Lecture 4 — Lines and Slope

1. Everyday idea

Slope is just the steepness of a line.
Think of walking on a hill:

- If it's very steep → lots of rise for a little run.
- If it's gentle → little rise for lots of runs.

Formula:
Slope = rise ÷ run

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2. On the xy-plane

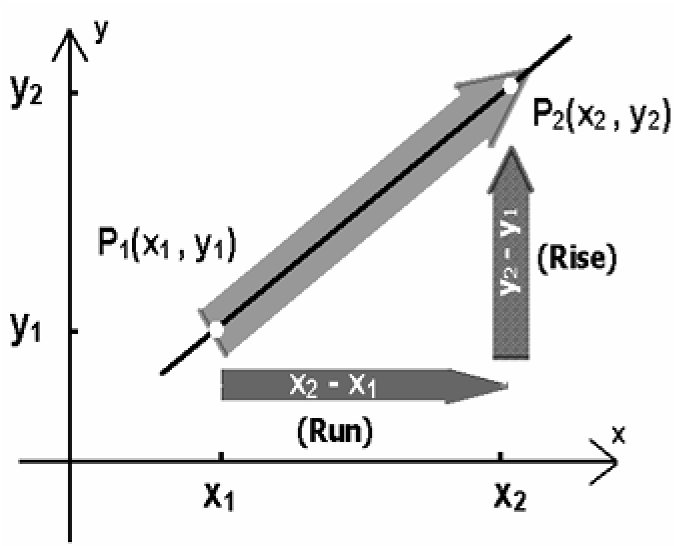
Take two points on a line:

- P1(x1, y1)
- P2(x2, y2)

Then:

- Rise = y2 – y1
- Run = x2 – x1

So slope is:
m = (y2 – y1) / (x2 – x1)



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3. Special cases

- If x2 – x1 = 0 → slope is undefined (vertical line).
- If y2 – y1 = 0 → slope = 0 (horizontal line).
- Switching the order of points does not change the slope.

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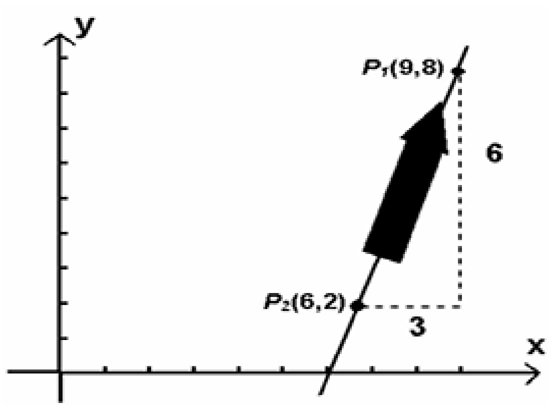
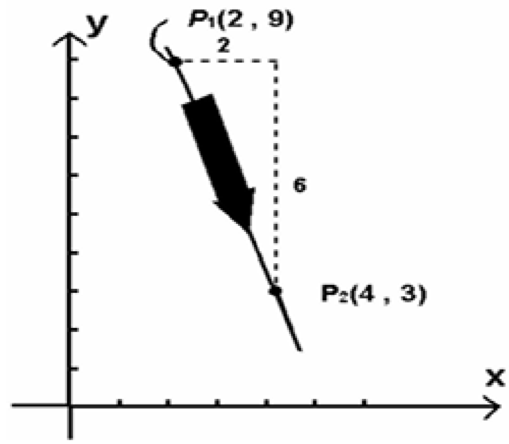
4. Examples

(a) Points (6,2) and (9,8)
Rise = 8 – 2 = 6
Run = 9 – 6 = 3
m = 6 / 3 = 2

(b) Points (2,9) and (4,3)
Rise = 3 – 9 = –6
Run = 4 – 2 = 2
m = –6 / 2 = –3





(c) Points (–2,7) and (5,7)

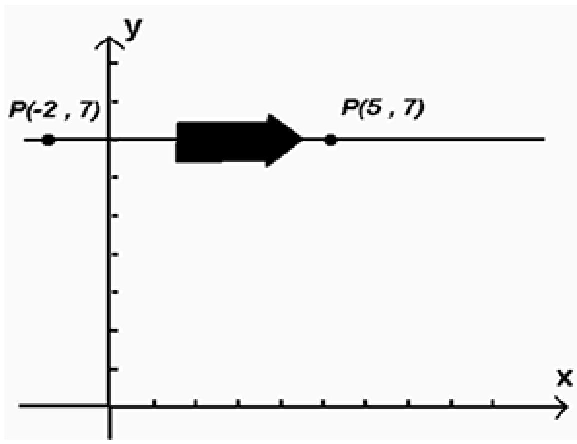
Rise = 7 – 7 = 0

Run = 5 – (–2) = 7

m = 0 / 7 = 0

m=2
Traveling left to right, a point on the line rises two units for each unit it moves in the positive x-direction.

m = -3
Traveling left to right, a point on the line falls three units for each unit it moves in the positive x-direction.



m = 0
Traveling left to right, a point on the line neither rises nor falls

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5. Interpretation

- Positive slope → line goes up to the right (↗).
- Negative slope → line goes down to the right (↘).
- Zero slope → line is flat (→).
- Undefined slope → line is vertical (↑).

