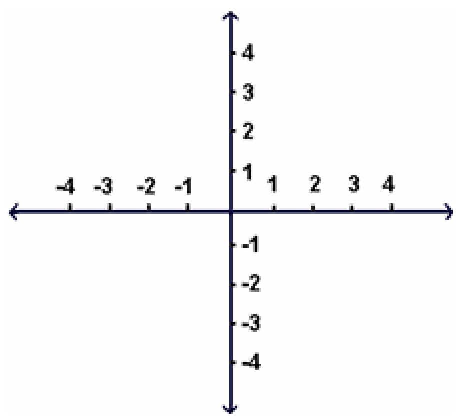
Lecture 3 – Coordinate Planes and Graphs

The Coordinate Plane

A **plane** is like a flat surface that extends forever in all directions.
We build the coordinate plane by taking two number lines:

- One goes left–right (x-axis)
- One goes up–down (y-axis)
  and placing them so they cross each other at 90°.



Every point in this plane can be described using an **ordered pair (a, b)**.

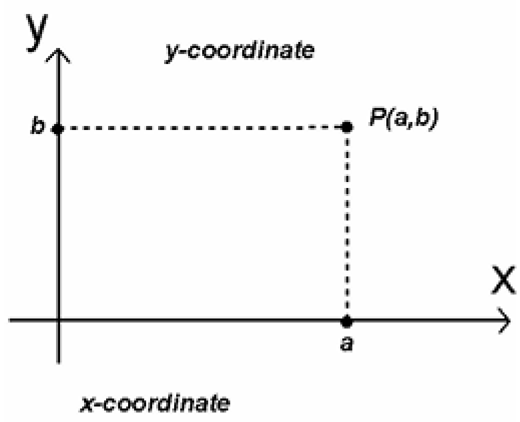
- The first number, **a**, tells you how far to go left or right along the x-axis.
- The second number, **b**, tells you how far to go up or down along the y-axis.

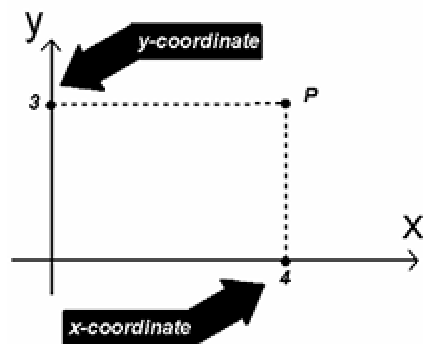
👉 Example: The point (4, 3) means move 4 steps to the right, and 3 steps up.

To find the coordinates of a point P in the plane, we can draw:

- A vertical line from P to the x-axis.
- A horizontal line from P to the y-axis.
  The numbers where the lines meet are the coordinates of P.

This system of describing points with ordered pairs is called the **rectangular coordinate system**.





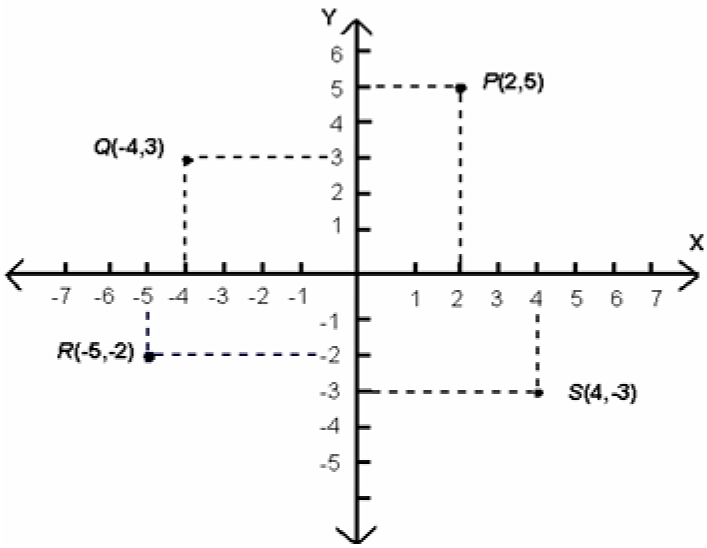
Plotting Points

To **plot** a point P(a, b), just locate its x-coordinate on the x-axis, then its y-coordinate on the y-axis.

