What Makes a Function Have a Max or Min?

Mathematicians discovered something beautiful —  
called the **Extreme Value Theorem**.

It says:

If a function is continuous on a closed interval  $[a, b]$ ,  
then it must have both an **absolute maximum** and an **absolute minimum**.

Plain English:

- Continuous  $\rightarrow$  no breaks or jumps.
- Closed interval  $\rightarrow$  both ends included.

Then your graph is guaranteed to have both highest and lowest points somewhere between  $a$  and  $b$ .

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#### **Step 4: When Extrema Don't Exist**

If one of these conditions fails, extrema may disappear!

Example  1

If the interval is open, like  $(0, 3)$ ,  
the function might never reach the end value —  
so no absolute max or min.


Example  2

If the function isn't continuous, it might jump —  
so again, no extrema on that interval.

You can't find a highest or lowest if the graph breaks off!

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#### **Step 5: The Procedure to Find Absolute Extrema**

Here's the simple 3-step recipe 

##### **Step 1:**

Find the critical points — where  
 $f'(x) = 0$  or  $f'(x)$  does not exist.

##### **Step 2:**

Evaluate  $f(x)$  at:

- The critical points
- The endpoints of the interval

##### **Step 3:**

Compare all the values.

- The largest  $\rightarrow$  Absolute Maximum
- The smallest  $\rightarrow$  Absolute Minimum

That's it!

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### Example

Find the absolute extrema of  
 $f(x) = 2x^3 - 15x^2 + 36x$   
on the interval  $[1, 5]$ .

**Step 1:** Find derivative

$$f'(x) = 6x^2 - 30x + 36$$

$$f'(x) = 6(x - 2)(x - 3)$$

So, critical points are  $x = 2$  and  $x = 3$ .

**Step 2:** Evaluate  $f(x)$

$$\text{At } x = 1 \rightarrow f(1) = 23$$

$$\text{At } x = 2 \rightarrow f(2) = 28$$


$$\text{At } x = 3 \rightarrow f(3) = 27$$

$$\text{At } x = 5 \rightarrow f(5) = 55$$

**Step 3:** Compare

Minimum value = 23 at  $x = 1$

Maximum value = 55 at  $x = 5$

 So the function has:

Absolute Minimum at  $x = 1$

Absolute Maximum at  $x = 5$

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### Step 6: Extrema on Infinite Intervals

Sometimes, we study functions over the entire real line, like  $(-\infty, \infty)$ .

In that case, we look at how the function behaves as  $x \rightarrow \pm\infty$ .

Example:

$$f(x) = x^4 + 3x^3 - 2x + 1$$

As  $x \rightarrow \pm\infty$ ,  $f(x) \rightarrow +\infty$ .

So, the function has **no maximum**,

but might have a **minimum** where the slope changes from negative to positive.

We find it by setting  $f'(x) = 0$  and checking where the sign of slope changes — just like before!

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## **Step 7: The Feynman Way to Think About It**

Don't memorize — **imagine**.

Think of walking along a landscape described by your function:

- The slope ( $f'(x)$ ) tells you whether you're going up or down.
- When slope = 0, you might be at a hilltop or a valley.
- If the road ends (endpoint of interval), check there too.

Then compare which is the highest and which is the lowest — that's literally what the function is doing.

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## **Final Summary**

Situation	What to Check	Meaning
$f'(x) = 0$ or DNE	Critical point	Possible peak or valley
Continuous on $[a, b]$	Guaranteed extrema	Theorem applies
Open interval / discontinuity	May have none	Edges missing or broken
Compare values at critical + endpoints	Find largest & smallest	Absolute max & min

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## **In Feynman's Words**

"You don't need to guess where the highest hill is — just measure the slopes.  
When every path around you goes downhill,  
you're already standing at the top." 