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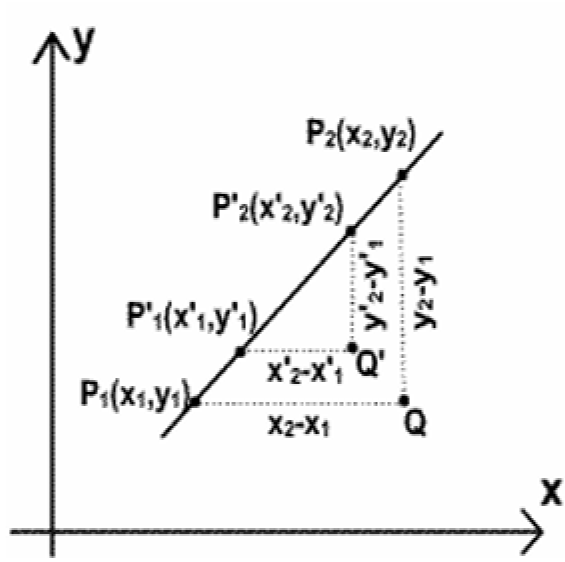
6. Big idea

Slope = **rate of change of y with respect to x**.
It tells you how much y changes for every 1 unit change in x.



Angle of Inclination, Parallel/Perpendicular Lines, and Equations of Lines

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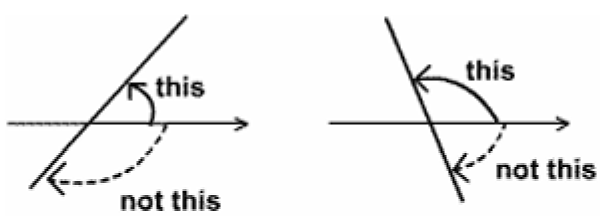
1. Angle of Inclination

Think of slope as a number. But slope also has a **geometric meaning**: it's connected to the **angle** a line makes with the positive x-axis.

👉 Angle of inclination (φ):

- It's the smallest angle measured **counterclockwise** from the positive x-axis to the line.
- Range: $0° \le \varphi \le 180°$ (or $0 \le \varphi \le \pi$ radians).
- If the line is flat (parallel to x-axis) → $\varphi = 0$.
- If the line is vertical → $\varphi = 90°$ (or $\pi/2$ radians).



Angles of inclination are measured counterclockwise from the x-axis

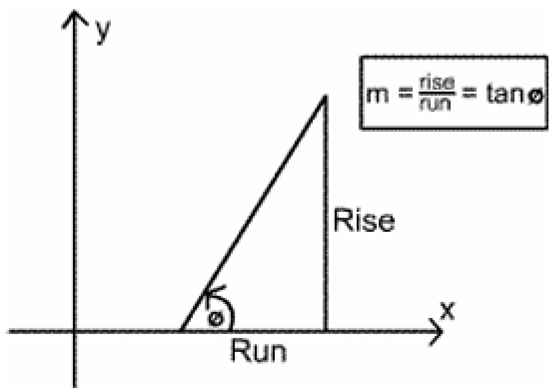
Relationship:
$m = \tan(\varphi)$

where m is slope, φ is angle of inclination.

- If line is vertical, $\tan(\varphi)$ is undefined → slope undefined.

Examples:

- If slope $m = 1 \rightarrow \tan(\varphi) = 1 \rightarrow \varphi = 45°$ ($\pi/4$).
- If slope $m = -1 \rightarrow \tan(\varphi) = -1 \rightarrow \varphi = 135°$ ($3\pi/4$).



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2. Parallel and Perpendicular Lines

Two lines L1 and L2 with slopes m1 and m2:

- **Parallel** if m1 = m2
- **Perpendicular** if m1 × m2 = –1

That second condition means their slopes are **negative reciprocals** of each other.

Example:

- Slope 2 and slope –1/2 → product = –1 → lines are perpendicular.

Example Problem

Points: A(1,3), B(3,7), C(7,5).

