# ⭐ Lecture 27: Sigma Notation

## 🧠 Big Idea First

Sigma notation is **not new mathematics**.

It is just a **short-hand language** for writing long sums without wasting ink, space, or patience.

Think of Sigma (Σ) as saying:

> "Add this expression again and again while the index changes."

That's all.

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## 🔍 Why Do We Need Sigma Notation?

Imagine writing this:

$1^2 + 2^2 + 3^2 + 4^2 + 5^2$

This is fine for small sums,
but what if we want to add **100 terms** or **1000 terms**?

Sigma notation lets us write the same idea **compactly**.

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## 📦 What Is Sigma (Σ)?

- Σ is the **capital Greek letter Sigma**
- It means **summation**
- It tells us to **add terms**

General form:

Σ (expression involving an index)

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# 🎯 Understanding a Simple Example

Consider the sum:

$1^2 + 2^2 + 3^2 + 4^2 + 5^2$

Each term is:

- a square
- of a counting number
- from 1 to 5

Let’s give those numbers a name.

Call them **k**.

Then each term becomes:

$k^2$

Now we say:

$\Sigma\, k^2$, where k goes from 1 to 5

Written mathematically:

$\sum_{\square=1}^{5} k^2$

This single line **replaces five terms**.

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# 🧩 Parts of Sigma Notation

$\sum_{\square=1}^{5} k^2$

- **k** → index of summation

- **1** → lower limit
- **5** → upper limit
- **$k^2$** → the rule for each term

Meaning:

> "Add $k^2$ for k = 1, 2, 3, 4, 5"

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# 🔁 The Index Can Be Anything

The index does NOT have to be k.

These are all equivalent:

$\sum_{i=1}^{4} 1$
$\sum_{\square=1}^{4} 1$
$\sum_{\square=1}^{4} 1$

They all mean:

1 + 1 + 1 + 1

The letter does not matter —
**the counting does**.

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# 🎯 Special Cases

## Case 1: Same Upper and Lower Limit

$\sum_{\square=3}^{3} k^2$

Only one value of k exists.

So the sum is simply:

$3^2 = 9$

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### Case 2: Expression Does NOT Depend on Index

Example:

$\sum_{i=1}^{5} 3$

This means:

3 + 3 + 3 + 3 + 3

Answer:

5 × 3 = 15

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## 🔄 Changing the Limits of Summation

The **same sum** can be written in different sigma forms.

Example: Sum of first five even numbers

2 + 4 + 6 + 8 + 10

One way to write it:

$\sum_{k=1}^{5} 2k$

Another way:

$\sum_{k=0}^{4} 2(k + 1)$

Different forms —
**same total sum**.

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# 🔁 Changing the Index (Very Important Skill)

Sometimes we want to rewrite a sigma sum using **different limits**.

### Example

Rewrite:

$\sum_{k=3}^{7} (5k - 2)$

So that the lower limit becomes **0**.

### Step 1: Define a new index

Let:

$j = k - 3$
So:
$k = j + 3$

### Step 2: Change the limits

- When $k = 3 \rightarrow j = 0$
- When $k = 7 \rightarrow j = 4$

### Step 3: Substitute

$\sum_{j=0}^{4} [5(j + 3) - 2]$

This represents **the same sum**, just re-indexed.

👉 Always check by plugging values to confirm.

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## 📘 General Sum Notation

A sum like:

$a_1 + a_2 + a_3$

Can be written as:

$\sum_{i=1}^{3} a_i$

A sum with **n terms**:

$a_1 + a_2 + a_3 + \dots + a_n$

Is written as:

$\sum_{k=1}^{n} a_k$

This is called a **general sum**.

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# 🧠 Open Form vs Closed Form

### Open Form

Shows all terms:

$1 + 2 + 3 + \ldots + n$

### Closed Form

A formula that gives the result directly:

$n(n + 1) / 2$

Sigma notation connects these two ideas.

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## 🧪 Example (Sum of Squares)

$\sum_{k=1}^{30} k^2$

Instead of adding 30 numbers, we use the formula:

$\sum_{k=1}^{n} k^2 = n(n + 1)(2n + 1) / 6$

For $n = 30$:

$30 \times 31 \times 61 / 6 = 9920$

That’s the **power of sigma notation**.

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# 🧠 Final Insight (Feynman Style)

Sigma notation does **not** change mathematics.

It only changes how we **talk about repetition**.

Think of Σ as a machine that:

1. Takes a formula
2. Changes the index
3. Adds everything together

Once you understand the language,
sigma notation becomes **your best friend** in calculus.