👉 Example:

- P(2, 5) → 2 right, 5 up
- Q(–4, 3) → 4 left, 3 up
- R(–5, –2) → 5 left, 2 down
- S(4, –3) → 4 right, 3 down

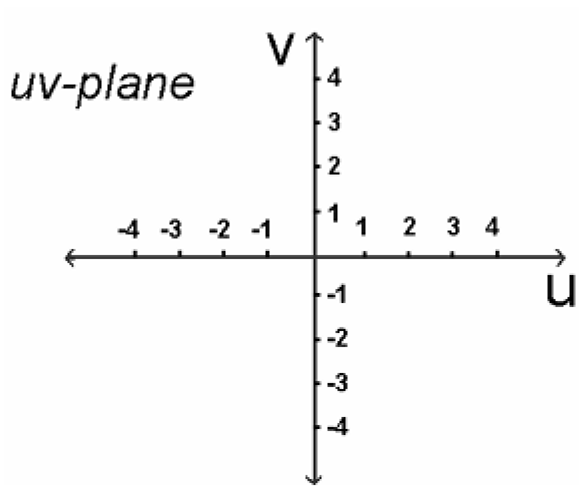
By plotting such points, we can start to see algebraic equations turn into geometric curves. And the reverse is also true: we can describe geometric curves using algebraic equations.

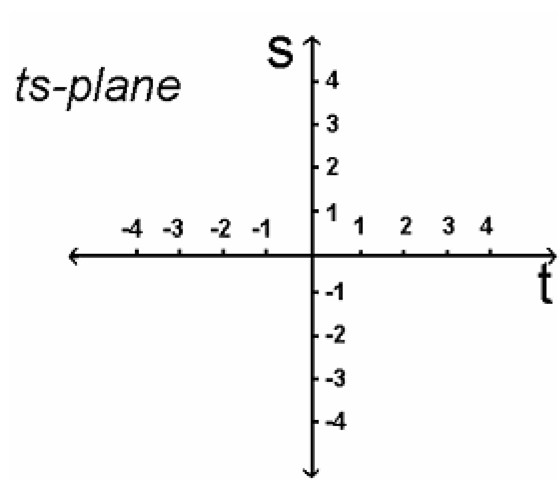


Naming the Plane

We usually call the horizontal axis **x** and the vertical axis **y**. Together, this makes the **xy-plane**.

But in applications, we don't have to stick to x and y. We can use other letters.
👉 Example: a **uv-plane** (horizontal = u, vertical = v), or a **ts-plane** (horizontal = t, vertical = s).
The first letter always names the horizontal axis, and the second letter names the vertical one.





Equations and Solutions

Equations in x and y can be “tested” with ordered pairs to see if the pair is a solution.

👉 Example: Check if (3, 2) solves **6x – 4y = 10**
Substitute: 6(3) – 4(2) = 18 – 8 = 10 ✅ True!
So (3, 2) is a solution.

Now check (2, 0):
6(2) – 4(0) = 12 ≠ 10 ❌ Not true.
So (2, 0) is not a solution.

Graph of an Equation

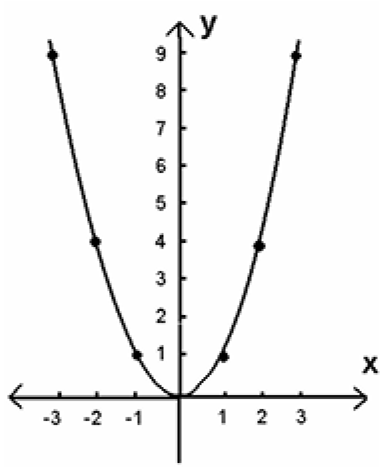
Definition: The graph of an equation in x and y is the set of all points (x, y) in the plane that satisfy the equation.

Example – Graph of $y = x^2$

x	$y = x^2$	(x, y)
0	0	(0, 0)
1	1	(1, 1)
2	4	(2, 4)
3	9	(3, 9)
-1	1	(-1, 1)
-2	4	(-2, 4)
-3	9	(-3, 9)

Pick some x values: –2, –1, 0, 1, 2.
Compute $y = x^2$ → gives 4, 1, 0, 1, 4.

Plot these points: (–2, 4), (–1, 1), (0, 0), (1, 1), (2, 4).



When we connect them smoothly, we get a curve called a **parabola**.