- Slope AB = (7 – 3) / (3 – 1) = 4 / 2 = 2
- Slope BC = (5 – 7) / (7 – 3) = –2 / 4 = –1/2
- m1 × m2 = 2 × (–1/2) = –1 → AB ⊥ BC

👉 Triangle ABC is a right triangle.

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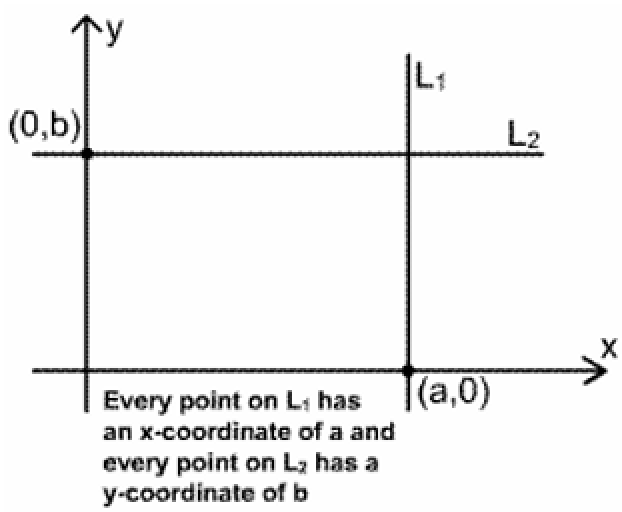
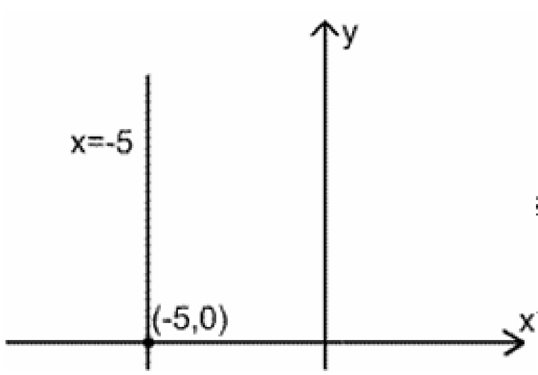
3. Lines Parallel to Axes

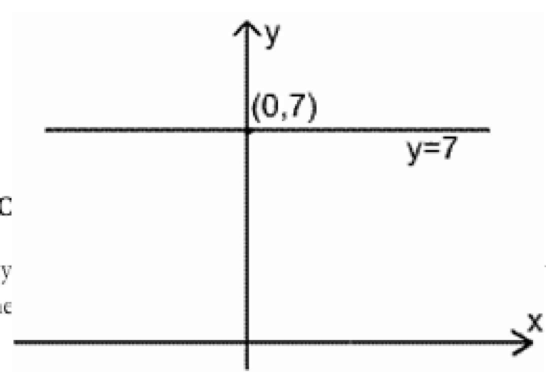
- Vertical line through (a,0): **x = a**
- Horizontal line through (0,b): **y = b**



Examples:

- x = –5 → vertical line through (–5,0)
- y = 7 → horizontal line through (0,7)





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4. Point-Slope Form

Equation of a line passing through (x_1,y_1) with slope m:

$y - y_1 = m(x - x_1)$

Example 1:
Line through (2,3) with slope –3/2
$y - 3 = -3/2\ (x - 2)$
Simplify → $y = -3/2\ x + 6$

Example 2:
Line through (–2, –1) and (3,4)
Slope $m = (4 - (-1)) / (3 - (-2)) = 5 / 5 = 1$
Equation: $y - (-1) = 1(x - (-2))$
$y + 1 = x + 2 \rightarrow y = x + 1$

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5. Slope-Intercept Form

Equation of line with slope m and y-intercept b:

$y = mx + b$

Example 1:
$y = 2x - 5$ → slope = 2, y-intercept = –5

Example 2:
Line with slope –9, crossing y-axis at (0,–4):
$y = -9x - 4$

Example 3:
Line through (3,4) and (–2,–1)
Slope = $(4 - (-1)) / (3 - (-2)) = 5/5 = 1$
Equation: $y = x + 1$

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6. General Equation of a Line

Any line can be written as:
$Ax + By + C = 0$
(A, B not both zero)

Example:
$8x + 5y = 20$
$\rightarrow 5y = -8x + 20$
$\rightarrow y = -8/5\ x + 4$

Slope = –8/5, y-intercept = 4

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7. Real-Life Importance

- Roads, stairs, roofs → slope tells steepness.
- In physics → motion in straight lines.
- In calculus → slope = rate of change.
- In everyday conversions → Fahrenheit vs Celsius is a line:
  $F = (9/5)C + 32$

👉 So slope connects numbers, geometry, and real-world change. Angle of inclination is just another way of seeing the same slope — as an angle instead of a fraction.