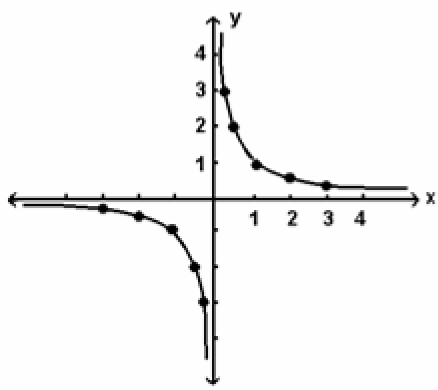
⚠️ Important: When we draw graphs by plotting points, the curve we sketch is only an **approximation**. The exact shape is determined by the equation itself, not by our rough drawing.

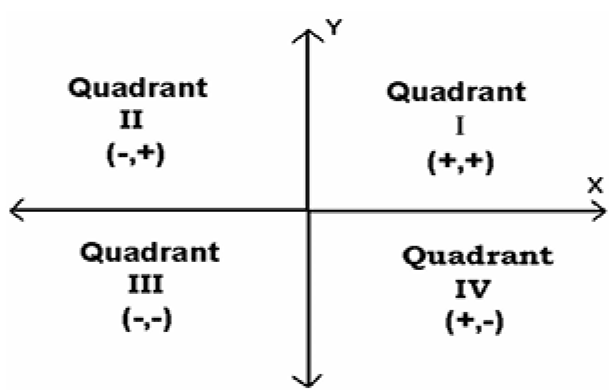
Example : Sketch the graph of

y = 1/x

x	y = 1/x	(x , y)
1/3	3	(1/3 , 3)
1/2	2	(1/2 , 2)
1	1	(1 , 1)
2	1/2	(2 , 1/2)
3	1/3	(3 , 1/3)
-1/3	-3	(-1/3 , -3)
-1/2	-2	(-1/2 , -2)
-1	-1	(-1 , -1)
-2	-1/2	(-2, -1/2)
-3	-1/3	(-3 , -1/3)

Because 1/x is undefined when x=0, we can plot only points for which x=0





Intercepts

When we draw a graph, some of the most important points are where the graph **crosses the axes**.
These crossing points are called **intercepts**.

---

1. x-intercept

This is where the graph cuts the **x-axis**.
On the x-axis, the y-coordinate is always 0.

👉 So every x-intercept looks like **(a, 0)**.

The number **a** is the x-intercept.

---

2. y-intercept

This is where the graph cuts the **y-axis**.
On the y-axis, the x-coordinate is always 0.

👉 So every y-intercept looks like **(0, b)**.

The number **b** is the y-intercept.

Example

Suppose we are given an equation.

- To find the **x-intercept** → set y = 0 and solve for x.
- To find the **y-intercept** → set x = 0 and solve for y.

x-intercepts

Set $y = 0$

$1/x = 0$ ⇒ **x** is undefined

No x-intercept

y-intercepts

Set $x = 0$

$y = 1/0$ ⇒ **y** is undefined

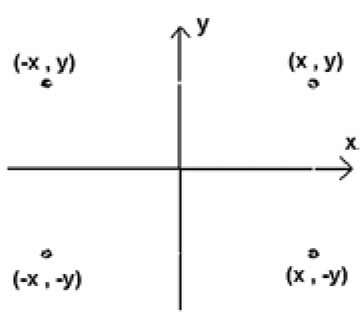
No y-intercept

That gives us the points where the graph touches the axes.

---

3. Symmetry Connection (Preview)

Sometimes, when we plot points like (x, y), (–x, y), (x, –y), (–x, –y),
we see that they form the **corners of a rectangle**.



This hints at the **symmetry** of graphs (mirror-like properties),
which we'll explore next.

👉 In short:

- Intercepts are simply the "meeting points" of a graph with the axes.
- x-intercept → graph meets x-axis (y = 0).
- y-intercept → graph meets y-axis (x = 0).

Example: Find all intercepts of

(a) $3x + 2y = 6$

(b) $x = y^2 - 2y$

(c) $y = 1/x$

Solution

$3x + 2y = 6$

x-intercepts

Set $y = 0$ and solve for x

$3x = 6$ or $\mathbf{x = 2}$

is the required x-intercept

$3x + 2y = 6$

y-intercepts

Set $x = 0$ and solve for y

$2y = 6$ or $\mathbf{y = 3}$

is the required y-intercept

Similarly you can solve part (b), the part (c) is solved here

$y = 1/x$

Symmetry

What is Symmetry?

In math (and in nature), symmetry means something looks the same even after you flip it or rotate it.
It’s like when you look in a mirror — the image looks the same, just reversed.

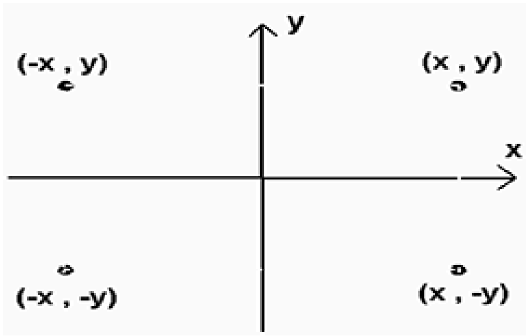
On the coordinate plane, points can be **symmetric** with respect to the axes or the origin:

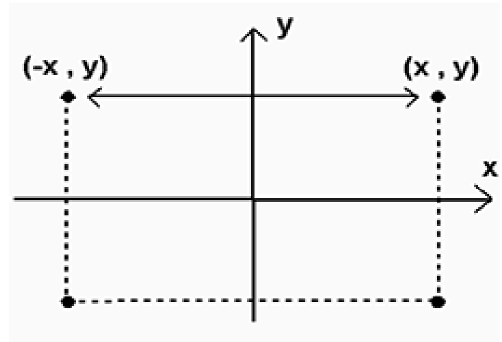
- The points **(x, y)** and **(x, –y)** are symmetric about the **x-axis**.
  (flip across the x-axis → same distance, opposite side)

- The points **(x, y)** and **(–x, y)** are symmetric about the **y-axis**.
  (flip across the y-axis)

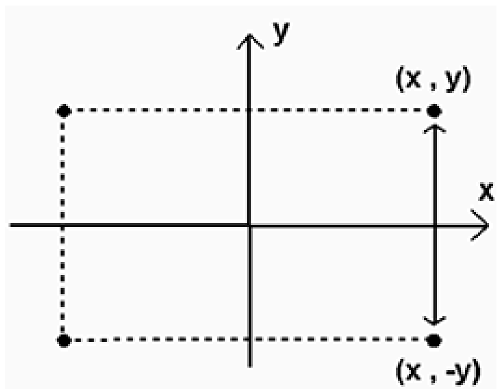
- The points **(x, y)** and **(–x, –y)** are symmetric about the **origin**.
  (rotate 180° around the origin)

If you plot all four points (x, y), (–x, y), (x, –y), (–x, –y),
they form the **corners of a rectangle**.





Symmetric about the y-axis



Symmetric about the x-axis

Why Symmetry Helps in Graphing

When a graph has symmetry, you don't have to calculate every single point.
You can find a few points in one part of the plane, and then "reflect" them to get the rest.
This makes graphing much easier.

Example 1

Equation: **$y = (1/8)(x^2 – 4)$**

Check symmetry:
Replace x with $–x \rightarrow y = (1/8)((–x)^2 – 4) = (1/8)(x^2 – 4)$.
This is the **same equation**.

👉 That means the graph is **symmetric about the y-axis**.

So:

- Just calculate y-values for positive x ($x \geq 0$).

- For each point (x, y), also plot (–x, y).

You only do half the work, symmetry gives you the rest.

Example 2

Equation: $\mathbf{x^2 = y^2}$

Solve for y:
y = x or y = –x.

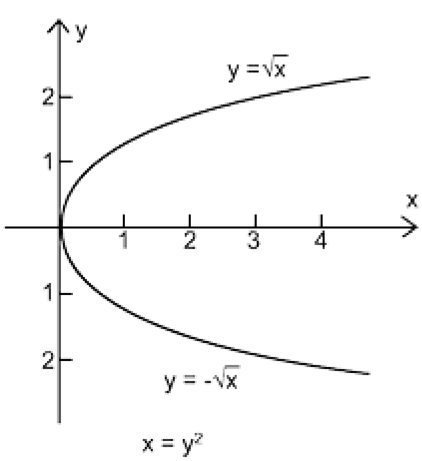
So the graph has two lines:

- y = x (the diagonal line through quadrants I & III)
- y = –x (the diagonal line through quadrants II & IV)

Check symmetry:

- Replace y with –y → still the same equation.
  So the graph is **symmetric about the x-axis**.

👉 You can draw one line (y = x), then flip it across the x-axis to get the second line (y = –x).



In short:

- Symmetry is when a graph mirrors itself across an axis or the origin.
- Types:
  - x-axis symmetry → flip across x-axis.
  - y-axis symmetry → flip across y-axis.
  - origin symmetry → rotate 180°.
- Use symmetry to save time: compute fewer points, then reflect them.